

MATH 307

Individual Homework 12

Instructions: Read textbook pages 64 to 66 before working on the homework problems. Show all steps to get full credits.

1. Find the inverse of matrix

$$A = \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$$

by finding the matrix B such that $AB = I$ and then double check $BA = I$.

2. Let A represent the matrix corresponding to rotate a 2-dimensional vector for 45 degree ($\frac{\pi}{4}$), i.e.,

$$A = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix},$$

given vector

$$b = \begin{pmatrix} 0 \\ 2\sqrt{2} \end{pmatrix},$$

find the solution to $Ax = b$ using the following three different approaches. You should reach to the same answers for all approaches.

- (a) Expand the linear equation system into two equations with two unknowns and then solve for x using elimination, substitution (what you learned in algebra class).
- (b) Find x directly by using the fact that A applied on a vector is to rotate the vector for 45 degree. Think about how to reverse the transformation.
- (c) Prove A is an orthogonal matrix first and use the fact to find the inverse of A , and then find x by calculating $A^{-1}x$.