Dynamic Competition in Electricity Markets with Mixed Hydroelectric and Thermal Generation

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Abstract

The behaviour of profit–maximizing hydroelectric generators in an unregulated electricity market is not fully understood due to the complexity introduced by the dynamics of water availability. We study dynamic competition between hydro and thermal electricity generators in a stochastic environment in which the hydro generator is constrained by water availability and the thermal generator by capacity. We compute the Feedback equilibrium over an infinite horizon and show that there can be strategic withholding of water by the hydro generator when the water inflow is sufficiently high. However, when the inflow of water is relatively low the hydro generator may actually use more water than socially optimal. This variation in the usage of water relative to the optimal level implies that there is not a simple relationship between market structure, price volatility and the frequency of price spikes. We find that the lack of social optimality of the market outcome is tempered by the capacity constraints: for a large range of possible thermal production capacities and water flow levels, welfare loss under the duopoly market structure is much less than would occur in the absence of water and capacity constraints.

Keywords: Electricity markets; Hydroelectricity; Imperfect Competition; Price Volatility

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1 Introduction

A common feature of many electricity markets is the coexistence of a variety of generation technologies, such as hydro, nuclear and thermal (coal, oil, gas) generation. In some jurisdictions, hydroelectric power generation is the dominant source of electricity. It accounts for 80% of generation in New Zealand, 97% in Brazil, 90% in Quebec, and 98% in Norway (Crampes and Moreaux (2001)). In other jurisdictions, such as Ontario and the Western United States, it is a significant source of electricity, but not as dominant. It is not uncommon to observe a large hydro producers competing with thermal generators. For example, in Honduras, large state-owned hydro generation facilities coexist with privately owned thermal generators.[1] Colombia has a similar structure, a large hydro operator with 64% of the installed capacity coexists with a thermal production sector with the rest of the capacity.[2]

We investigate the implications of the coexistence of hydro and thermal generation technologies for price and water dynamics when the producers are imperfectly competitive. One characteristic of hydroelectric power generation that makes this situation particularly interesting is that it is constrained by the dynamics of water availability. For hydro generators that use a reservoir to store water, generating more electricity in one period reduces the available water in the next period, and hence constrains generation at that time. In addition, these alternative generation technologies have rather different cost structures. Hydroelectric generation can be characterized by low marginal cost when operating, but subject to the availability of water to drive the turbines. In contrast, thermal generation units have more flexibility in the sense that their inputs (gas, coal, etc.) are not subject to the same constraints as water in a reservoir, however the marginal cost of generation is higher as generators need to purchase the fuel inputs.

The dynamic management of hydroelectric facilities takes on added importance when we consider that both the flow of water and the state of demand can fluctuate randomly. A common feature of restructured electricity markets is price volatility, so it is important to understand how the dynamic management of hydro generation affects the volatility of prices. One reason for the relatively high volatility of electricity prices is the inability to store electricity at a scale that would enable the smoothing of prices by agents storing electricity between periods of low and high price. However, the ability to store water behind a hydro dam does allow for some degree of price smoothing. A hydro operator may benefit from withholding water in periods with low prices in order to have more available for use in periods with high

[1] Installed generation capacities are approximately two-thirds hydro and one-third thermal in Honduras (see ENEE at www.enee.hn).

[2] See www.creg.gov.co.

prices. In a perfectly competitive market, it is likely that the hydro operators would choose their water release in a socially optimal way. However, in most jurisdictions, hydroelectric generators tend to be rather large producers, in which case there is no guarantee that water will be released optimally in an unregulated environment. We investigate this issue by a dynamic game between a hydro generator and a thermal generator in a fully unregulated environment. Comparing the equilibrium of this game to the socially optimal outcome allows us to determine the potential for suboptimal water use in an imperfectly competitive market for electricity.

The fact that hydro producers have relatively large market shares in many jurisdictions has led some authors to examine the issue of the use of market power by hydro producers. The papers most relevant to our work are Bushnell (2003) and Crampes and Moreaux (2001) as they examine models of Cournot competition in which the hydro producer behaves strategically.[3] Bushnell (2003) examines a Cournot oligopoly with fringe producers in which each producer controls both hydro and thermal generation facilities. Both hydro and thermal units face capacity constraints and the producers must decide how to allocate a fixed quantity of water over a number of periods. He solves the model with parameters calibrated to the western United States electricity market and finds that the dynamic allocation of water under imperfectly competitive conditions is not the efficient one. In particular, firms tend to allocate more water to off-peak periods than is efficient. Crampes and Moreaux (2001) model a Cournot duopoly in which a hydro producer uses a fixed stock of water over two periods while facing competition from a thermal producer. They find that hydro production is tilted towards the second period in the closed-loop equilibrium relative to the open-loop, hence there is strategic withholding of water in the first period by the hydro producer. Our model differs from these two in a couple of ways. In both Bushnell (2003) and Crampes and Moreaux (2001), a fixed stock of water is allocated across a finite number of periods as might be the case in a "within–year" model with wet periods followed by dry periods in which hydro reservoirs are exhausted.[4] In contrast, we examine an infinite horizon setting with regular water inflows, which can be viewed as modeling competition at a lower frequency, or as a situation in which there is not a clear dry season in which

[3]Scott and Read (1996) and Barroso et.al. (2002) also examine the behaviour of imperfectly competitive hydroelectric producers, but do not consider the dynamic aspects of the strategic behaviour of hydro producers. Garcia et.al. (2001) examine a strategic pricing game between two hydro producers who have a capacity constraint on their reservoirs, demonstrating that the Bertrand paradox of marginal cost pricing is mitigated as firms incorporate the opportunity cost of using water today rather than in a future period.

[4]Haddad (2011) studies a hydro management problem for a monopoly over two periods who faces alternating low and high water inflows and needs to allocate the available water over the two periods. His main focus is on determining the optimal reservoirs size.

reservoirs are exhausted. We allow for stochastic inflows, so there are "wet" and "dry" periods in our model, but these occur randomly. By also allowing for stochastic demand and a longer time horizon we are able to examine the implications of market power and water storage for the distribution of electricity prices.

Determining the optimal allocation of water for electricity generation is a daunting task due to the intertemporal constraints on hydro production, even in the absence of uncertainty. Borenstein and Bushnell (1999) acknowledge the difficulty of computing the optimal pattern of hydroelectric generation and instead implement a "peak-shaving" method in which hydro production is allocated to periods of peak demand. While tractable, this approach may understate or overstate the degree of market power depending on how the hydro producer's marginal revenue varies with demand.[5] We are interested in finding the equilibrium allocation of water by a profit-maximizing hydro producer and comparing that to the optimal allocation. The peak-shaving method of allocating hydro production is more closely related to the socially optimal solution in our model, as that results in generally using as much water as possible unless demand is relatively low.

We solve for the optimal pattern of generation in a model of dynamic quantity competition between a hydro and a thermal generator using a stochastic, dynamic game over an infinite time horizon. The hydro generator is constrained by water availability and the thermal generator is constrained by its capacity. We compute both the Feedback equilibrium as well as the socially optimal solution of the model using collocation techniques in order to compute approximations to the value function for the hydro producer. Specifically, we use a finite number of Chebyshev polynomials to approximate the expectation of the value function.

Using our computed solutions, we demonstrate that the hydro producer may engage in strategic withholding of water by comparing the Feedback and the myopic Cournot equilibria of the game. In addition, we compute the socially optimal solution to examine the departure from efficiency caused by market power in this setting. Our simulations show that, conditional on thermal capacity and water inflow not being too large, the outcome can be close to the social optimum. This result is interesting in light of the empirical work of Kauppi and Liski (2008) who find only small welfare losses for the Nordic power market even though they uncover evidence of the use of market power for hydro producers.

We turn next to a description of the basic aspects of the model for both the non-cooperative game between the hydro and thermal producers as well as the efficient outcome through the solution to a social planner's problem.

[5] Borenstein and Bushnell (1999, pg. 301).

In the third section we present our results via simulations of our numerical solution to both the game and the planner's problem. We do this for fixed thermal capacity and then allow capacity to vary in order to demonstrate how our model could be used to examine the incentives that the thermal player may have for investing in capacity.

2 The model

There are two types of technologies used in the industry: a hydroelectric generator uses water held behind dams to generate electricity and a thermal generator uses thermal units that burn fossil fuel. Thermal generation costs are quadratic in production, $C(q) = c_1 q + (c_2/2)q^2$, and production is subject to a capacity constraint, $q \leq K$. This results in a linear marginal cost up to capacity which is a commonly used functional form for modelling thermal generation marginal cost.[6]

Assuming that the hydro producer does not have to pay for the water it uses, it has a zero marginal cost of production. The hydro producer's electricity generation, h_t, is determined by a one-to-one relation with the amount of water it releases from its reservoir.[7] Its output is constrained by the amount of water available for release, W_t. The transition equation governing the level of water in the reservoir is

$$W_{t+1} = (1 - \gamma)(W_t - h_t) + \omega_t, \tag{1}$$

where W_t is the level of the reservoir at the beginning of period t, γ is a parameter that determines the rate of evaporation/leakage in the reservoir over an interval of time, and ω_t is the rate of inflow into the reservoir over an interval of time. The rate of inflow is random with a distribution function $F_\omega(\omega)$ and is observed after period t decisions are made. We do not impose a fixed capacity for the reservoir, although evaporation/leakage limits the accumulation of water.

The behaviour of consumers of electricity in any period $t = 0, 1, 2, ..., \infty$ is summarized by the following inverse demand function:

$$P_t = \alpha_t - \beta(h_t + q_t), \qquad \beta > 0. \tag{2}$$

The demand intercept, α_t, is random with a distribution function $F_\alpha(\alpha)$.

[6] Green and Newbery (1992) and Wolfram (1999) used a similar form for marginal cost in their empirical analyses of the British electricity market.

[7] In order to keep the model relatively simple, we are ignoring issues such as water pressure which can be an important consideration that links reservoir volume to electricity output.

Producers choose their outputs simultaneously in each period and both producers discount future payoffs with the common discount factor, $\delta \in (0, 1)$. We next describe the game played by the duopoly, after which we describe the efficient solution.

2.1 Duopoly

Each producer is assumed to maximize the discounted present value of profits. We focus on the case in which producers use Feedback strategies, which are functions of the current state, (α_t, W_t), only. Denote the strategies of the two producers by $s^H(\alpha_t, W_t)$ and $s^T(\alpha_t, W_t)$. We assume that both producers observe W_t and α_t before making decisions in period t. We will search for the Feedback equilibrium, which is a Nash equilibrium in strategies that depend only on the current state, (α_t, W_t).

Given the hydro producer's strategy, $s^H(\alpha_t, W_t)$, the problem for the thermal producer is to maximize the expected discounted sum of profits:

$$\max_{\{q_t\}} E \sum_{t=0}^{\infty} \delta^t \left[(\alpha_t - \beta(s^H(\alpha_t, W_t) + q_t))q_t - c_1 q_t - (c_2/2){q_t}^2\right] \tag{3}$$

subject to

$$0 \leq q_t \leq K.$$

The thermal producer's problem is simplified by the fact that the thermal producer does not influence the future state through its actions. Since its production decision does not affect its continuation payoff, thermal production is governed by its "static" best response function in the case of an interior solution. Incorporating the capacity and non-negativity constraints, we have[8]

$$s^T(\alpha_t, W_t) = \max\left[0, \min\left[\frac{\alpha_t - c_1 - \beta s^H(\alpha_t, W_t)}{2\beta + c_2}, K\right]\right] \tag{4}$$

Given the thermal producer's strategy, $s^T(\alpha_t, W_t)$, the problem faced by the hydro producer is

$$\max_{\{h_t\}} E \sum_{t=0}^{\infty} \delta^t \left[(\alpha_t - \beta(h_t + s^T(\alpha_t, W_t)))h_t\right] \tag{5}$$

subject to

$$0 \leq h_t \leq W_t$$

[8] As the thermal producer plays its static best response, extending this model to allow more thermal producers is possible. However, it would add more capacity constraints as well, complicating the analysis, so we limit our analysis to the case of one thermal producer only.

and

$$W_{t+1} = (1-\gamma)(W_t - h_t) + \omega_t,$$

The hydro producer's best response to the thermal producer's strategy, s^T, is determined by the solution to a dynamic optimization problem. The Bellman equation for the hydro producer's problem is

$$V(\alpha_t, W_t) = \max_{h_t \in [0, W_t]} \left\{ (\alpha_t - \beta(h_t + s^T(\alpha_t, W_t)))h_t + \delta E_t V(\alpha_{t+1}, W_{t+1}) \right\} \tag{6}$$

subject to (1). The solution to this problem yields $s^H(\alpha_t, W_t)$.

Define $\psi(h_t)$ as the derivative of the objective in the maximization problem in (6) with respect to h_t, i.e.,

$$\begin{aligned} \psi(h_t) = \alpha_t - 2\beta h_t - \beta s^T(\alpha_t, W_t) \\ - \delta(1-\gamma) E_t V_W(\alpha_{t+1}, (1-\gamma)(W_t - h_t) + \omega_t). \end{aligned} \tag{7}$$

Let b_{0t} and b_{Wt} be the Lagrange multipliers on the non-negativity and water availability constraints for the maximization problem in (6). The necessary conditions for optimal hydro output are then

$$\psi(h_t) + b_{0t} - b_{Wt} = 0 \tag{8}$$

$$b_{Wt}(W_t - h_t) = 0, \quad b_{Wt} \geq 0, \quad (W_t - h_t) \geq 0 \tag{9}$$

and

$$b_{0t} h_t = 0, \quad b_{0t} \geq 0, \quad h_t \geq 0. \tag{10}$$

Solving (8) – (10) for h_t yields the hydro producer's equilibrium strategy, $s^H(\alpha_t, W_t)$.

We can illustrate the strategic effect of the hydro producer's optimal choice of output by expanding the $E_t V_W(\alpha_{t+1}, W_{t+1})$ term in (7). Evaluating (6) at $t+1$, taking the expectation and differentiating with respect to W_{t+1} yields[9]

$$\begin{aligned} E_t V_W(\alpha_{t+1}, W_{t+1}) = E_t \Big[(\psi(h_{t+1}) + b_{0t+1} - b_{Wt+1}) s^H_W(\alpha_{t+1}, W_{t+1}) \\ - \beta s^T_W(\alpha_{t+1}, W_{t+1}) s^H(\alpha_{t+1}, W_{t+1}) \\ + b_{Wt+1} + \delta(1-\gamma) E_{t+1} V_W(\alpha_{t+2}, W_{t+2}) \Big] \end{aligned} \tag{11}$$

[9] As we see from (4), $s^T(\alpha_t, W_t)$ is a continuous, but kinked function. Consequently, $s^T_W()$ is discontinuous but still integrable and taking the expectation is valid.

Using (8) this simplifies to

$$E_t V_W(\alpha_{t+1}, W_{t+1}) = E_t\Big[-\beta s_W^T(\alpha_{t+1}, W_{t+1}) s^H(\alpha_{t+1}, W_{t+1}) \\ + b_{W t+1} + \delta(1-\gamma) E_{t+1} V_W(\alpha_{t+2}, W_{t+2})\Big] \quad (12)$$

Applying the same process for $V_W(\alpha_{T+2}, W_{t+2})$, $V_W(\alpha_{T+3}, W_{t+3})$, ... yields

$$E_t V_W(\alpha_{t+1}, W_{t+1}) = \\ E_t\Big[-\beta \sum_{i=0}^{\infty} \delta^i (1-\gamma)^i s^H(\alpha_{t+1+i}, W_{t+1+i}) s_W^T(\alpha_{t+1+i}, W_{t+1+i}) \\ + \sum_{i=0}^{\infty} \delta^i (1-\gamma)^i b_{W t+1+i}\Big]. \quad (13)$$

The strategic effect works through the influence of hydro output on future thermal output via future water availability. Since we expect $s_W^T(\alpha, W) \leq 0$, the hydro producer produces less output in the Feedback equilibrium relative an Open Loop equilibrium[10], for which the s_W^T terms are absent. This will result in more water available in future periods and hence lower thermal output in those future periods.

2.2 Social optimality

We wish to compare the outcome under duopoly to what is socially optimal. To this end, we solve the problem faced by a social planner choosing thermal and hydro generation with the objective of maximizing the expected present value of the stream of total surplus:

$$\max_{\{h_t, q_t\}} \sum_{t=0}^{\infty} \delta^t \left(\alpha_t(h_t + q_t) - \frac{\beta}{2}(h_t + q_t)^2 - c_1 q_t - \frac{c_2}{2} {q_t}^2 \right) \quad (14)$$

subject to

$$W_{t+1} = (1-\gamma)(W_t - h_t) + \omega_t,$$
$$0 \leq h_t \leq W_t,$$
$$0 \leq q_t \leq K.$$

[10] Open Loop equilibria are difficult to analyse in stochastic games. In the results presented below, we compare the Feedback equilibrium outcome to what occurs if the hydro producer behaved myopically.

The planner's value function then satisfies the Bellman equation:

$$V^P(\alpha_t, W_t) = \max_{h_t, q_t} \Big\{ \alpha_t(h_t + q_t) - \frac{\beta}{2}(h_t + q_t)^2 - c_1 q_t - \frac{c_2}{2} {q_t}^2 \\ + \delta E_t V^P(\alpha_{t+1}, W_{t+1}) \Big\} \tag{15}$$

subject to the above constraints.

The necessary conditions for the maximization problem are

$$\alpha_t - \beta(h_t + q_t) - \delta(1-\gamma) E_t V_W^P(\alpha_{t+1}, W_{t+1}) - b_t^W + b_t^0 = 0 \tag{16}$$

and

$$\alpha_t - \beta(h_t + q_t) - c_1 - c_2 q_t - a_t^K + a_t^0 = 0 \tag{17}$$

where b_t^W and b_t^0 are the Lagrange multipliers on hydro production's capacity and non-negativity constraints and a_t^K and a_t^0 are the multipliers on thermal production's capacity and non-negativity constraints. Equations (16) and (17) imply

$$a_K - a_0 + c_1 + c_2 q_t = \delta(1-\gamma) E_t V_W^P(\alpha_{t+1}, W_{t+1}) + b_W - b_0 \tag{18}$$

which for an interior solution simplifies to

$$\delta(1-\gamma) E_t V_W^P(\alpha_{t+1}, W_{t+1}) = c_1 + c_2 q_t, \tag{19}$$

the marginal value of retained water is equated with the marginal cost of thermal production.

2.3 Numerical solution algorithm

We now describe the algorithm we use to solve both the duopoly problem and the social planner's problem. As we use the same method for both, we just describe the method in terms of the duopoly problem here.

In order to solve (6), rather than approximate $V(\alpha_t, W_t)$ directly, we solve the problem by approximating $E_t V(\alpha_{t+1}, W_{t+1})$. We define $\widehat{W}_{t+1}$ to be the deterministic part of the transition equation (1): $\widehat{W}_{t+1} = (1-\gamma)(W_t - h_t)$, which is the quantity of water transferred from period t to period $t+1$ before the new inflow. We wish to find a function of $\widehat{W}_{t+1}$ that provides a good approximation to $E_t V(\alpha_{t+1}, W_{t+1})$. This approach has two benefits. First, it allows us to approximate a function of one variable only ($\widehat{W}_{t+1}$), whereas the value function itself is a function of two variables. Second, this approach has the added advantage that the expected value function will likely be a smooth function of $\widehat{W}_{t+1}$, while the value function itself may be kinked due to the constraints on both the hydro and thermal producers production.

We approximate the hydro producer's expected value function using the collocation method,[11] which approximates an unknown function with a linear combination of known basis functions at a known set of points. In particular,

$$E_t V(\alpha_{t+1}, \widehat{W}_{t+1} + \omega_t) \approx \sum_{i=1}^{n} d_i \phi_i(\widehat{W}_{t+1}) \equiv \widetilde{V}(\widehat{W}_{t+1}) \tag{20}$$

where the ϕ_i are known basis functions. Collocation proceeds by determining the d_i, $i = 1, ...n$, in order for the approximation to hold exactly at n collocation nodes, $\widehat{W}^1_+, \widehat{W}^2_+, ..., \widehat{W}^n_+$. For our application, ϕ_i is the i^{th} Chebyshev polynomial and the $\widehat{W}^i_+$ are the Chebyshev nodes. The algorithm we use to find the d_i is described as follows:

0. Choose a starting approximation $\widetilde{V}^0(\widehat{W}_+)$, i.e., starting values d_i^0, $i = 1, 2, ..., n$.

1. Given the current approximation, $\widetilde{V}^k(\widehat{W}_+)$, define $V^k(\alpha, W)$ as the solution to the maximization problem in (6) for a given value of α with $\widetilde{V}^k(\widehat{W})$ replacing the expectation of the next period's value function. This solution uses (4) to solve (8). A root-finding algorithm is used to find the optimal hydro production in the case of an interior solution.

2. For each of the collocation nodes, $\widehat{W}^1_+, \widehat{W}^2_+, ..., \widehat{W}^n_+$, integrate $V^k(\alpha, \widehat{W}^j_+ + \omega)$ numerically over possible demand states and water flows. Solve for the updated values, d_i^{k+1}:

$$\sum_{i=1}^{n} d_i^{k+1} \phi_i(\widehat{W}^j_+) = \iint V^k(\alpha, \widehat{W}^j_+) dF_\alpha(\alpha) dF_\omega(\omega) \qquad j = 1, 2, ..., n \tag{21}$$

   where $F_\alpha(\alpha)$ is the distribution of α and $F_\omega(\omega)$ the distribution of ω. As the ϕ_i are known functions and the $\widehat{W}^j_+$ known values, (21) is linear in the d's and so they are straightforward to compute once the values on the right-hand side of (21) are computed via numerical integration[12]. The updated approximation is then

$$\widetilde{V}^{k+1}(\widehat{W}) = \sum_{i=1}^{n} d_i^{k+1} \phi_i(\widehat{W}). \tag{22}$$

3. If $||d^{k+1} - d^k||$ is sufficiently small, stop. Else, return to step 1.

[11] Judd (1998), Chapter 11.

[12] In the solutions we present below, only one of α or ω is stochastic at a time, so only a single integration is required.

The parameter n is chosen so that the resulting approximation has relatively low residual error. The residual error is a measure of how well the approximation performs at points other than the collocation nodes. We plot the residual error for one of our scenarios in the appendix.

3 Results

We analyse the model by computing solutions numerically for particular parameter values, focusing on demand uncertainty for the presentation of our main results and then examining what changes when the source of uncertainty is the water inflow. The thermal cost function parameters used are $c_1 = 10$ and $c_2 = 0.025$, which are chosen to be roughly consistent with the ratio of these two parameters that was used in Green and Newbery (1992). The demand parameters are chosen so that there is a relatively small demand elasticity when used to compute the unconstrained (Cournot) solution. Setting $\beta = 20$ and $\mu_\alpha = 200$ (the mean of α_t) gives a demand elasticity of 0.54 at the Cournot solution.[13] Finally we choose $\delta = 0.9$, and $\gamma = 0.3$.

A useful benchmark to keep in mind is the equilibrium in an unconstrained situation. If neither producer were ever constrained (i.e., K and ω sufficiently large) the model would be a simple repeated Cournot game with random demand and firms having asymmetric costs. For these parameters, the hydro producer would produce approximately 3.5 units and the thermal producer 3 units on average resulting in an average price of 70.

We also present results for the myopic Cournot equilibrium to gauge the effects of the dynamics of water availability on the behaviour of the hydro producer. In a myopic Cournot equilibrium, the producers are able to respond to the realization of the demand state (α_t) and the hydro producer may be constrained by water availability, but the hydro producer does not strategically adjust future water levels (essentially, the $E_t V_W$ term in (7) is absent), so the strategies are functions of the demand shocks only ($h_t = s^H(\alpha_t)$ and $q_t = s^T(\alpha_t)$).

3.1 Demand uncertainty

We first examine the case in which $\sigma_\omega = 0$, so $\omega_t = \mu_\omega \, \forall t$, while the demand intercept is distributed normally: $\alpha_t \sim N(\mu_\alpha, \sigma_\alpha^2)$. The standard deviation for α_t is chosen to be 10% of the mean, so we have $\sigma_\alpha = 20$. We present results for alternative values of μ_ω and a large capacity for the thermal producer ($K = 6.0$, which is twice the average level of thermal output in

[13] This is near the upper end of the range of demand elasticities examined by Green and Newbery (1992).

the unconstrained game). A relatively high thermal capacity is chosen in order to analyse the effects of the water constraint on the equilibrium in the absence of a binding thermal capacity. After presenting results for this level of capacity we will then demonstrate the effects of varying the thermal capacity on the equilibrium outcome of the game.

Table 1 displays statistics for variables of interest under the equilibrium strategies. The statistics are created by generating 100 simulations of the model over 1,000 periods each.[14] The values in Table 1 are averages over the 100 runs. To compute the equilibrium strategies we use Chebyshev polynomials for the ϕ_i functions and choose n to provide an acceptable approximation.[15] We report n as well as an estimate of the maximum approximation residual for each case in the last two rows of Table 1.

[14]An initial run of 100 periods precedes the 1,000 period sample to minimize any effects of starting values.

[15]The computations are done with C++ and make use of routines for Chebyshev approximation, numerical integration, and root finding from the Gnu Scientific Library (2006). Computational time is minimal with the approximation and simulation together taking less than 5 seconds for the examples reported. In the appendix, we present a plot of the approximation residuals for the model with medium levels of inflow.

Table 1: Simulated Descriptive Statistics[a]: Demand Uncertainty

	Low inflow ($\mu_\omega = 1.75$)		Medium inflow ($\mu_\omega = 3.5$)		High Inflow ($\mu_\omega = 7.0$)	
	Duopoly	Optimal	Duopoly	Optimal	Duopoly	Optimal
Quantities:						
h[b]	1.74	1.72	3.26	3.46	3.50	7.00
q	3.87	5.99	3.11	5.67	3.00	2.49
$\%(h = W)$[c]	81.20	78.36	1.04	46.85	0.00	100.00
b_W	9.99	17.33	0.04	6.67	0.00	3.80
$\%(q = K)$[d]	0.00	100.00	0.00	55.84	0.00	0.02
a_K	0.00	35.51	0.00	7.16	0.00	0.00
Price:						
p	87.58	45.66	72.36	17.30	70.00	10.06
st.dev.(p)	9.58	17.09	7.08	11.04	6.66	0.05
skew.(p)	0.18	0.64	0.09	1.79	0.00	4.37
Water:						
W	1.77	1.83	4.05	3.59	15.17	7.00
st.dev.(W)	0.05	0.19	0.26	0.08	0.32	0.00
min(W)	1.75	1.75	3.50	3.50	13.81	7.00
max(W)	2.37	3.77	5.27	3.69	16.51	7.00
Payoffs:						
Π^H	1533.72	807.69	2389.12	617.44	2479.76	705.68
Π^T	3061.57	2146.08	1973.80	449.80	1825.34	0.95
Welfare	7787.57	8918.52	8485.45	9444.03	8583.69	9848.04
Myopic Cournot:						
h	1.77		3.48		3.50	
q	3.86		3.01		3.00	
Approx.Res.	10^{-7}	10^{-5}	10^{-4}	10^{-5}	10^{-9}	10^{-9}
n	10	10	13	4	2	2

[a] Statistics are generated from 100 runs of 1000 periods each.
[b] Unless otherwise indicated, values given are the mean of the variable over the simulations.
[c] Percentage of time that the water constraint binds.
[d] Percentage of time that the thermal capacity constraint binds.

3.1.1 High inflow

The high inflow case represents a benchmark in which neither of the constraints (water or capacity) are binding for the duopoly. For this scenario, we choose an inflow of water that is double mean hydro production in the unconstrained game discussed above ($\mu_\omega = 7.0$). In this case the approximation residual is very small (of the order 10^{-9}) with $n = 2$, which is expected given that the value function is quadratic if the constraints do not bind.

Not surprisingly, the duopoly equilibrium outcomes are what occur in the repeated Cournot game. Neither producer operates at capacity, so we just have an interior solution that replicates the Cournot outcome.[16] This is not socially optimal, since the planner would like to use more of the low cost technology, having the hydro producer at capacity in all periods. This scenario results in the largest welfare loss of the three examined. The duopoly price is seven times the optimal level[17] and there is substantial under-utilization of water, the water level being more than double the optimal level on average. This results in a shadow price of water that is zero for the duopoly hydro producer but significant (38% of the price level) for the planner.

The comparison between the duopoly and socially optimal outcomes in this case represents a measure of the static effect of market power alone as water dynamics do not influence the hydro producer in any way. The myopic Cournot solution is the same as the Feedback one since there the constraint on water availability never binds.

3.1.2 Low inflow

In order to examine the other extreme of water flow that substantially constrains hydro output we examine a scenario with $\mu_\omega = 1.75$, which is half of the average level of hydro production in the unconstrained version of the game (the high inflow case above). In this example, we expect hydro production to be frequently constrained by water availability.

For this low water inflow case, the hydro producer exhausts the available water 81% of the time which is actually more frequent than is socially optimal (78%) and we see that the average hydro output is actually higher than optimal. The reason for this is that the planner places a substantially higher value on water: the shadow price of water (b_W) is 17.3 for the planner vs. 9.99 for the hydro duopolist. This results in the planner wishing to maintain a higher average reservoir level ($E(W)$ of 1.83 vs. 1.77) for which it needs to

[16] This can be seen by the rows labelled $\%(h = W)$ and $\%(q = K)$ in Table 1, which report the percentage of periods for which the water and capacity constraints bind. A value of zero means that the constraints never bind.

[17] A very high price relative to marginal cost is expected in imperfectly competitive markets with low demand elasticities, a common situation in electricity markets.

produce less hydro electricity on average. Since thermal production is always at capacity in the social optimum, only hydro production can vary to adjust to demand fluctuations. This adds to the value of water to the planner in this scenario as it wishes to use the water not just to reduce the price level, but also to smooth the substantial price volatility. The standard deviation of the water level is much higher for the planner here that for the duopoly (0.19 vs. 0.05), which provides additional evidence that optimal hydro usage puts more weight on use of water for price smoothing than occurs in the duopoly.

The low water inflow reduces hydro producer payoffs and welfare relative to the high-inflow scenario. The thermal producer is better off, as it produces more output at a higher price relative to the high inflow case.

Comparing the myopic Cournot hydro output with that of the Feedback equilibrium shows that there is some reduction in output when the hydro producer considers the effects on future water availability, but it is rather small. In this scenario, since the water constraint is binding in most periods, the outputs under the two alternatives are almost always the same. However, in the periods in which the hydro producer does use less than the entire stock of available water, it curtails its production more in the Feedback equilibrium than in the myopic Cournot equilibrium.

3.1.3 Medium inflow

We now examine a case in which the water inflow is at an intermediate level which we take to be the output level in the unconstrained duopoly game. We set $\mu_\omega = 3.5$ which is the hydro producer's average level of output in the unconstrained game, guaranteeing that there is enough water in any period for the hydro producer to produce the same output as it would in the myopic Cournot equilibrium. This scenario allows the sharpest view on the extent to which the hydro producer will strategically withhold water since, unlike the high inflow scenario, there is a possibility that the hydro producer will be constrained and, unlike the low inflow scenario, it is likely that the available water will not constrain hydro generation most of the time.

The results are presented in the two central columns of Table 1. The 100% increase in inflow from 1.75 to 3.50 results in a 101% increase in optimal hydro production and a 97% increase in myopic hydro production, whereas the output of the hydro producer in the Feedback equilibrium increases by only 87% (from 1.74 to 3.26). This results in the hydro producer draining its reservoir only 1% of the time, whereas it is optimal to do so 47% of the time, resulting in a higher than optimal average reservoir level. This withholding of water is also reflected in the shadow price of water being close to zero for the duopolist while the optimal shadow price of water is 6.67, which represents 39% of the average price level.

The behaviour of the hydro producer results in a reduction of output relative to the optimal solution of 0.2 units (3.46-3.26). It is important to note that this output reduction is small relative to the output lost due to market power. This can be seen by examining the output of the thermal producer, who produces 3.11 units of electricity in the Feedback Equilibrium compared to an optimal output of 5.67 units, a reduction of 2.56 units. Hence, while the total electricity generated in the Feedback equilibrium falls short of the optimal level by 2.76 units (30% of the optimal level), only 7% of this shortfall is due to the withholding of water by the hydro producer.

3.1.4 Price volatility

From the solutions presented in Table 1, we see that price volatility, as measured by the standard deviation in price, is lower under the duopoly than is socially optimal when the water inflow is relatively low. However, the reverse occurs in the high inflow case. One force at work is the under-use of water by the hydro producer. When it is optimal to be using as much hydro generation as possible, water is not used for price smoothing in a significant way in the optimal solution. When constrained by water availability, there is simply no role for using water to smooth demand fluctuations. In contrast, under the duopoly market structure, the hydro producer is less often constrained by water availability and so reacts more to demand fluctuations, resulting in smoother prices.

Balancing the effect of the water availability constraint on price volatility is the effect of market power on how producers adjust output. As shown in Thille (2006), in a repeated Cournot game with uncertainty, imperfectly competitive firms will not adjust output as much as is optimal in response to demand shocks. This effect dominates the effect of the constraint on price volatility in the high inflow case since the constraint never binds, resulting in the duopoly producing higher price variability than is optimal in that case. Consequently, the effect of market power on price volatility depends on the degree to which the water availability constraint binds for the hydro producer.

3.1.5 Effects of Thermal Capacity

The results presented in Table 1 were generated with a thermal capacity large enough that it never bound in the Feedback equilibrium. This is rather unrealistic, especially since it seems that capacity constraints for the system as a whole have played an important role following electricity market deregulation.[18] In order to analyse how thermal generation capacity, K, affects the equilibrium outcome under the two market structures, we examine the medium inflow case of section 3.1.3 allowing for different levels of K. We solve the model for 20 different capacities ranging between zero and five units. For each solution, we simulate the model as above and plot some of the resulting statistics in Figures 1 and 2.

The top row of Figure 1 plots the average outputs of each producer by market structure. At low levels of thermal capacity, the thermal producer is essentially always operating at capacity, which is socially optimal. At large levels of capacity the thermal producer reduces output below capacity more frequently, resulting in sub-optimal output at higher capacities. In contrast, the hydro producer's average output is below capacity for any level K, although the magnitude of the difference is not large.

The bottom left graph in Figure 1 demonstrates that price is very close to the optimal level until thermal capacity reaches approximately 2.5. After this point, price levels off and approaches the Cournot price of 70.0, whereas the planner has price falling until thermal capacity is beyond 5.0. The implications for price volatility are demonstrated in the bottom right graph in Figure 1. The duopoly prices are less volatile than is optimal for all capacities depicted in Figure 1.

We plot payoffs and social welfare in Figure 2. Each payoff is very close to the socially optimal one for thermal capacities less than 2.5. From the above discussion we know that this is because the thermal constraint frequently binds under both market structures and the hydro producer does not greatly reduce output relative to the optimal level, so the outcome is not far from optimal.

Due to the substantial effect that K has on how close the duopoly outcome is to the optimal one, it is interesting to consider what thermal capacity would be chosen if the thermal producer could choose capacity in a previous period. Consider allowing the thermal producer to make a one time investment in capacity before time 0. The slope of the thermal producer's payoff in Figure 2 measures the benefit to the producer of a marginal addition to capacity. The thermal producer would choose a level of capacity that results in a significant departure from social optimality only if the the marginal cost of capacity is relatively low. Notice that for the outcome to diverge significantly from optimality in this case would require a capacity investment that exceeds the "average" Cournot output of the thermal producer (3.0 in this case). Since the region of capacity levels for which the thermal producer's payoff is relatively steep coincides with the region where the equilibrium

[18] For example Borenstein (2002) demonstrates how binding capacity constraints were key to the high market price following deregulation in California.

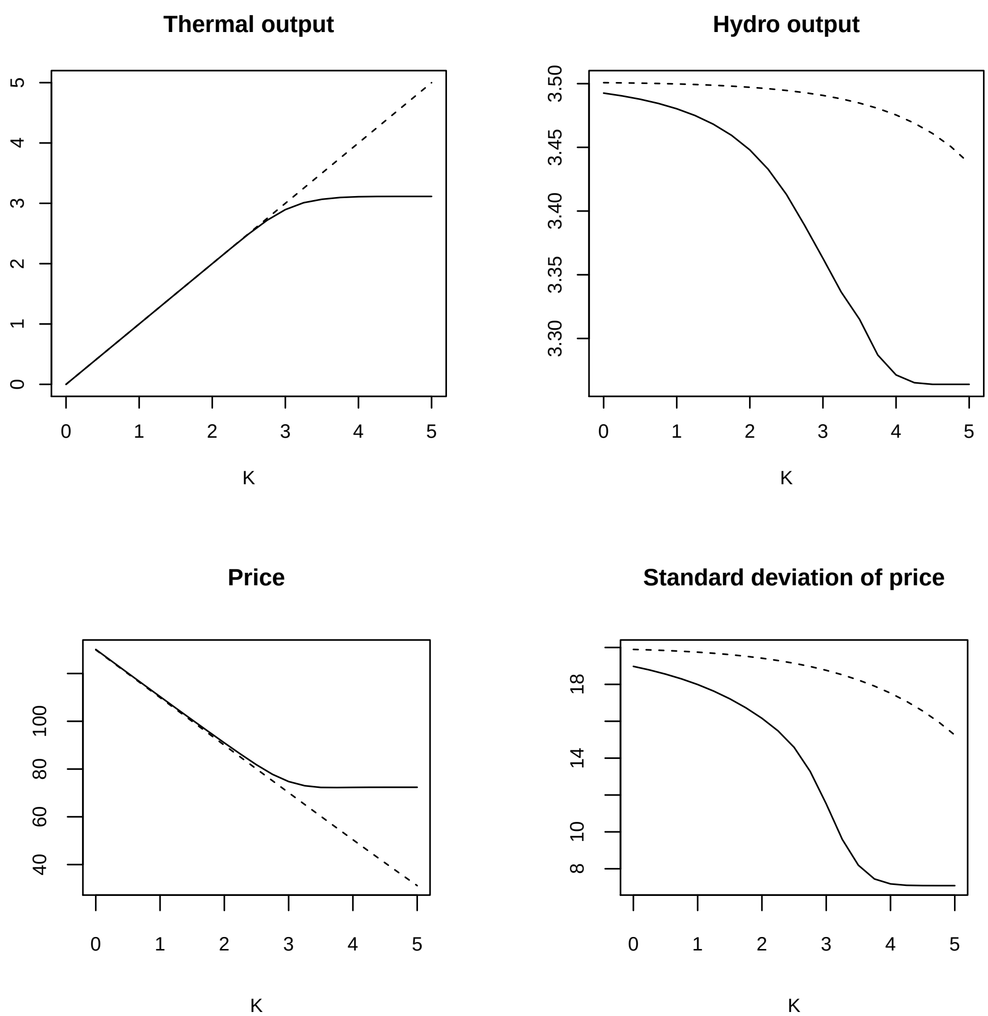

Figure 1: Average model values for alternative thermal capacities: Duopoly (solid line) and Socially Optimal (dashed line)

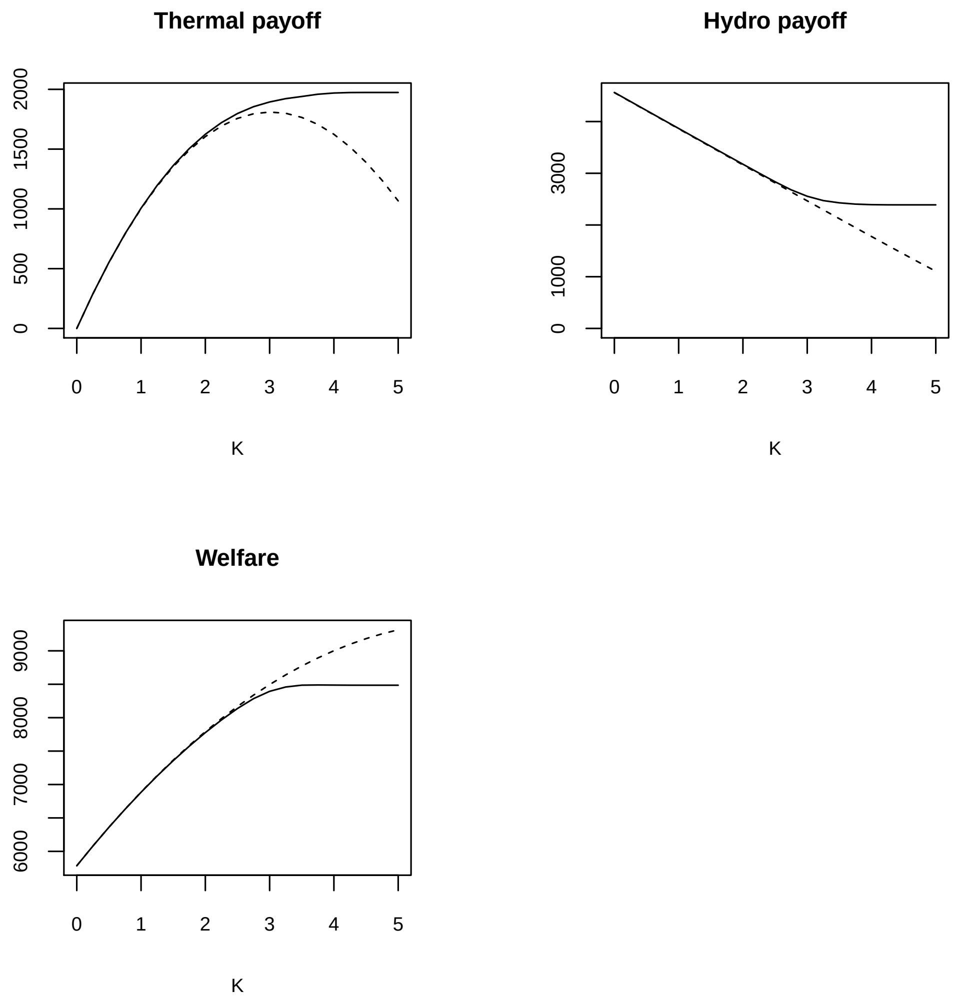

Figure 2: Payoffs for alternative thermal capacities: Duopoly (solid line) and Socially Optimal (dashed line)

is nearly optimal, we can suggest that *conditional on the level of capacity chosen*, the equilibrium in the dynamic duopoly game can be "close" to the optimal outcome if the marginal cost of capacity is not too low. Of course the planner may wish to choose a higher level of capacity, so an interesting extension of this game would be to examine optimal versus actual capacity choices.[19]

3.2 Water inflow uncertainty

An important characteristic of hydroelectric generation is that the flow of water into the system is often uncertain.[20] In order to examine the robustness of our results to this alternative source of uncertainty, we now examine the case in which demand is certain, but water inflows contain a random component: $\sigma_\alpha^2 = 0$ and $\sigma_\omega^2 > 0$. To model this case we now set $\mu_\omega = 3.5$,[21] $\sigma_\omega^2 = 0.35$, and $\sigma_\alpha^2 = 0$. All other parameter values remain the same as in the case with demand uncertainly only. The results are presented in Table 2.

With inflow uncertainty we see that most of the statistics are roughly similar to those in Table 1 with the exception of price skewness. In periods with a relatively high inflow realization, the amount of water available often exceeds the hydro producer's Cournot level of output. In these periods, neither producer varies output much with the water level, consequently price does not vary much. However, when there is a low realization of the random water inflow, the hydro producer becomes constrained and the adjustment of output to water level is more significant. These periods of low water availability result in a price "spike" which shows up as the high skewness in the price distribution.

The increased price skewness is not as pronounced for the optimal solution. As in the demand uncertainty case, it is optimal to use hydro generation as much as possible in most cases. This results in hydro generation constrained much more frequently than occurs under the duopoly. Hence, prices will be less skewed, although more volatile.

[19] The optimality of thermal investment decisions is examined for a two-period version of this game in Genc and Thille (2011). They find that equilibrium investment in thermal capacity may be higher or lower than optimal.

[20] See Bye et.al (2006) on the effects of inflow uncertainty on prices in the Nordic power market.

[21] We only examine the medium inflow scenario as this is the most interesting case in terms of the divergence from optimality.

Table 2: Simulated Descriptive Statistics: Inflow uncertainty

	Medium inflow ($\mu_\omega = 3.5$)	
	Duopoly	Optimal
Quantities:		
h	3.27	3.50
q	3.11	5.85
$\%(h = W)$	1.52	100.00
b_W	0.07	4.81
$\%(q = K)$	0.00	49.66
a_K	0.00	2.77
Price:		
p	72.38	12.91
st.dev.(p)	0.62	4.07
skew.(p)	3.06	1.64
Water:		
W	4.05	3.50
st.dev.(W)	0.44	0.35
min(W)	2.25	1.84
max(W)	5.98	4.93
Payoffs:		
Π^H	2364.37	441.50
Π^T	1940.79	172.30
Welfare	8378.38	9364.97
Myopic Cournot:		
h	3.48	
q	3.01	
Approx.Res.	10^{-4}	10^{-4}
n	14	14

4 Conclusion

Dynamic competition between thermal and hydroelectric producers under both demand and water inflow uncertainty has interesting implications for the distribution of electricity prices and the efficiency of market outcomes. When both thermal and hydro capacities bind frequently the duopoly outcome is not far from what is socially optimal given the capacity constraints. Our results illustrate that the worst case scenario occurs when neither thermal nor hydro constraints bind, suggesting that analyses of the costs of market power in electricity markets need to account for binding capacity constraints or else it will overestimate the degree of market failure.

A hydro producer does have an incentive to withhold water for sufficiently high water inflows. However, the reverse can occur when inflows are low: more water may be used than is socially optimal. This pattern of water usage by a hydro producer with market power has significant implications for price variability, since efficient water usage has storage of water driven by the desire to smooth price fluctuations. Prices are smoother than optimal for low to medium water inflows, but more volatile than optimal for high water inflows. In addition, the skewness of prices is also dependent on the usage of available water by the hydro producer and is affected by the source of uncertainty. Under demand uncertainty, we find that prices are less skewed than is optimal, while under water inflow uncertainty we find the reverse. As price skewness translates into the frequency of electricity price spikes, there is not a simple relationship between the frequency of price spikes and the structure of the market.

5 Appendix

Figure 3 shows the approximation residuals for the model in Table 1 with the largest approximation error, which is the duopoly with the medium inflow ($\mu = 3.5$). These residuals are the difference between our approximation, $\widetilde{V}(W_+)$, and the computed $EV(\alpha_+, W_+)$ for a set of W_+ 20 times larger than is used in computing the approximation. The residuals display the oscillations expected with Chebyshev approximation.

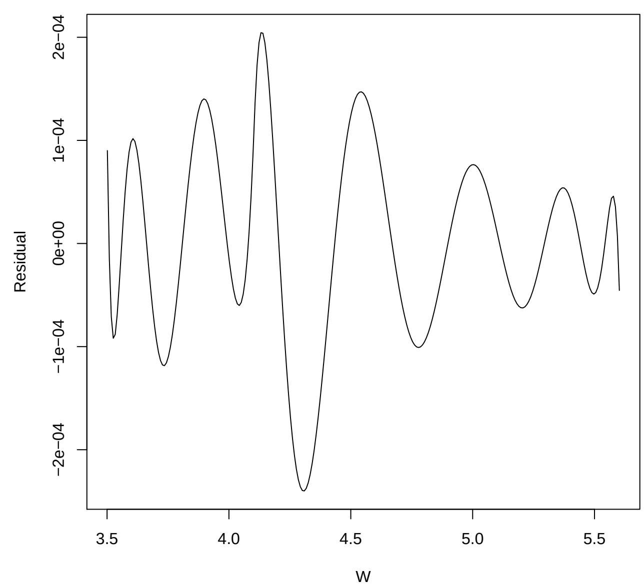

Figure 3: Approximation residual plot for case with demand uncertainty and medium inflow

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