1. Suppose that an input image is given by the following array and the 3x3 kernel used by a convolutional layer is shown below.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \end{bmatrix} \qquad \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}$$

Image $\qquad\qquad$ 3x3 kernel

(a) Find the output from the convolutional layer w/ zero-padding

(b) Suppose that the convolutional layer is followed by a max pooling layer w/ 2x2 filter.
What would be the output if stride of 2 is used for max pooling?

2. Suppose a dataset consists of the following six points (where each row corresponds to a data point).

$$\begin{bmatrix} 0 & 1 \\ 0 & -1 \\ 1 & 0 \\ -1 & 1 \\ -2 & 0 \\ -1 & -1 \end{bmatrix} = X$$

Find the two clusters w/ associated centroids after two iteration, starting w/ two initial centroids $\underline{c}_1 = [0 \quad 1]$ & $\underline{c}_2 = [-1 \quad 1]$, using k-means clustering.

3. Consider the following dataset.

$$X = \begin{bmatrix} 0.66 & 0.68 & 0.66 \\ 0.04 & 0.76 & 0.17 \\ 0.85 & 0.74 & 0.71 \\ 0.93 & 0.39 & 0.03 \end{bmatrix}$$

Find the 1-dimensional embedding / representation using classical multidimensional scaling.

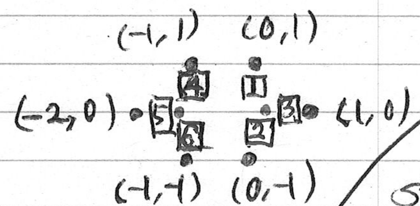
Solution

1a. $\begin{bmatrix} -1 & 2 & -2 & 2 \\ -0.5 & 2 & -2 & 2.5 \\ 1 & -1 & 1 & -0.5 \\ 1.5 & -2 & 2 & -1.5 \end{bmatrix}$

1b. $\begin{bmatrix} 2 & 2.5 \\ 1.5 & 2 \end{bmatrix}$

2. Iteration 1.

step 1. Assign each point to the closest centroid



Initially $C_1 = \{1, 2, 3\}$

$C_2 = \{4, 5, 6\}$

step 2: new centroids

$\underline{c}_1 = \frac{1}{3}\left([0\ 1] + [1\ 0] + [0\ -1]\right)$

$= \left[\frac{1}{3}\ 0\right]$

$\underline{c}_2 = \frac{1}{3}\left([-1\ 1] + [-2\ 0] + [-1\ -1]\right)$

$= \left[-\frac{4}{3}\ 0\right]$

Iteration 2.

step 1. Assign each point to the closest centroid

clearly, the assignments would not change because the first three data points are closer to $\underline{c}_1$, while the last three data points are closer to $\underline{c}_2$.

step 2. Since memberships have not changed the centroids are the same and the algorithm will terminate

3. To find the 1-dimensional embedding, we need to first compute the centered Gram matrix

$$B = HXX^TH$$

where $H = I_{4\times4} - \frac{1}{n}\underline{1}\,\underline{1}^T$

$$B = \begin{bmatrix} 0.0746 & -0.0783 & 0.0978 & -0.0940 \\ -0.0783 & 0.3997 & -0.1926 & -0.1288 \\ 0.0978 & -0.1926 & 0.1632 & -0.0684 \\ -0.0940 & -0.1288 & -0.0684 & 0.2913 \end{bmatrix}$$

Then, compute the eigenvalues & eigenvectors of B. Since we are interested in one-dimensional embedding, it is given by the eigenvector corresponding to the largest eigenvalue

$$\underline{v}^1 = [\,0.1793 \quad -0.8518 \quad 0.4261 \quad 0.2464\,]$$