603 Midterm Hairui Yin 121333988

1. (a) Consider Posterior Probability, denote issue as I, no issue as NI.

We know $P(I) = 0.05$, $P(NI) = 0.95$.

Suppose the probability that chooses NI is θ, choose I is $1-\theta$, then.

$P(error \mid x) = P(NI \mid x) + \theta(P(I \mid x) - P(NI \mid x))$.

We need to ~~decide~~ find $P(I \mid x) - P(NI \mid x)$ to minimize error rate.

$$P(I \mid x) = \frac{P(x \mid I)P(I)}{p(x)} = \frac{f_Z(x \mid \mu = 5, \sigma^2 = 5^2) \times 0.05}{p(x)}$$

$$= \frac{\frac{1}{\sqrt{2\pi}} \cdot \frac{1}{5} \exp\left(-\frac{(x-5)^2}{2 \times 5^2}\right) \times 0.05}{p(x)}$$

$$P(NI \mid x) = \frac{P(x \mid NI)P(NI)}{p(x)} = \frac{f_Z(x \mid \mu = -10, \sigma^2 = 5^2) \times 0.95}{p(x)}$$

$$= \frac{\frac{1}{\sqrt{2\pi}} \cdot \frac{1}{5} \exp\left(-\frac{(x+10)^2}{2 \times 5^2}\right) \times 0.95}{p(x)}$$

Compare $P(I \mid x)$ and $P(NI \mid x)$.

Let
$$\frac{P(NI \mid x)}{P(I \mid x)} = \frac{e^{-\frac{(x+10)^2}{2 \times 5^2}} \times 0.95}{e^{-\frac{(x-5)^2}{2 \times 5^2}} \times 0.05} = 19 \cdot e^{-\frac{6x+15}{10}}$$

① When $19 \cdot e^{-\frac{6x+15}{10}} \geq 1 \Leftrightarrow x \leq \frac{5}{3}\ln 19 - \frac{5}{2}$.

classify as work properly.

② When $19 \cdot e^{-\frac{6x+15}{10}} < 1 \Leftrightarrow x > \frac{5}{3}\ln 19 - \frac{5}{2}$.

classify as existing an issue.

Given $x = 2$. because $x \leq \frac{5}{3}\ln 19 - \frac{5}{2}$

So, reactor working properly.

(b) $\lambda(NI|I) = 20$, $\lambda(I|NI) = 5$

$\lambda(NI|NI) = 0$ $\lambda(I|I) = 0$.

$$R(NI|x) = \lambda(NI|I) P(I|x)$$
$$= 20 \times \frac{\frac{1}{\sqrt{2\pi}} \cdot \frac{1}{5} \cdot \exp(-\frac{(x-5)^2}{2\times 25}) \times 0.05}{p(x)}.$$

$$R(I|x) = \lambda(I|NI) P(NI|x)$$
$$= 5 \times \frac{\frac{1}{\sqrt{2\pi}} \cdot \frac{1}{5} \exp(-\frac{(x+10)^2}{2\times 25}) \times 0.95}{p(x)}.$$

Let $\frac{R(NI|x)}{R(I|x)} = 4 \times 19 \times e^{-\frac{6x+15}{10}} = 76\, e^{-\frac{6x+15}{10}}$.

① When $76\, e^{-\frac{6x+15}{10}} \geq 1 \iff x \leq \frac{5}{3} \ln 76 - \frac{5}{2}$.

classify as work properly

② When $76\, e^{-\frac{6x+15}{10}} < 1 \iff x > \frac{5}{3} \ln 76 - \frac{5}{2}$.

classify as possible issue.

It minimizes overall risk.

2\. (a) Find eigenvalues & eigenvectors of scatter matrix.

$$\det(\lambda I - S) = 0.$$

$$\Rightarrow \lambda_1 = 10, \quad \lambda_2 = 5, \quad \lambda_3 = 2.$$

eigenvectors : $(S - \lambda I) w = 0$.

$$\Rightarrow w_1 = [0.707, -0.707, 0]$$

$$W_2 = [0.707, 0.707, 0]$$

$$W_3 = [0, 0, 1].$$

So the first principle component is. $[0.707, -0.707, 0]$

(b) $a_K = W^T(x^k - m)$.

$$\cancel{= [0.707, -0.707, 0]([1.2, 1.7, -1.3]^T - m)}$$

$$\cancel{\tilde{x} = m + a_k W}$$

$$\cancel{= m + [0.707, -0.707, 0]([1.2, 1.7, -1.3]^T - m)}$$

$$\cancel{= [0.293, 1.707, 1]\, m - 0.3535}$$

We need Find the projection of x in w.

$$\alpha = \frac{\langle X \cdot w \rangle}{\|w\|} = -0.3536.$$

So $\tilde{X} = \alpha \cdot w = -0.3536 \times [0.707, -0.707, 0]$

$$= [-0.25, 0.25, 0]$$

3. (a).

$$\text{distance} = \frac{|g(x)|}{||w||}$$

$$= \frac{|[1,-0.5,2][1,-3,2]^T - 2|}{||[1,-0.5,2]||}$$

$$= 1.964.$$

$$g(x) = [1,-0.5,2][1,-3,2]^T - 2.$$

$$= 4.5 \geq +1$$

Therefore, x has label +1.

(b)

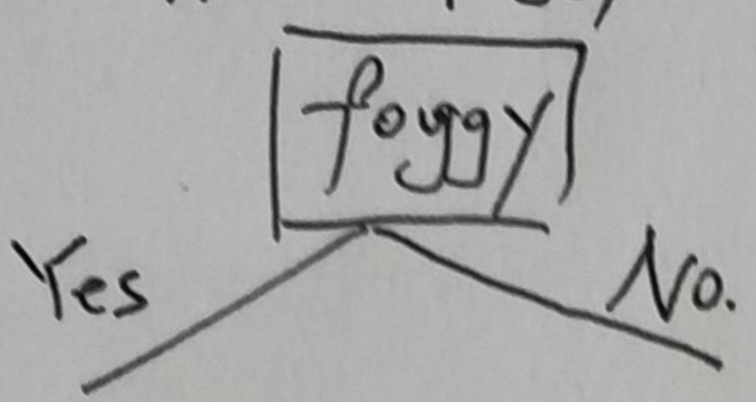
$$\text{margin} = \frac{2}{||w||}$$

$$= \frac{2}{||[1,-0.5,2]||}$$

$$= 0.873$$

4. (a)

① Suppose. foggy? first.



Total $GI = \frac{3}{7} \times \frac{4}{9} + \frac{4}{7} \times \frac{3}{8}$

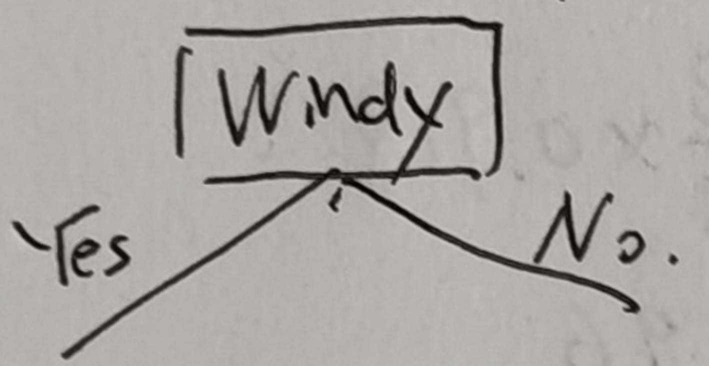
$= 0.405$.

2 Yes. 1 No. 3 Yes 1 No.

$GI = 1 - \sum_{j=1}^{2} p_j^2$. $GI = \frac{3}{8}$

$= \frac{4}{9}$

② Suppose windy? first.



Total $GI = \frac{3}{7} \times \frac{4}{9}$

$= 0.19$.

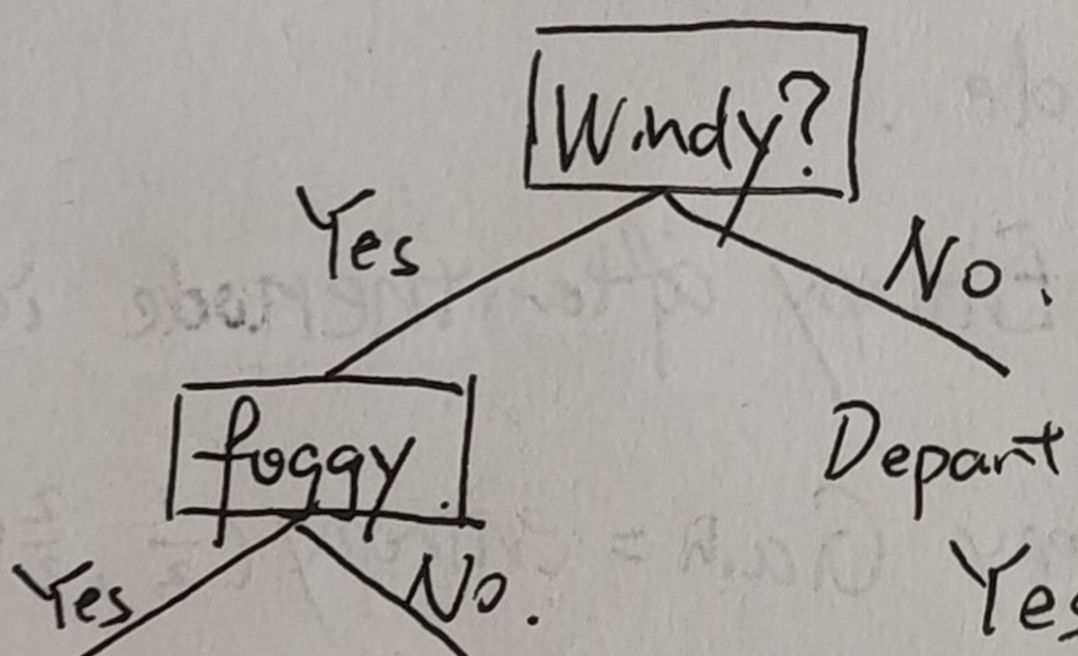
1 Yes 2 No 4 Yes.

$GI = \frac{4}{9}$

Because $0.19 < 0.405$, We choose windy? as first decision node

For second decision node, if with foggy $GI = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} < 1 - (\frac{2}{3})^2 - (\frac{1}{3})^2$.

So we add foggy as decision node after Windy Yes.



Depart on time. Yes.

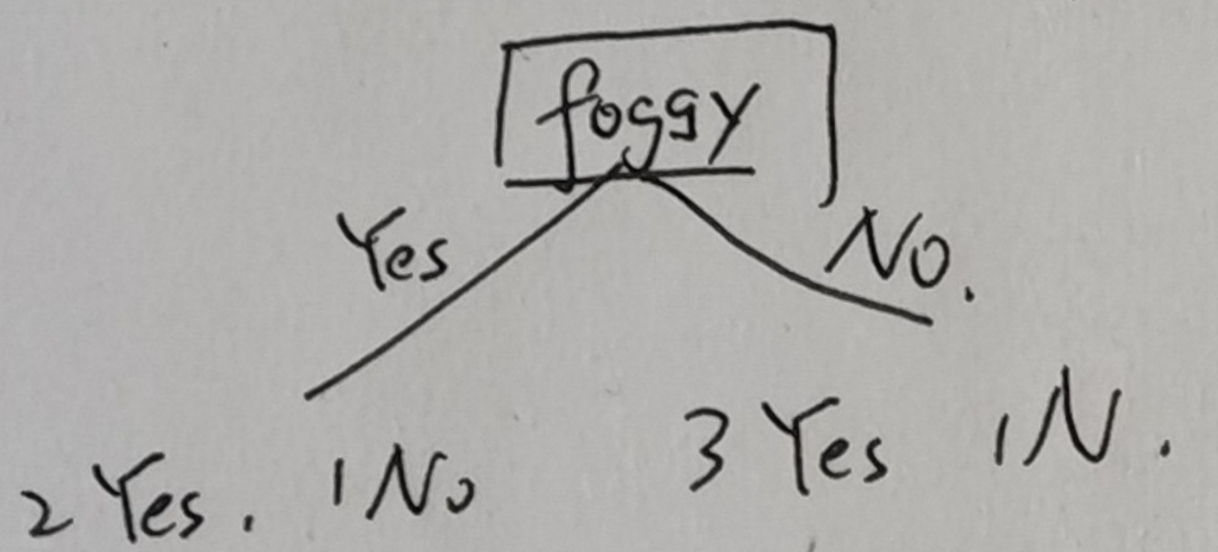
Depart on time. No

Depart on time. Yes/No. ← Since data has both depart on time and not on time in this case, we can randomly select one.

(b) Original entropy $E = -\sum_{i=1}^{2} p_i \log_2(p_i)$

$= 0.863.$

① Suppose Foggy? first

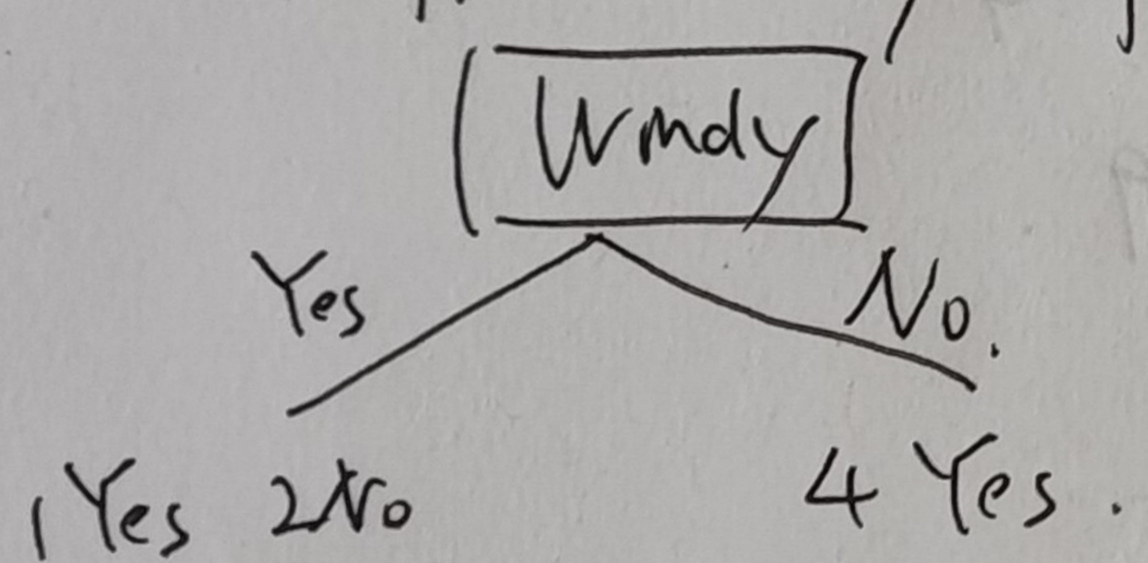


Entropy Gain $= E - \frac{3}{7}\times 0.918 - \frac{4}{7}\times 0.811$

$= 0.00598.$

$E = 0.918$ $\quad$ $E = 0.811.$

② Suppose Windy? first:



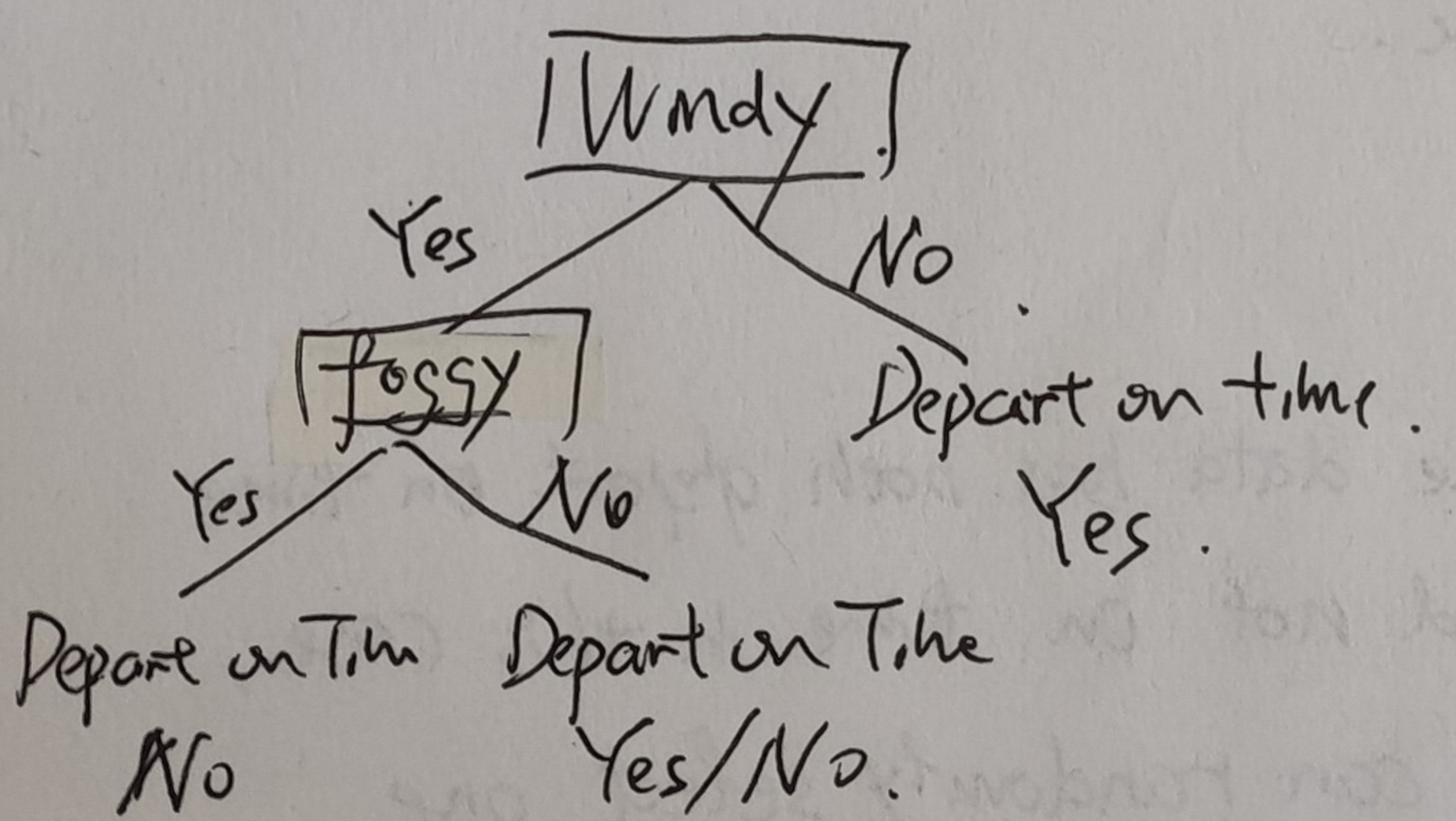
Entropy Gain $= E - \frac{3}{7}\times 0.918$

$= 0.4696.$

$E = 0.918$ $\quad$ $E = 0.$

Because Windy first entropy gain is larger, so we choose Windy? as first decision node.

For second decision node, if with foggy Entropy Gain is.



Entropy Gain $=$ entropy$(\frac{1}{3}, \frac{2}{3}) - \frac{2}{3}\times$ entroy$(\frac{1}{2}, \frac{1}{2})$

$= 0.252 > 0.$

So we add foggy as decision node after Windy Yes.

Same as question (a).

We can randomly pick one.