Sets

$J_m = \{1, 2, 3, 4....m\}$
$J = \{1, 2, 3.....\} = N^*$

DEF A set A is finite if A ~ J_m $(\exists f A \to J_m \text{ a bijection})$ for some $n \epsilon N^*$ or A = $\emptyset$

DEF A set A is countably infinite if A ~ J

DEF A set A is uncountable infinite if A is neither finite nor countably infinite.

Restatement of the definition of countably inifinite

A set A is countably infinite if its elements can be arranged in a sequence $\{x_1, x_2, x_3.....\}$ such that A = $\{x_1, x_2....\}$ this is another way of saying A is in bijective correspondence with J i.e $\exists f : A \to J$ a bijection namely A ~ J

Application of this statement

Z ~ N

- Indeed, we can write Z as a sequence since Z = {0,-1,-2,-3.....} so $Z \epsilon [N]$, Z is countably infinite like N
  - N ~ N*
  - Z ~ N
  - Such that by transitivity Z ~ N*, Z is countably infinite

Finite and infinite sets

The big difference between finite and infinite sets is that if:

Let A, B be finite sets, s.t $A \subsetneq B$, A is not in bijective correspondence with B since $\#(A) < \#(B)$. If $m \neq m$. Let A, B be infinite sets such that B is a subset of A and B is not a subset equal to A. It is possible that A ~ B. we saw this in hilberts hotel problem (Hilberts paradox of the grand hotel). N* is a subset of N, but N ~ N* via the bijection $f : N \to N\star$ given by $f(m) = n+1$.