MATH 307

Individual Homework 22

Read textbook pages 135 to 142, pages 126 to 128 before working on the homework problems. Show all steps to get full credits.

1. Let A be a square matrix with singular value decomposition $A = U\Sigma V^T$, prove that A is invertible if and only if all the singular values of A are nonzero.
2. Prove that the determinant of a square matrix is equal to the product of all its eigenvalues.
3. Use the result of the previous problem to prove that a square matrix is invertible if and only if its determinant is nonzero.