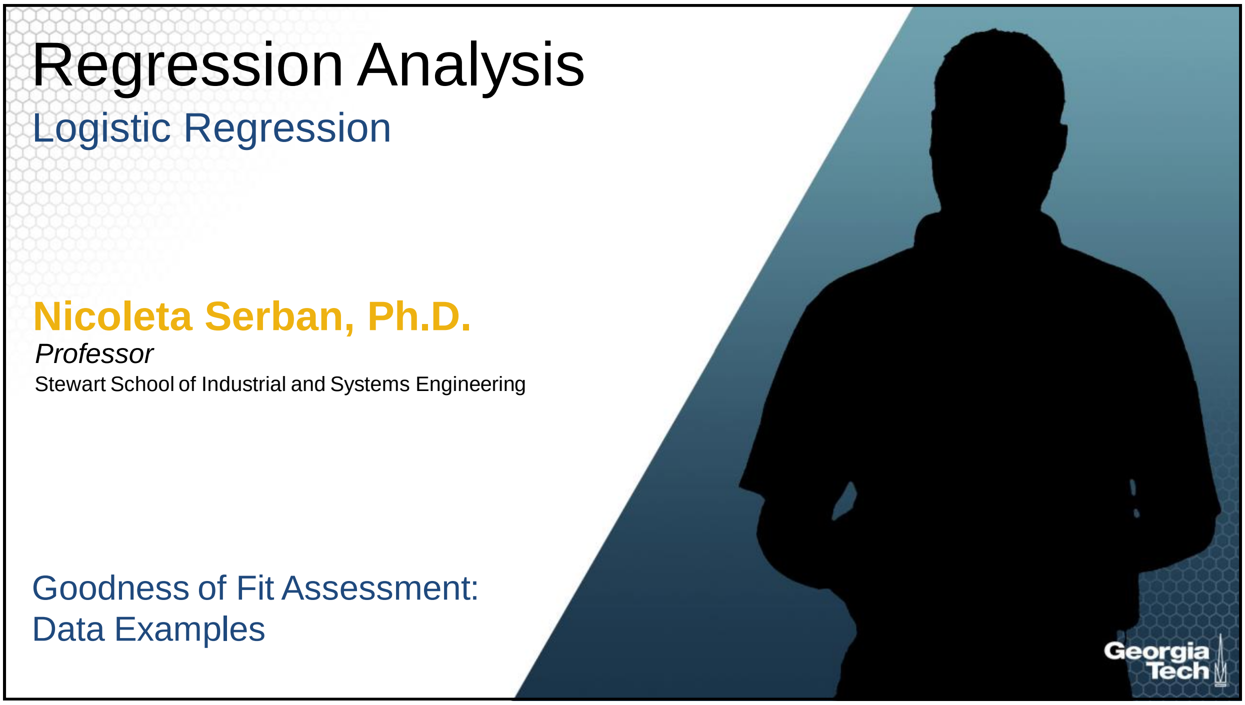

Regression Analysis

Logistic Regression

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Goodness of Fit Assessment:
Data Examples

About This Lesson

Data Example: Smoking

- Between 1972 and 1974, a survey was taken in Whickham, a mixed urban and rural district near Newcastle upon Tyne, United Kingdom.
  - Among the information obtained originally was whether a person was a smoker or not.
- Twenty years later a follow-up study was conducted.
  - 76.12% of the 582 smokers were still alive, while only 68.58% of 732 nonsmokers were still alive.

Smokers had a higher survival rate than nonsmokers!
Call Philip Morris, smoking leads to a longer life span!

Acknowledgement: This example was provided by Dr. Jeffrey Simonoff from New York University.

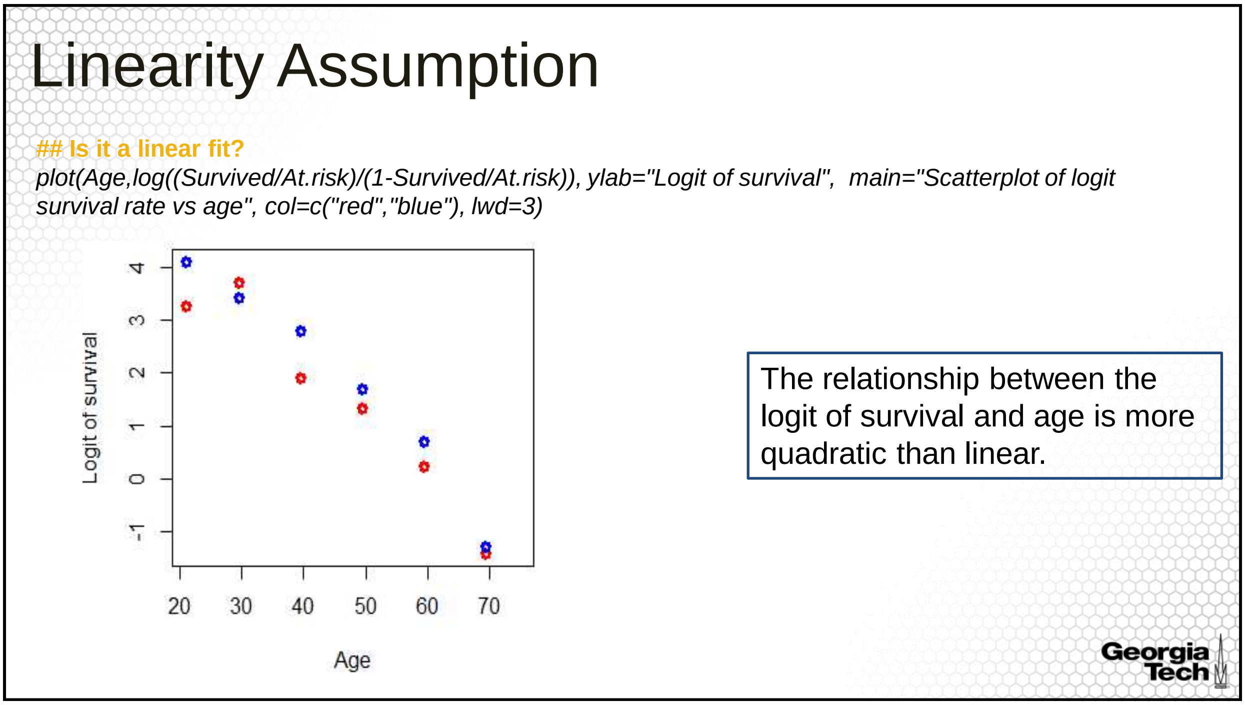
GOF Hypothesis Test

```
## Deviance Test for GOF using deviance residuals
c(deviance(smoke2), 1-pchisq(deviance(smoke2),11))
[1] 4.345918e+01 9.033325e-06
```

Test for goodness-of-fit:
- Using deviance residuals: P-value ≈ 0
- Reject the null hypothesis of good fit (thus NOT a good fit)

```
## GOF test using Pearson residuals
pearres2 = residuals(smoke2,type="pearson")
pearson.tvalue = sum(pearres2^2)
c(pearson.tvalue, 1-pchisq(pearson.tvalue,11))
[1] 36.751889370  0.000126796
```

Test for goodness-of-fit:
- Using Pearson residual: P-value ≈ 0.0001
- Reject the null hypothesis of good fit (thus NOT a good fit)



Linearity Assumption

## Is it a linear fit?

plot(Age,log((Survived/At.risk)/(1-Survived/At.risk)), ylab="Logit of survival", main="Scatterplot of logit survival rate vs age", col=c("red","blue"), lwd=3)

The relationship between the logit of survival and age is more quadratic than linear.

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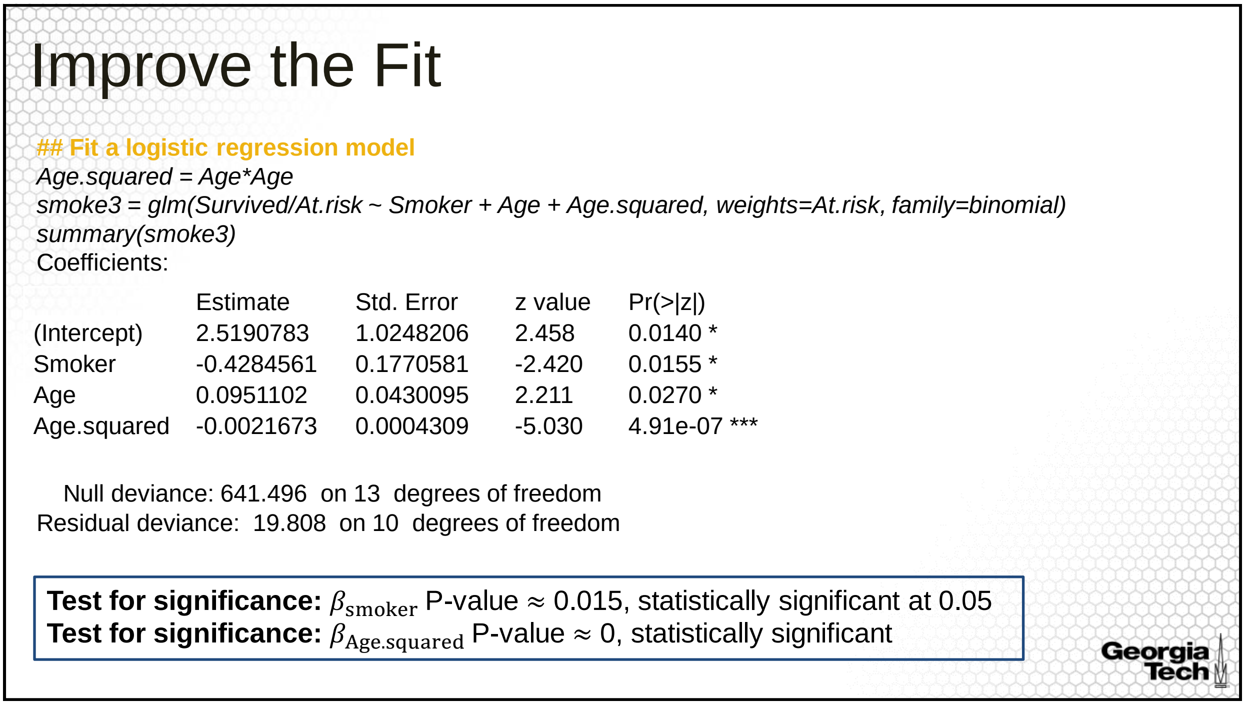

Improve the Fit

## Fit a logistic regression model

*Age.squared = Age*Age*
smoke3 = glm(Survived/At.risk ~ Smoker + Age + Age.squared, weights=At.risk, family=binomial)
summary(smoke3)

```
Coefficients:
              Estimate      Std. Error    z value    Pr(>|z|)
(Intercept)   2.5190783     1.0248206     2.458      0.0140 *
Smoker        -0.4284561    0.1770581     -2.420     0.0155 *
Age           0.0951102     0.0430095     2.211      0.0270 *
Age.squared   -0.0021673    0.0004309     -5.030     4.91e-07 ***

    Null deviance: 641.496  on 13  degrees of freedom
Residual deviance:  19.808  on 10  degrees of freedom
```

Test for significance: β_{smoker} P-value $\approx$ 0.015, statistically significant at 0.05
Test for significance: $\beta_{\text{Age.squared}}$ P-value $\approx$ 0, statistically significant

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GOF Test for Improved Model

```
## Test for goodness of fit
round(c(deviance(smoke3), 1-pchisq(deviance(smoke3),10)),2)
[1] 19.81  0.03

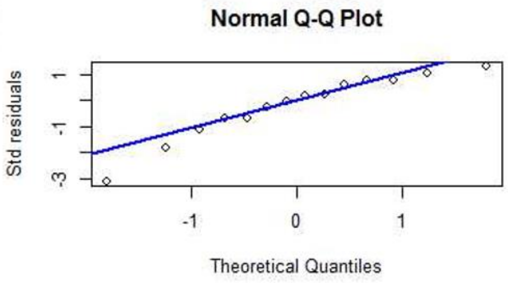
pearres3 = residuals(smoke3,type="pearson")
pearson = sum(pearres3^2)
round(c(pearson, 1-pchisq(pearson,10)),2)
[1] 14.79  0.14
```

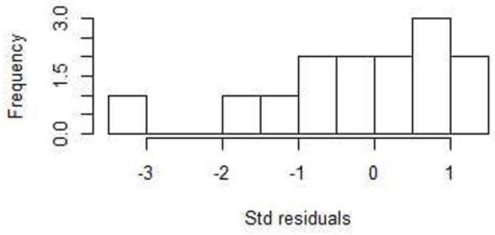
Does the goodness of fit improve?

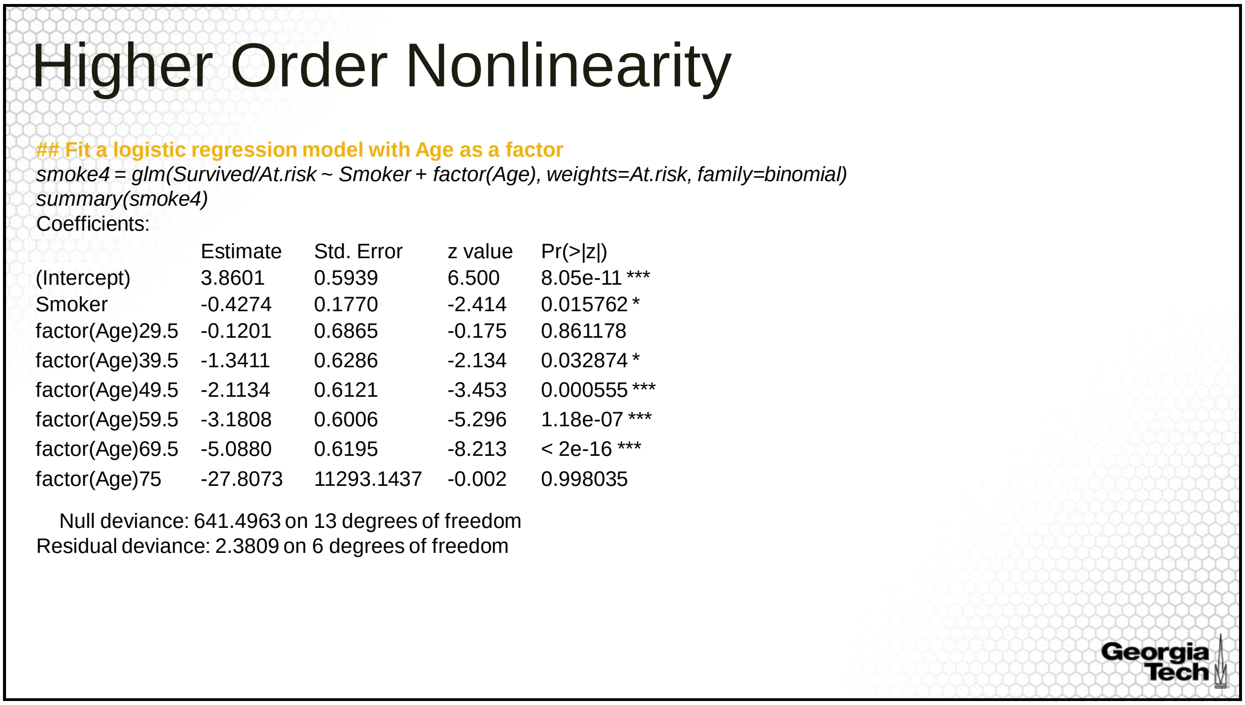
- Using deviance residuals: P-value = 0.03
- Using Pearson residual: P-value = 0.14
- Do not reject the null hypothesis of good fit using Pearson residuals, but do reject using Deviance residuals at the significance level 0.03 or higher.

Residual Analysis

```
## Residual Plots
res = resid(smoke3,type="deviance")
qqnorm(res, ylab="Std residuals")
qqline(res,col="blue",lwd=2)
hist(res,10,xlab="Std residuals", main="")
```





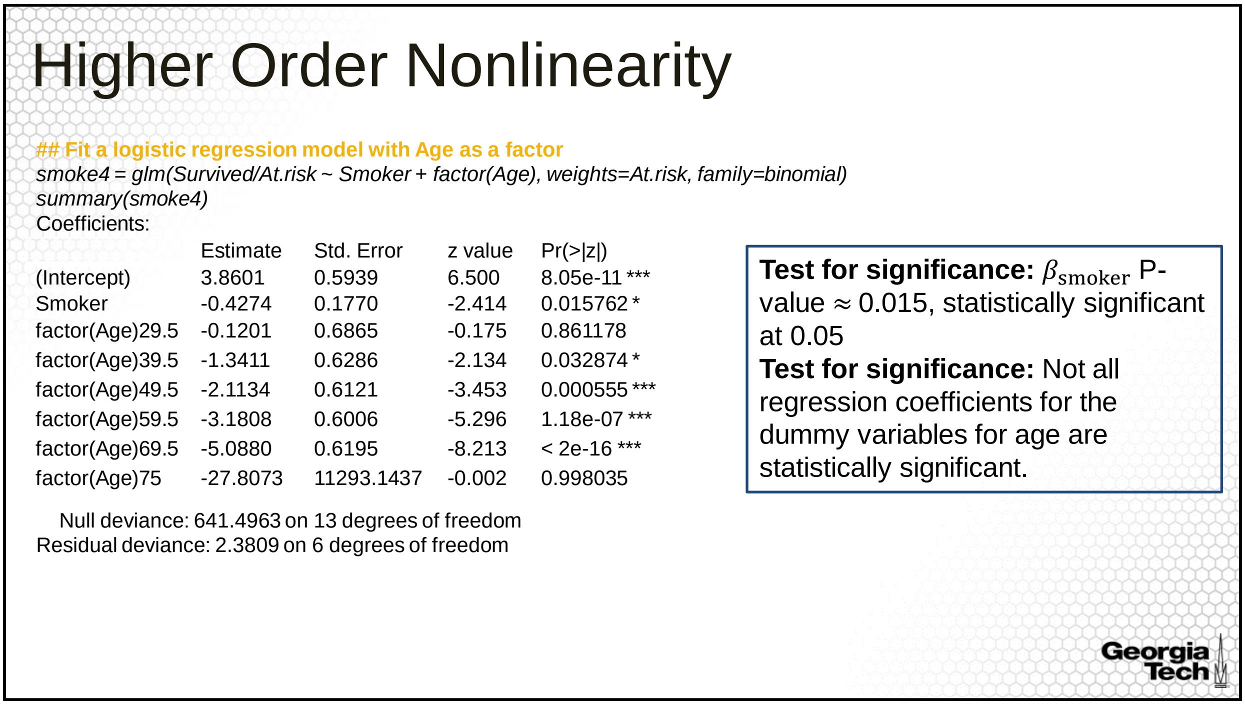


Higher Order Nonlinearity

## Fit a logistic regression model with Age as a factor

```
smoke4 = glm(Survived/At.risk ~ Smoker + factor(Age), weights=At.risk, family=binomial)
summary(smoke4)
Coefficients:
                Estimate   Std. Error  z value  Pr(>|z|)
(Intercept)     3.8601     0.5939      6.500    8.05e-11 ***
Smoker          -0.4274    0.1770      -2.414   0.015762 *
factor(Age)29.5 -0.1201    0.6865      -0.175   0.861178
factor(Age)39.5 -1.3411    0.6286      -2.134   0.032874 *
factor(Age)49.5 -2.1134    0.6121      -3.453   0.000555 ***
factor(Age)59.5 -3.1808    0.6006      -5.296   1.18e-07 ***
factor(Age)69.5 -5.0880    0.6195      -8.213   < 2e-16 ***
factor(Age)75   -27.8073   11293.1437  -0.002   0.998035

    Null deviance: 641.4963 on 13 degrees of freedom
Residual deviance: 2.3809 on 6 degrees of freedom
```



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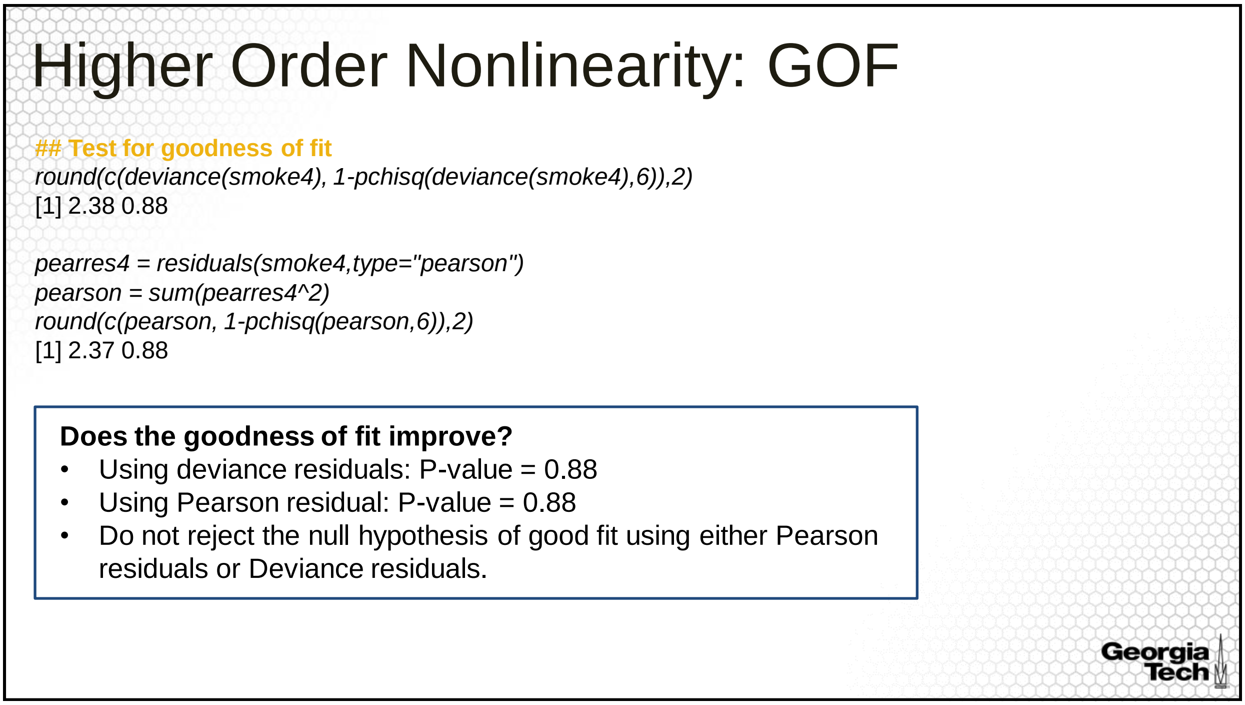
```

Test for significance: β_{smoker} P-value ≈ 0.015, statistically significant at 0.05

Test for significance: Not all regression coefficients for the dummy variables for age are statistically significant.



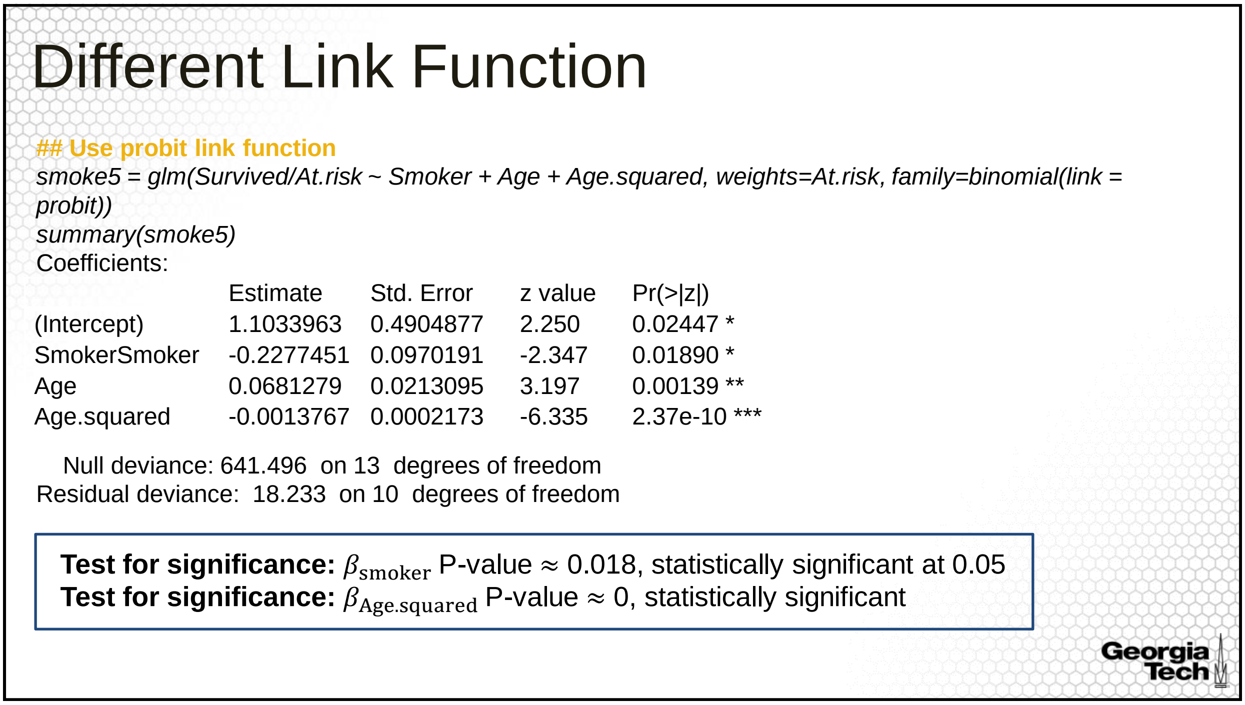
Higher Order Nonlinearity: GOF

```
## Test for goodness of fit
round(c(deviance(smoke4), 1-pchisq(deviance(smoke4),6)),2)
[1] 2.38 0.88

pearres4 = residuals(smoke4,type="pearson")
pearson = sum(pearres4^2)
round(c(pearson, 1-pchisq(pearson,6)),2)
[1] 2.37 0.88
```

Does the goodness of fit improve?

- Using deviance residuals: P-value = 0.88
- Using Pearson residual: P-value = 0.88
- Do not reject the null hypothesis of good fit using either Pearson residuals or Deviance residuals.



Different Link Function

```
## Use probit link function
smoke5 = glm(Survived/At.risk ~ Smoker + Age + Age.squared, weights=At.risk, family=binomial(link = 
probit))
summary(smoke5)
Coefficients:
              Estimate    Std. Error   z value   Pr(>|z|)
(Intercept)   1.1033963   0.4904877    2.250     0.02447 *
SmokerSmoker  -0.2277451  0.0970191    -2.347    0.01890 *
Age           0.0681279   0.0213095    3.197     0.00139 **
Age.squared   -0.0013767  0.0002173    -6.335    2.37e-10 ***

    Null deviance: 641.496  on 13  degrees of freedom
Residual deviance:  18.233  on 10  degrees of freedom
```

Test for significance: β_{smoker} P-value ≈ 0.018, statistically significant at 0.05
Test for significance: $\beta_{\text{Age.squared}}$ P-value ≈ 0, statistically significant

Different Link Function: GOF

```
## Test for goodness of fit
round(c(deviance(smoke5), 1-pchisq(deviance(smoke5),10)),2)
[1] 18.23  0.05

pearres5 = residuals(smoke5,type="pearson")
pearson = sum(pearres5^2)
round(c(pearson, 1-pchisq(pearson,10)),2)
[1] 14.00  0.17
```

Does the goodness of fit improve?

- Using deviance residuals: P-value = 0.05
- Using Pearson residual: P-value = 0.17
- Do not reject the null hypothesis of good fit using Pearson residuals or using deviance residuals at the significance level 0.01.

Simpson’s Paradox

Simpson’s paradox: Reversal of an association when looking at a marginal relationship versus a conditional relationship.

- Smoking is statistically significant with a negative estimated coefficient under the marginal model.
- Smoking has a positive estimated coefficient under the conditional model.

Marginal versus Conditional Relationship

- ***Marginal***: Capturing the association of a predicting variable to the response variable without consideration of other factors
- ***Conditional***: Capturing the association of a predicting variable to the response variable conditional on other predicting variables in the model

Summary