

1. Consider  $2 \times 2$  matrix  $\mathbf{B}$  given by

$$\mathbf{B} = \begin{bmatrix} 2.25 & -0.433 \\ -0.433 & 2.75 \end{bmatrix}.$$

- Find the eigenvalues of  $\mathbf{B}$ .
  - Find the eigenvectors corresponding to the eigenvalues and show that they are orthogonal. (Note that the eigenvectors are not unique.)
  - Compute the determinant of  $\mathbf{B}$  and verify that the determinant is equal to the product of the eigenvalues.
  - Verify that the trace of  $\mathbf{B}$  (i.e., the sum of diagonal elements) is equal to the sum of the eigenvalues.
2. Suppose that the joint distribution of the network state and the response time is shown in the following table.

	response time		
	fast	normal	slow
under attack	0.05	0.1	0.25
normal operation	0.2	0.3	0.1

- Compute the probability that the network is under attack.
  - Determine if the events  $A = \{\text{response time is slow}\}$  and  $B = \{\text{network is under attack}\}$  are independent.
  - Compute the conditional probability that the response time is slow given that the network is under attack.
  - Compute the conditional probability that the network is under attack given that the response time is slow, using the Bayes' rule (and the answer to part (c) above).
3. **[Suggested Problem - will not be graded]** Show that the eigenvalues of semidefinite matrix  $\mathbf{A} \in \mathbb{R}^{n \times n}$  are nonnegative.