Project #3B:
Solving Systems of Linear Equations

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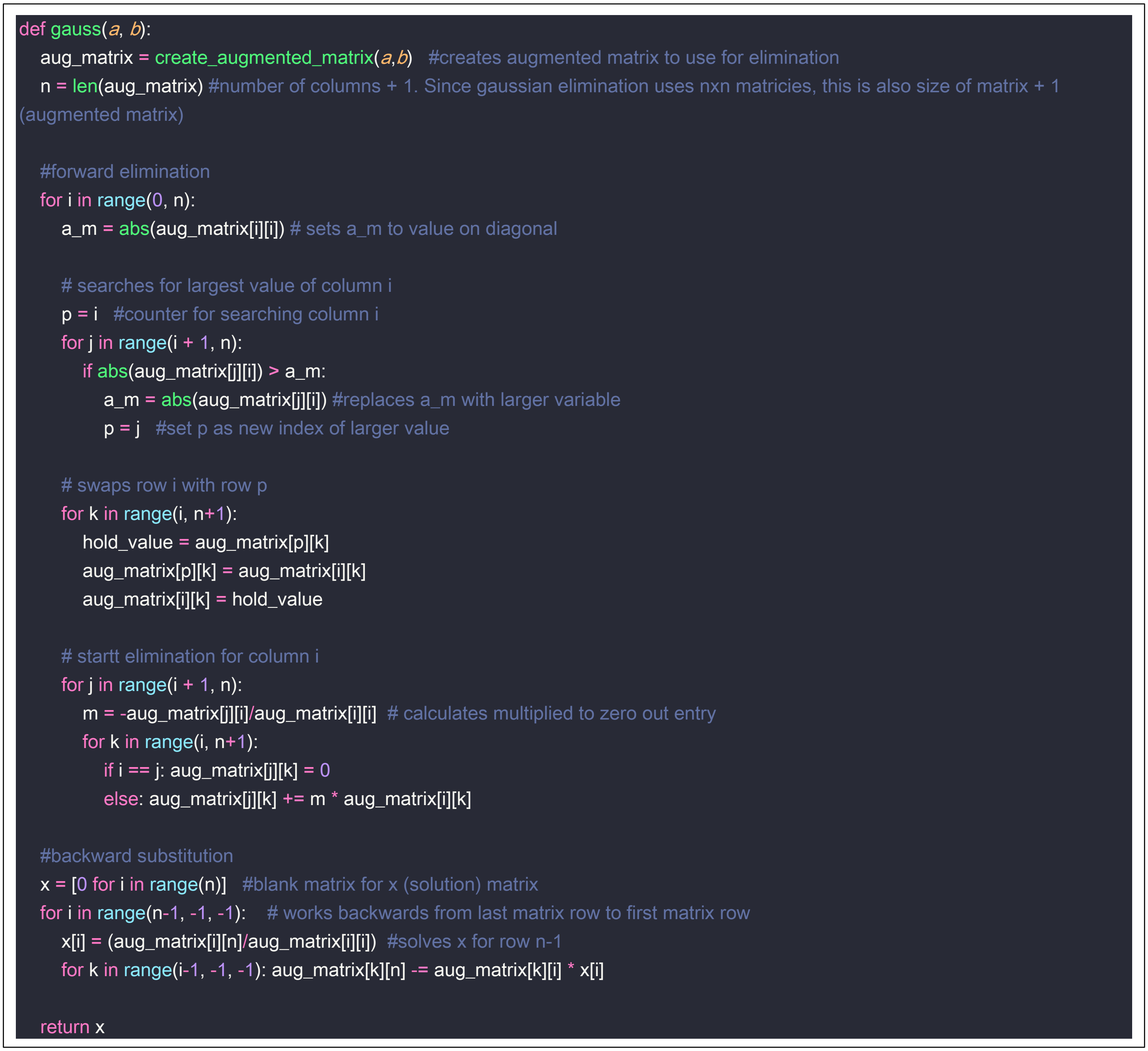
May 19, 2021

MA375

Dr. L. Aquino

1. Gaussian Elimination Code:

The snippet shown below is the method I created for gaussian elimination in Python. The script is based off of the pseudocode in Topic 7, Part 2 lecture notes (from Epperson). I modified the method so that it uses an augmented matrix rather than separate A and b matrices as it is seems more natural to use.

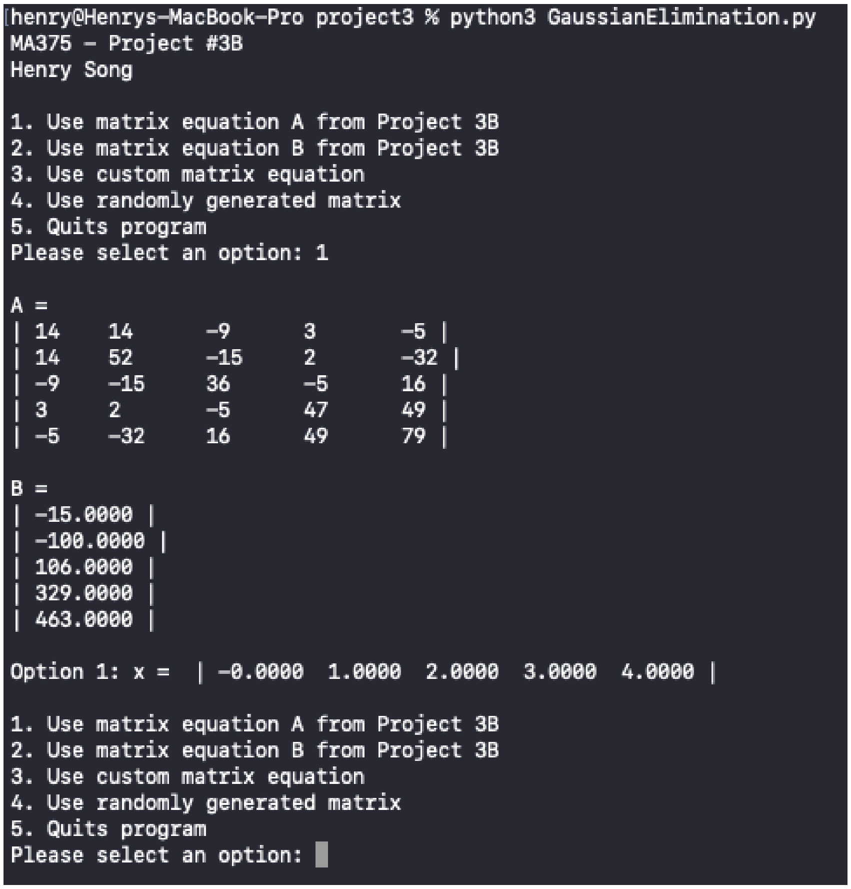


```
def gauss(a, b):
    aug_matrix = create_augmented_matrix(a,b)   #creates augmented matrix to use for elimination
    n = len(aug_matrix) #number of columns + 1. Since gaussian elimination uses nxn matricies, this is also size of matrix + 1
(augmented matrix)

    #forward elimination
    for i in range(0, n):
        a_m = abs(aug_matrix[i][i]) # sets a_m to value on diagonal

        # searches for largest value of column i
        p = i   #counter for searching column i
        for j in range(i + 1, n):
            if abs(aug_matrix[j][i]) > a_m:
                a_m = abs(aug_matrix[j][i]) #replaces a_m with larger variable
                p = j   #set p as new index of larger value

        # swaps row i with row p
        for k in range(i, n+1):
            hold_value = aug_matrix[p][k]
            aug_matrix[p][k] = aug_matrix[i][k]
            aug_matrix[i][k] = hold_value

        # startt elimination for column i
        for j in range(i + 1, n):
            m = -aug_matrix[j][i]/aug_matrix[i][i]  # calculates multiplied to zero out entry
            for k in range(i, n+1):
                if i == j: aug_matrix[j][k] = 0
                else: aug_matrix[j][k] += m * aug_matrix[i][k]

    #backward substitution
    x = [0 for i in range(n)]   #blank matrix for x (solution) matrix
    for i in range(n-1, -1, -1):   # works backwards from last matrix row to first matrix row
        x[i] = (aug_matrix[i][n]/aug_matrix[i][i])  #solves x for row n-1
        for k in range(i-1, -1, -1): aug_matrix[k][n] -= aug_matrix[k][i] * x[i]

    return x
```

2a. Matrix Equation A:

Using the given matrices for matrix equation A and the gaussian elimination method created, I was able to achieve the solution:

$$[0 \quad 1 \quad 2 \quad 3 \quad 4]^T$$



```
[henry@Henrys-MacBook-Pro project3 % python3 GaussianElimination.py
MA375 - Project #3B
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1. Use matrix equation A from Project 3B
2. Use matrix equation B from Project 3B
3. Use custom matrix equation
4. Use randomly generated matrix
5. Quits program
Please select an option: 1

A =
| 14    14      -9      3       -5 |
| 14    52      -15     2       -32 |
| -9    -15     36      -5      16 |
| 3     2       -5      47      49 |
| -5    -32     16      49      79 |

B =
| -15.0000 |
| -100.0000 |
| 106.0000 |
| 329.0000 |
| 463.0000 |

Option 1: x =  | -0.0000  1.0000  2.0000  3.0000  4.0000 |

1. Use matrix equation A from Project 3B
2. Use matrix equation B from Project 3B
3. Use custom matrix equation
4. Use randomly generated matrix
5. Quits program
Please select an option: 
```
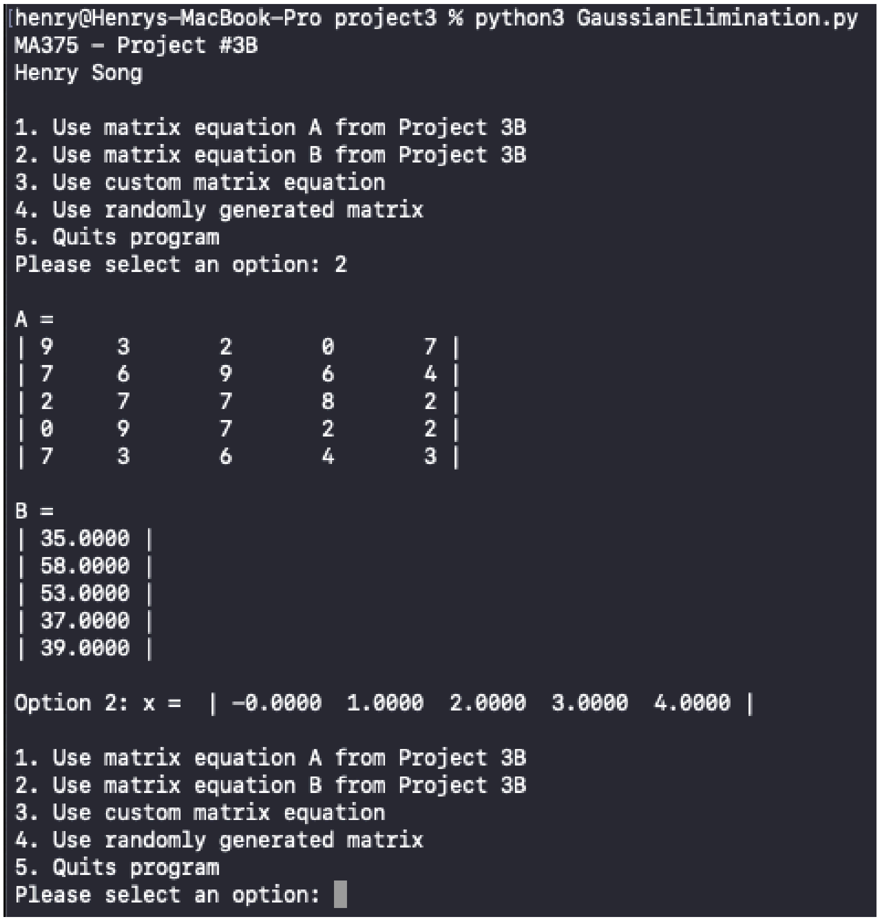

2b. Matrix Equation B:

Using the given matrices for matrix equation A and the gaussian elimination method created, I was able to achieve the solution:

$$[0 \quad 1 \quad 2 \quad 3 \quad 4]^T$$

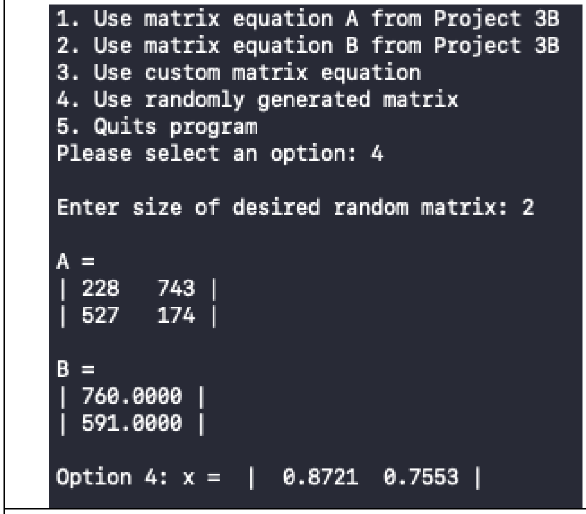


```
[henry@Henrys-MacBook-Pro project3 % python3 GaussianElimination.py
MA375 - Project #3B
Henry Song

1. Use matrix equation A from Project 3B
2. Use matrix equation B from Project 3B
3. Use custom matrix equation
4. Use randomly generated matrix
5. Quits program
Please select an option: 2

A =
| 9     3       2       0       7 |
| 7     6       9       6       4 |
| 2     7       7       8       2 |
| 0     9       7       2       2 |
| 7     3       6       4       3 |

B =
| 35.0000 |
| 58.0000 |
| 53.0000 |
| 37.0000 |
| 39.0000 |

Option 2: x =  | -0.0000  1.0000  2.0000  3.0000  4.0000 |

1. Use matrix equation A from Project 3B
2. Use matrix equation B from Project 3B
3. Use custom matrix equation
4. Use randomly generated matrix
5. Quits program
Please select an option: 
```

3. Solving other matrices:

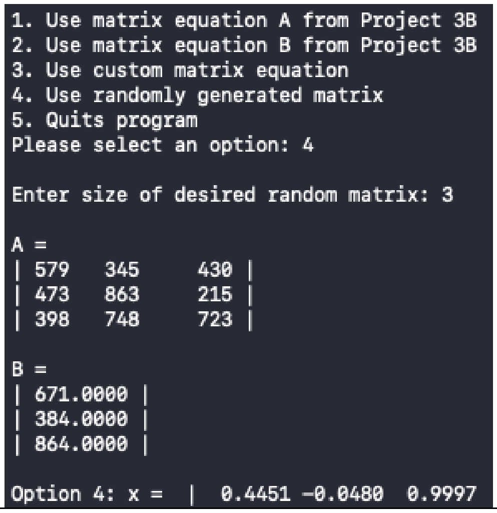
These matrices are generated using a manually entered size and filled with randomly generated integers. I apologize in advance for the tiny pictures, especially as the matrix gets larger. Normal resolution pictures are also included as stand-alone images in submission folder.

2x2 matrix:



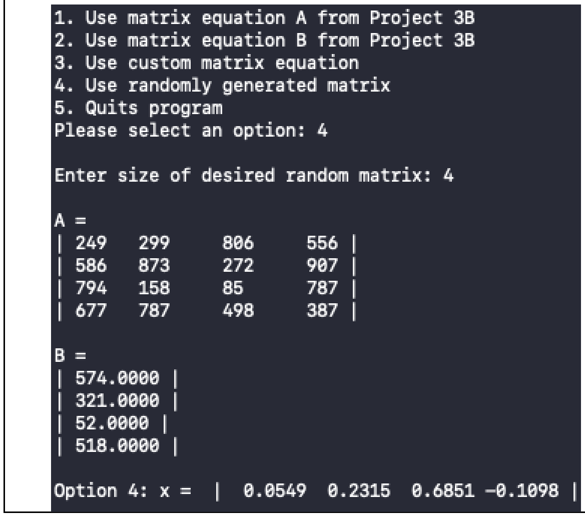
```
1. Use matrix equation A from Project 3B
2. Use matrix equation B from Project 3B
3. Use custom matrix equation
4. Use randomly generated matrix
5. Quits program
Please select an option: 4

Enter size of desired random matrix: 2

A =
| 228   743 |
| 527   174 |

B =
| 760.0000 |
| 591.0000 |

Option 4: x =  |  0.8721  0.7553 |
```

3x3 matrix:



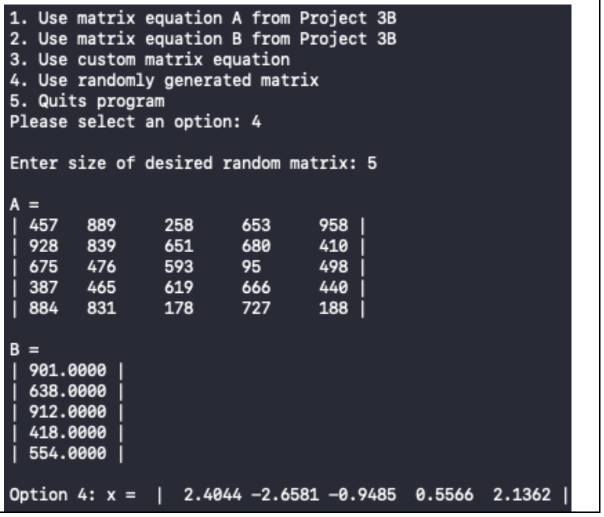
```
1. Use matrix equation A from Project 3B
2. Use matrix equation B from Project 3B
3. Use custom matrix equation
4. Use randomly generated matrix
5. Quits program
Please select an option: 4

Enter size of desired random matrix: 3

A =
| 579   345     430 |
| 473   863     215 |
| 398   748     723 |

B =
| 671.0000 |
| 384.0000 |
| 864.0000 |

Option 4: x =  |  0.4451 -0.0480  0.9997
```

4x4 matrix:



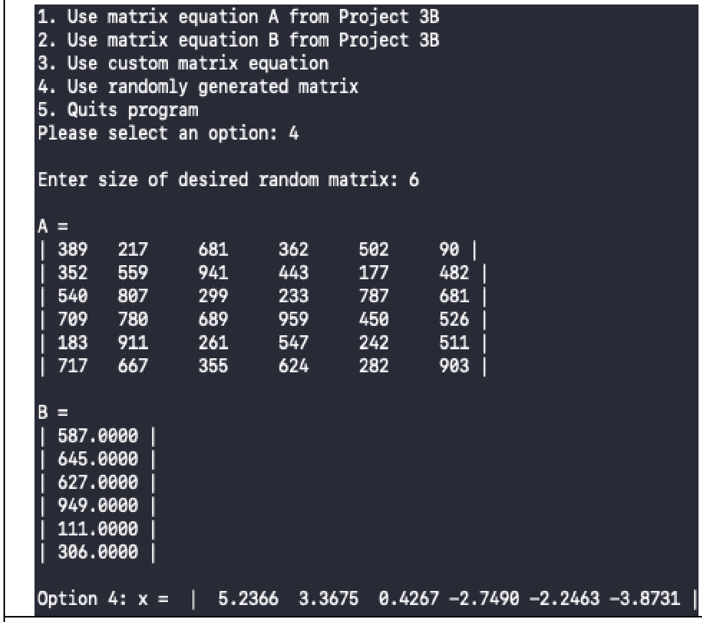
```
1. Use matrix equation A from Project 3B
2. Use matrix equation B from Project 3B
3. Use custom matrix equation
4. Use randomly generated matrix
5. Quits program
Please select an option: 4

Enter size of desired random matrix: 4

A =
| 249   299     806     556 |
| 586   873     272     907 |
| 794   158     85      787 |
| 677   787     498     387 |

B =
| 574.0000 |
| 321.0000 |
| 52.0000 |
| 518.0000 |

Option 4: x =  |  0.0549  0.2315  0.6851 -0.1098 |
```

5x5 matrix:



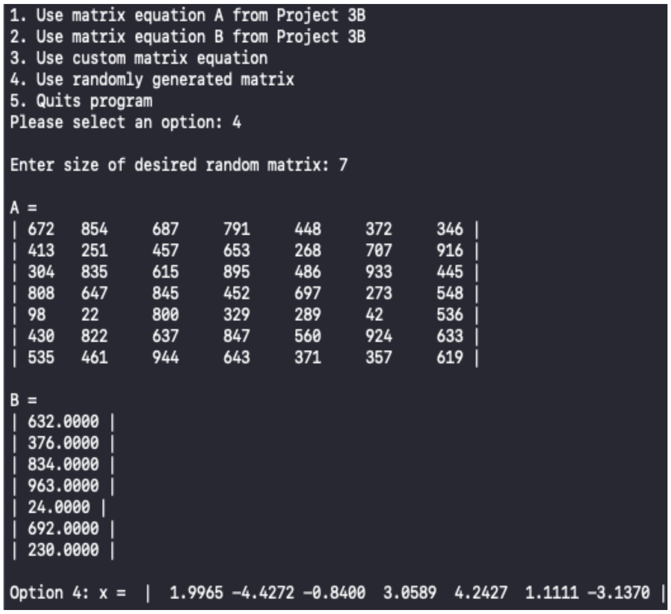
```
1. Use matrix equation A from Project 3B
2. Use matrix equation B from Project 3B
3. Use custom matrix equation
4. Use randomly generated matrix
5. Quits program
Please select an option: 4

Enter size of desired random matrix: 5

A =
| 457   889     258     653     958 |
| 928   839     651     680     410 |
| 675   476     593     95      498 |
| 387   465     619     666     440 |
| 884   831     178     727     188 |

B =
| 901.0000 |
| 638.0000 |
| 912.0000 |
| 418.0000 |
| 554.0000 |

Option 4: x =  |  2.4044 -2.6581 -0.9485  0.5566  2.1362 |
```

6x6 matrix:



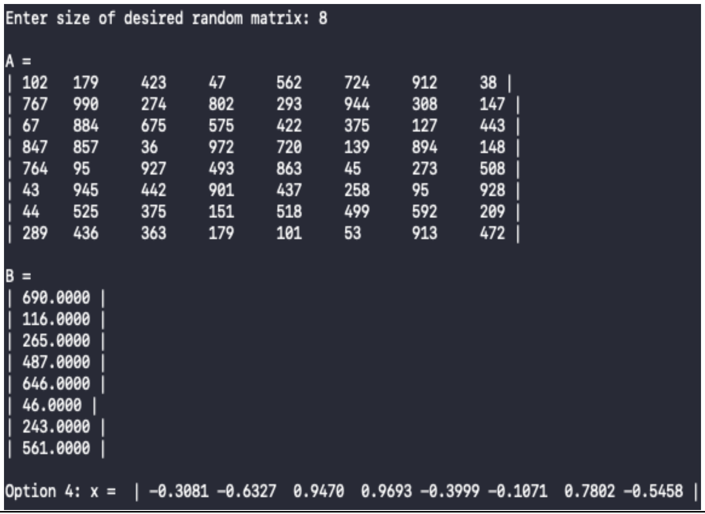
```
1. Use matrix equation A from Project 3B
2. Use matrix equation B from Project 3B
3. Use custom matrix equation
4. Use randomly generated matrix
5. Quits program
Please select an option: 4

Enter size of desired random matrix: 6

A =
| 389   217     681     362     502     90 |
| 352   559     941     443     177     482 |
| 540   807     299     233     787     681 |
| 709   780     689     959     450     526 |
| 183   911     261     547     242     511 |
| 717   667     355     624     282     903 |

B =
| 587.0000 |
| 645.0000 |
| 627.0000 |
| 949.0000 |
| 111.0000 |
| 306.0000 |

Option 4: x =  |  5.2366  3.3675  0.4267 -2.7490 -2.2463 -3.8731 |
```

7x7 matrix:



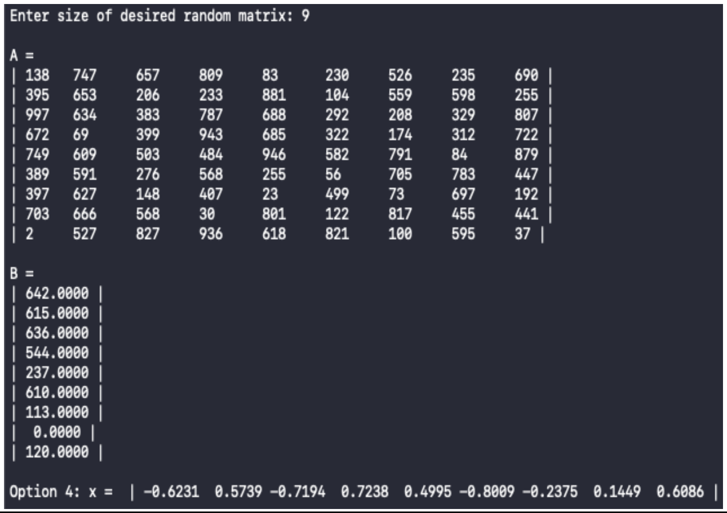
```
1. Use matrix equation A from Project 3B
2. Use matrix equation B from Project 3B
3. Use custom matrix equation
4. Use randomly generated matrix
5. Quits program
Please select an option: 4

Enter size of desired random matrix: 7

A =
| 672   854     687     791     448     372     346 |
| 413   251     457     653     268     707     916 |
| 304   835     615     895     486     933     445 |
| 808   647     845     452     697     273     548 |
| 98    22      800     329     289     42      536 |
| 430   822     637     847     560     924     633 |
| 535   461     944     643     371     357     619 |

B =
| 632.0000 |
| 376.0000 |
| 834.0000 |
| 963.0000 |
| 24.0000 |
| 692.0000 |
| 230.0000 |

Option 4: x =  |  1.9965 -4.4272 -0.8400  3.0589  4.2427  1.1111 -3.1370 |
```

8x8 matrix:



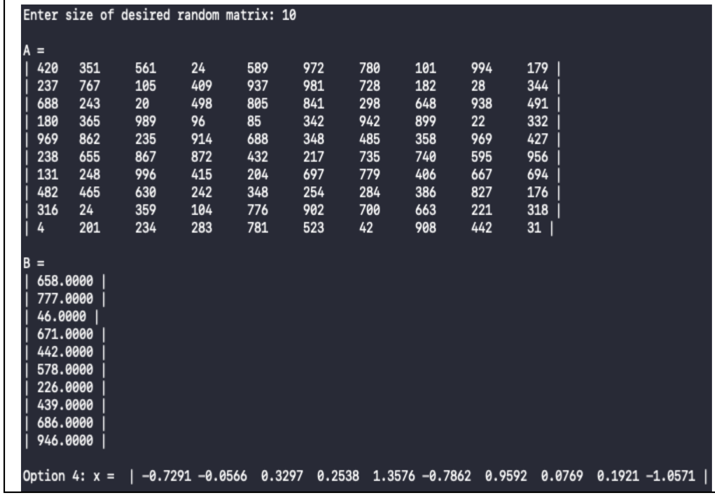
```
Enter size of desired random matrix: 8

A =
| 102   179     423     47      562     724     912     38 |
| 767   990     274     802     293     944     308     147 |
| 67    884     675     575     422     375     127     443 |
| 847   857     36      972     720     139     894     148 |
| 764   95      927     493     863     45      273     508 |
| 43    945     442     901     437     258     95      928 |
| 44    525     375     151     518     499     592     209 |
| 289   436     363     179     101     53      913     472 |

B =
| 690.0000 |
| 116.0000 |
| 265.0000 |
| 487.0000 |
| 646.0000 |
| 46.0000 |
| 243.0000 |
| 561.0000 |

Option 4: x =  | -0.3081 -0.6327  0.9470  0.9693 -0.3999 -0.1071  0.7802 -0.5458 |
```

9x9 matrix:



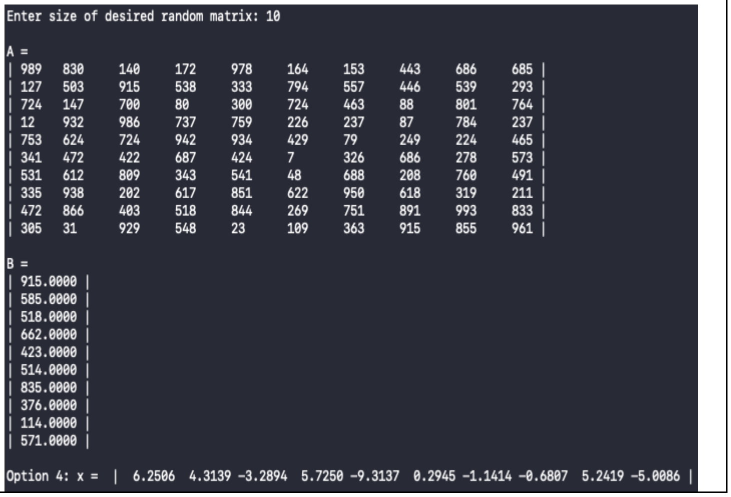
```
Enter size of desired random matrix: 9

A =
| 138   747     657     809     83      230     526     235     690 |
| 395   653     206     233     881     104     559     598     255 |
| 997   634     383     787     688     292     208     329     807 |
| 672   69      399     943     685     322     174     312     722 |
| 749   609     503     484     946     582     791     84      879 |
| 389   591     276     568     255     56      705     783     447 |
| 397   627     148     407     23      499     73      697     192 |
| 703   666     568     30      801     122     817     455     441 |
| 2     527     827     936     618     821     100     595     37 |

B =
| 642.0000 |
| 615.0000 |
| 636.0000 |
| 544.0000 |
| 237.0000 |
| 610.0000 |
| 113.0000 |
|  0.0000 |
| 120.0000 |

Option 4: x =  | -0.6231  0.5739 -0.7194  0.7238  0.4995 -0.8009 -0.2375  0.1449  0.6086 |
```

10x10 matrix:



```
Enter size of desired random matrix: 10

A =
| 420   351     561     24      589     972     780     101     994     179 |
| 237   767     105     409     937     981     728     182     28      344 |
| 688   243     20      498     805     841     298     648     938     491 |
| 180   365     989     96      85      342     942     899     22      332 |
| 969   862     235     914     688     348     485     358     969     427 |
| 238   655     867     872     432     217     735     740     595     956 |
| 131   248     996     415     204     697     779     406     667     694 |
| 482   465     630     242     348     254     284     386     827     176 |
| 316   24      359     104     776     902     700     663     221     318 |
| 4     201     234     283     781     523     42      908     442     31 |

B =
| 658.0000 |
| 777.0000 |
| 46.0000 |
| 671.0000 |
| 442.0000 |
| 578.0000 |
| 226.0000 |
| 439.0000 |
| 686.0000 |
| 946.0000 |

Option 4: x =  | -0.7291 -0.0566  0.3297  0.2538  1.3576 -0.7862  0.9592  0.0769  0.1921 -1.0571 |
```

10x10 matrix:



```
Enter size of desired random matrix: 10

A =
| 989   830     140     172     978     164     153     443     686     685 |
| 127   503     915     538     333     794     557     446     539     293 |
| 724   147     700     80      300     724     463     88      801     764 |
| 12    932     986     737     759     226     237     87      784     237 |
| 753   624     724     942     934     429     79      249     224     465 |
| 341   472     422     687     424     7       326     686     278     573 |
| 531   612     809     343     541     48      688     208     760     491 |
| 335   938     202     617     851     622     950     618     319     211 |
| 472   866     403     518     844     269     751     891     993     833 |
| 305   31      929     548     23      109     363     915     855     961 |

B =
| 915.0000 |
| 585.0000 |
| 518.0000 |
| 662.0000 |
| 423.0000 |
| 514.0000 |
| 835.0000 |
| 376.0000 |
| 114.0000 |
| 571.0000 |

Option 4: x =  |  6.2506  4.3139 -3.2894  5.7250 -9.3137  0.2945 -1.1414 -0.6807  5.2419 -5.0086 |
```

Source Code:

- Song_Henry_Project3.zip
    - Project Report.pdf
    - GaussianElimination.py
        - *Note: This file contains both the gaussian method and the driver scripts. To use, simply run ‘python3 GaussianElimination.py’ in local directory.
    - /matrix_calculation_images
        - 2x2.png
        - 3x3.png
        - 4x4.png
        - 5x5.png
        - 6x6.png
        - 7x7.png
        - 8x8.png
        - 9x9.png
        - 10x10_1.png
        - 10x10_2.png
- GitHub
    - https://github.com/henrysong671/MA375-Project3