Regression Analysis
Other Regression Methods
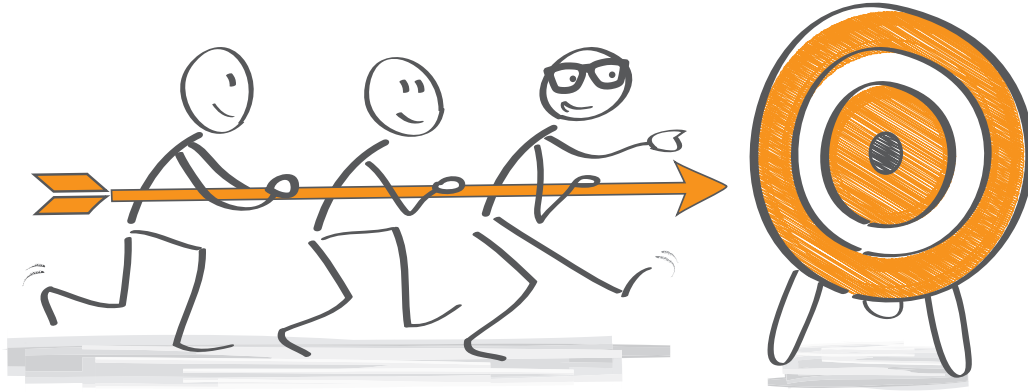
Nicoleta Serban, Ph.D.
Associate Professor

Stewart School of Industrial and Systems Engineering

Mixed Effects Models

GTx

About this lesson

ANOVA Model

Data: Y_{ij} for $j = 1, \ldots, n_i; i = 1, \ldots, k$

Model: $Y_{ij} = \mu_i + \varepsilon_{ij} = \mu + \tau_i + \varepsilon_{ij}$ where $\sum_{i=1}^{k} \tau_i = 0$

$\mu_i = $ i-th group mean decomposed into $\mu_i = \mu + \tau_i$

- In some designs, the categorical variable is "subject" or experimental setting

- Simplest example: repeated measures, where more than one (identical) measurement is taken on the same setting.

ANOVA Model: Random Effects

Data: Y_{ij} $j = 1, \quad ..., \quad n_i; i = 1,..., \quad k$

Are the assumptions the same as in ANOVA with fixed effects?
- In random effects model, the observations are no longer
- independent (even if the error terms are independent).

- $\varepsilon_{ij} \sim N(0, \sigma^2)$
- $\tau_i \sim N(0, \sigma_r^2)$
- We might be interested in the in the variability across subjects, i.e. σ_r^2. Is it zero?

GTx

ANOVA Model: Random Effects

When to use random effects?

- A "group" effect is random if we can think of the responses we observe in the group to be samples from a larger population.
- Example: if collecting data from different medical centers, "center" might be thought of as random.
- Example: if surveying students on different campuses, "campus" may be a random effect.

Regression Model: Mixed Effects

- In some studies, some factors can be thought of as fixed, others random.

- **If not all the X's are the same for each subject, or some observations are missing, things are more complicated.**
- **Covariance matrix of Y is more complicated; use a computer to estimate such models!**

- **Model**: $Y_{ij} = \mu + \tau_i + \alpha X_{ij} + \varepsilon_{ij}$

- $\varepsilon_{ij} \sim N(0, \sigma^2)$

- $\tau_i \sim N(0, \sigma_a^2)$

- $X_{ij} = j, j = 1, \dots, 7$ in this example

GTx

Summary