Tutorial 3

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Agenda

- Assignment 2
- Bezier
  - Bezier Curve
    - Tangent
  - Bezier Surface
    - Normal
    - Stitching
    - Sampling
    - Differential
- Skeleton Code
  - Tea Party
- Demo

Related Lecture : Lecture05

Assignment 2
Programming Requirements

- **Must**
  - [ ] Implementation of the basic iterative de Casteljau Bézier vertex evaluation algorithm is required. (25% **Must**)
  - [ ] Construction of Bézier Surface with normal evaluation at each mesh vertex is required. (40% **Must**)
  - [ ] Rendering a Bézier Surface in a OpenGL window based on vertices array is required. (5% **Must**)
  - [ ] Tea party! Stitching multiple Bézier Surface patchs together to create a complex meshes is required. (20% **Must**)
- **Bonus**
  - [ ] Adaptive mesh construction based on the curvature estimation. (15% **Optional**)
  - [ ] B-Spline / NURBS surfaces. (15% **Optional**)
  - [ ] Interactive editing (by selection) of control points. (15% **Optional**)
  - [ ] Make a Cut! Cut the bezier surface with a plane. (30% **Optional**)

Bézier Curve

Explicit definition

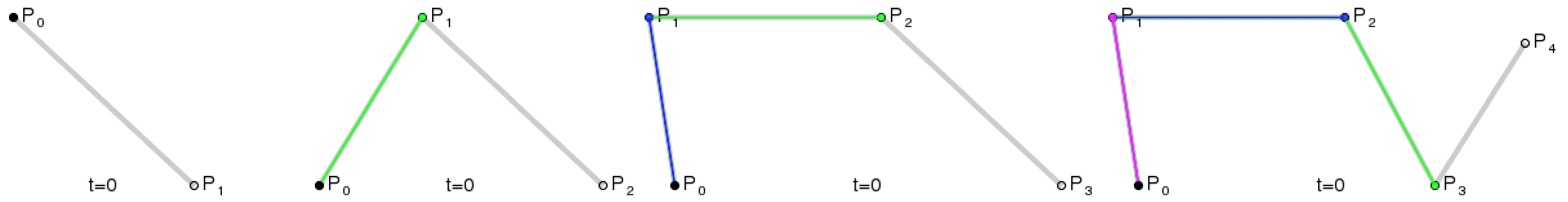
$$\mathbf{B}(t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i \mathbf{P}_i$$

$$= (1-t)^n \mathbf{P}_0 + \binom{n}{1} (1-t)^{n-1} t \mathbf{P}_1 + \cdots + \binom{n}{n-1} (1-t) t^{n-1} \mathbf{P}_{n-1} + t^n \mathbf{P}_n, \qquad 0 \leqslant t \leqslant 1$$

Recursive definition

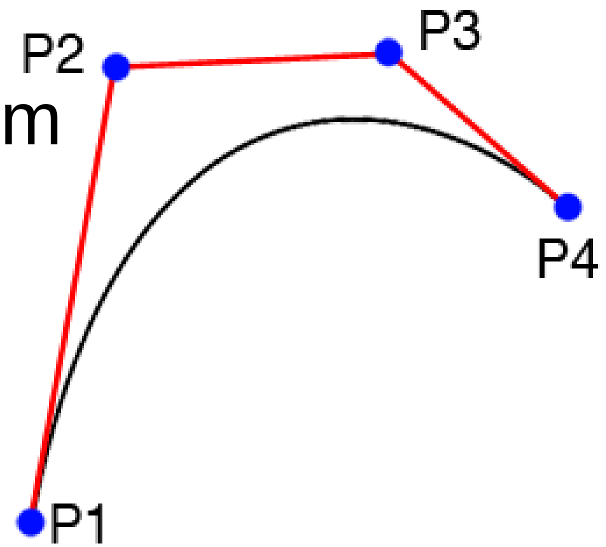
$\mathbf{B}_{\mathbf{P}_0}(t) = \mathbf{P}_0$, and

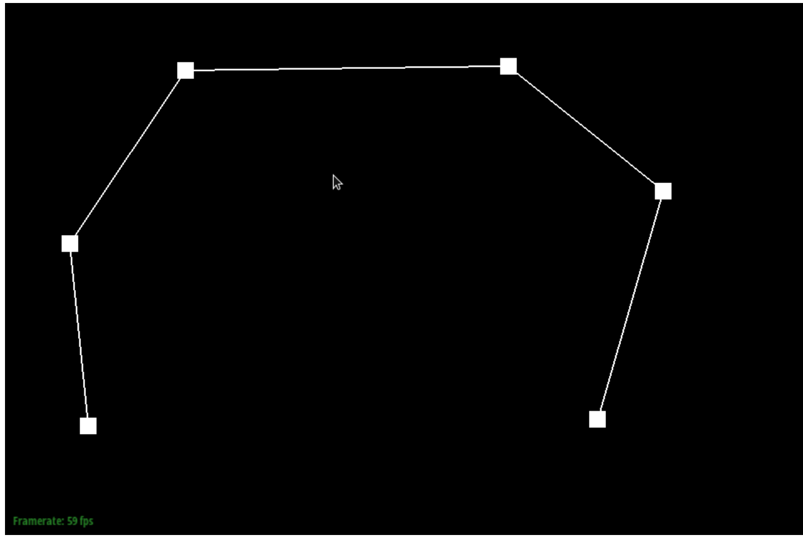
$$\mathbf{B}(t) = \mathbf{B}_{\mathbf{P}_0\mathbf{P}_1\ldots\mathbf{P}_n}(t) = (1-t)\mathbf{B}_{\mathbf{P}_0\mathbf{P}_1\ldots\mathbf{P}_{n-1}}(t) + t\mathbf{B}_{\mathbf{P}_1\mathbf{P}_2\ldots\mathbf{P}_n}(t)$$



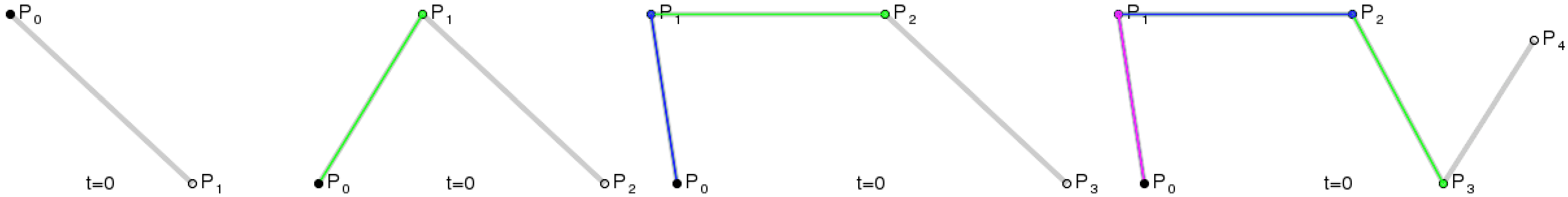
Bézier Curve Evaluation

- A simple way: Compute each term in the explicit defination
  - numerical errors
- De Casteljau's Algorithm
  - Fast and stable

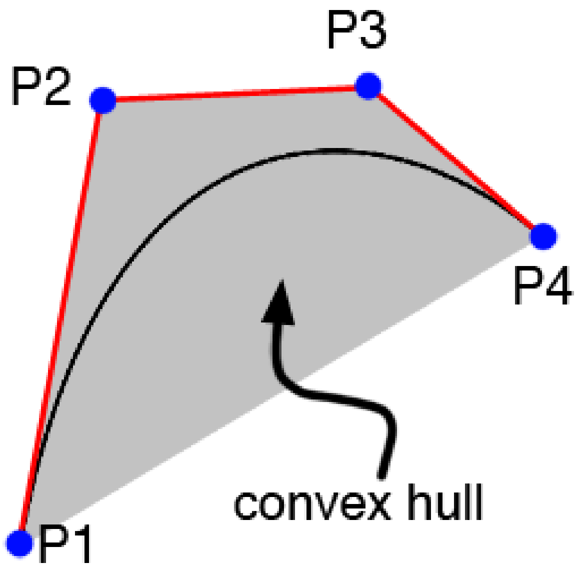


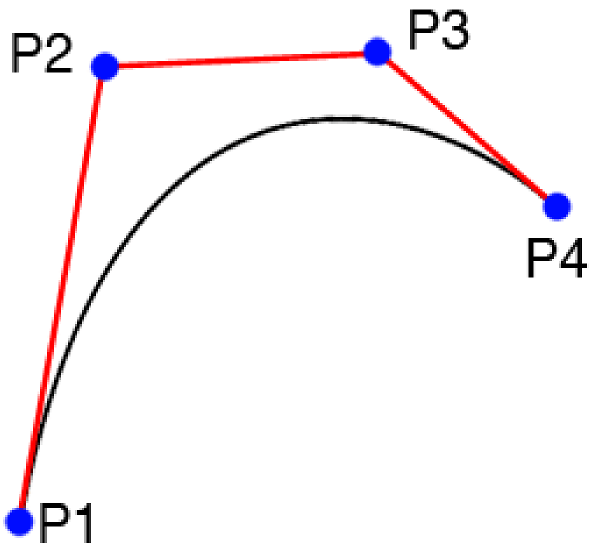


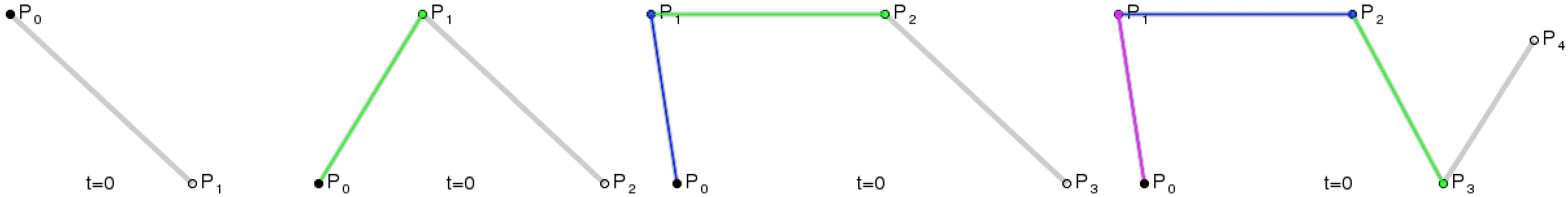
[√] Implementation of the basic iterative de Casteljau Bézier vertex evaluation algorithm is required. (25% **Must**)



Bézier Curve Tangent





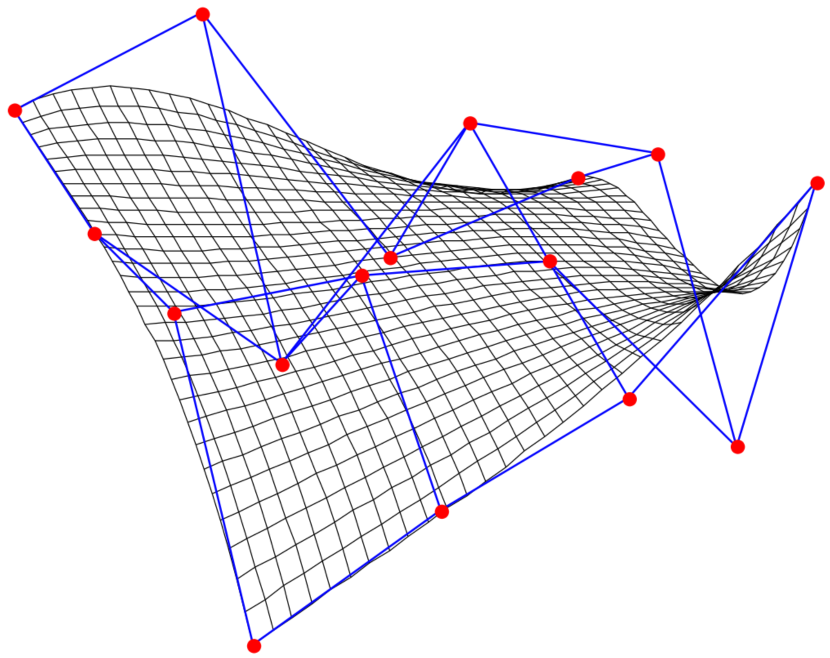


Bézier Surface

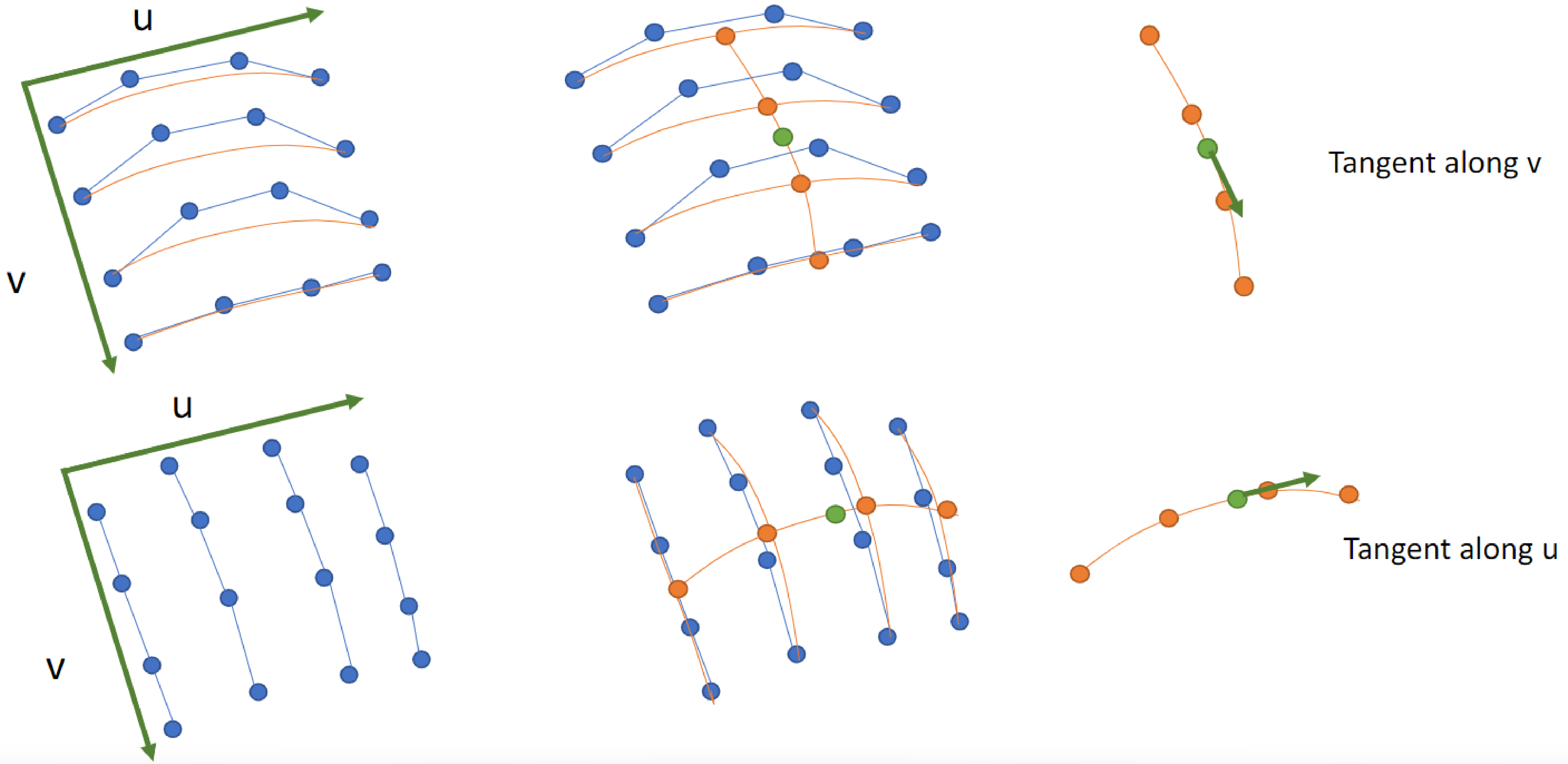
$$\mathbf{p}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} B_i^n(u)\, B_j^m(v)\, \mathbf{k}_{i,j}$$

Bernstein polynomial

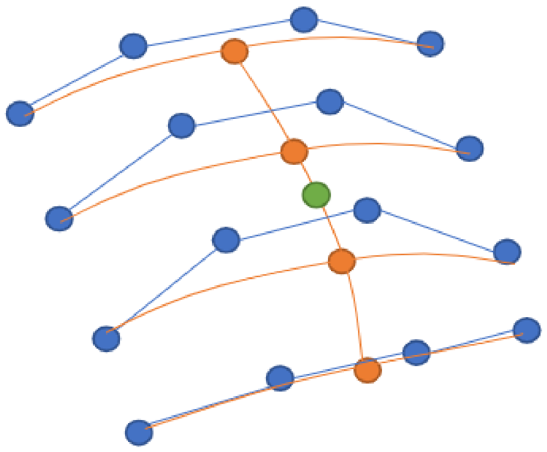
$$B_i^n(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

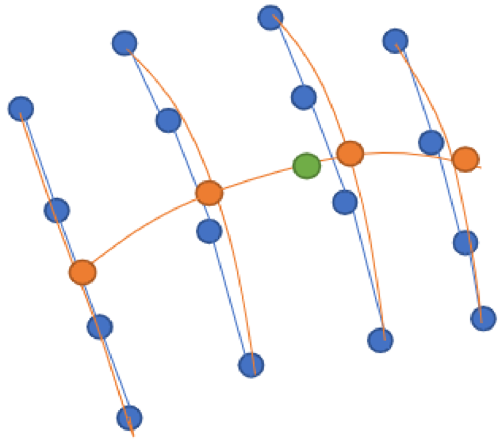


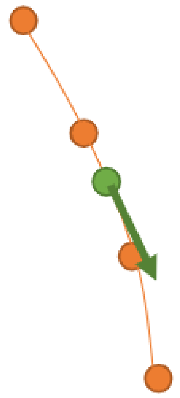
Bézier Surface Evaluation



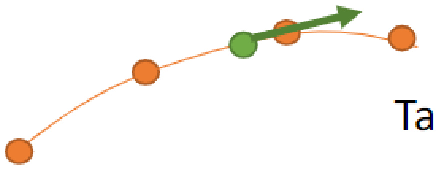
Bézier Surface Normal



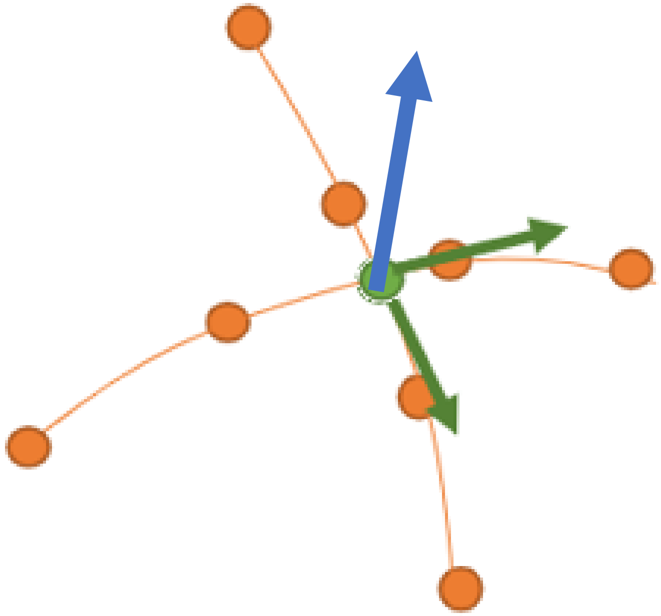




Tangent along v



Tangent along u

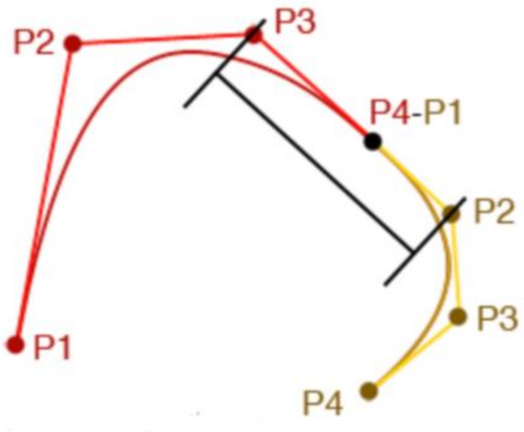


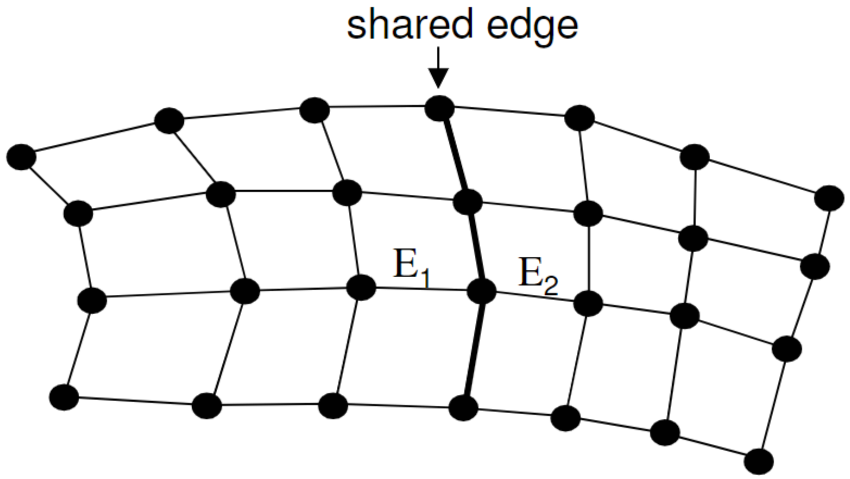
Surface normal:

$$\mathbf{m} = Normalize\left(\frac{\partial \mathbf{P}}{\partial u} \times \frac{\partial \mathbf{P}}{\partial v}\right)$$

Bézier Surface Stitching

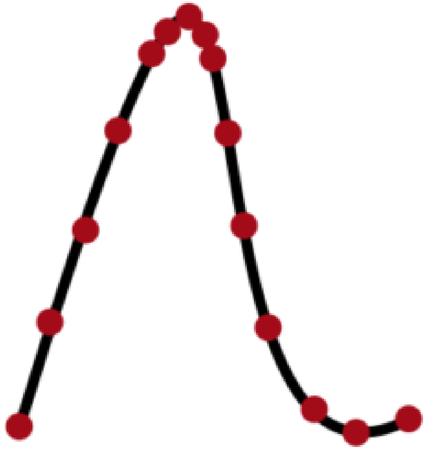
- For continuity of slope across two stitched patches, require:
  - Identical control points along common edge curves (0-order continuity)
  - Identical slopes of orthogonal control curves (1st -order continuity)
    - Co-linearity of control mesh edges that meet at a common edge control point,
      e.g. E1 and E2 (called geometric continuity), or
    - Equality of edge vectors(called parametric continuity)



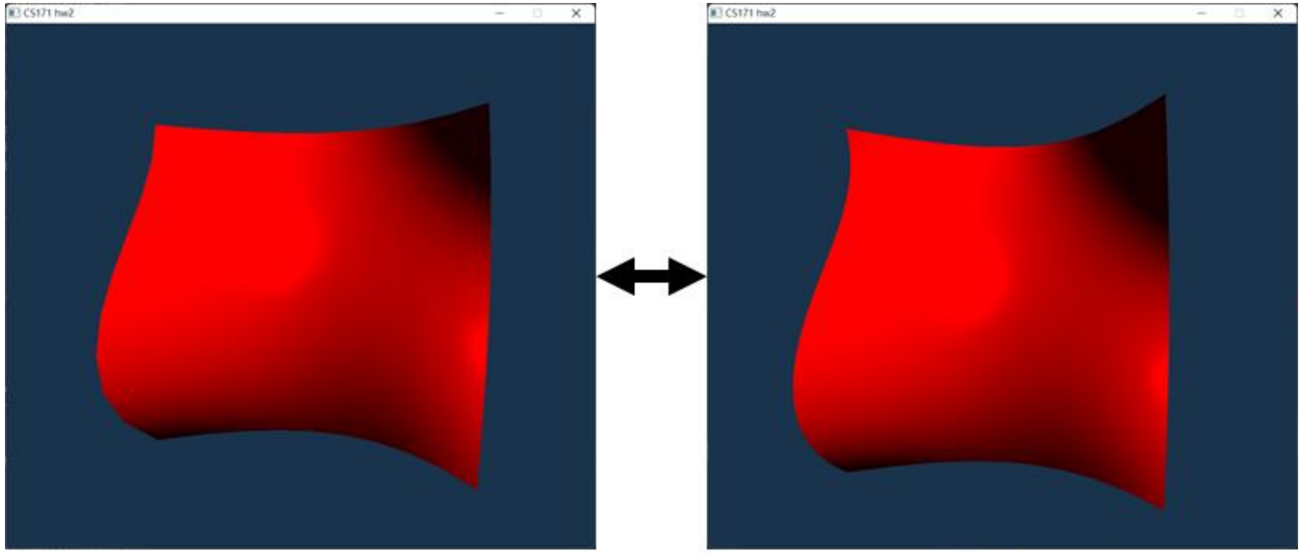


Bézier Surface Sampling

ordinary

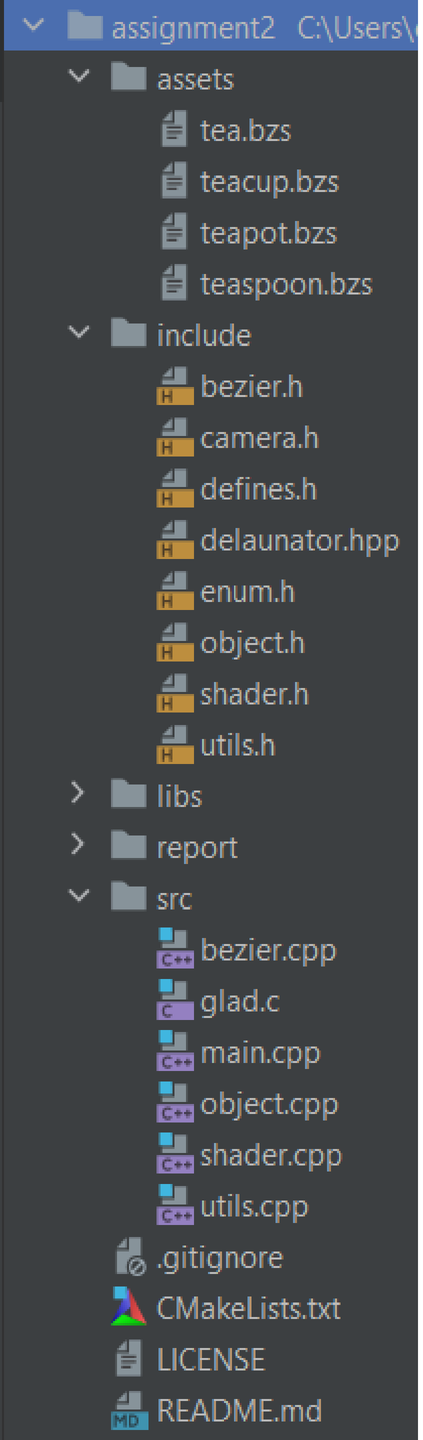


adaptive



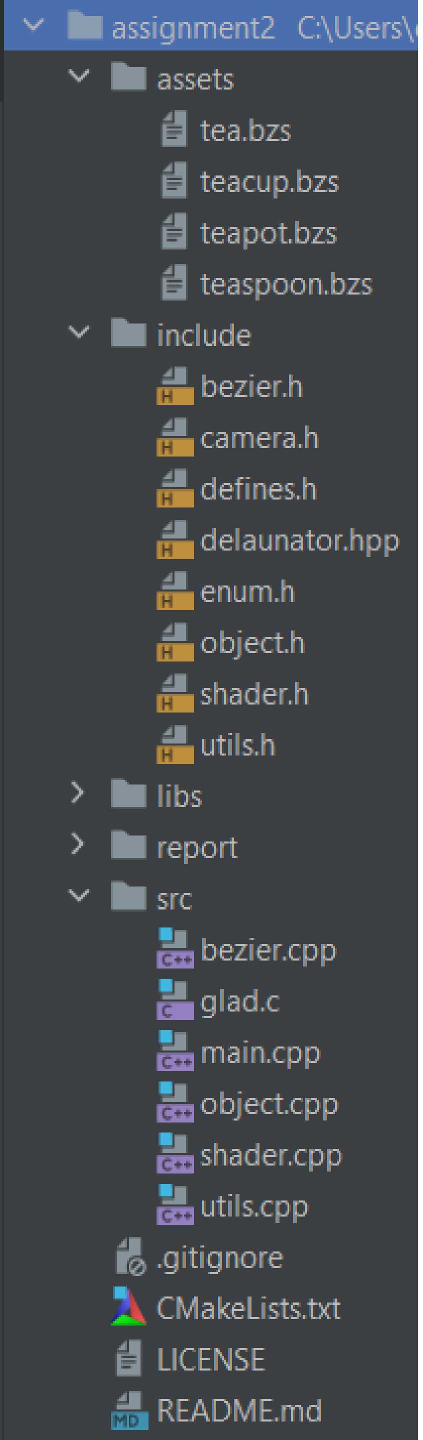
Uniform Sampling Adaptive Sampling

Curvature?
Deflection?
Length?



Skeleton Code

bezier.h/.cpp Implements of generate mesh from Bezier Control Points.
object.h/.cpp Implements of draw mesh in OpenGL.



Tea Party

b represents for number of Bezier Surface, p represents for control points number, m and n represents for the Bezier Surface's control point row number and column number. For the next b lines, each line includes m \times n integers ranging from 0 to b-1 telling the index of the control points of a single bezier surface. For the last p liens, each line have three float number telling the position of control points. we have offered teacup, teapot and teaspoon's bezier surface file separately and a tea_party.bzs file putting them together.

```
teaspoon.bzs

16 148 4 4
0 1 1 2 3 4 5 6 7 8 9 10 11 12 13 14
…
111 116 117 90 138 143 144 118 141 145 146 122 142 142 147 126
-0.000107143 0.205357 0.0
0.0 0.196429 -0.0178571
….
```