MATH 307: Individual Homework 11

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Problem 1

See HW instruction.

It is a linear mapping as it satisfies the following conditions (for $\lambda \in \mathbb{R}$):

$$\begin{aligned}
f(\lambda u) &= f(\lambda(a_0 + a_1x + a_2x^2 + a_3x^3) \\
&= f(\lambda a_0 + \lambda a_1 x + \lambda a_2 x^2 + \lambda a_3 x^3) \\
&= \lambda a_3 = \lambda f(u) \\
f(u+v) &= f(a_0 + a_1x + a_2x^2 + a_3x^3 + b_0 + b_1x + b_2x^2 + b_3x^3) \\
&= f((a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + (a_3 + b_3)x^3 \\
&= a_3 + b_3 = f(u) + f(v)
\end{aligned}$$

Problem 2

See HW instruction.

- A is diagonal, and therefore also upper-/lower-triangular, and symmatric. Since it is a matrix with elements in $\mathbb{R}$, it is also hermitian.
- B is not diagonal nor upper-triangular, but it is lower triangular. It is neither symmetric nor hermitian.
- C is not diagonal not lower-triangular, but it is upper-triangular. It is neither symmetric nor hermitian.
- D is not diagonal, not upper- or lower-triangular. It is symmatric but not hermitian as for row two in D, it should be equal to $[2, 4, i]$ but it is not.
- E is not diagonal, not upper- or lower-triangular. It is both symmatric and hermitian.
- F is not a square matrix so it is not diagonal, not upper-, and not lower-triangular. For the same reason it is also not symmatric nor hermitian.

For easy grading:

matrix	Diagonal	UT	LT	Symmatric	Hermitian
A	✓	✓	✓	✓	✓
B	✗	✗	✓	✗	✗
C	✗	✓	✗	✗	✗
D	✗	✗	✗	✓	✗
E	✗	✗	✗	✓	✓
F	✗	✗	✗	✗	✗

Problem 3

See HW instruction.

First start with LHS, known that the ij-th entry in $(\alpha A + \beta B)$ is $\alpha A_{ij} + \beta B_{ij}$. Thus, by the definition of transpose, the ij-th entry in $(\alpha A + \beta B)^T$ must be $\alpha A_{ji} + \beta B_{ji}$.

Now inspect the RHS, we have the ij-th entry in αA^T to be αA_{ji}; similiarily, the ij-th entry in βB^T is βB_{ji}. So the RHS equals to $\alpha A_{ji} + \beta B_{ji}$.

As the two sides are equal, the equality-in-question is therefore proven.

Problem 4

See HW instruction.

Being the diagonal entries of a Hermitian matrix A, each entry has to be real valued as we must have $A_{ij} = \overline{A_{ji}}$. This is only possible when the imaginary part of A_{ij} is $0i$, so it must be a real valued number.