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Discrete and Computational Geometry

Winter semester 2024/2025

Assignment 1

Problem 1: (6 Points)
Show that any set of $d+1$ points in $\mathbb{R}^d$ is linearly dependent.

Problem 2: (4+4 Points)
In the lecture, it was stated that each hyperplane can be represented either as the image of an affine mapping or as a solution of a linear equation. We are now interested in how to transform one representation into the other. Let $f : \mathbb{R}^{d-1} \to \mathbb{R}^d$ be an affine mapping with $f : y \to By + c$ where B is a $d \times (d-1)$ matrix and $c \in \mathbb{R}^d$. Let further $a \in \mathbb{R}^d \setminus \{0\}$ and $b \in \mathbb{R}$ such that

$$\text{Image}(f) = \{x \in \mathbb{R}^d \mid \langle a, x \rangle = b\}.$$

i) Given B and c, we want to find suitable a and b. Construct a system of $(d-1)$ linear equations that can be solved to determine a. How can we determine b given B, c and a?
ii) Given a and b, find suitable B and c.

Problem 3: (2+4 Points)

i) For the following sets determine if they are open or closed or both or neither:
(a) the empty set, (b) the interval $(0, 1]$, (c) the interval $[0, \infty)$

ii) Find an example of two disjoint closed convex sets that are not strictly separable.

Problem 4: (4+2 Points)
Each set $X \subset \mathbb{R}^2$ of diameter at most 1 (i.e., any 2 points have distance at most 1) is contained in some disc of radius $\frac{1}{\sqrt{3}}$.

i) Prove the statement for 3-element sets X.
(**Hint**: One could use the law of sines to prove this (Wikipedia))

ii) Prove the statement for all finite sets X.