Seventh Homework

1. Suppose that a random variable X has the density given by

$$f(x) = \begin{cases} 0, & x \leq 0 \\ cx, & x \in (0,2) \\ 0, & x \geq 2, \end{cases}$$

where c is a certain constant.

(a) Find c.
(b) Find the cumulative distribution function of X.
(c) Find $\mathrm{P}(X > 1)$.
(d) Find $\mathrm{E}X$
(e) Find $\mathrm{Var}(X)$.
(f) Find a formula for the density of $Y = e^X$.
(g) Find $\mathrm{E}e^X$.

2. A store-owner buys up to 100 liters of milk from a wholesaler at the beginning of the day with the price per liter equal to $2 - (x/400)$ dollars, where x is the total amount (in liters) that he buys. He then sells it during the day at 3 dollar per liter. Any unsold milk is wasted. The daily demand (in liters) is random, uniformly distributed on the interval $[0, 100]$. What amount of milk should the store-owener buy to maximize his expected profit?

3. The density of a random variable X is

$$f(x) = \begin{cases} a + bx^2, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find a and b if you know that $\mathrm{E}X = \frac{5}{8}$.