Regression Analysis

Simple Linear Regression

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Testing the Theory of Purchasing Power Parity (Part 1)

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About This Lesson

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Testing the Theory of Purchasing Power Parity

Acknowledgment: This example was made available by Dr. Jeffrey Simonoff from the New York University.

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Regression Variables

Response Variable: Average annual change in the exchange rate

$$\frac{\ln(\text{Exclange Rate for 2012}) - \ln(\text{Exchange Rate for 1975})}{\text{no. years}} \% = \textit{Annualized Percentage Change}$$

Predicting Variable: Average of the *difference in annual inflation rates* for a country vs U.S.

$$\frac{1}{\text{no. years}} \sum_{y=1975}^{2012} \left(\text{Inflation}_y(\text{U.S.}) - \text{Inflation}_y(\text{Country})\right)$$

Country	Inflation.difference	Exchange.rate.change	Developed
Australia	-1.2351	-3.1870	1
Austria	1.5508	1.4781	1
Belgium	1.0371	0.0395	1
Canada	0.0461	-1.6416	1
Chile	-18.4126	-20.6329	0

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Read the Data in R

 # Use read.table R command: pay attention to the file type to use the correct read file!#

ppp = read.table("ppp.dat",sep="\t", header=T, row.names=NULL)

How many countries?

dim(ppp)

[1] 40 4

Brazil is an outlier and it was not included in the data set initially; I am adding it back as follows

Addp = data.frame("Brazil",-76,-73,0)

names(addp) = names(ppp)

Save the data variables to be recognized by R as separate variables

ppp = data.frame(rbind(ppp,addp))

attach(ppp)

Re-label the 'Developed' column to differentiate between Developed and Developing countries

Developed[Developed==1] = "Developed"

Developed[Developed==0] = "Developing"

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Exploratory Data Analysis in R

 # Evaluate the Linear Relationship: Perform a scatter plot of the two variables

plot(Inflation.difference,Exchange.rate.change, main="Scatterplot of Exchange rate change vs Inflation difference", xlab="Inflation difference",ylab="Exchange rate change")

 # Evaluate differences between developed and developing countries

boxplot(Exchange.rate.change~as.factor(Developed), main="Boxplot of Exchange rate change by Developed vs Developing Countries",xlab="Country Class",ylab="Exchange rate change")

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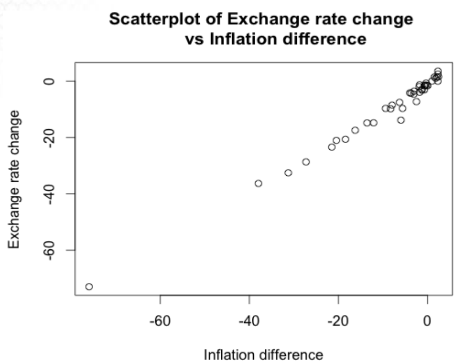
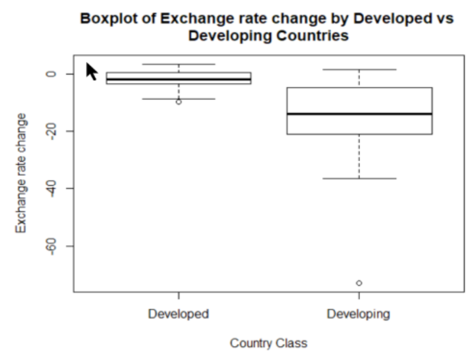
Exploratory Data Analysis in R





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Fitting Linear Regression in R

pppa = lm(Exchange.rate.change ~ Inflation.difference) ## regression model
summary(pppa)

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-1.51930	0.29415	-5.165	7.43e-06
Inflation.difference	0.96185	0.01781	53.991	< 2e-16

$\hat{\beta}_0$ = -1.5193, se($\hat{\beta}_0$) = 0.2941
$\hat{\beta}_1$ = 0.9618, se($\hat{\beta}_1$) = 0.0178
Test for statistical significance:
β_0: t-value = -5.165, p-value ≈ 0
β_1: t-value = 53.991, p-value ≈ 0

Residual standard error: 1.646 on 39 degrees of freedom
Multiple R-squared: 0.9868, Adjusted R-squared: 0.9865
F-statistic: 2915 on 1 and 39 DF, p-value: < 2.2e-16

$\hat{\sigma}$ = 1.646, n-2 = 39
R^2 = 98.7% variability explained

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Does the Theory Hold?

The principle of purchasing power parity (PPP) states:

Average annual change in the exchange rate = Difference in average annual inflation rates + Random error

The economic theory says that $\beta_0 = 0$, $\beta_1 = 1$.
The estimates for these coefficients are: $\hat{\beta}_0 = -1.519$, $\hat{\beta}_1 = -0.961$

Violations of PPP theory with respect to both the intercept and the slope.

Testing the theory:
$\beta_0 = 0$: Based on the t-test of statistical significance we find that β_0 is statistically different from zero.

$\beta_1 = 1$: We need to perform a t-test with this as the null hypothesis:
T-value $= \frac{\hat{\beta}_1 - 1}{\text{se}(\hat{\beta}_1)} = \frac{0.9618-1}{0.0178} = -2.1448$
p-value $= 2(1\text{-P}(T_{39} < 2.1448)) = 0.038$

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Hypothesis Testing in R

 # Perform the hypothesis test for slope coefficient H0: slope=1
 # use the library 'car' available in R (you need to install this library first then download it)

```
install.packages("car")
library(car)
linearHypothesis(pppa,c(0,1),rhs=1)
```

Alternatively, you can compute the t-value and p-value as follows:

```
tvalue = (0.9618−1)/0.01781
pvalue = 2*(1-pt(tvalue,39))
```

Use the help menu to learn more about the functions used above:

```
help(pt)
help(linearHypothesis)
```

P-value = 2P(T_{n-2} > |t-value|)
where

$$\text{t-value} = \frac{\hat{\beta}_1 - 1}{\sqrt{\hat{\sigma}^2 / S_{XX}}}$$

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