Effects of Overfishing on Coral Reef Ecosystems

Coral Reefsearchers

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1 Introduction

Coral reefs play a crucial role in the marine's ecosystem as it serves a purpose for an abundance of marine life. Additionally, healthy coral reefs benefit the economy as it provides jobs and businesses through tourism. Unfortunately, in the recent years the health of coral reefs have been declining due to several factors. According to a 2008 world coral reef status report, it predicts that 15% of all coral are in danger of disappearing within 10-20 years, and 20% within 20-40 years [7].

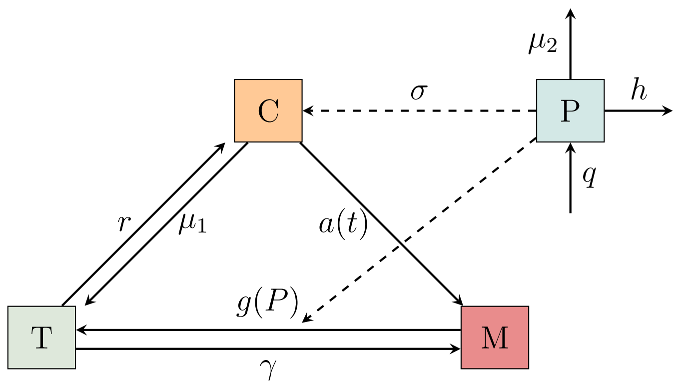
With climate change rates increasing, one of the prevalent factors affecting coral reefs is rising sea temperatures, which leads to mass bleaching of corals. Other destructive environmental factors include ocean acidification, nutrient flow from run-off [7]. Another factor that contributes to the decline of healthy coral reefs are due to human activities, such as exploitative fishing practices or pollution [3].

Because a handful of coral species are considered to be threatened, efforts in measuring the resiliency factors of a coral reef has been of interest to many. In one study, ecological factors were studied and scored in which resistance, recovery, and resilience were taken into account. It claimed the top three ecological factors that contribute a coral's resiliency is its species type, temperature variability, and nutrients for pollution run-off [8].

The objective of this paper is to analyze how Guam's reef ecosystem will change over the coming decades, focusing on the impact of overfishing of parrotfish. By setting up a compartment model and subsequent system of differential equations, we are able to model the dynamics of the ecosystem in response to different parameter and compartment values. This will allow us to analyze and predict the effect of overfishing on Guam's coral reef ecosystem. In addition, our analysis will include the application of education game theory in order to quantify the human factor in overfishing.

2 Modeling

2.1 Coral Reef Ecosystem Model



We assume that the (i) system is closed, (ii) it only consists of coral, macroalgae, and algal turfs, and (iii) macroalgae is the only predator for coral. Corals are assumed to (iv) recruit and overgrow algal turfs and that they are overgrown by macroalgae [3]. Macroalgae are also assumed (v) colonize dead coral by spreading vegetative over algal turfs [3]. In addition, (vi) corals do not die naturally and (vii) the maximum carrying capacity of parrotfish is equal to 1.

2.2 Differential Equations

C, T, and M are proportions of coral, algal turf, and macroalgae cover on the ocean floor, respectively, where $C + M + T = 1$ to signify the proportion of each population is a selected area. P is the population of the parrotfish that inhabit the coral reef ecosystem in proportion to the maximum carrying capacity. The coral reef dynamics are described as a system of nonlinear differential equations [2]:

$$\begin{aligned} \frac{dC}{dt} &= rTC + \sigma PC - (a(t)M + \mu_1)C \\ \frac{dP}{dt} &= qP\left(1 - \frac{P}{\beta C}\right) - P\left(h + \mu_2\right) \\ \frac{dT}{dt} &= \mu_1 C + \frac{g(P)M}{M+T} - T(rC + \gamma M) \\ \frac{dM}{dt} &= (a(t)C + \gamma T)M - \frac{g(P)M}{M+T} \end{aligned} \tag{1}$$

where:

$$g(P) = \frac{\alpha P}{\beta}, a(t) = |\frac{a_0(9\sin(\pi t) + 1)}{10}|$$

and

$\dfrac{g(P)M}{M+T}$ is the proportion of grazing that affects macroalgae[2].

2.3 Parameter Values

Parameter	Description	Rate	Units[6][3][2]
μ_1	natural death rate of coral reefs	0.15[9]	$year^{-1}$
μ_2	natural death rate of parrotfish	0.22[6]	$year^{-1}$
r	rate that coral recruit to overgrow algal turfs	10[9]	$year^{-1}$
γ	rate that macroalgae spread vegetative over algal turfs	0.8[11]	$year^{-1}$
q	intrinsic growth rate for parrotfish	0.47[6]	$year^{-1}$
h	harvesting rate for parrotfish	0.14[6]	$year^{-1}$
σ	rate that parrot fish bite coral	0.01*	$bites * year^{-1}$
α	maximum grazing intensity	1[2]	-
β	carrying capacity of parrotfish	1	-
a_0	control variable to simulate seasonal changes	0.99	-

Table 1: Model Parameters

** = estimated values*

2.4 Graphs

Using MatLab (See Appendix A), we were able to model the dynamics using our preliminary rates. This was achieved by changing the C, M, & T proportions. Below are the graphs that were were able to achieve:

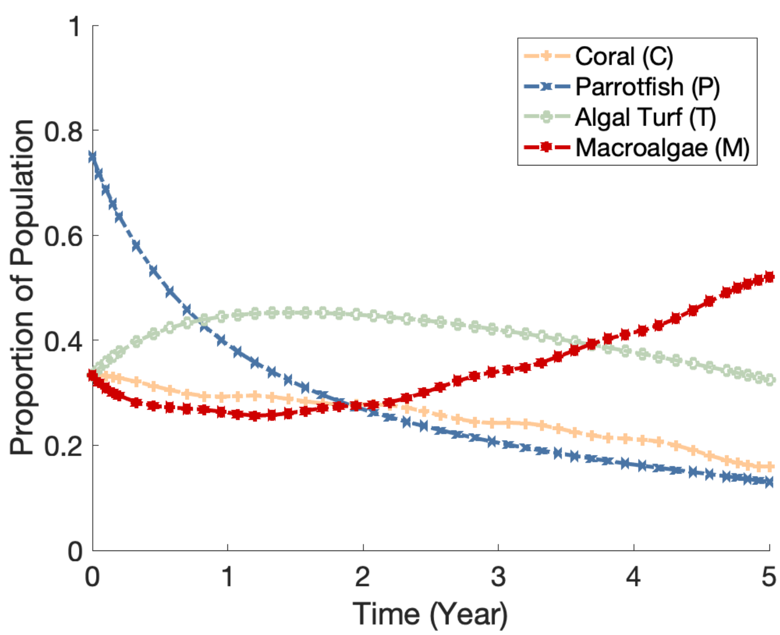


Figure 1: Initial Conditions: $C = T = M = \frac{1}{3}$, and $P = 1$

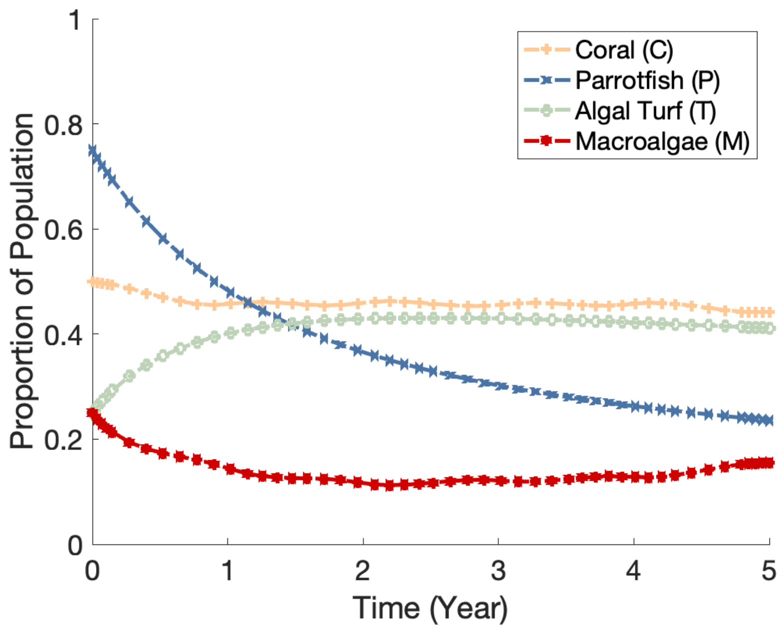


Figure 2: Initial Conditions: $C = \frac{1}{2}$, $T = M = \frac{1}{4}$, and $P = 1$

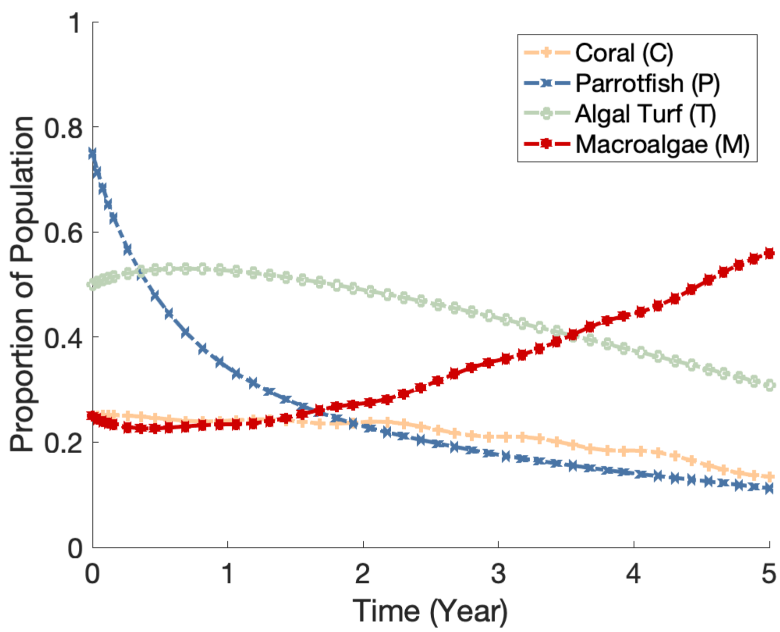


Figure 3: Initial Conditions: $T = \frac{1}{2}$, $C = M = \frac{1}{4}$, and $P = 1$

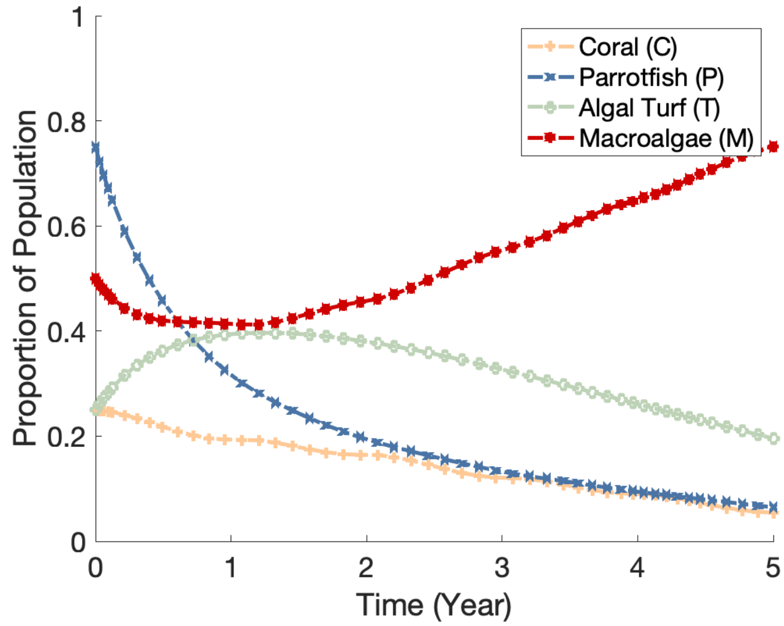


Figure 4: Initial Conditions: $M = \frac{1}{2}$, $C = T = \frac{1}{4}$, and $P = 1$

As we can see in Figures 1, 2, 3, & 4, as the parrot fish population decreases, the macroalgae proportion increases. In addition, as the macroalgae proportion increases, the coral proportion decreases, and subsequently the algal turf proportion decreases as well.

2.5 Disease-Free Equilibrium and $\mathscr{R}_0$

The disease-free equilibrium is the point at which no disease is present in the system, in which $M^0 = 0$. The system of equations becomes

$$\begin{aligned}
C^0 &= 1 - \frac{\mu_1}{r} \\
P^0 &= -\frac{\beta(1-\frac{\mu_1}{r})(h-\mu_2-q)}{q} \\
T^0 &= \frac{\mu_1}{r} \\
M^0 &= 0
\end{aligned}$$

and since C + T + M = 1, if $M^0 = 0$, then $C^0 + T^0 = 1$.Thus, our disease-free equilibrium is at $(1-\frac{\mu_1}{r}, -\frac{\beta(1-\frac{\mu_1}{r})(h-\mu_2-q)}{q}, 0, \frac{\mu_1}{r})$.

The basic reproduction number, $\mathscr{R}_0$, is defined as the number of secondary infections. For our model, macroalgae is considered to be our infection compartment. Thus,

$$\mathscr{F} = \left[aCM + \gamma MT\right] \text{ and } \mathscr{V} = \left[\frac{g(P)M}{M+T}\right]$$
$$\mathscr{R}_0 = \frac{\mu_1(ar - a\mu_1 + \gamma\mu_1)}{r^2 g(P)}$$

2.6 Endemic Equilibrium

$$
\begin{aligned}
P^* &= \beta C\left(\frac{q-(h+\mu_2)}{q}\right)\\
C^* &= rTC + \sigma PC - (a(t)M + \mu_1)C\\
T^* &= T^2(rC+\gamma M) + T(MrC + M^2 + \mu_1 C) - \left(\frac{\alpha P}{\beta}M + \mu_1 CM\right)\\
M^* &= \frac{\alpha P}{\beta(aC+\gamma T)} - T
\end{aligned}
$$

The Endemic Equilibrium tells us at what point will the disease not spread nor will it fully eradicate. Essentially it tells when the disease is stabilized. In order to find the endemic equilibrium, we have to set each differential equal to 0 and solve for each variable. The equations above are not yet finished as P^* is the only equation that is in terms of C. Every other equation will need to be computed in terms of C. After finding the equations, the next step would be to substitute the parameters in order to see at what point does each compartment need to be in order to have an Endemic Equilibrium. Note that $C = 1 - \frac{\mu_1}{r}$

Calculating P^* :

$$
\begin{aligned}
\frac{dP}{dT} &= qP\left(1-\frac{P}{\beta C}\right) - P(h+\mu_2)\\
0 &= qP\left(1-\frac{P}{\beta C}\right) - P(h+\mu_2)\\
P(h+\mu_2) &= qP\left(1-\frac{P}{\beta C}\right)\\
\frac{(h+\mu_2)}{q} &= \left(1-\frac{P}{\beta C}\right)\\
\frac{P}{\beta C} &= 1-\frac{h+\mu_2}{q}\\
P^* &= \beta C\left(\frac{q-(h+\mu_2)}{q}\right)
\end{aligned}
$$

3 Literature Review

Throughout this week, our group has dedicated a large portion of time to reviewing scholarly articles and research publications relevant to our areas of research. This aspect of performing our research is crucial as, through literature review, we are able to gather information, techniques, methods, data, results, and many other variables that we are able to use in our own research.

Our faculty mentors have graciously provided several research publications related to the overall study coral reef ecosystems in order to stimulate creativity in creating our own research topic. These papers are as follow:

- Assessing relative resilience potential of coral reefs to inform management[4]
- Model of coral population response to accelerated bleaching and mass mortality in a changing climate[8]
- Prioritizing Key Resilience Indicators to Support Coral Reef Management in a Changing Climate[5]
- Mathematical analysis of coral reef models[3]
- A Mathematical Model of Coral Reef Response to Destructive Fishing Practices with Predator-Prey Interactions[7]
- From bee species aggregation to models of disease avoidance: The Ben-Hur effect[10]
- Vaccination and the theory of games[1]
- The effect of fishing on hysteresis in Caribbean coral reefs [2]

These articles provide valuable insight in various areas of coral reef research from parameters and conditions to modeling and application. In particular, each of these papers gave us insight on how other researchers approached their problems, how they created and modified their methods, and how they produced results based on their models and equations.

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A MatLab Code

Below is the MatLab code used to calculate and verify the process of calculating $\mathscr{R}_0$:

```
% parameter values
mu1 = 0.15; % mortality rate of coral reefs
mu2 = 0.22; % natural death rate of parrotfish
q = 0.47; % intrinsic growth rate for parrotfish
alpha = 1; % %maximum grazing intensity
sigma = 0.01; % rate that parrotfish bite corals
r = 0.5; % rate that coral recruit to overgrow algal turfs
gamma = 0.8; %rate that macroalgae spread vegetative over algal turfs
beta = 1; % carrying capacity

a0 = 0.99; % rate that coral is overgrown by macroalgae
h = 0.1; %<———CONTROL VARIABLE FOR GAME THEORY

%grazing intensity 'g'
g = @(P) (alpha*P)/beta;

%sin function of
a = @(t) abs((a0*(9*sin(pi*t)+1))/(10));

% Compartment Initial Conditions
C = 1/4;
P = 3/4;
T = 1/4;
M = 1/2;


% set up DFE
% dC/dt = rTC + sigmaPC - C(aM + d) #ignore sigmaPC for now
% dP/dt = qP(1-P/betaC) + kapaP - (h + mu)P
% #remove kapaP, so = qP(1-P/betaC) - (h + mu)P
% dT/dt = dC + (g(P)M)/(M + T) - (rC + gammaM)T
% dM/dt = aMC + gammaMT - (g(P)M)/(M + T)
%
% C = y(1), P = y(2), T = y(3), M = y(4)
% f = @(t,y) [r*y(3)*y(1) + sigma*y(2)*y(1) - y(1)*(a(t)*y(4) + mu1),
%             q*y(2)*(1-(y(2)/(beta*y(1)))) - (h+mu2)*y(2),
%             mu1*y(1) +  (g(y(2))*y(4))/(y(4)+y(3)) - (r*y(1) + gamma*y(4))*
%             a(t) * y(4)*y(1) + gamma*y(4)*y(3) - (g(y(2))*y(4))/(y(4)+y(3))
%             y(1)+y(3)+y(4)];
%
% [t,ya] = ode45(f, [0 5], [C, P, T, M, 1]);
```

```
%
% figure
% hold on
% plot(t, ya(:,1), '+-.', 'Color', '#FFC996', 'Linewidth', 2.5)
% plot(t, ya(:,2), 'x-.', 'Color', '#4974A5', 'Linewidth', 2.5)
% plot(t, ya(:,3), 'o-.', 'Color', '#BDD2B6', 'Linewidth', 2.5)
% plot(t, ya(:,4), '*-.', 'Color', '#CF0000', 'Linewidth', 2.5)
% %legend('Coral (C)', 'Algal Turf (T)', 'Macroalgae (M)')
% set(gca, 'FontSize',18);
% ylim([0 1]);
% legend('Coral (C)', 'Parrotfish (P)', 'Algal Turf (T)', 'Macroalgae (M)')
% %text(0.25,0.05,txt, 'FontSize', 18);
% xlabel('Time (Year)')
% ylabel('Proportion of Population')

% For-loop for animating parameter change
for i = 1:length(t)
    a0 = i/length(t);
    a = @(t) abs((a0*(9*sin(pi*t)+1))/(10));

    % System of Differential Equations
    f = @(t,y) [r*y(3)*y(1) + sigma*y(2)*y(1) - y(1)*(a(t)*y(4) + mu1),
            q*y(2)*(1-(y(2)/(beta*y(1)))) - (h+mu2)*y(2),
            mu1*y(1) + (g(y(2))*y(4))/(y(4)+y(3)) - (r*y(1) + gamma*y(4))*y(3
            a(t) * y(4)*y(1) + gamma*y(4)*y(3) - (g(y(2))*y(4))/(y(4)+y(3)),
            y(1)+y(3)+y(4)];

    % Solve using ODE45
    [t,ya] = ode45(f, [0 5], [C, P, T, M, 1]);

    % Plot
    txt = ['a_0 = ' num2str(a0)]; % shows value of param value at current ite

    fig = figure;
    hold on
    plot(t, ya(:,1), '+-.', 'Color', '#FFC996', 'Linewidth', 2.5)
    plot(t, ya(:,2), 'x-.', 'Color', '#4974A5', 'Linewidth', 2.5)
    plot(t, ya(:,3), 'o-.', 'Color', '#BDD2B6', 'Linewidth', 2.5)
    plot(t, ya(:,4), '*-.', 'Color', '#CF0000', 'Linewidth', 2.5)

    set(gca, 'FontSize',18); % sets axis & legend font size to 18
    ylim([0 1]); % sets y-axis limit to always be 0-1
    legend('Coral (C)', 'Parrotfish (P)', 'Algal Turf (T)', 'Macroalgae (M)')
    text(0.25,0.05,txt, 'FontSize', 18); % displays text on plot
```

```
    xlabel('Time (Year)')
    ylabel('Proportion of Population')

    % automatically save figure into root directory (where this .m file is
    % stored)
    filename = append('Frame-', num2str(i)); %file name of current iteration
    saveas(fig, filename, 'png'); %save figure as .png
end
```

References

[1] Chris T Bauch and David J. D. Earn. "Vaccination and the theory of games". In: *PNAS* (July 2004).

[2] J. C. Blackwood, A. Hastings, and P. J. Mumby. "The effect of fishing on hysteresis in Caribbean coral reefs". In: *5* 1 (Nov. 2010), pp. 105–114. DOI: 10.1007/s12080-010-0102-0.

[3] Xiong Li et al. "Mathematical analysis of coral reef models". In: *Journal of Mathematical Analysis and Applications* 416.1 (2014), pp. 352–373. ISSN: 0022-247X. DOI: https://doi.org/10.1016/j.jmaa.2014.02.053. URL: https://www.sciencedirect.com/science/article/pii/S0022247X14001917.

[4] Jeffrey A. Maynard et al. "Assessing relative resilience potential of coral reefs to inform management". In: *Biological Conservation* 192 (2015), pp. 109–119. DOI: 10.1016/j.biocon.2015.09.001.

[5] Tim R. Mcclanahan et al. "Prioritizing Key Resilience Indicators to Support Coral Reef Management in a Changing Climate". In: *PLoS ONE* 7.8 (2012). DOI: 10.1371/journal.pone.0042884.

[6] Marc O. Nadon. 2019, pp. 1–100.

[7] Sarai Quintero et al. "A Mathematical Model of Coral Reef Response to Destructive Fishing Practices with Predator-Prey Interactions". In: (July 2016).

[8] Bernhard M. Riegl and Samuel J. Purkis. "Model of coral population response to accelerated bleaching and mass mortality in a changing climate". In: *Ecological Modelling* 220.2 (Jan. 2009), pp. 192–208. DOI: 10.1016/j.ecolmodel.2008.09.022.

[9] Eric Wolanski, Robert H. Richmond, and Laurence Mccook. "A model of the effects of land-based, human activities on the health of coral reefs in the Great Barrier Reef and in Fouha Bay, Guam, Micronesia". In: *Journal of Marine Systems* 46.1-4 (2004), pp. 133–144. DOI: 10.1016/j.jmarsys.2003.11.018.

[10] K. E. Yong, E. Díaz Herrera, and C. Castillo-Chavez. "From Bee Species Aggregation to Models of Disease Avoidance: The Ben-Hur effect". In: *Mathematical and Statistical Modeling for Emerging and Re-emerging Infectious Diseases* (2016), pp. 169–185. DOI: 10.1007/978-3-319-40413-4_11.

[11] R N Zikkah, N Anggriani, and A K Supriatna. "Mathematical model of coral reefs with fish harvesting effect". In: *Journal of Physics: Conference Series* 1657 (2020), p. 012062. DOI: 10.1088/1742-6596/1657/1/012062.