Thirteenth homework

Problem 1. Let $X_1, \dots, X_n$ be a random sample from a distribution X with the density function

$$f(x;\theta) = \begin{cases} \frac{1}{2}\left(1+\theta x\right), & x \in [-1,1], \\ \\ 0, & x \notin [-1,1], \end{cases}$$

that depends on the parameter θ. Find k such that $k\bar{X}_n$ is an unbiased estimator for θ, where

$$\bar{X}_n = \frac{X_1 + \cdots + X_n}{n}.$$

Problem 2. Let $X_1, \dots, X_n$ be a random sample from a distribution X with density function (that depends on (λ, θ))

$$f(x;\theta,\lambda) = \begin{cases} \lambda e^{-\lambda(x-\theta)}, & x > \theta, \\ \\ 0, & x \leq \theta, \end{cases}$$

with $\lambda > 0$ and $\theta \in \mathbb{R}$.

1. Find the methods of maximum likelihood estimator for (λ, θ).
2. Find the corresponding estimate of (λ, θ) when $n = 10$ and $x_1 = 3$, $x_2 = 0.5$, $x_3 = 2.5$, $x_4 = 2$, $x_5 = 5$, $x_6 = 3.5$, $x_7 = 10$, $x_8 = 9$, $x_9 = 18$, $x_{10} = 1.5$.

Problem 3. Let $x_1, x_2, \dots, x_n$ be independent and identically disputed samples from uniform distribution on the set $[0,\theta]$. (These values might look like $x_1 = 2.325, x_2 = 1.1242, x_3 = 9.262$, etc...) What is the MLE of θ ?