Regression Analysis

Multiple Linear Regression

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Testing for Subsets of Regression Parameters

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About This Lesson

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Testing Overall Regression

Analysis of Variance (ANOVA) for multiple regression:

Variability Source	DF	Sum of Squares	Mean SS	F-Statistic
Regression	p	SSReg	SSReg / p	MSSReg / MSE
Residual	n-p-1	SSE	SSE / (n-p-1)	
Total	n-1	SST		

$$\mathrm{SSReg} = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 \qquad \mathrm{SSE} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \qquad \mathrm{SST} = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

Null hypothesis: All predictor coefficients are 0, i.e., $\mathbf{H_0}: \beta_1 = \beta_2 = \cdots = \beta_p = 0$.

Reject $\mathbf{H_0}$ if F-statistic is large (> $F_{\alpha,\, p,\, n-p-1}$ for α significance level, p and n-p-1 df).

- At least one of the coefficients is different from zero at the α significance level.

p-value = Prob($F_{p,\, n-p-1}$ > F-statistic) for F-distribution with p and n-p-1 df.

- Reject $\mathbf{H_0}$ if p-value is small.

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Testing Subsets of Coefficients

Analysis of Variance (ANOVA):

$$\mathrm{SST}(X_1, \cdots, X_p) = \mathrm{SSReg}(X_1, \cdots, X_p) + \mathrm{SSE}(X_1, \cdots, X_p)$$

$$\mathrm{SSReg}(X_1, \cdots, X_p) = \mathrm{SSReg}(X_1) + \mathrm{SSReg}(X_2|X_1) + \mathrm{SSReg}(X_3|X_1, X_2) + \cdots + \mathrm{SSReg}(X_p|X_1, \cdots, X_{p-1})$$

$\mathbf{SSReg}(X_1)$: Sum of squares (SS) explained using only X_1

$\mathbf{SSReg}(X_2|X_1)$: ***Extra*** SS explained using X_2 in addition to X_1

$\mathbf{SSReg}(X_3|X_1, X_2)$: ***Extra*** SS explained using X_3 in addition to X_1 and X_2

$\mathbf{SSReg}(X_p|X_1, \cdots, X_{p-1})$: ***Extra*** SS explained using X_p in addition to X_1 , X_2 … X_{p-1}

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Testing Subsets of Coefficients

- Does X_1 alone significantly aid in predicting Y?
  - SSReg(X_1) vs. SSE(X_1)
- Does the addition of X_2 significantly contribute to the prediction of Y after accounting (controlling) for the contribution of X_1?
  - SSReg(X_2 | X_1) vs. SSE(X_1, X_2)
- Does the addition of X_3 significantly contribute to the prediction of Y after accounting (controlling) for the contribution of X_1 and X_2?
  - SSReg(X_3 | X_1, X_2) vs. SSE(X_1, X_2, X_3)
- Does the addition of X_p significantly contribute to the prediction of Y after accounting (controlling) for the contribution of X_1,..., X_{p-1}?
  - SSReg(X_p | X_1, ..., X_{p-1}) vs. SSE(X_1, X_2, ..., X_p)

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Testing Subsets of Coefficients

Partial F-test:

- Consider a full model with two sets of predictors, X_1, ..., X_p (perhaps controlling factors) and (Z_1, ..., Z_q) (perhaps additional explanatory factors):

$$Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p + \alpha_1 Z_1 + \cdots + \alpha_q Z_q + \varepsilon$$

- Test whether any of the Z factors add explanatory power to the model:

$$\mathbf{H_0}: \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0 \quad \textbf{vs.} \quad \mathbf{H_a}: \alpha_\mathrm{i} \neq 0 \text{ for at least one } \alpha_\mathrm{i}, \mathrm{i} = 1, \cdots, q$$

$$\text{F}-\text{statistic} = \text{F}_{partial} = \frac{\text{SSReg}(Z_1, \ldots, Z_q | X_1, \ldots, X_p)/q}{\text{SSE}(Z_1, \ldots, Z_q, X_1, \ldots, X_p)/(n-p-q-1)}$$

- Reject $\mathbf{H_0}$ if F-statistic is large (F-statistic $> \text{F}_{\alpha,\, q,\, n-p-q-1}$)
  - At least one coefficient is different from zero at the α significance level

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Testing for Statistical Significance

- Consider a full model with the set of predictors, X_1, ..., X_p and an additional predicting variable Z:

$$Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p + \alpha Z + \varepsilon$$

- Test whether Z has explanatory or predictive power:

$$\mathbf{H_0}: \alpha = 0 \text{ vs } \mathbf{H_a}: \alpha \neq 0$$

$$\text{F-statistic} = \text{F}_{partial} = \frac{\text{SSReg}(\text{Z}|X_1, \ldots, X_p)/1}{\text{SSE}(Z, X_1, \ldots, X_p)/(n-p-2)}$$

- Reject $\mathbf{H_0}$ if F-statistic is large (F-statistic > $\text{F}_{\alpha, 1, n-p-2}$)

This is equivalent to testing for statistical significance using the t-test

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Testing for Statistical Significance

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- Reject $\mathbf{H_0}$ if F-statistic is large (F-statistic > $\text{F}_{\alpha, 1, n-p-2}$)

This is equivalent to testing for statistical significance using the t-test

- Interpretation of the t-test for statistical significance is conditional on other predicting variables being in the model.
- The relationship between Y and X is statistically significant given all other predicting variables being in the model.

Do not perform variable selection based on the p-values of the t-tests!

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