Tenth Homework

1. A child gets a weekly allowance (a certain non-negative number of dollars). The amount (in dollars) is random with expectation equal to 6 and variance equal to 6. Let's denote this random variable by X. I want to estimate (from above) the chances that the child gets 12 dollars or more on a given week. In other words, I want to find c such that $P(X \geq 12) \leq c$, with c that is as small as possible.
   (a) Use the Markov inequality to find c such that $P(X \geq 12) \leq c$.
   (b) Use the Chebyshev inequality for the variance to find c such that $P(X \geq 12) \leq c$.
   (c) Use the one-sided Chebyshev inequality to find c such that $P(X \geq 12) \leq c$.

2. If X_1, X_2, X_3, and X_4 are (pairwise) uncorrelated random variables, each having mean 0 and variance 1, compute the correlations of
   (a) $X_1 + X_2$ and $X_2 + X_3$;
   (b) $X_1 + X_2$ and $X_3 + X_4$.

3. A certain component is critical to the operation of an electrical system and must be replaced immediately upon failure. If the mean lifetime of this type of component is 100 hours and its standard deviation is 30 hours, how many of these components must be in stock so that the probability that the system is in continual operation for the next 2000 hours is at least .95 ?