ex In a factory there are 100 units of a certain product, 5 of which are defective. We pick 3 units from the 100 at random. What is the prob that none of them are defective?

Define A_i as the event that the ith chosen unit is not defective $i = 1, 2, 3$

Want

$P(A_1 \cap A_2 \cap A_3)$ use tower prop

Know:

$$P(A_1) = \frac{95}{100}$$

$$P(A_2 | A_1) = \frac{94}{99}$$

$$P(A_3 | A_2 \cap A_1) = \frac{93}{98}$$

$$\Rightarrow P(A_1 \cap A_2 \cap A_3) = P(A_1) P(A_2 | A_1) P(A_3 | A_2 \cap A_1)$$

$$= \frac{95}{100} \frac{94}{99} \frac{93}{98} = 0.8560$$

ex A class is taking an exam one question is mult. choice, w/ 5 options.

$\frac{1}{2}$ of the class knows the answer (get it right)

other $\frac{1}{2}$ will guess randomly from the 5 choices.

If a student answered correctly, what is the prob that they actually knew the answer?

A: know answer

B: answer correctly

$P(A) = \frac{1}{2}$ $P(A^c) = \frac{1}{2}$

disjoint, exhaustive ↓ cover the entire space

Want: $P(A|B)$

$P(B|A) = 1$

$P(B|A^c) = \frac{1}{5}$

$$\Rightarrow P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

$$= \frac{1 \cdot \frac{1}{2}}{1 \cdot \frac{1}{2} + \frac{1}{5} \cdot \frac{1}{2}} = \frac{5}{6} \approx .833$$

ex Assume all cats are either lucky or unlucky. A lucky cat has a 40% chance of winning lottery & an unlucky cat has 20% of winning the lottery. If 30% of all cats are lucky, what is the prob a random cat wins the lottery?

A: is lucky (A^c: unlucky)

B: win the lottery

Know: $P(B|A) = \frac{4}{10}$ $P(A) = \frac{3}{10}$

$P(B|A^c) = \frac{2}{10}$ $P(A^c) = \frac{7}{10}$

Want: $P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$

$= \frac{4}{10}\frac{3}{10} + \frac{2}{10}\frac{7}{10} = \frac{13}{50} = 26\%$

ex I toss a coin until I observe the first tails at which point I stop.

Let X be the total number of coin tosses. Find $P(X=5)$

* $X=5$ → first 4 tosses are H
5th is a T

all coin tosses are independent of each other

$$P(X=5) = P(HHHHT) = P(H)P(H)P(H)P(H)P(T)$$

$$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$$

$$= \frac{1}{32}$$