

# MATH 307

## Individual Homework 23

Read textbook pages 135 to 142, pages 126 to 128 before working on the homework problems. Show all steps to get full credits.

1. Let  $A = \begin{pmatrix} -2 & 1 \\ -1 & 2 \end{pmatrix}$ ,  $b = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ , solve  $Ax = b$  using Cramer's rule and verify your answer is correct by checking whether  $Ax = b$  is satisfied.
2. Let  $A$  be a  $n \times n$  matrix, prove the following three statements are all equivalent:
  - (a)  $Ax = 0$  has nontrivial solutions (solutions other than 0).
  - (b) The determinant of  $A$  is zero.
  - (c) 0 is an eigenvalue of  $A$ .
3. Let  $A \in F^{m \times n}$ ,  $m \geq n$  with  $F = \mathbb{R}$  or  $\mathbb{C}$  be of full rank, prove that the normal equation  $A^*Ax = A^*b$  to the least squares problem  $\min \|Ax - b\|_2$  has a unique solution for any  $b \in F^n$ .