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University of Bonn
Institute of Computer Science II
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Computational Photography

Assignment Sheet 2

– due by 05 May 2024, 12:15 pm –

Please typeset your solution of the theoretical part in LaTeX and compile a PDF. Use the provided python files for your solution of the practical exercises. Upload a zip archive containing the pdf and both python files to eCampus.

1 Theoretical Part

1.1 *Photon Statistics (11 points)*

Given is a sensor with 10×7 pixels that is uniformly illuminated. Assume that each pixel receives an average of $n = 100$ photons per millisecond, which are perfectly counted (quantum efficiency $QE = 100\%$). After exposing the sensor for an integration time T, the photon count at each pixel follows a Poisson distribution,

$$p(x = k) = \frac{(nT)^k}{k!} e^{-nT}.$$

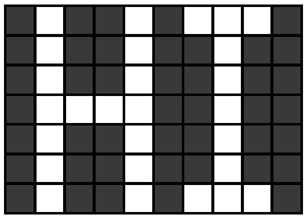


Figure 1: How likely will random photon noise result in this image?

a) Compute the probability that exactly N ($0 \leq N \leq 70$) arbitrary pixels receive no photon during a 0.01 millisecond exposure. (3 points)

b) What is the probability of seeing exactly the image in Figure 1 after 0.01 millisecond exposure? (black pixels = 0 photons; white pixels = 1 or more photons) (3 points)

c) Prove that the mean value (the expectation) of the Poisson distribution, $m = E(x)$, is nT, and that the variance $\sigma^2 = E\left[(x - E(x))^2\right]$ is also nT. (5 points)

1.2 *Rolling Shutter (6 points)*

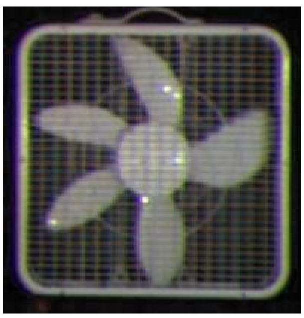
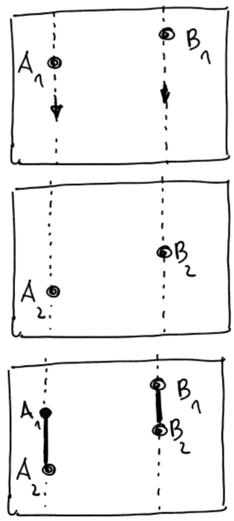


Figure 2: A rotating fan as seen by a rolling shutter camera.

a) It is known that the fan shown in Figure 2 rotates clockwise. What is the direction of shutter movement? Explain your choice. (2 points)

On the right, we see two frames showing two balls A and B which are freely falling in front of the camera. It is known that A and B were released simultaneously from the same height.



b) What is the direction of shutter movement? (2 points)
c) Do you think $A_1A_2 = B_1B_2$? If not, which segment is larger - and why? (2 points)

2 Practical Part

2.1 *Noisy Images (3 points)*

Load the image `yosemite.png` in grayscale and add Gaussian and Poisson distributed noise to it using numpy functions. Experiment with different standard deviations of the Gaussian and compare the results. Take care of proper value ranges when plotting the noisy images.

2.2 *Skellam Separation (12 points)*

When subtracting two images I_1 and I_2 that contain Poisson distributed shot noise, the shot noise of the difference image is characterized by a so called Skellam distribution. For the individual images, according to the Poisson characteristics, the mean value of each pixel equals its variance, thus

$$I_{1,2} = (\sigma_{1,2})^2 = \mu_{1,2}. \tag{1}$$

For the difference image that means that the mean and variance of each pixel is formed as

$$\begin{aligned} \mu_{\text{diff}} &= \eta\,(\mu_1 - \mu_2) \\ \sigma^2_{\text{diff}} &= \eta^2\left(\sigma_1^2 + \sigma_2^2\right) \end{aligned} \tag{2}$$

where η is a hardware parameter of the difference imager.

The images `parallel.png` and `perp.png` show the same scene illuminated with differently polarized light. In the first case, the polarization angle is parallel to an analyzing polarization filter on the camera, in the second case it's perpendicular. With suitable hardware it is possible to directly measure the difference of the two images.

a) Create 100 instances of the given images and add Poisson distributed noise to each of them independently and subtract them, thus simulating the acquisition of 100 difference images of the same scene that differ only in random shot noise.
b) Calculate the mean and variance values for each pixel over the 100 instances of the difference image.
c) Rewrite equations (2) as a matrix-vector product and solve for the vector $(\mu_1, \mu_2)^{\mathrm{T}}$. Use the resulting relation to reconstruct the input images for perpendicular and parallel illumination from the difference image noise statistics. Use $\eta = 1.1$.