CSUEB – STAT 6305 – Winter 2017 - Prof Yan Zhou

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February 09, 2017

HW 4: 14.11, 14.12, 14.13, 14.20, 14.21

14.11

14.11 Commercially produced ice cream is made from a mixture of ingredients:

- a minimum of 10% milk fat,
- 9–12% milk solids: this component, also known as the serum solids, contains the proteins (caseins and whey proteins) and carbohydrates (lactose) found in milk
- 12–16% sweeteners: usually a combination of sucrose and/or glucose-based corn syrup sweeteners
- 0.2–0.5% stabilizers and emulsifiers, e.g., agar or carrageenan extracted from seaweed
- 55%–64% water which comes from milk solids or other ingredients

Air is incorporated with the above ingredients during the mixing process. Less expensive ice creams contain lower-quality ingredients and more air is incorporated during the mixing process. The finest ice creams have between 3% and 15% air. Because most ice cream is sold by volume, it is economically advantageous for producers to reduce the density of the product in order to cut costs. A food scientist is investigating how varying the amounts of the above ingredients impacts the sensory rating of the final product. The scientist decides to use three levels of milk fat: 10%, 12%, 15%; three amounts of air: 5%, 10%, 15%; and two levels of sweeteners: 12%, 16%. Three replications of each of the formulations were produced and the sensory ratings (0–40) obtained, a higher number implies a more favorable sensory rating. The data and Minitab output are given here.

a) **Identify the design and treatment structure for this study.**

The experiment is setup as a CRD factorial treatment design with the following factors: sweetener percentage, milk fat percentage and air percentage. The design uses three replications for each combination.

b) **Write a model for this study, identifying all terms in the model.**

Factorial design model:

$$\boldsymbol{y_{ijkm} = \mu + \tau_i + \beta_j + \gamma_k + \tau\beta_{ij} + \tau\gamma_{ik} + \beta\gamma_{jk} + \tau\beta\gamma_{ijk} + \varepsilon_{ijkm}}$$

$$i = 1,2;\ j = 1,2,3;\ k = 1,2,3;\ m = 1,2,3$$

Using a factorial treatment structure with:

μ – overall sensory rating mean

y_{ijkm} – the sensory rating from each sweetener-milk fat-air combination: 54 observations from 3 replications of the 18 treatment combinations

τ_i – the effect due to the percentage of sweetener: 2 treatment levels

β_j – the effect due to the percentage of milk fat: 3 treatment levels

γ_k – the effect due to the percentage of air: 3 treatment levels

$\tau\beta_{ij}$ – the effect due to the two-way interaction between the levels of sweetener and milk fat factors: 6 interactions

$\tau\gamma_{ik}$ – the effect due to the two-way interactions between the levels of sweetener and air factors: 6 interactions

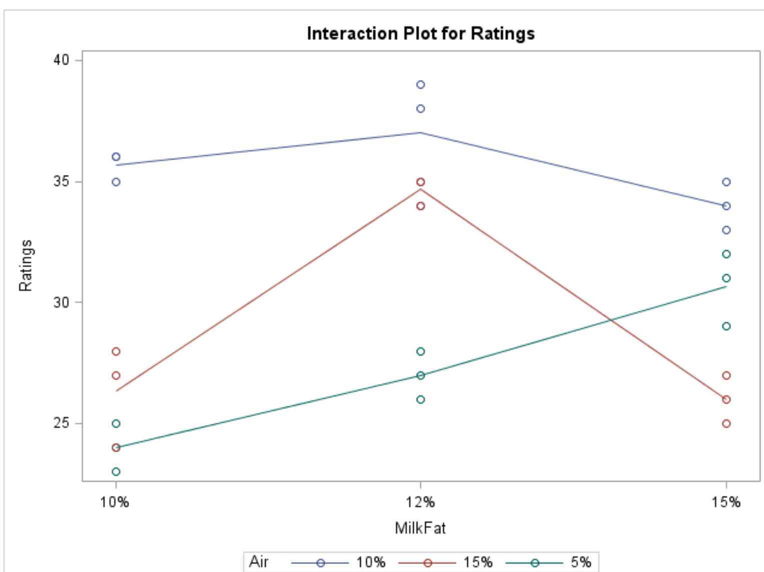
$\beta\gamma_{jk}$ – the effect due to the two-way interactions between the levels of milk fat and air factors: 9 interactions

$\tau\beta\gamma_{ijk}$ – the effect due to the three-way interactions between the levels of sweetener, milk fat and air factors: 18 interactions

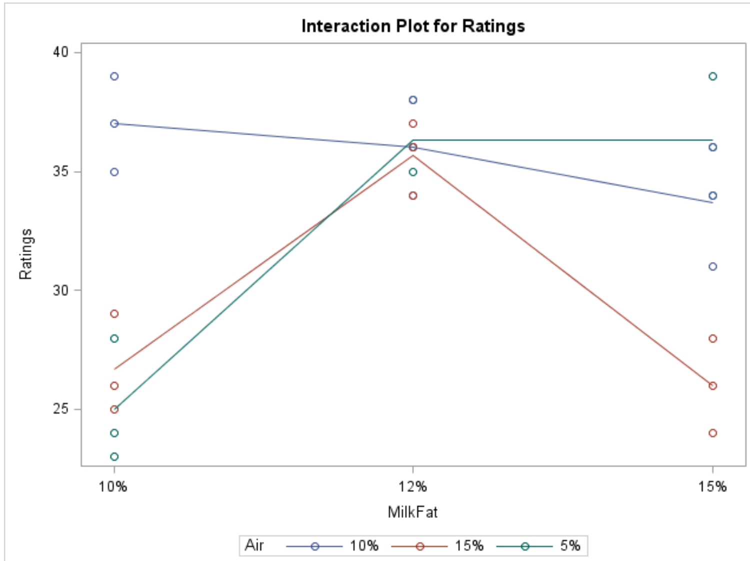
ε_{ijkm} – random error associated with each sweetener-milk fat-air combination: 54 residual errors

c) For each of the two levels of sweetener, draw profile plots of the effects of the percentage of air and milk fat on the sensory rating of ice cream.

Profile plot for the interaction between milk fat and air percentage for **sweetener at 12%**.



Profile plot for the interaction between milk fat and air percentage for **sweetener at 16%.**



d) From the profile plots, does there appear to be a three-way interaction between the effects percentage of sweetener, air, and milk fat in ice cream on the mean sensory rating?

Examining each profile plot – one for each level of sweetener – separately, it is clear that milkfat and air content interact as the lines cross each other and are not even close t parallel. Moreover, we observe comparing the two plots that the lines (especially the blue line, air = 5) has a different profile between different levels of sweetener. Thus, we can infer that air and sweetener factors probably interact in this experiment. However, the profiles of milkfat do not change as much between the two profile plots, which leads us to believe this interaction between milkfat and sweetener might not be as significant.

14.12

14.12 Refer to the study described in Exercise 14.11.

a) **Using the output given above, perform appropriate *F*-tests and draw conclusions from these tests concerning the effect of the percentage of sweetener, air, and milk fat on the sensory rating of ice cream. Use $\alpha = 0.05$.**

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Sweetener	1	50.0740741	50.0740741	15.28	0.0004
MilkFat	2	261.3333333	130.6666667	39.86	<.0001
Air	2	436.0000000	218.0000000	66.51	<.0001
Sweetener*MilkFat	2	11.2592593	5.6296296	1.72	0.1939
Sweetener*Air	2	78.8148148	39.4074074	12.02	0.0001
MilkFat*Air	4	355.6666667	88.9166667	27.13	<.0001
Sweetener*MilkFat*Air	4	46.1851852	11.5462963	3.52	0.0159

From the AOV table above, we have a p-value of 0.0159 for the three-way interaction, implying that it has a significant effect on the mean sensory rating. We also have a p-value of 0.001 for both the Milk Fat-Air interaction and the Sweetener-Air interaction. Therefore, we would conclude that the three-way and two-way interactions must be included in the model to explain the variance of the data.

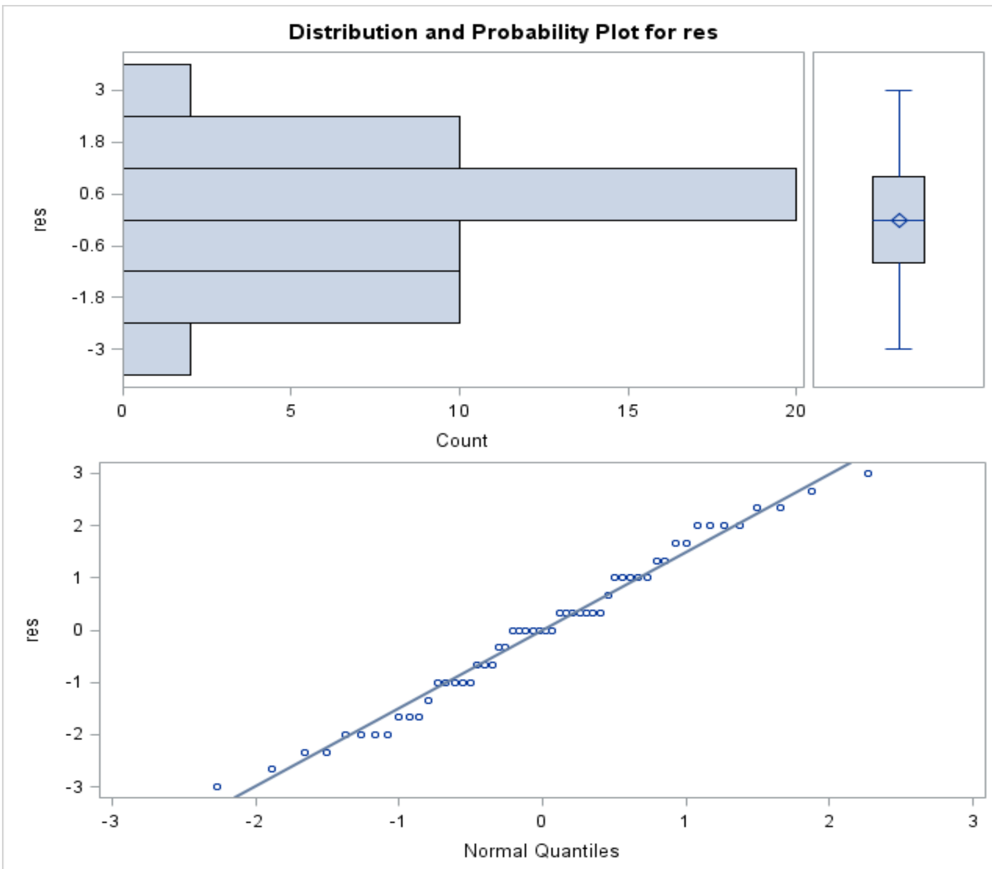
b) **Are the conclusions from the *F*-test consistent with your observations from the profile plots?**

Yes, the conclusion from the F-test in part (a) agree with the conclusion made from examining the profile plots in question 14.11 that there is a three-way interaction between the three factors. Moreover, we had observed that while there was probably a two-way factor interaction between sweetener-air and milkfat-air, the one between milkfat-sweetener was not as strong, which we could confirm with non-significant p-value.

14.13

14.13 Refer to the study described in Exercise 14.11. The following plots were obtained from the residuals from the fitted model.

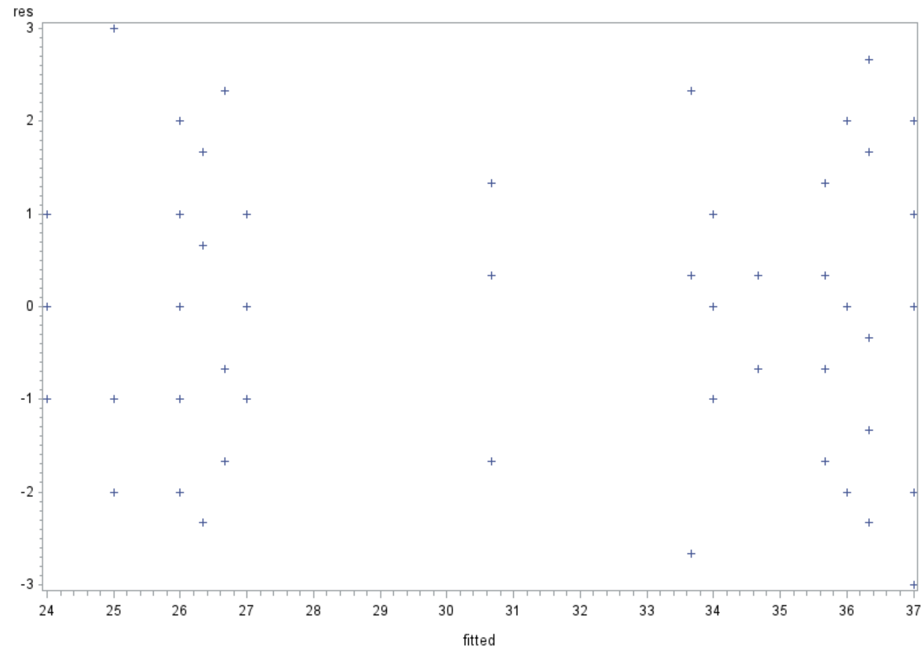
a) **Is there significant evidence that the residuals have a non-normal distribution?**



Tests for Normality				
Test	Statistic		p Value	
Shapiro-Wilk	W	0.97742	Pr < W	0.3978

We see from the Normal QQ-plot of the residuals that there is very little deviation from the line. Further, we have a *p*-value for the normality test of 0.3978. **Thus, we do not reject non-normality and conclude that the residuals follow a normal distribution.**

b) Is there significant evidence that the residuals do not have constant variances?



We see from the residuals vs. fitted values plot above that, although the fitted values are more concentrated on the extremes, there is no particularly strong vertical pattern in the residuals. To examine further, we run a Levene test on the data set where we consider each combination of Sweetener-Milk Fat-Air to be different levels of one factor.

Levene's Test for Homogeneity of ratings Variance ANOVA of Squared Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
treatment	17	121.3	7.1329	1.35	0.2165
Error	36	189.6	5.2654		

We have from the table above a *p*-value of 0.2165, implying that we fail to reject the null hypothesis of equal variances.

c) How could we assess whether the residuals are independently distributed?

We could review the conditions under which the experiment was performed, to make sure the treatments were truly randomly assigned. Besides what we already did, which is examining the fitted vs. residuals for patterns, we could also plot the response data vs the residuals and make sure that has no pattern.

14.20

14.20 A study was conducted to compare the effect of four manganese rates (from $MnSO_4$) and four copper rates (from $CuSO_4 \, 5H_2O$) on the yield of soybeans. A large field was subdivided into 32 separate plots. Two plots were randomly assigned to each of the 16 factor–level combinations (treatments) and the treatments were applied to the designated plot. Soybeans were then planted over the entire field in rows 3 feet apart. The yields from the 32 plots are given here (in kilograms/hectare).

a) **Identify the design for this experiment.**

The experiment follows a CRD factorial treatment design with the following factors: manganese (Mn) and copper rates (Cu). The design uses two replications for each combination.

b) **Write an appropriate statistical model for this experiment.**

Factorial design model: $\boldsymbol{y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \varepsilon_{ijk}}$

$$i = 1,2,3,4;\ j = 1,2,3,4;\ k = 1,2$$

μ – overall soybean yield mean

y_{ijk} – the yield of soybeans from each Mn-Cu combination: 32 observations from 3 replications of the 16 treatment combinations

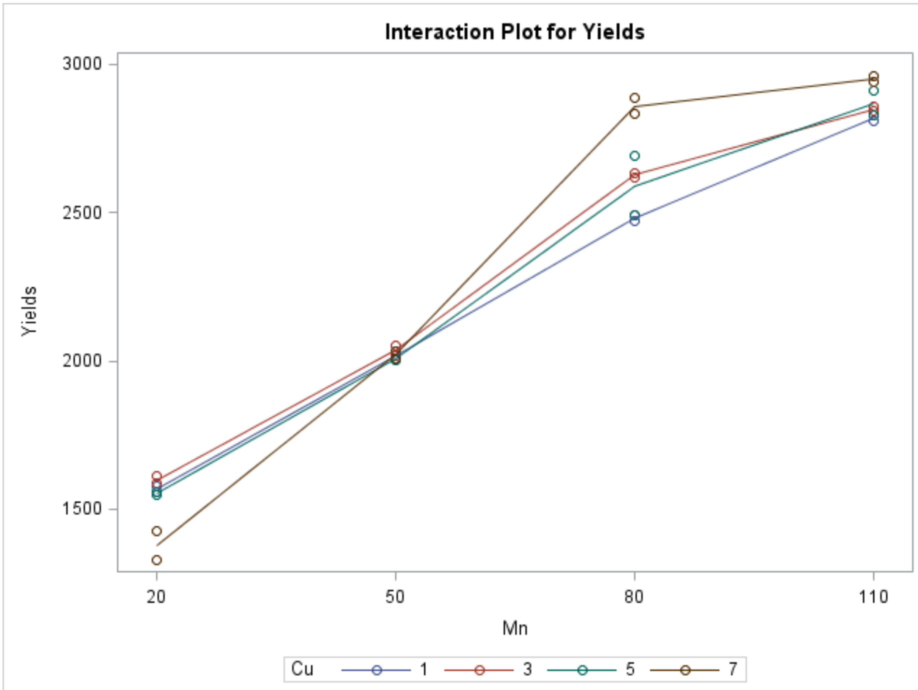
τ_i – the effect due to the manganese rate: 4 treatment levels

β_j – the effect due to the copper rate: 4 treatment levels

$\tau\beta_{ij}$ – the effect due to the two-way interaction between the levels of manganese and copper factors: 16 interactions

ε_{ijk} – random error associated with each Mn-Cu combination: 32 residual errors

c) Construct a profile plot and describe what this plot says about the effect of Mn and Cu on soybean yield.



From the profile plot above, we see that the profiles are not parallel to one another and, in fact, intersect. Thus, we conclude there is likely a two-way interaction between the Mn and Cu, and the interaction is non-orderly.

14.21

14.21 Refer to Exercise 14.20.

a) Using the computer printout given here, test for an interaction between the effect of $\boldsymbol{Mn}$ and $\boldsymbol{Cu}$ on soybean yield. Use $\boldsymbol{\alpha = 0.05}$.

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Mn	3	8935108.09	2978369.365	1486.70	<.0001
Cu	3	28199.344	9399.781	4.69	0.0155
Mn*Cu	9	204399.281	22711.031	11.34	<.0001

From the AOV table above, we have a p-value of less than 0.0001 for the two-way interaction Mn*Cu Thus, we conclude that the interaction has a significant effect on soybean yield.

b) What level of Mn appears to produce the highest yield?

Parameter	Estimate		Standard Error	t Value	Pr > |t|
Intercept	2859.500000	B	31.64920023	90.35	<.0001
Mn 110	91.000000	B	44.75872820	2.03	0.0590
Mn 20	-1482.000000	B	44.75872820	-33.11	<.0001
Mn 50	-839.000000	B	44.75872820	-18.74	<.0001
Mn 80	0.000000	B	.	.	.

By examining the profile plot, the maximum yield is achieved at $Mn = 110$. Further, this is confirmed by the largest parameter estimate above of 91.00 at this manganese level.

c) What level of Cu appears to produce the highest yield?

Parameter	Estimate		Standard Error	t Value	Pr > |t|
Intercept	2859.500000	B	31.64920023	90.35	<.0001
Cu 1	-379.500000	B	44.75872820	-8.48	<.0001
Cu 3	-233.500000	B	44.75872820	-5.22	<.0001
Cu 5	-269.500000	B	44.75872820	-6.02	<.0001
Cu 7	0.000000	B	.	.	.

By examining the profile plot, the maximum yield is achieved at $Cu = 7$. Further, this is confirmed by the parameter estimate above of 0.00 at this copper level.

d) What combination of Cu-Mn appears to produce the highest yield?

So this is interesting.

Parameter	Estimate		Standard Error	t Value	Pr > |t|
Intercept	2950.500000	B	31.64920023	93.23	<.0001
Mn*Cu 20 1	321.000000	B	63.29840045	5.07	0.0001
Mn*Cu 20 3	322.500000	B	63.29840045	5.09	0.0001
Mn*Cu 20 5	257.000000	B	63.29840045	4.06	0.0009
Mn*Cu 20 7	0.000000	B	.	.	.
Mn*Cu 50 1	128.000000	B	63.29840045	2.02	0.0602
Mn*Cu 50 3	115.000000	B	63.29840045	1.82	0.0880
Mn*Cu 50 5	66.500000	B	63.29840045	1.05	0.3091
Mn*Cu 50 7	0.000000	B	.	.	.
Mn*Cu 80 1	-249.000000	B	63.29840045	-3.93	0.0012
Mn*Cu 80 3	-133.500000	B	63.29840045	-2.11	0.0510
Mn*Cu 80 5	-189.000000	B	63.29840045	-2.99	0.0087
Mn*Cu 80 7	0.000000	B	.	.	.
Mn*Cu 110 1	0.000000	B	.	.	.
Mn*Cu 110 3	0.000000	B	.	.	.
Mn*Cu 110 5	0.000000	B	.	.	.
Mn*Cu 110 7	0.000000	B	.	.	.

By examining the profile plot, the maximum yield is achieved at $Mn * Cu = (110,7)$. But, this is actually not confirmed by the maximum parameter estimate above of 322.50 at $Mn * Cu = (20,3)$.

SAS code:

```
* 14.11
* input data;
data icecream;
        input Ratings Sweetner$ MilkFat$ Air$;
        cards;
23      12%     10%     5%
24      12%     10%     5%
25      12%     10%     5%
36      12%     10%     10%
35      12%     10%     10%
36      12%     10%     10%
28      12%     10%     15%
24      12%     10%     15%
27      12%     10%     15%
27      12%     12%     5%
```

```
28      12%     12%     5%
26      12%     12%     5%
34      12%     12%     10%
38      12%     12%     10%
39      12%     12%     10%
35      12%     12%     15%
35      12%     12%     15%
34      12%     12%     15%
31      12%     15%     5%
32      12%     15%     5%
29      12%     15%     5%
33      12%     15%     10%
34      12%     15%     10%
35      12%     15%     10%
26      12%     15%     15%
27      12%     15%     15%
25      12%     15%     15%
24      16%     10%     5%
23      16%     10%     5%
28      16%     10%     5%
37      16%     10%     10%
39      16%     10%     10%
35      16%     10%     10%
26      16%     10%     15%
29      16%     10%     15%
25      16%     10%     15%
38      16%     12%     5%
36      16%     12%     5%
35      16%     12%     5%
34      16%     12%     10%
38      16%     12%     10%
36      16%     12%     10%
36      16%     12%     15%
37      16%     12%     15%
34      16%     12%     15%
34      16%     15%     5%
36      16%     15%     5%
39      16%     15%     5%
34      16%     15%     10%
36      16%     15%     10%
31      16%     15%     10%
28      16%     15%     15%
26      16%     15%     15%
24      16%     15%     15%
;
run;

proc print data=icecream;
run;

* run factorial design anova test;
proc glm data=icecream;
        class Sweetner MilkFat Air;
        model ratings = Sweetner MilkFat Air Sweetner*MilkFat Sweetner*Air MilkFat*Air
Sweetner*MilkFat*Air;
        output out=residuals r=res p=fitted;
run;
quit;

* plot residuals qq-plot;
proc univariate normal plot data = residuals;
        var res;
run;
quit;

* plot residuals vs. fitted values to check for equal variances;
proc gplot data=residuals;
        plot res*fitted;
        symbol i=none v=star;
run;
quit;

* time series graph to look at independence of residuals;
proc gplot data=residuals;
        symbol i=line v=star;
```

```
        plot res*ratings=1;
run;
quit;

* re-enter data set so as to have one factor: the three factor combination;
data icecream_oneway;
        length treatment $12;
        input ratings treatment$;
        cards;
23      12%10%5%
24      12%10%5%
25      12%10%5%
36      12%10%10%
35      12%10%10%
36      12%10%10%
28      12%10%15%
24      12%10%15%
27      12%10%15%
27      12%12%5%
28      12%12%5%
26      12%12%5%
34      12%12%10%
38      12%12%10%
39      12%12%10%
35      12%12%15%
35      12%12%15%
34      12%12%15%
31      12%15%5%
32      12%15%5%
29      12%15%5%
33      12%15%10%
34      12%15%10%
35      12%15%10%
26      12%15%15%
27      12%15%15%
25      12%15%15%
24      16%10%5%
23      16%10%5%
28      16%10%5%
37      16%10%10%
39      16%10%10%
35      16%10%10%
26      16%10%15%
29      16%10%15%
25      16%10%15%
38      16%12%5%
36      16%12%5%
35      16%12%5%
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34      16%12%15%
34      16%15%5%
36      16%15%5%
39      16%15%5%
34      16%15%10%
36      16%15%10%
31      16%15%10%
28      16%15%15%
26      16%15%15%
24      16%15%15%
;
run;

proc print data=icecream_oneway;
run;

* run one-anova procedure with levene test to test for equal variances;
proc glm data=icecream_oneway;
        class treatment;
        model ratings = treatment;
        means treatment / hovtest=levene;
run;
```

```
quit;

* create new data set for only values associated with sweetner=12%;
data icecream_sweetner12; set icecream;
        if Sweetner = '12%';
run;

proc print data=icecream_sweetner12;
run;

* run two facotr factorial design anova test for sweetner=12% to look at interaction plot;
proc glm data=icecream_sweetner12;
        class MilkFat Air;
        model ratings = MilkFat Air  MilkFat*Air;
run;
quit;

* create new data set for only values associated with sweetner=16%;
data icecream_sweetner16; set icecream;
        if Sweetner = '16%';
run;

proc print data=icecream_sweetner16;
run;

* run two factor factorial design anova test for sweetner=16% to look at interaction plot;
proc glm data=icecream_sweetner16;
        class MilkFat Air;
        model ratings = MilkFat Air  MilkFat*Air;
run;
quit;

* 14.20
* input data;
data soybeans;
        input Yields Mn Cu;
        cards;
1558    20      1
1578    20      1
1590    20      3
1610    20      3
1558    20      5
1550    20      5
1328    20      7
1427    20      7
2003    50      1
2033    50      1
2020    50      3
2051    50      3
2003    50      5
2010    50      5
2010    50      7
2031    50      7
2490    80      1
2470    80      1
2620    80      3
2632    80      3
2490    80      5
2690    80      5
2887    80      7
2832    80      7
2830    110     1
2810    110     1
2860    110     3
2841    110     3
2830    110     5
2910    110     5
2960    110     7
2941    110     7
;
run;

proc print data=soybeans;
run;
```

```
* run factorial treatment ANOVA test;
proc glm data = soybeans;
        class Mn Cu;
        model Yields = Mn Cu Mn*Cu / solution;
run;
quit;
```