Regression Analysis

Simple Linear Regression

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Regression Concepts:
Assumptions and Diagnostics

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About This Lesson

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Simple Linear Regression: Model

Data: $\{(x_1,y_1),...,(x_n,y_n)\}$
Model: $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \; i = 1,...,n$

Assumptions:

- *Linearity/Mean Zero Assumption:* $E(\varepsilon_i) = 0$
- *Constant Variance Assumption*: $Var(\varepsilon_i) = \sigma^2$
- *Independence Assumption* $\{\varepsilon_1,...,\varepsilon_n\}$ *are independent random variables*
- (*Later we assume* $\varepsilon_i \sim$ *Normal)*

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Residual Analysis

Residual Values: $\varepsilon_i \rightarrow \hat{\varepsilon}_i = y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)$

Graphical display: **Plot of the residuals** $\boldsymbol{\varepsilon}_i$

If the scatter of ε_i is **not random around zero line**, it could be that
- ➢ The relationship between X and Y is not linear
- ➢ Variances of error terms are not equal
- ➢ Response data are not independent

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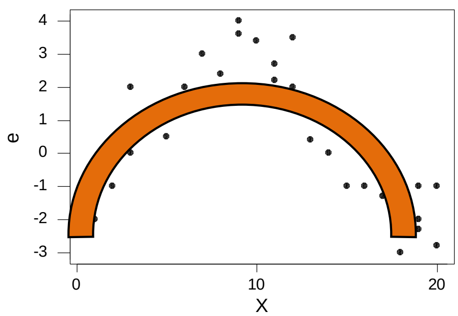
Checking Assumptions: Residual Analysis

Linearity Assumption:

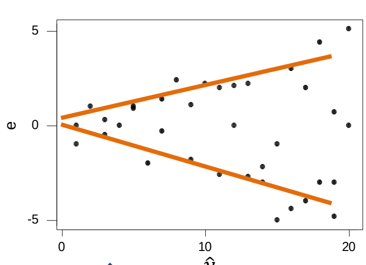
This shows that there may be a non-linear relationship between X and Y.



Checking Assumptions: Residual Analysis

Constant Variance Assumption:

The residuals show larger variance as the predicting variable increases.

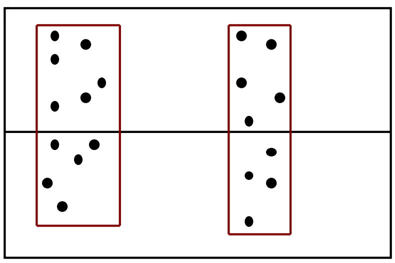


Here, it could be that σ^2 is not constant.

Checking Assumptions: Residual Analysis

Independence Assumption:

There are clusters of residuals: the independence assumption does not hold.



- **Using residual analysis, we check for uncorrelated errors but not independence.**
- **Independence is a more complicated matter. If the data are from a randomized trial, then independence is established, but most data are from observational studies.**

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Checking the Assumption of Normality

One way to check this assumption in a regression is using a **Normal Probability Plot**

x-axis:	$\Phi^{-1}\left(\frac{r_i - 3/8}{n + 1/4}\right)$
y-axis:	e_i

r_i = rank of e_i (between 1, n)

Φ = CDF of Normal Distribution

- Let the R statistical software do this for you!
- A straight line in normal probability plot implies assumption of normality is valid
- **Curvature** (especially at the ends) shows non-normality

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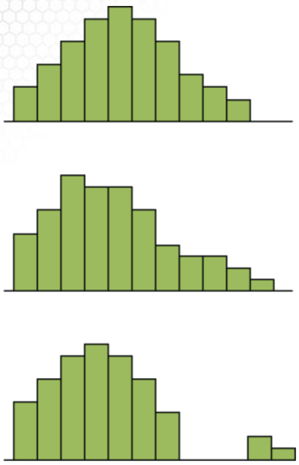
Checking the Assumption of Normality



A complementary approach to check for the normality assumption is by plotting the **histogram** of the residuals

Normality Assumption:
The residuals should have an approximately symmetric distribution, unimodal, and with no gaps in the data.

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Variable Transformation

- If the model fit is inadequate, it does not mean that a regression is not useful.
- One problem might be that the relationship between **X** and **Y** is ***not exactly linear***.
- To model the nonlinear relationship, we can transform **X** by some nonlinear function such as:

$$f(x) = x^a \text{ or } f(x) = \log(x)$$

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Normality Transformations

Problem: Normality or constant variance assumption does not hold.
Solution: Transform the response variable from y to y* via

$$y^* = y^{\lambda}$$

where the value of λ depends on how Var(Y) changes as X changes.

$$\sigma_y(x) \propto const \qquad \lambda = 1 \qquad \text{(don't transform)}$$

$$\sigma_y(x) \propto \sqrt{\mu_x} \qquad \lambda = 1/2$$

$$\sigma_y(x) \propto \mu_x \qquad \lambda = 0 \qquad y^* = \ln(y)$$

$$\sigma_y(x) \propto 1/\mu_x \qquad \lambda = -1$$

This is called Box-Cox Transformation: The parameter λ can be determined using R statistical software.

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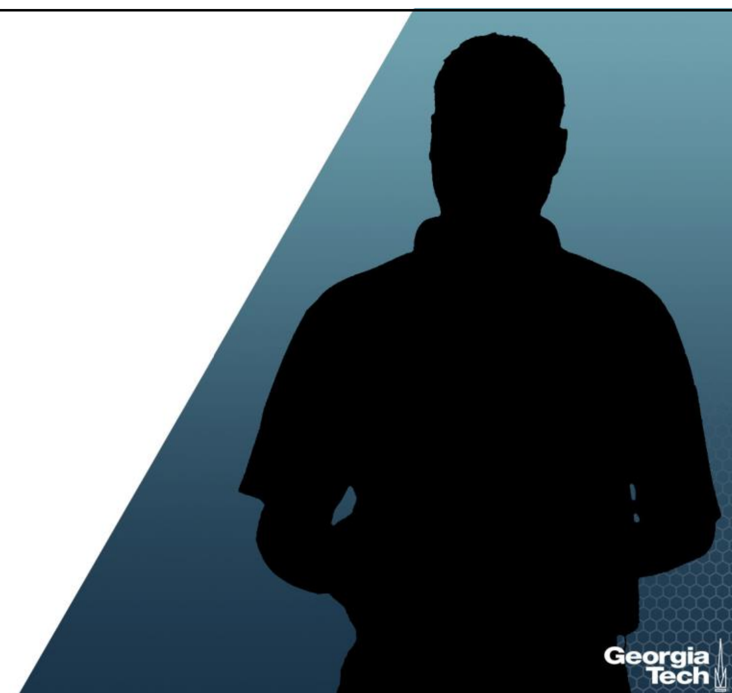
Summary



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