Table 0: Estimated N for MC and CB

d	Ball Volume	Cube Volume	p	q	pq	tol	t-value	M	MC N	Unit N	Est Time (s)	Est Time (h)	CB K	N	Unit N	Est Time (s)	Est Time (h)
1	2.000	2	1.000	0.000	0.000	0.001	3.25	10	0.00E+00	1.00E+06	0	0	2,000	2.00E+03	1.00E+07	0	0
2	3.142	4	0.785	0.215	0.169	0.001	3.25	10	2.85E+06	1.00E+06	14	0	6,283	3.95E+07	1.00E+07	4	0
3	4.189	8	0.524	0.476	0.249	0.001	3.25	10	1.69E+07	1.00E+06	84	0	12,566	1.98E+12	1.00E+07	198,440	55
4	4.935	16	0.308	0.692	0.213	0.001	3.25	10	5.77E+07	1.00E+06	288	0	19,739	1.52E+17	1.00E+07	1.52E+10	4,217,124
5	5.264	32	0.164	0.836	0.137	0.001	3.25	10	1.49E+08	1.00E+06	743	0	26,319	1.26E+22	1.00E+07	1.26E+15	3.51E+11
6	5.168	64	0.081	0.919	0.074	0.001	3.25	10	3.21E+08	1.00E+06	1,606	0	31,006	8.89E+26	1.00E+07	8.89E+19	2.47E+16
7	4.725	128	0.037	0.963	0.036	0.001	3.25	10	6.15E+08	1.00E+06	3,076	1	33,073	4.33E+31	1.00E+07	4.33E+24	1.20E+21
8	4.059	256	0.016	0.984	0.016	0.001	3.25	10	1.08E+09	1.00E+06	5,400	2	32,470	1.24E+36	1.00E+07	1.24E+29	3.43E+25
9	3.299	512	0.006	0.994	0.006	0.001	3.25	10	1.77E+09	1.00E+06	8,862	2	29,687	1.79E+40	1.00E+07	1.79E+33	4.97E+29
10	2.550	1,024	0.002	0.998	0.002	0.001	3.25	10	2.75E+09	1.00E+06	13,757	4	25,502	1.16E+44	1.00E+07	1.16E+37	3.23E+33
11	1.884	2,048	0.001	0.999	0.001	0.001	3.25	10	4.07E+09	1.00E+06	20,360	6	20,725	3.03E+47	1.00E+07	3.03E+40	8.42E+36
12	1.335	4,096	3.26E-04	1.000	3.26E-04	0.001	3.25	10	5.77E+09	1.00E+06	28,875	8	16,023	2.86E+50	1.00E+07	2.86E+43	7.96E+39
13	0.911	8,192	1.11E-04	1.000	1.11E-04	0.001	3.25	10	7.88E+09	1.00E+06	39,393	11	11,838	8.97E+52	1.00E+07	8.97E+45	2.49E+42
14	0.599	16,384	3.66E-05	1.000	3.66E-05	0.001	3.25	10	1.04E+10	1.00E+06	51,851	14	8,390	8.56E+54	1.00E+07	8.56E+47	2.38E+44
15	0.381	32,768	1.16E-05	1.000	1.16E-05	0.001	3.25	10	1.32E+10	1.00E+06	66,010	18	5,722	2.31E+56	1.00E+07	2.31E+49	6.40E+45
16	0.235	65,536	3.59E-06	1.000	3.59E-06	0.001	3.25	10	1.63E+10	1.00E+06	81,451	23	3,765	1.63E+57	1.00E+07	1.63E+50	4.53E+46
17	0.141	131,072	1.08E-06	1.000	1.08E-06	0.001	3.25	10	1.95E+10	1.00E+06	97,590	27	2,397	2.84E+57	1.00E+07	2.84E+50	7.89E+46
18	0.082	262,144	3.13E-07	1.000	3.13E-07	0.001	3.25	10	2.27E+10	1.00E+06	113,727	32	1,479	1.14E+57	1.00E+07	1.14E+50	3.17E+46
19	0.047	524,288	8.89E-08	1.000	8.89E-08	0.001	3.25	10	2.58E+10	1.00E+06	129,091	36	886	9.99E+55	1.00E+07	9.99E+48	2.77E+45
20	0.026	1,048,576	2.46E-08	1.000	2.46E-08	0.001	3.25	10	2.86E+10	1.00E+06	142,914	40	516	1.80E+54	1.00E+07	1.80E+47	5.00E+43

Footnote:
- d: dimension
- p = Ball Volume / Cube Volume = Ball's Surface Area / Cube's Surface Area, q = 1 - p
- tol: tolerance based on 4 digits of precision
- t-value: based on alpha = 1%, degree of freedome = M - 1 = 9
- M: number of runs in MC
- N: number of sample points per run in MC and number of sample cubes in CB
- Unit N: how many operations a computer can handle per second
- Est Time (s): estimated run time in terms of seconds
- Est Time (h): estimated run time in terms of hours
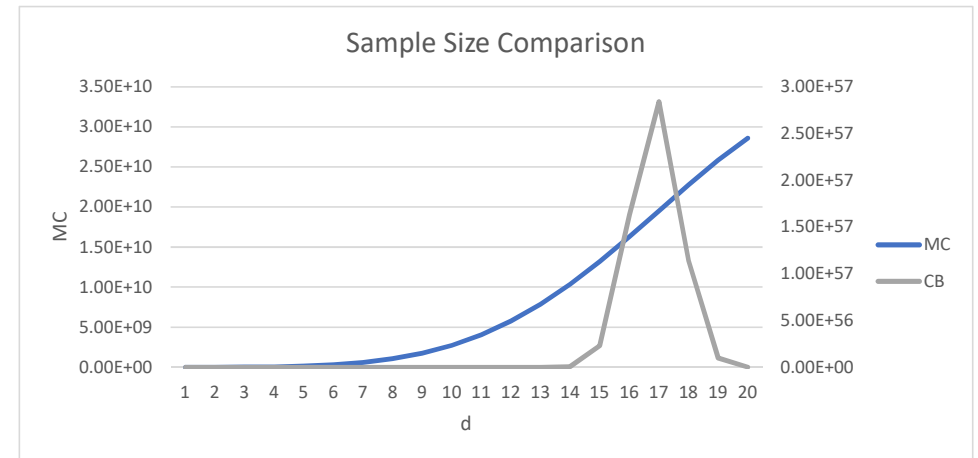- K: number of segments per dimension

Sample Size Comparison

Table 1: Part 1's Result

Precision	4	Value Comparison:			Accuracy Comparison:				Efficiency Comparison:			
alpha	1%	MC	CB	Diffs	MC	CB	Winner	Diffs	MC	CB	Winner	Diffs
d	Exact Answer	Answer	Answer		Accuracy	Accuracy			Run Time	Run Time		
1	2.000	2.000	2.000	0.00E+00	0.00E+00	0.00E+00	PAR	0.00E+00	0	0	CB	0
2	3.142	3.141	3.142	1.14E-04	1.14E-04	3.25E-08	CB	1.14E-04	2	2	CB	0
3	4.189	4.189	4.189	6.10E-06	6.15E-06	4.87E-08	CB	6.10E-06	36	64,913	MC	64,877
4	4.935	4.935	4.935	4.73E-04	4.91E-04	1.79E-05	CB	4.73E-04	84	70,605	MC	70,521
5	5.264	5.263	5.264	1.15E-03	4.72E-04	6.79E-04	MC	2.07E-04	2,907	73,591	MC	70,685
6	5.168	5.167	5.176	8.58E-03	5.82E-04	7.99E-03	MC	7.41E-03	3,095	79,862	MC	76,767
7	4.725	4.725	4.770	4.55E-02	1.47E-04	4.57E-02	MC	4.55E-02	24,836	85,949	MC	61,114
8	4.059	4.058	4.221	1.62E-01	3.40E-04	1.62E-01	MC	1.62E-01	18,360	97,780	MC	79,420

			MC's Result Details					
d	Exact Answer	Answer	Error	Lower Bound	Upper Bound	N	M	Run Time
1	2.000	2.000	0.000	2.000	2.000	1.00E+03	10	0
2	3.142	3.141	0.001	3.141	3.142	5.85E+06	10	2
3	4.189	4.189	0.001	4.188	4.189	6.34E+07	10	36
4	4.935	4.935	0.001	4.935	4.936	1.08E+08	10	84
5	5.264	5.263	0.001	5.262	5.264	2.37E+08	10	2,907
6	5.168	5.167	0.001	5.166	5.168	4.35E+08	10	3,095
7	4.725	4.725	0.001	4.724	4.726	1.14E+09	10	24,836
8	4.059	4.058	0.001	4.058	4.059	1.08E+09	10	18,360

Efficiency Comparison
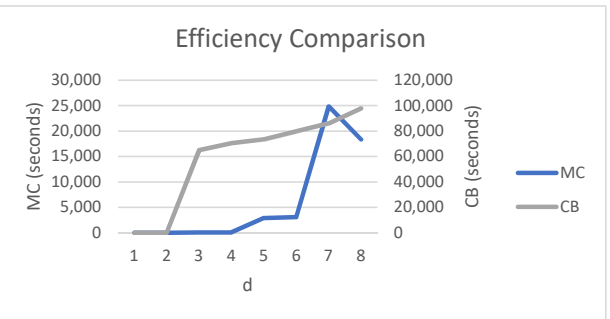
			CB's Result Details					
d	Exact Answer	Answer	Error	Lower Bound	Upper Bound	N	K	Run Time
1	2.000	2.000	0.000	2.000	2.000	1.00E+03	1,000	0
2	3.142	3.142	0.001	3.141	3.143	6.40E+07	8,000	2
3	4.189	4.189	0.002	4.187	4.191	1.00E+12	10,000	64,913
4	4.935	4.935	0.034	4.901	4.968	1.00E+12	1,000	70,605
5	5.264	5.264	0.196	5.071	5.462	1.00E+12	251	73,591
6	5.168	5.176	0.630	4.568	5.828	1.00E+12	100	79,862
7	4.725	4.770	1.389	3.498	6.277	1.00E+12	52	85,949
8	4.059	4.221	2.398	2.215	7.011	1.00E+12	32	97,780

Accuracy Comparison
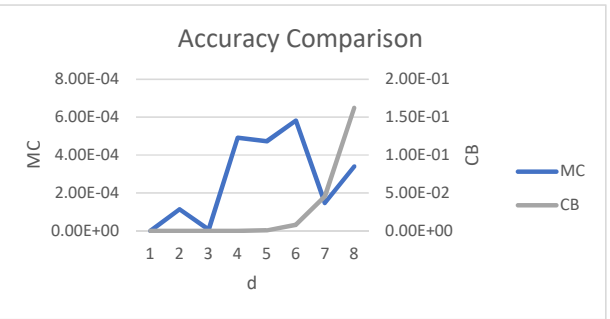
Footnote:
- accuracy: difference between the estimated answer and the exact answer
- PAR: have equal results
- run time: in terms of seconds

Table 2: Part 2's Result

When N = 1,000,000

		Value Comparison:			Accuracy Comparison:			
		MC	CB	Diffs	MC	CB	Winner	Diffs
d	Exact Answer	Answer	Answer		Accuracy	Accuracy		
1	2	2	2	0.00E+00	0.00E+00	0.00E+00	PAR	0.00E+00
2	3.141593	3.141552	3.141591	3.86E-05	4.07E-05	2.06E-06	CB	3.86E-05
3	4.18879	4.17936	4.189226	9.87E-03	9.43E-03	4.36E-04	CB	8.99E-03
4	4.934802	4.943552	4.952691	9.14E-03	8.75E-03	1.79E-02	MC	9.14E-03
5	5.263789	5.233344	5.435351	2.02E-01	3.04E-02	1.72E-01	MC	1.41E-01
6	5.167713	5.122432	5.984077	8.62E-01	4.53E-02	8.16E-01	MC	7.71E-01
7	4.724766	4.695552	6.737115	2.04E+00	2.92E-02	2.01E+00	MC	1.98E+00
8	4.058712	4.1344	9.286454	5.15E+00	7.57E-02	5.23E+00	MC	5.15E+00
9	3.298509	3.178496	10.056667	6.88E+00	1.20E-01	6.76E+00	MC	6.64E+00
10	2.550164	2.527232	26.905244	2.44E+01	2.29E-02	2.44E+01	MC	2.43E+01
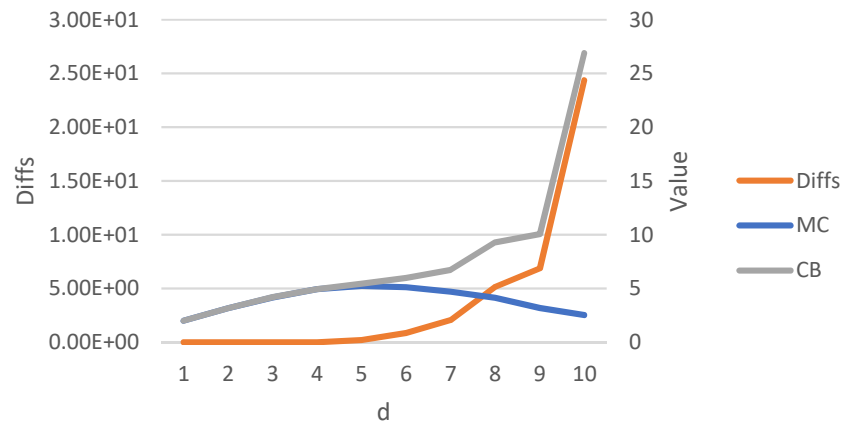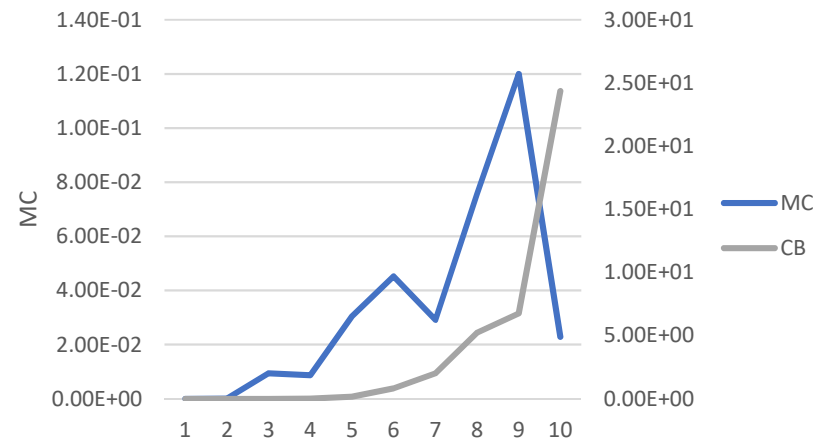
Table 3: Part 3's Result

		When N =	100		Min:	1.81E-02			When N =	10,000		Min:	1.98E-04		
		Value Comparison:			Accuracy Comparison:				Value Comparison:			Accuracy Comparison:			
		MC	CB	Diffs	MC	CB	Winner	Diffs	MC	CB	Diffs	MC	CB	Winner	Diffs
d	Exact Answer	Answer	Answer		Accuracy	Accuracy			Answer	Answer		Accuracy	Accuracy		
2	3.141593	3.36	3.123506	2.36E-01	2.18E-01	1.81E-02	CB	2.00E-01	3.12	3.141395	2.14E-02	2.16E-02	1.98E-04	CB	2.14E-02
3	3.141593	3.72	3.195135	5.25E-01	5.78E-01	5.35E-02	CB	5.25E-01	3.1632	3.148214	1.50E-02	2.16E-02	6.62E-03	CB	1.50E-02
4	3.141593	3.2	3.467915	2.68E-01	5.84E-02	3.26E-01	MC	2.68E-01	3.098387	3.194209	9.58E-02	4.32E-02	5.26E-02	MC	9.41E-03
5	3.141593	2.683282	3.883025	1.20E+00	4.58E-01	7.41E-01	MC	2.83E-01	3.043682	3.348677	3.05E-01	9.79E-02	2.07E-01	MC	1.09E-01
6	3.141593	2.485786	3.992412	1.51E+00	6.56E-01	8.51E-01	MC	1.95E-01	3.183246	3.547721	3.64E-01	4.17E-02	4.06E-01	MC	3.64E-01
7	3.141593	2.561158	6.821974	4.26E+00	5.80E-01	3.68E+00	MC	3.10E+00	3.149735	4.484429	1.33E+00	8.14E-03	1.34E+00	MC	1.33E+00
8	3.141593	3.329433	6.826947	3.50E+00	1.88E-01	3.69E+00	MC	3.50E+00	3.066927	4.52593	1.46E+00	7.47E-02	1.38E+00	MC	1.31E+00
9	3.141593	0	8.425732	8.43E+00	3.14E+00	5.28E+00	MC	2.14E+00	3.060166	5.040667	1.98E+00	8.14E-02	1.90E+00	MC	1.82E+00
10	3.141593	0	8.277444	8.28E+00	3.14E+00	5.14E+00	MC	1.99E+00	3.343428	5.03273	1.69E+00	2.02E-01	1.89E+00	MC	1.69E+00

		When N =	1,000,000		Min:	2.06E-06			When N =	100,000,000		Min:	2.08E-08		
		Value Comparison:			Accuracy Comparison:				Value Comparison:			Accuracy Comparison:			
		MC	CB	Diffs	MC	CB	Winner	Diffs	MC	CB	Diffs	MC	CB	Winner	Diffs
d	Exact Answer	Answer	Answer		Accuracy	Accuracy			Answer	Answer		Accuracy	Accuracy		
2	3.141593	3.14232	3.141591	7.29E-04	7.27E-04	2.06E-06	CB	7.25E-04	3.142033	3.141593	4.40E-04	4.40E-04	2.08E-08	CB	4.40E-04
3	3.141593	3.142956	3.141919	1.04E-03	1.36E-03	3.27E-04	CB	1.04E-03	3.141805	3.141609	1.96E-04	2.12E-04	1.61E-05	CB	1.96E-04
4	3.141593	3.146584	3.147282	6.97E-04	4.99E-03	5.69E-03	MC	6.97E-04	3.14217	3.142166	4.30E-06	5.78E-04	5.73E-04	CB	4.30E-06
5	3.141593	3.139898	3.192379	5.25E-02	1.69E-03	5.08E-02	MC	4.91E-02	3.140954	3.149659	8.71E-03	6.39E-04	8.07E-03	MC	7.43E-03
6	3.141593	3.144734	3.299004	1.54E-01	3.14E-03	1.57E-01	MC	1.54E-01	3.142518	3.174913	3.24E-02	9.26E-04	3.33E-02	MC	3.24E-02
7	3.141593	3.136983	3.536018	3.99E-01	4.61E-03	3.94E-01	MC	3.90E-01	3.141188	3.28E+00	1.36E-01	4.04E-04	1.36E-01	MC	1.36E-01
8	3.141593	3.144741	3.863806	7.19E-01	3.15E-03	7.22E-01	MC	7.19E-01	3.141015	3.434285	2.93E-01	5.78E-04	2.93E-01	MC	2.92E-01
9	3.141593	3.169692	4.151299	9.82E-01	2.81E-02	1.01E+00	MC	9.82E-01	3.141154	3.757372	6.16E-01	4.39E-04	6.16E-01	MC	6.15E-01
10	3.141593	3.123112	5.03273	1.91E+00	1.85E-02	1.89E+00	MC	1.87E+00	3.143806	3.793518	6.50E-01	2.21E-03	6.52E-01	MC	6.50E-01

Footnote:
- min: the best accuracy given a fixed N

When N = 100 | When N = 10000 | When N = 1000000 | When N = 100000000