MATH 307

Group Homework 9

Instructions: Read textbook pages 93 to 100 before working on the homework problems. Show all steps to get full credits.

1. Let $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $V = span(e_1, e_2 - e_1), W = span(e_2)$ be two subspaces of $\mathbb{C}^2$, prove that $\mathbb{C}^2 = V + W$ but it is not a direct sum.
2. Let $V = \text{span}\{e_1, e_2\}, W = \text{span}\{e_2, e_3\}$, where e_1, e_2, e_3 are vectors in $\mathbb{R}^3$, prove $\mathbb{R}^3 = V + W$. Is the sum a direct sum? Justify your answer.
3. Let A be a $m \times n$ complex valued matrix, use SVD to prove that $\mathbb{C}^m = range(A) \oplus null(A^*)$.