

Regression Analysis

Simple Linear Regression

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Regression Concepts: Estimation
Example



1

About This Lesson



2

Example in R

A company, which sells medical supplies to hospitals, clinics, and doctor's offices, had considered the effectiveness of a new advertising program. Management wants to know if the advertising is related to sales. This company intends to increase the sales with an effective advertising program.

Which are the response and the predicting variables?

$Y = \text{Sales}$ and $X = \text{Advertising Expenditure}$



3

Example in R: Estimation

- A. Fit a linear regression. What are the estimated regression coefficients and the estimated regression line?
- B. Interpret the coefficients.
- C. What does the model predict sales as the advertising expenditure increases for an additional **\$1,000**?
- D. What sales would you predict for an advertising expenditure of **\$30,000**?
- E. What is the estimate of the error variance?
- F. What could you say about the sales for an advertising expenditure of **\$100,000**?



4

Example in R (cont'd)

```
## Read Data in R
data = read.table("meddcor.txt", sep="", header = FALSE)
## Response & Predicting Variable
sales = data[, 1]
adv = data[, 2]
## Fit a linear regression model
model = lm(sales ~ adv)
summary(model)
Coefficients:
```

Estimated Model Parameters:

$$\hat{\beta}_0 = -157.3301$$

$$\hat{\beta}_1 = 2.7721$$

$$\hat{\sigma} = 101.4$$

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-157.3301	145.1912	-1.084	0.29
adv	2.7721	0.2794	9.921	8.87e-10 ***

Residual standard error: 101.4 on 23 degrees of freedom
 Multiple R-squared: 0.8106, Adjusted R-squared: 0.8024
 F-statistic: 98.43 on 1 and 23 DF, p-value: 8.873e-10



5

Example in R (cont'd)

- A. Fit a linear regression. What are the estimated regression coefficients and the estimated regression line?

Solution: Estimates (β_0, β_1) are $(-157.33, 2.77)$ and the *regression equation is:*

$$\text{Sales} = -157.33 + 2.77 \text{ Adv Expenditure}$$

- B. Interpret the coefficients.

Solution: The sales increase by \$2770 with each \$100 additional expenditure in advertisement. Or the sales increase with \$27.7 with each dollar invested in advertisement expenditure.

- C. What does the model predict as the advertising expenditure increases for an additional \$1,000?

Solution: The increase in sales is $10 \times 2.77 = 27.7$ thousands.



6

Example in R (cont'd)

- D. What sales would you predict for an advertisement expenditure of \$30,000?

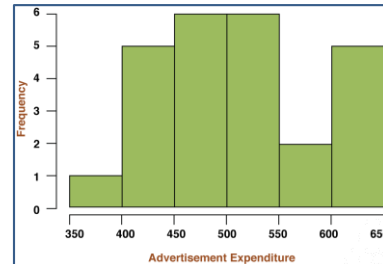
Solution: The predicted sales is
 $-157.33 + 300 \times 2.77 = 673.67$ thousands

- E. What is the estimate of the error variance?

Solution: Estimate σ^2 with $MSE = 10,281.96$

- F. What could you say about the sales for an advertising expenditure of \$100,000?

Solution: An advertisement expenditure of \$100,000 or 1000 units is outside of the observed range and thus we cannot predict the sales since this is extrapolation.



Summary

