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Mechanical Design 444

System Simulation Notes

Rock Bed Heat Storage

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List of symbols

Variables

a	surface area	$[\mathrm{m}^2]$
A	area	$[\mathrm{m}^2]$
c_p	specific heat capacity	$[\mathrm{J/(kg{\cdot}K)}]$
d	diameter	$[\mathrm{m}]$
G	mass flow rate per unit area	$[\mathrm{kg/(s{\cdot}m^2)}]$
h_v	volumetric heat transfer coefficient	$[\mathrm{W/(m^3{\cdot}K)}]$
H	height	$[\mathrm{m}]$
k	thermal conductivity	$[\mathrm{W/(m{\cdot}K)}]$
L	length	$[\mathrm{m}]$
$\dot{m}$	air mass flow rate	$[\mathrm{kg/s}]$
Nu	Nusselt number	$[—]$
Pr	Prandtl number	$[—]$
q	heat transfer	$[\mathrm{W/s}]$
Q	air volume flow rate	$[\mathrm{m^3/s}]$
Re	Reynolds number	$[—]$
T	temperature	$[\mathrm{K}]$
u	velocity	$[\mathrm{m/s}]$
v	volume	$[\mathrm{m^3}]$
W	width	$[\mathrm{m}]$
$\mathfrak{f}$	friction factor	$[—]$
α	thermal diffusivity	$[\mathrm{m^2/s}]$
η	loss factor	$[—]$
$\bar{\varepsilon}$	void ratio	$[—]$
ψ	sphericity	$[—]$
ρ	density	$[\mathrm{kg/m^3}]$
μ	dynamic viscosity	$[\mathrm{kg/(m{\cdot}s)}]$
ν	kinematic viscosity	$[\mathrm{m^2/s}]$
ζ	temporary variable	

Subscripts

b	packed bed
d	dimensionless parameter properties based on particle diameter
r	air properties through the rock bed heat store
p	particle element

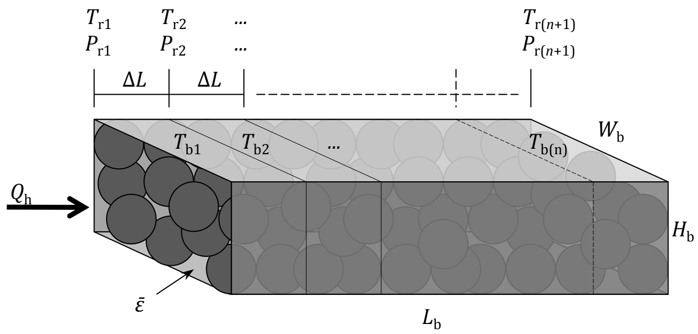

Figure 1: Packed bed heat store

1. Packed bed material properties

Consider the packed bed heat store shown in figure 1. It is filled with spherical stones. Define the average *void fraction* $\bar{\varepsilon}$ as

$$\bar{\varepsilon} = \frac{\text{volume of empty space}}{\text{volume of solid media}} \tag{1}$$

Assume that the flow is uniform and one-dimensional through the bed. The projected flow area A_b perpendicular to the flow direction is

$$A_\mathrm{b} = W_\mathrm{b}\, H_\mathrm{b} \tag{2}$$

with W_b and H_b the bed width and height.

Table 1: Fractions of air and solids in packed bed

	Air	Solids
Fraction	$\bar{\varepsilon}$	$(1-\bar{\varepsilon})$
Volume	$\bar{\varepsilon}\, A_\mathrm{b}\, L_\mathrm{b}$	$(1-\bar{\varepsilon})\, A_\mathrm{b}\, L_\mathrm{b}$
Mass	$\bar{\varepsilon}\, A_\mathrm{b}\, L_\mathrm{b}\, \rho_\mathrm{r}$	$(1-\bar{\varepsilon})\, A_\mathrm{b}\, L_\mathrm{b}\, \rho_\mathrm{p}$

The different fractions of solids and air are given in table 1. Figure 2 gives the average void fraction for same-sized spherical particles in cylindrical container.

Let v_p be the volume and a_p the surface area of a particle element in the bed. For none spherical particle the *equivalent particle diameter* is defined as

$$d_\mathrm{p} = \left(\frac{6}{\pi} v_\mathrm{p}\right)^{1/3} \tag{3}$$

The *sphericity* of the particle ψ is defined as the ratio of the particle surface area to that of a sphere with the same volume

$$\psi = \frac{a_\mathrm{p}}{\pi d_\mathrm{p}^2} \tag{4}$$

Define the the *superficial air velocity* u_r' in the bed as the flow velocity without any obstacles in its path

$$u_\mathrm{r}' = \frac{Q_\mathrm{r}}{A_\mathrm{b}} \tag{5}$$

with Q_r the volume flow rate. The mass flow rate of air per unit area, G_r, through the bed is

$$G_\mathrm{r} = \rho_\mathrm{r} u_\mathrm{r}' = \frac{\rho_\mathrm{r} Q_\mathrm{r}}{A_\mathrm{b}} \tag{6}$$

with ρ_r the air density and μ_r the air kinematic viscosity in the bed.

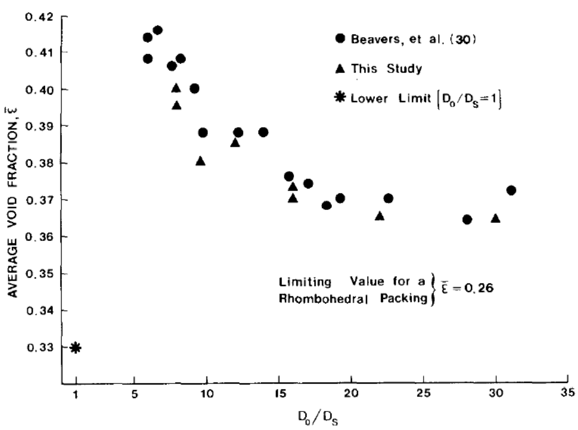

Figure 2: Average void fraction as a function of container diameter to particle diameter for uniform spheres. (Beasley and Clark, 1984)

Define the dimensionless groups based on the particle diameter d_p

$$Re_\mathrm{d} = \frac{G_\mathrm{r} d_\mathrm{p}}{\mu_\mathrm{r}} \qquad \text{Reynolds number} \tag{7}$$

$$Nu_\mathrm{d} = \frac{h'_\mathrm{v} d_\mathrm{p}^2}{k_\mathrm{r}} \qquad \text{Nusselt number} \tag{8}$$

$$\mathfrak{f}_\mathrm{d} = \frac{\Delta P}{\Delta L} \frac{\rho_\mathrm{r} d_\mathrm{p}}{G_\mathrm{r}^2} \qquad \text{friction factor} \tag{9}$$

Singh *et al.* (2006) performed a wide range of experiments for heat transfer and pressure loss through packed beds and found the following empirical relationships for the Nusselt number and friction factor

$$Nu_\mathrm{d} = 0.437\,(Re_\mathrm{d})^{0.75}\,(\psi)^{3.35}\,(\bar{\varepsilon})^{-1.62}\ \exp\!\left[29.03\,(\ln\psi)^2\right] \tag{10}$$

$$\mathfrak{f}_\mathrm{d} = 4.466\,(Re_\mathrm{d})^{-0.2}\,(\psi)^{0.696}\,(\bar{\varepsilon})^{-2.945}\ \exp\!\left[11.85\,(\ln\psi)^2\right] \tag{11}$$

The equations above holds for sphericity: $0.55 \le \psi \le 1.00$, void fraction: $0.306 \le \bar{\varepsilon} \le 0.63$ and mass velocity: $0.155\,\mathrm{kg/(s{\cdot}m^2)} \le G_\mathrm{r} \le 0.266\,\mathrm{kg/(s{\cdot}m^2)}$.

2. Heat transfer between the bed and the air

The heat transfer from the air to the bed in control volume i is

$$q_{\mathrm{b}(i)} = \dot{m}\,c_{p_\mathrm{r}}\left(T_{\mathrm{r}(i)} - T_{\mathrm{r}(i+1)}\right) = h'_\mathrm{v}\,A_\mathrm{b}\,\Delta L\,\Delta T_{\mathrm{eff}(i)} \tag{12}$$

with $T_{\mathrm{r}(i)}$ and $T_{\mathrm{r}(i+1)}$ the average in and outlet air temperatures of control volume i. The logarithmic effective temperature difference is

$$\Delta T_{\mathrm{eff}(i)} = \frac{T_{\mathrm{r}(i)} - T_{\mathrm{r}(i+1)}}{\ln\left(\frac{T_{\mathrm{r}(i)} - T_{\mathrm{b}(i)}}{T_{\mathrm{r}(i+1)} - T_{\mathrm{b}(i)}}\right)} \tag{13}$$

The mass flow rate through the control volume is

$$\dot{m} = \rho_\mathrm{r}\,Q_\mathrm{r} = G_\mathrm{r}\,A_\mathrm{b} \tag{14}$$

The outlet temperature $T_{\mathrm{r}(i+1)}$ in equations (12) and (13) is unknown. Solve for $T_{\mathrm{r}(i+1)}$, then

$$T_{\mathrm{r}(i+1)} = T_{\mathrm{b}(i)} - \left(T_{\mathrm{b}(i)} - T_{\mathrm{r}(i)}\right)\exp\left(-\frac{h'_\mathrm{v}\Delta L}{c_{p_\mathrm{r}} G_\mathrm{r}}\right) \tag{15}$$

3. Temperature change in the bed

The total mass of particles in control volume i is

$$m_\mathrm{b} = \rho_\mathrm{p}(1-\bar{\varepsilon})\,A_\mathrm{b}\,\Delta L \tag{16}$$

Assume that there is no temperature gradient inside the individual particles and that all the particle in a control volume is at the average bed temperature, $T_{\mathrm{b}(i)}$. The temperature change rate of the particles is then

$$m_\mathrm{b} c_{p_\mathrm{p}} \frac{dT_{\mathrm{b}(i)}}{dt} = q_{\mathrm{b}(i)} = \dot{m}\,c_{p_\mathrm{r}}\left(T_{\mathrm{r}(i)} - T_{\mathrm{r}(i+1)}\right) \tag{17}$$

or

$$\frac{dT_{\mathrm{b}(i)}}{dt} = \frac{c_{p_\mathrm{r}} G_\mathrm{r}}{\rho_\mathrm{p} c_{p_\mathrm{p}}(1-\bar{\varepsilon})\Delta L}\left(T_{\mathrm{r}(i)} - T_{\mathrm{r}(i+1)}\right) \tag{18}$$

4. Calculation procedure

With reference to figure 1, divide the packed bed in n control volumes with lengths $\Delta L = L_{\mathrm{b}}/n$. Let $T_{\mathrm{r}(i)}$ be the inlet air temperature for control volume i. For every control volume $i = 1, 2 \ldots, n$ and $T_{\mathrm{r}1} = T_{\mathrm{s}2}$

(a) Calculate average temperature over control volume

$$\bar{T}_i = \frac{1}{2}\left(T_{\mathrm{r}(i)} + T_{\mathrm{r}(i+1)}^{\mathrm{old}}\right)$$

(b) Calculate average air and flow properties in control volume

$$\rho_i = \rho_{\mathrm{air}}(\bar{T}_i)$$
$$\mu_i = \mu_{\mathrm{air}}(\bar{T}_i)$$
$$c_{p_i} = c_{p_{\mathrm{air}}}(\bar{T}_i)$$
$$k_i = k_{\mathrm{air}}(\bar{T}_i)$$
$$G_i = \rho_i Q_{\mathrm{r}}/A_{\mathrm{b}}$$

(c) Calculate parameters

$$Re_i = G_i d_{\mathrm{p}}/\mu_i$$
$$\mathfrak{f}_i = 4.466\,(Re_i)^{-0.2}\,(\psi)^{0.696}\,(\bar{\varepsilon})^{-2.945}\,\exp\left[11.85\,(\ln\psi)^2\right]$$
$$Nu_i = 0.437\,(Re_i)^{0.75}\,(\psi)^{3.35}\,(\bar{\varepsilon})^{-1.62}\,\exp\left[29.03\,(\ln\psi)^2\right]$$
$$h'_{\mathrm{v}i} = k_i\,Nu_i/d_{\mathrm{p}}^2$$

(d) Calculate air temperature change and pressure drop

$$T_{\mathrm{r}(i+1)} = T_{\mathrm{b}(i)} - \left(T_{\mathrm{b}(i)} - T_{\mathrm{r}(i)}\right)\exp\left(-\frac{h'_{\mathrm{v}}\Delta L}{c_{p_{\mathrm{r}}} G_{\mathrm{r}}}\right)$$

$$\Delta P = -\mathfrak{f}_i \frac{G_i^2\,\Delta L}{\rho_i d_{\mathrm{p}}}$$

(e) Calculate rate of change of rock bed temperature

$$\frac{dT_{\mathrm{b}(i)}}{dt} = \frac{c_{p_i} G_i\left(T_{\mathrm{r}(i)} - T_{\mathrm{r}(i+1)}\right)}{\rho_{\mathrm{p}} c_{p_{\mathrm{p}}}(1-\bar{\varepsilon})\Delta L}$$

Summate the pressure loss across all the control volumes and add an additional loss factor η_{r} for the total heat store system. The total pressure loss is then

$$\Delta P_{\mathrm{r}} = \eta_{\mathrm{r}} \sum_{i=1}^{n} \Delta P_i \tag{19}$$

5. Simulation input values

For preliminary simulation purpose use the following input values for rock

Bed void fraction	$\bar{\varepsilon}$ = 0.45
Particle diameter	d_{p} = 10 cm
Particle sphericity	ψ = 1.00
Particle density	ρ_{p} = 2640 kg/m^3
Particle heat capacity	$c_{p_{\mathrm{p}}}$ = 820 J/(kg·K)
Pressure loss factor	η_{r} = 1.2

References

Beasley, D.E. and Clark, J.A. (1984). Transient response of a packed bed for thermal energy storage. *International Journal of Heat and Mass Transfer*, vol. 21, no. 9, pp. 1659–1669.

Singh, R., Saini, R.P. and Saini, J.S. (2006). Nusselt number and friction factor correlations for packed bed solar energy storage system having large sized elements of different shapes. *Solar Energy*, vol. 80, pp. 760–771.