MATH 307: Individual Homework 13

Shaochen (Henry) ZHONG, sxz517@case.edu

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Problem 1

See HW instruction.

Say we have the angle between Qx and Qy being α and the angle between x and y being β, we have:

$$\begin{aligned}
\cos\beta &= \frac{< x, y >}{\|x\| \, \|y\|} \\
&= \frac{< x, y >}{\sqrt{< x, x >}\sqrt{< y, y >}} \\
\cos\alpha &= \frac{< Qx, Qy >}{\|Qx\| \, \|Qy\|} \\
&= \frac{< Qx, y >}{\sqrt{< Qx, Qx >}\sqrt{< Qy, Qy >}} = \frac{(Qy)^* Qx}{\sqrt{(Qx)^* Qx}\sqrt{(Qy)^* Qy}} \\
&= \frac{y^* Q^* Qx}{\sqrt{(Qx)^* Qx}\sqrt{(Qy)^* Qy}} = \frac{y^* x}{\sqrt{(Qx)^* Qx}\sqrt{(Qy)^* Qy}} \\
&= \frac{< x, y >}{\sqrt{< x, x >}\sqrt{< y, y >}} = \cos\beta
\end{aligned}$$

This shows the unitary matrix does not affect the angle vetween two vectors. Then we may find the angle as:

$$\begin{aligned}
\cos\alpha = \cos\beta &= \frac{< x, y >}{\|x\| \, \|y\|} \\
&= \frac{-3 + 0 + 3}{\|x\| \, \|y\|} = 0 \\
\alpha &= \frac{\pi}{2}
\end{aligned}$$

Problem 2

See HW instruction.

For $N(A) = \{x \in F^n \mid Ax = 0\}$, we have:

- $0 \in N(A)$, as $A0 = 0$.

- If $x, y \in N(A)$, we also have $x + y \in N(A)$ as $A(x + y) = Ax + Ay = 0 + 0 = 0$.
- If $x \in N(A)$ and $c \in F$, we have $cx \in N(A)$ as $A(cx) = cAx = c \cdot 0 = 0$.

Thus, $N(A)$ is a subspace of F^n.

Problem 3

See HW instruction.

Conduct $R2 - R1$, we have $rref(A) = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ and we know that the pivot column is the first column. Therefore, we have $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ to be the basis for A.

Problem 4

See HW instruction.

For $v \in range(AB)$, we must have $ABx = v$ for any v, where $ABx = A(Bx) = v$; so such v will also be in $\in range(A)$.