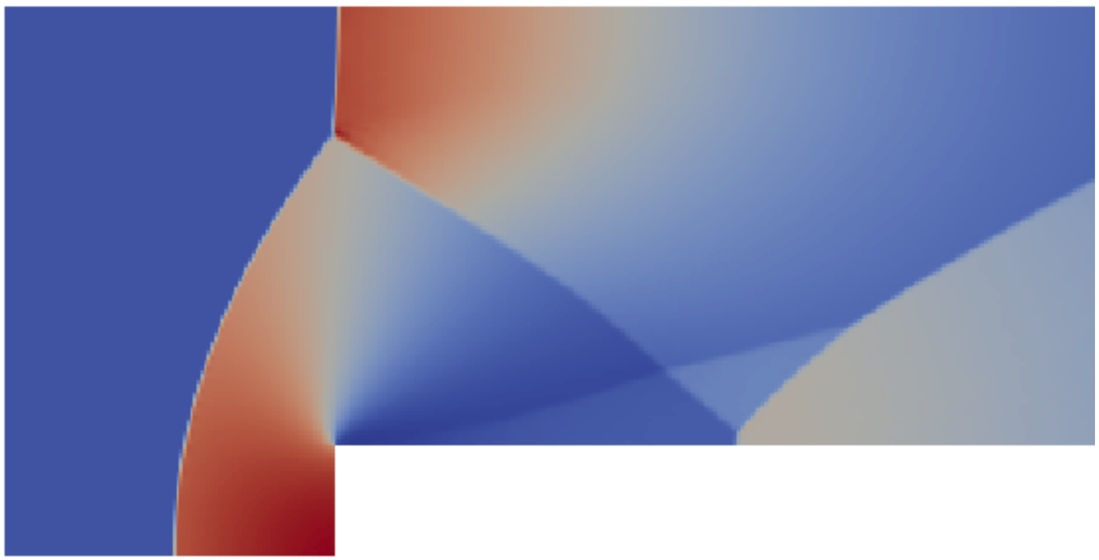
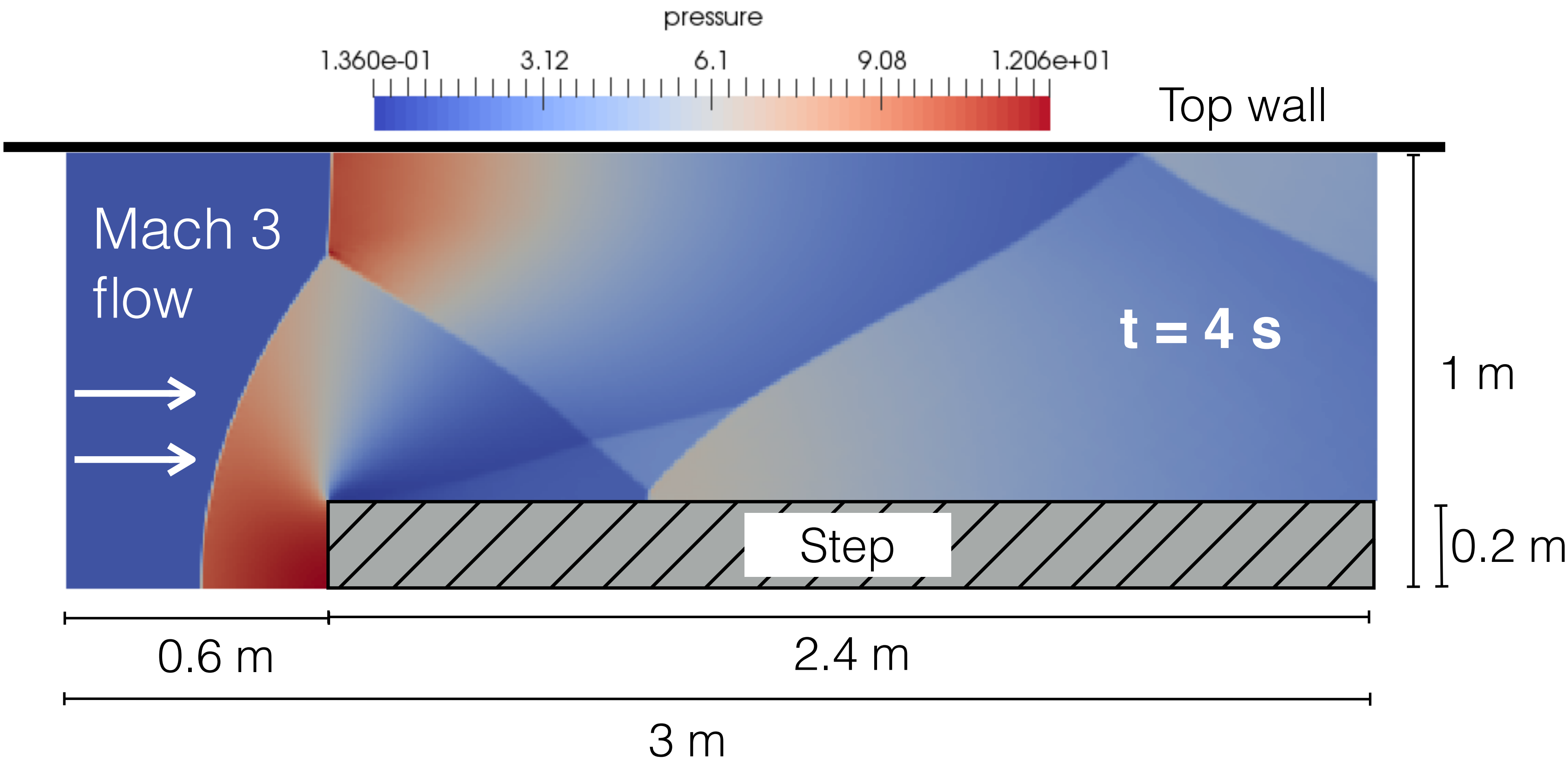
The “Mach 3 wind tunnel with a step”



- A canonical, **two-dimensional**, compressible flow
- In principle, fully characterized by the geometry and the approach Mach number.

$$\mathrm{Ma} = U/a$$

Overview of the flow



Inlet conditions

$$p = 1$$

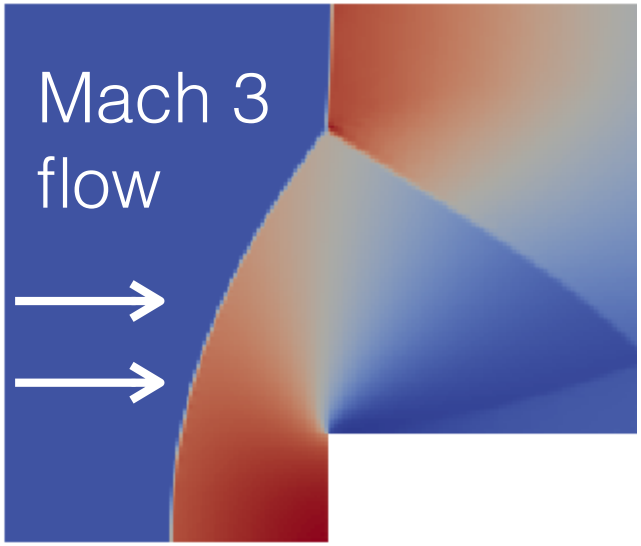
$$\rho = \gamma = 1.4$$

$$(u, v) = (U, 0)$$

$$a = (\gamma p/\rho)^{1/2} = 1$$

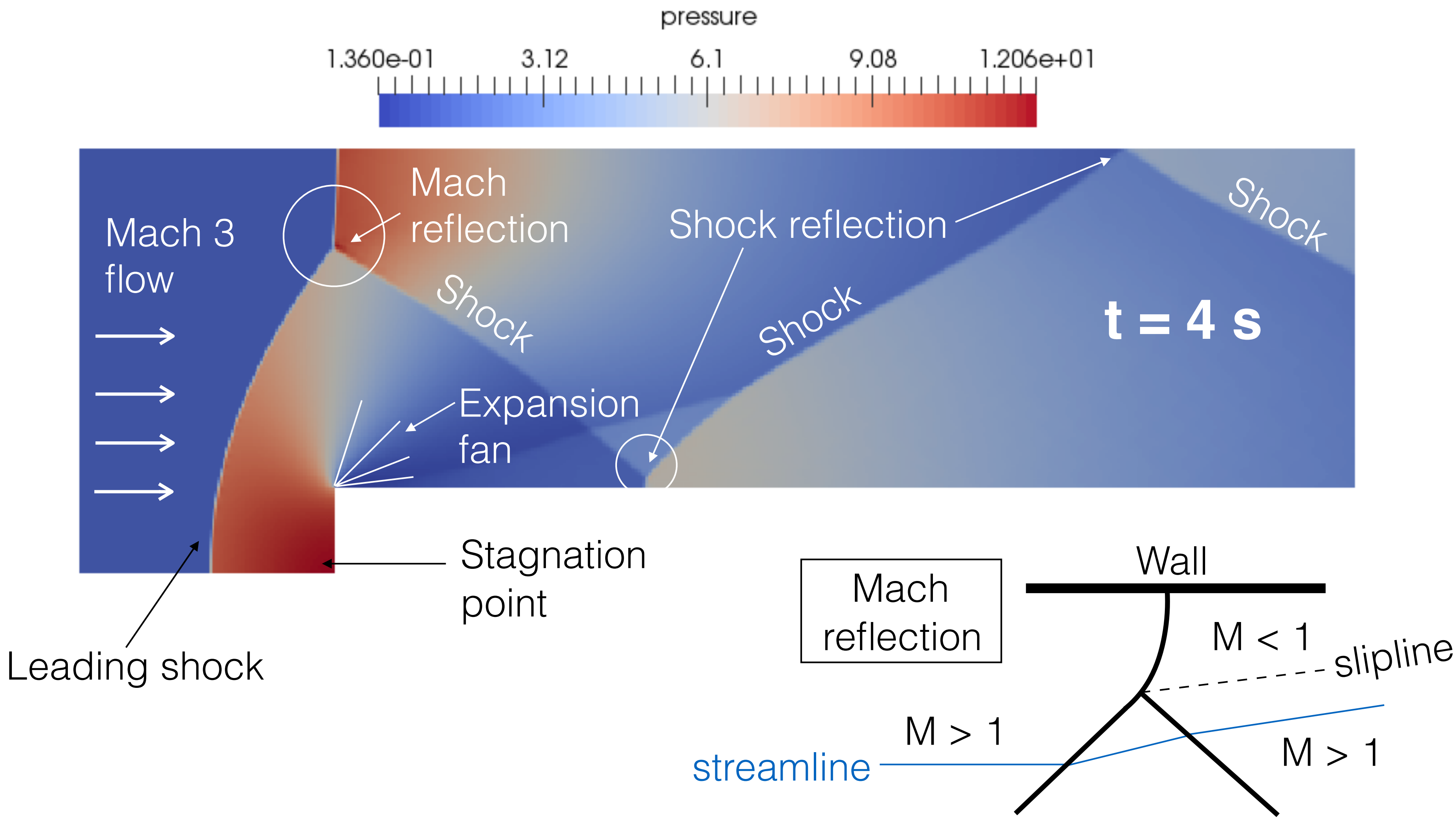
$$M = U/a = 3$$

$$U = 3$$

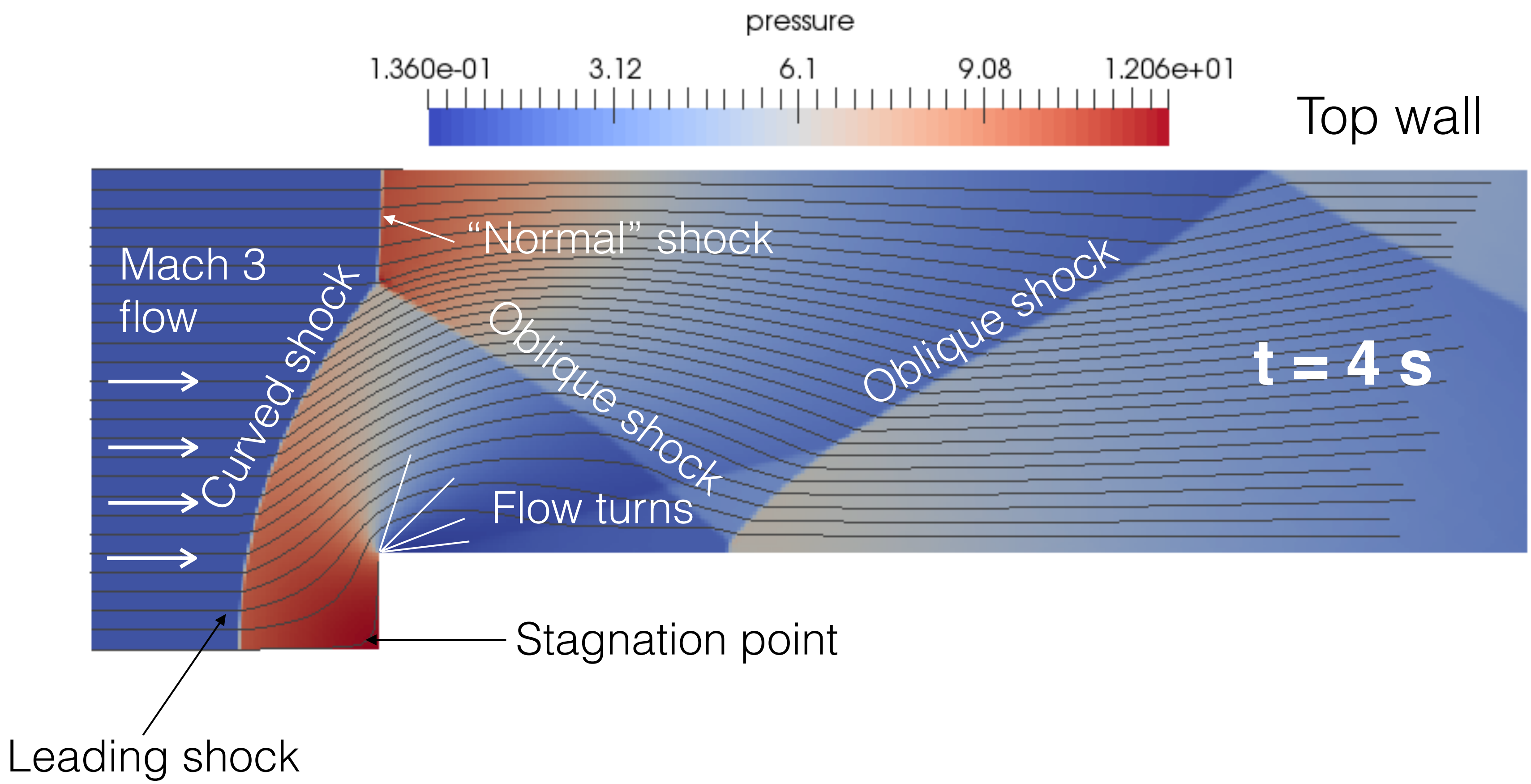


All in base SI units: m, s, Pa, K, kg/m3, etc.

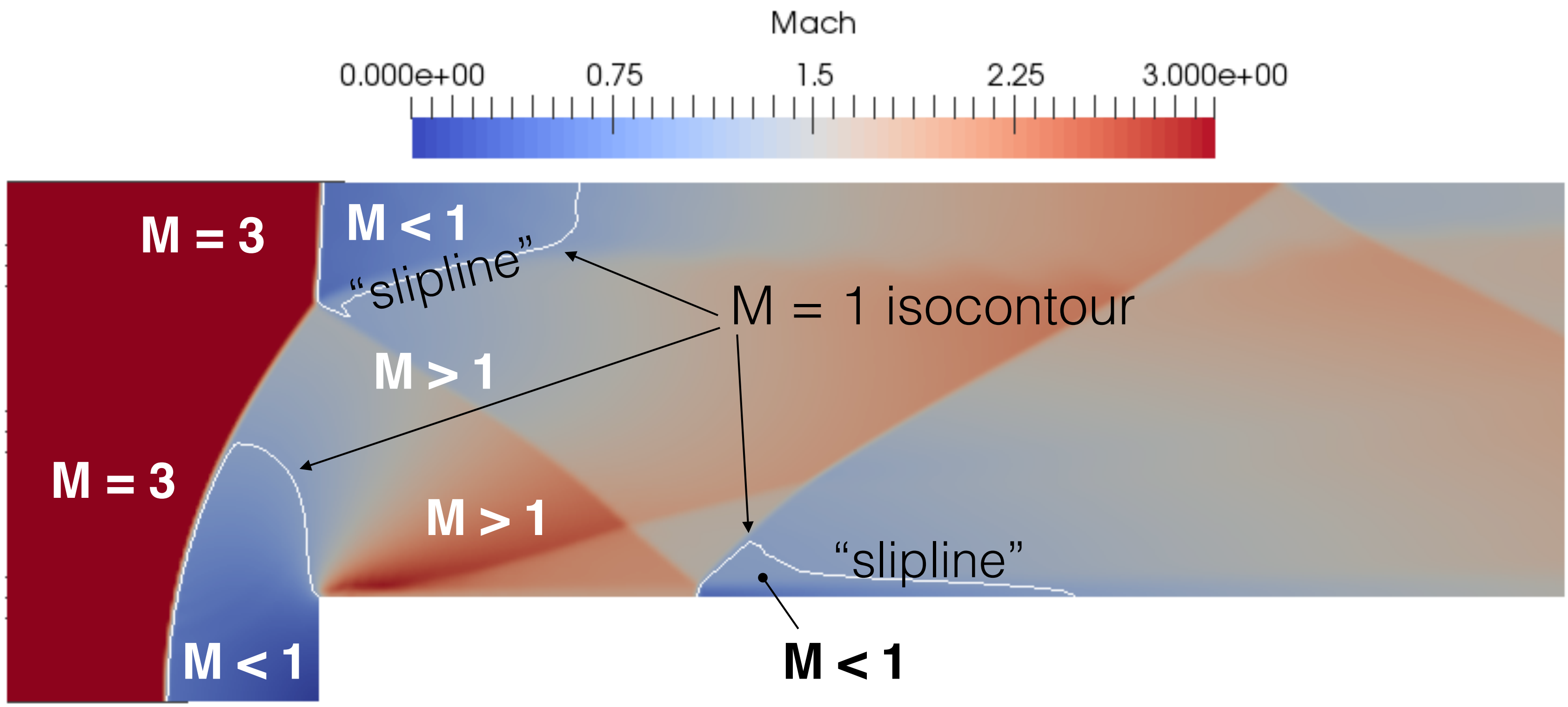
Some details



Flow streamlines



Mach nr

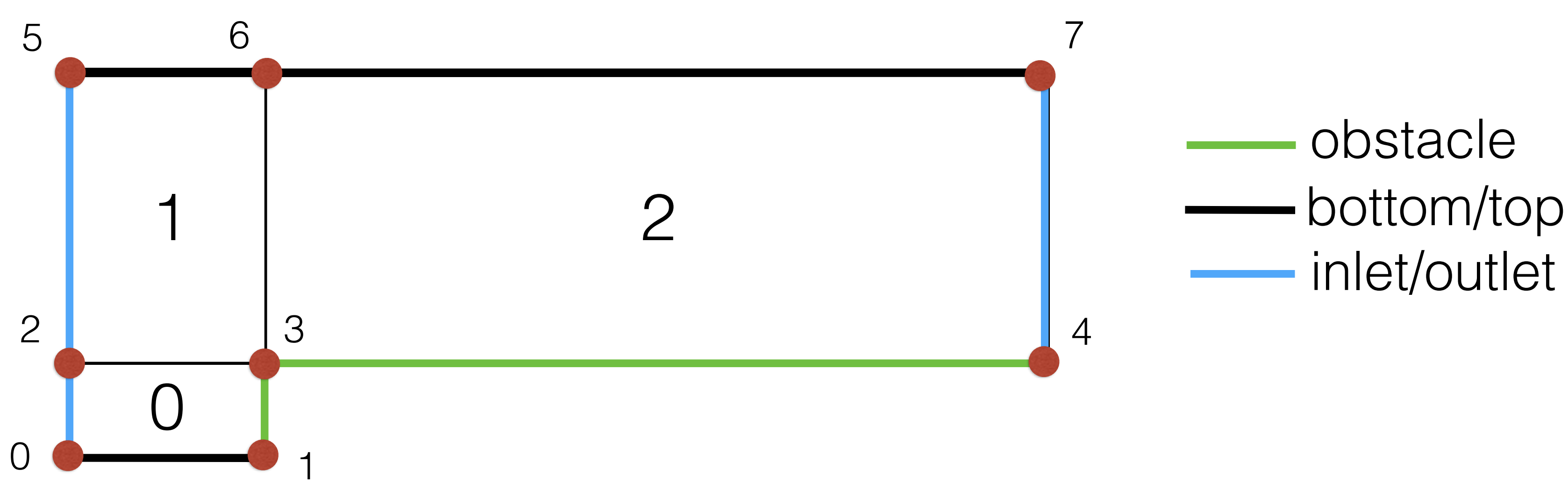


$$M = |\mathbf{u}|/a = |\mathbf{u}|/\sqrt{\gamma p/\rho}$$

Movie

- See Movie online

Meshing strategies



- 16 points (8 x 2) and 3 blocks are sufficient
- Uniform meshes are required, since shocks are “everywhere”
- Make sure to identify faces for boundary conditions

Solver

- You will be using the `rhoCentralFoam` solver available in either OpenFOAM 2.4.0 or 2.3.0
- I’ve posted a discussion on how to install OpenFOAM 2.3.0 on TACC’s Lonestar5 (`ls5.tacc.utexas.edu`), a Cray XC40 supercomputer.
- Solver implements “fast” high-order difference methods for hyperbolic conservation laws, i.e. the **Euler equations**

Field variables

- Velocity vector field, $U = (u,v)$
- Temperature scalar field, T
- Pressure scalar field, p

“Normalized fluid”

- An inviscid fluid with “desirable properties”
- Ratio of specific heats: $\gamma = 7/5 = 1.4$
- Temperature is 1 K for sound speed 1 m/s:

$$a = \sqrt{\gamma p/\rho} = \sqrt{\gamma R T} = \sqrt{\gamma R} = 1 \rightarrow \gamma R = 1$$

$$\gamma \times 8314\,(\mathrm{J/kmol-K}))/W\ (\mathrm{kg/kmol}) = 1$$

$$\rightarrow W = 8314 \times \gamma = 11640\,\mathrm{g/mol}$$

$$\rightarrow c_p = R/(1-1/\gamma) = 2.5\,(\mathrm{J/kg-K})$$

Inlet conditions

- Exercise caution due to the “normalized fluid” and make sure all conditions are consistent to provide a Mach 3 inlet flow

- U = (3, 0) m/s
  T = 1 K
  p = 1 Pa

- Since the sound speed at the inlet is a = 1 m/s,
  this is a Mach = 3 flow.

- Also notice that density at the inlet is $\rho = \gamma p / a^2 = \gamma$

Discretization schemes

- More information is available in Kurganov & Tadmor, JCP 2000