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Mechanical Design 444

System Simulation Notes

Yogurt Incubator

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List of symbols

Variables

A	area	[m^2]
c_p	specific heat capacity	[J/(kg·K)]
D	cannister diameter	[m]
f	friction coefficient	[—]
F	Prandtl number correction factor	[—]
H	total cannister height	[m]
k	thermal conductivity	[W/(m·K)]
$\dot{m}$	air mass flow rate	[kg/s]
N	number of cannisters	
Nu	Nusselt number	[—]
P	pressure	[Pa]
Pr	Prandtl number	[—]
q	heat transfer	[W/s]
Q	air volume flow rate	[m^3/s]
Re	Reynolds number	[—]
S	pitch between cannisters	[m]
t	time	[s]
T	temperature	[K]
u	velocity	[m/s]
ρ	density	[kg/m^3]
μ	dynamic viscosity	[kg/(m·s)]
ν	kinematic viscosity	[m^2/s]
η	loss factor	[—]
χ	friction correction factor	[—]

Subscripts

L	longitudinal direction
T	transverse direction
s	air properties through incubator stack
y	yogurt containers

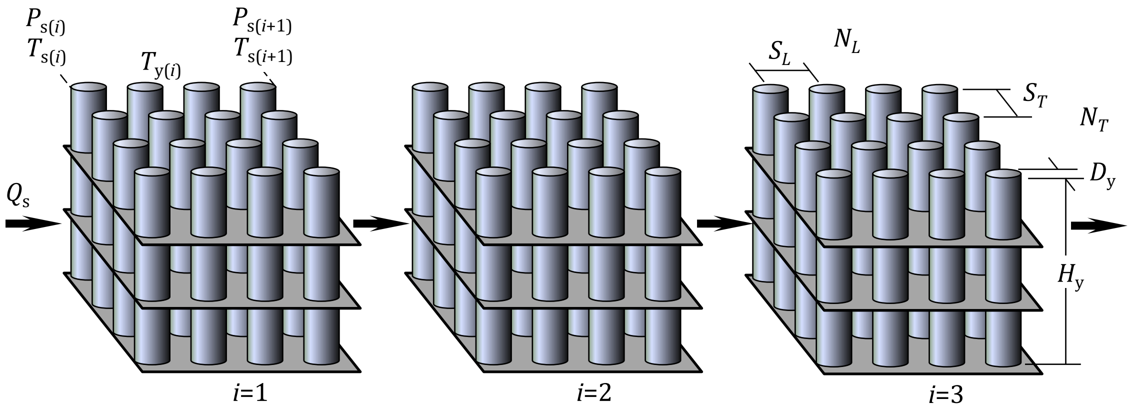

Figure 1: Yogurt incubator stack

1. Flow through yogurt cannister stacks

1.1. Air properties

The yogurt cannisters are arranged as shown in figure 1 in an inline configuration in the flow direction. Define the *longitudinal pitch* S_T and *transverse pitch* S_L. For a total yogurt stack height H_y the projected incubator stack flow area is

$$A_\mathrm{s} = N_T\, S_T\, H_\mathrm{y} \tag{1}$$

For a volume flow rate Q_s through the stack, define the superficial air velocity u'_s in the stack as the flow velocity without any obstacles in its path

$$u'_\mathrm{s} = \frac{Q_\mathrm{s}}{A_\mathrm{s}} \tag{2}$$

Define the average temperature in stack j as

$$\bar{T}_i = (T_{\mathrm{s}(i)} + T_{\mathrm{s}(i+1)})/2 \tag{3}$$

Calculate all the air properties except the exit Prandtl number $Pr_{\mathrm{s},e}$ using the average stack temperature

$$\rho_\mathrm{s} = \rho_\mathrm{air}(\bar{T}_i) \tag{4}$$
$$\mu_\mathrm{s} = \mu_\mathrm{air}(\bar{T}_i) \tag{5}$$
$$c_{p_\mathrm{s}} = c_{p_\mathrm{air}}(\bar{T}_i) \tag{6}$$
$$k_\mathrm{s} = k_\mathrm{air}(\bar{T}_i) \tag{7}$$
$$Pr_\mathrm{s} = Pr_\mathrm{air}(\bar{T}_i) \tag{8}$$

and

$$Pr_{\mathrm{s},e} = Pr_\mathrm{air}(T_{\mathrm{s}(i+1)}) \tag{9}$$

Define the following dimensionless groups based on the cannister diameter D_y

$$Re_\mathrm{D} = \frac{u'_\mathrm{s} D_\mathrm{y}}{\nu_\mathrm{s}} \qquad \text{Reynolds number} \tag{10}$$

$$Nu_\mathrm{D} = \frac{h_\mathrm{s} D_\mathrm{y}}{k_\mathrm{s}} \qquad \text{Nusselt number} \tag{11}$$

The *Nusselt number* for the flow through stack i is (Žukauskas, 1987)

$$Nu_{\mathrm{D}} = \begin{cases} 0.27\, F_{\mathrm{s}}\, Re_{\mathrm{D}}^{0.63} Pr_{\mathrm{s}}^{0.36} (Pr_{\mathrm{s}}/Pr_{\mathrm{s},e})^{0.25} & (10^3 < Re_{\mathrm{D}} < 2\times10^5) \\ 0.033\, F_{\mathrm{s}}\, Re_{\mathrm{D}}^{0.8}\ \ Pr_{\mathrm{s}}^{0.4}\ (Pr_{\mathrm{s}}/Pr_{\mathrm{s},e})^{0.25} & (2\times10^5 < Re_{\mathrm{D}} < 2\times10^6) \end{cases} \tag{12}$$

with F_{s} a correction factor for $N_L < 16$, which are given in table 1.

Table 1: Correction factor F_{s} for $N_L < 16$

N_L	1	2	3	4	5	7	10	13
F_{s}	0.7	0.8	0.86	0.90	0.93	0.96	0.98	0.99

1.2. Change in air temperature

Assume that all the canisters in stack i are at the same temperature, $T_{\mathrm{y}(i)}$. The heat transfer from the air to the canisters is

$$q_{\mathrm{s}(i)} = \dot{m}_{\mathrm{s}} c_{p_{\mathrm{s}}} \left(T_{\mathrm{s}(i+1)} - T_{\mathrm{s}(i)}\right) = h_{\mathrm{s}} A_{\mathrm{y}} \Delta T_{\mathrm{lm}} \tag{13}$$

with heat transfer coefficient from equations (11) and (12)

$$h_{\mathrm{s}} = \frac{Nu_{\mathrm{D}} k_{\mathrm{s}}}{D_{\mathrm{y}}} \tag{14}$$

mass flow rate

$$\dot{m}_{\mathrm{s}} = \rho_{\mathrm{s}} Q_{\mathrm{s}} \tag{15}$$

total yogurt canister outside area

$$A_{\mathrm{y}} = \pi D_{\mathrm{y}}\, H_{\mathrm{y}}\, N_L N_T \tag{16}$$

and the *log mean temperature difference* between the air and the canisters

$$\Delta T_{\mathrm{lm}} = \frac{T_{\mathrm{s}(i)} - T_{\mathrm{s}(i+1)}}{\ln\left(\frac{T_{\mathrm{s}(i)} - T_{\mathrm{y}(i)}}{T_{\mathrm{s}(i+1)} - T_{\mathrm{y}(i)}}\right)} \tag{17}$$

The exit temperature of the air through stack i can then be determined from equations (13) and (17)

$$T_{\mathrm{s}(i+1)} = T_{\mathrm{y}(i)} - \left(T_{\mathrm{y}(i)} - T_{\mathrm{s}(i)}\right) \exp\left(-\frac{A_{\mathrm{y}} h_{\mathrm{s}}}{\dot{m}_{\mathrm{s}} c_{p_{\mathrm{s}}}}\right) \tag{18}$$

1.3. Heat transfer to yogurt

The total mass of yogurt in stack i is

$$m_{\mathrm{y}} = \tfrac{\pi}{4}\, \rho_{\mathrm{y}} D_{\mathrm{y}}^2 H_{\mathrm{y}}\, N_L N_T \tag{19}$$

Assume that there is no temperature gradient inside the yogurt cannisters and that all the cannisters are at the same temperature $T_{\mathrm{y}(i)}$ inside stack i. The temperature change rate of the yogurt is then

$$m_{\mathrm{y}} c_{p_{\mathrm{y}}} \frac{\mathrm{d}T_{\mathrm{y}(i)}}{\mathrm{d}t} = -q_{\mathrm{s}(i)} = -\dot{m}_{\mathrm{s}} c_{p_{\mathrm{s}}} \left(T_{\mathrm{s}(i+1)} - T_{\mathrm{s}(i)}\right) \tag{20}$$

or

$$\frac{\mathrm{d}T_{\mathrm{y}(i)}}{\mathrm{d}t} = \frac{\dot{m}_{\mathrm{s}} c_p}{m_{\mathrm{y}} c_{p_{\mathrm{y}}}} \left(T_{\mathrm{s}(i)} - T_{\mathrm{s}(i+1)}\right) \tag{21}$$

Assume the following properties for the yogurt (Kim and Bhowmik, 1997)

$$c_{p_{\mathrm{y}}} = 3520\,\mathrm{J/(kg{\cdot}K)}$$
$$\rho_{\mathrm{y}} = 1050\,\mathrm{kg/m^3}$$

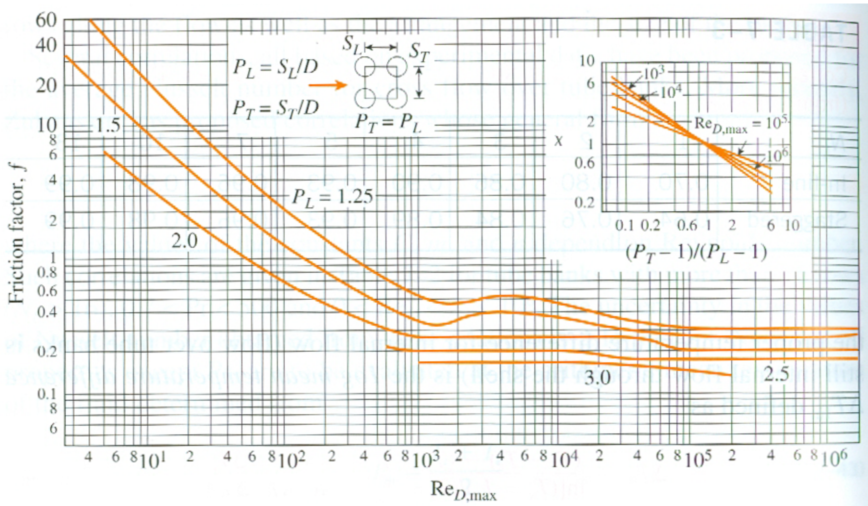

Figure 2: Friction factor f for tube banks (Çengel and Ghajar, 2011)

1.4. Pressure change through stack

The pressure change though stack i is given by

$$\Delta P_{\mathrm{s}(i)} = -N_L f \chi \frac{\rho_{\mathrm{s}} u_{\mathrm{s}}^{\prime 2}}{2} \tag{22}$$

The friction factor f and correction factor χ are given in figure 2. Note for a square layout ($S_T = S_L$) is $\chi = 1$. Further more if $Re_{\mathrm{D}} > 10^3$ then $f \approx$ const.

Summate the pressure loss through all the yogurt stacks and add an additional loss factor η_{s} for the total yogurt system. The total pressure loss is then

$$\Delta P_{\mathrm{s}} = \eta_{\mathrm{s}} \sum_{i=1}^{n_{\mathrm{s}}} \Delta P_{\mathrm{s}(i)} \tag{23}$$

References

Çengel, Y.A. and Ghajar, A.J. (2011). *Heat and mass transfer: Fundamentals and applications.* 4th edn. McGraw-Hill, New York.

Kim, S.S. and Bhowmik, S.R. (1997). Thermophysical properties of plain yogurt as functions of moisture content. *Journal of Food Engineering*, vol. 32, no. 1, pp. 109–124.

Žukauskas, A.A. (1987). Convective heat transfer in cross flow. In: Kakaç, S., Shah, R.K. and Aung, W. (eds.), *Handbook of Single-Phase Convective Heat Transfer*, chap. 6. Wiley, New York.