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Discrete and Computational Geometry

Winter semester 2024/2025

Assignment 7

CITE YOUR SOURCES!

Problem 1: (7 Points)

Suppose you are given an arrangement $A(L)$ of the set L of lines in the plane. Consider a line l not in L and the faces of $A(L)$ that it intersects. The set of these faces is called the 'zone' of l. The complexity of a face is the number of edges on its boundary. The complexity of a zone is the sum of the complexities of the faces in the zone.

Assume l is horizontal and that the set L is in general position (no two lines parallel, no three intersect at a point).

(Note that $O(n^2)$ is easy to see—there are $O(n)$ faces and each could have at most $O(n)$ complexity.)

(a) Consider the north-west chain NW of some face F in the zone of l (assuming that the face is oriented counter-clockwise, the chain starting at the topmost vertex of F and ending at the leftmost). There is one line l_t in L that may form the topmost edge of NW (that is, incident upon the highest vertex).

Apart from l_t, the chain NW may have "non-top" edges formed by other lines from L. Show that any of these lines cannot contribute to the complexity of the north-west chain of any other face in the zone of l.

(b) Using part (a) or otherwise, prove that zone complexity of l is in $O(n)$.

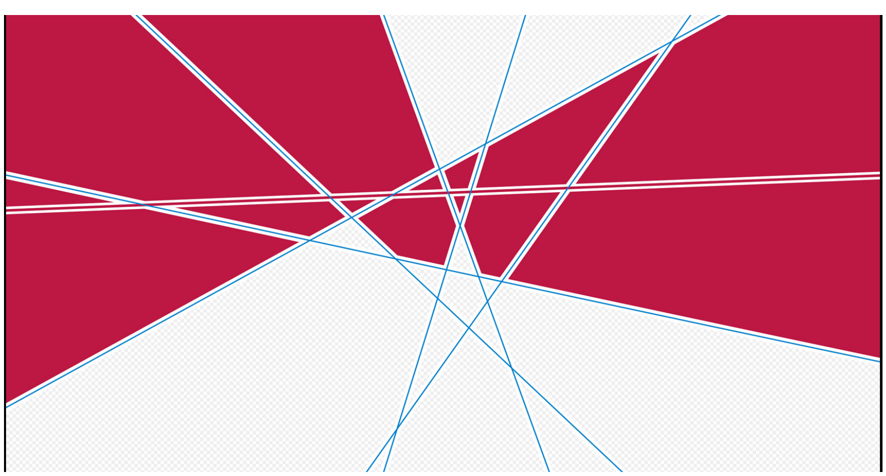

Figure 1: The faces in the zone of the red line (with respect to the arrangement of the blue lines) are shaded red.

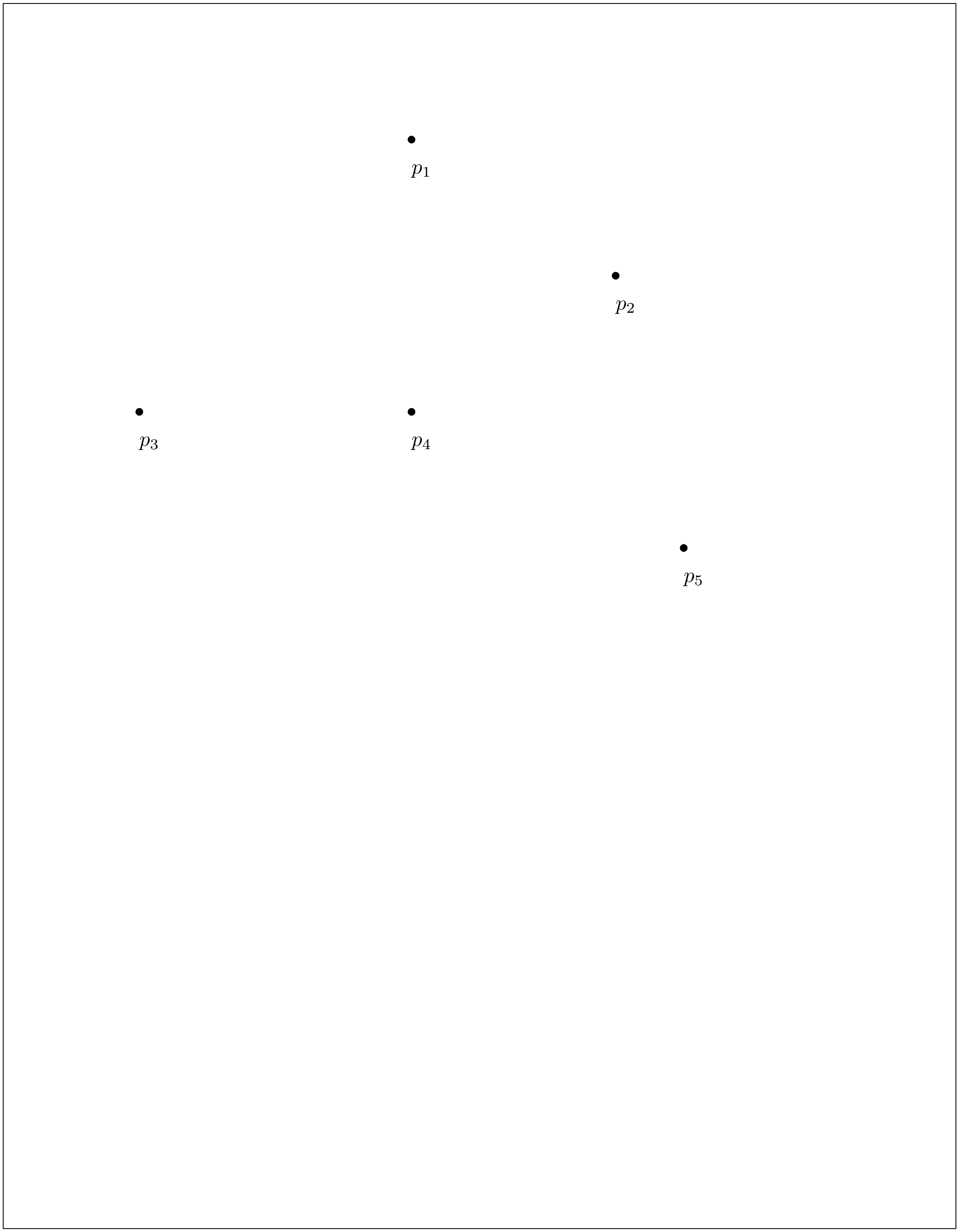

Figure 2: Set of 5 points in the plane.

Problem 2: (5 Points)

Consider the example of 5 points in Figure 2 on the next page. Draw the Voronoi diagram of order $k = 2$. Which subsets of points have a non-empty Voronoi region?

Problem 3: (5 Points)

Let P be a set of n points in the plane in general position (no three points on a line, no four points on a circle). Show that the graph of the order-$(n-1)$ Voronoi diagram of P has $O(n)$ Voronoi vertices and Voronoi edges.