MATH 307: Individual Homework 17

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Problem 1

See HW instruction.

Known that $A = U\Sigma V^*$ with U, V being orthogonal (implies $U^{-1} = U^*$ and $V^{-1} = V^*$)

$$\begin{aligned} A &= U\Sigma V^* \\ A^{-1} &= (U\Sigma V^*)^{-1} = (V^*)^{-1}\Sigma^{-1}U^{-1} \\ &= V\Sigma^{-1}U^* \end{aligned}$$

Problem 2

See HW instruction.

Known that $A = U\Sigma V^*$, and we know that $\Sigma^*\Sigma = \Sigma\Sigma^* = \Sigma^2$ since Σ is a square diagonal matrix with nonzero diagonal entries. We have:

$$\begin{aligned} A^*A &= (U\Sigma V^*)^*(U\Sigma V^*) \\ &= V\Sigma^*U^*U\Sigma V^* \\ &= V\Sigma^*\Sigma V^* \\ &= V\Sigma^2V^* \end{aligned}$$

$$\begin{aligned} AA^* &= (U\Sigma V^*)(U\Sigma V^*)^* \\ &= U\Sigma V^*V\Sigma^*U^* \\ &= U\Sigma\Sigma^*U^* \\ &= U\Sigma^2U^* \end{aligned}$$

Since Σ^2 is a square diagonal matrix with nonzero diagonal entries, from the pervious **Question 1**, we know both A^*A and AA^* are invertible.