Data601 Final Exam

Instructions: The exam will take place from 4:00 PM to 6:30 PM. You will have an additional 15 minutes, from 6:30 PM to 6:45 PM, to submit your work. You may submit your work at any time before this window. Submissions made after 6:45 PM will have a penalty point deduction for each minute late.

Show all supporting work for full credit. Your work should be neatly handwritten and scanned. Then upload your work to Gradescope.

Resources allowed: Course notes and textbook.

Resources NOT allowed: Working in groups and online resources.

1. From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
   (a) (1 point) there are no restrictions on the committees?
   (b) (1 point) 2 of the men refuse to serve together?
   (c) (1 point) 2 of the women refuse to serve together?
   (d) (1 point) 1 man and 1 woman refuse to serve together?

2. The random variables X and Y have joint density function
$$f(x, y) = 12xy(1 - x) \quad 0 < x < 1, 0 < y < 1$$
   and equal to 0 otherwise.
   (a) (2 points) Are X and Y independent? Justify and show supporting work.
   (b) (1 point) Find $E[X]$.
   (c) (2 points) Find $\text{Var}(Y)$.

3. Allie flips a fair coin five times. She earns 1 point for each turn that the outcome is the same as the previous turn. On the first coin flip she does not earn any points. For example if her outcomes are HHHTT, then she earns 1 point for each of the second, third and fifth turns, for a total of 3 points.
   (a) (2 points) What is the probability that Allie earns at least one point?
   (b) (2 points) Let X be the total points Allie earns. Find the expectation of X. (Hint: Let X_i be the point she earns on the i-th turn. $X = X_2 + X_3 + X_4 + X_5$. Then, use linearity of expectation.)
   (c) (2 points) Are the two events $X_2 = 1$ and $X_3 = 1$ independent? Justify and show supporting work!

4. The probability density function of a random variable X, depending on a parameter θ (where θ is between $1/3$ and $2/3$), is given by

$$f(x) = \begin{cases} 2\theta - \frac{2}{3} + \frac{2}{3}x - \frac{4}{3}\theta x & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) (2 points) Using the method of maximum likelihood and the random sample $x_1 = 1, x_2 = 2$ find an estimate for θ.

(b) (2 points) Repeat part (a), but this time use the method of moments.

5. Sid, a professional chess player, plays lots of games of chess every day. The outcome of each game for Sid is a win, a loss or a draw. If Sid wins a game he will win, draw or lose in the next game with probabilities $\frac{1}{2}, \frac{1}{2}$ and zero, respectively. If he draws a game, he wins, draws or loses the next game with probabilities $\frac{1}{3}, \frac{1}{2}$ and $\frac{1}{6}$, respectively. If Sid loses a game, in the next game his probabilities of win, draw or loss are $\frac{1}{10}, \frac{1}{5}$ and $\frac{7}{10}$, respectively.

(a) (2 points) Draw a state diagram for the situation.

(b) (2 points) Write down the transition matrix.

(c) (2 points) What is Sid's long term win ratio? (i.e. the ratio of his total win to total number of games.)