

MATH 307: Individual Homework 13

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Problem 1

See HW instruction.

Say we have the angle between Qx and Qy being α and the angle between x and y being β , we have:

$$\begin{aligned}\cos \beta &= \frac{\langle x, y \rangle}{\|x\| \|y\|} \\ &= \frac{\langle x, y \rangle}{\sqrt{\langle x, x \rangle} \sqrt{\langle y, y \rangle}} \\ \cos \alpha &= \frac{\langle Qx, Qy \rangle}{\|Qx\| \|Qy\|} \\ &= \frac{\langle Qx, y \rangle}{\sqrt{\langle Qx, Qx \rangle} \sqrt{\langle Qy, Qy \rangle}} = \frac{(Qy)^* Qx}{\sqrt{(Qx)^* Qx} \sqrt{(Qy)^* Qy}} \\ &= \frac{y^* Q^* Qx}{\sqrt{(Qx)^* Qx} \sqrt{(Qy)^* Qy}} = \frac{y^* x}{\sqrt{(Qx)^* Qx} \sqrt{(Qy)^* Qy}} \\ &= \frac{\langle x, y \rangle}{\sqrt{\langle x, x \rangle} \sqrt{\langle y, y \rangle}} = \cos \beta\end{aligned}$$

Thus, the unitary matrix does not affect the angle between two vectors.

Problem 2

See HW instruction.

For $N(A) = \{x \in F^n \mid Ax = 0\}$, we have:

- $0 \in N(A)$, as $A0 = 0$.
- If $x, y \in N(A)$, we also have $x + y \in N(A)$ as $A(x + y) = Ax + Ay = 0 + 0 = 0$.
- If $x \in N(A)$ and $c \in F$, we have $cx \in N(A)$ as $A(cx) = cAx = c \cdot 0 = 0$.

Thus, $N(A)$ is a subspace of F^n .

Problem 3

See HW instruction.

Conduct $R2 - R1$, we have $rref(A) = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ and we know that the pivot column is the first column. Therefore, we have $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ to be the basis for A .

Problem 4

See HW instruction.

For $v \in \text{range}(AB)$, we must have $ABx = v$ for any v , where $ABx = A(Bx) = v$; so such v will also be in $\in \text{range}(A)$.