Regression Analysis

Multiple Linear Regression

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Ranking States by SAT Performance: Model Fit

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About This Lesson

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Residual Analysis

To evaluate assumptions:

- ***Constant variance & uncorrelated errors***
  - Response variable or fitted values vs residuals
- ***Linearity***
  - Predicting variables vs residuals
- ***Normality***
  - Histogram and QQ normal plot

To evaluate outliers:

- Cook's distance plots

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Residual Analysis

```
## Residual analysis for the reduced model
res = stdres(reduced.line)
cook = cooks.distance(reduced.line)
par(mfrow = c(2,3))
plot(sat, res, xlab = "SAT  Score", ylab = "Residuals", pch = 19)
abline(h = 0)
plot(takers, res, xlab = "Percent of Students  Tested", ylab = "Residuals", pch = 19)
abline(h = 0)
plot(rank, res, xlab = "Median Class Ranking Percentile", ylab = "Residuals", pch = 19)
abline(h = 0)
hist(res, xlab="Residuals", main= "Histogram of Residuals")
qqnrom(res)
qqline(res)
plot(cook,type="h",lwd=3,  ylab = "Cook's Distance")
```

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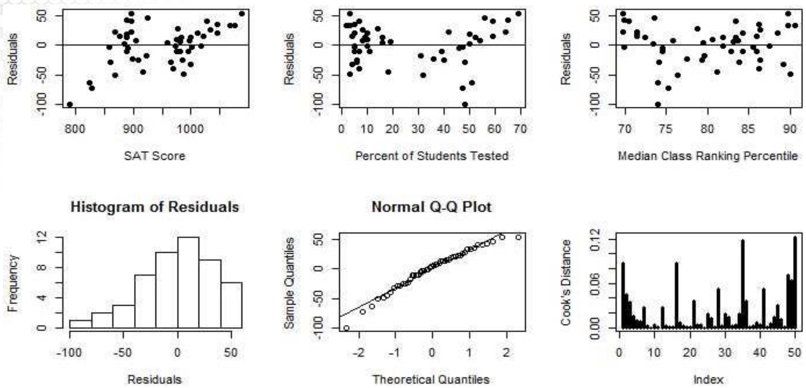
Residual Analysis



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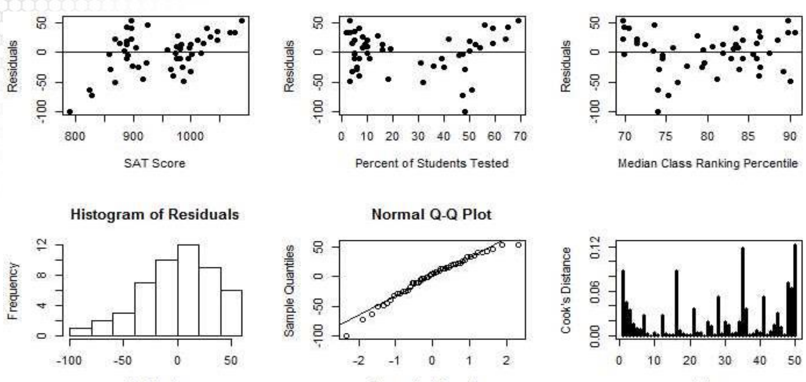
Residual Analysis



- Transform the predicting variable Percent of Students Tested (*takers*)
- Reanalyze heavy tailed residuals and outliers after transformation

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Linear Regression Analysis in R

```
regression.line = lm(sat ~
log(takers)+rank+income+years+public+expend)
summary(regression.line)
Coefficients:
              Estimate   Std. Error  t value   Pr(>|t|)
(Intercept) 407.53990   282.76325    1.441   0.15675
log(takers) -38.43758    15.95214   -2.410   0.02032 *
rank          4.11427     2.50166    1.645   0.10734
income       -0.03588     0.13011   -0.276   0.78407
years        17.21811     6.32007    2.724   0.00928 **
public       -0.11301     0.56239   -0.201   0.84168
expend        2.56691     0.80641    3.183   0.00271 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 24.86 on 43 degrees of freedom
Multiple R-squared:  0.8919,      Adjusted R-squared:  0.8769
F-statistic: 59.15 on 6 and 43 DF,  p-value: < 2.2e-16
```

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Test for statistical significance:

$\hat{\beta}_{\log(takers)}$ Pr(>|t|) ≈ 0.0203 < 0.1
$\hat{\beta}_{rank}$ Pr(>|t|) ≈ 0.1073 > 0.1
$\hat{\beta}_{income}$ Pr(>|t|) ≈ 0.7840 > 0.1
$\hat{\beta}_{years}$ Pr(>|t|) ≈ 0.0093 < 0.1
$\hat{\beta}_{public}$ Pr(>|t|) ≈ 0.8417 > 0.1
$\hat{\beta}_{expend}$ Pr(>|t|) ≈ 0.0027 < 0.1

$\hat{\sigma} = 24.86$, df = n-p-1 = 43
$R^2 \approx 0.892 \Rightarrow$ 89.2% of variability explained

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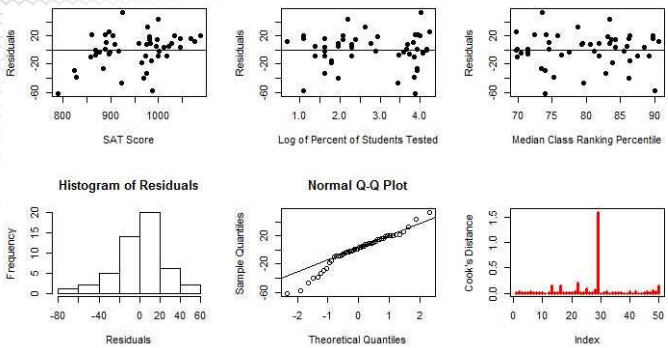
Residual Analysis



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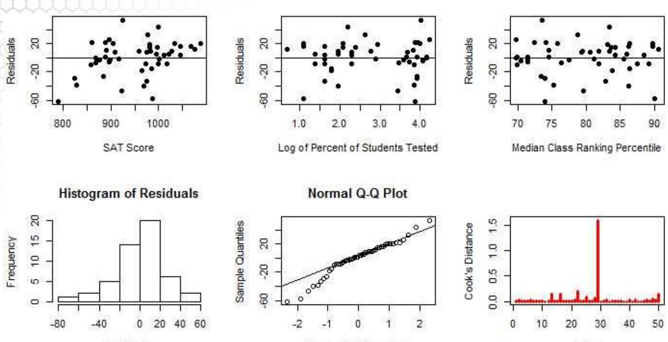
Residual Analysis



- Transformation has improved on the linearity assumption
- Heavy tailed residuals remain
- Cook's distance
  - Alaska is an outlier/influential point for the model

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State SAT Performance: Findings

- Given all other predictors in the model:
  - Percent of students taking SAT from a public school and family income of test takers are not statistically significantly associated to SAT score
  - A $100,000 increase in the expenditure on secondary schools results in a 2.56-point increase in the SAT score
  - One additional year that test takers had in social sciences, natural sciences, and humanities leads to a 17.2-point increase in the SAT score
- The predictors in the model explain close to 90% of the variability in SAT score
- We find that the relationship between state average SAT score and the percent of students taking SAT to be nonlinear
- Ranking changes after controlling for the bias selection factors
  - For example, Connecticut moves from 35th to 1st, Massachusetts from 41st to 4th, and New York from 36th to 5th

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Summary

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