4160 Lecture 4

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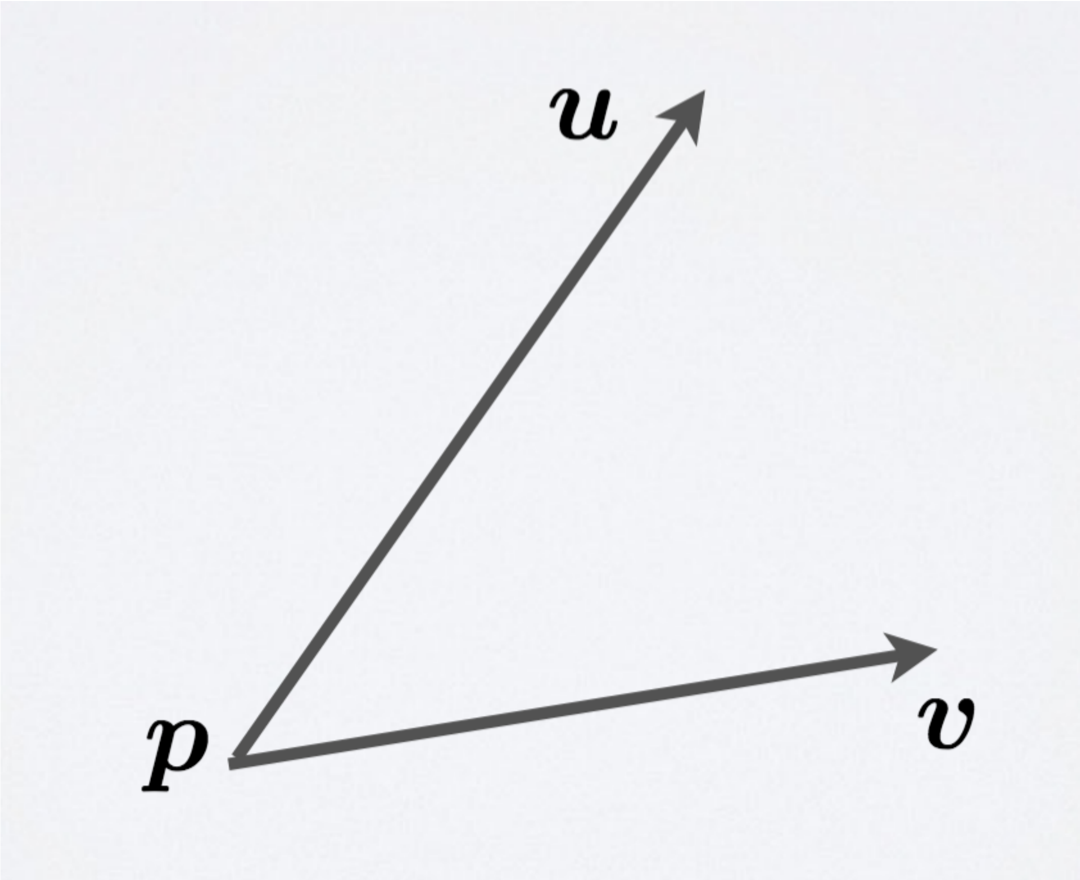
- Composing Transformation
  - Rotate about the particular point c counterwise θ
    - Translate $-c$
    - Rotate θ
    - Translate c
  - Scale along a particular line
- Directions vs Points
  - homogenous coordinate of directions $\begin{bmatrix} d \\ 0 \end{bmatrix}$
  - homogenous coordinate of points $\begin{bmatrix} p \\ 1 \end{bmatrix}$
- Affine Change of Coordinate
  - Use World Frame of Reference (**FoR**) as intermedia
  - 

  - The position in World FoR = $\begin{bmatrix} u & v & p \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_x \\ c_y \\ 1 \end{bmatrix}$

  - So take $M = \begin{bmatrix} u & v & p \\ 0 & 0 & 1 \end{bmatrix}$, we have
  - $c^{'} = M^{'-1}Mc$

- 3D coordinate
  - $\begin{bmatrix} u & v & w & p \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_x \\ c_y \\ c_z \\ 1 \end{bmatrix}$

- Transforming Normal Vectors
  - Use the Inverse of transpose matrix

- General Rotation Matrices
  - 2D: Around a point
  - 3D: Around an axis
  - Properties
    - $RR^T = R^T R = I_{3\times 3}$
    - Right-handed coordinate system: $\det(R) = 1$
    - Rotation in 3D is $SO(3)$

  - Representing Rotation
    - 2D: Just an angle
    - 3D: A unit vector, an angle
    - Transform Matirx: $R = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \sin\theta + (I - uu^T)\cos\theta uu^T$
    - Alternative2: skew-symmetric Matrix: $A = \begin{bmatrix} 0 & -z\theta & y\theta \\ z\theta & 0 & -x\theta \\ -y\theta & x\theta & 0 \end{bmatrix}$

      $R = e^A$
    - Alternative3: Euler Angles (Problem: Gimbal lock)