First Homework

1. Two dice are thrown. This experiment is modeled by the sample space

$$\Omega = \{(\omega_1, \omega_2), \quad \omega_i \in \{1, ..., 6\}\},$$

where ω_1 is the result on the first die, and ω_2 on the second one. Let E be the event that the sum of the dice is odd, let F be the event that at least one of the dice lands on 1, and let G be the event that the sum is 5. How many elementary outcomes are there in the events $E \bigcap F$, $E \bigcup F$, $F \bigcap G$, $E \setminus F$, and $E \bigcap F \bigcap G$?

2. There are n socks, 3 of which are red, in a drawer. What is the value of n if, when 2 of the socks are chosen randomly, the probability that they are both red is $1/2$?

3. Recall our definition of a σ-algebra: $\mathcal{F}$, a collection of subsets of Ω is said to be a σ-algebra if
(a) $\Omega \in \mathcal{F}$,
(b) If $A \in \mathcal{F}$, then $\Omega \setminus A \in \mathcal{F}$,
(c) If $A_1, A_2, ... \in \mathcal{F}$, then $\bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$.

Prove that if $A_1, A_2, ... \in \mathcal{F}$, then $\bigcap_{i=1}^{\infty} A_i \in \mathcal{F}$.