MATH 307

Individual Homework 11

Instructions: Read textbook pages 57 to 59 before working on the homework problems. Show all steps to get full credits.

1. Let $f : P^3 \to \mathbb{R}$ be a mapping with $f(a_0 + a_1x + a_2x^2 + a_3x^3) = a_3$ for all $a_0 + a_1x + a_2x^2 + a_3x^3$ in P^3. Prove that f is a linear mapping.

2. For each of the following matrices

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -8 \end{pmatrix}, B = \begin{pmatrix} -1 & 0 & 0 \\ 3 & 2 & 0 \\ 4 & 5 & 3 \end{pmatrix}, C = \begin{pmatrix} \sqrt{2} & i & 1-2i \\ 0 & 2-3i & 2+i \\ 0 & 0 & 1 \end{pmatrix},$$

$$D = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & -i \\ 3 & -i & 0 \end{pmatrix}, E = \begin{pmatrix} 1 & 1+i & 2-i \\ 1-i & 2 & 4 \\ 2+i & 4 & 3 \end{pmatrix}, F = \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & -2 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

   specify whether it is diagonal, upper-triangular, lower-triangular, symmetric or hermitian. Note one matrix might have more than one structures. For instance, a diagonal matrix is also upper-triangular. Moreover, a matrix is symmetric if $A = A^T$. It applies to complex matrices as well.

3. Prove that for two matrices A, B of the same size and α, β some coefficients, we have $(\alpha A + \beta B)^T = \alpha A^T + \beta B^T$. Note, to prove two matrices are equal, it suffices to prove the ij-th entry of the two matrices are equal for all legal indices i, j.

4. Prove that diagonal entries of Hermitian matrices have to be real valued.