

Regression Analysis

Other Regression Methods

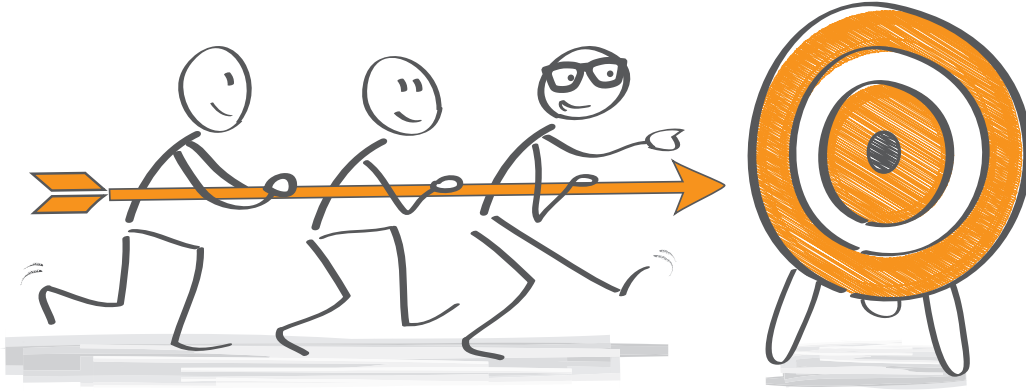
Nicoleta Serban, Ph.D.

Associate Professor

Stewart School of Industrial and Systems Engineering

Spatial Regression

About this lesson



Multiple Linear Regression

Data: $\{(x_{11}, \dots, x_{1p}), Y_1\}, \dots, \{(x_{n1}, \dots, x_{np}), Y_n\}$

What if uncorrelated errors assumption does not hold?

- **Degrees of freedom:** not equal to the sample size
- **Higher variability or uncertainty** than estimated thus less reliable statistical inference

- *Linearity/Mean Zero Assumption:* $E(\varepsilon_i) = 0$
- *Constant Variance Assumption:* $\text{Var}(\varepsilon_i) = \sigma^2$
- *Independence Assumption:* $\{\varepsilon_1, \dots, \varepsilon_n\}$ are independent random variables
- *Normality Assumption:* $\varepsilon_i \sim \text{Normal}$

Example: Spatial Processes

Correlation in space:

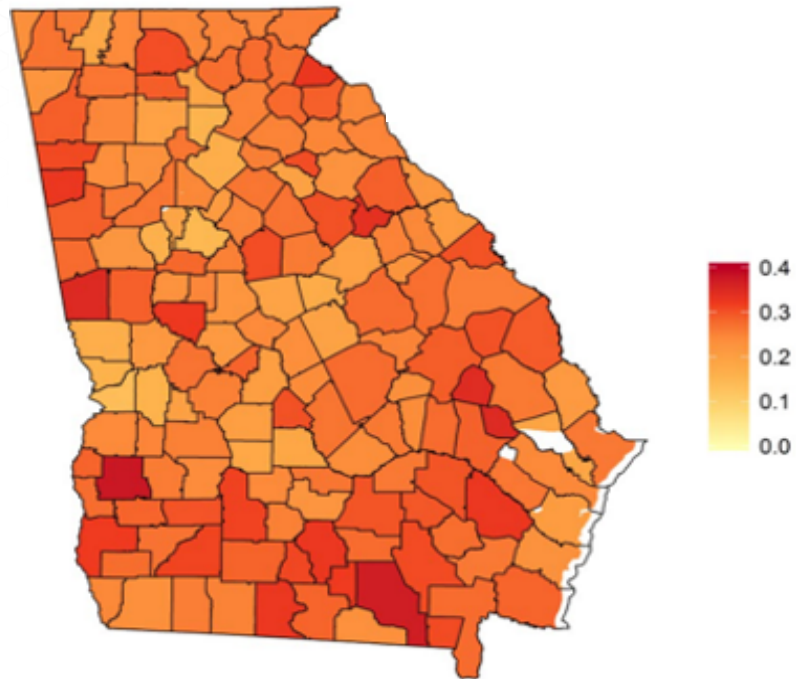
- Travel distance to a primary care provider evaluated at the community level
- The number emergency department visit per member per-month
- Locations of outbreak of a disease, for example, Zika
- Functional Magnetic Resonance Imaging (fMRI)
- Trajectory of the bison in the landscape

Spatial Process: Characteristics

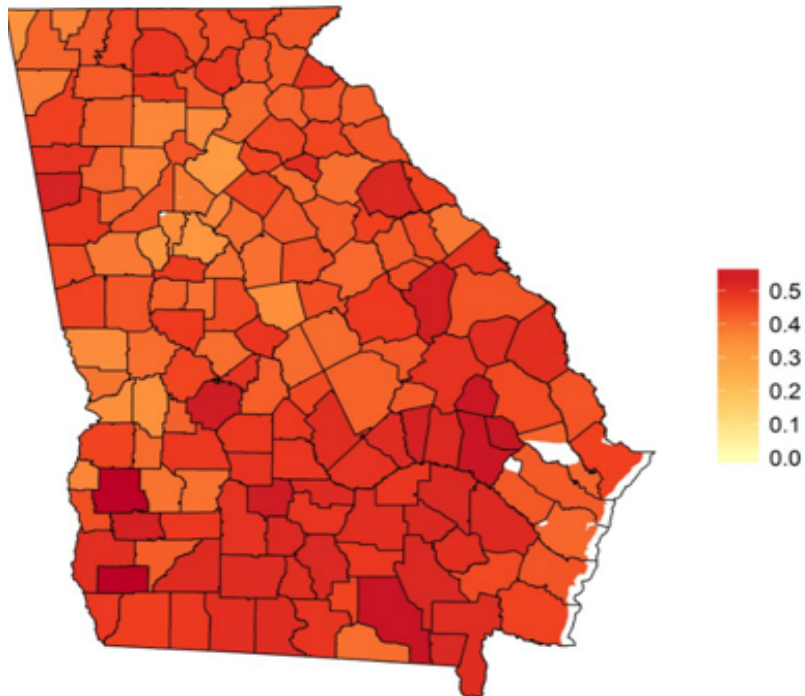
- **Trend:** long-distance increase or decrease in the data over space
- **Smoothness:** observations change slowly as the distance between their location increases
- **Heteroskedasticity:** varying variance with space
- **Continuous vs. Discrete:** at each location observe a numeric value versus a binary response versus a count response
- **Regular vs. irregular design:** the spatial process can be observed within regular or irregular division of the space

Example 1: ED Care Utilization

Percentage of **Children** with an ED visit



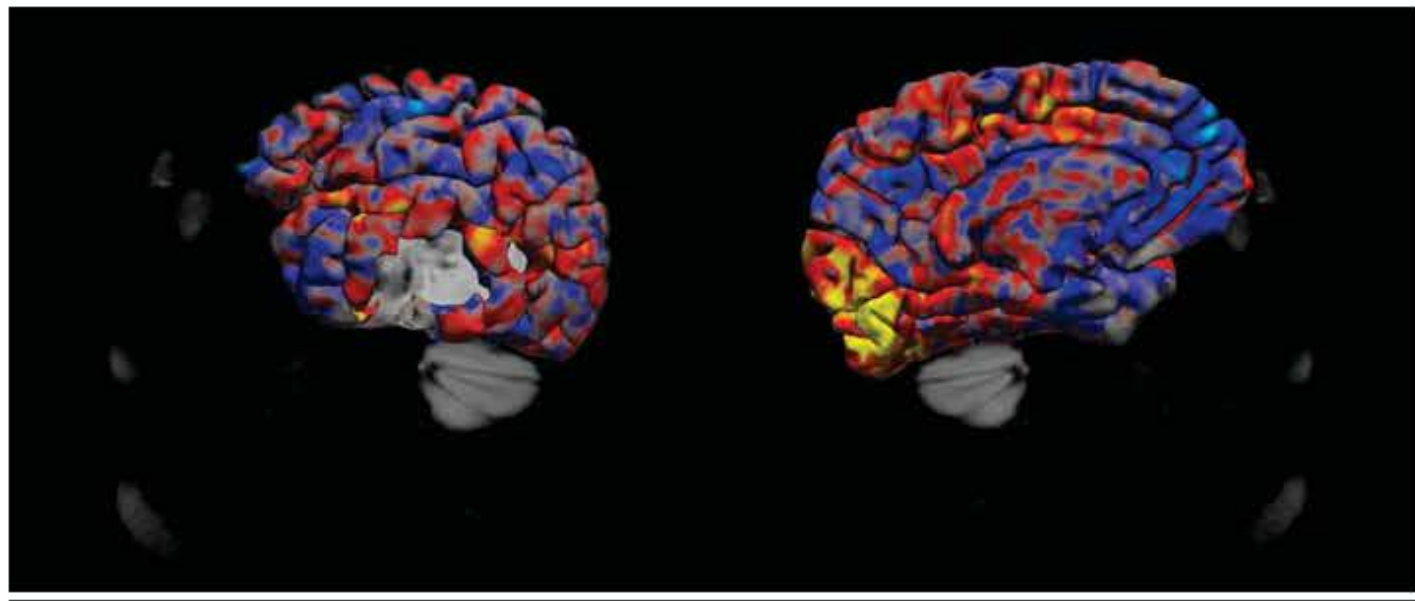
Percentage of **Adults** with an ED visit



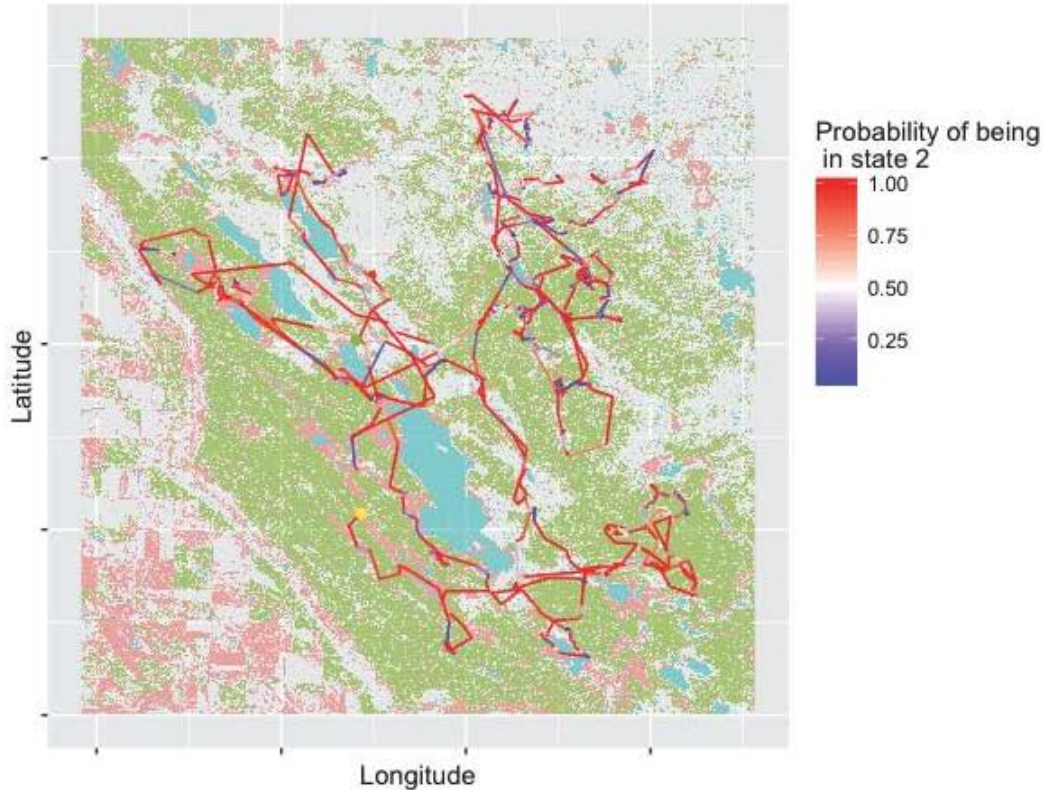
Example 2: fMRI

Lateral

Medial



Example: Bison Trajectory



Spatial Analysis: Basics

Data: Y_s , where s indexes space, e.g. voxels, census tract

Model:

- Y_s numeric response vs. point process
- Y_s stationary vs. non-stationary
- Y_s isotropic process
- $s \in S$ where S is the space domain: irregular vs regular observation grid
- Large scale vs. small-scale spatial dependence

Summary

