# Size of sets

We wrote down a set A with m elements as $A = a_1, a_2, a_3, ....a_m$. This notation means *The process of counting* a1 is the first element, a2 is the second element and so on. The mth element of A. In other words, another way of saying A is a set of m elements is that there exists a bijective function $f : A \to 1, 2, 3, ....m$

Let $J_m = 1, 2, 3, ....m$

- **DEF** A set A has m elements $\iff f : A \to J_m$ is a bijection
  - **NB** This definition works $\forall m \geq 1, m \epsilon N^*$
- **Notation** $\exists f : A \to J_m$ a bijection i, is denoted $A\ J_m$

More generally A ~ B means $\exists f : A \to B$ on bijection, and it is a solution on sets in fact it is an equivalence relation. $[J_m]$ is the equivalence class of all sets A of size m i.e $\#(A) = m$

- **DEF** A set A is **finite**, if A ~ $J_m$ for some $n \epsilon N^*$
- **DEF** A set A is infinite if A is not finite, Examples N, Q, R etc.

to understand sizes of infinite sets, generalize the construction abive let $J = N^* = \{1, 2, 3......\}$

- **DEF** A set A is countably infinite if A ~ J, where J = $N^*$ is a set of counting numbers.
- **DEF** A set A is uncountably infinite if A is neither finite nor countably infinite.

In fact we often treat together the cases A is finite or A is countably infinite since in both of these cases, the counting mechanism that is familiar to us works. Therefore we have the following definition.

- **DEF** A set A is *countable* if A is finite (A ~ $J_m$ or $A = \emptyset$ or A is countably infinite A ~ J.

Difference of terminology regarding countability between CS sources [textbooks ect] and maths sources.

> CS - Countable ||| MATHS - at most countable
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> CS - Countably infinite ||| MATHS - countable
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> CS - uncountably infinite ||| MATHS - uncountable

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## Processes

- **GOAL** Characterize [N], the equivalence class of countably infinite sets, and [R], the equivalence class of uncountably infinite sets, the same size as R.
- **BAD NEWS** Both [N] and [R] consist of infinite sets.
- **GOOD NEWS** we only can abstract these two equivalence classes.
- **NB** There are uncountable infinite sets of a size bigger than [R] that can be obtained from the power set construction, but ignore those.

To characterise [N] we need to recall the notion of a sequence:

- **DEF** A sequence is a set of elements $\{x_1, x_2, x_3.....\}$ indexed by J, i.e $\exists f : J \to \{x_1, x_2, ....\}$ s.t $f(m) = X_m$ $\forall m \epsilon J(m \geq 1, m \epsilon N)$

Sequences and their limits appear in the definitions of various notions in calculus (differentiation, integration, etc)

Calculators also use sequences in order to computer with rational and irrational numbers.

### Examples

1. $\pi \approx 3.14159.....$ i.e instead of $\pi$, we can work with the following sequence of rational numbers. $x_1 = 3, x_2 = 3.1, x + 3 = 3.14, ..... \lim_{m \to e} x_m = \pi$
2. $\frac{1}{3} \approx 0.333333.....$ means we can set up the sequence of rational just as we did above with pi.

**RE-DEF Countably infinite**. A set A is countably infinite if its elements can be arranged in a sequence $\{x_1, x_2, x_3....\}$ such that A = $\{x_1, x_2...\}$. This is another way of saying A is in bijective correspondence with J. i.e $\exists f : A \to J$ a bijection, namely A ~ J.