MATH 307

Group Homework 6

Instructions: Read textbook pages 59 to 62 before working on the homework problems. Show all steps to get full credits.

1. Find the matrix for the linear transformation which reflects every 2-dimensional vector across the y axis and hen rotate by an angle of $\pi/4$.
2. Is $(A+B)^2 = A^2 + 2AB + B^2$ true for two square matrices A, B of the same sizes? Justify your answer.
3. Let A, B be matrices of appropriate sizes, prove that $(AB)^* = B^*A^*$.
4. Let A, B be two square upper-triangular matrices with the same size, prove that AB is also upper-triangular. The same conclusion applies for lower-triangular matrices.
5. Let $F = \mathbb{R}$ or $\mathbb{C}$, prove that $A \in F^{n\times n}$ is invertible if and only if all the columns of A are linearly independent. (Hint: to show A is invertible, it is enough to show that there exists a matrix B such that $AB = I$ as $BA = I$ will follow from $AB = I$.