Countability of sets 4

Argument that R is uncounably infinite in 2 steps

1. R ~ (0,1)
2. Set up a correspondance between (0,1) and the set A of all sequences of 0's and 1's via a binary representation. To each $x \in (0,1)$, we want to associate $0, x_1, x_2, x_3, \ldots.$ where often decimal $\{x_1, x_2, \ldots\}$ is a squence of 0's and 1's. In other words we are gicing a binary expansion of every $x \in (0,1)$ as $0, x_1, X_2, x_3 \cdots = 0 + x_1 \frac{1}{2} + x_2 + \frac{1}{4} \cdots = \sum_{m=1}^{\infty} x_m \frac{1}{2^m}$

Similarly any $x \in (1,0) that is a sum of the form $\frac{1}{2^{p_1}} + \frac{1}{2^{p_2}}$ for, $

Q

How can we represent $x = \frac{1}{1^{p}1} \ldots\ldots$ in an easier to understand form.

A

Do some algebra:

$x = \frac{1}{2^{p}1} \ldots. = \frac{2^{pk-1}}{2^{pk-p1} - 2^{p}2} \ldots\ldots. + \frac{1}{2^{p}k} = frac2^{pk-p1} + 2^{pk-p2} \ldots. 2^{p}k = \frac{\text{odd natual numbers}}{\text{powers of two}} = \frac{m}{2^m} form \in N \text{ and } m \in N^{\star} \text{ as } p_1 < p_2 < \ldots p_k$ So the difference between $p_k - p\ldots$, as all positive integers.

So the sequence in (0,1) that has this discussed binary expansion is B = $\{frac12 + frac14 + \frac{3}{4} + \frac{1}{8} \ldots\}$ NOTE that B is countably infinite as each set $B_m = \{0 < \frac{odd}{2^m} <\}$ is finite, $B = U_{m=1}^{\infty} B_m$ is countable by one corolloary , and the countably infinite sequence {1/2,1/4,1/8....} subset of B which means.