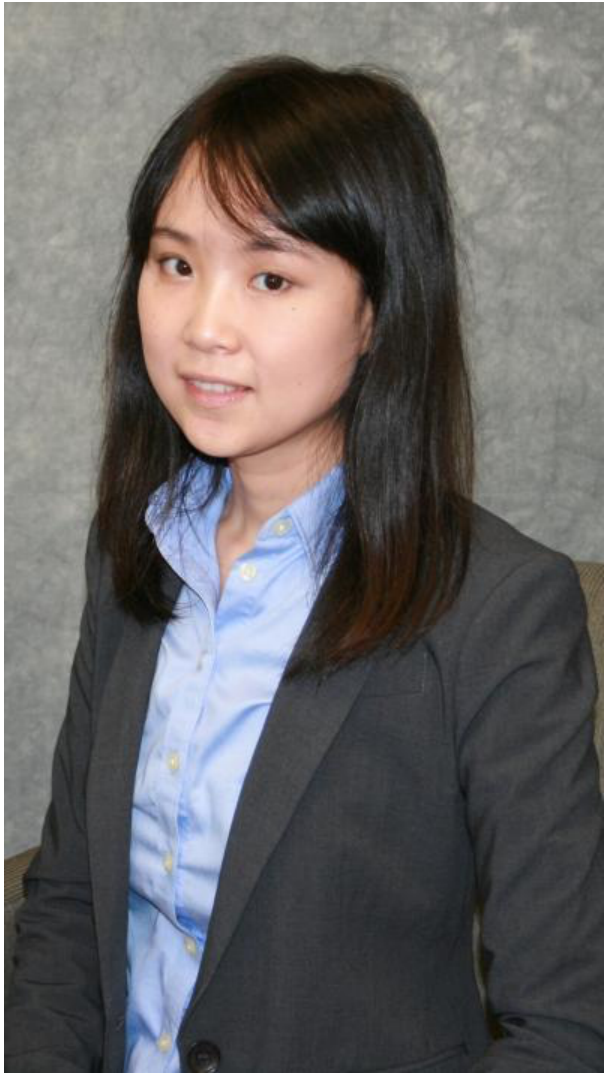
Computational Data Analysis

Machine Learning



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Associate Professor

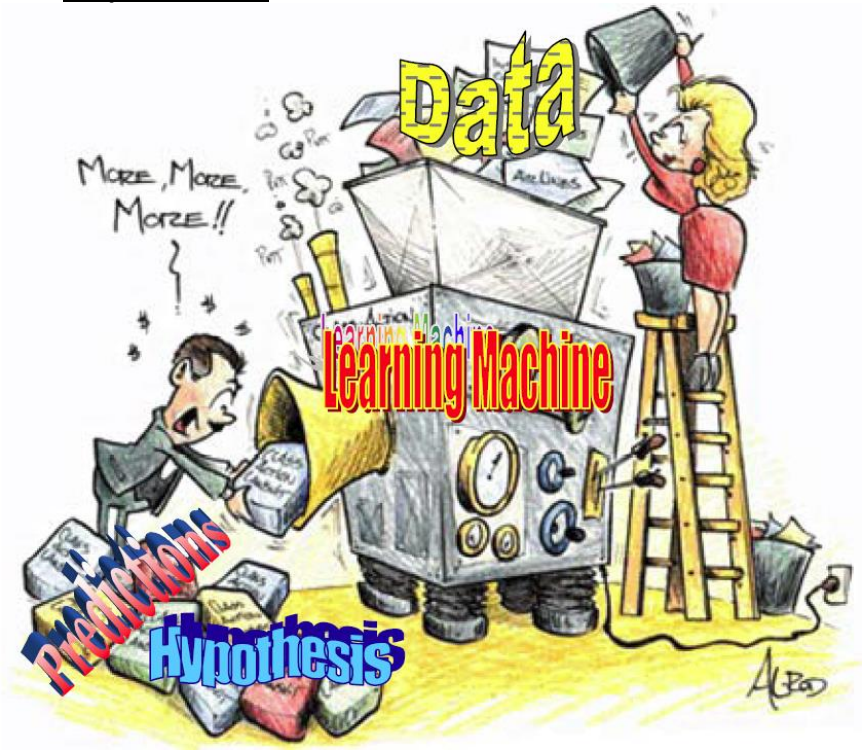
Harold R. and Mary Anne Nash Early Career Professor

H. Milton Stewart School of Industrial and Systems Engineering

Introduction

What is machine learning (ML)

- Study of algorithms that improve their <u>performance</u> at some <u>task</u> with <u>experience</u>



Common to industrial scale problems

13 million wikipedia pages

facebook

800 million users

flickr

6 billion photos

340 million tweets per day

YouTube

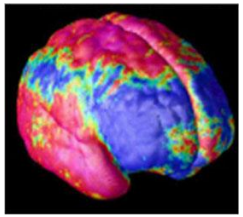
24 hours video uploaded per minutes

Google

> 1 trillion webpages

Increasingly relevant to science problems

Brain

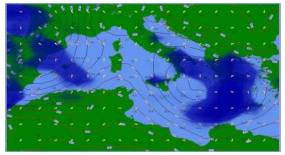


Galaxy

Self-driving car

Robot

Genome

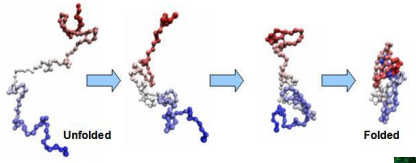


Weather

Finance

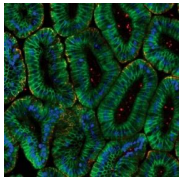
Music

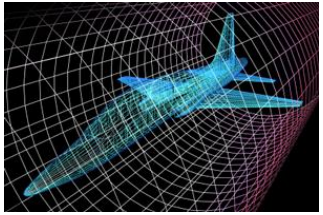
Sustainability



Protein

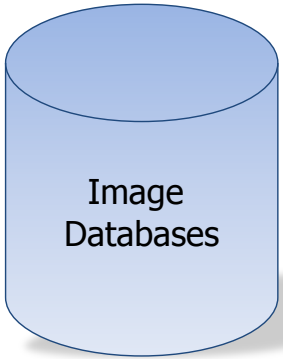
Cell





Design

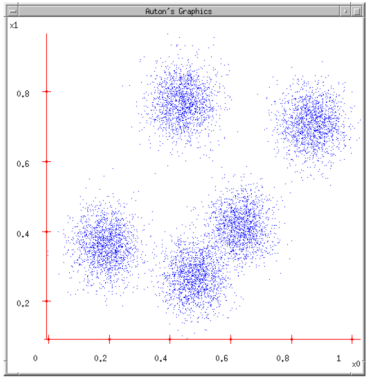
Organizing Images

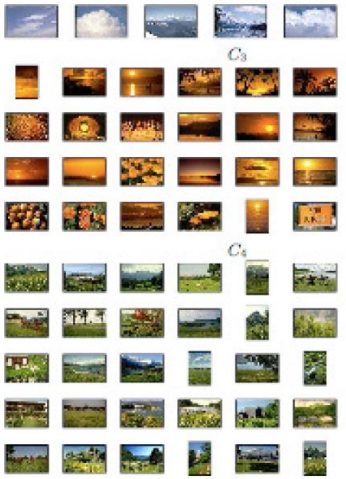


What are the desired outcomes?

What are the inputs (data)?

What are the learning paradigms?



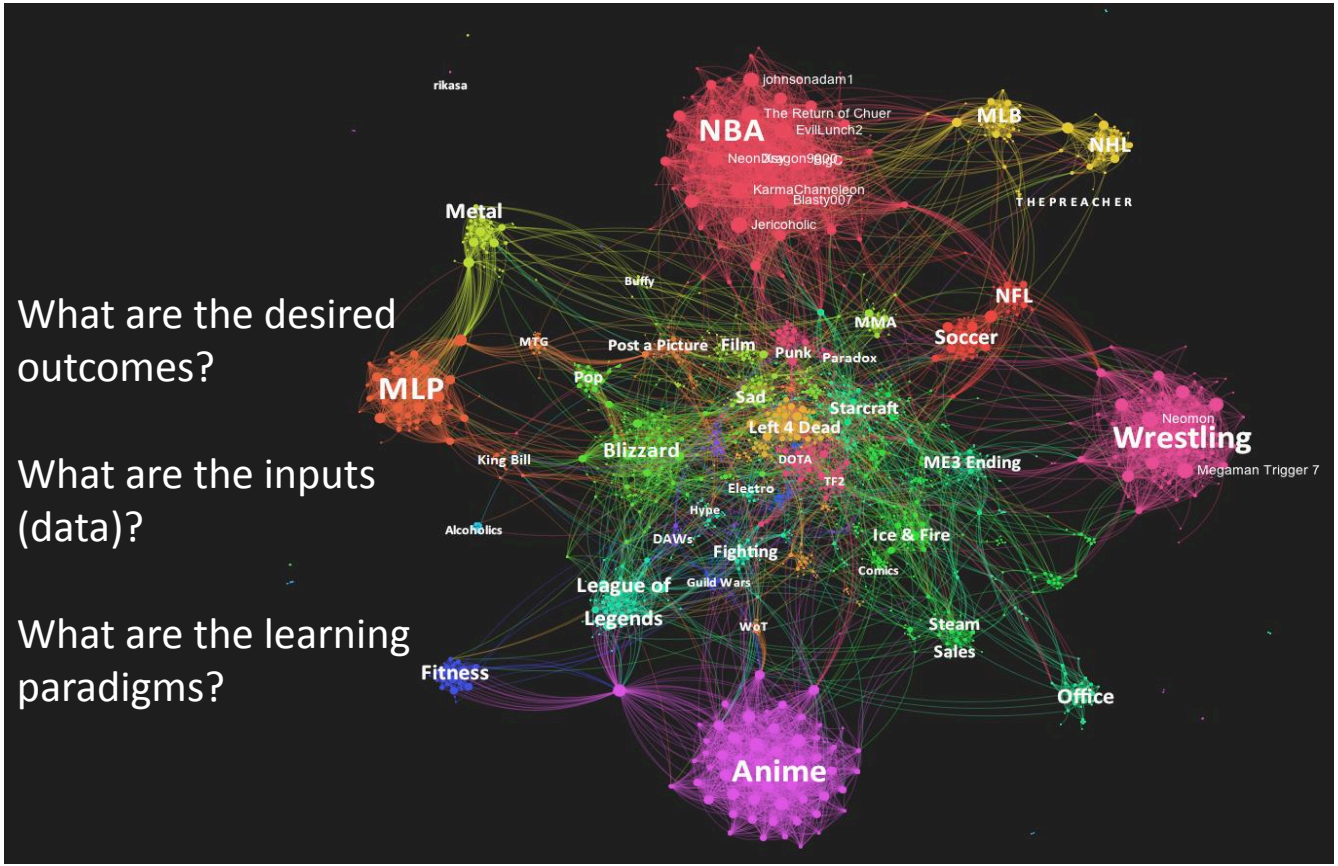


Find community in social networks

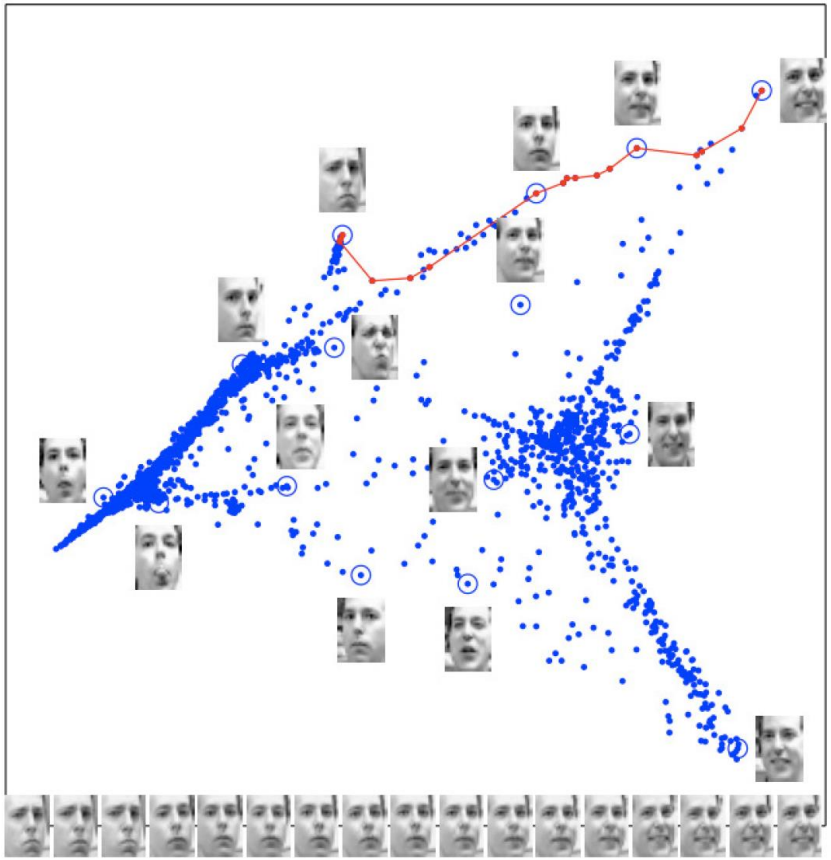
What are the desired outcomes?

What are the inputs (data)?

What are the learning paradigms?



Visualize Image Relations



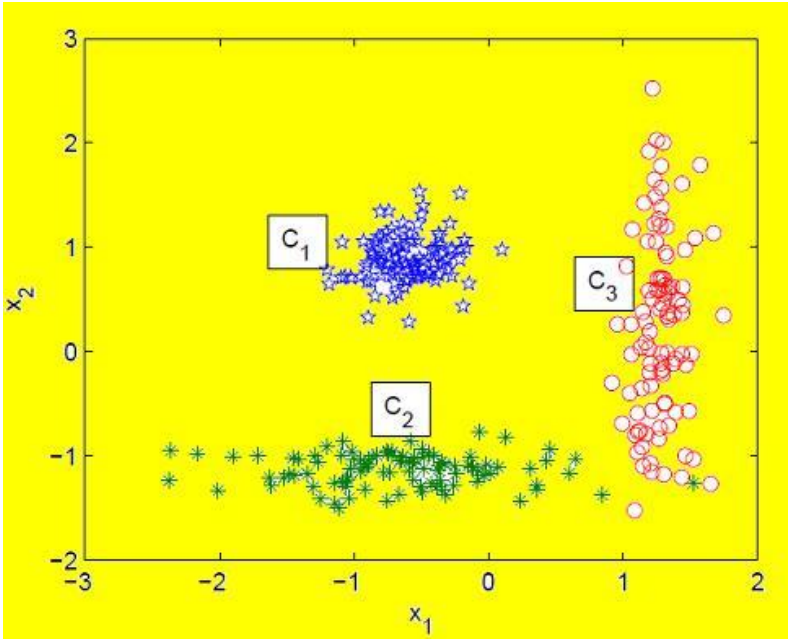
Each image has thousands or millions of pixels.

What are the desired outcomes?

What are the inputs (data)?

What are the learning paradigms?

Feature selection



What are the desired outcomes?

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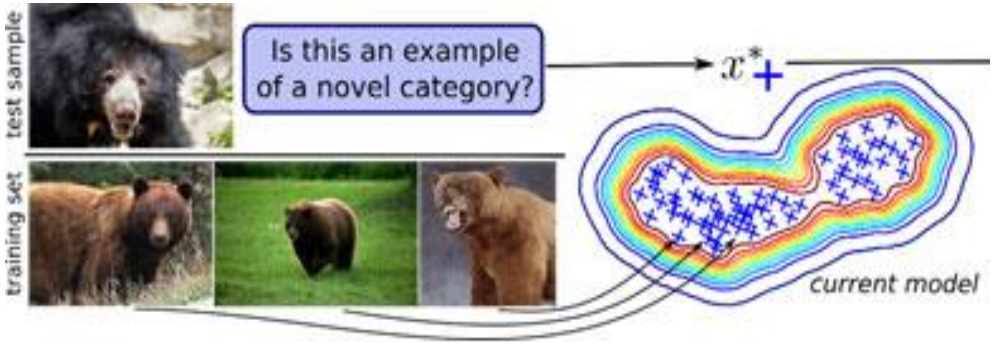
What are the learning paradigms?

Novelty/abnormality detection



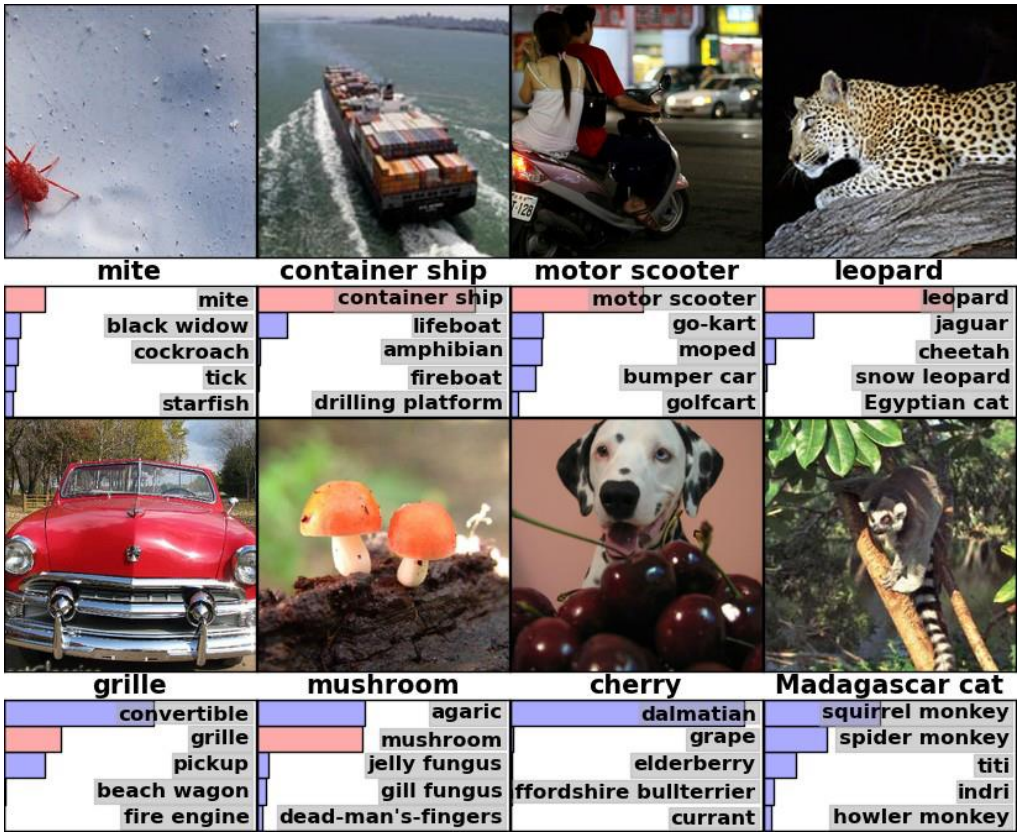
Find abnormal object

What are the desired outcomes?

What are the inputs (data)?

What are the learning paradigms?

Image classification



What are the desired outcomes?

What are the inputs (data)?

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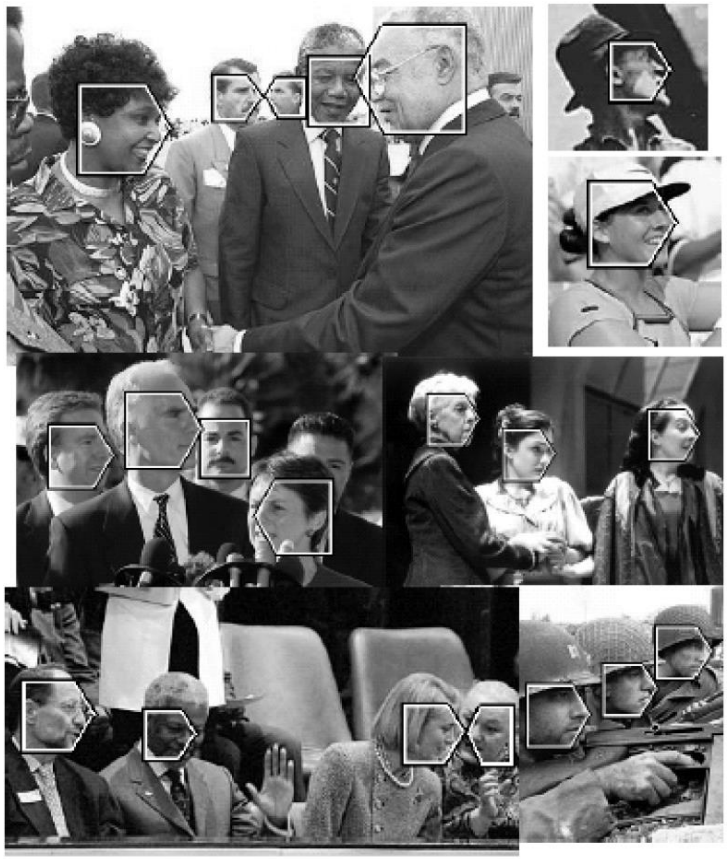
Face Detection

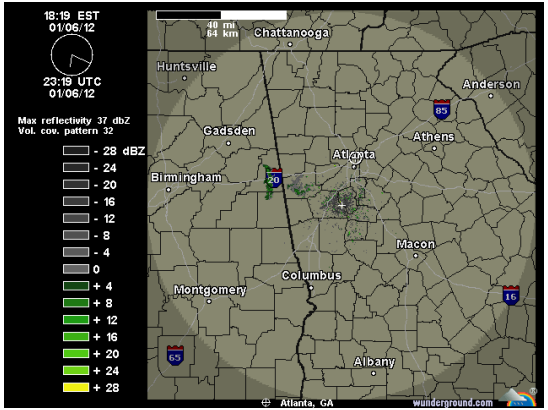


What are the desired outcomes?

What are the inputs (data)?

What are the learning paradigms?

Weather Prediction



Predict

Numeric values:
40 F
Wind: NE at 14 km/h
Humidity: 83%

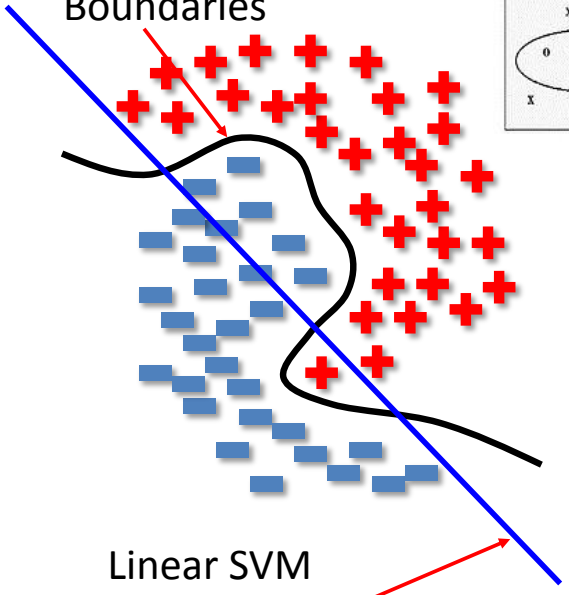
Predict

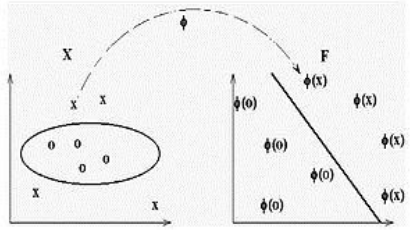
What are the desired outcomes?

What are the inputs (data)?

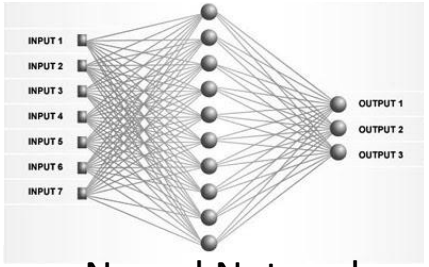
What are the learning paradigms?

Nonlinear classifier





Kernel methods



Neural Networks

What are the desired outcomes?

What are the inputs (data)?

What are the learning paradigms?

Spam Filtering

What are the desired outcomes?

What are the inputs (data)?

What are the learning paradigms?

Handwritten digit recognition/text annotation

Inter-character dependency

Inter-word dependency

Aoccdrnig to a sudty at Cmabrigde Uinervtisy, it deosn't mttaer in waht oredr the ltteers in a wrod are, the olny iprmoetnt tihng is taht the frist and lsat ltteer be at the rghit pclae. The rset can be a ttoal mses and you can sitll raed it wouthit a porbelm. Tihs is bcuseae the huamn mnid deos not raed ervey lteter by istlef, but the wrod as a wlohe.

What are the desired outcomes?

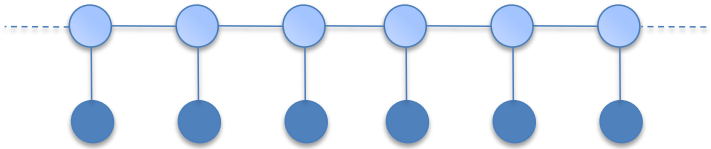
What are the inputs (data)?

What are the learning paradigms?

Speech recognition

Models

Hidden Markov Models



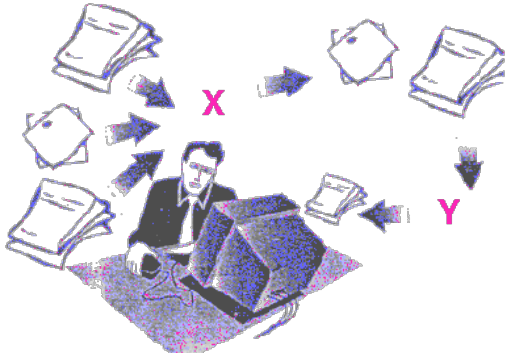
Text

“Machine Learning is the preferred method for speech recognition …”

Audio signals

Organizing documents

- Reading, digesting, and categorizing a vast text database is too much for human!



- **We want:**

"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHER
ACTOR	NEW	SAYS	BENNETT
FIRST	STATE	FAMILY	MANIGAT
YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
THEATER	PROGRAMS	PERCENT	PRESIDENT
ACTRESS	GOVERNMENT	CARE	ELEMENTARY
LOVE	CONGRESS	LIFE	HAITI

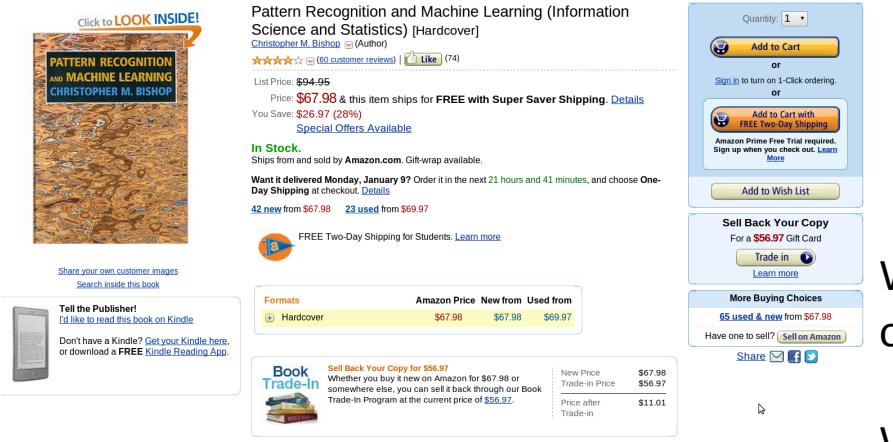
The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.

What are the desired outcomes?

What are the inputs (data)?

What are the learning paradigms?

Product Recommendation



What are the desired outcomes?

What are the inputs (data)?

What are the learning paradigms?

Robot Control

- Now cars can find their own ways!

What are the desired outcomes?

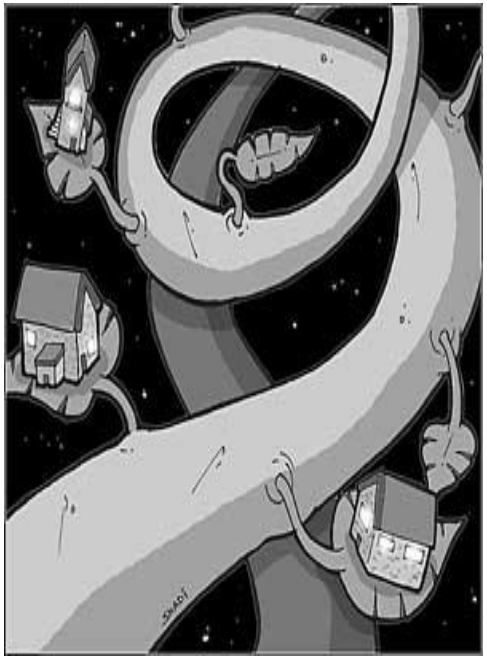
What are the inputs (data)?

What are the learning paradigms?

Basics/Prerequisites

- Probabilities
  - Distributions, densities, marginalization, conditioning
- Statistics
  - Mean, variance, maximum likelihood estimation
- Linear algebra
  - Vector, matrix, multiplication, inversion, eigen-decomposition
- Algorithms and Programming
  - Matlab, Basic data structures, computational complexity
- Convex optimization
  - Basics will be covered during lecture

Machine learning for apartment hunting



- Suppose you are to move to Atlanta
- And you want to find the **most reasonably priced** apartment satisfying your **needs:**

  square-ft., # of bedroom, distance to campus …

Living area (ft^2)	# bedroom	Rent ($)
230	1	600
506	2	1000
433	2	1100
109	1	500
…		
150	1	?
270	1.5	?

Linear Regression Model

- Assume y is a linear function of x (features) plus noise ϵ

$$y = \theta_0 + \theta_1 x_1 + \cdots + \theta_n x_n + \epsilon$$

where ϵ is an error model as Gaussian $N(0, \sigma^2)$ ← Probability

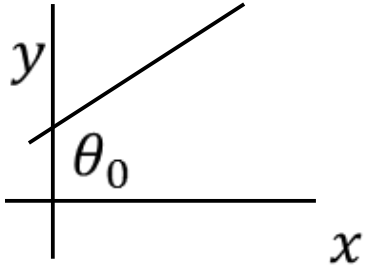
- Let $\theta = (\theta_0, \theta_1, \ldots, \theta_n)^\top$, and augment data by one dimension

Linear algebra

$$x \leftarrow (1, x)^\top$$

Then $y = \theta^\top x + \epsilon$

Linear algebra



Least mean square method

- Given m data points, find θ that minimizes the mean square error

$$\hat{\theta} = argmin_{\theta}\, L(\theta) = \frac{1}{m}\sum_{i}^{m}\left(y^i - \theta^{\top}x^i\right)^2$$

Optimization

Statistics

- Set gradient to 0 and find parameter

Optimization

Linear algebra

$$\frac{\partial L(\theta)}{\partial \theta} = -\frac{2}{m}\sum_{i}^{m}\left(y^i - \theta^{\top}x^i\right)x^i = 0$$

$$\Leftrightarrow -\frac{2}{m}\sum_{i}^{m} y^i x^i + \frac{2}{m}\sum_{i}^{m} x^i x^{i\top}\theta = 0$$

Statistics

Statistics

Matrix version of the gradient

- Define $X = (x^1, x^2, \ldots x^m), y = (y^1, y^2, \ldots, y^m)^\top$, gradient becomes

Linear algebra →

$$\frac{\partial L(\theta)}{\partial \theta} = -\frac{2}{m} Xy + \frac{2}{m} XX^\top \theta$$

Linear algebra →

$$\Rightarrow \hat{\theta} = (XX^\top)^{-1} Xy$$

Algorithms Programming →

- Matrix inversion in $\hat{\theta} = (XX^\top)^{-1} Xy$ expensive to compute
  - Gradient descent

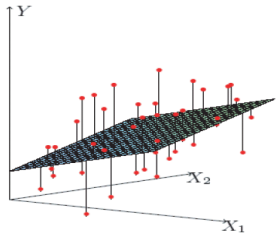
$$\hat{\theta}^{t+1} \leftarrow \hat{\theta}^t + \frac{\alpha}{m} \sum_i^m \left(y^i - \hat{\theta}^{t\top} x^i \right) x^i$$

Optimization ←

Probabilistic Interpretation of LMS

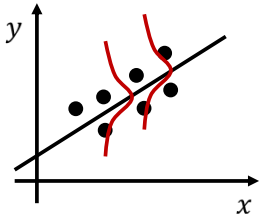
- Assume y is a linear in x plus noise ϵ

$$y = \theta^\top x + \epsilon$$



- Assume ϵ follows a Gaussian $N(0,\sigma^2)$

$$p(y^i|x^i;\theta) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(y^i-\theta^\top x^i)^2}{2\sigma^2}\right)$$



- By independence assumption, likelihood is $L(\theta)$

$$= \prod_i^m p(y^i|x^i;\theta) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^m \exp\left(-\frac{\sum_i^m (y^i-\theta^\top x^i)^2}{2\sigma^2}\right)$$

Probability

Probabilistic Interpretation of LMS, cont.

- Hence the log-likelihood is:

$$\log L(\theta) = m \log \frac{1}{\sqrt{2\pi}\sigma} - \frac{1}{2\sigma^2} \sum_i^m \left(y^i - \theta^\top x^i\right)^2$$

- LMS is equivalent to MLE of θ !

Statistics

$$LMS: \quad \frac{1}{m} \sum_i^m \left(y^i - \theta^\top x^i\right)^2$$

- How to make it work in real data?

Algorithms
Programming


SUMMARY