Minimal Spanning Trees

A minimum spanning tree (MST) for a connected graph $G = (V, E)$ is an acyclic subset of edges that spans all vertices with minimum total edge weight.

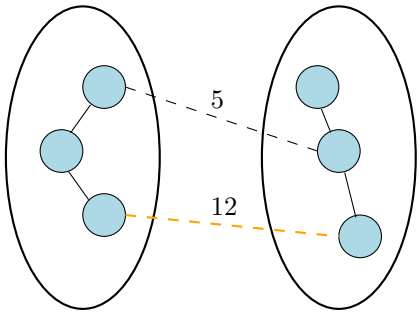
In general, MST's have the following properties:

- n nodes on $n - 1$ edges
- Adding an edge introduces a cycle
- Removing any edge in such a cycle restores the spanning tree

Given a cut $(S, V - S)$, it respects a subset of edges if none of those edges crosses the cut.

Claim If $(S, V - S)$ is any cut that respects some viable MST solution denoted as a set of edges A, then the lightest edge that crosses this cut is always "safe" to add.

Proof By contradiction, imagine 2 cuts of a part of a viable solution that we add a non-minimum crossing edge to:



However, we can swap out the expensive edge for a cheaper crossing edge and still maintain a spanning tree with lower weight, so therefore the lightest crossing edge is always a safe edge to add. Call this the **safe edge lemma**