

Regression Analysis

Simple Linear Regression

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Example 1: Testing the Theory of
Purchasing Power Parity
(Part 1)



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About This Lesson

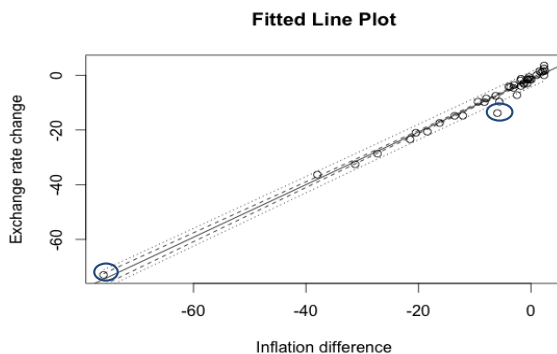


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Confidence Bands in R

```
# Function for fitted line plot: See ppp-revised.R for this function
# regplot.confbands.fun = function(x, y, confidencelevel=.95, Clmean=T, Pl=T,
# Clregline=F, legend=F){
#   ##### Modified from a function written by Sandra McBride, Duke University
#   ....}
```

```
regplot.confbands.fun(Inflation.difference, Exchange.rate.change)
```



The fitted line plot shows several lines:

- The continuous line is the fitted regression line.
- The wider interrupted line band is the prediction confidence band.
- The narrower interrupted line band is the confidence band.
- The circles correspond to outliers.



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Confidence and Prediction Intervals

Confidence and prediction intervals for new observation

Create new data point

```
newppp = data.frame(Inflation.difference = c(-0.68))
```

Specify whether a confidence or prediction interval

```
predict(pppa, newppp, interval=c("confidence"))
```

```
fit      lwr      upr
1 -2.173351 -2.756818 -1.589884
```

```
predict(pppa, newppp, interval=c("prediction"))
```

```
fit      lwr      upr
1 -2.173351 -5.554071 1.207369
```

Why are the intervals different?

Interpretation of the two intervals:

- The 95% confidence limits of the average exchange rate change for all countries inflation difference equal to -0.68 are (-2.757, -1.590);
- The 95% confidence limits for the exchange rate change for one country with inflation difference equal to -0.68 are (-5.554, 1.207).



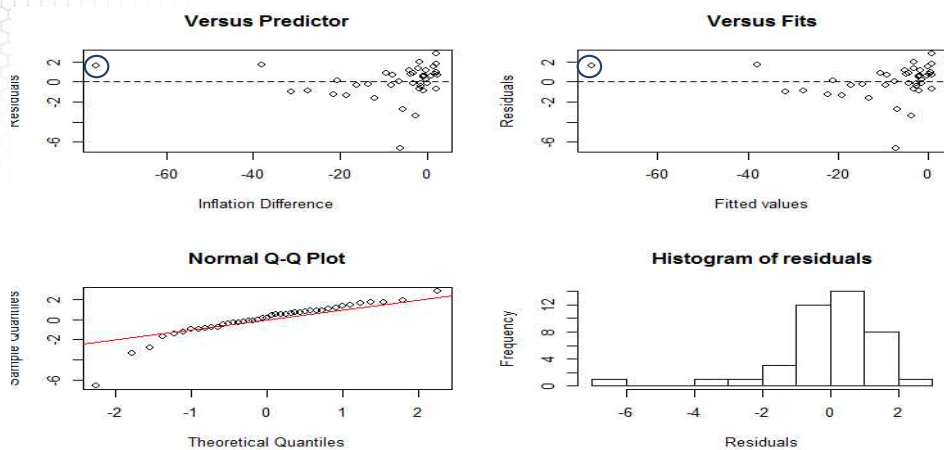
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Residual Analysis in R

```
par(mfrow=c(2,2))
plot(Inflation.difference, residuals(pppa), xlab="Inflation
Difference", ylab="Residuals", main="Versus Predictor")
abline(h=0, lty=2)
plot(fitted(pppa), residuals(pppa), xlab="Fitted values", ylab="Residuals", main="Versus Fits")
abline(h=0, lty=2)
qqnorm(residuals(pppa))
abline(0, 1, lty=1, col="red")
hist(residuals(pppa), main="Histogram of residuals", xlab="Residuals")
```

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Residual Analysis in R



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Residual Analysis in R

Leverage Points: The isolated point in residual plots is Brazil. Why is Brazil a leverage point?

- Brazil had a period of hyperinflation from 1980 to 1994, a time period during which prices went up by a factor of roughly 1 trillion.

Why do we care about leverage points?

- It can have a strong effect on the fitted regression, drawing the line away from the bulk of the points. It also can affect measures of fit like R-squared and t-statistics.



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Influential Points in Regression Analysis

Repeat Analysis: Omit Brazil

remove the data row corresponding to Brazil

```
newppp = ppp[ppp$Country!="Brazil",]
attach(newppp)
```

Fit Linear Regression

```
pppn = lm(Exchange.rate.change ~ Inflation.difference)
summary(pppn)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-1.37222	0.30517	-4.497	6.31e-05
Inflation.difference	0.99152	0.02626	37.757	< 2e-16

Residual standard error: 1.62 on 38 degrees of freedom
Multiple R-squared: 0.974, Adjusted R-squared: 0.9734

Test whether the slope is equal to 1 (PPP theory)

```
tvalue = (0.9915-1)/ 0.02626
pvalue = 2*(1-pt(tvalue,38))
```

$\hat{\beta}_0 = -1.372$, $se(\hat{\beta}_0) = 0.305$
Statistical significance for β_0 :
t-value= -4.497, p-value ≈ 0

$\hat{\beta}_1 = 0.9915$, $se(\hat{\beta}_1) = 0.02626$
Test the null hypothesis $\beta_1 = 1$:
p-value = 0.748

We are seeing violations of PPP with respect to intercept only.



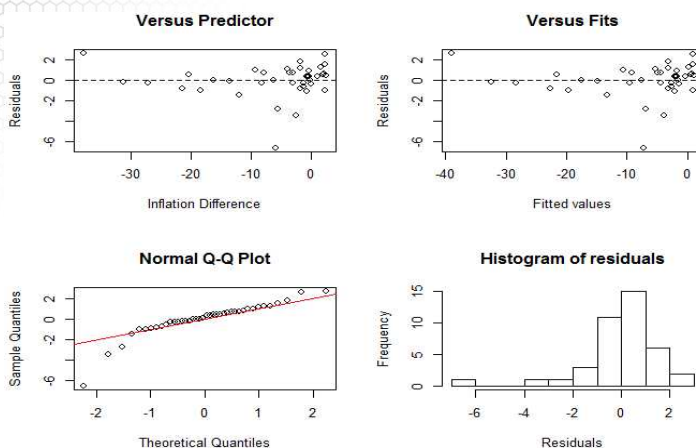
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Residual Analysis: Model without Brazil

```
par(mfrow=c(2,2))
plot(Inflation.difference, residuals(pppn), xlab="Inflation Difference", ylab="Residuals",
main="Versus Predictor")
abline(h=0, lty=2)
plot(fitted(pppn), residuals(pppn), xlab="Fitted values", ylab="Residuals", main="Versus Fits")
abline(h=0, lty=2)
qqnorm(residuals(pppn))
abline(0, 1, lty=1, col="red")
hist(residuals(pppn), main="Histogram of residuals", xlab="Residuals")
```

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Residual Analysis: Model without Brazil



Assumptions:

Linearity: No pattern in the residuals with respect to the predicting variable.

Constant Variance: The variance is higher for higher fitted values. Does not hold.

Uncorrelated Errors: No grouping of the residuals

Normality: Except for the presence of an outlier, it is reasonably symmetric.

Outliers (observations for which the residual value is away from the range):

The isolated point in the residual plots is Indonesia. Would omitting Indonesia change anything? The strength of the relationship would increase, but so the rejection of PPP.

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Testing the Theory of Purchasing Power Parity

Findings:

- Support is decidedly mixed
- Developed countries:
 - Changes in inflation difference do seem to be balanced by exchange rate changes
 - One outlier: Greece
- Developing countries:
 - The case for PPP is considerably weaker;
 - Brazil and Indonesia
- PPP is not robust to unusual economic or political conditions

Summary

