1. Consider a unconstrained optimization problem

$$\text{minimize}_{\mathbf{x}\in\mathbf{R}^2} \; f(\mathbf{x}) \; ,$$

where

$$f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{P}\mathbf{x} + \mathbf{q}^T\mathbf{x} + \log\left(e^{-2x_1} + e^{-x_2}\right)$$

with

$$\mathbf{P} = \begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix} \text{ and } \mathbf{q} = \begin{bmatrix} -2 \\ 4 \end{bmatrix} .$$

Use the initial point $\mathbf{x}^0 = [1 \;\; 2]^T$ and the stopping condition $\|\nabla f(\mathbf{x})\| < 10^{-2}$ for problems 1 and 2.

(a) Solve the optimization using the gradient descent method with exact line search. Plot the sequence of solutions $\mathbf{x}^k$, $k = 0, 1, \ldots$.

Ans: Figure 1 plots the sequence $\mathbf{x}_k$ and the value of the objective function $f(\mathbf{x}_k)$.

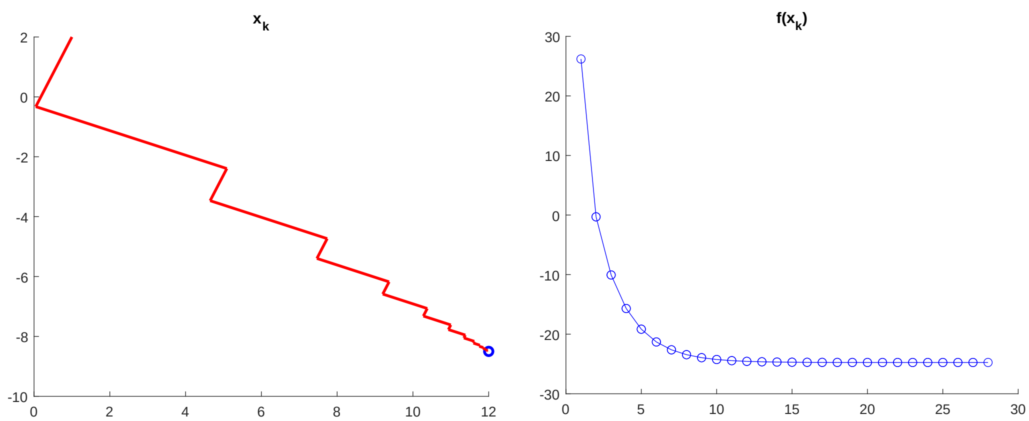

Figure 1: Plot of $\mathbf{x}_k$ and $f(\mathbf{x}_k)$ using the gradient descent method with exact line search.

(b) Solve the optimization using the gradient descent method with backtracking line search with the parameter $\alpha_{init} = 0.15, \gamma = 0.7$ and $\beta = 0.8$. Plot the sequence of solutions $\mathbf{x}^k$, $k = 0, 1, \ldots$.

Ans: Figure 2 plots the sequence $\mathbf{x}_k$ and the value of the objective function $f(\mathbf{x}_k)$.

2. Repeat problem 1 with a new matrix

$$\mathbf{P} = \begin{bmatrix} 5.005 & 4.995 \\ 4.995 & 5.005 \end{bmatrix} .$$

Ans: Figures 3 and 4 plots the sequence $\mathbf{x}_k$ and the value of the objective function $f(\mathbf{x}_k)$ for both algorithms. It is clear from the plots that with the new matrix $\mathbf{P}$, the convergence of both algorithms is much slower. This is due to a large condition number (ratio of the eigenvalues of $\mathbf{P}$).

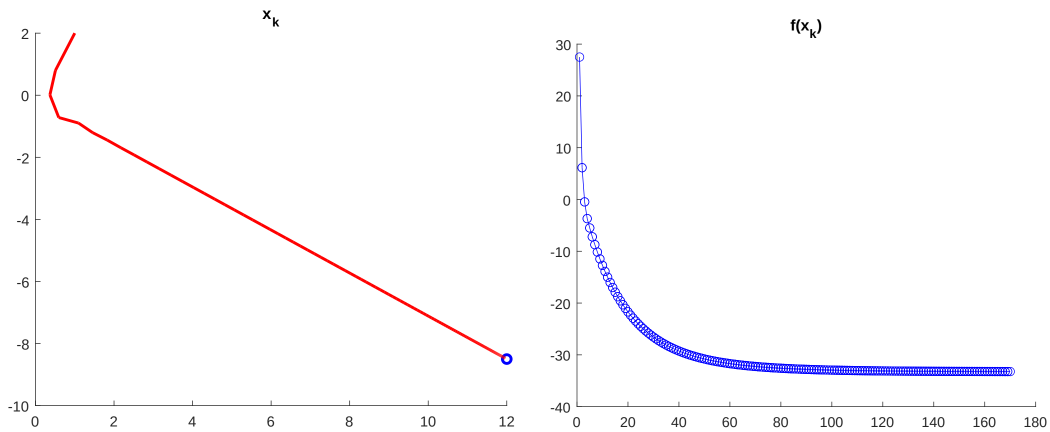

Figure 2: Plot of $\mathbf{x}_k$ and $f(\mathbf{x}_k)$ using the gradient descent method with backtracking line search.

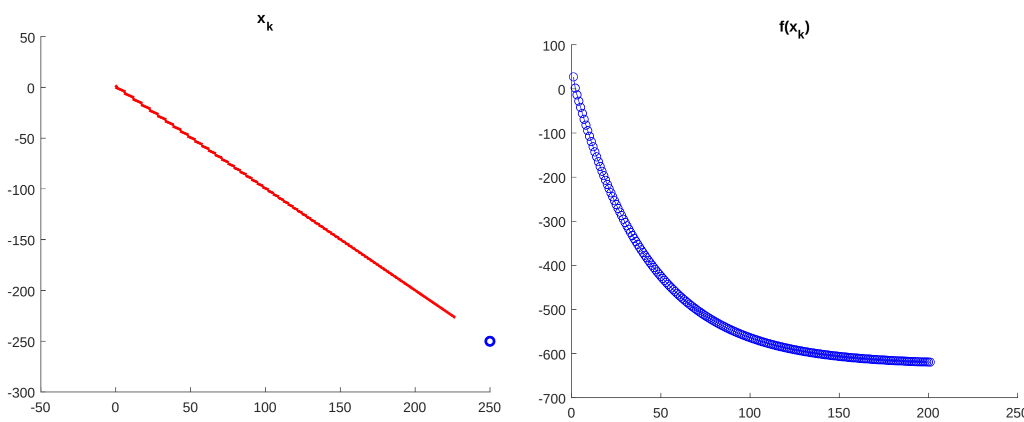

Figure 3: Plot of $\mathbf{x}_k$ and $f(\mathbf{x}_k)$ using the gradient descent method with exact line search.

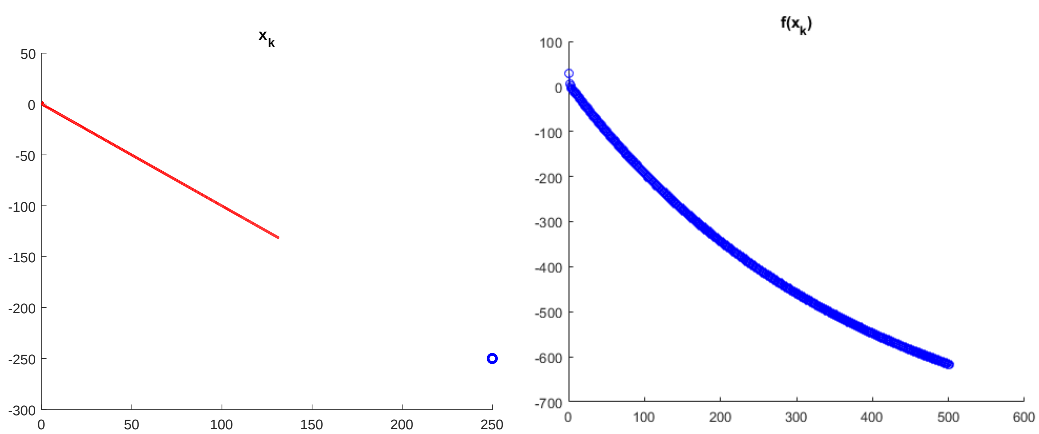

Figure 4: Plot of $\mathbf{x}_k$ and $f(\mathbf{x}_k)$ using the gradient descent method with backtracking line search.