CS168 Miniproject 7

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1. A.

All of the graphs are irreducible because they are all connected so starting from any state, we can reach any other state through random walks through the graph.

(1), the circle graph with n = 10, is not aperiodic. To see this, consider how many steps it could take to walk from node 1 back to node 1. It could just take two steps, 1 → 2 → 1 (or 1 → n → 1, but we will focus on moving toward increasing numbers for this example. Just keep in mind that this is symmetric in the other direction around the circle). When considering nodes 1, 2, and 3, it could take 4 steps, 1 → 2 → 3 → 2 → 1, or any multiple of two past 4 (1 → 2 → 3 → 2 → X \* (3 → 2 →) 1 for any X). Basically, from node 1 to another node x and back to node 1, it will take 2 \* (x – 1) steps to get there and back, plus 2 \* (number of times we bounce back and forth between nodes). Finally, if we go all the way around the circle to get back to 1, this will take 10 steps + 2 \* (number of times we bounce back and forth between nodes). This is true for any node in the circle, and in either direction around the circle. Thus, the gcd of the number of steps is 2 rather than 1, making (1) periodic.

(2), however, is aperiodic. Everything we said about (1) is true for (2), except that going all the way around the circle will take 9 steps + 2 \* (number of times we bounce back and forth between nodes) rather than 10, which means that going all the way around the circle back to a given node is not divisible by 2. Therefore, the gcd of the number of steps is 1, making (2) periodic.

Its stationary distribution is [0.33, 0.33, 0.33, 0.33, 0.33, 0.33, 0.33, 0.33, 0.33].

(3) is also aperiodic. Everything we said about (2) is true for (3), except additionally, using the shortcut between nodes 1 and 5 will take either 5 steps + 2 \* (number of times we bounce back and forth between nodes) or 6 steps + 2 \* (number of times we bounce back and forth between nodes). The gcd of 9, 5 + 2x, 6 + 2x, and 2y + 2x is 1, making (3) periodic.

Its stationary distribution is [-0.44232587, -0.29488391, -0.29488391, -0.29488391, -0.44232587, -0.29488391, -0.29488391, -0.29488391, -0.29488391].

B.

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