

Discrete and Computational Geometry

Winter semester 2024/2025

Assignment 2

Problem 1: (6 Points)

We consider n points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ in \mathbb{R}^2 which are chosen uniformly and independently at random from $[0, 1]^2$, i.e. each coordinate is chosen uniformly and independently at random from $[0, 1]$. Show that the expected number of vertices of the convex hull is $O(\log n)$.

Hint: You may use that all sorted orders of the x_i (resp. y_i) are equally likely.

Problem 2: (6 Points)

A *line transversal* for a set of *vertical* line segments in the plane is a line that intersects every segment. Show that if $I_1, \dots, I_n, n \geq 3$ are vertical segments in the plane such that every three of them have a line transversal, then all of them have a line transversal.

Problem 3: (8 Points)

Let A be a set of n points on the plane. Given a point q on the plane, describe an algorithm that computes $\text{depth}_A(q)$ in $O(n \log n)$ time. Show the correctness and analyse the running time of your algorithm.