Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Mathematical model of coral reefs with fish harvesting effect

To cite this article: R N Zikkah *et al* 2020 *J. Phys.: Conf. Ser.* **1657** 012062

Mathematical model of coral reefs with fish harvesting effect

R N Zikkah, N Anggriani and A K Supriatna

Universitas Padjadjaran, Jl. Raya Bandung Sumedang KM 21, Jatinangor, Indonesia

E-mail: riska16001@mail.unpad.ac.id

Abstract. The degradation of coral reefs is becoming increasingly alarming. If left unchecked, there will be an imbalance of the marine ecosystem. Many factors are killing the coral reefs. There is nature factor such as global warming and human activities factor such as destructive fishing. In this paper, we proposed model coral reefs with a system of the nonlinear ordinary differential equation between five populations, namely coral reefs, parrotfish, macroalgae, algal turfs, and crown of thorns starfish in the Great Barrier Reef, Coral Sea. We consider biological interactions such as predator-prey relationships between the crown of thorns starfish and coral reefs, and macroalgae grazers on coral reefs. We consider destructive fishing that damage the coral. Using the standard dynamic system theory, we determined the stability of the system with and without fishing. We show numerical simulation and sensitivity analysis to evaluate the investigation results to conclude the effect of the destructive fishery on coral reefs.

1. Introduction

Coral reefs are the center of biodiversity in the world's oceans, home to all life at sea [1], and have many functions for life. Coral reefs provide shelter and food for organisms that live in the vicinity. There are so many functions of coral reefs besides being a home for marine ecosystems, coral reefs function as spawning grounds for marine biota before going to the open seas, and protecting the coast from waves and strong currents [2,3]. Besides, coral reefs are also very beneficial to the community in economic aspects, especially in terms of fisheries and tourism [3].

The status report of the world's coral reefs in 2008 stated that 15% of all coral reefs in the world threatened extinction in the next 10-20 years, and 20% in the next 20-40 years [1]. The extinction of these coral reefs causes severe problems for the entire marine ecosystem. Several factors caused this damage, such as damage by nature (e.g. coral bleaching [4,5], ocean acidification [6], global warming [5,7], etc.) and humans by destructive fishing [1].

Although many factors affect coral reef degradation, this research will focus on the effects of destructive fishing compared to other natural factors. The use of explosives in harvesting fish is considered one of the most damaging threats to coral reef ecosystems with very harmful impacts [8]. P.J Mumby et al. [8] used a complete parameter simulation model combined with a simple analytic model to show the dynamic behavior of systems that dramatically different between the high and low levels of microalgae erosion performed by parrots [9]. Furthermore, by including control overfishing pressure, Blackwood et al. [10] broaden the model and treat microalgae erosion as a dynamic process and consider two different coral recovery scenarios that identify the level of fishing effort that allows for coral recovery. Li et al. [11] modified the model in [8] using DDE (Delay Differential Equation). Furthermore, Quintero et al. examined the response of coral reefs to destructive fishing using a predator model in the waters of Raja Ampat.

This study will focus on the effects of blast fishing or poison fishing on herbivorous fish by explicitly including grazer dynamics in the Li et al. model [11] and predator-prey relationships of coral reefs from the Quintero et al. [1], making it possible to develop recommendations for the management of coral reef ecosystems with better fishing methods so that it does not cause damage to coral reef ecosystem.

2. Mathematical Model of Coral Reef

Populations that play a role in this model are populations of coral reefs, crown-of-thorns starfish (CoTS), algae turf, macroalgae, and parrotfish. We assume that (i) coral is the only prey for CoTS, (ii) algal turfs grow from grazed macroalgae and natural morality of coral, (iii) coral overgrow algal turfs, (iv) macroalgae overgrow on coral, (v) macroalgae colonize dead coral by spreading vegetative over algal turf, and (vi) we use the year as a time dimension as shown in Figure 1.

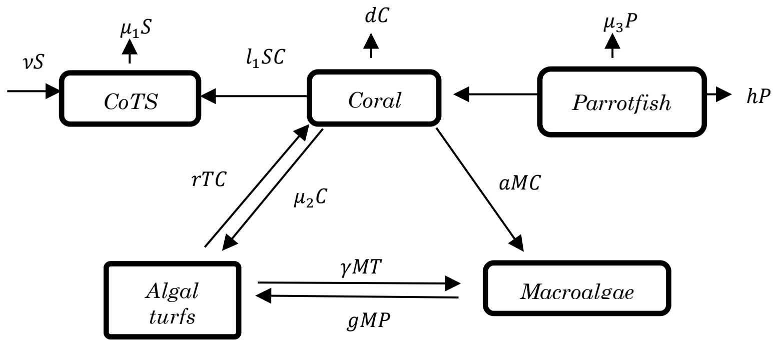


Figure 1. Ecosystem Flow of Coral Reefs

Let $M(t)$ be the density of macroalgae population, $C(t)$ for coral reefs population density, $T(t)$ for algal turf, $P(t)$ for parrotfish, and $S(t)$ for crown-of-thorns starfish (CoTS), the mathematical model of coral reefs:

$$\begin{aligned}
\frac{dM}{dt} &= aMC - gMP + \gamma MT \\
\frac{dC}{dt} &= rTC - aMC - \mu_2 C - l_1 SC - dC \\
\frac{dT}{dt} &= gMP - \gamma MT - rTC + \mu_2 C \\
\frac{dP}{dt} &= qP\left(1 - \frac{P}{\beta C}\right) - \mu_3 P - hP \\
\frac{dS}{dt} &= vS\left(1 - \frac{S}{k}\right) - \mu_1 S + l_2 SC
\end{aligned} \tag{1}$$

All parameters in the system (1) are non-negative and the model will be analyzed in a feasible region $D = \{(M, C, T, P, S) \in \mathbb{R}_+^5, M(0) = M_0, C(0) = C_0, T(0) = T_0, S(0) = S_0, P(0) = P_0$. Table 1 shows the description of each parameter.

Table 1. Parameter Description

Parameter	Description
$\boldsymbol{\mu_1}$	The natural death rate of CoTS
$\boldsymbol{\mu_2}$	The natural death rate of coral reefs
$\boldsymbol{\mu_3}$	The natural death rate of parrotfish
$\boldsymbol{a}$	Rate that coral is overgrown by macroalgae
$\boldsymbol{r}$	Rate that coral recruit to and overgrow algal turfs
$\boldsymbol{g}$	Grazing rate that parrotfish graze macroalgae without distinction from algae turfs
$\boldsymbol{\gamma}$	Rate that macroalgae spread vegetative over algal turfs
$\boldsymbol{q}$	Intrinsic growth rate for parrotfish

$\boldsymbol{\nu}$	Intrinsic growth rate for CoTS
$\boldsymbol{k}$	Carrying capacity of CoTS
$\boldsymbol{\beta}$	Carrying capacity of parrotfish
$\boldsymbol{l_1}$	Predation rate from CoTs on coral
$\boldsymbol{l_2}$	Benefit rate from CoTS predation on coral
$\boldsymbol{h}$	Harvesting rate for parrotfish
$\boldsymbol{d}$	Intrinsic destruction rate for coral from harvesting

3. Mathematical Analysis

Assume that coral reefs cover the entire seabed, macroalgae, and algae turf with M + C + T = 1. Then by assuming T = 1-C-M, the following model is obtained:

$$\begin{aligned}
\frac{dM}{dt} &= M[-gP + (a-\gamma)C - \gamma(M-1)] \\
\frac{dC}{dt} &= C[r(1-C) - (r+q)M - \mu_2 - l_2 S - d] \\
\frac{dP}{dt} &= qP\left(1 - \frac{P}{\beta C}\right) - \mu_3 P - hP \\
\frac{dS}{dt} &= \nu S\left(1 - \frac{S}{k}\right) - \mu_1 S + l_2 C
\end{aligned} \tag{2}$$

The equilibrium point is obtained when all populations have no change or can be written as $\frac{dM}{dt} = \frac{dC}{dt} = \frac{dP}{dt} = \frac{dS}{dt} = 0$, so the equilibrium point is obtained as presented in Table 2. For each equilibrium point, a stability analysis using the eigenvalue method and the Routh-Hurwitz criterion.

Table 2. Stability of System

Equilibrium	**Existence**	**Stability**
$\boldsymbol{E_1(M,C,P,S) = \left(0, \frac{B}{r}, 0, 0\right)}$	If $B > 0$	Stable if $d + \mu_2 < r, q + \mu_3 < h$, and $\nu > \lambda_1$
$\boldsymbol{E_2(M,C,P,S) = \left(0, \frac{Q}{A}, 0, \frac{F}{A}\right)}$	If $Q > 0$ and $F > 0$	Stable if $\gamma\left(1 + \frac{b_0}{b_1}\right) < \frac{ab_0}{b_1}, q < h + \mu_3$ and $A_0 > 0$
$\boldsymbol{E_3(M,C,P,S) = \left(0, \frac{B}{r}, \frac{\beta BR}{rq}, 0\right)}$	If $B > 0$ and $R > 0$	Stable if $aqB + g\beta R > B\gamma q + q$
$\boldsymbol{E_4(M,C,P,S) = \left(0, \frac{Q}{A}, \frac{BQR}{Aq}\right)}$	If $B > 0, Q > 0$ and $R > 0$	Stable if $q\gamma < g\beta + qa, h + \mu_3 < q$ and $A_0 > 0$
$\boldsymbol{E_5(M,C,P,S) = \left(\frac{T}{aG}, \frac{\gamma(a+d+\mu_2)}{aG}, 0, 0\right)}$	If $T > 0$ and $G > 0$	Stable if $q < h + \mu_3, B_0 < 0, B_1 < 0, B_2 < 0$ and $C_0 > 0$
$\boldsymbol{E_6(M,C,P,S) = \left(\frac{\gamma J}{D}, \frac{K}{D}, 0, \frac{kL}{D}\right)}$	If $J, K, L, D > 0$	Stable if $q < h + \mu_3, D_0 < 0, D_1 < 0$ and $D_2 < 0$
$\boldsymbol{E_7(M,C,P,S) = \left(\frac{Y}{X}, \frac{\gamma q(a+d+\mu_2)}{X}, \frac{Z}{X}, 0\right)}$	If $X, Y, Z > 0$	Stable if $F_1 > 0, F_2 > 0, F_3 > 0$
$\boldsymbol{E_8(M,C,P,S) = \left(\frac{W}{V}, \frac{\gamma qJ}{V}, \frac{\gamma\beta N}{V}, \frac{kE}{V}\right)}$	If $W, V, E, J, N > 0$	Stable if $J_1 > 0, J_2 > 0, J_3 > 0$ and $J_4 > 0$

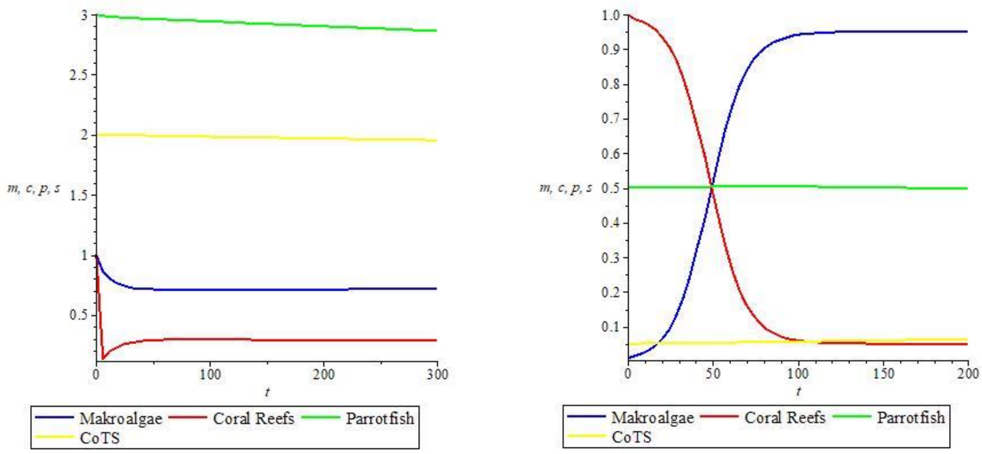
4. Numerical Analysis

We will numerically simulate the dimensionless model (2) in this section using Maple 18. By providing the parameter values in Table 3.

Table 3. Parameter Value

Parameter	Description	Value	Reference
μ_1	The natural death rate of CoTS	1×10^{-4}	Estimation
μ_2	The natural death rate of coral reefs	8.2×10^{-8}	Estimation
μ_3	The natural death rate of parrotfish	0.25×10^{-6}	Estimation
a	Rate that coral is overgrown by macroalgae	0.1	[2]
r	Rate that coral recruit to and overgrow algal turfs	65	Estimation
g	Grazing rate that parrotfish graze macroalgae without distinction from algae turfs	0.01	Estimation
γ	Rate that macroalgae spread vegetative over algal turfs	0.8	[2]
q	Intrinsic growth rate for parrotfish	$1{,}25 \times 10^{-4}$	Estimation
ν	Intrinsic growth rate for CoTS	0.0013	[1]
k	Carrying capacity of CoTS	2	Estimation
β	Carrying capacity of parrotfish	5	Estimation
l_1	Predation rate from CoTs on coral	0.007215	[1]
l_2	Benefit rate from CoTS predation on coral	0.5×10^{-5}	Estimation
h	Harvesting rate for parrotfish	0.8×10^{-5}	Estimation
d	Intrinsic destruction rate for coral from harvesting	0.5×10^{-6}	Estimation

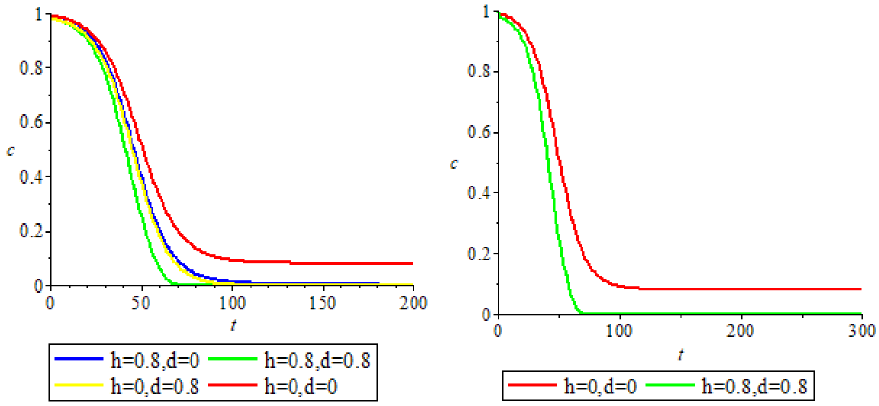
After providing all the parameter values that can be seen in Table 3, the simulation will be carried out without fishing with values $h = 0$ and $d = 0$.



(i) (ii)

Figure 2. Coexistence simulation with the absence of harvest and destruction with varying initial conditions (i) $M(0) = 1, C(0) = 1, P(0) = 2$, dan $S(0) = 3$ and (ii) $M(0) = 0.01, C(0) = 1, P(0) = 0.5$, and $S(0) = 0.05$

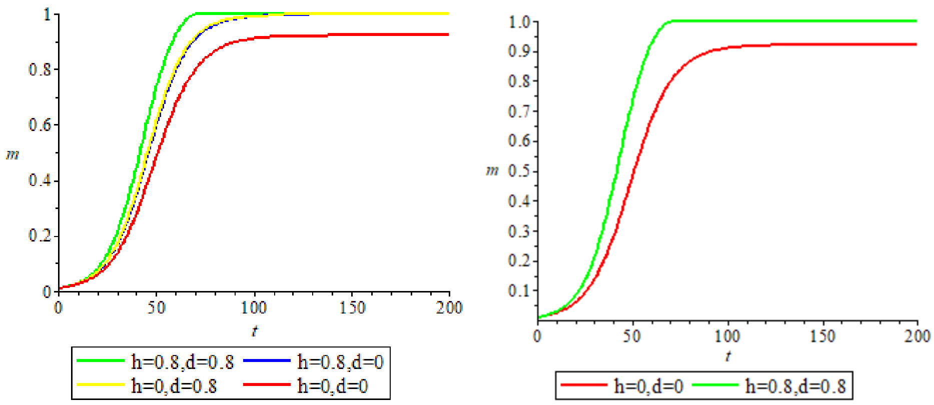
Figure 2 shows that all four species can coexist. This simulation is carried out using the values in Table 2 by comparing the initial population values to see whether this affects the system, but the two graphs in Figure 2 (i) and Figure 2 (ii) show that the population is heading for equilibrium. Furthermore, numerical simulations will be carried out with the destructive effects of fishing on each population.



(i) (ii)

Figure 3. Comparison on harvesting and destruction effects on coral biomass

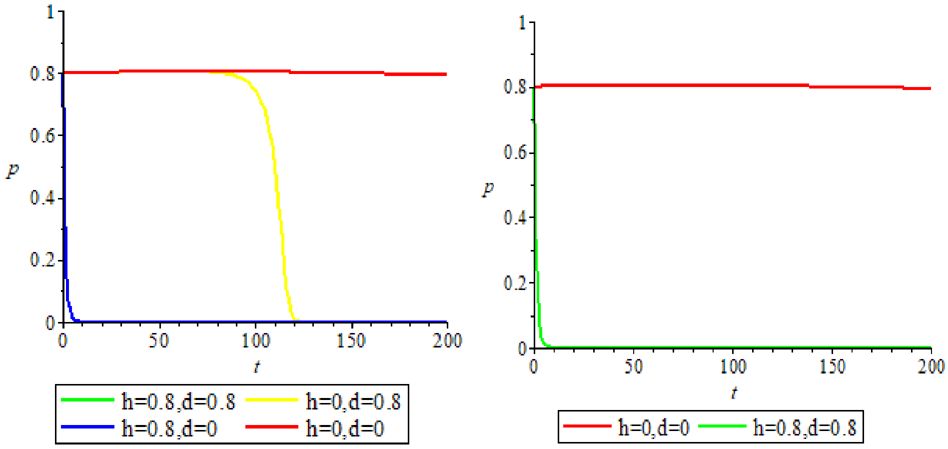
Figure 3 (i) and (ii) show the effect of fishing on coral reefs. When comparing the impact of fisheries and damage, Figure 3 (i) shows that destructive fishing has a more significant impact on coral reefs than fishing only, although not very significant. Figure 3 (ii) shows that the combination of destructive and fishing does not give a huge significant difference with only destructive.



(i) (ii)

Figure 4. Comparison on harvesting and destruction effects on macroalgae biomass

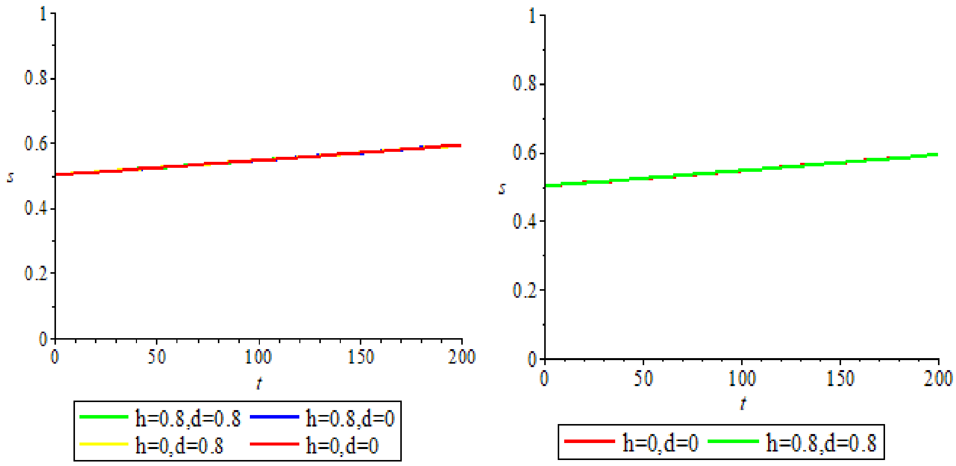
Figures 4 (i) and (ii) show the effects of destructive fishing on the macroalgae population. Figure 4 (i) shows that the single impact of fisheries and the consequences of damage each have no difference. Whereas in Figure 4 (ii), the combination of the two effects of fishing and destruction has a positive impact on macroalgae because of the missing of macroalgae predator, parrotfish.



(i) (ii)

Figure 5. Comparison on harvesting and destruction effects on parrotfish biomass

Figure 5 (i) and (ii) show the effects of destructive fishing on the parrotfish population. In Figure 5 (i), the comparison between the single impact of fisheries and the only effect of damage to parrotfish is very different, namely that fishing itself directly affects parrots. In Figure 5 (ii), the combination of both regular and destructive fishing does not have a significant difference with only the fishing effect due to the direct fishing has an impact on the parrotfish as its object.



(i) (ii)

Figure 6. Comparison on harvesting and destruction effects on CoTS biomass

Figures 6 (i) and (ii) show the effects of destructive fishing on the crown-of-thorns starfish (CoTS) population. We can see that Figures 6 is very different from Figures 2, 3, and 4 because CoTS are not directly related to parrotfish harvesting.

5. Conclusion

Coral reefs around the world decline by many factors, one of the factors is destructive fishing that straightly damages the coral. Our numerical analysis showed that increases in parrotfishes fishing lead to increases in macroalgae biomass which is overgrown the coral. The increase in macroalgae biomass also leads to a decline in coral biomass. Therefore, parrotfish fishing indirectly impact to the depletion of coral reef. Besides, the destructive fishing directly damage the coral by explosion and poison fishing practice. In conclusion, destructive fishing practices are harmful to the coral reef system. Limiting these practices by enforcing the law against illegal, unreported, and unregulated (IUU) fishing practices would give a great benefit to the coral reef system.

6. Acknowledgments
The work was supported by Kementrian Riset dan Teknologi/Badan Riset dan Inovasi Nasional 2020, with contract number 1827/UN6.3.1/LT/2020 through Penelitian Dasar.

7. References
[1] Quintero S, Machuca V, Cotto H, Bradley M and Rios-Soto K 2016 A mathematical model of coral reef response to destructive fishing practices with predator-prey interactions.
[2] Arini D I D 2013 Potensi terumbu karang indonesia "tantangan dan upaya konservasinya" *INFO BPK Manado* **3** 147.
[3] Maragos J, Crosby M and McManus J 1996 Coral Reefs and Biodiversity: A Critical and Threatened Relationship *Oceanography* **9** 83.
[4] Anna Scott, D L D 2016 Reef fishes can recognize bleached habitat during settlement: sea anemone bleaching alters anemonefish host selection *Proc. R. Soc. B.* **283** 20152694.
[5] Ryan E J, K H, P S K 2019 Massive Corals Maintain A Positive Carbonate Budget of a Maldivian Upper Reef Platform Despite Major Bleaching Event *Scientific Reports.*
[6] Steve S and Doo P J 2019 Ocean Acidification Effects on In Situ Coral Reef Metabolism *Scientific Reports*.
[7] Terry P and Hughes J T 2019 Global Warming Impairs Stock-recruitment Dynamics of Corals.
[8] Mumby P J, Hastings A, and Edwards H J 2007 Thresholds and the resilience of Caribbean coral reefs *Nature* **450**.
[9] Pet-Soede C, Cesar H S J, and Pet J S 1999 An economic analysis of blast fishing on Indonesian coral reefs *Environmental Conservation* **26** 83.
[10] Blackwood J C, Hastings A, and Mumby P J 2010 The effect of fishing on hysteresis in Caribbean coral reefs *Springerlink.*
[11] Li X, Wang H, Zhang Z, and Hastings A 2014 Mathematical analysis of coral reef models *Journal of Mathematical Analysis and Applications* 352.