Sixth Homework

1. Suppose that a biased coin that lands on heads with probability p is flipped 10 times. Given that a total of 6 heads results, find the conditional probability that the first 3 outcomes are H, T, T (meaning that the first flip results in heads, the second in tails, and the third in tails).

2. Each cereal box comes with a coupon inside. There are 5 types of coupons, and each type is equally likely to be included in a box (independently of other boxes). A person buying these boxes needs to collect at least one coupon of each type in order to get a prize. (This way, a person may need to buy more than 5 boxes in order to get the prize since getting multiple coupons of the same type doesn't advance him on the way to getting the prize). How many boxes will he need to buy, on average, in order to get the prize?

(Hint: consider the random variables $X_1, ..., X_5$, where $X_1 = 1$ is the number of boxes needed to get the first coupon, X_2 is the number of additional boxes needed to get a coupon of a type different from the first one, X_3 is the number of additional boxes needed to get a coupon of a type different from the first two types he already got, etc.)

3. Suppose that, in a certain desert, the probability of having at least one rainy day in a given year is 1/2. Calculate (approximately) the probability that it rains exactly three times in the next 250 days?

(Hint: you need to realize that the number of rainy days in a given year is approximated well by a Poisson random variable.)