

Eighth Homework

1. Suppose that X is an exponential random variable with parameter $\lambda = 1$. Let $Y = [X]$ (i.e., Y is the integer part of X).

- (a) Write a formula for the probability mass function of Y .
- (b) Calculate $E(Y)$.

2. In the following problem, you should use the normal approximation to the binomial. Use a calculator, a computer program, or an online resource to evaluate (approximately) the required integrals. The answers will be approximate.

Supposed that a game is played where you win each round with probability equal to $1/4$.

- (a) If you play 1,000,000 rounds, what is your probability (approximately) to win at least 250,100 rounds?
- (b) If you play 1,000,000 rounds, find such n that the probability of winning at least n rounds is around 90 percent?
- (c) How many rounds should you play in order to have your chances of winning 24 percent of the rounds equal to 90 percent?

3. X and Y are independent random variables uniformly distributed over $(0, 1)$. Compute the probability that the larger of the two is at least three times as large as the other one.

4. The random variables X and Y have joint density

$$p(x, y) = \begin{cases} cxy(1-x) & \text{if } 0 < x < 1, \quad 0 < y < 1, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a positive constant.

- (a) Find c .
- (b) Are X and Y independent?
- (c) Find EY .
- (d) Find $\text{Var}(X)$.