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Discrete and Computational Geometry

Winter semester 2024/2025

Assignment 10

Based on Lecture 17 and Lecture 18

Problem 1: (5 Points)

Consider the points $p_1 = (0.05, 0.68)$, $p_2 = (0.07, 0.68)$, $p_3 = (0.12, 0.55)$, $p_4 = (0.3, 0.36)$, $p_5 = (0.63, 0.01)$, $p_6 = (0.68, 0.01)$ as depicted in Figure 1 on the next page. Find the WSPD of this point set with separation ratio $s = 3$, which is obtained by the construction algorithm presented in class (lecture 18).

Problem 2: (8 Points)

For any $p \in \mathbb{R}^2$, $r > 0$, let $B(p, r)$ be the Euclidean ball of radius r, centered at p. Consider a set P of n points in $\mathbb{R}^2$, stored in a compressed quadtree. Design an algorithm which, given a query point $q \in \mathbb{R}^2$, radius $r > 0$, and approximation parameter $1 \geq \epsilon > 0$, returns an integer m such that

$$|B(q, r) \cap P| \leq m \leq |B(q, (1 + \epsilon)r) \cap P|.$$

You can assume that the compressed quadtree of P stores for each node the number of leaves of its subtree. The query algorithm must be adaptive to ϵ, meaning that larger values of ϵ should lead to faster running time. Analyse the running time of your algorithm.

Problem 3: (5 Points)

Prove the following statement (Lemma 18.5): Let P be a set of n points in $\mathbb{R}^d$. For any two distinct points $p, q \in P$, the algorithm WellSeparatedPairDecomposition from the lecture outputs exactly one s-well-separated pair that covers $\{p, q\}$.

Problem 4: (5 Points)

Define the weight of a WSPD $\{\{A_1, B_1\}, \ldots, \{A_m, B_m\}\}$ as the quantity

$$\sum_{i=1}^{m} |A_i| + |B_i|$$

Show that there exists a set of n points in $\mathbb{R}^1$, such that any WSPD with separation ratio at least 2 has weight $\Omega(n^2)$.

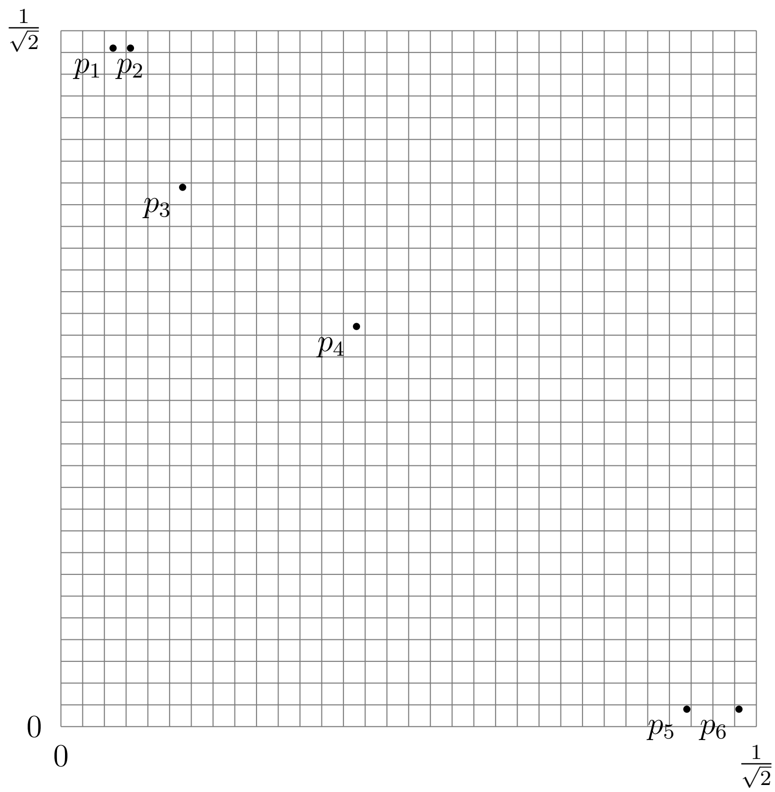

Figure 1