CSCI HW 1 - ROHAN KARNAWAT - 2102695224

1. Given : A line $l \equiv (a, b, c)$ in the image ⇒ normalized coordinate system

∴ l can be represented as $\boxed{a\hat{x} + b\hat{y} + c = 0}$ — (1)

$\hat{x}$ and $\hat{y}$ are the coordinate axes of the image plane

Consider a point $(\hat{x}, \hat{y})$ in the $\hat{x}-\hat{y}$ plane. Let this be the projection for a point (X, Y, Z) in the camera coordinate system. Then we have:

$$\hat{X} = \frac{Xd}{Z} = \frac{X}{Z} \qquad (d = 1)$$

$$\hat{Y} = \frac{Yd}{Z} = \frac{Y}{Z} \quad \text{— (2)}$$

Our goal is to reverse map a line from the image plane to the 3D plane in the camera coordinate system.

From (1) ⇒ $a\hat{x} + b\hat{y} + c = 0$.

From (2) ⇒ $a\frac{X}{Z} + b\frac{Y}{Z} + c = 0$

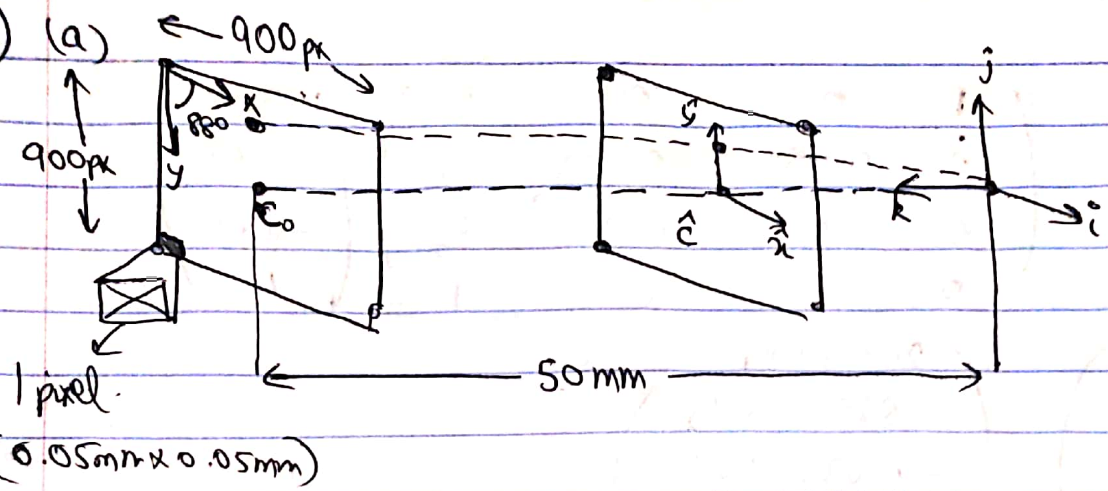
$$\Rightarrow \boxed{aX + bY + cZ = 0}$$

This is the equation of plane the actual 3D line / structure lies on for the line's image.

In homogeneous coordinates the plane can be represented as

$$(a, b, c, 0)^T$$

2) (a)



The magnification parameters α and β are kf and lf.

$$\therefore \alpha = \beta = \frac{1}{0.05} \times 50 = \underline{\underline{1000 \text{ px}}}.$$

$\theta = \underline{\underline{-88^\circ}}$. x_0 and y_0 - From center to left corner on top.

$$\therefore (x_0, y_0) = (450, 450),$$

$\therefore$ We have K (intrinsic camera calibration matrix) as follows

$$K = \begin{pmatrix} \alpha & -\alpha \cot\theta & x_0 \\ 0 & \beta/\sin\theta & y_0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{bmatrix} 1000 & 34.92 & 450 \\ 0 & -1000.609 & 450 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, camera is rotated by $+10^\circ$ about X_c. (parallel to x axis / $\hat{i}$)

$$\therefore R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 10 & -\sin 10 \\ 0 & \sin 10 & \cos 10 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.984 & -0.173 \\ 0 & 0.173 & 0.984 \end{bmatrix}$$

Now, the image of world origin O_w is $O_c = (2m, 3m, -5m)$
$= (2000, 3000, -5000)$

We know the $O_c = \begin{pmatrix} R & t \\ 0^T & 1 \end{pmatrix} O_w$

$$\therefore \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 0.984 & -0.173 & t_2 \\ 0 & 0.173 & 0.984 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2000 \\ 3000 \\ -5000 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2000 + t_1 \\ 3822.7 + t_2 \\ -4403.09 + t_3 \\ 1 \end{pmatrix}$$

$$\vec{t} = [-2000, -3822.7, 4403.09]$$

$$\therefore M = K(R\,t) =$$

$$\begin{pmatrix} 1000 & 134.92 & 450 \\ 0 & -1000.609 & 450 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -2000 \\ 0 & 0.984 & -0.173 & -3822.7 \\ 0 & 0.173 & 0.984 & 4403.09 \end{pmatrix}$$

$$M = \begin{bmatrix} 1000 & 112.21 & 436.76 & -152098.18 \\ 0 & -906.75 & 615.90 & 5806418.52 \\ 0 & 0.173 & 0.984 & 4403.09 \end{bmatrix}$$

(b) The infinity point for a vertical line $\in \vec{Y}_w$ would be $[0 \ 1 \ 0 \ 0]^T$

$$\therefore P = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$x = \frac{m_1 P}{m_3 P} = \frac{112.21}{0.173} = 648.61 \text{ px}$$

$$y = \frac{m_2 \cdot P}{m_3 P} = \frac{-906.75}{0.173} = -5241.32 \text{ px}$$

$\therefore$ Vanishing point $= (648.61, -5241.32)$

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(c) For a set of parallel lines in horizontal plane $(X_w Z_w)$:
~~$(x, 0, z)^T$~~ gives the direction
$(x, 0, z, 0)^T$

$$\therefore x = \frac{m_1 \cdot P}{m_3 \cdot P} = \frac{1000x + 436.76z}{0.984z}$$

$$y = \frac{m_2 P}{m_3 P} = \frac{615.90\cancel{z}}{0.984\cancel{z}} = 625.91$$

$\therefore$ The VP of the set of lines in image coordinates is

$$\left(\frac{1000x + 436.76z}{0.984z}, 625.91 \right)$$

(d) <u>Horizon line</u>

All points from the $X_w Z_w$ plane lie on the line $y = 625.91$, as shown above, in part (c).

$$\text{For all } x_i ~~\text{and } z~~ = \frac{1000x + 436.76z}{0.984z}$$

$$x_i = 1016 \frac{x}{z} + 443.86$$

$$x_i \in X \text{ paxis} \Rightarrow 1016 \frac{x}{z} + 443.86 \in X$$

$\therefore$ $\boxed{y = 625.91}$ is the equation of the horizon line.

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