Henry Shi
AST 390 Final Project
Spring 2020

The Effect of Planet X on Kuiper Belt Objects

Motivation

In 2014, astronomers hypothesized the existence of a possible “Planet 9” far beyond the orbit of Neptune. Their hypothesis was based on an unusual property of the perihelia of Kuiper Belt objects with semimajor axes between 150AU and 250AU. Namely, the perihelia were clustered about ω=318 degrees, which could be explained by the existence of a massive gas-giant planet on a wide, highly inclined orbit with a perihelion argument in the opposite direction.

The possible existence of a massive planet in the far reaches of the solar system has long fascinated me. The possibility of life there is limited, but if its existence is confirmed then it could offer insight into the formation of the solar system as well as the properties of the neighborhood around our Sun. With better measurements of its orbital and physical properties, scientist could determine whether Planet 9 formed around our Sun or was captured from interstellar space.

Through this computational project I hope to evaluate the evidence for the existence of Planet 9 by simulating its hypothetical interactions with the outer Solar System, in particular the Kuiper Belt. The current evidence for the planet’s existence is tenuous. If the perihelia clustering is observed through repeated simulations with different properties for Planet 9, then the case for Planet 9 is strengthened and observations of Kuiper Belt objects’ orbital clustering can possibly be used to infer such a planet.

Methodology

This project uses the REBOUND N-body simulation code. I began by setting up an orbital system with the Sun in the center, orbited by the four gas giants: Jupiter, Saturn, Uranus, and Neptune. I added in the well-known Kuiper Belt objects of Pluto, Eris, and Sedna. I preserved all the physical and orbital properties of these objects. I neglected the terrestrial inner planets and asteroid belt due to their close-in orbits and small mass.

After adding the actual solar system objects, I generated the Kuiper belt as 100 objects with a total mass of $1.97 * 10^{-2}$ Earth masses. Therefore, each object has an average mass of approximately $5.916 * 10^{-10}$. Each object has a randomly generated semimajor axis between 30AU and 50AU, and a randomly generated eccentricity between 0.05 and 0.15. Each object has a randomly generated inclination up to 5 degrees. I assumed a radius of 0 for each object.

I generated 100 random scattered disc objects, each with semimajor axis between 150AU and 550AU. The anomalous clustering of orbital perihelia has been observed for scattered disc objects in 150AU < a < 550AU, so this distance will be investigated. The objects of interest have Each object has a random eccentricity between 0 and 0.8. With the minimum possible semimajor axis of 150 AU, an eccentricity of 0.8 would result in a perihelion distance of (150AU)(1-0.8) = 30AU, which is the distance of Neptune from the Sun. Since all objects of interest are just outside the orbit of Neptune, the eccentricity of any given object is capped at 0.8. The scattered disc objects have orbital inclinations up to 40 degrees. Their masses are assumed to be negligible so their masses and radii are each taken to be zero.

In accordance with Batygin & Brown (1), I set up the properties of Planet 9 as follows: a=700 AU, e= 0.6, m= $10M_\oplus$, and i= 30 deg and ω = 150 deg. In the control setup, I did not add planet 9. Otherwise, both systems had identical properties.

I set up the following systems:

System	Experimental setup	Control setup
Outer planets + Kuiper Belt + Scattered Disc	Planet 9 included	Planet 9 excluded
Outer planets only	Planet 9 included	Planet 9 excluded
Kuiper Belt only	Planet 9 included	Planet 9 excluded
Scattered Disc only	Planet 9 included	Planet 9 excluded

For each system, I integrated it forward for 10,000,000 years, simulating the system on a 10-million-year timescale. After the integration, I plotted graphs of the system to get a pictorial, qualitative view of the system. I tried to observe qualitative differences in the two post-integration systems (the experimental system and the control system).

From the objects in the systems, I also created arrays of orbital properties: semimajor axis a, eccentricity e, inclination i, and perihelion parameter ω. Using these arrays, I generate histograms of i and ω distributions, and a plot of e vs a. I tried looking for a clustering of ω values about 0, as this would be expected from the gravitational influence of Planet 9. In addition, I tried looking for overall differences in i distribution and e vs a distribution between the experimental system and the control system.

I calculated mean and standard deviation of a, e, i, and ω for both systems and compared them to determine whether or not they were in agreement. A lack of agreement between these properties and—in the case of the Scattered Disc—a clustering of ω would support the hypothetical existence of Planet 9.

Henry Shi
AST 390 Final Project
Spring 2020

Results

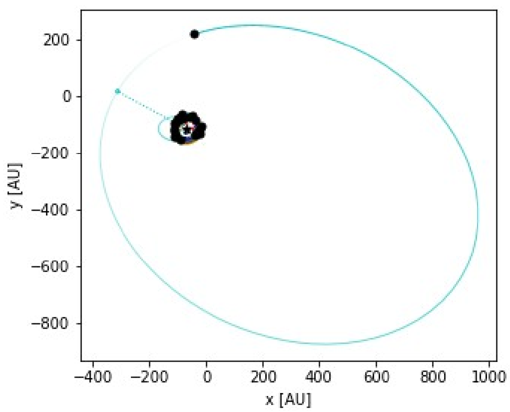
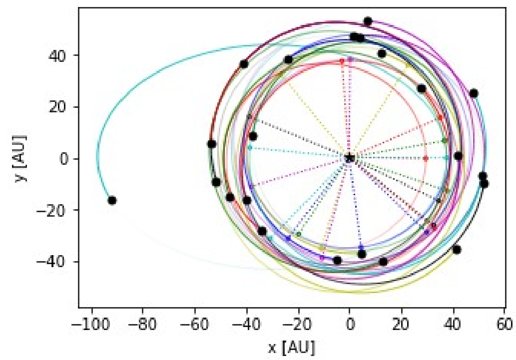
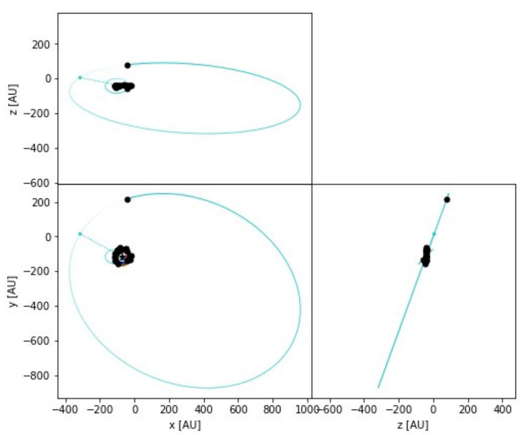
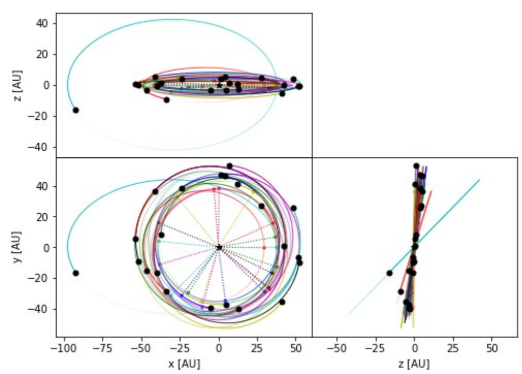
System 1: Outer planets + Kuiper Belt + Scattered Disc

[unable to complete simulation]

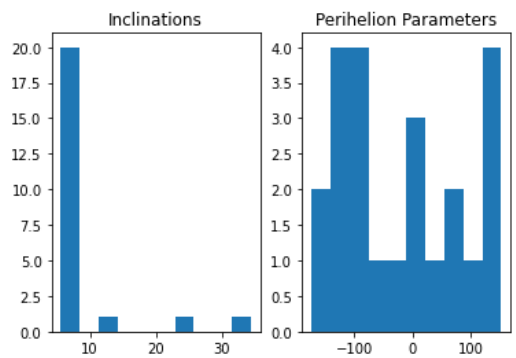
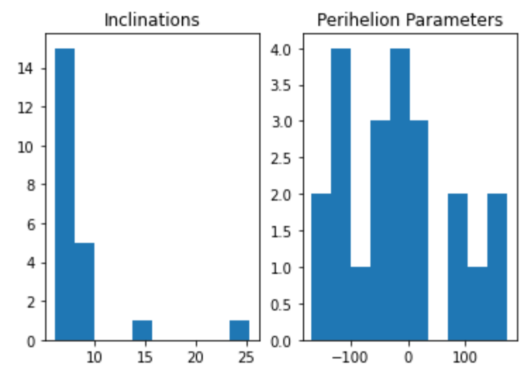
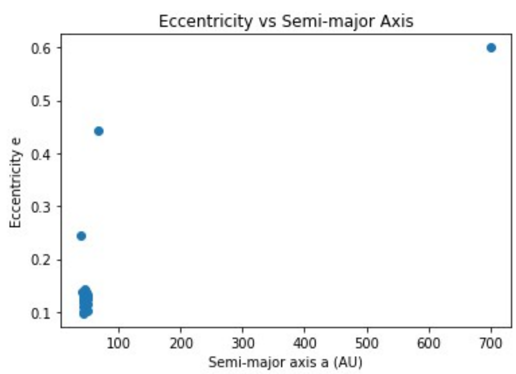
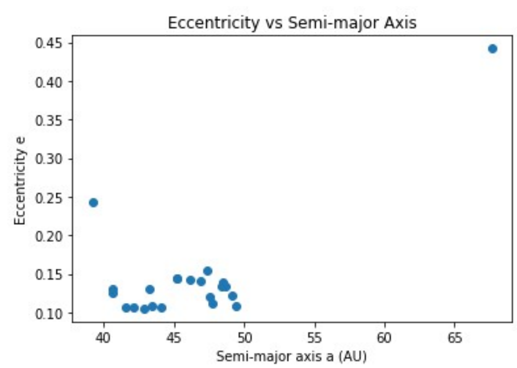
System 2: Outer planets only

[unable to complete simulation]

System 3: Kuiper Belt only

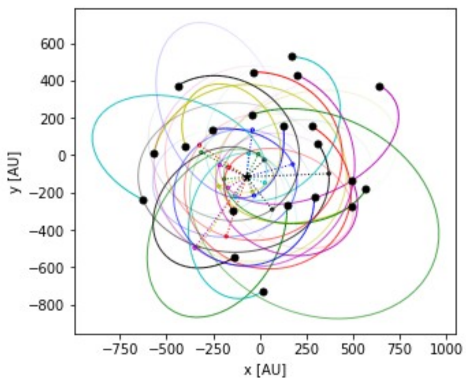
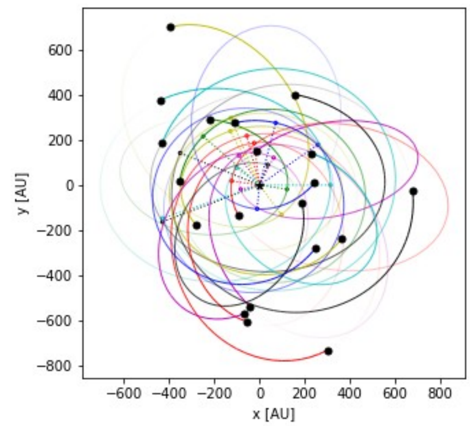
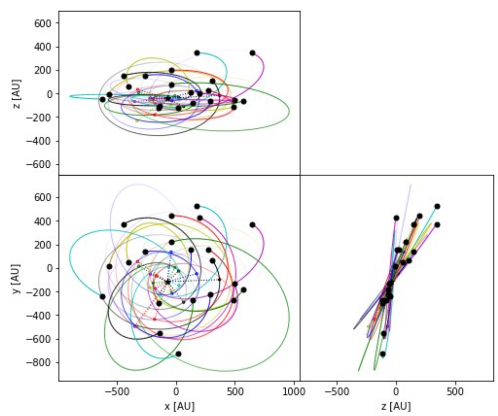
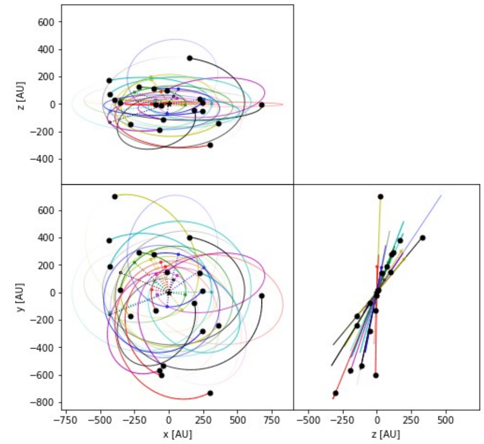
Property	Experimental	Control
Orbit plot		
Orbital inclination plot		
Semimajor axis	a = 74.811110671677 +/- 133.40474846651 AU	a = 46.170482467397 +/- 5.58253259269267AU
Eccentricity	e = 0.1612151946245 +/- 0.11691092076958	e = 0.14562383866296 +/- 0.07074139750142

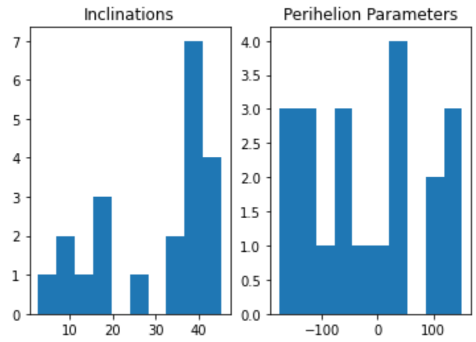
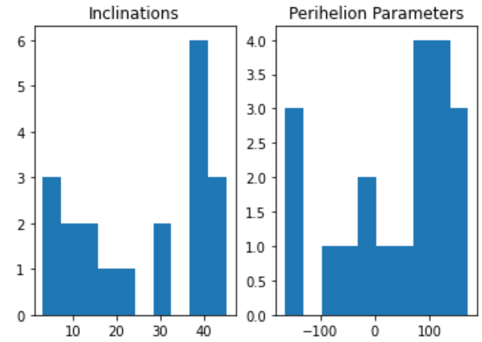
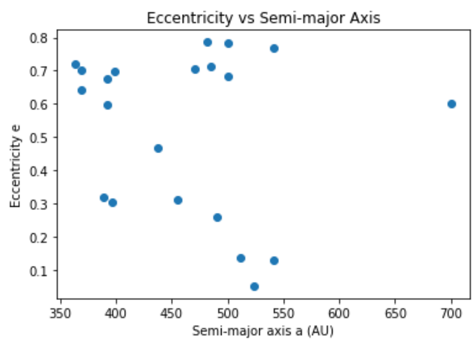
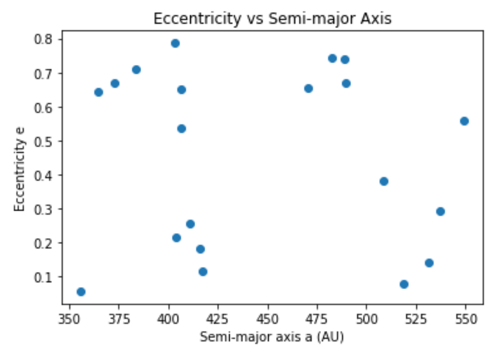
Henry Shi
AST 390 Final Project
Spring 2020

Inclination	i = 9.236950245363973 +/- 6.698502339085103 degrees	i = 8.343631351881774 +/- 4.053183513688653 degrees
Perihelion parameter	ω = -18.8470004717246 +/- 104.078262363090 degrees	ω = -16.94717621113129 +/- 97.1720002441316 degrees
i, omega histograms		
e vs a plot		

Henry Shi
AST 390 Final Project
Spring 2020

System 4: Scattered Disc only

Property	Experimental	Control
Orbit plot		
Orbital inclination plot		
Semimajor axis	a = 462.42623554406225 +/- 79.14540798328889	a = 445.9187927705657 +/- 61.170454080565385 AU
Eccentricity	e = 0.5261300516185567 +/- 0.23672476791028385	e = 0.45421130288714895 +/- 8.2523236909920256
Inclination	i = 30.145031432743448 +/- 13.56333010747321 degrees	i = 26.02439066257194 +/- 14.457082565006216
Perihelion parameter	ω = -18.86201212463621 +/- 103.8989893942102 degrees	ω = 38.29649167038268 +/- 106.92698335476904

<table>
<tr><td>i, omega histograms</td><td>
</td><td>
</td></tr>
<tr><td>e vs a plot</td><td>
</td><td>
</td></tr>
</table>

Discussion

Kuiper Belt only

For the Kuiper Belt only system, there does not appear to be any notable visual differences in the orbits of the Kuiper Belt objects. However, it is difficult to ascertain any differences between the two systems since the orbit of Planet 9 in the experimental setup causes the experimental orbital plot to have a different scale. The orbit of Eris appears to have the same position, eccentricity, and semimajor axis in both systems. Due to the difference in scales of the orbital inclination plots, it is also difficult to ascertain any differences in orbital inclination with a visual inspection.

The raw semimajor axis of the experimental Kuiper Belt system is 74.811110671677 AU. When adjusted for Planet 9's semimajor axis, this becomes:

$a_{adj} = ((74.811110671677) * 101 – 700) / 100$

$a_{adj} = 68.559221778$ AU

Since I am unable to determine an uncertainty value for a_{adj}, I assume its uncertainty to be 0 when calculating its z-score with respect to semimajor axis a_{cont} from the control setup.

Henry Shi
AST 390 Final Project
Spring 2020

$$z = \frac{(a_{adj} - a_{cont})}{\sqrt{\sigma^2_{a_{adj}} + \sigma^2_{a_{cont}}}}$$

$$z = \frac{68.559221778 - 46.170482467397}{\sqrt{0^2 + 5.58253259269267^2}}$$

$$z = 4.010498629$$

This z-score corresponds to a p-value of 0.000061, which is less than my chosen significance level of α=0.05. Therefore, there is a significant difference in average semimajor axis of the Kuiper Belt objects in the presence of Planet 9 compared to the absence of Planet 9.

The raw average eccentricity of the experimental setup is 0.1612151946245. When adjusted for Planet 9's eccentricity, this becomes:

e_{adj} = ((0.1612151946245) * 101 – 0.6) / 100

e_{adj} = 0.156827346570745

As with semimajor axis, I am unable to determine an uncertainty value for e_{adj}. Therefore, I assume its uncertainty to be 0 when calculating its z-score with respect to control eccentricity e_{cont}.

$$z = \frac{e_{adj} - e_{cont}}{\sqrt{\sigma^2_{e_{adj}} + \sigma^2_{e_{cont}}}}$$

$$z = \frac{0.156827346570745 - 0.14562383866296}{\sqrt{0^2 + 0.07074139750142^2}}$$

We obtain a z-score of z=0.1583727252145, which corresponds to a p-value of 0.874142. This p-value is greater than the significance level of α=0.05. Therefore, there is no significant difference in average eccentricity of the Kuiper Belt objects in the presence of Planet 9 compared to the absence of Planet 9.

The adjusted average orbital inclination of the experimental setup can be determined as follows:

i_{adj} = ((9.236950245363973) * 101 – 30) / 100

i_{adj} = 9.02931974781761273 degrees

Its z-score can be determined as follows:

$$z = \frac{9.02931974781761273 - 8.343631351881774}{\sqrt{0^2 + 4.054183513688653^2}}$$

z = 0.169131070071363

This z-score corresponds to a p-value of 0.865718, which is greater than the signifcance level of α=0.05. Therefore, there is no significant difference in average orbital inclination of the Kuiper Belt objects in the presence of Planet 9 compared to the absence of Planet 9.

Based on the histograms, there does not appear to be a significant difference in the orbital inclinations of the Kuiper Belt objects in the presence of Planet 9 compared to the absence of Planet 9. In both plots, the objects' inclinations appear to cluster around 5-10^{o}. There also does not appear to be a qualitative difference in the perihelion parameter distribution between the experimental setup and control setup.

In the e vs a plots, qualitative differences are difficult to ascertain due to the difference in scale between the experimental plot and control plot. However, in the experimental plot, there is an object with eccentricity between 0.4 and 0.5. Said object does not exist in the control plot. This could be a programming bug. Further simulations are necessary to determine whether this anomaly remains.

Scattered Disc only

For the Scattered Disc only system, there appears to be slight visual differences in the orbits of the objects. In the experimental setup, there appear to be 3 objects with aphelion distances that exceed 800AU. However, in the control setup, no object's aphelion exceeds 800AU.

The raw average semimajor axis of the experimental Scattered system is a = 462.42623554406225 AU. When adjusted for Planet 9's semimajor axis, this becomes:

a_{adj} = ((462.42623554406225) * 101 – 700) / 100

a_{adj} = 460.0504978995028725 AU

Since I am unable to determine an uncertainty value for a_{adj}, I assume its uncertainty to be 0 when calculating its z-score with respect to semimajor axis a_{cont} from the control setup.

$$z = \frac{(a_{adj} - a_{cont})}{\sqrt{\sigma^2_{a_{adj}} + \sigma^2_{a_{cont}}}}$$

$$z = \frac{460.0504978995028725 - 445.9187927705657}{\sqrt{0^2 + 61.170454080565385^2}}$$

z = 0.231021746386332

This z-score corresponds to a p-value of 0.817315, which is greater than the significance level of α=0.05. Therefore, there is no significant difference is average orbital semimajor axis of the Scattered Disc whether Planet 9 is present or absent.

The raw average eccentricity of the experimental setup is 0.5261300516185567. When adjusted for Planet 9's eccentricity, this becomes:

e_{adj} = ((0.5261300516185567) * 101 – 0.6) / 100

e_{adj} = 0.525391352134742267

As with semimajor axis, I am unable to determine an uncertainty value for e_{adj}. Therefore, I assume its uncertainty to be 0 when calculating its z-score with respect to control eccentricity e_{cont}.

$$z = \frac{e_{adj} - e_{cont}}{\sqrt{\sigma^2_{e_{adj}} + \sigma^2_{e_{cont}}}}$$

$$z = \frac{0.525391352134742267 - 0.45421130288714895}{\sqrt{0^2 + 8.2523236909920256^2}}$$

z = 0.0086254553157301

This corresponds to a p-value of 0.993138. This p-value is greater than the significance level of α=0.05. Therefore, there is no significant difference in average eccentricity of the Scattered Disc objects in the presence of Planet 9 compared to the absence of Planet 9.

The adjusted average orbital inclination of the experimental setup can be determined as follows:

i_{adj} = ((30.145031432743448) * 101 – 30) / 100

i_{adj} = 30.14648174707088248 degrees

Its z-score can be determined as follows:

$$z = \frac{30.14648174707088248 - 26.02439066257194}{\sqrt{0^2 + 14.457082565006216^2}}$$

z = 0.285126066477381

This z-score corresponds to a p-value of 0.775568, which is greater than the signifcance level of α=0.05. Therefore, there is no significant difference in average orbital inclination of the Scattered Disc objects in the presence of Planet 9 compared to the absence of Planet 9.

Based on the histograms, there does not appear to be a significant difference in the distribution of orbital inclinations of the Kuiper Belt objects in the presence of Planet 9 compared to the absence of Planet 9. In both plots, the objects' inclinations appear to cluster around i=40^{o}.

However, there does appear to be a qualitative difference in the distribution of perihelion parameter ω between the experimental and control setups. In the experimental setup the ω values are distributed evenly, whereas in the control setup the ω values are clustered around $\omega=100^{o}$.

In the e vs a plots, the only notable qualitative difference between the experimental and control setups is the distribution of objects with eccentricity $e<0.4$. In the experimental setup, these objects' semimajor axes range from 400AU to 550AU. In the control setup, these objects occupy a similar semimajor range of 350AU to 550AU, but there is a clump of 4 objects between 400AU and 425AU. Further simulation with more objects is necessary to determine whether these patterns hold.

Conclusion

Kuiper Belt

The average semimajor axis of Kuiper Belt objects was significantly greater in the presence of Planet 9 than in its absence. However, there is no significant difference in average eccentricity or average orbital inclination of the objects, as determined from z-score and p-value calculations and qualitative analysis of histograms. Aside from an anomaly with eccentricity $0.4<e<0.5$, there is also no qualitative difference between the e vs a plots of the experimental and control systems. Overall, the presence of Planet 9 appears to significantly increase the average orbital semimajor axis of the Kuiper Belt but does not appear to affect any other orbital parameters.

For the Scattered Disc objects, there is no significant difference in average semimajor axis, average eccentricity, or average orbital inclination, whether in the presence or absence of Planet 9. This conclusion is determined from both z-score and p-value calculations and qualitative analysis of histograms. In the e vs a plots, there is a notable qualitative difference between the experimental and control setups in the distribution of objects with low eccentricity ($e < 0.4$), namely the clumping of objects with semimajor axis $400AU < a < 425AU$. Overall, the presence of Planet 9 does not appear to affect any orbital parameters of the Scattered Disc. This experiment also failed to support the hypothesis that Planet 9 would cause clumping of the orbital perihelion parameters ω of Scattered Disc objects.

Further testing with more objects ($N > 100$) or on longer timescales ($t > 10^7$ year) is needed to determine whether these results hold. Unfortunately, my simulation size and integration time were limited by my computer's processing power. With a more powerful computer I could re-do this investigation in greater detail, and with a full outer Solar System (including gas giants, Kuiper Belt, Scattered Disc, and Oort Cloud).

Henry Shi
AST 390 Final Project
Spring 2020

Sources

EVIDENCE FOR A DISTANT GIANT PLANET IN THE SOLAR SYSTEM, Konstantin Batygin & Michael E. Brown <https://arxiv.org/pdf/1601.05438.pdf>

The Planet Nine Hypothesis, Batygin et. al. <https://arxiv.org/pdf/1902.10103.pdf>

Sedna (dwarf planet) <https://www.cs.mcgill.ca/~rwest/wikispeedia/wpcd/wp/9/90377_Sedna.htm>

Eris (dward planet) <https://www.cs.mcgill.ca/~rwest/wikispeedia/wpcd/wp/e/Eris_%2528dwarf_planet%2529.htm>

Kuiper Belt <https://solarsystem.nasa.gov/solar-system/kuiper-belt/in-depth/>

Kuiper Belt Dynamics <http://www.scholarpedia.org/article/Kuiper_belt_dynamics>

The Solar System <https://imagine.gsfc.nasa.gov/features/cosmic/solar_system_info.html>