Algorithms

TASK: Use Hilbert's 10th problem to give an example of something that is Turing recognizable but not Turing decidable.
The continuum hypothesis of Cantor was the 1st of Hilbert's 23 problems in 1900 at the international congress of mathematicians

Hilbert's 10th problem

Find a procedure that tests whether a polynomial in several variables with integer coefficients has integer rules.
EXAMPLE: $p(x, y) = 2x^2 - xy - y^2$ is a polynomial in 2 variables (x, y) with integer coefficients (2, -1, -1) that lack integer roots.
$p(1, 1) = 2(1)^2 - 1(1) - 1^2 = 0$ so $X = 1, Y \in Z$ is a a solution.

Hilbert's problem asked how to find integer roots via a set procedure. In 1936 independently Alonzo Church invented Λ calculus to define algorithms, while Alan Turing invented Turing machines. Church's definition was show to be equivalent to Turing's. This equivalence says:

> **Intuitive notion of algorithms = Turing machine algorithms**

and is known as the Church-Turing thesis.

It led to the formal definition of an algorithm and eventually in resolving in the negative Hilbert's 10th problem. Yuri Matijasevic proves in 1970 that there does not exist an algorithm which can decide when a polynomial has integer roots using previous work Martin Davis, Hillary Putnam and Julia Robinson. Hilbert's 10th problem is an example of a problem that is Turing-recognizable but not Turing decidable.