PRACTICE PROBLEMS

The following information relates to questions 1–8

Megan Beade and Hanna Müller are senior analysts for a large, multi-divisional money management firm. Beade supports the institutional portfolio managers, and Müller does the same for the private wealth portfolio managers.

Beade reviews the asset allocation in Exhibit 1, derived from a mean–variance optimization (MVO) model for an institutional client, noting that details of the MVO are lacking.

Exhibit 1 Asset Allocation and Market Weights (in percent)

Asset Classes	Asset Allocation	Investable Global Market Weights
Cash	0	—
US bonds	30	17
US TIPS	0	3
Non-US bonds	0	22
Emerging market equity	25	5
Non-US developed equity	20	29
US small- and mid-cap equity	25	4
US large-cap equity	0	20

The firm's policy is to rebalance a portfolio when the asset class weight falls outside of a corridor around the target allocation. The width of each corridor is customized for each client and proportional to the target allocation. Beade recommends wider corridor widths for high-risk asset classes, narrower corridor widths for less liquid asset classes, and narrower corridor widths for taxable clients with high capital gains tax rates.

One client sponsors a defined benefit pension plan where the present value of the liabilities is $241 million and the market value of plan assets is $205 million. Beade expects interest rates to rise and both the present value of plan liabilities and the market value of plan assets to decrease by $25 million, changing the pension plan's funding ratio.

Beade uses a surplus optimization approach to liability-relative asset allocation based on the objective function

$$U_m^{LR} = E\left(R_{s,m}\right) - 0.005\lambda\sigma^2\left(R_{s,m}\right)$$

where $E(R_{s,m})$ is the expected surplus return for portfolio m, λ is the risk aversion coefficient, and $\sigma^2(R_{s,m})$ is the variance of the surplus return. Beade establishes the expected surplus return and surplus variance for three different asset allocations, shown in Exhibit 2. Given $\lambda = 1.50$, she chooses the optimal asset mix.

Exhibit 2 Expected Surplus Return and Volatility for Three Portfolios

	Return	Standard Deviation
Portfolio 1	13.00%	24%
Portfolio 2	12.00%	18%
Portfolio 3	11.00%	19%

Client Haunani Kealoha has a large fixed obligation due in 10 years. Beade assesses that Kealoha has substantially more funds than are required to meet the fixed obligation. The client wants to earn a competitive risk-adjusted rate of return while maintaining a high level of certainty that there will be sufficient assets to meet the fixed obligation.

In the private wealth area, the firm has designed five subportfolios with differing asset allocations that are used to fund different client goals over a five-year horizon. Exhibit 3 shows the expected returns and volatilities of the subportfolios and the probabilities that the subportfolios will exceed an expected minimum return. Client Luis Rodríguez wants to satisfy two goals. Goal 1 requires a conservative portfolio providing the highest possible minimum return that will be met at least 95% of the time. Goal 2 requires a riskier portfolio that provides the highest minimum return that will be exceeded at least 85% of the time.

Exhibit 3 Characteristics of Subportfolios

Subportfolio	A	B	C	D	E
Expected return, in percent	4.60	5.80	7.00	8.20	9.40
Expected volatility, in percent	3.46	5.51	8.08	10.80	13.59
Required Success Rate	**Minimum Expected Return for Success Rate**				
99%	1.00	0.07	–1.40	–3.04	–4.74
95%	2.05	1.75	1.06	0.25	–0.60
90%	2.62	2.64	2.37	2.01	1.61
85%	3.00	3.25	3.26	3.19	3.10
75%	3.56	4.14	4.56	4.94	5.30

Müller uses a risk parity asset allocation approach with a client's four–asset class portfolio. The expected return of the domestic bond asset class is the lowest of the asset classes, and the returns of the domestic bond asset class have the lowest covariance with other asset class returns. Müller estimates the weight that should be placed on domestic bonds.

Müller and a client discuss other approaches to asset allocation that are not based on optimization models or goals-based models. Müller makes the following comments to the client:

Comment 1 An advantage of the "120 minus your age" heuristic over the 60/40 stock/bond heuristic is that it incorporates an age-based stock/bond allocation.

Comment 2 The Yale model emphasizes traditional investments and a commitment to active management.

Comment 3 A client's asset allocation using the 1/*N* rule depends on the investment characteristics of each asset class.

1 The asset allocation in Exhibit 1 *most likely* resulted from a mean–variance optimization using:

A historical data.

B reverse optimization.

C Black–Litterman inputs.

2 For clients concerned about rebalancing-related transactions costs, which of Beade's suggested changes in the corridor width of the rebalancing policy is correct? The change with respect to:

A high-risk asset classes.

B less liquid asset classes.

C taxable clients with high capital gains tax rates.

3 Based on Beade's interest rate expectations, the pension plan's funding ratio will:

A decrease.

B remain unchanged.

C increase.

4 Based on Exhibit 2, which portfolio provides the greatest objective function expected value?

A Portfolio 1

B Portfolio 2

C Portfolio 3

5 The asset allocation approach most appropriate for client Kealoha is *best* described as:

A a surplus optimization approach.

B an integrated asset–liability approach.

C a hedging/return-seeking portfolios approach.

6 Based on Exhibit 3, which subportfolios *best* meet the two goals expressed by client Rodríguez?

A Subportfolio A for Goal 1 and Subportfolio C for Goal 2

B Subportfolio B for Goal 1 and Subportfolio C for Goal 2

C Subportfolio E for Goal 1 and Subportfolio A for Goal 2

7 In the risk parity asset allocation approach that Müller uses, the weight that Müller places on domestic bonds should be:

A less than 25%.

B equal to 25%.

C greater than 25%.

8 Which of Müller's comments about the other approaches to asset allocation is correct?

A Comment 1

B Comment 2

C Comment 3

The following information relates to questions 9–13

Investment adviser Carl Monteo determines client asset allocations using quantitative techniques such as mean–variance optimization (MVO) and risk budgets. Monteo is reviewing the allocations of three clients. Exhibit 1 shows the expected return and standard deviation of returns for three strategic asset allocations that apply to several of Monteo's clients.

Exhibit 1 Strategic Asset Allocation Alternatives

	Adviser's Forecasts	
Asset Allocation	Expected Return (%)	Standard Deviation of Returns (%)
A	10	12.0
B	8	8.0
C	6	2.0

Monteo interviews client Mary Perkins and develops a detailed assessment of her risk preference and capacity for risk, which is needed to apply MVO to asset allocation. Monteo estimates the risk aversion coefficient (λ) for Perkins to be 8 and uses the following utility function to determine a preferred asset allocation for Perkins:

$$U_m = E(R_m) - 0.005\lambda\sigma_m^2$$

Another client, Lars Velky, represents Velky Partners (VP), a large institutional investor with $500 million in investable assets. Velky is interested in adding less liquid asset classes, such as direct real estate, infrastructure, and private equity, to VP's portfolio. Velky and Monteo discuss the considerations involved in applying many of the common asset allocation techniques, such as MVO, to these asset classes. Before making any changes to the portfolio, Monteo asks Velky about his knowledge of risk budgeting. Velky makes the following statements:

Statement 1 An optimum risk budget minimizes total risk.

Statement 2 Risk budgeting decomposes total portfolio risk into its constituent parts.

Statement 3 An asset allocation is optimal from a risk-budgeting perspective when the ratio of excess return to marginal contribution to risk is different for all assets in the portfolio.

Monteo meets with a third client, Jayanta Chaterji, an individual investor. Monteo and Chaterji discuss mean–variance optimization. Chaterji expresses concern about using the output of MVOs for two reasons:

Criticism 1: The asset allocations are highly sensitive to changes in the model inputs.

Criticism 2: The asset allocations tend to be highly dispersed across all available asset classes.

Monteo and Chaterji also discuss other approaches to asset allocation. Chaterji tells Monteo that he understands the factor-based approach to asset allocation to have two key characteristics:

Characteristic 1 The factors commonly used in the factor-based approach generally have low correlations with the market and with each other.

Characteristic 2 The factors commonly used in the factor-based approach are typically different from the fundamental or structural factors used in multifactor models.

Monteo concludes the meeting with Chaterji after sharing his views on the factor-based approach.

9 Based on Exhibit 1 and the risk aversion coefficient, the preferred asset allocation for Perkins is:

A Asset Allocation A.

B Asset Allocation B.

C Asset Allocation C.

10 In their discussion of the asset classes that Velky is interested in adding to the VP portfolio, Monteo should tell Velky that:

A these asset classes can be readily diversified to eliminate idiosyncratic risk.

B indexes are available for these asset classes that do an outstanding job of representing the performance characteristics of the asset classes.

C the risk and return characteristics associated with actual investment vehicles for these asset classes are typically significantly different from the characteristics of the asset classes themselves.

11 Which of Velky's statements about risk budgeting is correct?

A Statement 1

B Statement 2

C Statement 3

12 Which of Chaterji's criticisms of MVO is/are valid?

A Only Criticism 1

B Only Criticism 2

C Both Criticism 1 and Criticism 2

13 Which of the characteristics put forth by Chaterji to describe the factor-based approach is/are correct?

A Only Characteristic 1

B Only Characteristic 2

C Both Characteristic 1 and Characteristic 2

SOLUTIONS

1 A is correct. The allocations in Exhibit 1 are most likely from an MVO model using historical data inputs. MVO tends to result in asset allocations that are concentrated in a subset of the available asset classes. The allocations in Exhibit 1 have heavy concentrations in four of the asset classes and no investment in the other four asset classes, and the weights differ greatly from global market weights. Compared to the use of historical inputs, the Black–Litterman and reverse-optimization models most likely would be less concentrated in a few asset classes and less distant from the global weights.

2 A is correct. Theoretically, higher-risk assets would warrant a narrower corridor because high-risk assets are more likely to stray from the desired strategic asset allocation. However, narrow corridors will likely result in more frequent rebalancing and increased transaction costs so in practice corridors width is often specified to be proportionally greater the higher the asset class's volatility. Thus, higher-risk assets should have a wider corridor to avoid frequent, costly rebalancing. Beade's other suggestions are not correct. Less liquid asset classes should have a wider, not narrower, corridor width. Less liquid assets should have a wider corridor to avoid frequent rebalancing. For taxable investors, transactions trigger capital gains in jurisdictions that tax them. For such investors, higher tax rates on capital gains should be associated with wider (not narrower) corridor widths.

3 A is correct. The original funding ratio is the market value of assets divided by the present value of liabilities. This plan's ratio is \$205 million/\$241 million = 0.8506. When the assets and liabilities both decrease by \$25 million, the funding ratio will decrease to \$180 million/\$216 million = 0.8333.

4 B is correct. The objective function expected value is $U_m^{LR} = E\left(R_{s,m}\right) - 0.005\lambda\sigma^2\left(R_{s,m}\right)$. λ is equal to 1.5, and the expected value of the objective function is shown in the rightmost column below.

Portfolio	$E(R_{s,m})$	$\sigma^2(R_{s,m})$	$U_m^{LR} = E(R_{s,m}) - 0.005(1.5)\sigma^2(R_{s,m})$
1	13.00	576	8.68
2	12.00	324	9.57
3	11.00	361	8.29

Portfolio 2 generates the highest value, or utility, in the objective function.

5 C is correct. The hedging/return-seeking portfolios approach is best for this client. Beade should construct two portfolios, one that includes riskless bonds that will pay off the fixed obligation in 10 years and the other a risky portfolio that earns a competitive risk-adjusted return. This approach is a simple two-step process of hedging the fixed obligation and then investing the balance of the assets in a return-seeking portfolio.

6 A is correct. Goal 1 requires a success rate of at least 95%, and Subportfolio A has the highest minimum expected return (2.05%) meeting this requirement. Goal 2 requires the highest minimum expected return that will be achieved 85% of the time. Subportfolio C meets this requirement (and has a minimum expected return of 3.26%).

7 C is correct. A risk parity asset allocation is based on the notion that each asset class should contribute equally to the total risk of the portfolio. Bonds have the lowest risk level and must contribute 25% of the portfolio's total risk, so bonds must be overweighted (greater than 25%). The equal contribution of each asset class is calculated as:

$$w_i \times \text{Cov}\left(r_i, r_p\right) = \frac{1}{n}\sigma_p^2$$

where

w_i = weight of asset i
$\text{Cov}(r_i, r_p)$ = covariance of asset i with the portfolio
n = number of assets
σ_p^2 = variance of the portfolio

In this example, there are four asset classes, and the variance of the total portfolio is assumed to be 25%; therefore, using a risk parity approach, the allocation to each asset class is expected to contribute (1/4 × 25%) = 6.25% of the total variance. Because bonds have the lowest covariance, they must have a higher relative weight to achieve the same contribution to risk as the other asset classes.

8 A is correct. Comment 1 is correct because the "120 minus your age" rule reduces the equity allocation as the client ages, while the 60/40 rule makes no such adjustment. Comments 2 and 3 are not correct. The Yale model emphasizes investing in alternative assets (such as hedge funds, private equity, and real estate) as opposed to investing in traditional asset classes (such as stock and bonds). The 1/*N* rule allocates an equal weight to each asset without regard to its investment characteristics, treating all assets as indistinguishable in terms of mean returns, volatility, and correlations.

9 C is correct. The risk aversion coefficient (λ) for Mary Perkins is 8. The utility of each asset allocation is calculated as follows:

Asset Allocation A:

$$U_A = 10.0\% - 0.005(8)(12\%)^2$$
$$= 4.24\%$$

Asset Allocation B:

$$U_B = 8.0\% - 0.005(8)(8\%)^2$$
$$= 5.44\%$$

Asset Allocation C:

$$U_C = 6.0\% - 0.005(8)(2\%)^2$$
$$= 5.84\%$$

Therefore, the preferred strategic allocation is Asset Allocation C, which generates the highest utility given Perkins's level of risk aversion.

10 C is correct. Less liquid asset classes—such as direct real estate, infrastructure, and private equity—represent unique challenges when applying many of the common asset allocation techniques. Common illiquid asset classes cannot be readily diversified to eliminate idiosyncratic risk, so representing overall asset class performance is problematic. Furthermore, there are far fewer indexes that attempt to represent aggregate performance for these less liquid asset classes than indexes of traditional highly liquid asset classes. Finally, the risk and return

characteristics associated with actual investment vehicles—such as direct real estate funds, infrastructure funds, and private equity funds—are typically significantly different from the characteristics of the asset classes themselves.

11 B is correct. The goal of risk budgeting is to maximize return per unit of risk. A risk budget identifies the total amount of risk and attributes risk to its constituent parts. An optimum risk budget allocates risk efficiently.

12 A is correct. One common criticism of MVO is that the model outputs, the asset allocations, tend to be highly sensitive to changes in the model. Another common criticism of MVO is that the resulting asset allocations tend to be highly concentrated in a subset of the available asset classes.

13 A is correct. The factors commonly used in the factor-based approach generally have low correlations with the market and with each other. This results from the fact that the factors typically represent what is referred to as a zero (dollar) investment or self-financing investment, in which the underperforming attribute is sold short to finance an offsetting long position in the better-performing attribute. Constructing factors in this manner removes most market exposure from the factors (because of the offsetting short and long positions); as a result, the factors generally have low correlations with the market and with one another. Also, the factors commonly used in the factor-based approach are typically similar to the fundamental or structural factors used in multifactor models.