4160 Lecture 3

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Solving Least-square System

- When solve linear system

  $$Ax = b$$

  If A is a $m \times n, m > n$ matrix, then we may not have a solution. Instead, we define

  $$f(x) = ||Ax - b||$$

  So take gradient of f, we have

  $$\nabla f = \nabla \left(x^T A^T Ax - x^T A^T b - b^T Ax + b^T b\right) = 2A^T Ax - 2A^T b = 0$$

  Then we only have to solve square linear system

  $$A^T Ax = A^T b$$

- We can also use **Single Value Decomposition**

  $$A = USV^T$$

Curves

- Explicit(parametric) representation
  - $f(t) \rightarrow x$, here x is a vector
- Implicit representation in **2D**
  - $g(x, y) = 0$ e.g. $x^2 + y^2 - 4 = 0$
  - In **3D**: We need two equations. *1 equation kill 1 degree of freedom.*

  $$\begin{cases} f(x, y, z) = 0 \\ g(x, y, z) = 0 \end{cases}$$

- Jacobian for vector-value vector function

$$J = \nabla g = \begin{bmatrix} \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} & \frac{\partial g_1}{\partial z} & \cdots \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} & \frac{\partial g_2}{\partial z} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

- Taylor expansion
  - Scalar funtion
    $f(x + \Delta x) = f(x) + \frac{1}{1!} f^{'}(x)\Delta x + \frac{1}{2!} f^{'}(x)\Delta^2 x + \cdots$
  - Scalar-value vector function
    $f(x + \Delta x) = f(x) + \nabla f(x)^T \Delta x + \cdots$

- Normal direction $\boldsymbol{n}$

$$\nabla g \propto n$$

  - Proof:

$$\begin{cases} g(x) = 0 \\ g(x + \epsilon t) = 0 \end{cases}$$

- Geometric Transformation

$$S \rightarrow \{T(v) | v \in S\}$$

  - Explicit representation
    $\{T(f(t)) | t \in D\}$
  - Implicit representation
    $\{T(v) | g(v) = 0\} \rightarrow \{u | g(T^{-1}(u)) = 0\}$
  - Composing Transformation
    $v \rightarrow T(v) \rightarrow S(T(v)) = (S \circ T)(v)$
  - Commutative in composing
    Rotate is not commutative in 3D

- Homogeneous Coordinates
  - To unify Transformation and Translation
  - Transformation $\boldsymbol{M}$ and Translation $\boldsymbol{u}$
  - A homogeneous transformation: $\begin{bmatrix} M & u \\ 0_{1\times 2} & 1 \end{bmatrix}$
  - **Transform Then Translate**

- Affine Transformation
  - Straight lines preserved
  - Parallel lines preserved
  - Ratio of length along lines preserved

- Rigid Motion
  Only Rotate and Translate can be called as Rigid Motion
- **Question**: What's the inverse of a homogeneous coordinate? (my answer) translate first then rotate