Regression Analysis Regression Analysis in Practice

Nicoleta Serban, Ph.D.

Professor

Stewart School of Industrial and Systems Engineering

Emergency Department Healthcare Costs: Variable Selection

About This Lesson



Lasso Regression

```
predictors = as.matrix(dataAdult[, -c(1, 2, 3, 4, 5, 10, 13, 18)])

# Set up indicator (dummy) variables for State and Urbanicity
# Leave out one indicator (dummy) variable for each group

#AL= rep(0, length(State))
AR = rep(0, length(State))
LA = rep(0, length(State))
NC = rep(0, length(State))
#AL[as.numeric(State)==1] = 1
AR[as.numeric(State)==2] = 1
LA[as.numeric(State)=3] = 1
NC[as.numeric(State)=4] = 1

#rural = rep(0, length(Urbanicity))
suburban = rep(0, length(Urbanicity))
urban = rep(0, length(Urbanicity))
# rural[as.numeric(Urbanicity)=1] = 1
suburban[as.numeric(Urbanicity)=2] = 1
urban[as.numeric(Urbanicity)=3] = 1

predictors = cbind(predictors, AR, LA, NC, suburban, urban)
```

Georgia Tech

Lasso Regression

10-fold CV to find the optimal lambda

lassomodel.cv = cv.glmnet(predictors, log(EDCost.pmpm), alpha=1) nfolds=10)

Fit lasso model with 100 values for lambda

lassomodel = glmnet(predictors, log(EDCost.pmpm), alpha=1) nlambda=100)

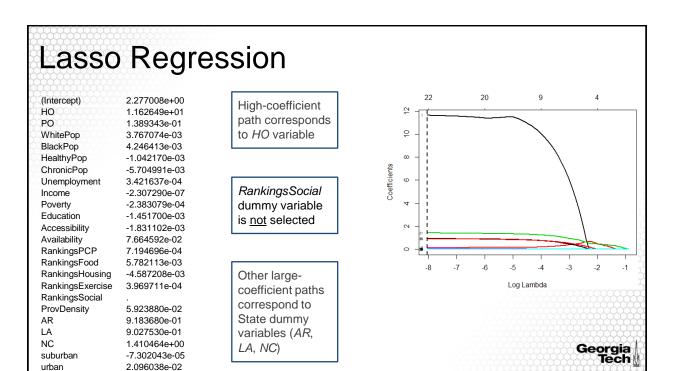
Plot coefficient paths

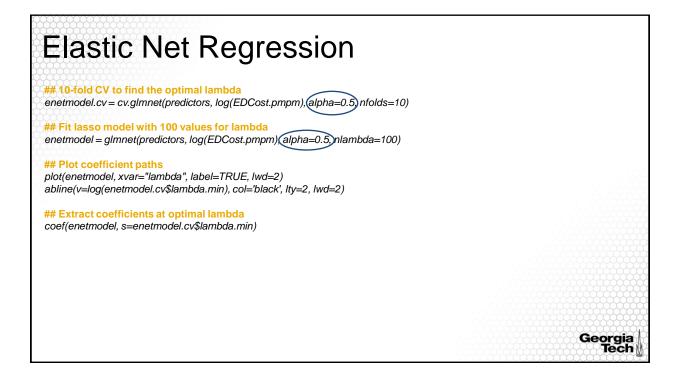
plot(lassomodel, xvar="lambda", label=TRUE, lwd=2) abline(v=log(lassomodel.cv\$lambda.min), col='black', lty=2, lwd=2)

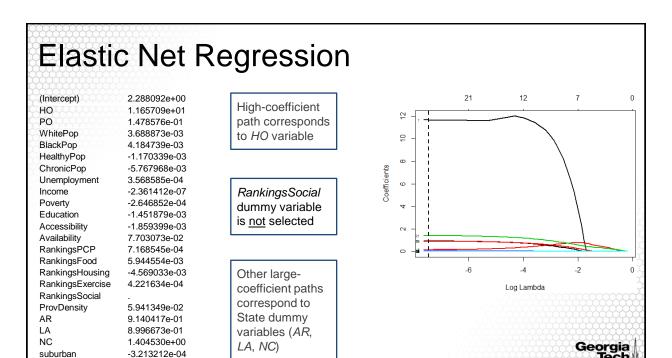
Extract coefficients at optimal lambda

coef(lassomodel, lassomodel.cv\$lambda.min)

Georgia Tech







Stepwise Regression

2.105330e-02

urban

full = Im(log(EDCost.pmpm) ~ HealthyPop + ChronicPop + State + Urbanicity + HO + PO +

BlackPop + WhitePop + Unemployment + Income + Poverty+ Education +

Accessibility + Availability + ProvDensity +

RankingsPCP + RankingsFood + RankingsExercise + RankingsSocial)

minimum = Im(log(EDCost.pmpm) ~ HealthyPop + ChronicPop)

Forward Stepwise Regression

forward.model = step(minimum, scope=list(lower=minimum, upper=full), direction="forward") summary(forward.model)

Backward Stepwise Regression

backward.model = step(full, scope=list(lower=minimum, upper=full), direction = "backward") summary(backward.model)

Forward-Backward Stepwise Regression

both.min.model = step(minimum, scope=list(lower=minimum, upper=full), direction = "both") summary(both.min.model)



Stepwise Regression

Observations

- Some variables were not selected by any method
 - Unemployment, Income, Poverty, RankingExercise
- UrbanicitySuburban was not statistically significant in any model
- Variables selected first by forward stepwise regression, in order
 - State dummy variables (StateAR, StateLA, StateNC)
 - Number of claims per-member-per-month (HO, PO)

Georgia

Stepwise Regression Model

- Two models to consider
 - Forward stepwise regression selected RankingsSocial

summar	y(forward	d.model)

Julinia y (101 warani	ouci,			
	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2.397e+00	1.008e-01	23.789	< 2e-16 ***
HealthyPop	-2.143e-03	8.196e-04	-2.615	0.008945 **
ChronicPop	-7.301e-03	2.085e-03	-3.501	0.000467 ***
StateAR	1.011e+00	1.348e-02	75.041	< 2e-16 ***
StateLA	9.231e-01	1.228e-02	75.145	< 2e-16 ***
StateNC	1.482e+00	1.304e-02	113.630	< 2e-16 ***
НО	9.896e-04	9.043e-05	10.943	< 2e-16 ***
PO	-3.928e-05	4.952e-06	-7.933	2.61e-15 ***
Education	-1.635e-03	2.436e-04	-6.715	2.09e-11 ***
ProvDensity	6.105e-02	1.581e-02	3.861	0.000114 ***
RankingsPCP	7.457e-04	1.641e-04	4.543	5.67e-06 ***
Availability	1.006e-01	1.942e-02	5.180	2.30e-07 ***
Accessibility	-2.551e-03	7.118e-04	-3.584	0.000342 ***
RankingsFood	1.571e-02	4.241e-03	3.705	0.000214 ***
RankingsSocial	1.014e-03	1.351e-03	0.750	0.453169
UrbanicitySuburban	9.041e-03	1.384e-02	0.653	0.513725
UrbanicityUrban	2.708e-02	1.278e-02	2.118	0.034197 *
BlackPop	4.159e-03	5.871e-04	7.084	1.60e-12 ***
WhitePop	4.059e-03	5.833e-04	6.959	3.87e-12 ***

Residual standard error: 0.2357 on 5000 degrees of freedom Multiple R-squared: 0.8437, Adjusted R-squared: 0.8431 F-statistic: 1499 on 18 and 5000 DF, p-value: < 2.2e-16

summary(backward.model)

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2.403e+00	1.005e-01	23.905	< 2e-16 ***
HealthyPop	-2.172e-03	8.187e-04	-2.653	0.008015 **
ChronicPop	-7.228e-03	2.083e-03	-3.470	0.000524 ***
StateAR	1.011e+00	1.344e-02	75.199	< 2e-16 ***
StateLA	9.209e-01	1.194e-02	77.157	< 2e-16 ***
StateNC	1.482e+00	1.303e-02	113.694	< 2e-16 ***
UrbanicitySuburban	9.280e-03	1.384e-02	0.671	0.502523
UrbanicityUrban	2.550e-02	1.261e-02	2.022	0.043193 *
НО	9.913e-04	9.040e-05	10.966	< 2e-16 ***
PO	-3.933e-05	4.951e-06	-7.943	2.42e-15 ***
BlackPop	4.269e-03	5.686e-04	7.507	7.10e-14 ***
WhitePop	4.180e-03	5.606e-04	7.456	1.04e-13 ***
Education	-1.642e-03	2.434e-04	-6.746	1.69e-11 ***
Accessibility	-2.538e-03	7.116e-04	-3.566	0.000366 ***
Availability	1.005e-01	1.941e-02	5.177	2.34e-07 ***
ProvDensity	6.172e-02	1.579e-02	3.910	9.36e-05 ***
RankingsPCP	7.736e-04	1.599e-04	4.838	1.35e-06 ***
RankingsFood	1.508e-02	4.158e-03	3.628	0.000288 ***

Residual standard error: 0.2357 on 5001 degrees of freedom Multiple R-squared: 0.8437, Adjusted R-squared: 0.8432 **Georgia** F-statistic: 1588 on 17 and 5001 DF, p-value: < 2.2e-16

Stepwise Regression Model

- Two models to consider
 - Forward stepwise regression selected RankingsSocial

summary(forward.model)							
	Estimate	Std. Error	t value	Pr(> t)			
(Intercept)	2.397e+00	1.008e-01	23.789	< 2e-16 ***			
HealthyPop	-2.143e-03	8.196e-04	-2.615	0.008945 **			
ChronicPop	-7.301e-03	2.085e-03	-3.501	0.000467 ***			
StateAR	1.011e+00	1.348e-02	75.041	< 2e-16 ***			
StateLA	9.231e-01	1.228e-02	75.145	< 2e-16 ***			
StateNC	1.482e+00	1.304e-02	113.630	< 2e-16 ***			
HO	9.896e-04	9.043e-05	10.943	< 2e-16 ***			
PO	-3.928e-05	4.952e-06	-7.933	2.61e-15 ***			
Education	-1.635e-03	2.436e-04	-6.715	2.09e-11 ***			
ProvDensity	6.105e-02	1.581e-02	3.861	0.000114 ***			
RankingsPCP	7.457e-04	1.641e-04	4.543	5.67e-06 ***			
Availability	1.006e-01	1.942e-02	5.180	2.30e-07 ***			
Accessibility	-2.551e-03	7.118e-04	-3.584	0.000342 ***			
RankingsFood	1.571e-02	4.241e-03	3.705	0.000214 ***			
RankingsSocial	1.014e-03	1.351e-03	0.750	0.453169			
UrbanicitySuburban	9.041e-03	1.384e-02	0.653	0.513725			
UrbanicityUrban	2.708e-02	1.278e-02	2.118	0.034197 *			
BlackPop	4.159e-03	5.871e-04	7.084	1.60e-12 ***			

5.833e-04

Residual standard error: 0.2357 on 5000 degrees of freedom Multiple R-squared: 0.8437 Adjusted R-squared: 0.8431 F-statistic: 1499 on 18 and 5000 DF, p-value: < 2.2e-16

Both models explain the same amount of variance (about 84%). Prefer the smaller model.

summary(backward.model)						
	Estimate	Std. Error	t value	Pr(> t)		
(Intercept)	2.403e+00	1.005e-01	23.905	< 2e-16 ***		
HealthyPop	-2.172e-03	8.187e-04	-2.653	0.008015 **		
ChronicPop	-7.228e-03	2.083e-03	-3.470	0.000524 ***		
StateAR	1.011e+00	1.344e-02	75.199	< 2e-16 ***		
StateLA	9.209e-01	1.194e-02	77.157	< 2e-16 ***		
StateNC	1.482e+00	1.303e-02	113.694	< 2e-16 ***		
UrbanicitySuburban	9.280e-03	1.384e-02	0.671	0.502523		
UrbanicityUrban	2.550e-02	1.261e-02	2.022	0.043193 *		
НО	9.913e-04	9.040e-05	10.966	< 2e-16 ***		
PO	-3.933e-05	4.951e-06	-7.943	2.42e-15 ***		
BlackPop	4.269e-03	5.686e-04	7.507	7.10e-14 ***		
WhitePop	4.180e-03	5.606e-04	7.456	1.04e-13 ***		
Education	-1.642e-03	2.434e-04	-6.746	1.69e-11 ***		
Accessibility	-2.538e-03	7.116e-04	-3.566	0.000366 ***		
Availability	1.005e-01	1.941e-02	5.177	2.34e-07 ***		
ProvDensity	6.172e-02	1.579e-02	3.910	9.36e-05 ***		
RankingsPCP	7.736e-04	1.599e-04	4.838	1.35e-06 ***		
RankingsFood	1.508e-02	4.158e-03	3.628	0.000288 ***		

Residual standard error: 0.2357 on 5001 degrees of freedom Multiple R-squared: 0.8437 Adjusted R-squared: 0.8432 F-statistic: 1588 on 17 and 5001 DF, p-value: < 2.2e-16

Georgia Tech

Stepwise Regression Model

3.87e-12 ***

- Two models to consider
 - Forward stepwise regression selected RankingsSocial

summary(forward.model)						
	Estimate	Std. Error	t value	Pr(> t)		
(Intercept)	2.397e+00	1.008e-01	23.789	< 2e-16 ***		
HealthyPop	-2.143e-03	8.196e-04	-2.615	0.008945 **		
ChronicPop	-7.301e-03	2.085e-03	-3.501	0.000467 ***		
StateAR	1.011e+00	1.348e-02	75.041	< 2e-16 ***		
StateLA	9.231e-01	1.228e-02	75.145	< 2e-16 ***		
StateNC	1.482e+00	1.304e-02	113.630	< 2e-16 ***		
НО	9.896e-04	9.043e-05	10.943	< 2e-16 ***		
PO	-3.928e-05	4.952e-06	-7.933	2.61e-15 ***		
Education	-1.635e-03	2.436e-04	-6.715	2.09e-11 ***		
ProvDensity	6.105e-02	1.581e-02	3.861	0.000114 ***		
RankingsPCP	7.457e-04	1.641e-04	4.543	5.67e-06 ***		
Availability	1.006e-01	1.942e-02	5.180	2.30e-07 ***		
Accessibility	-2.551e-03	7.118e-04	-3.584	0.000342 ***		
RankingsFood	1.571e-02	4.241e-03	3.705	0.000214 ***		
RankingsSocial	1.014e-03	1.351e-03	0.750	0.453169		
UrbanicitySuburban	9.041e-03	1.384e-02	0.653	0.513725		
UrbanicityUrban	2.708e-02	1.278e-02	2.118	0.034197 *		
BlackPop	4.159e-03	5.871e-04	7.084	1.60e-12 ***		
WhitePop	4.059e-03	5.833e-04	6.959	3.87e-12 ***		

Residual standard error: 0.2357 on 5000 degrees of freedom Multiple R-squared: 0.8437, Adjusted R-squared: 0.8431 F-statistic: 1499 on 18 and 5000 DF, p-value: < 2.2e-16

UrbanicitySuburban is not statistically significant at $\alpha =$ 0.05.

summary(backward.model)

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	2.403e+00	1.005e-01	23.905	< 2e-16 ***	
HealthyPop	-2.172e-03	8.187e-04	-2.653	0.008015 **	
ChronicPop	-7.228e-03	2.083e-03	-3.470	0.000524 ***	
StateAR	1.011e+00	1.344e-02	75.199	< 2e-16 ***	
StateLA	9.209e-01	1.194e-02	77.157	< 2e-16 ***	
StateNC	1.482e+00	1.303e-02	113.694	< 2e-16 ***	
UrbanicitySuburban	9.280e-03	1.384e-02	0.671	0.502523	
UrbanicityUrban	2.550e-02	1.261e-02	2.022	0.043193 *	
НО	9.913e-04	9.040e-05	10.966	< 2e-16 ***	
PO	-3.933e-05	4.951e-06	-7.943	2.42e-15 ***	
BlackPop	4.269e-03	5.686e-04	7.507	7.10e-14 ***	
WhitePop	4.180e-03	5.606e-04	7.456	1.04e-13 ***	
Education	-1.642e-03	2.434e-04	-6.746	1.69e-11 ***	
Accessibility	-2.538e-03	7.116e-04	-3.566	0.000366 ***	
Availability	1.005e-01	1.941e-02	5.177	2.34e-07 ***	
ProvDensity	6.172e-02	1.579e-02		9.36e-05 ***	
RankingsPCP	7.736e-04	1.599e-04		1.35e-06 ***	
RankingsFood	1.508e-02	4.158e-03	3.628	0.000288 ***	

Residual standard error: 0.2357 on 5001 degrees of freedom Multiple R-squared: 0.8437, Adjusted R-squared: 0.8432 **Georgia** F-statistic: 1588 on 17 and 5001 DF, p-value: < 2.2e-16 **Tech**

Stepwise Regression Model

- Two models to consider
 - Forward stepwise regression selected RankingsSocial

**
**

**
**
**
**

*

Access to primary care (Accessibility and Availability) is statistically significantly associated to ED cost.

summary(backward.model)						
	Estimate	Std. Error	t value	Pr(> t)		
(Intercept)	2.403e+00	1.005e-01	23.905	< 2e-16 ***		
HealthyPop	-2.172e-03	8.187e-04	-2.653	0.008015 **		
ChronicPop	-7.228e-03	2.083e-03	-3.470	0.000524 ***		
StateAR	1.011e+00	1.344e-02	75.199	< 2e-16 ***		
StateLA	9.209e-01	1.194e-02	77.157	< 2e-16 ***		
StateNC	1.482e+00	1.303e-02	113.694	< 2e-16 ***		
UrbanicitySuburban	9.280e-03	1.384e-02	0.671	0.502523		
UrbanicityUrban	2.550e-02	1.261e-02	2.022	0.043193 *		
НО	9.913e-04	9.040e-05	10.966	< 2e-16 ***		
PO	-3.933e-05	4.951e-06	-7.943	2.42e-15 ***		
BlackPop	4.269e-03	5.686e-04	7.507	7.10e-14 ***		
WhitePop	4.180e-03	5.606e-04	7.456	1.04e-13 ***		
Education	-1.642e-03	2.434e-04	-6.746	1.69e-11 ***		
Accessibility	-2.538e-03	7.116e-04	-3.566	0.000366 ***		
Availability	1.005e-01	1.941e-02	5.177	2.34e-07 ***		
ProvDensity	6.172e-02	1.579e-02	3.910	9.36e-05 ***		
RankingsPCP	7.736e-04	1.599e-04	4.838	1.35e-06 ***		
RankingsFood	1.508e-02	4.158e-03	3.628	0.000288 ***		

Residual standard error: 0.2357 on 5001 degrees of freedom Multiple R-squared: 0.8437, Adjusted R-squared: 0.8432 Georgia F-statistic: 1588 on 17 and 5001 DF, p-value: < 2.2e-16

Stepwise Regression Vs Full Models

Compare full model to selected model

Residual standard error: 0.2357 on 5000 degrees of freedom

Multiple R-squared: 0.8437, Adjusted R-squared: 0.8431 F-statistic: 1499 on 18 and 5000 DF, p-value: < 2.2e-16

reg.step = Im(log(EDCost.pmpm) ~ HealthyPop + ChronicPop + State + Urbanicity + HO + PO + BlackPop + WhitePop + Education + Accessibility + Availability + ProvDensity + RankingsPCP + RankingsFood)

anova(reg.step, full)

Res.Df RSS Df Sum of Sq F Pr(>F)277.80 1 5001 2 4996 277.69 5 0.11778 0.4238 (0.8324)

- P-value large
 - Do not reject the null hypothesis (reduced model)
- The reduced model is plausibly as good in terms of explanatory power as the full model

Georgia

Residual Analysis: Outliers & Normality

red.resid = residuals(reg.step)
red.cook = cooks.distance(reg.step)

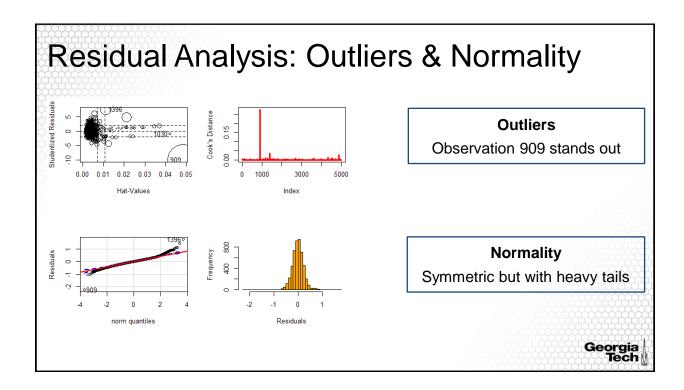
Check outliers

influencePlot(reg.step)
plot(red.cook,type="h",lwd=3,col="red", ylab = "Cook's Distance")

Check normality

qqPlot(red.resid, ylab="Residuals", main = "") qqline(red.resid, col="red", lwd=2) hist(red.resid, xlab="Residuals", main = "", nclass=30, col="orange")

> Georgia Tech



Removing Outlier

eummary	(backward.model)
Summary	Dackwaru.IIIouer)

summary(backward.model.no.out)

	Estimate	Std. Error	t value	Pr(> t)		Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2.403e+00	1.005e-01	23.905	< 2e-16 ***	(Intercept)	1.9356344	0.0991296	19.526	< 2e-16 ***
HealthyPop	-2.172e-03	8.187e-04	-2.653	0.008015 **	HealthyPop	0.0003798	0.0007824	0.485	0.627430
ChronicPop	-7.228e-03	2.083e-03	-3.470	0.000524 ***	ChronicPop	-0.0010849	0.0020519	-0.529	0.597031
StateAR	1.011e+00	1.344e-02	75.199	< 2e-16 ***	StateAR	0.9379139	0.0154403	60.745	< 2e-16 ***
StateLA	9.209e-01	1.194e-02	77.157	< 2e-16 ***	StateLA	0.8989533	0.0117596	76.444	< 2e-16 ***
StateNC	1.482e+00	1.303e-02	113.694	< 2e-16 ***	StateNC	1.4282364	0.0156224	91.422	< 2e-16 ***
UrbanicitySuburban	9.280e-03	1.384e-02	0.671	0.502523	UrbanicitySuburban	-0.0006647	0.0135555	-0.049	0.960895
UrbanicityUrban	2.550e-02	1.261e-02	2.022	0.043193 *	UrbanicityUrban	0.0222961	0.0123314	1.808	0.070654.
НО	9.913e-04	9.040e-05	10.966	< 2e-16 ***	HO	11.5397384	0.7193214	16.043	< 2e-16 ***
PO	-3.933e-05	4.951e-06	-7.943	2.42e-15 ***	PO	0.1338608	0.0403440	3.318	0.000913 ***
BlackPop	4.269e-03	5.686e-04	7.507	7.10e-14 ***	BlackPop	0.0050502	0.0005547	9.105	< 2e-16 ***
WhitePop	4.180e-03	5.606e-04	7.456	1.04e-13 ***	WhitePop	0.0044178	0.0005478	8.064	9.14e-16 ***
Education	-1.642e-03	2.434e-04	-6.746	1.69e-11 ***	Education	-0.0017147	0.0002292	-7.480	8.72e-14 ***
Accessibility	-2.538e-03	7.116e-04	-3.566	0.000366 ***	Accessibility	-0.0018658	0.0006940	-2.688	0.007205 **
Availability	1.005e-01	1.941e-02	5.177	2.34e-07 ***	Availability	0.0755848	0.0189930	3.980	7.00e-05 ***
ProvDensity	6.172e-02	1.579e-02	3.910	9.36e-05 ***	ProvDensity	0.0654339	0.0154862	4.225	2.43e-05 ***
RankingsPCP	7.736e-04	1.599e-04	4.838	1.35e-06 ***	RankingsPCP	0.0007560	0.0001564	4.835	1.37e-06 ***
RankingsFood	1.508e-02	4.158e-03	3.628	0.000288 ***	RankingsFood	0.0162198	0.0040412	4.014	6.07e-05 ***

Residual standard error: 0.2357 on 5001 degrees of freedom Multiple R-squared: 0.8437 Adjusted R-squared: 0.8432 F-statistic: 1588 on 17 and 5001 DF, p-value: < 2.2e-16

Residual standard error: 0.2301 on 5000 degrees of freedom
Multiple R-squared: 0.8504) Adjusted R-squared: 0.8499
F-statistic: 1672 on 17 and 5000 DF, p-value: < 2.2e-16 Georgia

Model Interpretation: State Differences

Comparing 2011 ED Costs by Location (AL, AR, LA, and NC)

- Controlling for utilization, access, and socioeconomics
 - In AR versus AL
 - ED cost PMPM is exp(0.938) = \$2.55 higher
 - ED cost per member per year is \$30.65 higher
 - In LA versus AL
 - ED cost PMPM is exp(0.899) = \$2.46 higher
 - ED cost per member per year is \$29.49 higher
 - In NC versus AL
 - ED cost PMPM is exp(1.428) = \$4.17 higher
 - ED cost per member per year is \$50.04 higher

Overall Interpretation: Controlling for many potential factors contributing to ED costs, North Carolina pays significantly more while Alabama pays significantly less per member on emergency care than do Louisiana and Arkansas. Georgia

Model Interpretation: Utilization

Healthcare Utilization

- PO
 - Proxy of regular care utilization
 - Number of claims reimbursed for care in a physician's office
- HC
 - Proxy of inpatient care utilization
 - Number of claims reimbursed for hospital care

Interpretation

- An increase of 1 claim PMPM for regular care results in a 0.133 increase in log of ED cost PMPM, given all other predictors fixed
- An increase of 1 claim PMPM for inpatient care results in a 11.54 increase in log of ED cost PMPM, given all other predictors fixed



Model Interpretation: Access to Care

Access to primary care

- Availability
 - Proxy of wait times for appointment
 - Takes values between 0 (low wait time) and 1 (high wait time)
- Accessibility
 - · Travel distance to primary care providers, measured in miles

Interpretation

- An increase of 1% in lack of availability of primary care providers results in 0.0755 unit increase in log(ED cost PMPM) given all other predictors fixed
- A reduction of 1 mile in travel distance to primary care providers results in 0.002 unit increase in log(ED cost PMPM) given all other predictors fixed
- The correlation between the two measures is 0.696. If *Availability* is discarded from the model, *Accessibility* is not statistically significant.



