Coding Project 5: Background Subtraction through Dynamic Mode Decomposition

Hao Li
Department of Applied Mathematics
University of Washington
haoli25@uw.edu

Abstract

This project presents an analysis of the dynamics of a mechanical system consisting of a ball rolling along a specially-designed landscape, which simulates the two-bounce resonance of solitary wave collisions [1]. We utilize dynamic mode decomposition (DMD) to extract underlying modes and demonstrate that DMD is effective in subtracting the background from the data and identifying the movement and motion of objects within the video. The results of the study highlight the potential of DMD as a useful tool for analyzing complex mechanical systems.

1 Introduction

In the original paper, the authors used the the background subtraction method (BSM) to exact the motion of the ball [1]. In the project, we will use dynamic mode decomposition (DMD) to see if we can get the same results. More specifically, after extracting the video frames, we can decompose the data to extract its underlying modes to capture the spatiotemporal coherent structures in the video data. The background will then be identified by choosing the modes that has slowest temporal frequencies. Hence, we can capture the object's motion by performing simple subtraction from the background. In the next section, we will briefly introduce the mathematical background for DMD and algorithms will be described in section 3. All results are shown in section 4 and result videos are also uploaded to Canvas.

2 Theoretical Background

Dynamic mode decomposition (DMD) is a dimensionality reduction algorithm developed by Peter Schmid in 2008 [3]. The data takes the form of video frames:

$$V_1^N = v_1, v_2, \ldots, v_N,$$

where $v_i \in \mathbb{R}^M$ is the i-th frame of the flow field, and $V_1^N \in \mathbb{R}^{M\times N}$ is a data matrix whose columns are the individual frames. These frames are assumed to be related via a linear mapping that defines a linear dynamical system. Written in matrix form, this implies that

$$V_2^N = AV_1^{N-1} + re_{N-1}^T,$$

where r is the vector of residuals that accounts for behaviors that cannot be described completely by A, $e_{N-1} = \{0, 0, \ldots, 1\} \in \mathbb{R}^{N-1}$, $V_1^{N-1} = \{v_1, v_2, \ldots, v_{N-1}\}$, and $V_2^N = \{v_2, v_3, \ldots, v_N\}$. Regardless of the approach, the output of DMD is the eigenvalues and eigenvectors of A, which are referred to as the DMD eigenvalues and DMD modes respectively [3].

3 Numerical Methods

The algorithms are fully described in Nathan's book [2]:

(i) Sample data at N prescribed locations M times. The data snapshots should be evenly spaced in time by a fixed Δt. This gives the data matrix X.

(ii) From the data matrix X, construct the two sub-matrices X_{M-1} and X_M.

(iii) Compute the SVD decomposition of X_{M-1}.

(iv) The matrix $\tilde{S}$ can then be computed and its eigenvalues and eigenvectors found.

(v) Project the initial state of the system onto the DMD modes using the pseudo-inverse.

(vi) Compute the solution at any future time using the DMD modes along with their projection to the initial conditions and the time dynamics computed using the eigenvalue of $\tilde{S}$.

4 Results

Here are three screenshots of the original and resulting videos:

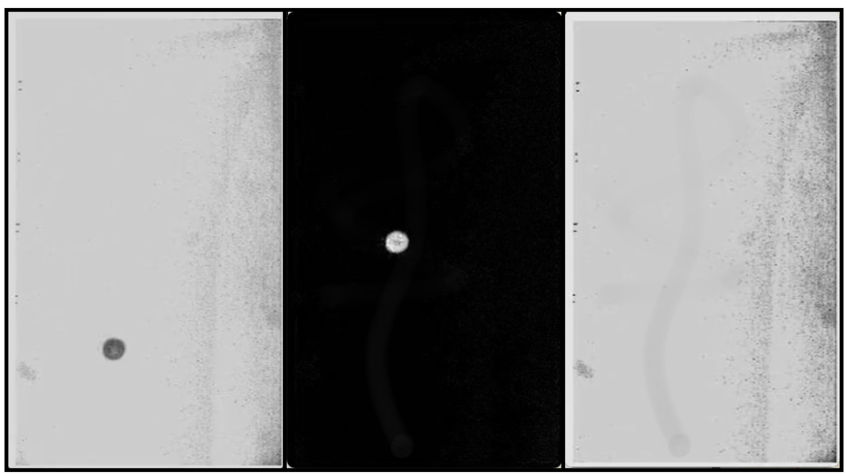

Figure 1: screenshots of original (left), foreground (middle), and background (left) videos

Clearly as we see from the plot, all the backgrounds are set to black and in contrast, the moving ball is set to white. Also by looking at the background, DMD has clearly moved the moving object but somehow we can see a little motion track in the background. In general, DMD has done a very good job of extracting the moving object. The resulting videos are uploaded to Canvas and we see a better results of the performance of DMD.

5 Conclusion

In conclusion, this project has demonstrated the effectiveness of dynamic mode decomposition (DMD) in analyzing the dynamics of a mechanical system. By using DMD to extract the underlying modes from video data, we were able to successfully subtract the background from the data and identify the motion of objects within the video. Future work could involve further refining the DMD algorithm to improve its accuracy in extracting the moving object and minimizing the background motion track.

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References

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[3] Wikipedia contributors. Dynamic mode decomposition — Wikipedia, the free encyclopedia. `https://en.wikipedia.org/w/index.php?title=Dynamic_mode_decomposition&oldid=1137083479`, 2023. [Online; accessed 14-March-2023].