DYNAMIC SOCIAL NETWORKS BASED ON MOVEMENT

Supplement A

For all fits, we acquire 200,000 iterations on a single computing node, discard the first 100,000 draws as burnin, and thin by a factor of 5 to yield a sample of size 20,000.

We employ diffuse priors for most parameters in both the simulation and application.

1 Priors

Prior distributions are selected to be conjugate where possible.

$$\sigma^2 \sim \mathrm{IG}(a_\sigma, b_\sigma) \tag{1}$$

$$[\alpha] \propto (1+\alpha)^{\alpha_\alpha - 1}(1-\alpha)^{\beta_\alpha - 1}\mathbf{1}_{(-1,1)}(\alpha) \tag{2}$$

This is the kernel for the density of a random variable $X = 2Y - 1$ where $Y \sim \mathrm{Beta}(\alpha_\alpha, \beta_\alpha)$. It is the same shape in a sense as the Beta, but spread over the support $[-1, 1]$, so that we allow for the possibility of anti-alignment. See Section 1.1 for details.

$$\beta \sim \mathcal{N}(0, \sigma_\beta^2) \tag{3}$$

$$c \sim \mathrm{IG}(a_c, b_c) \tag{4}$$

$$\phi \sim \mathrm{Beta}(\alpha_\phi, \beta_\phi) \tag{5}$$

$$p_1 \sim \mathrm{Beta}(\alpha_{p_1}, \beta_{p_1}) \tag{6}$$

1.1 Prior for the strength of the alignment effect

The relevant support for α is the real interval $(-1, 1)$. Negative values for α correspond to the case when connected individuals tend to move in parallel, but opposite, directions. We generally expect α to be positive, but to allow for the possibility of anti-alignment behavior we specify a prior on the full support. To specify a flexible family of prior distributions with support $(-1, 1)$, we shift and scale a Beta distribution to align with $(-1, 1)$. In fitting our model with an MCMC algorithm, α is sampled with a Metropolis-Hastings step, therefore we only need the kernel of the full-conditional

$$[\alpha|\alpha_\alpha, \beta_\alpha] \propto (1+\alpha)^{\alpha_\alpha - 1}(1-\alpha)^{\beta_\alpha - 1} I_{(-1,1)}(\alpha). \tag{7}$$

2 Full conditionals

Define the following variables as:

$$n \equiv \text{number of individuals in study population} \tag{8}$$

$$T \equiv \text{number of discrete time steps in mean position process} \tag{9}$$

It is convenient to also define

$$\mathbf{K}(t) \equiv (\mathbf{W}_+^c(t) - \alpha \mathbf{W}(t)) \otimes \mathbf{I}_2 \tag{10}$$

so that

$$\mathbf{Q}(t) = \sigma^{-2}\mathbf{K}(t). \tag{11}$$

2.1 σ: variability around mean step process

Gibbs step.

$$\sigma^2 | \cdot \sim \mathrm{IG}(a^*, b^*) \tag{12}$$

$$a^* \equiv a_\sigma + nT \tag{13}$$

$$b^* \equiv b_\sigma + \frac{1}{2} \sum_{t=2}^{T} \left(\boldsymbol{\mu}(t) - \boldsymbol{\mu}(t-1) - \beta \tilde{\boldsymbol{\mu}}(t-1)\right)' \mathbf{K}(t) \left(\boldsymbol{\mu}(t) - \boldsymbol{\mu}(t-1) - \beta \tilde{\boldsymbol{\mu}}(t-1)\right) \tag{14}$$

2.2 W: network

Gibbs step.

$$
\begin{align}
\mathbb{P}(w_{ij}(1)=1|\cdot) &\propto [\boldsymbol{\mu}_i(2)|\ldots,w_{ij}(1)=1]\left[\boldsymbol{\mu}_j(2)|\ldots,w_{ij}(1)=1\right] \tag{15}\\
&\times p_{1|1}^{w_{ij}(2)}p_{0|1}^{1-w_{ij}(2)}p_1 \tag{16}\\
\mathbb{P}(w_{ij}(1)=0|\cdot) &\propto [\boldsymbol{\mu}_i(2)|\ldots,w_{ij}(1)=0]\left[\boldsymbol{\mu}_j(2)|\ldots,w_{ij}(1)=0\right] \tag{17}\\
&\times p_{1|0}^{w_{ij}(2)}p_{0|0}^{1-w_{ij}(2)}(1-p_1) \tag{18}\\
\tag{19}
\end{align}
$$

$$
\begin{align}
\mathbb{P}(w_{ij}(t)=1|\cdot) &\propto [\boldsymbol{\mu}_i(t)|\ldots,w_{ij}(t)=1]\left[\boldsymbol{\mu}_j(t)|\ldots,w_{ij}(t)=1\right] \tag{20}\\
&\times [\boldsymbol{\mu}_i(t+1)|\ldots,w_{ij}(t)=1]\left[\boldsymbol{\mu}_j(t+1)|\ldots,w_{ij}(t)=1\right] \tag{21}\\
&\times p_{1|1}^{w_{ij}(t+1)}p_{0|1}^{1-w_{ij}(t+1)}p_{1|0}^{1-w_{ij}(t-1)}p_{1|1}^{w_{ij}(t-1)} \tag{22}\\
\mathbb{P}(w_{ij}(t)=0|\cdot) &\propto [\boldsymbol{\mu}_i(t)|\ldots,w_{ij}(t)=0]\left[\boldsymbol{\mu}_j(t)|\ldots,w_{ij}(t)=0\right] \tag{23}\\
&\times [\boldsymbol{\mu}_i(t+1)|\ldots,w_{ij}(t)=0]\left[\boldsymbol{\mu}_j(t+1)|\ldots,w_{ij}(t)=0\right] \tag{24}\\
&\times p_{1|0}^{w_{ij}(t+1)}p_{0|0}^{1-w_{ij}(t+1)}p_{0|1}^{w_{ij}(t-1)}p_{0|0}^{1-w_{ij}(t-1)} \tag{25}\\
\tag{26}
\end{align}
$$

$$
\begin{align}
\mathbb{P}(w_{ij}(T)=1|\cdot) &\propto [\boldsymbol{\mu}_i(T)|\ldots,w_{ij}(T)=1]\left[\boldsymbol{\mu}_j(T)|\ldots,w_{ij}(T)=1\right] \tag{27}\\
&\times p_{1|0}^{1-w_{ij}(T-1)}p_{1|1}^{w_{ij}(T-1)} \tag{28}\\
\mathbb{P}(w_{ij}(T)=0|\cdot) &\propto [\boldsymbol{\mu}_i(T)|\ldots,w_{ij}(T)=0]\left[\boldsymbol{\mu}_j(T)|\ldots,w_{ij}(T)=0\right] \tag{29}\\
&\times p_{0|1}^{w_{ij}(T-1)}p_{0|0}^{1-w_{ij}(T-1)} \tag{30}
\end{align}
$$

2.3 α: alignment/anti-alignment

M-H step. Kernel of full conditional density is

$$[\alpha|\cdot] \propto \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\sum_{t=2}^{T} w_{i+}(t)\left(\boldsymbol{h}_i(t) - \alpha\overline{\boldsymbol{h}}_i(t)\right)'\left(\boldsymbol{h}_i(t) - \alpha\overline{\boldsymbol{h}}_i(t)\right)\right\} \tag{31}$$

$$\times (1+\alpha)^{\alpha_\alpha - 1}(1-\alpha)^{\beta_\alpha - 1}\mathbf{1}_{(-1,1)}(\alpha) \tag{32}$$

where

$$\boldsymbol{h}_i(t) \equiv \boldsymbol{\mu}_j(t) - \boldsymbol{\mu}_j(t-1) - \beta\tilde{\boldsymbol{\mu}}_j(t-1) \tag{33}$$

$$\overline{\boldsymbol{h}}_i(t) \equiv \sum_{j \neq i} \frac{w_{ij}(t)}{w^c_{i+}(t)}\boldsymbol{h}_j(t) \tag{34}$$

and $\overline{\boldsymbol{h}}_i(t) = 0$ if animal i is completely unconnected at time t. Proposal distribution is normal:

$$\mathcal{N}\left(\alpha_{\text{iter}-1}, \sigma^2_{\alpha\text{-tune}}\right) \tag{35}$$

2.4 β: attraction/repulsion

Gibbs step.

$$\beta|\cdot \sim \mathcal{N}(\mu^*_\beta, {\sigma^*_\beta}^2) \tag{36}$$

$${\sigma^*_\beta}^2 \equiv \left(\sum_{t=2}^{T}\tilde{\boldsymbol{\mu}}(t-1)'\mathbf{Q}(t)\tilde{\boldsymbol{\mu}}(t-1) + \frac{1}{\sigma^2_\beta}\right)^{-1} \tag{37}$$

$$\mu^*_\beta \equiv {\sigma^*_\beta}^2\left(\sum_{t=2}^{T}\tilde{\boldsymbol{\mu}}(t-1)'\mathbf{Q}(t)\left(\boldsymbol{\mu}(t) - \boldsymbol{\mu}(t-1)\right) + \mu_\beta\right) \tag{38}$$

2.5 ϕ: network stability

M-H step. Kernel of full conditional density is

$$[\phi|\cdot] \propto (p_{0|0})^{g_{0|0}}(p_{1|0})^{g_{1|0}}(p_{0|1})^{g_{0|1}}(p_{1|1})^{g_{1|1}}(\phi)^{\alpha_\phi-1}(1-\phi)^{\beta_\phi-1} \tag{39}$$

$$g_{0|0} \equiv \sum_{i<j}\sum_{t=2}^{T}(1-w_{ij}(t-1))(1-w_{ij}(t)) \tag{40}$$

$$g_{1|0} \equiv \sum_{i<j}\sum_{t=2}^{T}(1-w_{ij}(t-1))w_{ij}(t) \tag{41}$$

$$g_{0|1} \equiv \sum_{i<j}\sum_{t=2}^{T}w_{ij}(t-1)(1-w_{ij}(t)) \tag{42}$$

$$g_{1|1} \equiv \sum_{i<j}\sum_{t=2}^{T}w_{ij}(t-1)w_{ij}(t) \tag{43}$$

with proposals coming from a Beta distribution centered on ϕ_{iter-1}:

$$\text{Beta}\left(\frac{\beta_{\phi\text{-tune}}\phi_{\text{iter}-1}}{1-\phi_{\text{iter}-1}}, \beta_{\phi\text{-tune}}\right) \tag{44}$$

2.6 p_1: network density

M-H step. Kernel of full conditional density is

$$[p_1|\cdot] \propto (p_1)^{\sum_{i<j} w_{ij}(1)}(1-p_1)^{\sum_{i<j}(1-w_{ij}(1))}(p_{0|0})^{g_{0|0}}(p_{1|0})^{g_{1|0}}(p_{0|1})^{g_{0|1}}(p_{1|1})^{g_{1|1}}(p_1)^{\alpha_{p_1}-1}(1-p_1)^{\beta_{p_1}-1} \tag{45}$$

with proposals coming from a Beta distribution centered on $p_{1_{iter-1}}$:

$$\text{Beta}\left(\frac{\beta_{p_1\text{-tune}}p_{1_{\text{iter}-1}}}{1-p_{1_{\text{iter}-1}}}, \beta_{p_1\text{-tune}}\right) \tag{46}$$

2.7 c: precision for unconnected animals

Gibbs step.

$$c|\cdot \sim \mathrm{IG}\left(a_c + \sum_{i=1}^{n}\sum_{t=2}^{T}\mathbf{1}_{\{w_{i+}(t)=0\}},\ b_c + \frac{1}{2}\sum_{i=1}^{n}\sum_{t=2}^{T}\mathbf{1}_{\{w_{i+}(t)=0\}}\boldsymbol{h}_i(t)'\boldsymbol{h}_i(t)\right) \tag{47}$$

3 Hyper-/tuning-parameters

parameter	hyper parameters		tuning
α	$\alpha_\alpha = 1$	$\beta_\alpha = 1$	$\sigma^2_{\alpha-tune} = 0.1^2$
β	$\mu_\beta = 0$	$\sigma^2_\beta = 1000$	conjugate
p_1	$\alpha_{p_1} = 1$	$\beta_{p_1} = 1$	$\beta_{p_1-tune} = 24$
ϕ	$\alpha_\phi = 100$	$\beta_\phi = \frac{100}{9}$	$\beta_{\phi-tune} = 7$
c	$a_c = 3.5$	$b_c = 1.5$	conjugate
σ	$a_\sigma = 0.1$	$b_\sigma = 0.001$	conjugate

Table 1: Simulation

parameter	hyper parameters		tuning
α	$\alpha_\alpha = 1$	$\beta_\alpha = 1$	$\sigma^2_{\alpha-tune} = 0.01^2$
β	$\mu_\beta = 0$	$\sigma^2_\beta = 1000$	conjugate
p_1	$\alpha_{p_1} = 1$	$\beta_{p_1} = 1$	$\beta_{p_1-tune} = 10$
ϕ	$\alpha_\phi = 100$	$\beta_\phi = \frac{100}{9}$	$\beta_{\phi-tune} = 5$
c	$a_c = 3.5$	$b_c = 1.5$	conjugate
σ	$a_\sigma = 0.1$	$b_\sigma = 0.001$	conjugate

Table 2: Killer whales