CSCE-629 Homework 5

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September 18, 2020

Exercise 17.1-2

Given a k bits sequence $A = 1\overbrace{000\cdots000}^{k-1}$, k bits flip when we perform the DECREMENT operation. Following the first operation, we perform the INCREMENT operation, and all k bits flip again. If we continually perform INCREMENT and DECREMENT in turns for n times, the total cost will be $n \times k$, which is also the worst case. Therefore, n operations could cost as much as $\theta(nk)$ time.

Exercise 17.2-1

- If MULTIPOP is included in the stack operation as in the text book, each of the three operations need to account for one more unit cost. Therefore, the amortized cost is: {PUSH 3, POP 1, MULTIPOP 1}.
- If MULTIPOP is not included, we should assign: {PUSH 2, POP 2}.

For either case, we pay 1 dollar more for the later COPY operation. Since the stack size never exceeds k, and we make a COPY after k operations, we have always charged enough, and we have ensured that the amount of credit is always non-negative. Thus, the total amortized cost is $O(n)$ which is also the upper bound of total actual cost.

Exercise 17.3-1

- Let $\Phi'(D_i) = \Phi(D_i) - \Phi(D_0)$
- $\Phi'(D_i) \geq 0$, because $\Phi(D_i) - \Phi(D_0) \geq 0$
- $\Phi'(D_0) = \Phi(D_0) - \Phi(D_0) = 0$
- The amortized cost: $\hat{c}_i' = c_i + \Phi'(D_i) - \Phi'(D_{i-1})$
  $= c_i + (\Phi(D_i) - \Phi(D_0)) - (\Phi(D_{i-1}) - \Phi(D_0))$
  $= c_i + (\Phi(D_i) - \Phi(D_{i-1})) - (\Phi(D_0) - \Phi(D_0)) = \hat{c}_i$

  Therefore, the amortized costs using Φ' are the same as the amortized costs using Φ

Exercise 17.3-2

Let the potential function $\Phi(D_i) = 2i - 2^{(1+\lfloor(lg(i))\rfloor)}$, for $i > 0$, and $\Phi(D_0) = 0$

- For $i = 1 = 2^0, \hat{c}_i = c_i + \Phi(D_i) - \Phi(D_{i-1}) = 1 + (2-2) - (0) = 1$
- For $i = 2^k, k \geq 1, \hat{c}_i = 2^k + (2(2^k) - 2^{k+1}) - (2(2^k - 1) - 2^k) = 2$
- For $i \neq 2^k, i \geq 3, \hat{c}_i = 1 + (2i - 2^{(1+\lfloor(lg(i))\rfloor)}) - (2(i-1) - 2^{(1+\lfloor(lg(i-1))\rfloor)})$
  $= 1 + 2i - 2(i-1) = 3$

Thus, the amortized cost of each of operations is $O(1)$.