Vairants of Turing machines

-- MISSING NOTES HERE --

Non-deterministic turing machines

Theorem: Every non-deterministic Turing machine has an equivalent deterministic turing machine

Corollary: A language is Turing-recognizable $\iff$ some non-deterministic Turing machine recognizes it.

Proof: $" \rightarrow "$ A deterministic Turing machine is a non-deterministic turing machine.
$" \leftarrow "$ follows the previous theorem

Enumerators

As we saw, a Turing-recognizable language is called in some textbooks a recursively enumerable language. The term came from a variant of Turing machine called an enumerator, largely, an enumerator is a Turing machine with an attached printer.

The enumerator prints out the language L it accepts as a sequence of strings. The enumerator can print out the strings of the language in any order and possibly with repetitions.

Theorem: A language L is Turing recognizable $\iff$ some enumerator enumerates. (Turing recognizable if it can be enumerated by an enumerator)

PROOF: Let E be the enumerator. We construct the following Turing Machine M.
M on input w:

1. We run E. Every time that E outputs a string, compare it with w.
2. If w even appears in the output of E, accept w.

Thus M accepts exactly the strings on E's list and no others, hence L.\

$" \rightarrow "$ let M be a Turing machine that recognizes L. We would to construct an enumerator E that outputs L. Let A be the alphabet of L. i.e $L \subset A*$. In the unit on countability, we proved A* is countably infinite. (The alphabet is always assumed to be finite). So A^* has an enumeration as a sequence. $A^* = \{w_1, w_2, ...\}$.
E = ignore the input

1. Repeat the following for i = 1,2,3,......
2. Run M for i steps on each input $w_1, w_2, ..., w_i$
3. If any computations accept, print out the corresponding wj.

Every string accepted by M will eventually appear on the list of E, and once it does it will appear infinitely many times because M runs from the beginning on each string for each repetition or step 1.

> Each string accepted by M is accepted in some finite number of steps, say l steps, so this string will be printed on E's list for every $i \geq \max(l, k)$ where the string $w = w_k$ in the enumeration $\tilde{A} = \{u_1, u_2, ...\}$

MORAL OF THE STORY

The single tape turing machine we first introduced is as powerful as any variant of turing machine we can think of.