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Mechanical Design 444
System Simulation Notes

Fan Calculations

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2017

Version 1.0

List of symbols

Variables

a	polynomial coefficient	
D	fan diameter	[m]
H	power	[W]
$\dot{m}$	air mass flow rate	[kg/s]
n_f	number of fans	
N	fan speed	[rpm]
P	air pressure	[Pa]
Q	total air volume flow rate	[$\mathrm{m^3/s}$]
T	air temperature	[K]
ρ	air density	[$\mathrm{kg/m^3}$]

subscripts

0	initial/base value
a, b, i, m, n	counter
f	fan
min	minimum
max	maximum
sys	system

subscripts

*	Reference value
′	Values for one fan

1. Fan laws

The following relationships are valid for dimensional equivalent fans:

$$\begin{aligned} &\text{volume flow:} && Q \propto N D^3 \\ &\text{mass flow:} && \dot{m} \propto \rho\, N D^3 \\ &\text{pressure increase:} && \Delta P \propto \rho\, N^2 D^3 \\ &\text{power:} && H \propto \rho\, N^3 D^5 \end{aligned} \tag{1}$$

2. Fan calculations

The pressure increase ΔP across the fan can be calculated for any system volume flow Q and air density ρ with the aid of the fan laws in equation (1).

Consider a single fan, then D=const. From the fan characteristic curves, e.g. Appendix A, select a reference curve ΔP^* with reference fan speed N^* and reference air density ρ^*. Fit a polynomial or any other appropriate curve through it, for example

$$\Delta P^*(Q^*) = a_n\, {Q^*}^n + a_{n-1}\, {Q^*}^{n-1} + \cdots + a_1 Q^* + a_0 \quad \text{[Pa]} \tag{2}$$

Calculate the reference volume flow Q^* and the reference pressure increase ΔP^* with equation (2)

$$Q' = Q/n_\mathrm{f} \tag{3}$$

$$Q^* = Q' \,\frac{N^*}{N} \qquad \text{with} \quad N_\mathrm{min} \le N \le N_\mathrm{max} \tag{4}$$

$$\Delta P^* = \Delta P^*(Q^*) \qquad \text{with} \quad Q^*_\mathrm{min} \le Q^* \le Q^*_\mathrm{max} \tag{5}$$

With ΔP^* known, calculate the pressure increase from the fan laws

$$\Delta P = \Delta P^* \,\frac{\rho}{\rho^*} \left(\frac{N}{N^*}\right)^2 \tag{6}$$

The calculation procedure is shown in figure 1.

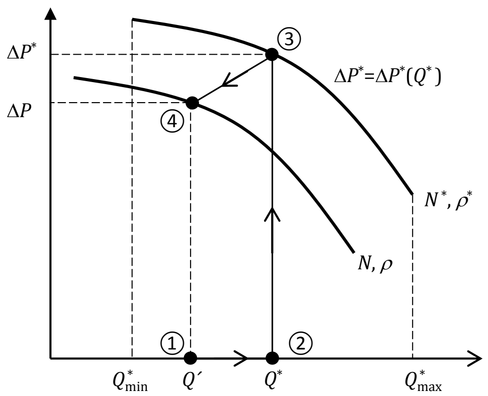


Figure 1: Calculation of ΔP

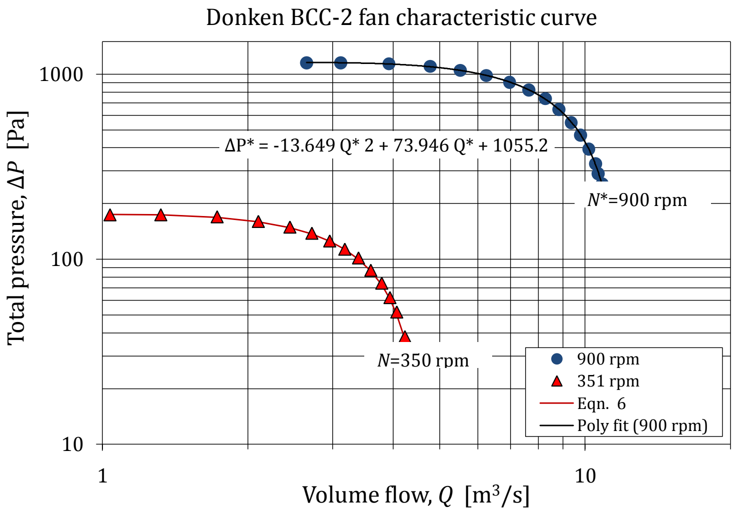
3. Example calculation

As an example use the Donken BCC-2 fan in Appendix A with reference speed $N^* = 900$ rpm. The digitization of the characteristic curve is given in figure 2. A polynomial curve fit of the points gives

$$\Delta P^*(Q^*) = -13.649\, {Q^*}^2 + 73.946\, Q^* + 1055.2 \quad \text{[Pa]} \tag{7}$$

and $N^* = 900$ rpm $\quad Q^*_{min} = 2.6\ \text{m}^3/\text{s} \quad N_{min} = 350$ rpm
$\rho^* = 1.2\ \text{kg/m}^3 \quad Q^*_{max} = 11.0\ \text{m}^3/\text{s} \quad N_{max} = 1337$ rpm

The second curve at 350 rpm in figure 2 is a check to verify the calculation procedure.



Q^*	ΔP^*
2.647	1152.0
3.118	1152.3
3.921	1138.1
4.773	1102.7
5.512	1047.9
6.242	983.2
6.976	904.9
7.644	822.2
8.267	737.6
8.824	645.0
9.357	546.2
9.791	468.5
10.179	394.2
10.513	327.5
10.648	290.0
10.856	255.2

Figure 2: Digitization of Donken BCC-2 fan characteristic curve at 900 rpm

4. Calculations

4.1. Fan work point

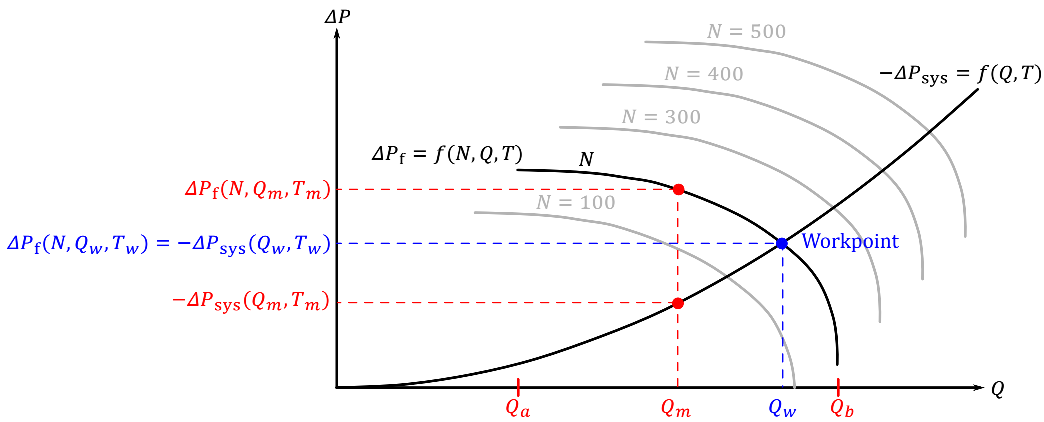


Figure 3: Work point calculations

The work point of a fan running at a selected speed N is the flow rate $Q = Q_w$ with balanced temperature $T = T_w$ where the pressure increase through the fan $\Delta P_{\mathrm{f}} = f(N, Q_w, T_w)$ is in equilibrium with the pressure loss through the system, $\Delta P_{\mathrm{sys}} = f(Q_w, T_w)$.

A simple method to obtain the work point numerically is the method of *interval halving*[1]:

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function WORKPOINT(N)
  $Q_a \leftarrow n_{\mathrm{f}}\, Q^*_{\min}\, N/N^*$ — *// Search bracket lower limit*
  $Q_b \leftarrow n_{\mathrm{f}}\, Q^*_{\max}\, N/N^*$ — *// Search bracket upper limit*
  $Q_m \leftarrow (Q_a + Q_b)/2$ — *// Midpoint*
  Balance system temperatures at Q_m to get T_m — *// Required for calculation of correct pressures*
  $i \leftarrow 1$ — *// Incrementor*
  while $|(Q_b - Q_a)/Q_m| > \varepsilon$ and $i \le i_{\max}$ **do**
    $\Delta P_{\mathrm{f}} \leftarrow \Delta P_{\mathrm{f}}(N, Q_m, T_m)$
    $\Delta P_{\mathrm{sys}} \leftarrow \Delta P_{\mathrm{sys}}(Q_m, T_m)$
    if $|\Delta P_{\mathrm{f}}| > |\Delta P_{\mathrm{sys}}|$ **then**
      $Q_a \leftarrow Q_m$ — *//*
    else — *// Halve the search interval*
      $Q_b \leftarrow Q_m$ — *//*
    end if
    $Q_m \leftarrow (Q_a + Q_b)/2$
    Balance system temperatures at Q_m to get T_m
    $i \leftarrow i + 1$
  end while
  return $Q \leftarrow Q_m$ — *//* Q_w
end function

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[1] Please note that there are more efficient algorithms for zero finding such as *Ridder's* method.

Appendix A Donken BCC-2 fan curve