PROGRESS REPORT OF FEEDBACK CONTROL OF ROBOTIC GYMNAST

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JULY 26, 2018

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1 Introduction

The feedback control of a robotic gymnast attempt to model a gymnast swinging from the hanging position to the inverted balancing position on a bar as a double pendulum connected with a hinge. The lower pendulum in the stable equilibrium position is actuated by a motor to model the behaviour of the swinging legs. This system describe in the mathematical sense lends itself to be a underactuated system. Underactuated systems are where the control input cannot command an instantaneous acceleration in any direction of the state variables describing the system. The robotic gymnast needs to use the coupling between the actuated pendulum and unactuated pendulum to swing and balance the gymnast from it's stable equilibrium position to the unstable equilibrium position [1].

The field of underactuated robotics are becoming increasingly more important due to multiple fields such as the rocket,satellite,aerospace and the consumer products relying more on control systems which needs to control a underactuated system. Examples such as the James Webb Telescope for the satellite industries, SpaceX landing of their rockets, and drones for consumers products are easily media attention seekers that are underactuated systems.

The double pendulum that consist of a swing-up and balancing parts, is a underactuated problem, that employs fundamental concepts which exist in most underactuated problems. An complex non-linear problem of the swing-up part and the well defined linearised system of balancing the inverted double pendulum. The double pendulum is a great introductory problem to solve to step into the world of underactuated robotics.

The process of achieving the swing up and balance of the robotic gymnast consist of a sequence of steps that is critical to the success of the project. The project starts of by doing a literature study to learn and understand new concepts and the design paradigms towards the characteristic properties of a underactuated system. The literature study equips the reader to solve problems by understanding the fundamental behaviour of the system. Simulation of the system is done to verify the newly learned concepts, learn the impact of system properties on system behaviour and test different design paradigms. This will entail the use of information technology and engineering tools to implement the system on a simulation package and make use of scientific and engineering knowledge to distinguish between conflicting and authentic behaviour describe by models and supported by the literature study. The mechanical design of the system is to follow based on the requirements and specification determined during simulation and to verify the accuracy of the model. The electronic design will occur simultaneously to measure the state variables and perform signal conditioning allowing the control system to be implemented. These designs will require the synthesis of components, system and procedural design. The project will come to life by integrating the mechanical and electronic designs to allow the verification of the experimental data with simulation data. Problem solving and the application of engineering knowledge will be key to identify and verify any hypothesis in behaviour of the system. Reporting on the project will occur during the various phases described above and will demonstrate the competence to communicate effectively in writing. The project will be supervised by a researcher who will give critical feedback on the student's performance and project task. The relationship will illustrate the individual, team and multidisciplinary working during the project.

This report will document the process as describe in the preceding paragraph. The first chapter will provide a review of the essential concepts in control theory. This will be followed by the model describing the robotic gymnast and the assumptions made during the derivation of the model. Next the report describe the design paradigms to solving the non-linear swing-up of the gymnast. The report then describe the balancing of the gymnast in the unstable equilibrium position. Next the report discuss how the transition between the non-linear swing-up and the linearised will function.

2 Derivation of Double Pendulum

The robotic gymnast is modeled as two pendulums connected together with a hinge. Each pendulum is modeled as having their mass distributed arbitrary along their axis and a torque actuating the lower pendulum. Friction is modeled as proportional to the angular velocity of the pendulums. The friction that develops at the hinge connecting the 2 pendulums are a function of the relative motion between the two pendulums. The angle, ϕ was purposefully chosen relative to angle, θ. Figure 1 displays the free body diagram of the robotic gymnast.

Deriving the equation of motion of the robotic gymnast can be approached on different methods, but by exploring the system it can be shown that the system's energy is easily defined. The energy in the system are the potential energy of the 2 pendulums, the rotational kinetic energy of the underactu-

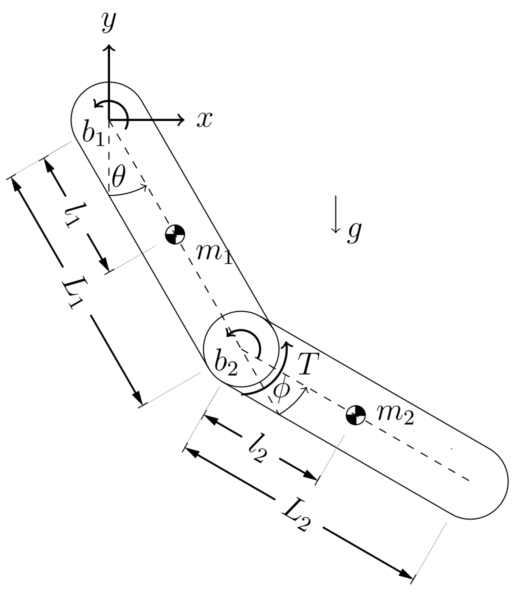

Figure 1: Free Body Diagram of the Double Pendulum

ated pendulum and the velocity- and rotational kinetic energy of the actuated pendulum. The system's energy is easily defined and for this reason the Lagrange-Euler equation is used to derive the differential equation describing the system dynamics. The derivation of the differential equation can be found in Appendix A.

The equation of motion for the robotic gymnast is non-linear. It contains terms that is a function of squared angular velocity, sine and more. These non-linearity can be linearised around a working point, but during the swing-up of the robotic gymnast, where the gymnast will operate in a large operating region linearisation will cause dramatic errors and thus these non-linearities need to be taken into account. How to overcome these non-linearities is discussed in the following section.

3 Swing Up of the Double Pendulum

It has been shown that it is not possible to linearise the dynamics of the gymnast by means of static state feedback and non-linear transformation [2], but it is possible to achieve a linear response from one of the state variables by implementing a non-linear feedback. This non-linear feedback is the partial feedback linearisation, where any of the 2 responses of the state variables can be linearised.

Collocated linearisation of the non-linear model is done to linearise the response of the actuated pendulum resulting in being proportional to the input of the system. The derivation of the collocated linearisation is shown in appendix C. This input can be selected to force the actuated pendulum to follow a desired trajectory [3].

To force the actuated pendulum to follow a desired trajectory, it is possible to pump energy into the system. The desired trajectory for ϕ^d is chosen as

$$\phi^d = \alpha \arctan(\dot{\theta})$$

[3]. The α coefficient constrains the actuated pendulum to stay within a interval of $\phi \in [-\alpha, \alpha]$ [3]. This provides better control over the system to stay within the null controllability region when the system reaches the unstable inverted position to switch over to the linearised model.

4 Balancing of the Double Pendulum

The balancing of the gymnast will be achieved by catching the gymnast when the swing-up control algorithm has brought the gymnast to the null controllability region. The null controllability region is the set of states that can be steered to inverted unstable equilibrium position in a fixed time with a constrained control input [4].

Within the null controllability region the gymnast can be approximated as a linear system. Linearising the model at

$$\vec{Q}_s = [\vec{q_s}, \dot{\vec{q_s}}, \ddot{\vec{q_s}}]^T = [\pi, 0, 0, 0, 0, 0]$$

the unstable inverted equilibrium position using the Taylor Series Expansion the model can be written in the state space form. The linearisation of the gymnast model is shown in Appendix B. The state space variables are chosen as Δq and $\Delta \dot{q}$ which results in the state space representation as:

$$\dot{\vec{x}} = \boldsymbol{A}\vec{x} + \boldsymbol{B}u$$

and

$$\vec{y} = \boldsymbol{D}\vec{x} + \boldsymbol{0}u$$

.

When the linearised system is at rest, any disturbance will result in a theoretically infinite growth of the state variables, but this behaviour can be controlled by introducing feedback. The instability of the system can be identified by examining the poles of the systems. The linearised model contains 4 real, 2 positive and 2 negative poles, and these poles will be moved to the desired position by using the method of dominant poles. The method of dominate poles chooses a pair of the poles for the closed-loop system and select the other open-loop poles to have real parts.

References

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Appendices

A Derivation of the Double Pendulum

$$x_1 = l_1 \cos(\theta)$$
$$y_1 = -l_1 \sin(\theta)$$

$$x_2 = L_1 \sin(\theta) + l_2 \sin(\theta + \phi)$$
$$y_2 = -L_1 \cos(\theta) - l_2 \cos(\theta + \phi)$$

$$\dot{x_2} = L_1 \cos(\theta)\dot{\theta} - l_2 \cos(\theta + \phi)(\dot{\theta} + \dot{\phi})$$
$$\dot{y_2} = L_1 \sin(\theta)\dot{\theta} + l_2 \sin(\theta + \phi)(\dot{\theta} + \dot{\phi})$$

$$x_2^2 = L_1^2 \cos(\theta)^2 \theta^2 + l_2^2 \cos(\theta + \phi)^2 (\dot{\theta} + \dot{\phi})^2 + 2L_1 l_2 \dot{\theta}(\dot{\theta} + \dot{\theta}) \cos(\theta) \cos(\theta + \phi)$$
$$y_2^2 = L_1^2 \sin(\theta)^2 \theta^2 + l_2^2 \sin(\theta + \phi)^2 (\dot{\theta} + \dot{\phi})^2 + 2L_1 l_2 \dot{\theta}(\dot{\theta} + \dot{\theta}) \sin(\theta) \sin(\theta + \phi)$$

$$x_2^2 + y_2^2 = L_1^2 \theta^2 [\cos(\theta)^2 + \sin(\theta)^2] + l_2^2 (\dot{\theta} + \dot{\phi})^2 [\cos(\theta + \phi)^2 + \sin(\theta + \phi)^2] +$$
$$2L_1 l_2 \dot{\theta}(\dot{\theta} + \dot{\phi})[\cos(\theta) \cos(\theta + \phi) + \sin(\theta) \sin(\theta + \phi)]$$

Using the following trigonometric identities

$$\cos(\gamma)^2 + \sin(\gamma)^2 = 1$$

$$\cos(\gamma)\cos(\alpha) + \sin(\gamma)\sin(\alpha) = \cos(\gamma - \alpha)$$

the above equation resolves to:

$$V_2^2 = L_1 \dot{\theta}^2 + l_2^2 (\dot{\theta} + \dot{\phi})^2 + 2L_1 l_2 (\dot{\theta} + \dot{\phi}) \dot{\theta} \cos(\phi)$$

The kinetic energy in the system consist of the fixed rotation of the under-actuated pendulum and the rotation and velocity of the actuated pendulum.

$$T = \frac{1}{2} I_A \dot{\theta}^2 + \frac{1}{2} I_B (\dot{\theta} + \dot{\phi})^2 + \frac{1}{2} m_2 V_2^2$$

$$T = \frac{1}{2} I_A \dot{\theta}^2 + \frac{1}{2} I_B (\dot{\theta} + \dot{\phi})^2 + \frac{1}{2} m_2 [L_1 \dot{\theta}^2 + l_2^2 (\dot{\theta} + \dot{\phi})^2 + 2L_1 l_2 (\dot{\theta} + \dot{\phi}) \dot{\theta} \cos(\phi)]^2$$

The potential energy in the system is defined as

$$V = -m_1 g l_1 \cos(\theta) - m_2 g[L_1 \cos(\theta) + l_2 \cos(\theta + \phi)]$$

The Lagrange is defined as

$$\mathcal{L} = T - V$$

$$\mathcal{L} = \frac{1}{2} I_A \dot{\theta}^2 + \frac{1}{2} I_B (\dot{\theta} + \dot{\phi})^2 + \frac{1}{2} m_2 [L_1 \dot{\theta}^2 + l_2^2 (\dot{\theta} + \dot{\phi})^2 + 2 L_1 l_2 (\dot{\theta} + \dot{\phi}) \dot{\theta} \cos(\phi)]^2 + m_1 g l_1 \cos(\theta) + m_2 g[L_1 \cos(\theta) + l_2 \cos(\theta + \phi)]$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = -m_1 g l_1 \sin(\theta) - m_2 g L_2 \sin(\theta) - m_2 g l_2 \sin(\theta + \phi)$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = I_A \ddot{\theta} + I_B \ddot{\theta} + I_B \ddot{\phi} + m_2 L_1^2 \ddot{\theta} + m_2 l_2^2 \ddot{\theta} + m_2 l_2 \ddot{\phi} + 2 m_2 L_1 l2 \ddot{\theta} \cos(\phi) - 2 m_2 L_1 l_2 \dot{\theta} \dot{\phi} \sin(\phi) + m_2 L_1 l_2 \ddot{\phi} \cos(\phi) - m_2 L_1 l_2 \dot{\phi}^2 \sin(\phi)$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m_2 L_1 l_2 (\dot{\theta} + \dot{\phi}) \dot{\theta} \sin(\phi) - m_2 g l_2 \sin(\theta + \phi)$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = I_B \ddot{\theta} + I_B \ddot{\phi} + m_2 l_2^2 \ddot{\theta} + m_2 l_2^2 \ddot{\phi} + m_2 L_1 l_2 \ddot{\theta} \cos(\phi) - m_2 L_1 l_2 \dot{\theta} \dot{\phi} \sin(\phi)$$

The differential equation describing the dynamics of the system is

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\vec{q}}} - \frac{\partial \mathcal{L}}{\partial q} = B(\dot{q}) + \tau(q)$$

where $q = \begin{bmatrix} \theta \\ \phi \end{bmatrix}$

B Linearisation of the Double Pendulum

The system will be linearised using the Taylor Series Expansion around the operating point

$$\vec{Q}_s = [\vec{q}_s, \dot{\vec{q}}_s, \ddot{\vec{q}}_s]^T = [\pi, 0, 0, 0, 0, 0]$$

and approximate the system as

$$F([\vec{q}, \dot{\vec{q}}, \ddot{\vec{q}}]^T) = F(\vec{Q}) \approx F(\vec{Q}_s) + [\Delta\vec{Q} \cdot \nabla F(\vec{Q}_s)]$$

where $\Delta\vec{Q} = \vec{Q} - \vec{Q}_s$. Resulting in 2 linear dependent equations:

$$\begin{split}\Delta\ddot{\theta}(I_A + I_B + m_2 l_2^2 + m_2 L_1^2 + 2m_2 l_2 L_1) + \Delta\ddot{\phi}(I_B + m_2 l_2^2 + m_2 L_1 l_2) + \\ \Delta\theta(-m_1 g l_1 - m_2 g L_1 - m_2 g l_2) + \Delta\phi(-m_2 g l_2) = \Delta\dot{\theta} b_1\end{split} \quad (1)$$

$$\begin{split}\Delta\ddot{\theta}(I_B + m_2 l_2^2 + m_2 L_1 l_2) + \Delta\ddot{\phi}(I_B + m_2 l_2^2) + \Delta\theta(-m_2 g l_2) + \\ \Delta\phi(-m_2 g l_2) = \tau + (\Delta\dot{\theta} + \Delta\dot{\phi}) b_2\end{split} \quad (2)$$

Equation (1) and (2) can be rewritten in state-space form by substituting the 2 equations into each other to remove the angular acceleration term of the other respectable angle.

The state space variables are chosen as Δq and $\Delta\dot{q}$ which results in the state space representation as:

$$\begin{bmatrix}\Delta\dot{\theta}\\ \Delta\dot{\phi}\\ \Delta\ddot{\theta}\\ \Delta\ddot{\phi}\end{bmatrix} = \begin{bmatrix}x & x & x & x\\ x & x & x & x\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix}\Delta\theta\\ \Delta\phi\\ \Delta\dot{\theta}\\ \Delta\dot{\phi}\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\\ 0\end{bmatrix} \tau$$

$$\begin{bmatrix}\Delta\theta\\ \Delta\phi\\ \Delta\dot{\theta}\\ \Delta\dot{\phi}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix}\Delta\theta\\ \Delta\phi\\ \Delta\dot{\theta}\\ \Delta\dot{\phi}\end{bmatrix} + \begin{bmatrix}0\end{bmatrix} \tau$$

The compact form will be used as

$$\dot{\vec{x}} = \boldsymbol{A}\vec{x} + \boldsymbol{B}\vec{u}$$

and

$$\vec{y} = \boldsymbol{D}\vec{x} + \boldsymbol{0}\vec{u}$$