MATH 307

Individual Homework 21

Instructions: Read textbook pages 114 to 117 before working on the homework problems. Show all steps to get full credits.

1. Let A be a $n \times n$ matrix with $a_{11} \neq 0$, the first step in LU decomposition is to introduce zeros below the first diagonal a_{11}. This can be done by multiplying A by a lower triangular matrix L_1 that is equal to the $n \times n$ diagonal matrix except the first column looks like $\ell_1 = \begin{pmatrix} 1 \\ -l_{21} \\ -l_{31} \\ \vdots \\ -\ell_{n1} \end{pmatrix}$ with $l_{j1} = \frac{a_{j1}}{a_{11}}, j = 2, 3, \cdots n$. It is obvious that $L_1 = I + \ell_1 e_1^T$. Prove $L_1^{-1} = I - \ell_1 e_1^T$. This is the first stroke of luck in LU decomposition: find the inverse of L_1 can be done by simply negativing the entries below the first diagonal.

2. Find the general solution to $Ax = b$ with $A = \begin{pmatrix} 1 & 2 & 1 & 3 \\ -3 & 2 & 1 & 0 \\ 3 & 2 & 1 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ -5 \\ 2 \end{pmatrix}$. You may use some of the information from the previous problem.

3. Given the matrix $A = \begin{pmatrix} -1 & 2 & 1 & 0 & 2 \\ 2 & 0 & 0 & 3 & -1 \\ -1 & 6 & 3 & 3 & 5 \end{pmatrix}, b = \begin{pmatrix} -1 \\ 0 \\ c \end{pmatrix}, c \in \mathbb{R}$,

   (a) For which value of c does the equation $Ax = b$ have a solution?

   (b) After choosing c so that the system has a solution, find a particular solution to $Ax = b$.