RL Homework 1

October 15, 2019

```
In [1]: import random
        import math
        import numpy as np
        import matplotlib.pyplot as plt
```

1 Part 1

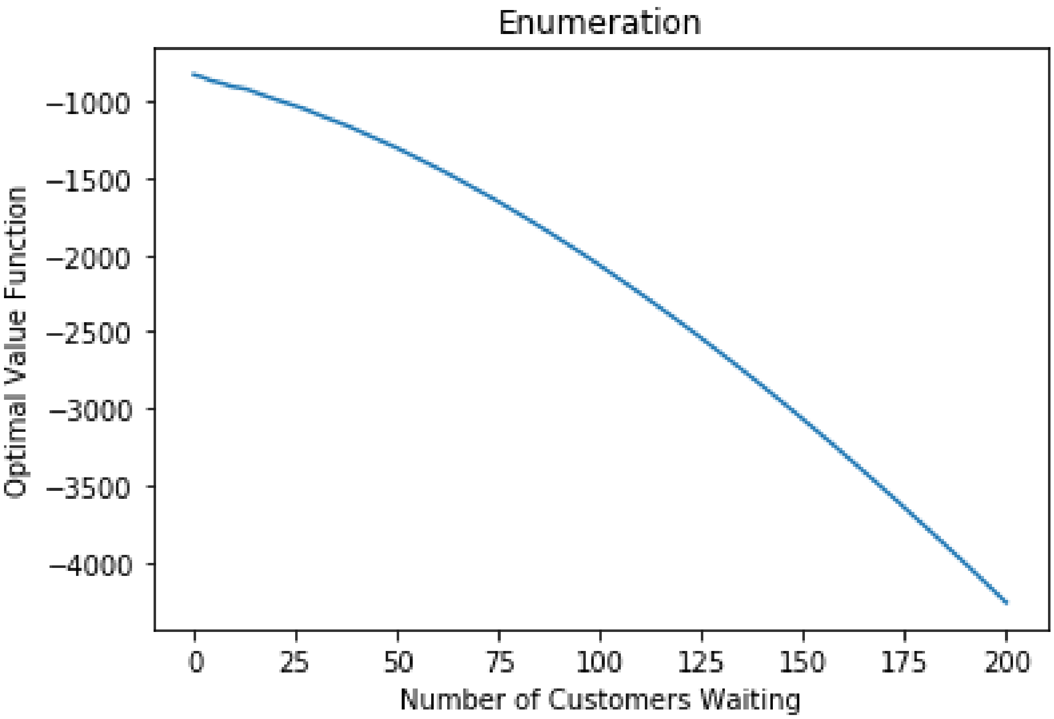
1.0.1 Hyung C. Park (hcp084)

1.0.2 Enumeration

```
In [2]: val = [0]*201
        next_val = [0]*201
        discount_rate = 0.95

        for iteration in reversed(range(0, 501)):
            for state in range(0,201):
                possible_values_0 = []
                possible_values_1 = []
                for i in range(1,6):
                    possible_values_0 = []
                    possible_next_state = state + i
                    if (possible_next_state >= 0) and (possible_next_state <= 200):
                        possible_values_0.append(discount_rate * val[possible_next_state])
                    possible_next_state -= 15
                    if possible_next_state < 0:
                        possible_next_state = 0
                    possible_values_1.append(discount_rate * val[possible_next_state])
                if (len(possible_values_0) > 0) and (len(possible_values_1) > 0):
                    next_val[state] = max([-2 * state + sum(possible_values_0) / len(possible_v
                                          -2 * state - 100 + sum(possible_values_1) / len(possible_
                elif len(possible_values_0) > 0:
                    next_val[state] = -2 * state + sum(possible_values_0) / len(possible_values
                elif len(possible_values_1) > 0:
                    next_val[state] = -2 * state - 100 + sum(possible_values_1) / len(possible_
            val = next_val.copy()
```

```
In [3]: plt.plot(range(0,201), val)
        plt.title("Enumeration")
        plt.xlabel('Number of Customers Waiting')
        plt.ylabel('Optimal Value Function')
```

```
Out[3]: Text(0, 0.5, 'Optimal Value Function')
```



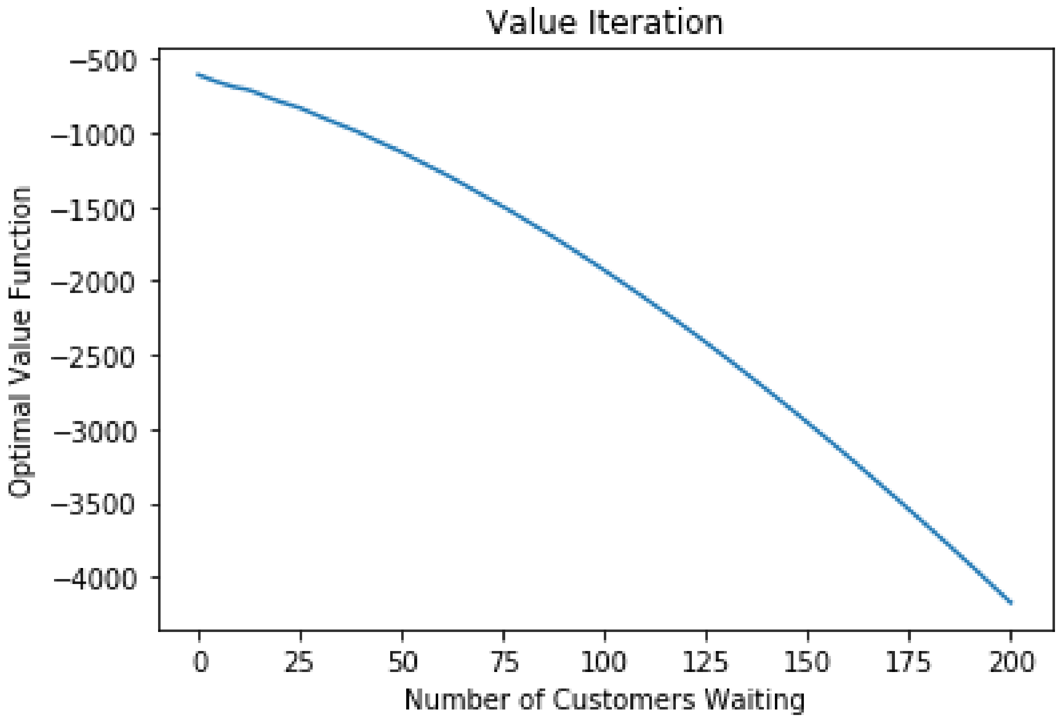
```
In [4]: val = [0]*201
        next_val = [0]*201
        discount_rate = 0.95
        epsil = 1e-15

        iteration = 0

        while True:
        #     for state in range(0,201):
        #         possible_values_0 = []
        #         possible_values_1 = []
        #         for i in range(1,6):
        #             possible_values_0 = []
        #             possible_next_state = state + i
        #             if (possible_next_state >= 0) and (possible_next_state <= 200):
        #                 possible_values_0.append(discount_rate * val[possible_next_state])
        #             possible_next_state -= 15
        #             if possible_next_state >= 0:
        #                 possible_values_1.append(discount_rate * val[possible_next_state])
        #         if (len(possible_values_0) > 0) and (len(possible_values_1) > 0):
        #             next_val[state] = max([-2 * state + sum(possible_values_0) / len(possibl
        #                                   -2 * state - 50 + sum(possible_values_1) / len(possibl
        #         elif len(possible_values_0) > 0:
        #             next_val[state] = -2 * state + sum(possible_values_0) / len(possible_val
        #         elif len(possible_values_1) > 0:
        #             next_val[state] = -2 * state - 50 + sum(possible_values_1) / len(possibl
        #     if (np.sum(np.abs(np.array(val) - np.array(next_val))) < epsil):
        #         break
        #     val = next_val

            for state in range(0,201):
                possible_values_0 = []
                possible_values_1 = []
                for i in range(1,6):
                    possible_next_state = state + i
                    if (possible_next_state >= 0) and (possible_next_state <= 200):
                        possible_values_0.append(discount_rate * val[possible_next_state])
                    possible_next_state -= 15
                    if possible_next_state < 0:
                        possible_next_state = 0
                    possible_values_1.append(discount_rate * val[possible_next_state])
                if (len(possible_values_0) > 0) and (len(possible_values_1) > 0):
                    next_val[state] = max([-2 * state + sum(possible_values_0) / len(possible_v
                                          -2 * state - 100 + sum(possible_values_1) / len(possible_
                elif len(possible_values_0) > 0:
                    next_val[state] = -2 * state + sum(possible_values_0) / len(possible_value
                elif len(possible_values_1) > 0:
                    next_val[state] = -2 * state - 100 + sum(possible_values_1) / len(possible_
            if (np.sum(np.abs(np.array(val) - np.array(next_val))) < epsil):
                break
            val = next_val.copy()
            iteration += 1
        # val
```

In [5]:

```
plt.plot(range(0,201), val)
plt.title("Value Iteration")
plt.xlabel('Number of Customers Waiting')
plt.ylabel('Optimal Value Function')
```

Out[5]:

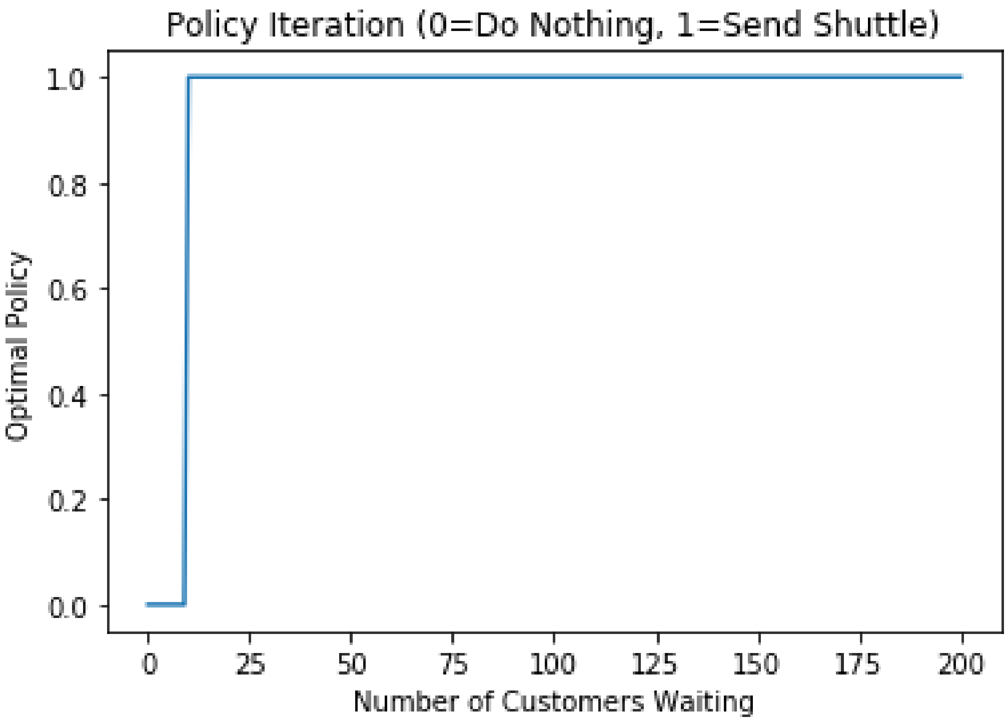
```
Text(0, 0.5, 'Optimal Value Function')
```



```
In [6]: val = [0]*201
        old_action = [0]*201
        new_action = [0]*201
        discount_rate = 0.95

        iteration = 0

        while True:
            for state in range(0,201):
                possible_values_0 = []
                possible_values_1 = []
                for i in range(1,6):
                    possible_next_state = state + i
                    if (possible_next_state >= 0) and (possible_next_state <= 200):
                        possible_values_0.append(discount_rate * val[possible_next_state])
                    possible_next_state -= 15
                    if possible_next_state < 0:
                        possible_next_state = 0
                    possible_values_1.append(discount_rate * val[possible_next_state])
                if (len(possible_values_0) > 0) and (len(possible_values_1) > 0):
                    val[state] = max([-2 * state + sum(possible_values_0) / len(possible_value
                                      -2 * state - 100 + sum(possible_values_1) / len(possible_
                    new_action[state] = (-2 * state + sum(possible_values_0) / len(possible_va
                                      -2 * state - 100 + sum(possible_values_1) / len(possible_
                elif len(possible_values_0) > 0:
                    val[state] = -2 * state + sum(possible_values_0) / len(possible_values_0)
                    new_action[state] = 0
                elif len(possible_values_1) > 0:
                    val[state] = -2 * state - 100 + sum(possible_values_1) / len(possible_valu
                    new_action[state] = 1

            if (new_action == old_action) or iteration > 100:
                break

            iteration += 1

            old_action = new_action.copy()
```

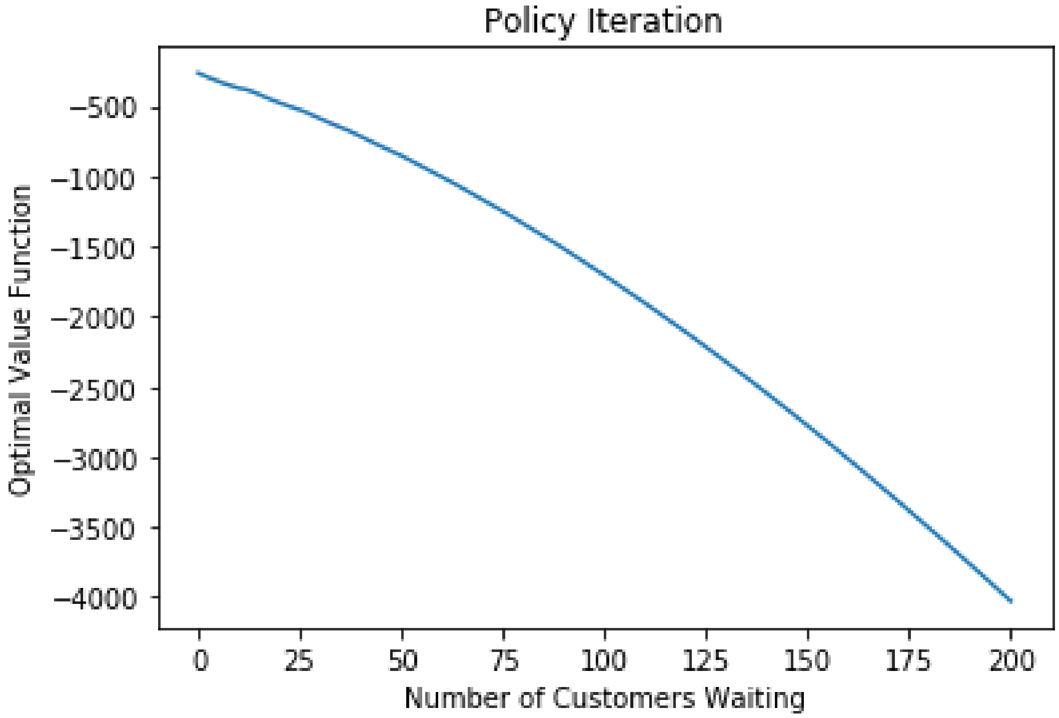
```
In [7]: plt.plot(range(0,201), new_action)
        plt.title("Policy Iteration (0=Do Nothing, 1=Send Shuttle)")
        plt.xlabel('Number of Customers Waiting')
        plt.ylabel('Optimal Policy')
```

```
Out[7]: Text(0, 0.5, 'Optimal Policy')
```



```
In [8]: plt.plot(range(0,201), val)
        plt.title("Policy Iteration")
        plt.xlabel('Number of Customers Waiting')
        plt.ylabel('Optimal Value Function')
```

```
Out[8]: Text(0, 0.5, 'Optimal Value Function')
```



We can see that the enumeration, value iteration, and policy iteration functions all converge to the same optimal value function, which we expected. Also, the optimal policy is to start sending shuttles from the 11 or more people standing in line

2 Part 2

Assume that there are 5 types of customers with ch = {1, 1.5, 2, 2.5, 3} and each type can have maximum 100 people of each class waiting for shuttle and At for each class follows same distribution. Capacity of the shuttle is K = 30.

I had to interrupt because it couldn't even run through one iteration. Even for 1 iteration, it has to go through all of the states (101^5) and calculate the value/policy for each possible state. So we small changes to the problem has an enormous impact on computational complexity.

2.0.1 Enumeration

```
In [12]: next_val = np.zeros((101, 101, 101, 101, 101))
         val = np.zeros((101, 101, 101, 101, 101))

         discount_rate = 0.95
```

```
for iteration in reversed(range(0, 501)):
    for ind, item in np.ndenumerate(val):
        possible_next_states = []
        possible_values_0 = []
        possible_values_1 = []
        current_state = list(ind)

        for j in range(0,5):
            possible_next_state = list(ind)
            possible_next_state[j] = possible_next_state[j] + 1
            if (possible_next_state[j] >= 0) and (possible_next_state[j] <= 100):
                possible_next_states.append(possible_next_state)
                possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for ind, item in
                + discount_rate * next_val[tuple(possible_next_state)])
            if ((possible_next_state[j] - 15) >= 0):
                possible_next_state[j] = possible_next_state[j] - 30
                possible_next_states.append(possible_next_state)
                possible_values_1.append(sum([(ind[0]+2)/-2.0 * item for ind, item in
                + discount_rate * next_val[tuple(possible_next_state)] - 100)
        for i in range(1,5):
            poss_next_states = possible_next_states.copy()
            possible_next_states = []
            for j in range(0,5):
                for item in poss_next_states:
                    possible_next_state = item.copy()
                    possible_next_state[j] = possible_next_state[j] + 1
                    if (possible_next_state[j] >= 0) and (possible_next_state[j] <= 10
                        possible_next_states.append(possible_next_state)
                        possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for ind,
                        + discount_rate * next_val[tuple(possible_next_state)])
                    if ((possible_next_state[j] - 15) >= 0):
                        possible_next_state[j] = possible_next_state[j] - 30
                        possible_next_states.append(possible_next_state)
                        possible_values_1.append(sum([(ind[0]+2)/-2.0 * item for ind,
                        + discount_rate * next_val[tuple(possible_next_state)] - 100)
        if ((len(possible_values_1) > 0) and (len(possible_values_0) > 0)):
            next_val[ind] = max((sum(possible_values_1)/len(possible_values_1)),
                                (sum(possible_values_0)/len(possible_values_0)))
        elif (len(possible_values_1) > 0):
            next_val[ind] = (sum(possible_values_1)/len(possible_values_1))
        elif (len(possible_values_0) > 0):
            next_val[ind] = (sum(possible_values_0)/len(possible_values_0))
    val = next_val
    print(iteration)
```

```
---------------------------------------------------------------------------

KeyboardInterrupt                         Traceback (most recent call last)

<ipython-input-12-4587357bdcc6> in <module>
 32                         if (possible_next_state[j] >= 0) and (possible_next_state[j] <
 33                             possible_next_states.append(possible_next_state)
---> 34                             possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for i
 35                             + discount_rate * next_val[tuple(possible_next_state)])
 36                         if ((possible_next_state[j] - 15) >= 0):


<ipython-input-12-4587357bdcc6> in <listcomp>(.0)
 32                         if (possible_next_state[j] >= 0) and (possible_next_state[j] <
 33                             possible_next_states.append(possible_next_state)
---> 34                             possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for i
 35                             + discount_rate * next_val[tuple(possible_next_state)])
 36                         if ((possible_next_state[j] - 15) >= 0):


KeyboardInterrupt:
```

```
In [11]: next_val = np.zeros((101, 101, 101, 101, 101))
         val = np.zeros((101, 101, 101, 101, 101))

         discount_rate = 0.95

         for iteration in reversed(range(0, 501)):
             for ind, item in np.ndenumerate(val):
                 possible_next_states = []
                 possible_values_0 = []
                 possible_values_1 = []
                 current_state = list(ind)

                 for j in range(0,5):
                     possible_next_state = list(ind)
                     possible_next_state[j] = possible_next_state[j] + 1
                     if (possible_next_state[j] >= 0) and (possible_next_state[j] <= 100):
                         possible_next_states.append(possible_next_state)
                         possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for ind, item in
                         + discount_rate * next_val[tuple(possible_next_state)])
                     if ((possible_next_state[j] - 15) >= 0):
                         possible_next_state[j] = possible_next_state[j] - 30
                         possible_next_states.append(possible_next_state)
                         possible_values_1.append(sum([(ind[0]+2)/-2.0 * item for ind, item in
                         + discount_rate * next_val[tuple(possible_next_state)] - 100)
                 for i in range(1,5):
```

```
            poss_next_states = possible_next_states.copy()
            possible_next_states = []
            for j in range(0,5):
                for item in poss_next_states:
                    possible_next_state = item.copy()
                    possible_next_state[j] = possible_next_state[j] + 1
                    if (possible_next_state[j] >= 0) and (possible_next_state[j] <= 1
                        possible_next_states.append(possible_next_state)
                        possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for ind,
                        + discount_rate * next_val[tuple(possible_next_state)])
                    if ((possible_next_state[j] - 15) >= 0):
                        possible_next_state[j] = possible_next_state[j] - 30
                        possible_next_states.append(possible_next_state)
                        possible_values_1.append(sum([(ind[0]+2)/-2.0 * item for ind,
                        + discount_rate * next_val[tuple(possible_next_state)] - 100)
        if ((len(possible_values_1) > 0) and (len(possible_values_0) > 0)):
            next_val[ind] = max((sum(possible_values_1)/len(possible_values_1)),
                                (sum(possible_values_0)/len(possible_values_0)))
        elif (len(possible_values_1) > 0):
            next_val[ind] = (sum(possible_values_1)/len(possible_values_1))
        elif (len(possible_values_0) > 0):
            next_val[ind] = (sum(possible_values_0)/len(possible_values_0))
    if (np.sum(np.abs(val - next_val)) < epsil):
        break
    val = next_val
    print(iteration)
    iteration += 1
```

```
    ---------------------------------------------------------------------------

    KeyboardInterrupt                         Traceback (most recent call last)

    <ipython-input-11-7e6e38f6a729> in <module>
         32                     if (possible_next_state[j] >= 0) and (possible_next_state[j] <
         33                         possible_next_states.append(possible_next_state)
    ---> 34                     possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for i
         35                         + discount_rate * next_val[tuple(possible_next_state)])
         36                     if ((possible_next_state[j] - 15) >= 0):


    ~/anaconda3/lib/python3.7/site-packages/numpy/lib/index_tricks.py in __init__(self, ar
        579
        580     def __init__(self, arr):
    --> 581         self.iter = asarray(arr).flat
        582
        583     def __next__(self):
```

```
~/anaconda3/lib/python3.7/site-packages/numpy/core/numeric.py in asarray(a, dtype, orde
    536
    537     """
--> 538     return array(a, dtype, copy=False, order=order)
    539
    540


KeyboardInterrupt:
```

```
In [13]: next_val = np.zeros((101, 101, 101, 101, 101))
         val = np.zeros((101, 101, 101, 101, 101))

         discount_rate = 0.95
         iteration = 0

         while True:
             for ind, item in np.ndenumerate(val):
                 possible_next_states = []
                 possible_values_0 = []
                 possible_values_1 = []
                 current_state = list(ind)

                 for j in range(0,5):
                     possible_next_state = list(ind)
                     possible_next_state[j] = possible_next_state[j] + 1
                     if (possible_next_state[j] >= 0) and (possible_next_state[j] <= 100):
                         possible_next_states.append(possible_next_state)
                         possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for ind, item in
                         + discount_rate * next_val[tuple(possible_next_state)])
                     if ((possible_next_state[j] - 15) >= 0):
                         possible_next_state[j] = possible_next_state[j] - 30
                         possible_next_states.append(possible_next_state)
                         possible_values_1.append(sum([(ind[0]+2)/-2.0 * item for ind, item in
                         + discount_rate * next_val[tuple(possible_next_state)] - 100)
                 for i in range(1,5):
                     poss_next_states = possible_next_states.copy()
                     possible_next_states = []
                     for j in range(0,5):
                         for item in poss_next_states:
                             possible_next_state = item.copy()
                             possible_next_state[j] = possible_next_state[j] + 1
                             if (possible_next_state[j] >= 0) and (possible_next_state[j] <= 1
                                 possible_next_states.append(possible_next_state)
                                 possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for ind,
                                 + discount_rate * next_val[tuple(possible_next_state)])
```

```
                    if ((possible_next_state[j] - 15) >= 0):
                        possible_next_state[j] = possible_next_state[j] - 30
                        possible_next_states.append(possible_next_state)
                        possible_values_1.append(sum([(ind[0]+2)/-2.0 * item for ind,
                        + discount_rate * next_val[tuple(possible_next_state)] - 100)
        if ((len(possible_values_1) > 0) and (len(possible_values_0) > 0)):
            next_val[ind] = 1*((sum(possible_values_1)/len(possible_values_1)) > \
                                (sum(possible_values_0)/len(possible_values_0)))
        elif (len(possible_values_1) > 0):
            next_val[ind] = 1
        elif (len(possible_values_0) > 0):
            next_val[ind] = 0
    val = next_val
    if np.array_equal(next_val, val):
        break
    val = next_val
    print(iteration)
    iteration += 1
```

```
    ---------------------------------------------------------------------------

    KeyboardInterrupt                         Traceback (most recent call last)

    <ipython-input-13-b8fc5ca8a9d5> in <module>
     33                         if (possible_next_state[j] >= 0) and (possible_next_state[j] <=
     34                             possible_next_states.append(possible_next_state)
---> 35                             possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for in
     36                             + discount_rate * next_val[tuple(possible_next_state)])
     37                         if ((possible_next_state[j] - 15) >= 0):


    <ipython-input-13-b8fc5ca8a9d5> in <listcomp>(.0)
     33                         if (possible_next_state[j] >= 0) and (possible_next_state[j] <=
     34                             possible_next_states.append(possible_next_state)
---> 35                             possible_values_0.append(sum([(ind[0]+2)/-2.0 * item for in
     36                             + discount_rate * next_val[tuple(possible_next_state)])
     37                         if ((possible_next_state[j] - 15) >= 0):


    KeyboardInterrupt:
```