

Stat ST485/685, Project 3
Due: Thursday, October 28

Prediction

1. (30 points)

- (a) Simulate a 200 point sample $\{x_t\}_{t=1}^{200}$ from an AR(4) process $\{X_t\}$ with $\phi_1 = .8$, $\phi_2 = .2$, $\phi_3 = -.2$, $\phi_4 = -.1$ and with $\{Z_t\} \sim \text{WN}(0, 1)$ with $Z_t \sim N(0, 1)$. (see Example 3.4.3). Plot the sample with appropriately labeled axes.
- (b) Plot the sample acf versus lag. Are there significant linear dependencies for lags greater than 1? Explain your answer.
- (c) Compare one step predictions from the original data: Using the data $\{x_t\}_{t=1}^{200}$, compute estimated one step predictions

$$P \left(X_{n+1} \middle| \begin{pmatrix} X_n \\ X_{n-1} \\ X_{n-2} \\ X_{n-3} \end{pmatrix} \right), \quad 4 \leq n \leq 200,$$

and plot the predictions and the original data using different colors for the two series on the same graph (see Figures 1 and 2 for Example 3.4.2).

- (d) Compute predictions into the future: Using a software package that computes the best linear MSE predictor, compute the predictions

$$P_{200}X_{200+h} = P \left(X_{200+h} \middle| \begin{pmatrix} X_{200} \\ \vdots \\ X_1 \end{pmatrix} \right), \quad 1 \leq h \leq 10.$$

Plot the predictions together with the original data.