MATH 307: Group Homework 11

Group 8
Shaochen (Henry) ZHONG, Zhitao (Robert) CHEN, John MAYS, Huaijin XIN
{sxz517, zxc325, jkm100, hxx200}@case.edu

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Problem 1

See HW instruction.

For A, B, C, D we only have one entry on the first row bing non-zero, so we can only calculate the part leading by the non-zero entry.

$$\begin{aligned}\det(A) &= -1(1 \cdot 1) = -1\\ \det(B) &= 1(3 \cdot 1) = 3\\ \det(C) &= 1(1 \cdot 1) = 1\\ \det(D) &= 2(-5 \cdot 3) = -30\end{aligned}$$

For D, we have:

$$\det(D) = 1(1 \cdot 1) - 4(-1 \cdot 1) - 1(-1 \cdot 2) = 1 + 4 + 2 = 7$$

Problem 2

See HW instruction.

Known that $AA^{-1} = I$, so there must be $\det(AA^{-1}) = \det(I) = 1$. Know that determinant of the product is just product of determinants, we have $\det(A)\det(A^{-1}) = 1 \implies \det(A^{-1}) = \frac{1}{\det(A)}$.

Problem 3

See HW instruction.

Known that $\det(A) = \det(A^T)$ assuming $\det(A) \neq 0$ (ref.), we have:

$$\det(A^{-1}(B^T)^2) = \det(A^{-1})\det((B^T)^2) = \det(A^{-1})\det(B^T)^2 = \frac{1}{\det(A)}\det(B)^2 = \frac{1}{2}(-1)^2 = \frac{1}{2}$$

$$\det((B^T)^{-1}A^3) = \det((B^T)^{-1})\det(A)^3 = \det(B^{-1})\det(A)^3 = \frac{1}{\det(B)}\det(A)^3 = -1 \cdot (2)^3 = -8$$

Problem 4

See HW instruction.

For $QQ^T = I$ there must be $\det(QQ^T) = \det(I) = 1$, so we have:

$$\begin{aligned} 1 &= \det(QQ^T) = \det(Q^T)\det(Q) = \det(Q)\det(Q) \\ &= \det(Q)^2 \\ \implies \det(Q) &= \pm 1 \end{aligned}$$

Therefore the determinant of an orthonormal basis Q must be either 1 or -1.