Simulation of multi-lane traffic using intelligent driver model & cellular automata

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1 Introduction

Transporting of goods and people using cars, busses and trucks have been paramount to society for the past 100 years and will certainly continue to be important in the future. As long as humans, with all our imperfections, continue to drive the vehicles interruptions of the free flow of traffic will persist. A simple lane change on a highway could lead to a traffic congestion lasting for hours. Simulating traffic, even in a simple case, could be of interest to try to optimize the flow of the traffic so that less time is spent in transportation in the society as a whole. Simulating this will also show how self-driving cars can be programmed and implemented, as they will be programmed to follow similar rules as here.

2 Simulation & models

2.1 Intelligent driver model

To simulate this we will make use of the the continuum traffic model *intelligent driver model* (IDM) which describes the traffic as a time-continuous function of each vehicle. IDM consists of the two differential equations

$$\dot{x}_n = \frac{dx_n}{dt} = v_n \tag{1}$$

$$\dot{v}_n = \frac{dv_n}{dt} = a_n \left[1 - \left(\frac{v_n}{v_{n,0}} \right)^{\delta} - \left(\frac{s_n^*}{s_n} \right)^2 \right] \tag{2}$$

Where we have the definition of s_n^* as

$$s_n^* = s_{n,0} + v_n T_n + \frac{v_n \Delta v_n}{2\sqrt{a_n b_n}} \tag{3}$$

and here we have the definition of $\Delta v_n = v_n - v_{n-1}$. We also have the definition of s_n as

$$s_n = x_{n-1} - x_n - d_{n-1}. \tag{4}$$

In these differential equations we denote $n-1$ as the car in front of car n, where the first car is $n = 1$. In this model we have a couple of parameters. $s_{n,0}$ is the minimum distance from car n to car $n-1$. a_n is the maximum acceleration, $v_{n,0}$ is the velocity of car n in free traffic, T_n is the minimum safety time to the vehicle $n-1$, b_n is a comfortable deceleration and d_n is the length of the vehicle. The acceleration exponent δ describes how fast the acceleration decreases when approaching $v_{n,0}$ and is typically set to be 4[1]. For simplicity we will here consider most of the parameters of the model to be equal for each vehicle in the system, so that we only look at a highway consisting of the same type of car. That is, we set $a_n = a$, $b_n = b$, $T_n = T$, $d_n = d$, $s_{n,0} = s_0$.

The free traffic velocity $v_{n,0}$ is set to be different for each car to simulate different driving behaviours. We will also introduce random events that will slow down some cars at different times. These random events could be a temporary lapse of concentration of the driver or similar. Lane changes when looking at multiple lanes would occur when a car approaches another car at a larger velocity than the car in front and the overtaking car has room to change lane. The overtaking car will return to the original lane after overtaking all cars having lower velocity.

We will consider periodic boundary conditions so that for the N cars we have $\Delta v_1 = v_N - v_1$ and $s_1 = x_N - x_1 - l$. Lane switches will also be considered in the simple case when the velocity of the approaching car is greater than the car being approached when $\gamma_{max} d > s_n > \gamma_{min} d$ and there is room for lane change. This is when the approaching car is more than $\gamma_{min} + 1$ car lengths behind and less than $\gamma_{max} + 1$. Room for lane change is defined as the cars in the lane to be changed into are a safe distance away. Here we will mark safe as $\gamma_{safe} d$ in both directions. The car will also have a set time $t_{overtake}$ for when the overtaking car does not change back a lane.

[1] Treiber, Martin; Hennecke, Ansgar; Helbing, Dirk (2000), "Congested traffic states in empirical observations and microscopic simulations", Physical Review E62, , 1805-1824 (2000)

2.2 Cellular automaton

Simulation of traffic can also be done using a cellular automaton (CA) traffic model where one considers integer positions and velocities instead. We will use the notation of car i's position as x_i and speed as v_i. The rules for the simple cellular automaton on a road with one lane comes in four steps:

1. If $v_i < v_{max}$ then $v_i \leftarrow v_i + 1$.
2. If $x_{i-1} - x_i \leq v_i$ then $v_i \leftarrow x_{i-1} - x_i - 1$.
3. If $p \geq r$ and $v_i > 0$ then $v_i \leftarrow v_i - 1$.
4. $x_i \leftarrow x_i + v_i$.

Where $r \in U(0,1)$ and $p \in [0,1]$ is a set probability. v_{max} is the maximum velocity the cars hold on the road. The first step is modelling the acceleration of the car to the maximum velocity, the second step models how a car needs to break when catching up to another car. The third step is modelling discrepancies of the free flow of the traffic and the fourth step updates the position of the car with the new velocity. We are actually multiplying v_i by the time-step in the last step but since we look at integer solutions the time-step is set equal to 1. With periodic boundary conditions we have that $i-1$ represents car N when $i = 1$ and we have N cars.

Now that we want to simulate cars on a multi-lane highway we have to change the rules slightly. We define l_i as the lane car x_i is driving on, with $l_i = 1$ being the rightmost lane (for left-hand traffic) and counting inwards to the last lane M. We also want to simulate with periodic boundary conditions as earlier and define the road length L. We count the cars as the order in which they start. The choice of how the counting is done will result in different overall behaviour, this arbitrary choice will simplify the calculations. With these definitions the updated rules we will make use of here are:

1. If $v_i < v_{max}$ then $v_i \leftarrow v_i + 1$.
2. If $l_i < M$, $v_{i-k} < v_i$ where k is the smallest positive integer such that $l_{i-k} = l_i$, $x_{i+j} < x_i$ where j is the smallest positive integer such that $l_{i+j} = l_i + 1$ and $x_{i-k} - x_i < v_i$ then $l_i \leftarrow l_i + 1$.
3. If $l_i > 1$, $v_i < x_{i-n} - x_i$ where n is the smallest positive integer such that $l_{i-n} = l_i - 1$ and $x_{i+m} < x_i$ where m is the smallest positive integer such that $l_{i+m} = l_i - 1$ then $l_i \leftarrow l_i - 1$
4. If $x_{i-k} - x_i \leq v_i$ then $v_i \leftarrow x_{i-k} - x_i - 1$ where k is defined as above, but with the new l_i.
5. If $p \geq r$ and $v_i > 0$ then $v_i \leftarrow v_i - 1$.
6. $x_i \leftarrow x_i + v_i$.

With the same definitions as above. On step two we first check if the car is not on the leftmost lane, then if the velocity of the car ahead is lower. We then check if the position of the car behind on the next lane is behind the current car and if the distance to the car ahead is less than the velocity of the car we are looking at. For the third step we do a similar check but for the lane inside instead.

3 Problem

By adding more lanes of traffic on a highway the flow of the traffic will change. We will here try to simulate this and qualitatively inspect how the road capacity changes. Specifically we will want to answer the question "Can you expect the flow rate of a highway to double by having two lanes instead of one, and will adding a third lane triple it?"

We will also study the effect the maximum speed of the highway has on the flow rate.

As a third poinrt we will look into how different amount of lanes can have consequences on the convergence of the simulation and calculation of flow rates. Here we will try to answer the question "Will increasing the amount of lanes also increase the amount of simulations needed to be done for sufficient convergence?"

Lastly, we will analyze the differences between IDM and CA in regards to efficiency and accuracy of model. Specifically we want to answer the question "What is the qualitatively difference between an *intelligent driver model* and a *cellular automata* model when simulating multi-lane traffic on a highway?"

4 Method

To answer the questions posed different statistics of the a run of the simulation will be presented. The flow rate of the system will be plotted against time for a set initialization of the system. To get better understanding of how the flow rate depends on the number of lanes the fundamental diagram will also be plotted. To test the convergence of the simulation we will do a number of simulations and plot the standard error of the mean against the number of simulations, which will then be compared between the number of lanes. The maximum allowed velocity will also be changed and plotted against the fundamental diagram to see the behaviour of that.

Most of the things described will also be compared between the two models when applicable.

4.1 Intelligent driver model

For the IDM we will make use the Runge-Kutta method of the fourth order (RK4) to calculate the velocity and Euler's method to calculate the position. For RK4 we will assume that we have the velocity and position of the vehicle in front of the one we would like to calculate our new velocity for. That is we have the values of v_{n-1} and x_{n-1}. By inserting equation (3) into equation (2) we get

$$\frac{dv_n}{dt} = a\left[1 - \left(\frac{v_n}{v_{n,0}}\right)^{\delta} - \left(\frac{s_0 + v_n T + \left(v_n^2 - v_n v_{n-1}\right)/(2\sqrt{ab})}{s_n}\right)^2\right] := f(v_n) \tag{5}$$

Discretizing this equation to $v_n(k\Delta t) = v_{n,k}$ for a step-size Δt. RK4 then gives the next time step as

$$v_{n,k+1} = v_{n,k} + \frac{1}{6}\left(\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4\right)\Delta t \tag{6}$$

where

$$\begin{cases} \alpha_1 = f(v_{n,k}) & \alpha_2 = f\left(v_{n,k} + \alpha_1 \Delta t/2\right) \\ \alpha_3 = f\left(v_{n,k} + \alpha_2 \Delta t/2\right) & \alpha_3 = f\left(v_{n,k} + \alpha_3 \Delta t\right). \end{cases} \tag{7}$$

Since we have all the other parameters of the model this can be simply implemented. Our initial conditions we set as all cars being equally spaced around the highway and the velocities being set as $v_{n,1} = v_{n,0} - r$ where $r \in U(0, 0.2v_{n,0})$ and $v_{n,0}$ is defined above. Discretizing the position $x_n(k\Delta t) = x_{n,k}$ we then get the solution for $x_{n,k+1}$ using the implicit Euler's method as

$$x_{n,k+1} = x_{n,k} + v_{n,k+1}\Delta t + \frac{1}{2}\frac{dv_{n,k}}{dt}\Delta t \tag{8}$$

As described above in the model we will also implement lane changes, these will be done pretty similar to how it is done with CA, along with the added description of the γ terms above.

The choice to use RK4 is suitable since it has an accumulated error of order $\mathcal{O}(\Delta t^4)$, so we can choose relatively big Δt without offering to much error. This will also result in the simulation running quicker. That is needed when using IDM because we use it to simulate a real highway with velocities around 30 m/s, which needs longer road to be of interest and therefore longer time intervals.

To get some randomness the maximum speed of each car will slightly differ between cars. The initial velocity will both be tested to be set to 0 as well as to each car's maximum speed. The initial position of each car will be uniformly spread out across the road, with cars directly beside each other for roads with more than one lane.

Parameter	Value
a	0.73
δ	4
s_0	2
T	1.5
b	1.67
d	5
γ_{max}	10
γ_{min}	1
γ_{safe}	4
$t_{overtake}$	3

Table 1: Parameter values, IDM

The parameters used in this simulation will not
be tested for different values since that is not the object of study, as such the values are presented in table 1. The values of γ where chosen by testing how well the system developed. The other values were chosen to be the same as how the inventors of IDM chose[2].

4.2 Cellular automaton

For the CA model the rules stated earlier will be used directly. Different p will be chosen to get fair results, but also to get similar results to the IDM when comparing the models.

[2]Treiber, Martin; Hennecke, Ansgar; Helbing, Dirk (2000), "Congested traffic states in empirical observations and microscopic simulations", Physical Review E62, , 1805-1824 (2000)

5 Results

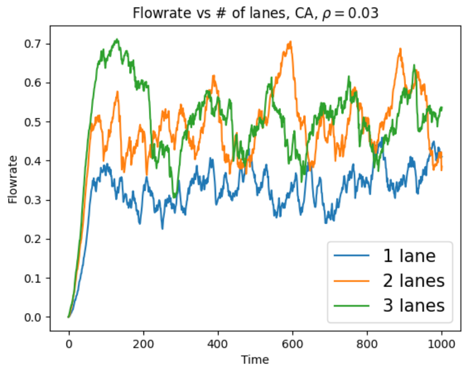

Figure 1: Flow rate of CA when road length $L = 1000$, $v_{max} = 30$ and $p = 0.5$

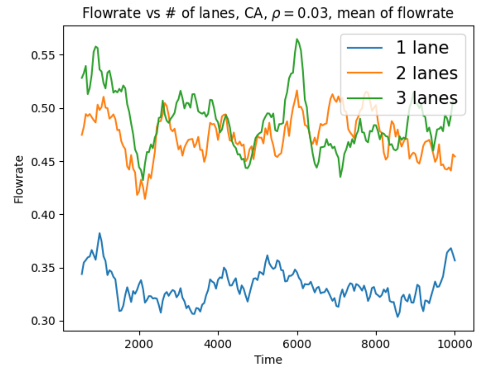

Figure 2: The mean of the flow rate of the last 50 values, $L = 1000$, $v_{max} = 30$ and $p = 0.5$.

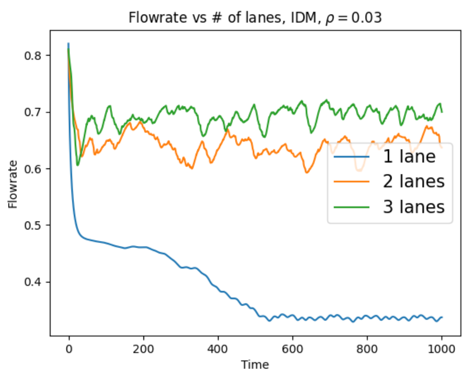

Figure 3: Flow rate when $L = 1000$, $v_{max} = 30$ and initial speed is $v_{0,n}$, $dt = 0.1$.

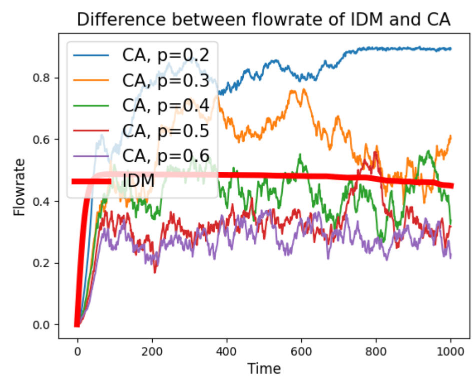

Figure 4: Flow rate comparison of CA and IDM, $L = 1000$, $v_{max} = 30$, 30 cars and $dt = 0.1$ for IDM.

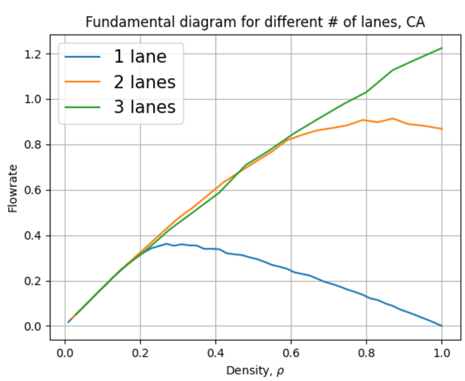

Figure 5: Fundamental diagram of CA with $L = 100$, $v_{max} = 2$, $p = 0.3$

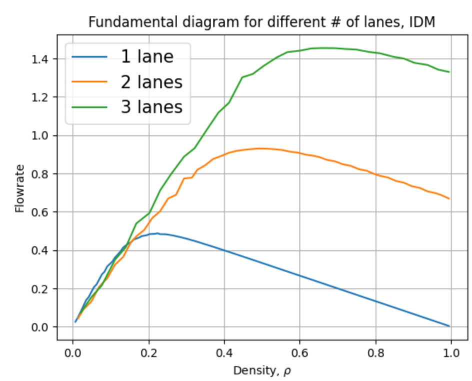

Figure 6: Fundamental diagram of IDM with $L = 1000$, $v_{max} = 30$, $dt = 0.1$.

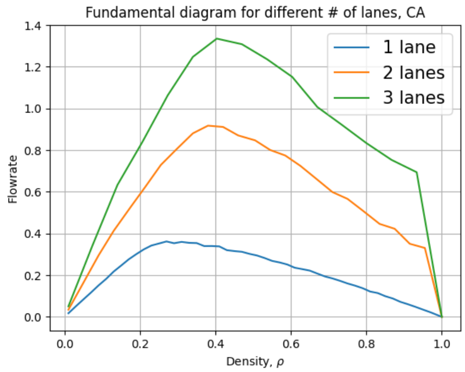

Figure 7: Fundamental diagram of CA with $L = 100$, $v_{max} = 2$, $p = 0.3$. Density defined to total road size.

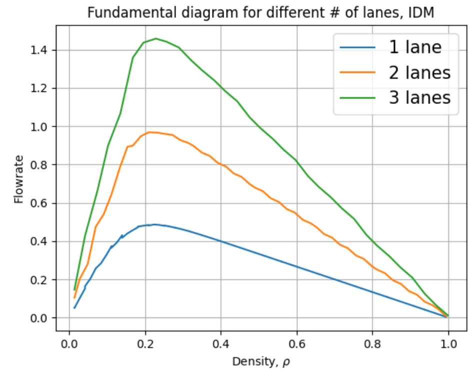

Figure 8: Fundamental diagram IDM with $L = 500$, $v_{max} = 30$. Density defined to total road length.

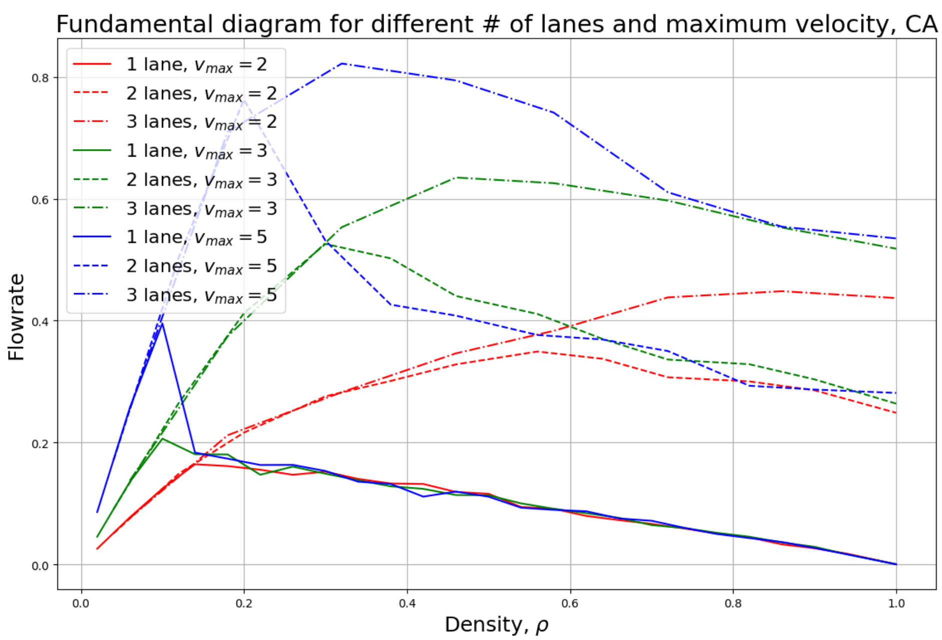

Figure 9: Fundamental diagram for different lanes and maximum velocities using CA.

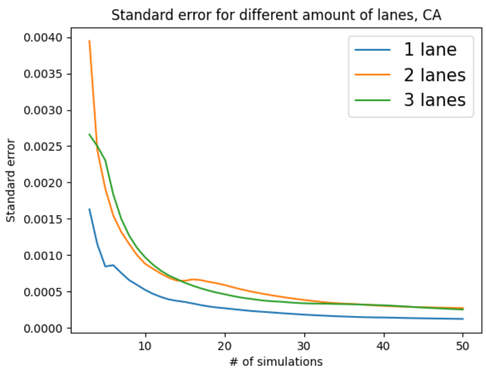

Figure 10: Standard error when $L = 50$, $v_{max} = 2$, $t_{max} = 1000$, 30 cars.
Flow rate calculated using last 500 values.

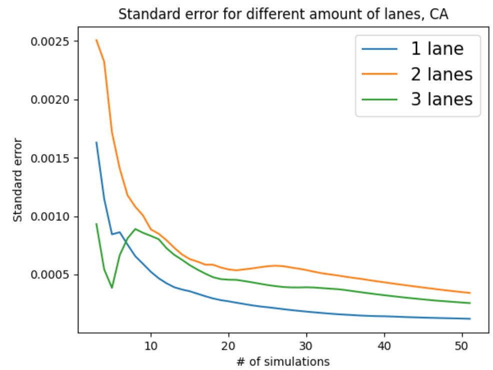

Figure 11: Standard error when $L = 50$, $v_{max} = 2$, $t_{max} = 1000$, 30 cars.
Flow rate calculated using last 500 values.

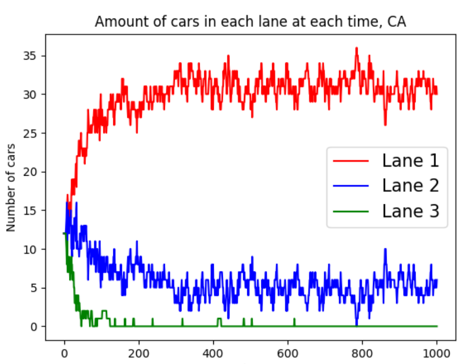

Figure 12: Count of amount of cars in each lane, $L = 100$, $v_{max} = 2$, $p = 0.3$, 12 cars.

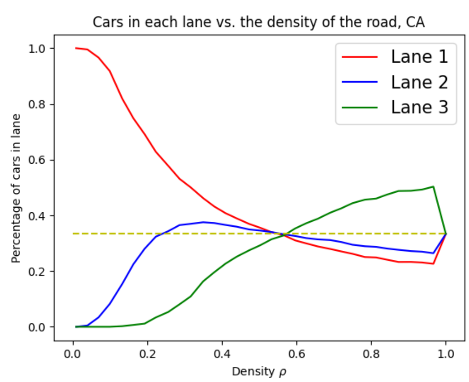

Figure 13: Percentage of cars in each lane for increasing density of cars on the road. $L = 100$, $v_{max} = 2$, $p = 0.3$. Amount of cars calculated at equilibrium.

6 Discussion & Analysis

6.1 Flow rates

In figures (1) to (4) we can see how the flow rate of the system changes with time both for IDM and CA. Setting the speed of the cars to their maximum for IDM at the beginning was tested in figure (3) and that resulted in somewhat chaotic behaviour. Instead setting them to zero we got smoother development of the flow rate, as can be seen in figure (4). In this figure different values of p for CA was tested to see how they compared to IDM. The velocity of for the CA was very big and not that representative of a real world simulation since the cars would jump unrealistically. But, we can still compare the two models as such and see that a probability of cars having to slow down between $p = 0.3$ and $p = 0.4$ is reasonable for a highway. That is because $v_{max} = 30$ (m/s) results in a highway speed limit of $v = 108$ km/h. As such we will set $p = 0.3$ for the rest of this paper since we are not interested in checking it's behaviour on the simulation.

In figures (5) and (6) the fundamental diagram for CA and IDM is plotted for 1, 2 and 3 lanes. Here it has been chosen so that the density of the traffic is calculated only with regards to the length of the road, and not regarding how many lanes their are. Counting with how many lanes there are is shown in figures (7) and (8). Both are reasonable ways to plot the fundamental diagram since the first way requires the density to be the same when equal amount of cars are on the road, whilst the second way shows the difference in the "actual" density. The peaks we can see in figure (7) is at 0.36, 0.92 and 1.34. We can not see as clearly in figure (5) how the specific values compare but for the same amount of cars the flow rate is much higher for more lanes when the density increases. For smaller density the graphs align because all cars ultimately drive on the outermost lane. When calculating the density with number of lanes counted we have 2 and 3 times more cars for the same density and that is a reason we see the increase of the flow rate close to those factors. The exact values gives an increase of $0.92/0.36 = 2.56$ and $1.34/0.36 = 3.72$ will be the result of faster cars being able to overtake slower cars instead of having to follow them. For the IDM we have the maximum values of 0.49, 0.97 and 1.46, which is within a safe approximation of a doubling and tripling of the flow rate.

In figure (9) the maximum velocity of the highway is compared along with how many lanes there are for the density calculated only with the road length. What can be noted is that for low densities the systems with the same maximum velocity behave similarly but for higher densities the amount of lanes govern how the flow rate develops. This is expected since for low densities the cars would all stay in the first lane and go at their maximum speed, which is the difference we see. For higher densities cars will begin to form traffic jams and as such the maximum velocity allowed will not matter as much since they all go at relatively the same speed.

6.2 Convergence

For figures (10) and (11) the standard error for different amount of lanes is plotted for two different runs and the parameters in the caption. Several similar runs were done and the same behaviour was noted. We clearly see that there is faster convergence for one lane but we don't see this faster convergence between two lanes and three lanes. The reason it is faster for one lane comes from the fact that the randomness has less effect on the simulation when the cars can not change lane. This is because for one lane the cars will have to slow down instead of changing lane when approaching a slower car.

To show the reliability of the simulation the amount of cars in each lane at each time step for a three lane highway is shown in figure (12). We can see that for a low density most cars drive at the outermost lane (lane one) whilst some cars are overtaking other cars. In figure (13) the percentage of the amount of cars in each lane is plotted against the density of the road. The first half of this graph is expected with more cars in lane one and two for low densities. Reaching the halfway point of $\rho = 0.5$ the equal proportion of cars in the lanes is also expected. The yellow dotted line is 1/3. For even higher densities the increase of cars in the innermost lane is a product of the condition set for changing lane after overtaking because we do not do step 3 if step 2 was done for the CA model. This means that most cars will stay in lane three. For higher densities the proportion of cars should be roughly equal in every lane which is something that can be improved if doing more thorough analysis, but here the simulation is adequate for the analysis being done.

6.3 CA vs. IDM

"What is the qualitatively difference between an *intelligent driver model* and a *cellular automata* model when simulating multi-lane traffic on a highway?" was a question we stated in section 3. First and foremost we can conclude that using IDM is far more time inefficient. For reasonable results a time step of around 0.1 can be used which takes a lot of time. Another qualitatively difference is the maximum speed of the cars in the models. Having larger v_{max} for the CA model results in similar behaviour to larger time steps for the IDM, that is more unreliable results due to the discrete nature of CA. Cars jumping forward 30 steps in one time step results in very chaotic behaviour which is seen in figure (2). Most of the plots are done for CA and that was chosen because the results are very similar between models regardless of the maximum speed being 30 for IDM and 2 for CA. As such a faster simulation was achieved by choosing CA. To get results from CA as universal as possible $p = 0.3$ was chosen as described in section 6.1. No integration accuracy for IDM was done explicitly but very similar results was found when decreasing the time step.