# Probability and Statistics Homework 1

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## Problem-1

Specifically, the sample space is

$$\begin{aligned}\Omega = \{&(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\ &(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\ &(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\ &(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ &(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\ &(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}\end{aligned}$$

Similarly, E, F, G can be written as:

$$\begin{aligned}E &= \{(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),(3,2),(3,4),(3,6),\\ &\quad (4,1),(4,3),(4,5),(5,2),(5,4),(5,6),(6,1),(6,3),(6,5)\}\\ F &= \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}\\ G &= \{(1,4),(2,3),(3,2),(4,1))\}\end{aligned}$$

With the definition of Union, Intersection and Difference, events are:

$$\begin{aligned}E \cap F &= \{(1,2),(1,4),(1,6),(2,1),(4,1),(6,1)\}\\ E \cup F &= \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,3),(2,5),\\ &\quad (3,1),(3,2),(3,4),(3,6),(4,1),(4,3),(4,5),(5,1),(5,2),\\ &\quad (5,4),(5,6),(6,1),(6,3),(6,5)\}\\ F \cap G &= \{(1,4),(4,1)\}\\ E \setminus F &= \{(2,3),(2,5),(3,2),(3,4),(3,6),(4,3),(4,5),(5,2),(5,4),(5,6),(6,3),(6,5)\}\\ E \cap F \cap G &= \{(1,4),(4,1)\}\end{aligned}$$

In conclusion, there are

- **6** elementary outcomes in $E \cap F$
- **23** elementary outcomes in $E \cup F$
- **2** elementary outcomes in $F \cap G$
- **12** elementary outcomes in $E \setminus F$
- **2** elementary outcomes in $E \cap F \cap G$

# Problem-2

When choosing the first sock, the probability that it's red is $\frac{3}{n}$.
After a red sock is picked out, the probability that the second sock is red is $\frac{2}{n-1}$.
Since the probability that both are red is $\frac{1}{2}$, the equation can be listed as

$$\frac{3}{n} \times \frac{2}{n-1} = \frac{1}{2}$$

The value of $n$ is $4$.

# Problem-3

*Proof.* 1. According to $\sigma$-algebra property (2), since $A_1, A_2, ... \in \mathcal{F}$, then $\Omega \setminus A_1, \Omega \setminus A_2, ... \in \mathcal{F}$

2. According to $\sigma$-algebra property (3), since $\Omega \setminus A_1, \Omega \setminus A_2, ... \in \mathcal{F}$, then $\cup_{i=1}^{\infty}(\Omega \setminus A_i) \in \mathcal{F}$

3. According to DeMorgan's Law $E^C \cup F^C = (E \cap F)^C$, since

$$\cup_{i=1}^{\infty}(\Omega \setminus A_i) = \Omega \setminus (\cap_{i=1}^{\infty} A_i)$$

   then $\Omega \setminus (\cap_{i=1}^{\infty} A_i) \in \mathcal{F}$

4. According to $\sigma$-algebra property (2), since $\Omega \setminus (\cap_{i=1}^{\infty} A_i) \in \mathcal{F}$, then $\cap_{i=1}^{\infty} A_i \in \mathcal{F}$

$\square$