

MATH 307

Individual HW 14

Instructions: Read textbook pages 67 to 68 before working on the homework problems. Show all steps to get full credits.

1. Let $A \in F^{m \times n}$, with $F = \mathbb{R}$ or \mathbb{C} , prove that $\text{range}(A)$ is orthogonal to $\text{null}(A^*)$, i.e., any arbitrary vector in $\text{range}(A)$ is orthogonal to an arbitrary vector in $\text{null}(A^*)$.
2. Use Gram-Schmidt method to find a QR factorization of the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

3. Consider a matrix $A \in F^{m \times n}$ with $m \geq n$ and all columns being orthogonal but not of unit length, what should its reduced QR decomposition look like?