MATH 307

Group HW 8

Instructions: Read textbook pages 149 to 155 before working on the homework problems. Show all steps to get full credits.

1. Let $A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$, determine whether A is nondefective, i.e., whether the algebra multiplicity and the geometric multiplicity are identical for all eigenvalues.

2. Let A be a Hermitian matrix, prove that eigenvectors corresponding to distinct eigenvalues of A are orthogonal. Note this is stronger than eigenvectors of distinct eigenvalues of a general matrix are linearly independent.

3. Let $A = U\Sigma V^*$ be a singular value decomposition of A, prove that $V(\Sigma^*\Sigma)V^*$ is an eigendecomposition of A^*A.

4. Use SVD to solve $Ax = b$. Let $A = U\Sigma V^T$ with $U = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}, \Sigma = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, V = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, b = \begin{pmatrix} -\frac{3}{2}\sqrt{2} \\ -\frac{\sqrt{2}}{2} \end{pmatrix}$.