1. Consider a unconstrained optimization problem

$$\text{minimize}_{\mathbf{x}\in\mathbf{R}^2}\ f(\mathbf{x})\ ,$$

where

$$f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{P}\mathbf{x} + \mathbf{q}^T\mathbf{x} + \log\left(e^{-2x_1} + e^{-x_2}\right)$$

with

$$\mathbf{P} = \begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix} \text{ and } \mathbf{q} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}\ .$$

Use the initial point $\mathbf{x}^0 = [1\ \ 2]^T$ and the stopping condition $\|\nabla f(\mathbf{x})\| < 10^{-2}$ for problems 1 and 2.

(a) Solve the optimization using the gradient descent method with exact line search. Plot the sequence of solutions $\mathbf{x}^k$, $k = 0, 1, \ldots$.

(b) Solve the optimization using the gradient descent method with backtracking line search with the parameter $\alpha_{init} = 0.15, \gamma = 0.7$ and $\beta = 0.8$. Plot the sequence of solutions $\mathbf{x}^k$, $k = 0, 1, \ldots$.

2. Repeat problem 1 with a new matrix

$$\mathbf{P} = \begin{bmatrix} 5.005 & 4.995 \\ 4.995 & 5.005 \end{bmatrix}\ .$$