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Mechanical Design 444
System Simulation Notes

Electric and Solar Heaters

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List of symbols

Variables

A	area	[m^2]
c_p	specific heat capacity	[J/(kg·K)]
D_H	hydraulic diameter	[m]
E	solar irradiance	[W/m^2]
H	collector height	[m]
L	collector length	[m]
$\dot{m}$	air mass flow rate	[kg/s]
p	pressure	[Pa]
q	heat transfer	[W/s]
Q	air volume flow rate	[m^3/s]
Re	Reynolds number	[−−]
t	time	[s]
t_s	solar time	[hr]
T	temperature	[K]
v	velocity	[m/s]
W	collector width	[m]
ε	surface roughness	[m]
$\mathfrak{f}$	D'Arcy friction factor	[−−]
θ	flow division	
η	thermal efficiency	
ρ	density	[kg/m^3]
μ	dynamic viscosity	[kg/(m·s)]
ν	kinematic viscosity	[m^2/s]

Subscripts

air	air parameters
h	electric heater unit
env	environment parameters
s	solar collector parameters

1. Electrical heater

The change of the air temperature through the electrical heater with a power input of q_h is

$$q_\mathrm{h} = \dot{m}_\mathrm{h} c_{p_\mathrm{air}} (T_\mathrm{h2} - T_\mathrm{h1}) \tag{1}$$

for a mass flow rate of

$$\dot{m}_\mathrm{h} = \rho_\mathrm{air}\, Q \tag{2}$$

The outlet temperature is

$$T_\mathrm{h2} = T_\mathrm{h1} + \frac{q_\mathrm{h}}{\dot{m}_\mathrm{h} c_{p_\mathrm{air}}} \tag{3}$$

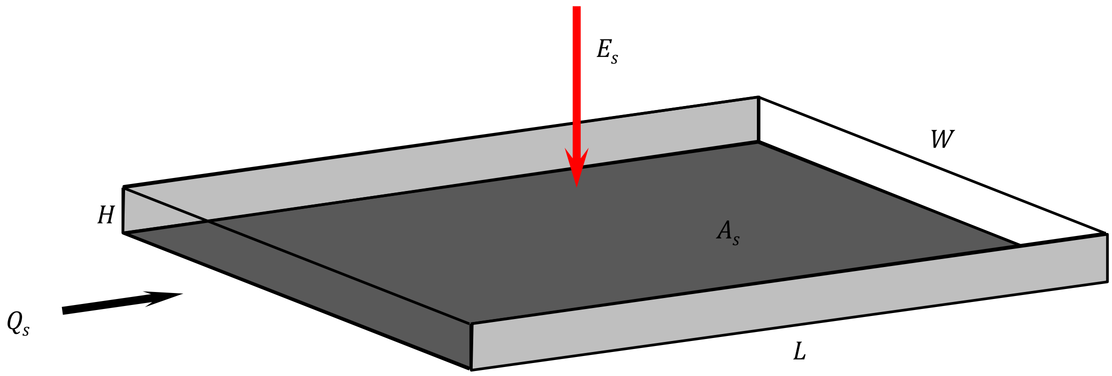

Figure 1: Flat plate solar collector

2. Flat plate solar collector

Consider the flat plate solar collector in figure 1. The flow through the solar collector is Q_s

2.1. Temperature change

The flat plate solar collector shown in figure 1 receives solar radiation of E_s. The heat transfer to the air is then

$$q_\mathrm{s} = \eta_\mathrm{s}\, E_\mathrm{s}\, A_\mathrm{s} = \dot{m}_\mathrm{s} c_{p_\mathrm{air}} (T_\mathrm{s2} - T_\mathrm{s1}) \tag{4}$$

with A_s the area of the collector. The thermal efficiency of the collector, η_s, is dependent on the difference between the environmental temperature T_env and the inlet temperature T_s1. It is usually defined as

$$\eta_\mathrm{s} = \eta_\mathrm{s0} - \eta_\mathrm{s1} \frac{T_\mathrm{s1} - T_\mathrm{env}}{E_\mathrm{s}} \tag{5}$$

with η_s0 and η_s1 proportional constants that depends on the construction and materials of the collector. Define an efficiency corrected solar irradiance value

$$E'_\mathrm{s} = \eta_\mathrm{s} E_\mathrm{s} = \eta_\mathrm{s0} E_\mathrm{s} - \eta_\mathrm{s1} (T_\mathrm{s1} - T_\mathrm{env}) \tag{6}$$

Note that E'_s can be negative when $E_\mathrm{s} < (\eta_\mathrm{s1}/\eta_\mathrm{s0})(T_\mathrm{s1} - T_\mathrm{env})$. This implies that the system are loosing heat to the environment. When this occurs a bypass valve will redirect the flow past the solar collector. For programming purposes make $E'_s \geq 0$ under all conditions.

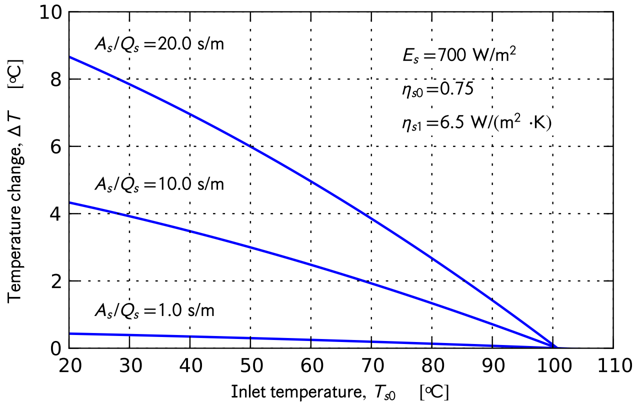

Figure 2: Temperature increase through a solar air heater

The mass flow rate through the collector is

$$\dot{m}_{\mathrm{s}} = \rho_{\mathrm{air}}\, Q_{\mathrm{s}} \tag{7}$$

The outlet temperature is then

$$T_{\mathrm{s2}} = T_{\mathrm{s1}} + \frac{E_{\mathrm{s}}'\, A_{\mathrm{s}}}{\dot{m}_{\mathrm{s}} c_{p\,\mathrm{air}}} \tag{8}$$

The change in air temperature is

$$\Delta T = \frac{E_{\mathrm{s}}'\, A_{\mathrm{s}}}{\dot{m}_{\mathrm{s}} c_{p\,\mathrm{air}}} = \frac{E_{\mathrm{s}}'}{\rho_{\mathrm{air}} c_{p\,\mathrm{air}}} \cdot \frac{A_{\mathrm{s}}}{Q_{\mathrm{s}}} \tag{9}$$

The ratio of the area over the volume flow rate $A_{\mathrm{s}}/Q_{\mathrm{s}}$ is the residence time of the air in the collector (unit [s/m]). In figure 2 a basic system analysis of the solar collector is performed.

2.2. Pressure loss

The pressure loss in a pipe, tube or duct can be expressed with the D'Arcy-Weisbach equation

$$\Delta p_{\mathrm{s}} = -\mathfrak{f} \left(\frac{L}{D_{\mathrm{H}}}\right)\left(\frac{\rho_{\mathrm{air}} v_{\mathrm{s}}^2}{2}\right) \tag{10}$$

The air velocity through the solar collector is

$$v_{\mathrm{s}} = \frac{Q_{\mathrm{s}}}{A} = \frac{Q_{\mathrm{s}}}{H \cdot W} \tag{11}$$

and hydraulic diameter given by

$$D_{\mathrm{H}} = 4 \times \frac{\text{flow area}}{\text{wetted perimeter}} = \frac{2H \cdot W}{H + W} \tag{12}$$

The Haaland (1983) approximation of the Darcy friction factor is given by

$$\frac{1}{\sqrt{\mathfrak{f}}} = -1.8 \log_{10}\left[\left(\frac{\varepsilon/D_{\mathrm{H}}}{3.7}\right)^{1.11} + \frac{6.9}{Re_{\mathrm{H}}}\right] \qquad (Re_{\mathrm{H}} > 4000) \tag{13}$$

with Reynolds dimensionless number

$$Re_{\mathrm{H}} = \frac{v_{\mathrm{s}} D_{\mathrm{H}}}{\nu_{\mathrm{air}}} = \frac{2Q_{\mathrm{s}}}{\nu_{\mathrm{air}}(H + W)} \tag{14}$$

2.3. Solar irradiance

Let t be the simulation time in seconds and t_s the solar time given in hours

$$t_\mathrm{s} = t_{\mathrm{s}0} + \frac{t}{3600} \tag{15}$$

with $t_{\mathrm{s}0}$ the simulation start time.

To account for the hourly change in solar irradiance E_s, we define a simple cosine solar irradiance model for an average day

$$E_\mathrm{s} = E_{\mathrm{smax}} \times \max\left[0, -\cos\left(\frac{\pi}{12}\, t_\mathrm{s}\right)\right] \tag{16}$$

with E_{smax} the maximum solar irradiance during the day.

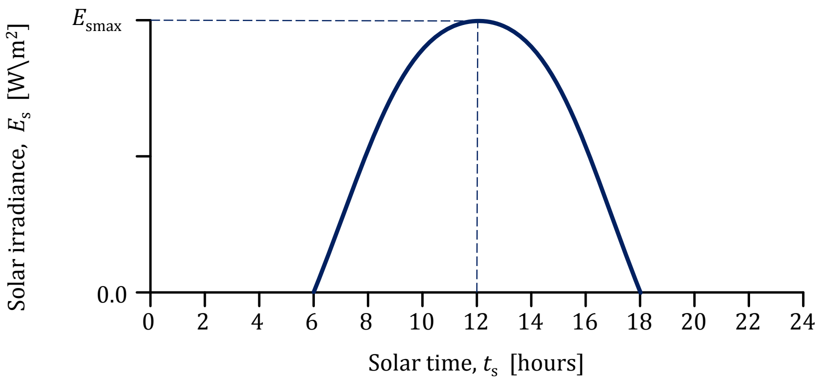

Figure 3: Solar irradiance model

References

Haaland, S.E. (1983). Simple and explicit formulas for the friction factor in turbulent flow. *Journal of Fluids Engineering*, vol. 105, no. 1, pp. 89–90.