Directed Graphs

- **Task**: Introduce a new category of graphs where the edges have directions and loops are allowed
- **DEF**: A *Directed graph* of *digraph* (V,E) consists of a finite set V together with a subset $E \text{ of } VxV$. The elements of V are the vertices of the digraph. Where the elements of E are the edges of the digraph.

Remark

1

Recall that when we disected undierected graphs (V,E), the set of the edges E was a subset of V_2 where V_2 was the set consisting of all subsets in V with *exactly* two elements. note that $w, w = w, v \epsilon V_2, \iff$
$v \text{ not equal to } w, \text{ whereas } (v, w) \text{ not equal to } (w, v) \epsilon VxV$. The pais in V xV are ordered, also $(v, v) \epsilon VxV$ loops are allowed as edges of a digraph, whereas they werent allowed as edges of an undirected graph.

- **DEF** : let $(v, u) \epsilon E$ be the edge of the digraph (V,E). we say v is the initial vertex and w is the terminal vertex of the edge. Furthermore we say that the vertex w is *adjacent from* the vertex v, and vertex v is *adjacent to* the vertex w, whereas the edge (v, w) is *incident from* the vertex v and incident to the vertex w.

Examples

Use arrows to indicate the directions of the edges of a digraph.

Just like an undirected graph, a directed graph has an adjacency matrix associated with it.

let (V,E) be a directed graph, and let vertices in V be ordered $v_1, v_2 \dots . v_m$. The adjacency matric of (V,E) is the $m \text{ x } m$ matrix (b_{ij})
Examples

2

The adjacency matrix of an undirected graph always has 0's on the diagonal. Where as the adjacency matrix of a directed graph could have some 1's on the diagonal done to the presence of loops.

Directed graphs and binary relations

- **Task** Describe the one to one correspondence between the directed graphs and binary relations on finite sets.
  - let V be a finite set,
  - **To every relation R on V, the corresponds a directed graph**
  - Indeed, set $E = (v, w) \epsilon V \text{ x } V | vRw$, then (V,E) is a directed graph
  - **To every directed graph (VE), there is a corresponding relation R on V**

Indeed we define the notation R on V as $\exists v, w \epsilon V \ vRw \iff (v, w) \epsilon E$ **Moral of the story** We can