Speculative storage in imperfectly competitive markets

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Abstract

Markets for many commodities are characterized by imperfectly competitive production as well as substantial speculative activity attracted by significant price volatility. We examine how speculative storage affects the behavior of an oligopoly producing a commodity for which demand is subject to random shocks. Speculators compete with consumers to purchase output and then subsequently compete with producers when selling their stocks, resulting in two opposing incentives: on the one hand, producers would like to increase production to capture future sales in advance by selling to speculators; while on the other hand, they would like to withhold production to deter speculation, thereby eliminating the additional supply from speculators in the next period. We find that these incentives are non-monotonic in the number of producers. Speculators are more active in a relatively concentrated oligopoly than in the extremes of monopoly or perfect competition. Furthermore, when production is concentrated the incentive to sell to speculators might result in a price that eliminates purchases by consumers entirely. Due to prices potentially being higher than in the Cournot equilibrium, the welfare effects of speculation are ambiguous.

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1 Introduction

Commodity market speculation and imperfect competition are two topics that have long been of interest to economists. Despite the attention given to each topic individually, little is known on how a market performs in circumstances where oligopolistic producers and competitive speculators interact, a topic that has recently been the subject of considerable public attention. For example, the world oil market has been the object of a recent surge in political interest, due to the sudden increase in speculation of the early 2000s. As pointed out in Smith (2009), this market is characterized by oligopolistic supply where OPEC is dominant along with speculation by both "commercial" and "non-commercial" traders. Similarly, the impact of speculation on price, the importance of inventories, the access to important resources for developing countries, and the overall economic performance of commodity markets have been the subject of several recent debates. For example, the U.S. Senate committee on Homeland Security and Governmental Affairs pointed out in 2006 that inventories of crude oil and natural gas have increased in the U.S. and in OECD countries[1], due to an overall increase in speculation that sustained high prices and gave financial incentives to agents to store. According to this report, the inventory-price relationship has been perturbed compared to the usual negative correlation historically observed[2]. Likewise, the impact of shortage risks on some strategic commodity markets has been the object of a recent communication[3] of the European Commission, who list 14 critical raw materials[4] for which concentration is high and supply lies in the hands of a few firms or in the control of a very small number of countries. The substitutability of these commodities with alternative materials is low in the short run, and for many of them recycling is also low. Finally, the formation of speculative bubbles on markets of vital or strategic importance for the development of emerging countries has attracted the attention of the United Nations Conference on Trade and Development[5], for their crucial consequences on economic development and on the risks populations face. These questions have triggered substantial academic interest investigating the relationship between inventories and speculative trading on commodity markets from an econometric point of view[6]. However little is known from a theoretical perspective on the performance of these markets in which oligopolistic producers face the constraints that competitive speculation

[1] *The role of market speculation in rising oil and gas prices: a need to put the cop back on the beat*, staff report by the Permanent Subcommittee on Investigations of the Committee on Homeland Security and Governmental Affairs, 109–65.

[2] *Ibid* p.15, figure 6

[3] "Tackling the Challenges in Commodity Markets and on Raw Materials" (EC COM(2011) 25 of 2.2.2011)

[4] Antimony, Beryllium, Cobalt, Fluorspar, Gallium, Germanium, Graphite, Indium, Magnesium, Niobium, Platinum Group Metals, Rare earths, Tantalum, and Tungsten.

[5] "Speculative Influence on Commodity Future Prices 2006-2008", by C. Gilbert (UNCTAD Discussion papers), where the markets studied are crude oil, three non-ferrous metals (aluminium, copper and nickel) and three agricultural commodities (wheat, corn and soybeans).

[6] See among others Frankel and Rose (2010), Kilian (2008, 2009), Kilian and Murphy (2011).

places on price dynamics. In this setting, strategic considerations[7] may be fundamental to production decisions which ultimately affect the level of inventories, the distribution of prices and social welfare. Hence, understanding the consequences of the interaction between imperfect competition and competitive speculation is an important issue that we seek to contribute to in this paper.

We analyze a model in which an oligopoly produces a good in each of two periods with demand subject to stochastic shifts. In addition to producers and consumers there are competitive agents who are able to store the good between periods. When storage by these speculators is possible, the net demand faced by producers is affected: on the one hand, speculators increase demand for the product when price is low as they buy the good to store for future sale. On the other hand, speculators increase the supply of the product when price is high and they wish to sell from their inventories. Hence speculators are competitors to producers when selling and competitors to consumers when buying, changing the effective demand faced by firms. This particular feature of speculation is magnified when producers are imperfectly competitive: do producers who have an individual influence on prices want to deter or encourage speculation? What are the consequences for the distribution of prices, for the levels of speculative inventories, and for social welfare?

We demonstrate that both demand uncertainty and the number of competitors are significant factors determining the nature of the equilibrium. Depending on expected demand growth and the number of producers of the commodity the equilibrium may entail either i) the deterrence of speculation, or ii) the accommodation or even encouragement of speculation, or iii) the complete exclusion of consumers from the first period market. The most interesting result we obtain is that speculative activity is non-monotonic in the number of firms in competition, being lower for the two extremes of monopoly and perfect competition and highest for an intermediate number of firms. This result is due to two countervailing forces in our model: first, producers have an incentive to sell to speculators in order to capture second period sales in advance; second, producers have an incentive to deter speculation in order to limit the future competition represented by speculative inventories. While the former incentive is absent in monopoly, but present in every oligopolistic market where a price differential allows speculators to store, the latter exists only when the price differential is limited, so that a small reduction of output reduces speculation. This may happen either under monopoly, where the firm reaps the entire benefit of deterring speculation, or under more competitive market structures, where prices are closer to marginal cost which limits the benefit of sales to speculators. This incentive to deter speculation can be quite strong. If first-period demand is relatively low firms may choose to produce no output at all in order to avoid purchases

[7] Much of the focus on imperfect competition in natural resource markets has been applied to the operation of cartels within these industries (see Teece et al. (1993)) in which strategic behavior is absent. In the case of non-renewable resources, the exhaustion constraint is also important. We do not consider exhaustion in our model in order to focus on the effects of speculation on its own.

by speculators. This type of equilibrium is also non-monotonic in the number of firms: it occurs under monopoly and for a relatively large oligopoly but is less likely in a relatively concentrated oligopoly.

Another interesting result we obtain is that, when expected demand growth is relatively large, speculators may purchase the entire first period output, resulting in zero first period consumption. This occurs when speculators value the good in the first period more highly than consumers do essentially out-bidding them for the current production.[8] This result is particularly interesting in connection with the political concerns mentioned above where speculation is thought to be damaging: speculation results in prices in the current period that are above what they would be in the absence of speculation, even to such an extent that consumers are squeezed out of the market. Furthermore, we demonstrate that this outcome occurs only for relatively concentrated market structures.

An important force comes from stochastic shifts in demand, which are shown to generate a degree of strategic complementarity in producer's best-response functions. Indeed uncertainty over future demand forces producers to take into account the risk that production will not be profitable in the future due to speculative inventories being sufficient to satisfy the entire demand. Since an increase in competitors' production lowers the current price it increases speculative inventories when speculators find it profitable to store. More inventories being carried in turn increases the likelihood of production not being profitable in the future due to the possibility that speculative inventories may be sufficient to meet all demand. This effect increases the incentive to produce and sell in the present relative to the future.

The combination of speculative storage and imperfect competition has important implications for our view of the welfare effects of speculation. We demonstrate that the overall effect of speculation on consumer surplus and total welfare is ambiguous, due to the possible occurrence of either the deterrence of speculation or the possibility of consumers being squeezed out of the first period market. Indeed the first-period price exceeds the Cournot price in these cases. Consequently, welfare may be reduced by speculation even though speculation reduces the volatility of prices.

We next provide a discussion of the relevant literature followed by a description of the model. We carefully discuss the implications of speculation for firms' residual demand in section 4 after which we examine the implication for marginal revenue and best-response functions in section 5. Section 6 contains the main theorem, in which we prove the potential existence five qualitatively different types of equilibrium and discuss the implications for welfare and the distribution of prices. Through a couple of computed examples we illustrate the existence of each of the alternative equilibria described in the theorem as well as demonstrate

[8]This equilibrium is related to the "entitlement failure" used by Sen (1981) to explain how famines can occur even when production is not particularly low and storehouses contain grain. In this situation, speculators' willingness to pay for the available supply exceeds that of consumers. This case is also relevant to the policy debate around anti-hoarding laws (see Wright and Williams (1984)).

the robustness of the results to a longer time horizon.

2 Relevant Literature

Modern approaches to the study of speculation in perfectly competitive markets was pioneered by Gustafson (1958) who was the first to formulate the problem in a dynamic programming context and highlight the important difficulty caused by the restriction that aggregate inventories cannot be negative for the solution of the model. Samuelson (1971) showed that the speculative storage problem could be reformulated as a social planner's problem. The theory pertaining to the rational expectations competitive storage model was further developed in a series of works including Newbery and Stiglitz (1981), Scheinkman and Schechtman (1983), Newbery (1984), Williams and Wright (1991), Deaton and Laroque (1992), and McLaren (1999). The focus in this literature is on the effects of storage on the distribution of prices caused by the movement of production across periods resulting from random production (harvest) shocks. As aggregate inventories cannot be negative, speculators smooth prices across periods only when positive inventories exist. Unexpectedly large prices result in stock-outs which leads to a breakdown of the price smoothing role of speculative storage. These occasional stock-outs lead to a skewed distribution of price.

Market power in commodity markets has been considered by examining imperfect competition in the storage function (Newbery (1984), Williams and Wright (1991), and McLaren (1999)), but production itself remains perfectly competitive in these works, hence there is no scope for strategic considerations on the part of producers. Another common element of these works is a focus on agricultural commodities, for which the assumption of perfectly competitive production is justifiable. However, for other commodities, such as metals, the assumption of perfectly competitive production is harder to justify. The problem of a monopoly producer facing competitive storage was touched on briefly in Newbery (1984), although he did not solve it. Mitraille and Thille (2009) examine a monopolist producer facing competitive storage using a model with capacity constrained speculators. They show that both the level and variance of prices are affected by speculation under monopolistic production. In particular, the presence of speculators may cause the monopolist to set a price higher than the monopoly one in order to eliminate competition from speculative sales in future periods, a result similar to that of limit pricing. In this paper we allow for an arbitrary number of firms and impose no constraint on speculative capacity, which allows us to examine the consequences of speculation for the functioning of an oligopolistic market.

Our paper lies at the cross-roads of two streams of research in industrial organization: imperfect competition under demand uncertainty on the one hand, and the literature studying strategic uses of storage on the other hand. Imperfect competition under demand uncertainty has not been studied much in industrial organization with the exception of the seminal article by Klemperer and Meyer (1986). They study how price or quantity competition interacts with demand uncertainty in a static game where decisions are taken before uncertainty is revealed in order to examine the preference of firms for price versus quantity competition. In our two-period model of Cournot competition, demand uncertainty can cause producers to manipulate speculation in either a pro- or anti-competitive manner, depending on current demand relative to future demand. Moreover, demand uncertainty introduces a degree of strategic complementarity in firms' strategies, despite being in a Cournot setting with perfect substitutes. To the best of our knowledge, these features have not been identified in either this earlier literature or by the literature on strategic storage. Indeed, although the speculative storage literature focuses on the storage function being performed by speculators, it is reasonable to consider storage being performed by producers or consumers themselves. Arvan (1985) and Thille (2006) examine the former case, in which producers store for strategic reasons, as inventories are accumulated in order to gain a cost advantage. Storage by consumers is more similar to the speculative storage that we examine in that storage is undertaken by competitive agents. Consumer storage with duopoly production is examined in Anton and Varma (2005) and Guo and Villas-Boas (2007). Whereas Anton and Das Varma show that consumer storage has a pro-competitive effect in a two-period duopoly, Guo and Villas-Boas show that horizontal differentiation may mitigate this effect. Consumer storage under monopoly production is examined by Dudine et al. (2006) who find that consumer storage can have an anti-competitive effect as the monopolist may raise price to limit storage. By examining a general n-firm oligopoly facing demand uncertainty, we are able to discuss under what conditions these pro- and anti-competitive effects occur. Furthermore, we show that the effect of speculation on the functioning and the performance of an oligopolistic market with respect to consumer surplus, social welfare and the distribution of prices, are non-monotonic in the number of competitors.

3 The Model

Consider the market for a homogeneous product consisting of producers, consumers, and speculators. We wish to focus on the effects of speculation alone, so we assume that all agents are risk-neutral. There are n producers in Cournot competition over two periods, indexed by $t \in \{1, 2\}$. Let $\delta \in (0, 1]$ be the agents' common discount factor. In each period t, firm $i \in \{1, ..., n\}$ chooses the quantity it wants to produce, $q_t^i \in \mathbb{R}^+$. All firms face the same quadratic cost function of $\frac{c}{2} {q_t^i}^2$. Convex production costs are not crucial to our results, however, as we discuss below, speculators' behavior can cause problems for the concavity of producer revenue. By allowing for sufficient convexity in the cost function we are able to simplify the exposition. We assume that firms are unable to store their production between the two periods. Clearly it would be more realistic to allow firms to also store, however that would add a substantial degree of complexity to the analysis potentially clouding the effects of independent speculators.[9] We denote the aggregate quantity produced in period t by Q_t, and the aggregate quantity produced by all firms but i by Q_t^{-i}, where $Q_t = \sum_{i=1}^{n} q_t^i$ and $Q_t^{-i} = \sum_{j=1,j\neq i}^{n} q_t^j$. Producer i's payoff, Π^i, is

$$\Pi^i = p_1 q_1^i - \frac{c}{2}{q_1^i}^2 + \delta E_1 \left(p_2 q_2^i - \frac{c}{2}{q_2^i}^2 \right). \tag{1}$$

In order for speculation to be potentially profitable, we require some price variation over time. We ensure this by having demand vary randomly across periods.[10] This assumption also has the implication that storage is potentially socially valuable in this model as it allows producers to smooth their convex production costs across periods. In each period t, consumers' aggregate demand for the product is given by,

$$D_t(p_t) = \max\{a_t - p_t, 0\} \tag{2}$$

where a_t is independently and identically distributed over the support $[0, A]$. We denote the continuous cumulative distribution function for a_t as $F(a)$, with $f(a)$ the associated density function. Random changes in a_t may be interpreted as random shocks affecting the distribution of income in the population of consumers from period to period, modifying in turn the willingness to pay for the product sold by firms and stored by speculators. The distribution of the intercept of market demand is common knowledge to both producers and speculators who learn the current state of demand, a_t, prior to any production or storage decisions in a period. Information is consequently symmetric between producers and speculators.

We model speculation in the same way as is done in the competitive speculative storage literature (Newbery (1984), Deaton and Laroque (1992, 1996)). In each period a large number of risk-neutral, price-taking agents exist who have access to a storage technology and face no borrowing constraints. The storage technology allows speculators to store the commodity at a unit cost of w between periods. The expected return to storing a unit of the commodity from the first to the second period is $\delta(E_1[p_2] - w) - p_1$. The aggregate behavior of these speculators ensures that $p_1 \geq \delta(E_1[p_2] - w)$ with a stock-out occurring if the inequality is strict. We use X_t to denote aggregate speculative sales in period t (purchases have $X_t < 0$) and H_t to denote beginning of period stocks. We assume that $H_1 = 0$ and so $H_2 = -X_1$ describes the inventory dynamics.[11] We also assume that inventories unsold at the end of the

[9]See Arvan (1985) and Thille (2006) for analyses of producer storage under imperfect competition.

[10]Having the source of variation be in demand means that for a sufficiently large number of producers speculation will not occur as price will tend to equality in the two periods.

[11]We seek to analyze the effects of the entry of speculators. Allowing positive initial stocks would introduce an exogenous source of supply, clouding the comparisons with the standard Cournot model. This means that stock-outs, the selling of all speculative stocks, cannot occur in the first period of our model. However, the situation in which zero inventories are carried into the second period is analogous to a stock-out.

second period can be disposed at no cost. To ensure that speculation is at least potentially profitable, we assume that consumers expected valuation exceeds the unit cost of storage, $E[a] \geq w$.

The timing of moves within a period is simply the standard Cournot timing with demand adjusted for net speculative supply: producers set outputs and then price adjusts to clear the market with demand now equal to the sum of consumer demand and speculators' net demand. We search for the subgame-perfect Nash equilibrium to this game with rational expectations on the part of speculators. We begin our analysis in the next section by examining the implications of speculation for producers' inverse demand.

4 Effects of Speculation on Net Demand

In this section we derive the inverse demand function faced by firms after the behaviour of speculators is taken into account. Since speculators' behaviour depends on the expected price in the second period, we start by deriving the second period equilibrium for a given level of speculative inventories. We then examine the effect of speculation on the oligopolist's first period inverse demand.

Since there is no salvage value to inventories held at the end of the second period, speculators will sell the quantity they have stored as long as $p_2 > 0$. In this case aggregate speculative sales are equal to $X_2 = H_2$. If on the other hand the aggregate quantity stored by speculators is such that the market price is equal to zero once H_2 is sold, then the aggregate speculative sales cannot exceed the maximal quantity consumers are ready to buy given the aggregate production of firms, $a_2 - Q_2$. In that case, aggregate speculative sales are simply $X_2 = a_2 - Q_2$ and the price is zero. Consequently, the inverse demand faced by producers in the second period is

$$P_2(H_2, Q_2) = \max\{a_2 - H_2 - Q_2, 0\}. \tag{3}$$

Given the residual inverse demand $P_2(H_2, Q_2)$ it faces, each producer chooses the quantity q_2^i to maximize its second period profit, $\pi_2^i = P_2(H_2, Q_2)q_2^i - \frac{c}{2}{q_2^i}^2$, with respect to the quantity produced q_2^i. For the ease of exposition we introduce the following notation:

Definition 1 *Let*

(a) $\beta = (1 + c)/(n + 1 + c) \in (0, 1)$.

(b) $\gamma = (2 + c)/2(n + 1 + c)^2 \in (0, 1)$.

For a given c, β reflects the impact of competition on the market price and γ reflects the impact of competition on each firm's profit. The larger the number of producers the closer β and γ are to zero. Using Definition 1, the unique and symmetric pure strategy Nash equilibrium for the second period subgame is given by the following functions of inventories and demand[12] (H_2, a_2):

$$q_2^*(H_2,a_2) = \frac{1}{n}(1-\beta)\max[a_2 - H_2, 0], \quad Q_2^*(H_2,a_2) = (1-\beta)\max[a_2 - H_2, 0], \tag{4}$$

$$p_2^*(H_2,a_2) = \beta \max[a_2 - H_2, 0], \tag{5}$$

$$X_2^*(H_2,a_2) = \min[H_2, a_2], \tag{6}$$

$$\pi_2^*(H_2,a_2) = \gamma(\max[a_2 - H_2, 0])^2. \tag{7}$$

It is important to note that an increase in speculative inventories H_2 reduces each firm's second period marginal profit by a factor which decreases with the number of producers. That is, the larger the number of producers, the less sensitive producers become to the competition of speculators in second period.

Turning now to the first period, we note that the non-negativity constraint on aggregate speculative inventories implies that speculators aggregate behaviour in the first period satisfies the complementarity condition

$$-X_1 \geq 0, \quad p_1 - \delta(E_1[p_2^*] - w) \geq 0, \quad -X_1(p_1 - \delta(E_1[p_2^*] - w)) = 0. \tag{8}$$

Either no inventories are carried and the return to storage is negative, or inventories are carried and the return to storage is zero. We will use $X_1^*(Q_1)$ to denote the equilibrium storage undertaken when producers sell Q_1 in aggregate. The market clearing price, $P_1(Q_1)$, must be such that, given Q_1, the total of consumer and speculative purchases satisfy

$$D_1(P_1(Q_1)) - X_1^*(Q_1) = Q_1. \tag{9}$$

There are three possible situations that we must concern ourselves with depending on who purchases first period output. First, speculators may buy nothing in the first period and producers sell only to consumers. Second, both speculators and consumers may purchase the good in the first period. We will refer to this case as the "smoothing" outcome, as it corresponds to the standard case of speculators smoothing prices across time. Third, consumers may buy nothing in the first period and producers sell only to speculators.

Consider first the case in which speculators do not purchase. Consumers will buy the entire industry supply if their marginal willingness to pay for it exceeds speculators expected return of storing, $p_1 = a_1 - Q_1 \geq \delta(\beta E[a] - w)$, as $E_1[p_2^*] = \beta E[a]$ when no inventories are carried into the second period. If this were not the case inventories could profitably be

[12] We will often suppress the arguments of these functions and simply refer to them as q_2^*, p_2^*, Q_2^* and π_2^* as convenient.

carried forward. Hence, a unique threshold output exists such that the first period price is independent of a_1 and equal to the future expected price when no inventories are carried forward, $p_1 = \delta(\beta E[a] - w)$. We denote this threshold output by Q_L, given by

$$Q_L = a_1 - \delta(\beta E[a] - w), \tag{10}$$

which is the level of industry output that just renders speculators indifferent between storing and not storing. If $0 \leq Q_1 < Q_L$, speculators do not find profitable to store the product, as consumers have a higher marginal willingness to pay for Q_1. If $Q_1 \geq Q_L$, speculators will purchase and store the product.

If Q_L is negative, that is if $a_1 < \delta(\beta E[a] - w)$, then speculators can never be excluded from the market. However in this case consumers may be excluded if consumers' willingness to pay is low relative to the expected return speculators may obtain by storing. In this case speculators may purchase the good at prices that exclude consumers in first period. An important effect to keep in mind here is that the lower is a_1, the higher is speculators' demand (if a low a_1 indeed corresponds to a low equilibrium p_1). Hence, speculator and consumer demands are negatively correlated, introducing the possibility that speculators squeeze consumers out of the market. Consumers will not purchase if $p_1 \geq a_1$ and in order for speculators to purchase $X_1 = -Q_1$ it must be the case that $p_1 = \delta(E[p_2^*(Q_1, a_2)] - w)$, as otherwise the market would not clear. We can define a threshold output, $\widehat{Q}$, such that speculators purchase the entire first period output for $Q_1 < \widehat{Q}$. This threshold is defined by the output at which consumer demand is just extinguished, $p_1 = a_1$, and speculators are willing to store the first period output, $p_1 = \delta(E[p_2^*(\widehat{Q}, a_2)] - w))$. Consequently, this threshold is defined by

$$\delta \left(\beta \int_{\widehat{Q}}^{A} (a - \widehat{Q}) dF(a) - w \right) - a_1 = 0. \tag{11}$$

If Q_1 exceeds $\widehat{Q}$ both speculators and consumers purchase the product in period one.

Finally, when both consumers and speculators purchase the commodity in the first period the quantity stored is determined implicitly by the relationship between first period and expected second period prices. We define the position of speculators when smoothing occurs at non-zero[13] p_1 to be $\widetilde{X}(Q_1)$, which is the solution in X to

$$a_1 - X - Q_1 = \delta \left(\beta \int_{-X}^{A} (a + X) dF(a) - w \right). \tag{12}$$

We summarize these results for speculators' behavior with the following lemma:

[13] There is also a possibility that speculators buy at a zero first period price, however, we do not discuss this case as it will not occur in equilibrium.

Lemma 1 *There exist unique Q_L, $\widehat{Q}$, and $\widetilde{X}(Q_1)$ solving (10), (11) and (12) with the properties*

(i)

$$\frac{d\widetilde{X}}{dQ_1} = -\frac{1}{1+\delta\ \beta(1-F(-\widetilde{X}(Q_1)))} < 0$$

(ii)

$$\widehat{Q} = 0 \Leftrightarrow Q_L = 0 \quad \text{and} \quad sign(\widehat{Q}) = -sign(Q_L).$$

Consequently, speculative sales are given by

$$X_1^*(Q_1) = \begin{cases} 0 & \text{if} \quad \widehat{Q} < 0 \leq Q_1 \leq Q_L \\ -Q_1 & \text{if} \quad Q_L < 0 \leq Q_1 \leq \widehat{Q} \\ \widetilde{X}(Q_1) & \text{otherwise.} \end{cases} \tag{13}$$

Proof. See appendix.‖

Part *(i)* of Lemma 1 establishes that speculative sales are monotonically decreasing in first period output (as $X_1 \leq 0$ this means that purchases are increasing). Part *(ii)* of the lemma establishes that one and only one of Q_L and $\widehat{Q}$ is positive, which implies that one and only one of the first two conditions in (13) is possible. The fact that unique Q_L and $\widetilde{X}(Q_1)$ exist simply confirms the finding of the competitive storage literature. Compared to that literature however, the existence of $\widehat{Q}$ is more novel, as in our setting the possibility that consumers may not be served at all must be considered. Studying this possibility is particularly important as we shall see after having presented our main theorem, because the incentives of oligopolists to produce when demand is low enough are entirely driven by the future expected price paid by speculators and not the value of consumers current demand.

The inverse demand that is faced by producers in period one is then

$$P_1(Q_1) = \begin{cases} a_1 - Q_1 & \text{if} \quad \widehat{Q} < 0 \leq Q_1 \leq Q_L \\ \delta\beta \int_{Q_1}^{A} (a - Q_1) dF(a) - \delta w & \text{if} \quad Q_L < 0 \leq Q_1 \leq \widehat{Q} \\ a_1 - \widetilde{X}(Q_1) - Q_1 & \text{otherwise.} \end{cases} \tag{14}$$

We can see in (14) that producers only will care about second period demand to the extent that they are able to make sales in the second period: the expectation in the second line of (14) is only over values of a_2 that exceed Q_1 (which is the inventories carried in this case), and in (12) where the expectation is only for values of a_2 larger than $-X$.

5 Effects of Speculation on Producer Behaviour

Using (13) and (14) we now describe how the addition of speculative storage affects the payoffs of producers. We will let $\Pi^i(q_1^i, Q_1^{-i})$ denote the total profit to firm i when it produces q_1^i and the other firms produce Q_1^{-i} in aggregate while incorporating the equilibrium of the second period game:

$$\Pi^i(q_1^i, Q_1^{-i}) = P_1(Q_1)q_1^i - \frac{c}{2}{q_1^i}^2 + \delta E_1[\pi_2^*] \tag{15}$$

with $Q_1 = q_1^i + Q_1^{-i}$. As speculators' position, $X_1^*(Q_1)$, and inverse demand, $P_1(Q_1)$, are continuous[14] piece-wise differentiable, individual profit is also continuous and piece-wise differentiable. The marginal profit of a firm choosing a level of output q_1^i in period one, where it exists, is equal to

$$\Pi_q^i(q_1^i, Q_1^{-i}) = P_1'(Q_1)q_1^i + P_1(Q_1) - c\ q_1^i + \delta\ \frac{\partial E_1[\pi_2^*]}{\partial Q_1} \tag{16}$$

where first period output affects expected second period profit because $H_2 = -X_1^*(Q_1)$, and $P_1'(Q_1)$ denotes the derivative of first period inverse demand with respect to the industry output Q_1. The effects of first period output choice on expected second period profit is summarized in Lemma 2.

Lemma 2 *A producer's expected second period profit is decreasing in aggregate first period production, Q_1, when speculators purchase in the first period:*

$$\frac{\partial E_1[\pi_2^*]}{\partial Q_1} = 2\gamma\ \frac{dX_1}{dQ_1} \int_{-X_1(Q_1)}^{A} (a + X_1(Q_1))dF(a) \leq 0 \tag{17}$$

where dX_1/dQ_1 is equal to -1 when speculators buy the entire production in first period, or to $d\widetilde{X}/dQ_1 \in (-1, 0)$ when both consumers and speculators buy in first period.

Proof. See appendix.‖

We are now in a position to describe the marginal profit of a producer's choice of first period output. When $a_1 \leq \delta(\beta E[a] - w)$ speculators are always active and consumers are excluded from the market when total output is lower than $\widehat{Q}$. The marginal profit of firm i for an output level $q_1^i \leq \widehat{Q} - Q_1^{-i}$ is equal to

$$\Pi_q^S(q_1^i, Q_1^{-i}) \equiv \delta(\beta - 2\gamma) \int_{Q_1}^{A} (a - Q_1)dF(a) - \delta w - \delta\beta(1 - F(Q_1))q_1^i - cq_1^i, \tag{18}$$

[14] Continuity follows from the definitions of Q_L, $\widehat{Q}$, and $\widetilde{X}(Q_1)$.

where we use an S superscript to denote case in which speculators purchase the entire first period output. If $q_1^i \geq \widehat{Q} - Q_1^{-i}$ marginal profit is equal to

$$\Pi_q^{CS}(q_1^i, Q_1^{-i}) \equiv a_1 - \widetilde{X}(Q_1) - Q_1 - \left(1 + \frac{d\widetilde{X}}{dQ_1}\right) q_1^i - cq_1^i \\ - \frac{2\gamma(a_1 - \widetilde{X}(Q_1) - Q_1 + \delta w)}{\beta(1 + \delta\beta(1 - F(-\widetilde{X}(Q_1)))}, \quad (19)$$

where we use a CS superscript to denote the case in which both consumers and speculators purchase in the first period.

When $a_1 \geq \delta(\beta E[a] - w)$ consumers are always active and speculators are excluded from the first period market when total output is lower than Q_L. The marginal profit of firm i for an output level $q_1^i \leq Q_L - Q_1^{-i}$ is equal to

$$\Pi_q^C(q_1^i, Q_1^{-i}) \equiv a_1 - 2q_1^i - Q_1^{-i} - cq_1^i, \quad (20)$$

where a C superscript denotes the case in which consumers purchase the entire first period. Finally, if $q_1^i \geq Q_L - Q_1^{-i}$ then the marginal profit is again given by equation (19). To summarize,

$$\Pi_q^i(q_1^i, Q_1^{-i}) = \begin{cases} \Pi_q^C(q_1^i, Q_1^{-i}) & \text{if } \widehat{Q} < 0 \leq Q_1 \leq Q_L \\ \Pi_q^S(q_1^i, Q_1^{-i}) & \text{if } Q_L < 0 \leq Q_1 \leq \widehat{Q} \\ \Pi_q^{CS}(q_1^i, Q_1^{-i}) & \text{if otherwise.} \end{cases} \quad (21)$$

The behaviour of a producer's marginal profit across the kinks in the inverse demand curve is important for our analysis of the equilibria of this game as it determines the behaviour of the best response functions. We establish the following result.

Lemma 3 *The marginal profit of producer i, $\Pi_q^i(q_1^i, Q_1^{-i})$,*

(i) jumps up at the output level such that consumers start to buy the product, $q_1^i = \widehat{Q} - Q_1^{-i}$, whenever this output level exists and is positive;

(ii) jumps up or down at the output level such that speculators start to buy the product, $q_1^i = Q_L - Q_1^{-i}$, whenever this output level exists and is positive.

Proof. See appendix.‖

The consequences of this lemma are the following. First, as is well known, the possibility of upward jumping marginal profit may generate multiple local solutions to the firm's optimization problem. Second, the possibility of an downward jump at $q_1^i = Q_L - Q_1^{-i}$ suggests that an equilibrium may exist in which all producers choose output levels such that the industry output is equal to Q_L. In this case speculators do not buy the product and the first period price is equal to $\delta(\beta E[a] - w)$. As Mitraille and Thille (2009) showed in this context for a monopoly, this limit outcome is a possibility in an oligopoly as well.

We now analyze the shape of the best-response functions in the regions where speculators buy and store the product in order to clearly see the strategic effects of speculation on producers' behaviour. Applying the implicit function theorem to the first order condition $\Pi^i_q(q^i_1, Q^{-i}_1) = 0$ (where the derivative exists) gives the slope of the reaction function $dq^i_1/dQ^{-i}_1 = -\Pi^i_{qQ}(q^i_1, Q^{-i}_1)/\Pi^i_{qq}(q^i_1, Q^{-i}_1)$, where $\Pi^i_{qq}(q^i_1, Q^{-i}_1)$ indicates the derivative of the marginal profit of firm i with respect to its own output, and $\Pi^i_{qQ}(q^i_1, Q^{-i}_1)$ indicates the derivative of firm i marginal profit with respect to competitors output Q^{-i}_1. This slope differs depending whether speculators buy the entire production Q_1 or not, and may be positive. If positive, producers output choices are strategic complements which is an unexpected feature in a game of Cournot competition. We first establish the lemma and then discuss this issue.

Lemma 4 *The slope of the best response of each producer to competitors output when speculators buy in period one are given by:*

(i) when only speculators buy the product, $Q_1 \leq \widehat{Q}$,

$$\frac{dq^S_1}{dQ^{-i}_1} = -\frac{\Pi^S_{qQ}}{\Pi^S_{qQ} - \delta\beta(1 - F(Q_1)) - c},$$

where

$$\Pi^S_{qQ} = \delta(2\gamma - \beta)(1 - F(Q_1)) + \delta\beta f(Q_1) q^i_1$$

(ii) when consumers and speculators buy the product, $Q_1 \geq \max[Q_L, \widehat{Q}]$,

$$\frac{dq^{CS}_1}{dQ^{-i}_1} = -\frac{\Pi^{CS}_{qQ}}{\Pi^{CS}_{qQ} - c - \left(1 + \frac{d\widetilde{X}}{dQ_1}\right)},$$

where

$$\begin{aligned}\Pi^{CS}_{qQ} = &\left(1 + \tfrac{d\widetilde{X}}{dQ_1}\right)\left[-1 + \tfrac{2\gamma}{\beta(1+\delta\beta(1-F(-\widetilde{X}(Q_1))))}\right] \\ &+ \tfrac{\delta\beta f(-\widetilde{X}(Q_1))}{(1+\delta\beta(1-F(-\widetilde{X}(Q_1)))^3}\left[q^i_1 - 2\tfrac{\gamma}{\beta}(P_1(Q_1) + \delta w)\right]\end{aligned}$$

Proof. See appendix.‖

As the first terms in Π^S_{qQ} and Π^{CS}_{qQ} are negative, the sign of these expressions will depend on the second terms in each of them. Consider first Π^S_{qQ}: clearly the second term is positive, and if it is large then this may very well force the marginal profit of each individual firm to increase with the output of its competitors, in contrast to the standard Cournot model. The sign of Π^S_{qQ} depends on i) the effect of an increase in output on the likelihood of facing a zero price in second period, $f(Q_1)$, ii) the size of q_1^i, and iii) the number of competitors through the effect of n on β. On the other hand Π^S_{qq} is equal to Π^S_{qQ} plus a negative term containing the slope of the marginal cost c. It c is large enough, this may be negative for some distributions $F()$ and model parameters. Consequently the slope of the reaction function may be positive for some level of output Q_1^{-i}, and negative for others, depending on the sign of Π^S_{qQ}. From the discussion above, this may be more likely for a small number of competitors and a density function, $f()$, large for specific values of a. The economic intuition is the following: when a firm's competitors increase output which is then purchased by speculators, the likelihood of a zero second period price is increased. This increases the incentive to sell more in the first period as the cost of doing so in terms of reduced expected second period profit is lower[15] and the firm may follow its competitors by increasing its output in which case there is strategic complementarity.

Similar logic can be applied to case (ii) of the lemma. However an upward sloping best-response function is less likely to occur in this case as the second term of Π^{CS}_{qQ} contains an additional negative term.[16]

6 Analysis of Equilibrium

We now turn to an analysis of the equilibria of this game, outlining regions of the parameter space in which the various equilibria occur. It is important to keep in mind that the concavity of the firms' profit functions is not granted: indeed when speculators are active, the linkage of the period one price to the period two price leads to the period one price being convex in Q_1. This raises the possibility that a firm's payoff function is not concave. The results we present in this section are conditional on the assumption that the equilibrium period one price is not so convex that it renders payoffs convex. Given this assumption, we first present results for a general distribution for a and then follow that with a computed example using a uniform distribution.

[15]There is no marginal effect of increased first period production on second period profit in the cases where speculative sales drive price to zero.

[16]In the context of duopoly and consumer storage Anton and Varma (2005) and Guo and Villas-Boas (2007) establish that best responses are decreasing but upward jumping at the point where consumers begin to store by assuming that the distribution of consumer valuations has a decreasing inverse hazard rate. In our model such an assumption is insufficient since demand uncertainty introduces the possibility that second period demand may be entirely satisfied by speculative stocks.

6.1 Equilibrium

We first describe the various equilibria that may occur and then state the main theorem. First, the Cournot equilibrium, which we label C, obtains for the obvious case in which speculators entry is blockaded: if the model parameters are such that the second period expected price without storage, $\beta E[a]$, is lower than the unit cost of storage, w, then speculators never find it profitable to store regardless of the level of demand in first period.[17] In this situation the unique symmetric Nash equilibrium in the first period is the Cournot outcome, with individual producer output of

$$q_C = \frac{1}{n}(1-\beta)a_1. \tag{22}$$

Note that as n increases, β falls, so for $w > 0$ there is a threshold number of producers above which the Cournot equilibrium is unique outcome for any value of a_1.

If $\beta E[a] > w$, then speculators might be active. However, the Cournot outcome can still be an equilibrium if i) speculators do not wish to purchase at the Cournot price and, ii) a firm has no incentive to increase output to a level that induces purchases by speculators when all other firms are producing q_C. This will be the case if $\Pi_q^C(q_1^i, Q_1^{-i}) < 0$ at the threshold where speculators enter and no individual producer can earn higher profits by increasing production to sell to speculators.

It is also possible that firms' choose outputs below the Cournot level in order to ensure that speculators do not purchase anything in the first period. We say that speculation is *deterred* in this case. Let $(q_1^{1*}, ..., q_1^{n*})$ denote the n-tuple of individual output such that this occurs, then speculation is deterred if

$$\sum_{i=1}^{n} q_1^{i*} = a_1 - \delta(\beta E[a] - w). \tag{23}$$

In this case the market price in first period is equal to $\delta(\beta E[a] - w)$, which is the minimum price consistent with zero storage. This will happen if $\Pi_q^C(q_1^i, Q_1^{-i}) > 0$ and $\Pi_q^{CS}(q_1^i, Q_1^{-i}) < 0$ at the equilibrium individual outputs produced by all firms. We will use an L subscript to denote this equilibrium.

The outcome involves smoothing if speculators are active and link prices between the two periods. This can happen with and without consumers active in the first period market. We use the label CS to denote an equilibrium in which both speculators and consumers purchase

[17] It is important to note that this situation is analogous to a stock-out in the competitive storage literature and is reliant on our assumption of zero initial speculative stocks. If positive initial speculative stocks were present, then speculators would sell their stocks in this situation and the equilibrium would differ from the Cournot case.

the good in the first period. Individual firm output, q_{CS}, is the solution of

$$\Pi_q^{CS}(q_{CS},(n-1)q_{CS})=0, \tag{24}$$

which exists only if $\Pi_q^{CS}(q_1^i,Q_1^{-i})>0$ at the threshold at which both consumers and speculators wish to buy. Depending on the model parameters this threshold is either Q_L or $\widehat{Q}$. In addition, no individual producer has an incentive to reduce production in order to drive the price up, causing speculative purchases to go to zero.

We denote the potential equilibrium in which consumers are excluded with an S label. In this equilibrium firms produce q_S, which is the solution to

$$\Pi_q^{S}(q_S,(n-1)q_S)=0. \tag{25}$$

A solution exists if $\Pi_q^{S}(q_1^i,Q_1^{-i})<0$ at the threshold at which consumers enter, $\widehat{Q}-(n-1)q_S$. In addition, this equilibrium requires that no individual firm has an incentive to increase production in order to induce sales to consumers.

Finally, we allow for the possibility that first period output is zero and denote this equilibrium with a NP subscript. Firms may find it preferable to produce nothing than to sell to speculators, which will occur if $\Pi_q^{S}(q_1^i,0)<0$. Since marginal profit jumps upward at $\widehat{Q}$, the NP equilibrium also requires $\Pi_q^{CS}(\widehat{Q},0)<0$, which ensures that marginal profit is negative for all levels of output.

The following theorem establishes that each one of these potential equilibria is indeed possible.

Theorem 1 *If marginal profit is well behaved, in the sense that (24) and (25) have solutions, then any of C, L, CS, S, and NP can be the equilibrium to the oligopoly game with speculation.*

Proof: See appendix.$\|$

In the proof of Theorem 1 we demonstrate the potential existence of the alternative equilibria without fully characterizing when each occurs. However, we are able to derive some information about the relative strength of the incentives to deter and to encourage speculation.

The deterrence of speculation occurs in the limit (L) and the no-production (NP) equilibria. The limit equilibrium, in which aggregate output is maintained at Q_L, requires a relatively low first period demand low enough (otherwise the C equilibrium is the outcome). First period price is precisely that at which speculators would obtain zero profits upon entry. To deter speculation, oligopolists choose to reduce their output compared to static Cournot competition to force the price to remain at a level higher than the expected second period price net of the cost of storage. The benefit to a firm from increasing output in order to make sales to speculators is insufficient to offset the loss it incurs due to the addition of speculative competition in the second period. This equilibrium also requires that the number of producers is either one, or is relatively large.[18]

The equilibrium with no production (NP) can occur if demand is very low. It also requires $w \leq (1+c)E(a)/(n+1+c)$ and $w \geq (n(1+c)+c^2+c-1)E(a)/(n+1+c)^2$ which can simultaneously hold if the number of firms is either small enough or large enough whenever the expected demand $E[a]$ is not too small. Consequently, when first period demand is very low, if NP is the equilibrium, it will tend to occur for either one firm, or for a relatively large number of firms. Just as in the limit equilibrium, the incentive to deter speculation is non-monotonic in the number of producers.

Encouragement of speculation is more likely for moderately concentrated production due to the strategic incentive to increase first-period sales by selling to speculators. In order to see clearly the strategic effect of competitive storage it is useful to consider the extreme situation of zero first period demand and zero cost of production, $a_1 = c = 0$. In this case, first period demand can only come from speculators, who buy any output produced as long as p_1 is not too high. To characterize the incentive for producers to sell to speculators in this setting, consider the marginal profit faced by a firm when industry output is in fact zero. Using $Q_1 = 0$ in (18) gives

$$\Pi_q^S(0,0) = \delta\left((\beta - 2\gamma)\ E[a] - w\right) \tag{26}$$

Note that for $c = 0$, $\beta - 2\gamma = (n-1)/(n+1)^2$, which is zero for a monopolist: even though speculators would be willing to buy output at a positive price, the monopolist will not supply them with output. This is not the case for an oligopoly. For $n > 1$, $\Pi_q^S(0,0)$ is strictly positive if the cost of storage is low enough and firms will want to produce positive output even with zero consumer demand. Furthermore, this effect is non-monotonic in n: $(n-1)/(n+1)^2$ reaches a maximum for $n = 3$ and stays above $1/9$ (its value when $n = 2$) for n between 2 and 5. It decreases to 0 as n goes to infinity. In this simple setting, we see the strategic effect of competitive storage: by selling to speculators in the first period, firms in effect capture some of the second period expected demand. This effect is less clear to see with c and a_1 positive, but the force is still there. To summarize this result we have

Corollary 1 *For a_1 close to zero and consumers excluded from buying, a firm's marginal profit when industry output is zero, $\Pi_q^S(0,0)$, is larger under oligopoly than under monopoly or perfectly competitive market structures.*

The incentive to sell to speculators is non-monotonic in the number of producers, suggesting that equilibria involving speculative activity may be more likely for moderately concentrated

[18] This is established in the proof of Theorem 1. The two conditions required for the limit equilibrium to occur, $w \geq (1+c)(n-1)E(a)/(n+1+c)^2$ and $w \leq (1+c)E(a)/(n+1+c)$, simplify to $w \leq (1+c)E(a)/(2+c)$ for $n = 1$, and require n large enough when $n > 1$.

market structures.

6.2 The effects of uncertainty

In this subsection we elaborate on the effects of uncertain future demand on first period residual demand since randomness in demand can have significant effects on the behavior of imperfectly competitive producers. Indeed, by equating marginal revenue to marginal costs, oligopolists' decisions are more sensitive to characteristics of market demand and hence more sensitive to the effects of demand uncertainty than occurs in a more competitive industry. Therefore demand uncertainty matters more to an oligopoly than it does to a perfectly competitive industry.

The volatility of consumers willingness to pay for the commodity sold by producers has two effects: first, as seen in the discussion of Theorem 1, variations in demand over the two periods changes the type of equilibrium of the game. Second, it also changes the likelihood of the different equilibrium regimes, depending on the characteristics of the demand distribution F. But, more interestingly as we now prove, it also increases the amount of speculative inventories and of oligopolists' production since it modifies the strategic responses of each oligopolist to the choices of its opponents.

We first characterize the effect of uncertainty over second period demand on first period residual demand. The benchmark case without uncertainty[19] has $a_2 = E[a]$ with probability one. From (10) we see that the threshold at which speculators become active, Q_L, does not depend on any other characteristic of the distribution of a_2 other than $E[a]$, so $Q_L^o = Q_L$. However, the threshold at which speculators purchase the entire first period output (defined by $\delta p_2^*(\widehat{Q}^o, E[a]) - \delta w = a_1$) becomes

$$\widehat{Q}^o = -Q_L/(\delta\beta). \tag{27}$$

Comparing $\widehat{Q}^o$ with $\widehat{Q}$ we see that uncertainty increases the range of first period output over which consumers are squeezed out by speculators:

Lemma 5 *$\widehat{Q}^o < \widehat{Q}$.*

Proof. See appendix.‖

This effect is due to the fact that speculators care about the future expected price only in cases where sales are profitable to them, that is in cases where the future price is positive. This modifies speculators' willingness to pay for Q_1 in first period, as they are ready to buy producers output at a price higher than what they are ready to pay without uncertainty: in the S equilibrium the expected price with demand uncertainty is given by $\beta \int_{Q_1}^{A} (a - Q_1) dF(a)$

[19] We will denote variables in this case with a o superscript.

which is strictly higher than $\beta(E[a] - Q_1)$, the second period price without uncertainty. Uncertainty over future demand leads to a higher first period price in this case.

When $Q_L > 0$, the position of speculators when future demand is certain is determined by a simplified[20] version of (12):

$$a_1 - X - Q_1 = \delta\beta E[a] + \delta\beta X - \delta w \tag{28}$$

giving

$$\widetilde{X}^o(Q_1) = \frac{Q_L - Q_1}{1 + \delta\beta}. \tag{29}$$

Using the lemma above, we can prove:

Proposition 1 *Compared to the case of certain future demand, demand uncertainty leads to more inventories being carried between periods and causes inventories to be more sensitive to first period production.*

Proof. See appendix.‖

Finally, Lemma 4 established that strategic complementarities in producers' first period choices were a possibility, caused by the terms involving $1 - F(Q_1)$ or $1 - F(-\widetilde{X}(Q_1))$ in Lemma 4, which measure the probability that demand will be high enough for speculators to be unable to drive price to zero. The converse of this probability measures the chance that speculative inventories are sufficient to meet the entire second period demand. This situation corresponds to the notion of a "speculative overhang" of inventories which cause worry about "speculative collapse" of a market.

6.3 Implications for the distribution of price and welfare

We now address the implications of competitive speculation for the distribution of prices and for welfare in an imperfectly competitive market.

6.3.1 Price distribution

We wish to examine the implications of this analysis for the ex-ante first and second moments of first period price and for the correlation between first and second period price. By "ex-ante", we are thinking about the distribution of first period price prior to knowledge of a_1. Admittedly, our two period model is not ideal for examining the effects of speculation on the distribution of price as the analysis is conditional on zero initial stocks, and so cannot account

[20] The future price in this case is $p_2^* = \beta \max[E[a] + X, 0]$, but we ignore the possibility of a zero second period price with certain demand as speculators would not wish to carry stocks if that were the case.

for longer term inventory dynamics. However, it does provide some insight into the type of effects that speculation can have on oligopoly price distributions.

Whether p_1^* is above or below the Cournot price depends on which equilibrium obtains. If the equilibrium is of type L, S, or NP, p_1^* exceeds the Cournot price, while if it is of type CS, p_1^* is lower than the Cournot price. In terms of variance, when p_1^* is higher than the static Cournot price (S, NP or L), p_1^* is independent of a_1 and therefore does not contribute to the ex-ante variance. When the p_1^* is strictly lower than the static Cournot price (CS), computing the variance also leads to a smaller contribution to the ex-ante variance than in the Cournot equilibrium. We summarize this in the following corollary

Proposition 2 *Compared to static Cournot competition, the presence of speculators:*

(i) can increase or decrease the ex-ante mean of the price p_1^* depending on the distribution of the demand intercept, $F()$,

(ii) unambiguously decreases the ex-ante variance of the price p_1^*.

The relationship between p_1^* and p_2^* is also dependent on which equilibrium obtains. When either CS or S is the equilibrium, prices are tied together due to speculation, $p_1^* = \delta(E[p_2^*]-w)$. As we demonstrate below with an example, the likelihood of the region in which prices are tied together between periods can be larger when competition is imperfect than when it is perfect or when there is a monopoly. This suggests that speculators may be more active under imperfectly competitive production than under perfectly competitive production. This finding is interesting in relation to Deaton and Laroque (1992), (1996), who amongst other results identify more price auto-correlation in their data than what their theoretical model predicts. Our results suggest that one possible explanation, realistic for many commodity markets, lies in imperfect competition.

6.3.2 Welfare

We now turn to an analysis of the welfare implications of speculation in a Cournot market. To compute discounted social welfare, we need to aggregate the discounted sum of consumers surplus, producers profits and speculators payoffs across the two periods, taking the expectation of second period payoffs with respect to the demand intercept a_2. Note that, although speculators' payoffs do contribute to second period welfare if the realized market price exceeds their marginal cost of storage, they do not contribute to discounted social welfare when aggregating across the two periods. Indeed given their equilibrium position in period one, X_1^*, speculators payoffs are equal to $(p_1^* - \delta E(p_2^*) + \delta w)X_1^*$. Either a stock-out occurs, $X_1^* = 0$, or speculators store and resell in second period, but in that case $p_1^* = \delta E(p_2^*) - \delta w$. In both cases their discounted expected payoff is zero. Consequently, we can ignore speculative profits in our welfare analysis, the cost of storage is accounted for through it effects on prices.

In the cases where speculators are inactive and do not store in the first period (C, L and NP), consumers surplus and producers profits are unaffected by speculation in the second period. In first period, either the quantity produced is equal to the Cournot outcome (C), or it is strictly lower (L, and NP). We deduce immediately:

Proposition 3 *When speculation is possible but no stocks are carried into the second period, the present value of consumer surplus and producer profits are no higher than and can be lower than the values they obtain under Cournot competition in the absence of speculation.*

The presence of speculators, even if they are inactive, can be detrimental to social welfare.

In the cases where speculators are active in period one (outcomes CS and S), consumer surplus increases in the second period. Indeed the total quantity sold on the market is equal to the sum of speculators position and producers output, $-X_1^* + Q_2^*(-X_1^*, a_2)$, which always exceeds the static Cournot output.[21] Consequently, second period consumer surplus with active speculators exceeds that without active speculators. Producer profits are clearly reduced by the release of speculative inventories on the market in the second period relative to profits in the absence of speculators. The net effect is that expected second period total surplus increases as the quantity consumed increases and production costs are lower.

We can now turn our attention to the first period. In the case where consumers are excluded by speculators on the market (S), consumers surplus is clearly reduced compared to Cournot competition. The loss in consumer surplus is precisely the consumer surplus available under the Cournot equilibrium. On the other hand producer profits are larger than their Cournot static counterpart: the equilibrium quantity is larger and the price is higher. Which effect dominates, the gain in producers profits or the loss in consumers surplus, depends on particular values of the model parameters.

When consumers can purchase the product and speculators store in first period (CS), consumers surplus increases due to speculation since the market price is lower than the static Cournot price. Consumers purchase a larger quantity at a lower price and consequently enjoy a larger surplus. Producer profit can increase or decrease compared to static Cournot competition. To summarize, we have established:

Proposition 4 *When speculators carry stocks the net effect on welfare is ambiguous:*

(i) in the CS equilibrium, consumer surplus increases in both periods, while producer profits can increase or decrease in period one and decrease in period two,

(ii) in the S equilibrium, consumers surplus decreases in period one and increases in period two, and producers profits increase in period one and decrease in period two.

[21] From (4), producers reduce their output by a factor $1 - \beta$ when speculators increase their first period inventories by 1 unit, and this factor is strictly lower than 1 as long as the number of producers is finite (see Definition 1).

In both cases, depending on the distribution $F()$ and the model parameters, the discounted social welfare computed across the two periods may increase or decrease compared to Cournot competition.

6.4 Uniform Distribution Example

In order to demonstrate that the alternative equilibria can in fact exist, we now compute an example using a uniform distribution for the demand uncertainty.

Assumption 1 (Uniform demand shock) $f(a) = \frac{1}{A}$, $a \in [0, A]$

For payoffs to be concave on the interior of the sets where CS and S are equilibria, we require costs to be sufficiently convex. A sufficient condition given by the following assumption:

Assumption 2 (Cost convexity) $c \geq \delta\beta$

We know from its definition that β is declining in n, so this condition is more constraining for more concentrated market structures. This implies that, if Assumption 2 holds for $n = 1$ it will hold for all n. We now have the following result:

Proposition 5 *Under assumptions 1 and 2, the marginal profits of firm i in the S and CS regions are both strictly decreasing in q_1^i, i.e.*

$$\Pi_{qq}^{S} < 0 \quad \textit{and} \quad \Pi_{qq}^{CS} < 0.$$

Proof. See appendix.‖

The implication of 5 is that it guarantees that a local solution to the first order condition exists in the S and CS regions. To determine the circumstances under which a certain type (C, CS, S, NP or L) equilibrium exists, it then suffices to check whether or not non-local deviations from a candidate equilibrium exist. That is, it suffices to check whether or not there exists a profitable deviation into a region with a different functional form for the marginal profit. Although closed form expressions for $\widehat{Q}_1$, $\widetilde{X}(Q_1)$, and marginal profit in the S and CS regions can be determined when demand shocks are uniformly distributed and Assumption 2 holds, they are rather complicated to work with and so we instead illustrate the results using numerical computations.

Combined with Proposition 3, Proposition 5 allows us to determine the symmetric equilibria to our game as follows. From Lemma 1, we can consider in turn the cases $Q_1^L \geq 0$ and $\widehat{Q}_1 \geq 0$, which define regions in the set of all possible a_1 and n. In each of these two cases, the conditions under which the alternative equilibria occur define regions in the set of all possible combinations of a_1 and n. When computing the equilibrium, we ensure that all possible combinations have been considered. We describe the details of the solution method in the appendix.

We now choose the values of the model parameters so that they satisfy the two assumptions above. We assume that $A = 20$, and we fix $\delta = 0.95$. A value of 0.6 is chosen for c, which ensures that Assumption 2 holds for $n \geq 1$. Consequently, we are certain that payoffs are concave apart from the discontinuities at Q_1^L and $\widehat{Q}_1$. We start with a value of 0.2 for w. We find the period one equilibrium as described above for a range of values for n and a_1, in particular, for $n = 1, 2, ..., 80$ and for 500 equally spaced values for $a_1 \in [0, 20]$.

We illustrate the nature of the equilibrium for each (n, a_1) pair in Figure 1, color coded to denote the type of equilibrium. The C equilibrium, colored in blue and pink, is the most common. It is the unique equilibrium (blue) for "large" first period demand and when there are more than 78 firms, speculation is blockaded and only the Cournot equilibrium is possible. The CS equilibrium, colored in red, pink and yellow, occurs for a large set of parameters. The red color illustrates the situations in which CS is the unique equilibrium. There are a substantial number of (n, a_1) pairs for which we have multiple equilibria of either C and CS (colored in pink). The extent to which speculation can occur when demand is expected to fall ($a_1 > 10$) is dramatic. The non-monotonic nature of the strategic effect leading to the CS equilibrium, discussed in Corollary 1, is clearly evident: the proportion of demand states in which speculation occurs is largest for an intermediate number of firms.

The consumer exclusion equilibrium (S), colored in orange and yellow, occurs for a fairly substantial range of parameters when the market is not very competitive. It is the unique equilibrium in the orange section. There are also a relatively small number of cases in which either S or CS can be the equilibrium (yellow). The (NP) equilibrium, colored in green, does occur in this example, but only for $a_1 = 0$ when there are more than 76 firms in the market and is rather hard to see (when there are fewer than 77 firms, the S equilibrium obtains when $a_1 = 0$). The limit equilibrium (L), colored in purple, occurs for a small number of demand states when there is one firm or when there are 76–78 firms.

The set of values of the demand state for which the CS equilibrium obtains in Figure 1 is non-monotonic in the number of firms. This is true even when we consider only the situations in which CS is the unique equilibrium. To examine the implications of non-monotonicity in the type of equilibrium for expected price and welfare, we compute speculative purchases, expected price and welfare for $n = 1, ..., 80$ and plot the results in Figure 2. The effect on prices and welfare are computed as the difference between the equilibrium values with speculation and those without, expressed as a proportion of the latter. We see that the level of speculative purchases is non-monotonic in the number of firms, peaking at around 10 firms and slowly declining to zero at over 70 firms. Expected second period price mirrors the level of stocks carried and clearly speculation has the effect of reducing second period prices.

However, the effect on first period price is dependent on market structure: with relatively few firms speculation causes an increase in expected first period price, due largely to the relatively high prices when the first period equilibrium is S. With more firms competing, however, the effect of speculation is to reduce expected price as the S equilibrium occurs only rarely. The implications of these effects on welfare are shown in the lower right panel of Figure 2 in which we plot the discounted sum of expected welfare (consumer surplus plus producer profit) against the number of producers. Again we see a non-monotonic effect: welfare increases for relatively concentrated market structures, but actually falls for relatively unconcentrated ones. The magnitude of the welfare effects of speculation in this example is not large (at most under 2% of expected welfare in the absence of speculation) but there is potential for substantial redistribution of welfare between consumers and producers within periods as seen by the effects on the expected price, which can be as high as 12%.

To see the effects of higher storage costs and to demonstrate that the limit equilibrium need not be as rare as we see in Figure 1, in Figure 3 we plot the equilibria that obtain with a value of $w = 1.1$ and all other parameters the same as in Figure 1. Not surprisingly, equilibria with storage occur much less frequently. The L equilibrium occurs somewhat more frequently for the monopoly and when there are 11 or 12 firms. The monopolist chooses zero production for values of a_1 within roughly the bottom 20% of the distribution.

6.5 Example with a longer time horizon

The two-period model in which the above results were generated has the benefit of being simple enough to produce some analytic results, however this comes at a cost of very simple behaviour for speculators in the second period: they simply sell any inventories they have if price is positive and can destroy any surplus stocks at no cost. Consequently, it is important to investigate the extent to which the above results generalize to longer time horizons. We expect these results to be robust to longer time horizons since the speculative behaviour that leads to discontinuous marginal payoffs for producers in the model is due to speculators buying behaviour in the first period, not the dumping of inventories in the second. Consequently, the alternative types of equilibria that obtain in the first period of our model are likely to still obtain in a model with a longer time horizon. In order to demonstrate this result we present the solution for a four period version of the example from the previous subsection.

Extending the model beyond two periods is conceptually straightforward, however analysis is complicated by the difficulty of obtaining closed-form solutions for the price and value functions. In addition, the possibility of multiple equilibria in the penultimate period means that an equilibrium selection assumption must be made in order to find the equilibrium in an earlier period. We overcome these complications by using cubic splines to approximate the expected price and value functions in periods two and three and by assuming that C is the equilibrium when both C and CS are possible and CS is the equilibrium when both CS and S are possible. We proceed by taking the solution in Figure 1 for period $T-1$ and numerically integrating the price and payoff over the possible values for the demand parameter for a fixed set of values for H_{T-1}. We then fit an approximation to the expected price and expected value functions via spline approximation. Given the approximations for period $T-1$, we repeat the procedure for periods $T-2$ and $T-3$. We then compute the frequency of the alternative equilibria for period $T-3$ with an zero initial speculative stocks and plot the results in Figure 4. Comparing Figure 4 to Figure 1, the only significant difference to the two-period model is that the CS equilibrium occurs somewhat less frequently, but the qualitative results are broadly similar.

One interesting feature of Figure 4 is that when there are fewer than seven firms, the Cournot equilibrium does not obtain for some values of a_1 greater than 10. In this situation, even though speculation is not profitable at the Cournot price, the strategic incentive to sell to speculators leads to the Cournot outcome not being an equilibrium: storage occurs even though demand is expected to fall.[22]

7 Conclusion

It is interesting to interpret our model and connect our findings to the recent public debates on commodity prices and speculation we evoked earlier in the introduction. The framework we build in this paper corresponds to an oligopolistic industry that sells to a competitive sector that can either consume the commodity or store it at some cost. Hence it is a fairly general model to tackle some of the questions in the air.

In much of the public discussion of recent events in commodity markets, there is a recognition, implicit or explicit, of the extreme form of competition speculators exert to other market participants, a competition that is often viewed as detrimental by neither "ensuring an efficient, economic and regular supply to consumers" nor "a fair return for those investing in the industry"[23]. In that respect, the example of developing countries struggling to satisfy their needs on the demand side, as emphasized by regulators or international agencies, is striking. In light the equilibrium in which consumers are excluded, this outcome is not surprising: in circumstances where current demand is low compared to expected future demand, which could correspond to the recent economic boom in Asia, the entire output may be purchased without being consumed and paid at the expected future price which is disconnected from current demand. In that case, the market price is "high", the inventories are "large", and the

[22]Of course this effect is also present in Figure 1, but only for states in which there were multiple equilibria. Here we see that it can be a unique equilibrium outcome for storage to occur in the face of falling demand.

[23]These objectives are part of the OPEC mission in the petroleum industry, as can be read on their website, http://www.opec.org/opec_web/en/about_us/23.htm

consumption is "nil".[24] As we discussed in the introduction, the occurrence of high prices with large inventories is a significant concern to regulators[25].

Our results indicate that speculation is more likely in oligopoly and that speculation can have a dramatic effect on the equilibrium in a non-cooperative setting. Of particular interest is the result that speculative activity can vary non-monotonically with market structure. We demonstrated with an example that this activity can cause non-monotonic effects on prices and welfare. Although the net effect on total welfare is modest, there are significant distributional effects of speculation between producers and consumers as well as over time and these effect vary with market structure. As many commodity markets feature oligopolistic production, our results suggest that the prevalence of concern about speculation may be warranted.

Given the interest in cartel activity in natural resource markets, an interesting extension of our analysis would be to see how speculation affects cartel stability. The uranium cartel dealt with unwanted competition from "middlemen" by selling directly to end-users only.[26] However, in most commodity markets the ability to discriminate between speculators and end-users is limited and it seems that speculation would have the potential to significantly constrain a cartel's activities.

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[24]Even without the full exclusion of consumption, when both consumers and speculators are active (the CS equilibrium in our model) current consumption is displaced by speculative purchases to some extent.

[25]U.S. Senate Staff report (*ibid*), pp.2–3).

[26]Teece et al. (1993), p. 1159.

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Appendix

Proof of Lemma 1

The uniqueness of Q_L is obvious from (10). The uniqueness of $\widehat{Q}$ is straightforward as the left-hand side of (11) is strictly decreasing in $\widehat{Q}$, equal to $\delta(\beta E[a] - w) - a_1 > 0$ for $\widehat{Q} = 0$ and equal to $-\delta w - a_1 < 0$ for $\widehat{Q} = A$.

To prove the uniqueness of $\widetilde{X}(Q_1)$, we first establish a bound to the aggregate quantity of inventories carried into the second period. Define $-X^0$ to be the quantity of inventories held that causes the expected second period revenue from speculation to be 0. Clearly, speculators would not carry more inventory than $-X^0$. To find X^0 we define

$$g_0(X) = \beta \int_{-X}^{A} (a + X)\ dF(a) - w, \tag{30}$$

for $X \in (-A, 0)$. We have $g_0'(X) = \beta(1 - F(-X)) > 0$, $g_0(0) = \beta E[a] - w > 0$, and $g_0(-A) = -w < 0$. Hence $g_0(X) = 0$ defines a unique X^0. We define another function,

$$g(X, Q_1) = a_1 - X - Q_1 - \delta \left(\beta \int_{-X}^{A} (a + X)\, dF(a) - w \right) \tag{31}$$

where the domain on which g is defined changes depending on which quantity amongst Q_L and $\widehat{Q}$ is positive. Equation (12) is equivalent to $g(X, Q_1) = 0$, which defines $\widetilde{X}(Q_1)$. The function g is continuous and differentiable in both X and Q, and decreasing in X given Q_1. Indeed:

$$\frac{\partial g}{\partial X}(X, Q_1) = -1 - \delta\beta(1 - F(-X)) < 0. \tag{32}$$

First consider the case in which speculators can be excluded from the market, $Q_L > 0$. In this case, $\widetilde{X} : [Q_L, \bar{Q}] \to [X^0, 0]$, where $\bar{Q} = a_1 - X^0$. As g is strictly decreasing in X, to prove that there exists a unique $\widetilde{X}(Q_1)$ it suffices to prove that $g(X^0, Q_1) \geq 0$ and $g(0, Q_1) \leq 0$. We have

$$\begin{aligned} g(X^0, Q_1) &= a_1 - X^0 - Q_1 - \delta\Big(\beta \textstyle\int_{-X^0}^{A} (a + X^0)\, dF(a) - w\Big) \\ &= a_1 - X^0 - Q_1 \end{aligned} \tag{33}$$

by definition of X^0. Since $Q_1 \leq \bar{Q} = a_1 - X^0$, clearly $g(X^0, Q_1) \geq 0$. Similarly,

$$\begin{aligned} g(0, Q_1) &= a_1 - Q_1 - \delta(\beta E[a] - w) \\ &= Q_L - Q_1 \end{aligned} \tag{34}$$

so we have $g(0, Q_1) \leq 0$ for $Q_1 \geq Q_L$. Hence, there is a unique value of $\widetilde{X}(Q_1)$ associated with each $Q_1 \in [Q_L, \bar{Q}]$.

Next consider the case in which consumers can be excluded from the market, $\widehat{Q} \geq 0$. In this case, $\widetilde{X} : [\widehat{Q}, \bar{Q}] \to [X^0, -\widehat{Q}]$. We now need to show $g(X^0, Q_1) \geq 0$ and $g(-\widehat{Q}, Q_1) \leq 0$ for a unique $\widetilde{X}(Q_1)$. The value for $g(X^0, Q_1) \geq 0$ is unchanged and

$$g(-\widehat{Q}, Q_1) = a_1 + \widehat{Q} - Q_1 - \delta\Big(\beta \int_{\widehat{Q}}^{A} (a - \widehat{Q})\, dF(a) - w\Big). \tag{35}$$

Using $g(-\widehat{Q}, \widehat{Q}) = 0$ by the definition of $\widehat{Q}$ and the fact that $\frac{\partial g}{\partial Q_1}(X, Q_1) = -1 < 0$, clearly $g(-\widehat{Q}, Q_1) < 0$ for $Q_1 \in [\widehat{Q}, \bar{Q}]$. Hence, there is a unique value of $\widetilde{X}(Q_1)$ associated with each $Q_1 > [\widehat{Q}, \bar{Q}]$ when $\widehat{Q} > 0$.

Applying the Implicit Function Theorem to $g(X, Q_1) = 0$ gives immediately part (i) of the lemma.

Finally, to prove part (ii) of the Lemma, note that $Q_L = 0$ is equivalent to $a_1 = \delta(\beta E[a] - w)$ by (10). Replacing this value of a_1 in (11) gives

$$\delta\beta \int_{\widehat{Q}}^{A} (a - \widehat{Q}) dF(a) - \delta w - \delta(\beta E[a] - w) = 0$$

or

$$\int_{\widehat{Q}}^{A} (a - \widehat{Q}) dF(a) = E[a] = \int_{0}^{A} a dF(a),$$

hence, $\widehat{Q} = 0$. From (10) it is clear that Q_L is strictly increasing with a_1, while it is straightforward to show that $\widehat{Q}$ is strictly decreasing with a_1. As we have just shown, $Q_L = \widehat{Q} = 0$ when $a_1 = \delta(\beta E[a] - w)$. Consequently $sign(\widehat{Q}) = -sign(Q_L)$ for $a_1 \neq \delta(\beta E[a] - w)$.

Proof of Lemma 2

The expected second period profit can be expanded as

$$E_1[\pi_2^*] = \gamma \left(\int_{-X_1^*(Q_1)}^{A} a^2 dF(a) + 2X_1^*(Q_1) \int_{-X_1^*(Q_1)}^{A} adF(a) + (X_1^*(Q_1))^2 \int_{-X_1^*(Q_1)}^{A} dF(a) \right).$$

Its derivative with respect to Q_1 is equal to:

$$\begin{aligned} \frac{\partial E_1[\pi_2^*]}{\partial Q_1} &= \left(\gamma(-X_1^*(Q_1))^2 + 2\gamma X_1^*(Q_1)(-X_1^*(Q_1)) + \gamma(X_1^*(Q_1))^2\right)\frac{dX_1^*}{dQ_1} f(-X_1^*(Q_1)) \\ &\quad +2\gamma \frac{dX_1^*}{dQ_1} \int_{-X_1^*(Q_1)}^{A} adF(a) + 2\gamma \frac{dX_1^*}{dQ_1} X_1^*(Q_1) \int_{-X_1^*(Q_1)}^{A} dF(a), \end{aligned}$$

which simplifies to

$$\begin{aligned} \frac{\partial E_1[\pi_2^*]}{\partial Q_1} &= 2\gamma \frac{dX_1^*}{dQ_1} \left(\int_{-X_1^*(Q_1)}^{A} adF(a) + 2\gamma X_1^*(Q_1) \int_{-X_1^*(Q_1)}^{A} dF(a) \right) \\ &= 2\gamma \frac{dX_1^*}{dQ_1} \int_{-X_1^*(Q_1)}^{A} (a + X_1^*(Q_1)) dF(a) \\ &= 2\frac{\gamma}{\beta} \frac{dX_1^*}{dQ_1} \frac{P_1(Q_1)+\delta w}{\delta} \\ &\leq 0 \end{aligned}$$

since dX_1^*/dQ_1 is equal to -1 when speculators buy the entire production in first period, or to $d\widetilde{X}/dQ_1 \in (-1, 0)$ when both consumers and speculators buy in first period.

Proof of Lemma 3

(i) For $0 < q_1^i < \widehat{Q} - Q_1^{-i}$, $p_1 > a_1$ and consumer demand is zero, while if $q_1^i > \widehat{Q} - Q_1^{-i}$, $p_1 < a_1$ and both consumer and speculative demand is positive. Consequently, the individual firm's first period inverse demand flattens as the threshold $\widehat{Q} - Q_1^{-i}$ is crossed, which results in the firm's profit being less sensitive to increases in output for q_1^i larger than $\widehat{Q} - Q_1^{-i}$ than for smaller levels of output. Hence, marginal profit must take an upward jump at $q_1^i = \widehat{Q} - Q_1^{-i}$.

(ii) In this case the discontinuity in marginal profit occurs at $q_1^i = Q_L - Q_1^{-i}$. Only consumers buy the product when the output of firm i is lower than this value, and consequently the marginal profit at the left of the discontinuity is

$$\Pi_q^o(Q_L - Q_1^{-i}, Q_1^{-i}) = a_1 - (2+c)Q_L + (1+c)Q_1^{-i}. \tag{36}$$

Since $\widetilde{X}(Q_L) = 0$, marginal profit at the right of the discontinuity is

$$\Pi_q^{CS}(Q_L - Q_1^{-i}, Q_1^{-i}) = a_1 - Q_L - \left(1 + c - \frac{1}{1+\delta\beta}\right)(Q_L - Q_1^{-i}) - \frac{2\gamma(a_1 - Q_L + \delta w)}{\beta(1+\delta\beta)}, \quad (37)$$

and the difference across the discontinuity is equal to

$$\Pi_q^{CS}(Q_L - Q_1^{-i}, Q_1^{-i}) - \Pi_q^{o}(Q_L - Q_1^{-i}, Q_1^{-i}) = \frac{Q_L - Q_1^{-i} - 2\ \delta\gamma\ E[a]}{1+\delta\beta}. \quad (38)$$

Even if Q_1^{-i} is such that $Q_L - Q_1^{-i} > 0$, this difference may be larger or smaller than $2\ \delta\gamma\ E[a]$ and consequently the marginal profit may upward or downward jumping at $Q_L - Q_1^i$.

Proof of Lemma 4

Denoting $P_1''(Q_1)$ the second derivative of first period inverse demand, the general expression of the derivative $\Pi_{qq}^i(q_1^i, Q_1^{-i})$ is given by:

$$\begin{aligned}\Pi_{qq}^i(q_1^i, Q_1^{-i}) &= P_1''(Q_1)q_1^i + 2P_1'(Q_1) - c \\ &+2\delta\gamma\left(\tfrac{d^2X_1^*}{d(Q_1)^2}\textstyle\int_{-X_1^*(Q_1)}^{A}(a + X_1^*(Q_1))dF(a) + \left(\tfrac{dX_1^*}{dQ_1}\right)^2(1 - F(-X_1^*(Q_1)))\right)\end{aligned} \quad (39)$$

Similarly the the general expression of the derivative $\Pi_{qQ}^i(q_1^i, Q_1^{-i})$ is given by:

$$\begin{aligned}\Pi_{qQ}^i(q_1^i, Q_1^{-i}) &= P_1''(Q_1)q_1^i + P_1'(Q_1) \\ &+\delta 2\gamma\left(\tfrac{d^2X_1^*}{d(Q_1)^2}\textstyle\int_{-X_1^*(Q_1)}^{A}(a + X_1^*(Q_1))dF(a) + \left(\tfrac{dX_1^*}{dQ_1}\right)^2(1 - F(-X_1^*(Q_1)))\right)\end{aligned} \quad (40)$$

Consider first case (i) in which $X_1^*(Q_1) = -Q_1$. The first period price is decreasing and convex in Q_1:

$$P_1'(Q_1) = -\delta\beta(1 - F(Q_1)) \leq 0 \text{ and } P_1''(Q_1) = \delta\beta f(Q_1) \geq 0. \quad (41)$$

and (39) simplifies to

$$\Pi_{qq}^S(q_1^i, Q_1^{-i}) = 2\delta(\gamma - \beta)(1 - F(Q_1)) - c + \delta\beta f(Q_1)q_1^i, \quad (42)$$

and similarly

$$\Pi_{qQ}^S(q_1^i, Q_1^{-i}) = \delta(2\gamma - \beta)(1 - F(Q_1)) + \delta\beta f(Q_1)q_1^i. \quad (43)$$

Second, consider case (ii) in which $X_1^*(Q_1) = \widetilde{X}(Q_1)$. The derivatives $P'(Q_1)$ and $P''(Q_1)$

are equal to:

$$P_1'(Q_1) = -1 - \frac{d\widetilde{X}}{dQ_1} \leq 0 \text{ and } P_1''(Q_1) = -\frac{d^2\widetilde{X}}{d(Q_1)^2} \tag{44}$$

From lemma 1,

$$\frac{d^2\widetilde{X}}{d(Q_1)^2} = \frac{\delta\beta \frac{d\widetilde{X}}{dQ_1} f(-\widetilde{X}(Q_1))}{[1+\delta\beta(1-F(-\tilde{X}(Q_1)))]^2} = -\frac{\delta\beta f(-\widetilde{X}(Q_1))}{[1+\delta\beta(1-F(-\tilde{X}(Q_1)))]^3} \tag{45}$$

which is negative. We have

$$\begin{aligned} &2\gamma \tfrac{d^2\widetilde{X}}{d(Q_1)^2} \textstyle\int_{-\widetilde{X}(Q_1)}^{A} (a+\widetilde{X}(Q_1))dF(a) + 2\gamma \left(\tfrac{d\widetilde{X}}{dQ_1}\right)^2 (1-F(-\widetilde{X}(Q_1))) \\ &= 2\gamma \tfrac{d^2\widetilde{X}}{d(Q_1)^2} \tfrac{a_1-\widetilde{X}(Q_1)-Q_1+\delta w}{\delta\beta} + 2\gamma \left(\tfrac{d\widetilde{X}}{dQ_1}\right)^2 \tfrac{(1+\delta\beta(1-F(-\widetilde{X}(Q_1))))\left(1+\frac{d\widetilde{X}}{dQ_1}\right)}{\delta\beta} \end{aligned} \tag{46}$$

and consequently (39) reduces to

$$\begin{aligned} &\Pi_{qq}^{CS}(q_1^i, Q_1^{-i}) = 2\left(1+\tfrac{d\widetilde{X}}{dQ_1}\right)\left[-1+\tfrac{\gamma}{\beta(1+\delta\beta(1-F(-\widetilde{X}(Q_1))))}\right] - c \\ &+ \tfrac{\delta\beta f(-\widetilde{X}(Q_1))}{(1+\delta\beta(1-F(-\widetilde{X}(Q_1)))^3} \left[q_1^i - 2\tfrac{\gamma}{\beta}(P_1(Q_1)+\delta w)\right] \end{aligned} \tag{47}$$

Similarly we have:

$$\begin{aligned} &\Pi_{qQ}^{CS}(q_1^i, Q_1^{-i}) = \left(1+\tfrac{d\widetilde{X}}{dQ_1}\right)\left[-1+\tfrac{2\gamma}{\beta(1+\delta\beta(1-F(-\widetilde{X}(Q_1))))}\right] \\ &+ \tfrac{\delta\beta f(-\widetilde{X}(Q_1))}{(1+\delta\beta(1-F(-\widetilde{X}(Q_1)))^3} \left[q_1^i - 2\tfrac{\gamma}{\beta}(P_1(Q_1)+\delta w)\right]. \end{aligned} \tag{48}$$

Proof of Theorem 1

We prove the theorem by demonstrating for each candidate equilibrium that a non-empty set of parameters exists for which the candidate is indeed an equilibrium.

C **equilibrium** The Cournot equilibrium obtains trivially when $\beta E[a] - w \leq 0$. In this case Q_L exceeds a_1 and $\widehat{Q} < 0$ so that no profitable speculation can occur over the range of productions $[0, a_1]$. For the case in which speculation is not blockaded, we use the necessary condition that $\Pi_q^o(Q_L - (n-1)q_C, (n-1)q_C) \leq 0$ must hold in a Cournot equilibrium. This condition is $a_1 - (2+c)Q_L + (1+c)(n-1)q_C \leq 0$, which upon substituting for Q_L and q_C we get

$$a_1 \geq \delta E[a] - \delta w/\beta. \tag{49}$$

In addition to this condition, a deviation to a higher level of output than q_C must not be profitable. A sufficient condition for this is that $\Pi_q^{CS}(Q_L - (n-1)q_C, (n-1)q_C) \leq 0$, from which we derive

$$a_1 \geq \frac{[((2+c)(1+\delta\beta)-1)(\delta\beta E[a]-\delta w)-2\delta\gamma E[a]]\, n(n+1+c)}{((1+c)(1+\delta\beta)-1)(n^2+1+c)}. \tag{50}$$

Since the right-hand sides of both (49) and (50) are finite, there exists parameters and distribution function for a_2 that satisfies both (49) and (50) and a Cournot equilibrium exists.

L **equilibrium.** In order to demonstrate the existence of an L equilibrium, it is sufficient to examine the symmetric case ($q_i = Q_L/n\ \forall i$) as it is easy to show that the symmetric equilibrium exists if any asymmetric ones do. An L equilibrium exists if $\Pi_q^o(Q_L/n, (n-1)Q_L/n) \geq 0$, $\Pi_q^{CS}(Q_L/n, (n-1)Q_L/n) \leq 0$ and $\beta E[a] - w \geq 0$. The first condition reduces to

$$a_1 \leq \delta E[a] - \delta w/\beta \tag{51}$$

which is the complement to (49). The second condition reduces to

$$a_1 \geq \left[\frac{n(1+\delta\beta)}{\delta\beta + c(1+\delta\beta)} + 1\right]\delta(\beta E[a] - w) - \frac{2n\delta\gamma E[a]}{\delta\beta + c(1+\delta\beta)} \tag{52}$$

Conditions (51) and (52) can both hold if

$$w \geq \frac{(1+c)(n-1)}{(n+1+c)^2}E[a]. \tag{53}$$

The condition $\beta E[a] - w \geq 0$ requires

$$w \leq \frac{1+c}{n+1+c}E[a], \tag{54}$$

so an L equilibrium exists as these two constraints on w can simultaneously hold.

CS **equilibrium.** Consider $a_1 \leq \delta(\beta E[a] - w)$, in which case marginal profit jumps upward when total output equals $\widehat{Q}$, so q_{CS} is an equilibrium if $\Pi_q^S(\widehat{Q} - (n-1)q_{CS}, (n-1)q_{CS}) \geq 0$ or

$$a_1 - [\delta\beta(1 - F(\widehat{Q})) + c]\ (\widehat{Q} - (n-1)q_{CS}) - \frac{2\gamma(a_1 + \delta w)}{\beta} \geq 0. \tag{55}$$

For $a_1 = 0$ this condition cannot hold, while for $a_1 = \delta(\beta E[a] - w)$, $\widehat{Q} = 0$ and (55) reduces to $(\delta\beta + c)(n-1)q_{CS} + \delta(\beta - 2\gamma)E[a] - \delta w \geq 0$. As $\beta - 2\gamma \geq 0$, this condition will hold for w small enough, which is enough to insure that the region of parameters in which the CS equilibrium exists is not empty.

S **equilibrium.** When $a_1 \leq \delta(\beta E[a] - w)$ so that $\widehat{Q} > 0$, by Lemma 3 we know that marginal profit is upward jumping at $q_1^i = \widehat{Q} - Q_1^{-i}$. Consequently an S equilibrium will exist if $\Pi_q^S(0,(n-1)q_S) \geq 0$ and $\Pi_q^{CS}(\widehat{Q} - (n-1)q_S, (n-1)q_S) \leq 0$ (which is sufficient for $\Pi_q^S(\widehat{Q} - (n-1)q_S, (n-1)q_S) \leq 0$ as marginal profit jumps up at this point).

Using the definition of q_S and the fact that $\int_Q^A (a-Q)dF(a)$ decreases with Q, we have

$$\begin{aligned}
\Pi_q^S(0,(n-1)q_S) &= \delta(\beta - 2\gamma)\int_{(n-1)q_S}^{A}(a-(n-1)q_S)dF(a) - \delta w \\
&\geq \delta(\beta - 2\gamma)\int_{nq_S}^{A}(a - nq_S)dF(a) - \delta w \\
&= \delta\beta(1 - F(nq_S))q_S + cq_S \\
&> 0
\end{aligned} \tag{56}$$

for $q_S > 0$. For q_S to be positive, it must be the case that $\Pi_q^S(0,0) > 0$, that is $\delta(\beta - 2\gamma)E[a] - \delta w > 0$. As $\beta > 2\gamma$ this condition can be satisfied.

$\Pi_q^{CS}(\widehat{Q} - (n-1)q_S, (n-1)q_S) \leq 0$ if

$$a_1 - \left[\frac{\delta\beta(1-F(\widehat{Q}))}{1+\delta\beta(1-F(\widehat{Q}))} + c\right](\widehat{Q} - (n-1)q_S) - \frac{2\gamma(a_1 + \delta w)}{\beta[1 + \delta\beta(1 - F(\widehat{Q}))]} \leq 0, \tag{57}$$

which is clearly satisfied as a_1 tends to 0.

NP **equilibrium.** An NP equilibrium requires $\widehat{Q} > 0$ or, $a_1 \leq \delta(\beta E[a] - w)$. In addition, marginal profit must be negative for any output, so we require $\Pi_q^{CS}(\widehat{Q}, 0) \leq 0$ and $\Pi_q^S(0,0) \leq 0$. We have

$$\Pi_q^{CS}(\widehat{Q}, 0) = a_1 - \left(\frac{\delta\beta(1-F(\widehat{Q}))}{1+\delta\beta(1-F(\widehat{Q}))}\right)\widehat{Q} - c\widehat{Q} - \frac{2\gamma(a_1+\delta w)}{\beta(1+\delta\beta(1-F(\widehat{Q})))} \tag{58}$$

so $\Pi_q^{CS}(\widehat{Q}, 0) \leq 0$ for a_1 small enough as the last three terms of this expression are negative.

$\Pi_q^S(0,0) = \delta((\beta - 2\gamma)E[a] - w)$, so $\Pi_q^S(0,0) < 0$ if $\beta - 2\gamma < w/E[a]$. At the same time, speculation must not be blockaded, so we require $w/E[a] < \beta$. As this condition does not depend on a_1, we have that NP can be an equilibrium for a_1 small and

$$\beta - 2\gamma < \frac{w}{E[a]} < \beta,$$

a condition which is feasible for some model parameters.

Proof of Lemma 5

Total differentiation of (11) yields $\frac{d\widehat{Q}}{da_1} = -\frac{1}{\delta\beta(1-F(\widehat{Q}))} < 0$, so we have $\frac{d\widehat{Q}}{da_1} < \frac{d\widehat{Q}^o}{da_1} = -\frac{1}{\delta\beta} < 0$. When $a_1 = \delta\beta E[a] - \delta w$, $\widehat{Q} = \widehat{Q}^o = 0$. For $a_1 < \delta\beta E[a] - \delta w$ (i.e. such that $Q_L < 0$), $\widehat{Q} > \widehat{Q}^o$ as $\widehat{Q}$ is increasing at a faster rate as a_1 declines. Consequently $\widehat{Q} > \widehat{Q}^o$ for any a_1 such that $Q_L < 0$.‖

Proof of Proposition 1

Comparing i) of Lemma 1 with (29) we have that

$$\frac{d\widetilde{X}}{dQ_1} = -\frac{1}{1 + \delta\beta(1 - F(\widetilde{X}(Q_1)))} < \frac{d\widetilde{X}^o}{dQ_1}. \tag{59}$$

We can then distinguish 4 cases:

- When $Q_1 \leq \widehat{Q}^o$, $X_1^o(Q_1) = X_1(Q_1) = -Q_1$, and both have the same sensitivity to an increase of output.
- When $\widehat{Q}^o \leq Q_1 \leq \widehat{Q}$ and $Q_L < 0$, then $X_1(Q_1) = -Q_1 < \widetilde{X}^o(Q_1) < 0$. Speculators buy less than first period output without uncertainty while they still buy Q_1 with demand uncertainty.
- When $Q_1 > \widehat{Q}$ and $Q_L < 0$, since $\widetilde{X}(Q_1)$ is negative, decreasing and steeper than $\widetilde{X}^o(Q_1)$, and since $\widetilde{X}^o(\widehat{Q}) > -\widehat{Q} = \widetilde{X}(\widehat{Q})$, we have $-Q_1 < \widetilde{X}(Q_1) < \widetilde{X}^o(Q_1) < 0$.
- When $Q_L > 0$, $\widetilde{X}(Q_L) = \widetilde{X}^o(Q_L) = 0$. For $Q_1 > Q_L$, since $\widetilde{X}(Q_1)$ is steeper than $\widetilde{X}^o(Q_1)$, we have $\widetilde{X}(Q_1) < \widetilde{X}^o(Q_1) < 0$, with equality at $Q_1 = Q_L$.

We can conclude that $X_1(Q_1)$ is always larger in absolute value under future demand uncertainty than when demand is certain.‖

Proof of Proposition 5

In region S

$$\begin{aligned}\Pi_{qq}^S =& \Pi_{qQ}^S - \delta\beta(1 - F(Q_1)) - c \\ =& \delta(2\gamma - \beta)(1 - F(Q_1)) + \delta\beta f(Q_1)q_1^i - \delta\beta(1 - F(Q_1)) - c \\ =& 2\delta(\gamma - \beta)(1 - F(Q_1)) + \delta\beta f(Q_1)q_1^i - c\end{aligned} \tag{60}$$

Using Assumption 1 we have

$$\Pi_{qq}^S = 2\delta(\gamma - \beta)(1 - Q_1/A) + \delta\beta q_1^i/A - c \tag{61}$$

which is negative since $\gamma < \beta$ and $\delta\beta q_1^i/A < c$ from Assumption 2 and $q_1^i/A < 1$.

In the CS region we have

$$\begin{aligned}\Pi_{qq}^{CS} =& \Pi_{qQ}^{CS} - \left(1 + \frac{d\widetilde{X}}{dQ_1}\right) - c \\ =& \left(1 + \frac{d\widetilde{X}}{dQ_1}\right)\left(-2 + \frac{2\gamma}{\beta(1 + \delta\beta(1 + \widetilde{X}(Q_1)/A))}\right) \\ &+ \frac{\delta\beta/A}{(1 + \delta\beta(1 + \widetilde{X}(Q_1)/A)^3}\left(q_1^i - 2\frac{\gamma}{\beta}(P_1(Q_1) + \delta w)\right) - c. \end{aligned} \tag{62}$$

The first term is negative due to $\gamma < \beta$, while under Assumption 2 we have

$$\delta\beta\frac{q_1^i/A}{(1 + \delta\beta(1 + \widetilde{X}(Q_1)/A)^3} \quad < \delta\beta \leq c. \tag{63}$$

Consequently, $\Pi_{qq}^{CS} < 0.\|$.

Solution method

We describe here the method we use for determining the equilibrium in the first period of the example. Let us start with $Q_L \geq 0$:

- The equilibrium is of type C, with an individual quantity q_C determined by $\Pi_q^C(q_C, (n-1)q_C) = 0$, if no individual firm has an incentive to deviate to a quantity that induces speculation, i.e., to $\tilde{q} > Q_1^L - (n-1)q_C$. If $\Pi_q^{CS}(Q_1^L - (n-1)q_C, (n-1)q_C) \geq 0$, then a deviation may exist and we check that the profit obtained from this deviation is lower than that in the candidate C equilibrium before declaring a C equilibrium to exist.

- The equilibrium is of type CS, with an individual quantity q_{CS} determined by $\Pi_q^{CS}(q_{CS}, (n-1)q_{CS}) = 0$, if no individual firm has an incentive to deviate to a quantity that precludes speculation, i.e. to $\tilde{q} < Q_1^L - (n-1)q_{CS}$. If $\Pi_q^C(Q_1^L - (n-1)q_{CS}, (n-1)q_{CS}) \leq 0$, a deviation potentially exists and we check that the profit obtained from this deviation is lower than that in the candidate CS equilibrium before declaring the existence of a CS equilibrium.

- The equilibrium is of type L, with an individual quantity given by $q_L = Q_1^L/n$, if the marginal profit is downward jumping at Q_1^L: $\Pi_q^C(Q_1^L - (n-1)q_L, (n-1)q_L) \geq 0$ and $\Pi_q^{CS}(Q_1^L - (n-1)q_L, (n-1)q_L) \leq 0$. Under Proposition 5 no profitable deviation is possible from an L equilibrium.

Where $\widehat{Q}_1 \geq 0$, we know from Lemma 3 that marginal profit jumps upward when the aggregate quantity produced equals $\widehat{Q}_1$, ruling out the possibility for $\widehat{Q}_1$ to be an equilibrium outcome. The possibilities are:

- The equilibrium is of type S, with an individual quantity q_S determined by $\Pi_q^S(q_S, (n-1)q_S) = 0$, if no individual firm has an incentive to deviate to a quantity that induces consumer purchases, i.e., to $\tilde{q} > \widehat{Q}_1 - (n-1)q_S$. If $\Pi_q^{CS}(\widehat{Q}_1 - (n-1)q_S, (n-1)q_S) \geq 0$, a deviation potentially exists and we declare an S equilibrium only if the profit obtained by the deviation is less than that in the S equilibrium.

- The equilibrium is of type CS, with an individual quantity q_{CS} determined by $\Pi_q^{CS}(q_{CS}, (n-1)q_{CS}) = 0$, if no individual firm has an incentive to deviate to a quantity that precludes consumer purchases, i.e. $\tilde{q} < \widehat{Q}_1 - (n-1)q_{CS}$. If $\Pi_q^S(\widehat{Q}_1 - (n-1)q_{CS}, (n-1)q_{CS}) \leq 0$, then a deviation is possible and we declare a CS equilibrium only if the profit obtained by the deviation is less than that in the CS equilibrium.

- Finally the equilibrium is of type NP, with an individual quantity produced equal to 0, if the marginal payoff of producing 0 is negative, $\Pi_q^S(0, 0) \leq 0$.

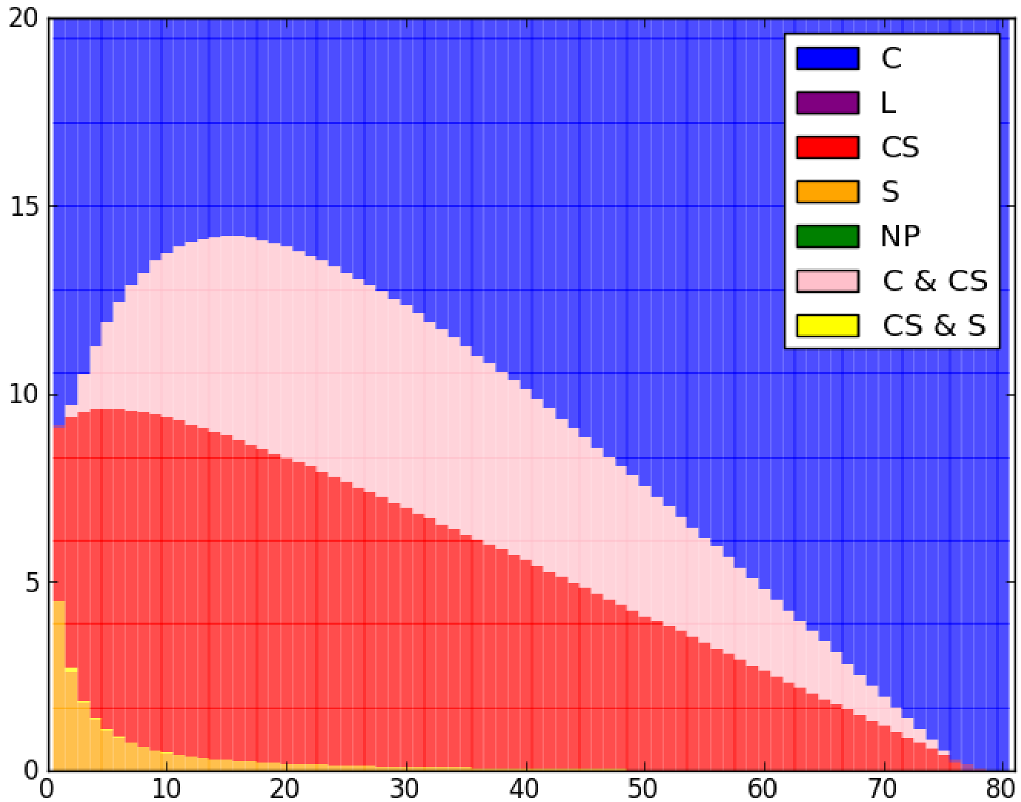

Figure 1: Equilibria in first period of two period game: $c = 0.6$, $w = 0.2$.

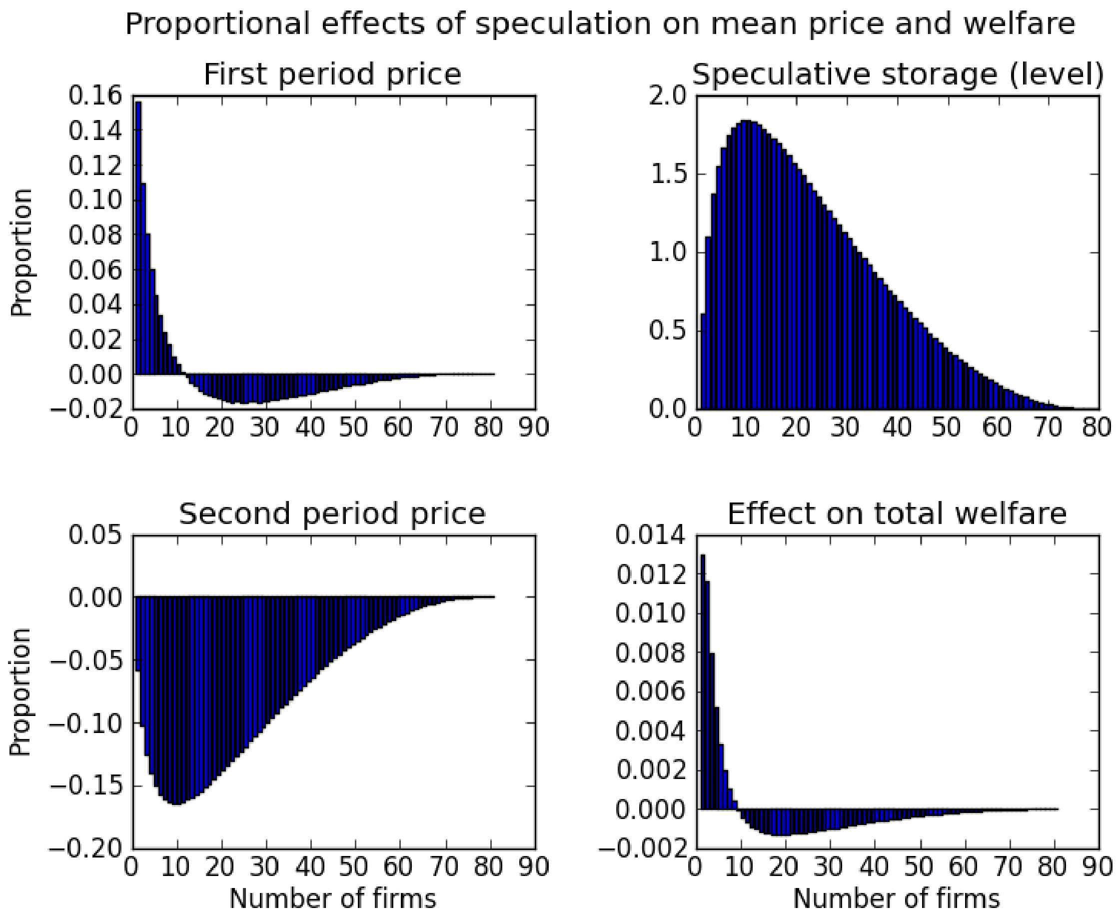

Figure 2: The effects of speculative storage on expected price and welfare relative to that in the absence of speculation: $c = 0.6$, $w = 0.2$.

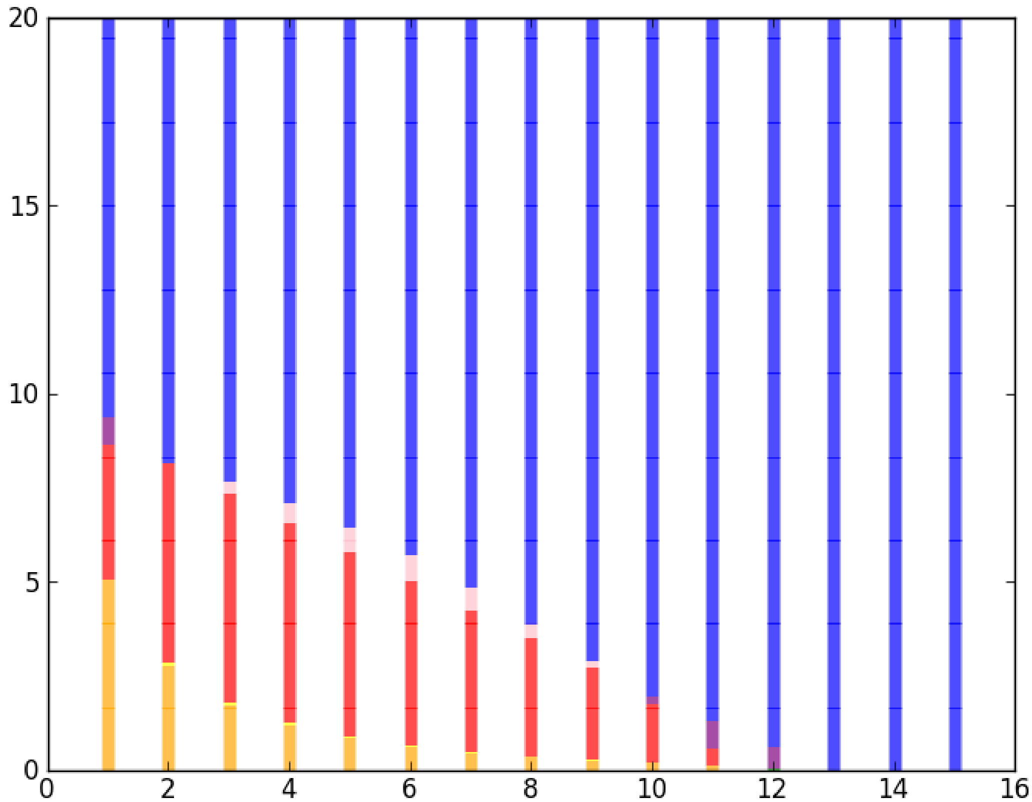

Figure 3: Equilibria in first period of two period game: $c = 0.6$, $w = 1.1$.

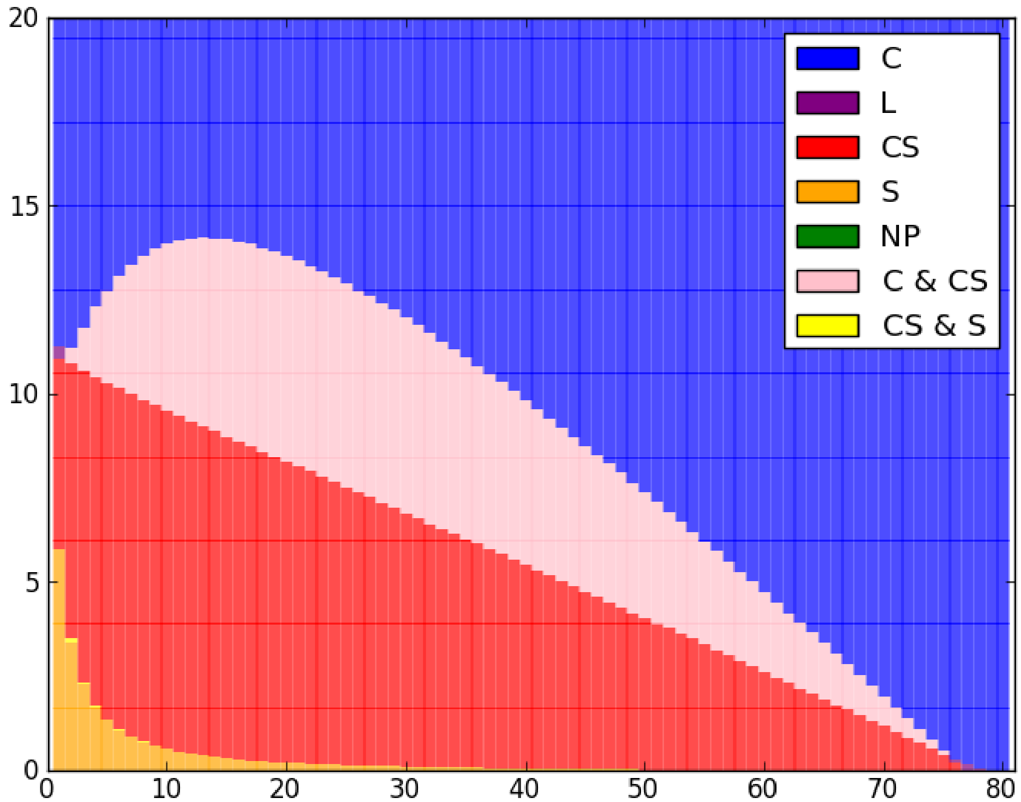

Figure 4: Equilibria in first period of four period game: $c = 0.6$, $w = 0.2$.