## Eleventh Homework

**1.** A trader invented a strategy where he can either win or lose a certain amount of money every day (say, $k$ dollars) with equal probability, independently of all other days. (In fact, such a strategy is not very difficult to implement - buy a very liquid asset whose price moves without major jumps and liquidate the position as soon as you are making or losing $k$ dollars). He then made the following statement: “If I use my strategy every day for the next 400 days, my chances of ending up within 10,000 dollars of the original account balance are equal to 1/3.

(a) What is $k$ (approximately)?

(b) What are his chances (approximately) of making over 20,000 with this strategy in 400 days?

**2.** You are trying to use a machine that only works on some days. If on a given day, the machine is working it will break down the next day with probability $0 < b < 1$, and works on the next day with probability $1 - b$. If it is not working on a given day, it will work on the next day with probability $0 < r < 1$ and not work the next day with probability $1 - r$.

(a) In this problem we will formulate this process as a Markov chain. First, let $X^{(t)}$ be a variable that denotes the state of the machine at time $t$. Then, define a state space $\mathcal{S}$ that includes all the possible states that the machine can be in. Lastly, for all $A, B \in S$ find $\mathrm{P}\left(X^{(t+1)} = A \mid X^{(n)} = B\right)$ ( $A$ and $B$ can be the same state).

(b) Suppose that on day 1, the machine is working. What is the probability that it is working on day 3?

**3.** Suppose that $X_1, ..., X_4$ are independent normal random variables with parameters $(a, \sigma^2)$. Find the density of the random variable $Y = X_1 - 2X_2 + 3X_3 - 4X_4$.