MATH 307

Individual Homework 18

Instructions: Read textbook pages 87 to 90 before working on the homework problems. Show all steps to get full credits.

1. Let $A \in F^{m\times n}$ with $F = \mathbb{R}$ or $\mathbb{C}$, find a basis for both $range(A)$ and $range(A^*)$ and then prove that the column rank of A is the same as the row rank of A.
2. Assume matrix $A \in F^{6\times 8}$ has singular value decomposition $A = U\Sigma V^*$ with singular values $21, 11, 6, 6, 0.2, 0$.
   (a) Find the row rank of A, i.e, the dimension of $range(A^*)$ and find an orthonormal basis of $range(A^*)$ in terms of the SVD of A and prove it.
   (b) Find the nullity A^*, i.e., the dimension of $null(A^*)$ and find an orthonormal basis of $null(A^*)$ in terms of the SVD of A and prove it. You may use the rank-nullity theorem without proving it.