

# MATH 307

## Group Homework 11

Instructions: Read textbook pages 135 to 142 before working on the homework problems. Show all steps to get full credits.

1. Find the determinants of the following matrices:  $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  (a permutation elementary row operation matrix),  $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  (a multiplication elementary row operation matrix),  $C = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  (an adding a multiple of one row to another row elementary row operation matrix),  $D = \begin{pmatrix} 2 & 0 & 0 \\ 1 & -5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ ,  $E = \begin{pmatrix} 1 & 4 & -1 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ .
2. Let  $A$  be an invertible matrix, one can prove that  $|A| \neq 0$ , find the determinant of  $A^{-1}$  in terms of  $|A|$ .
3. If  $|A| = 2$ ,  $|B| = -1$ , find  $|A^{-1}(B^T)^2|$ ,  $|(B^T)^{-1}A^3|$ .
4. Suppose that  $Q$  is a  $n \times n$  real orthonormal matrix, i.e.,  $QQ^T = I$ . Find the possible values for  $|Q|$ .