Maths - Graphs

LEGEND

> T == Theorm

Graph properties

Undirected graph (V,E) = finite set of V with a subset of E of $V2$

Graph is *trivial* if it consists of a single vertex

if $e = vv\prime$ for some $v\prime$ then v is said to be incident to e and e is said to be incident to v

v and v' are adjacent iff $v, v\prime \epsilon E$

Let (V,E) and (V,E`) be graphs. $(V\prime, E\prime) \subset (V, E)$ iff $V\prime \subset V$ and $E\prime \subset E$.

Movement through graphs

The degree of a vertex is the number of edges on the graph that are incident to that vertex

A vertex with a degree of 0 is an ***isolated vertex***

A vertex with a degree of one is an ***pendent vertex***

A **walk** is a traversal of a graph.

A ***trail*** is a walk that traverses any edge at most once.

A ***path*** is a walk that traverses any vertex at most once.

A ***graph*** is considered connected if there exists a path from v to u

A ***circuit*** is a non-trivial closed trail.

T If a graph has no isolated or pendant vertices then it contains at least one simple circuit

T Let u and v be vertices of a graph, where u is not equal to v. If there exists at least two distinct paths from u to v then the graph contains at least one simple circuit

Forests and Trees

A graph without a circuit is ***acyclic***

A ***forest*** is an acyclic graph

a ***tree*** is an acyclic graph

T *Every forest has at least one isolated or pendent vertex*

- If a graphs has no isolated or pendant vertices then it contains a circuit. Therefore a forest must have at least one isolated or pendent vertex

T *A non-trivial tree contains at least one pendent vertex*

- If a non-trivial graph has an isolated vertex then there does not exist a path or walk from that vertex to any other vertex of the graph, and therefore the graph is not connected. But a tree in a forest is by definition connected. Therefore a non-trivial tree cannot have any isolated vertex.

T *Let* (V, E) *be a tree. Then* $\#(E) = \#(V) - 1$

- Proof by induction.

  Every tree with m vertices has m - 1 edges. Let (V,E) be a tree with such properties. Let v be a pendent vertex and w adjacent to v. Let V' = V \ {v} and E' = E \ {v,w}. Then (V', E') is a subgraph of (V,E).

Spanning trees

A ***spanning tree*** in a graph is a subgraph of the graph that is a tree which includes every vertex of the original graph.

T *Every connected graph contains a spanning tree*