Juergen Gall

RANSAC and Image Alignment
MA-INF 2201 - Computer Vision
WS24/25

Motivation for feature-based alignment: Image mosaics

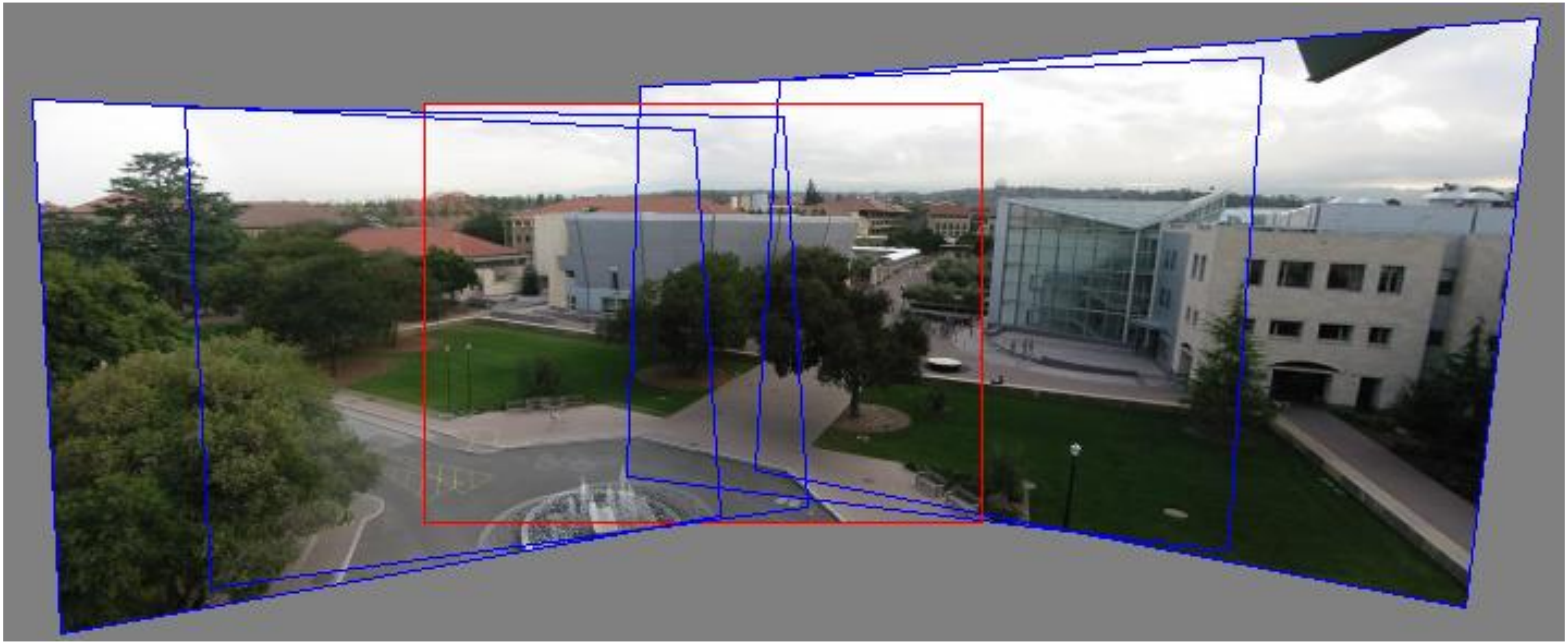
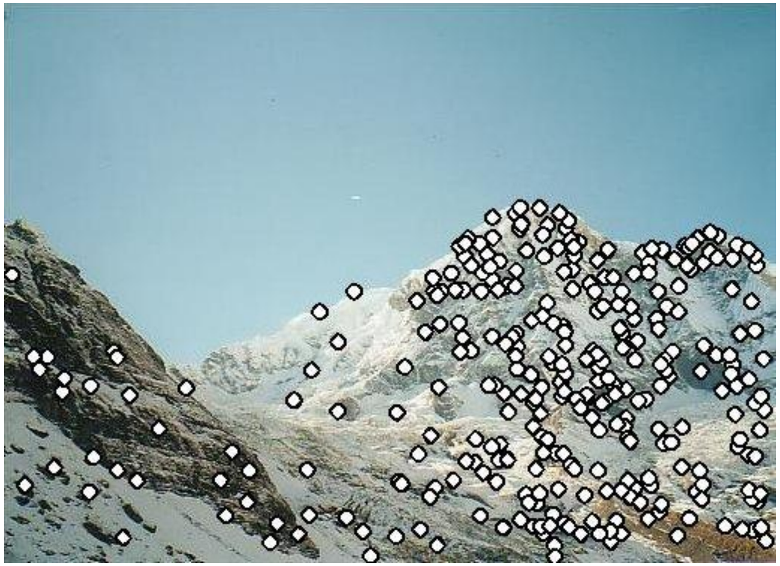
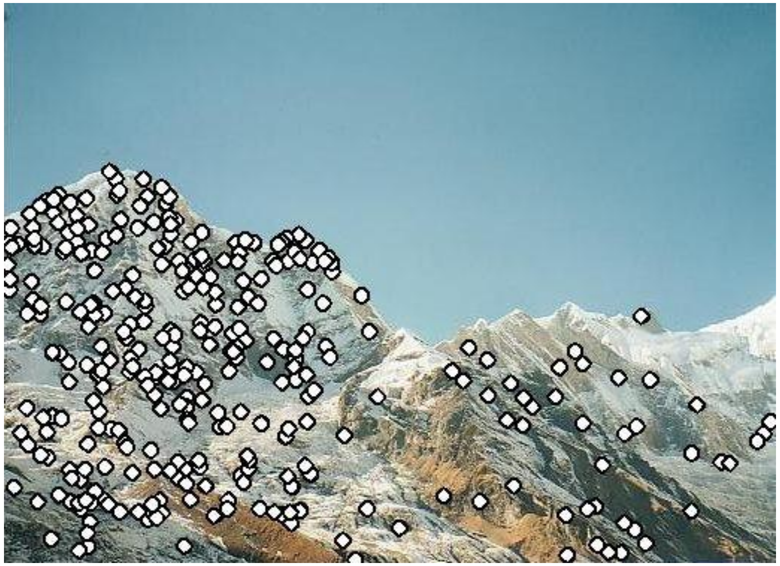


Image from http://graphics.cs.cmu.edu/courses/15-463/2010_fall/

Robust feature-based alignment





- Extract features

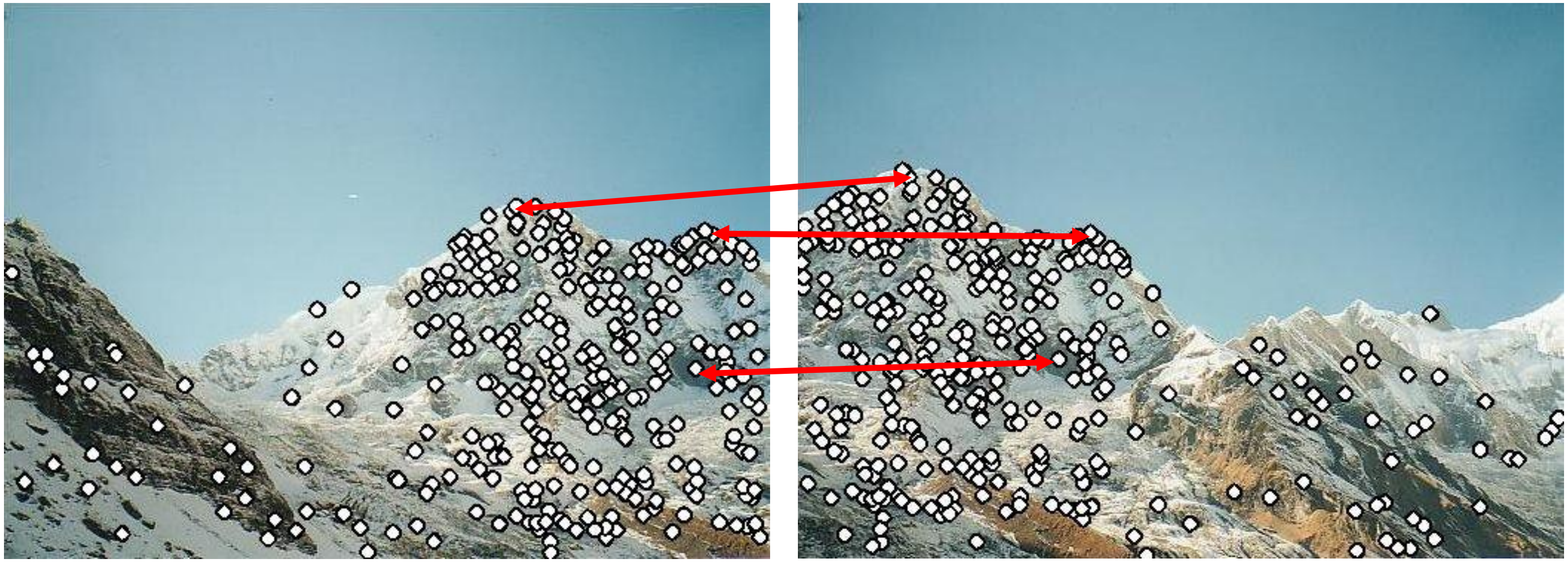
Source: L. Lazebnik

Robust feature-based alignment

- Extract features
- Compute *putative matches*

Source: L. Lazebnik

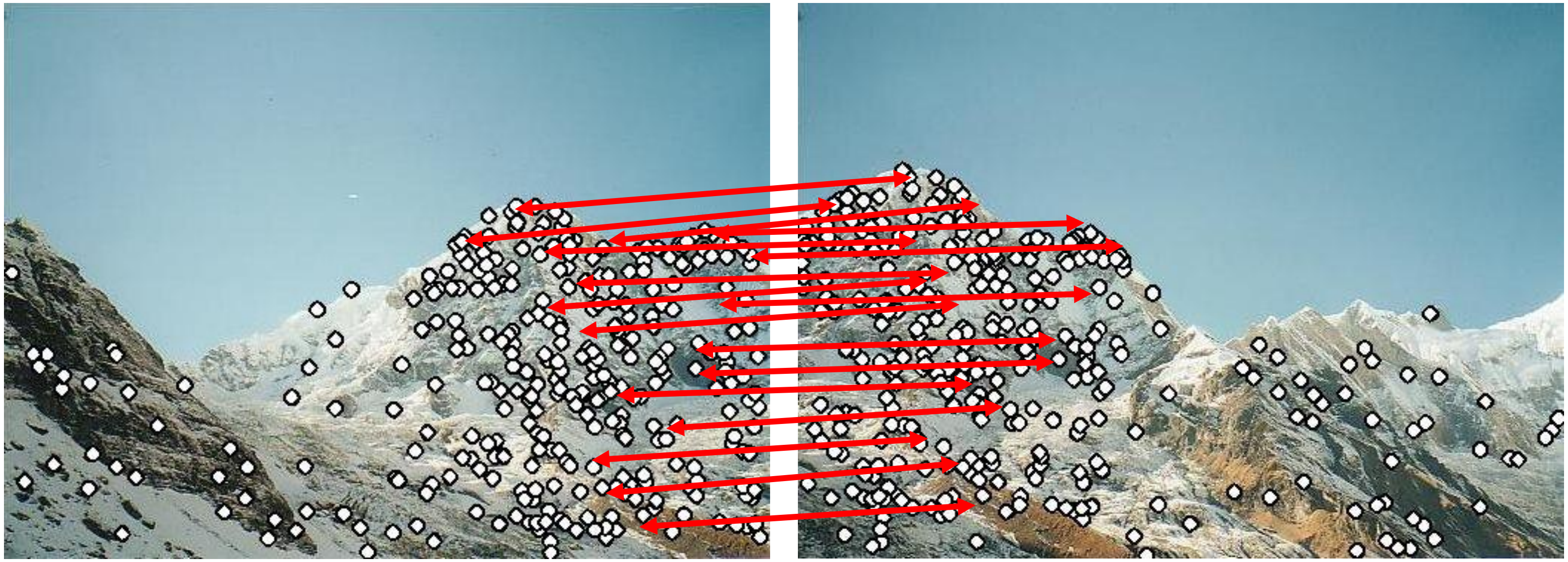
Robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)

Source: L. Lazebnik

Robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)
  - *Verify* transformation (search for other matches consistent with *T*)

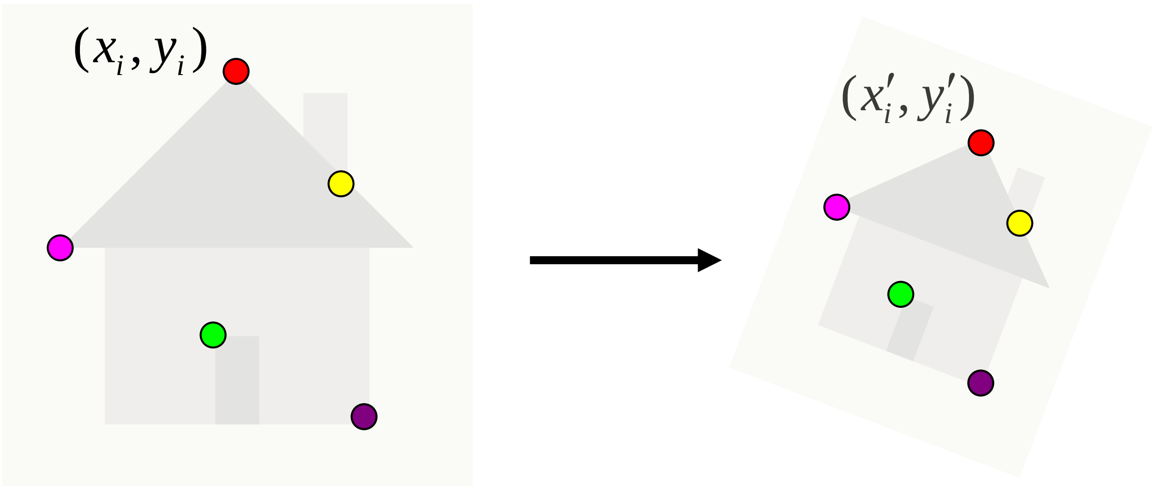
Source: L. Lazebnik

Robust feature-based alignment

- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)
  - *Verify* transformation (search for other matches consistent with *T*)

Source: L. Lazebnik

Recall: Fitting an affine transformation



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

Source: K. Grauman

Recall: Least squares problem

A *m x n* matrix of rank *n*:

$$Ax = b$$

Solution with pseudo-inverse:

$$A^T(Ax - b) = 0$$

$$x = (A^T A)^{-1} A^T b$$

Pseudo-inverse $\mathbf{A}^+$

Solution with SVD:

- SVD: $\mathbf{A} = \mathbf{UDV}^T$; $\mathbf{A}^+ = \mathbf{VD}^{-1}\mathbf{U}^T$
- $\mathbf{b}' = \mathbf{U}^T\mathbf{b}$
- $y_i = b'_i / D_{ii}$
- $\mathbf{x} = \mathbf{Vy}$

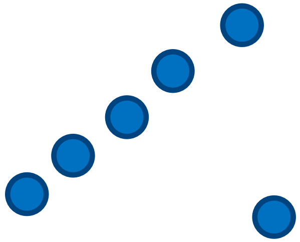
There are different ways to solve the problem. The optimal method depends on size and sparseness of **A**

Specific C++ linear algebra libraries exist!

Outliers

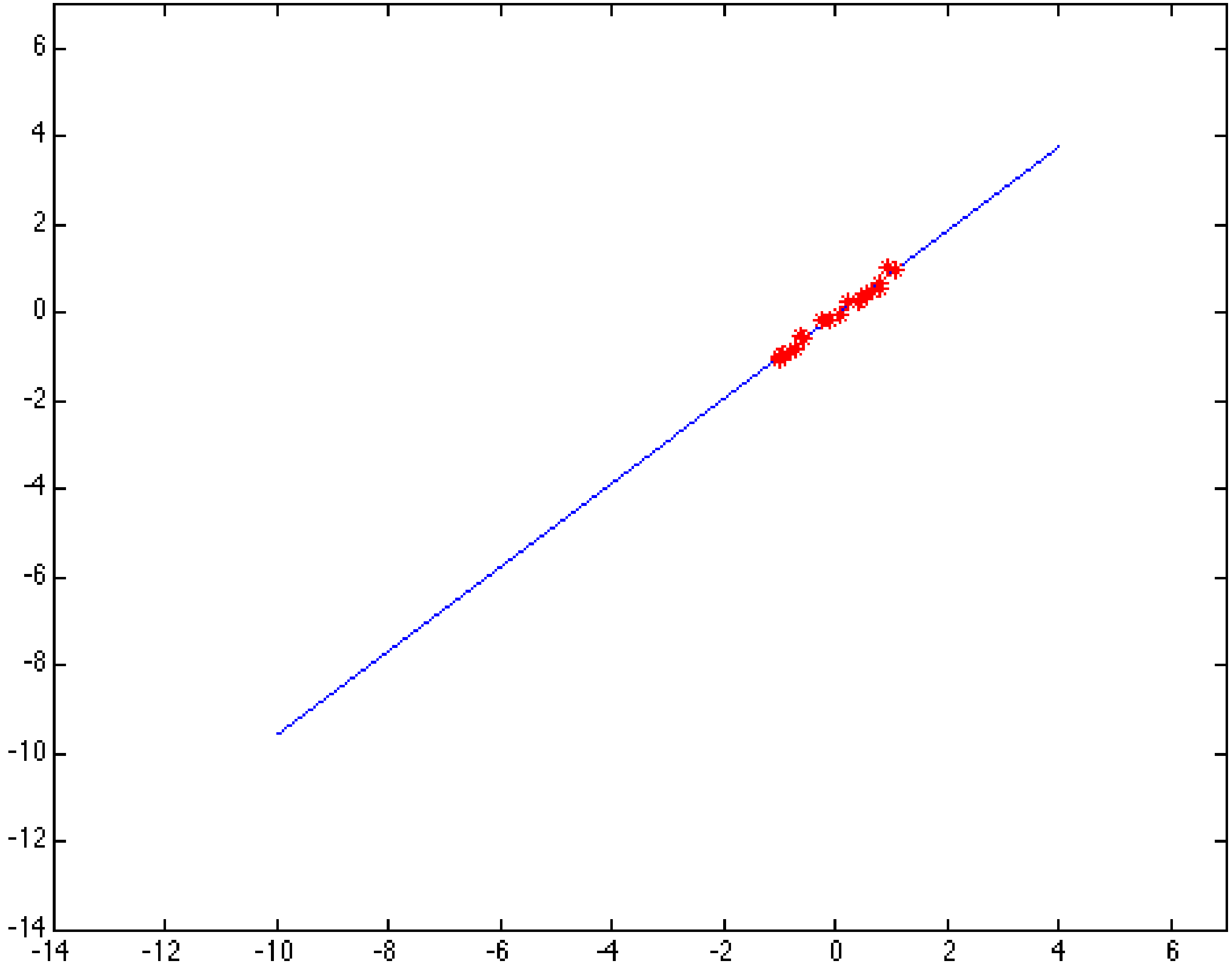
Outliers can hurt the quality of our parameter estimates, e.g.,

- an erroneous pair of matching points from two images
- an edge point that is noise, or doesn't belong to the line we are fitting.



Kristen Grauman

Outliers affect least squares fit



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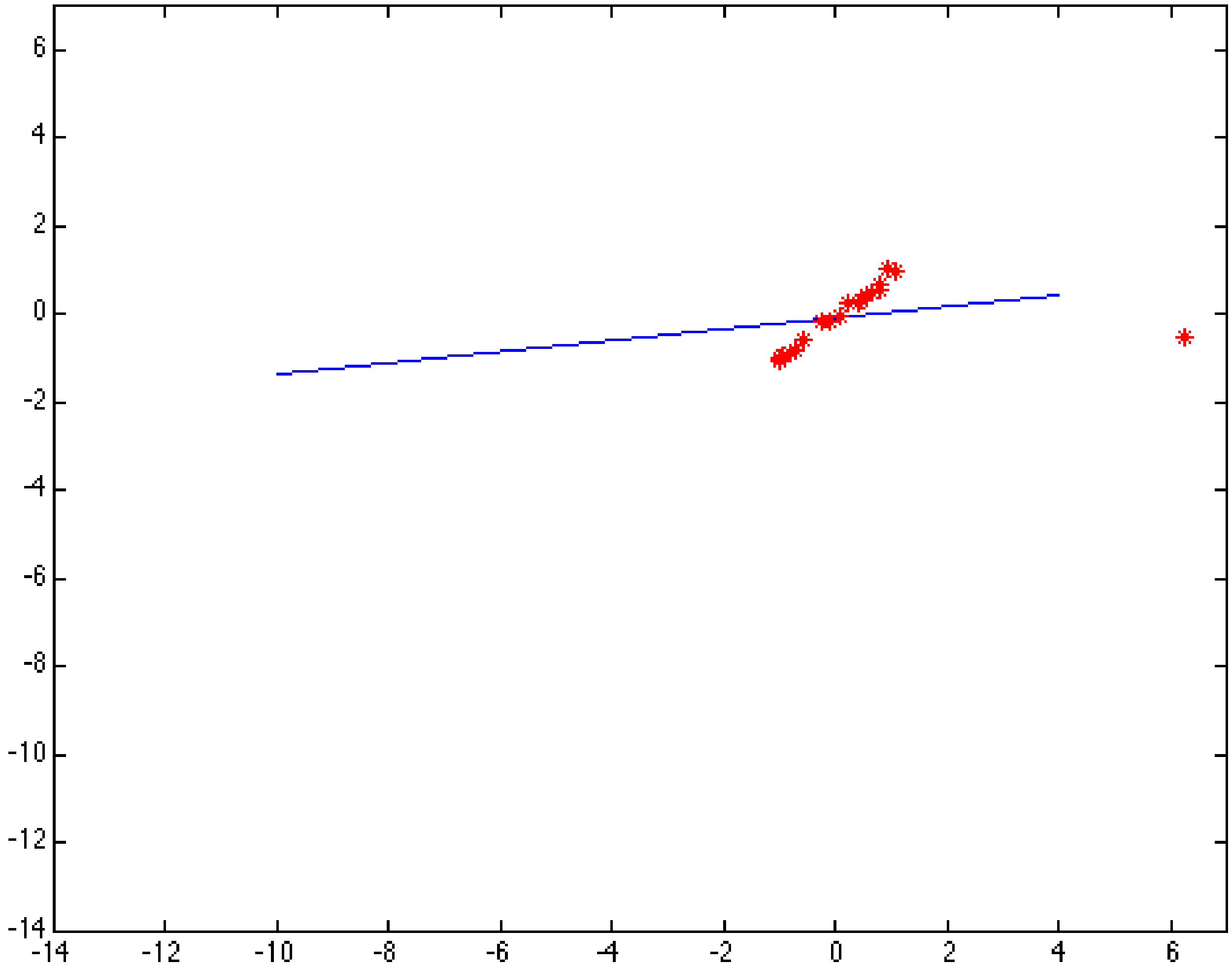
Outliers affect least squares fit



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RANdom Sample Consensus (RANSAC)

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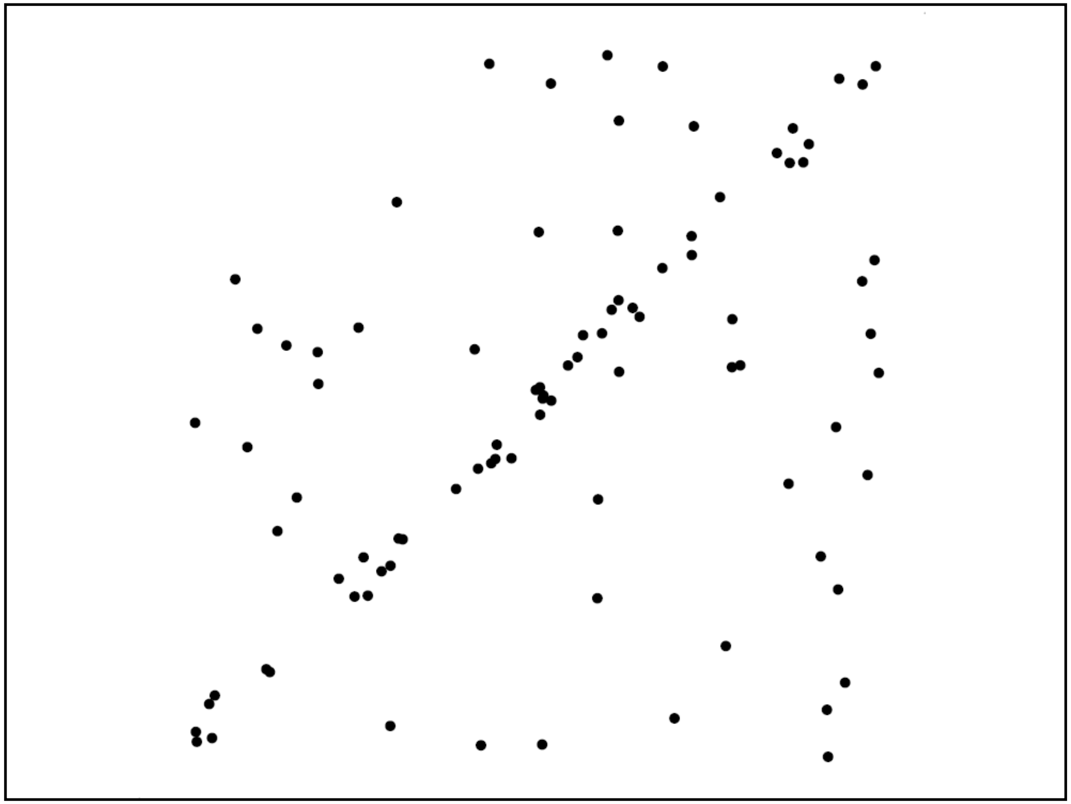
RANSAC loop:

1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute estimate of transformation on all of the inliers

Keep the transformation with the largest number of inliers

Kristen Grauman

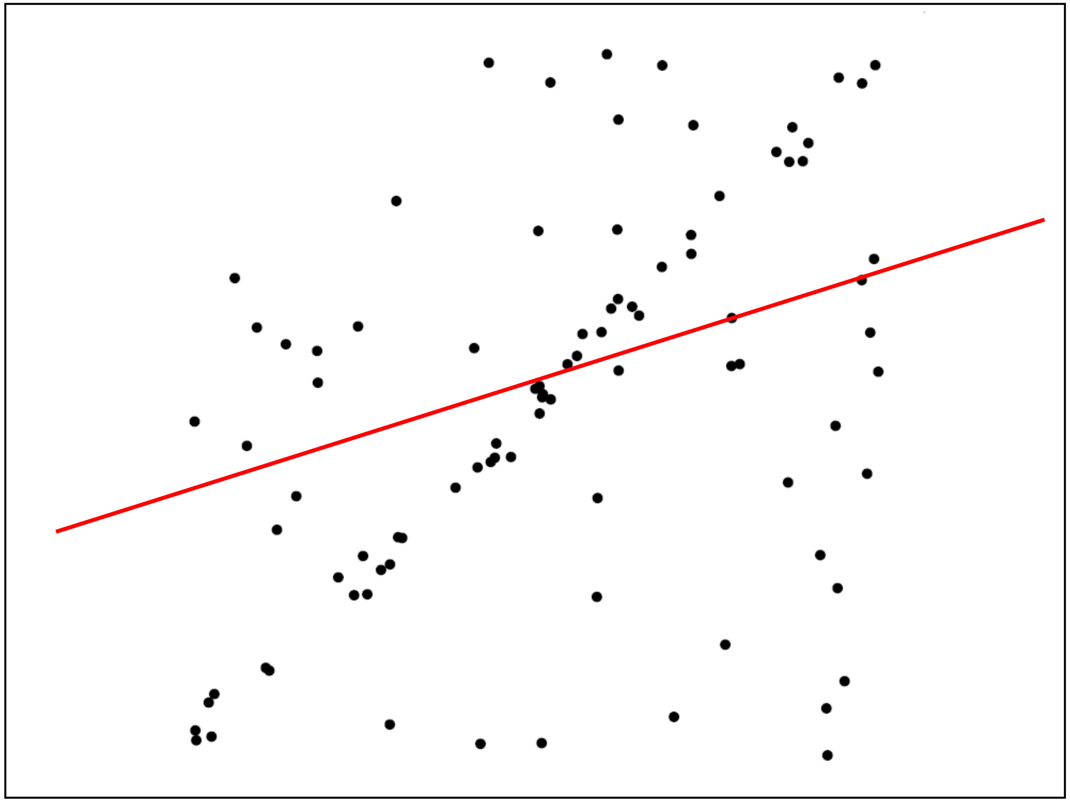
RANSAC for line fitting example



Source: R. Raguram

Lana Lazebnik

RANSAC for line fitting example



Least-squares fit

Source: R. Raguram

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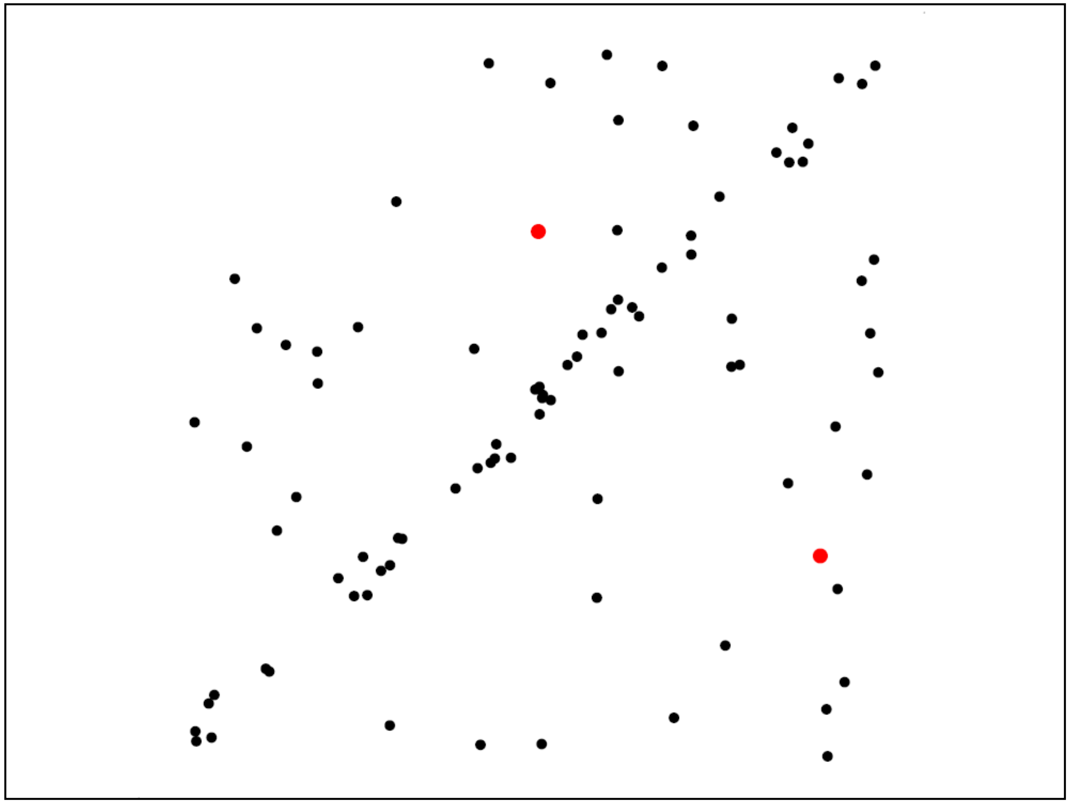
RANSAC for line fitting example



1. Randomly select minimal subset of points

Source: R. Raguram

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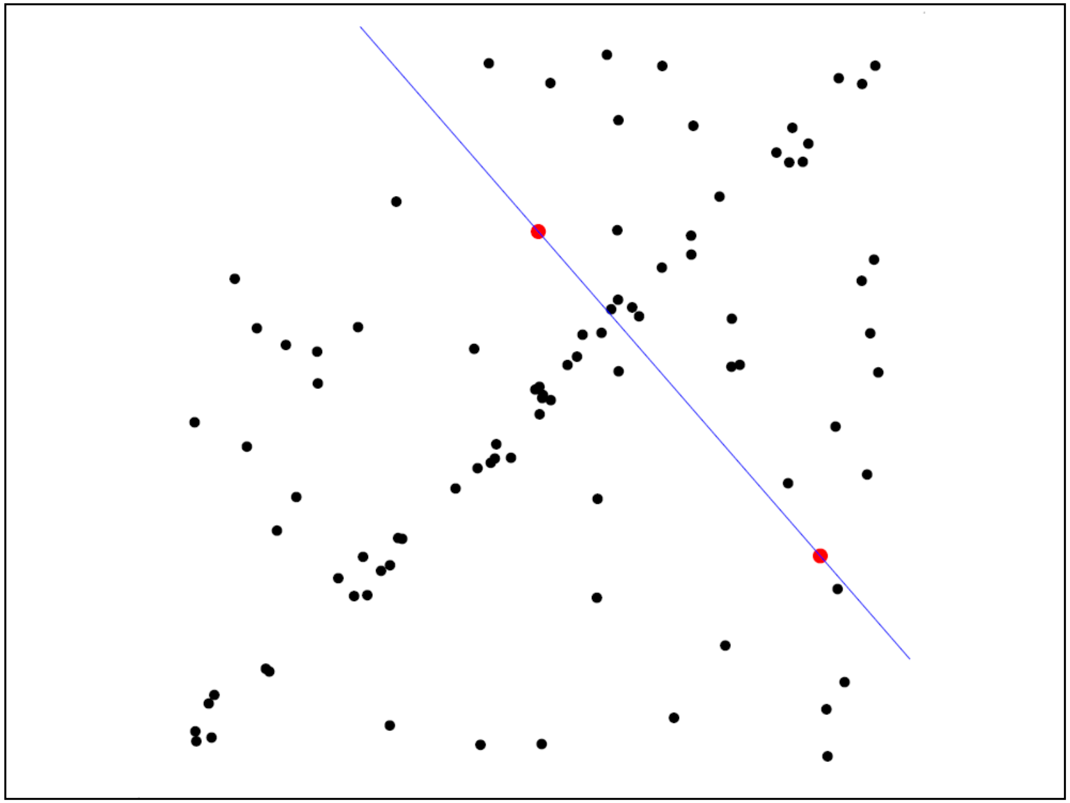
RANSAC for line fitting example

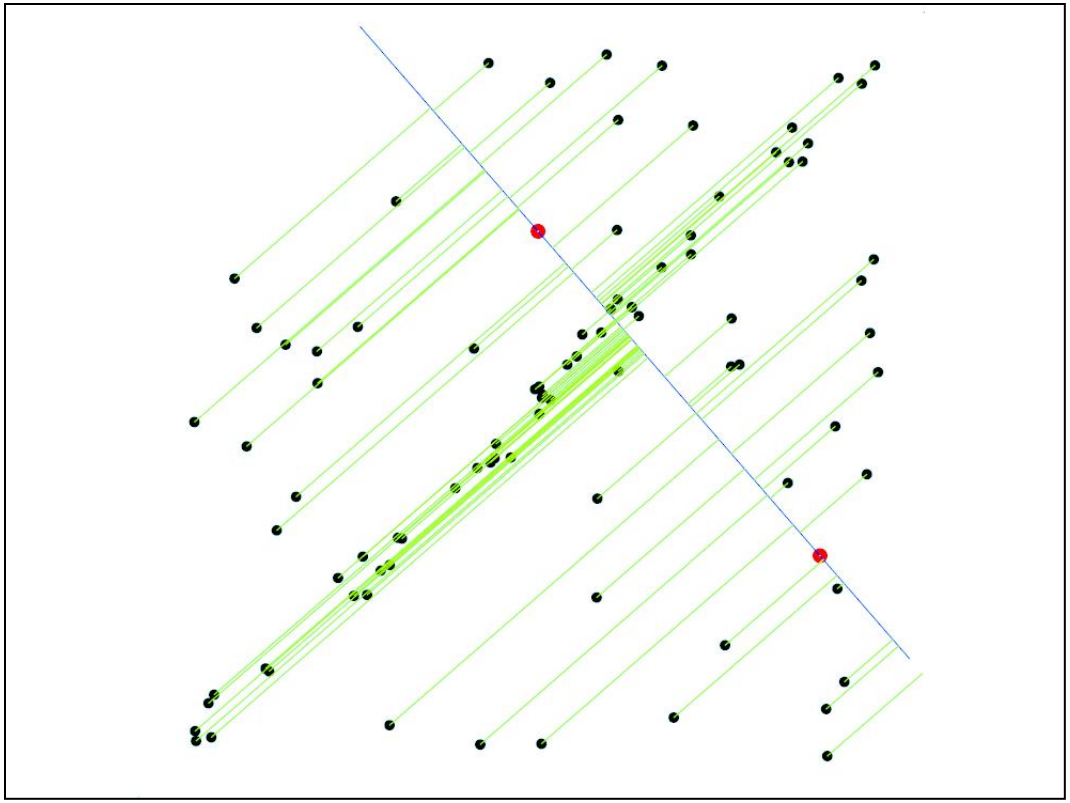


1. Randomly select minimal subset of points
2. Hypothesize a model

Source: R. Raguram

Lana Lazebnik

RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function

Source: R. Raguram

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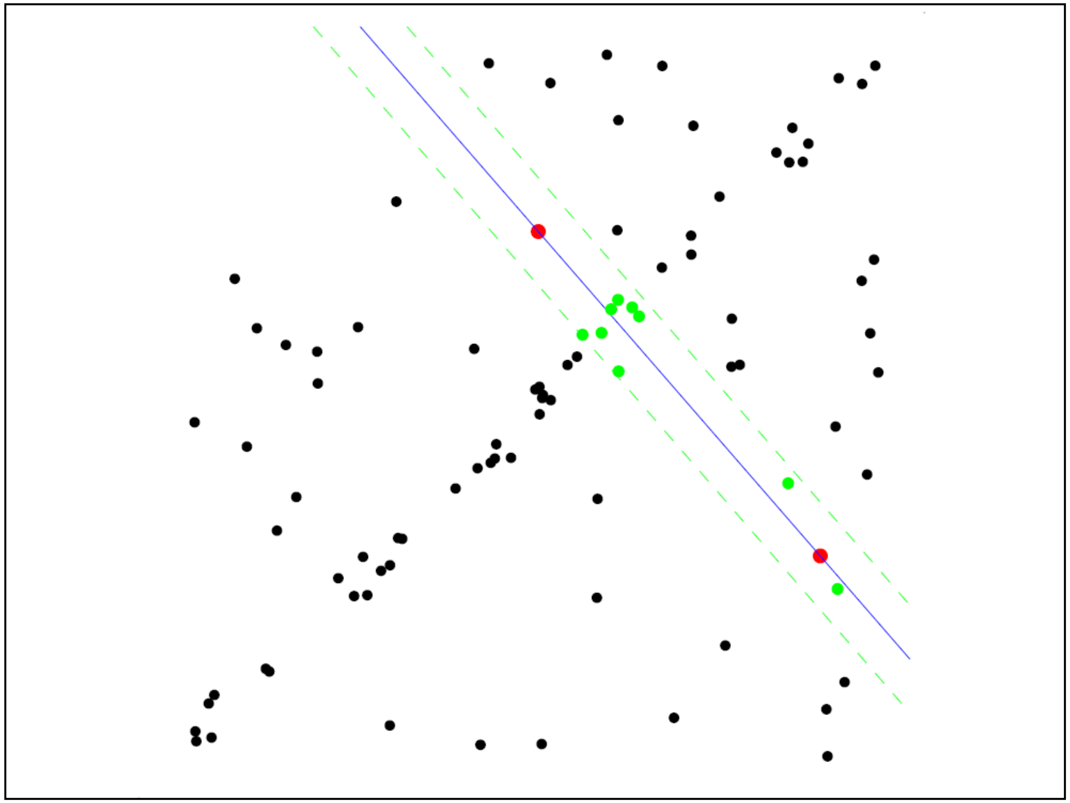
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model

Source: R. Raguram

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RANSAC for line fitting example

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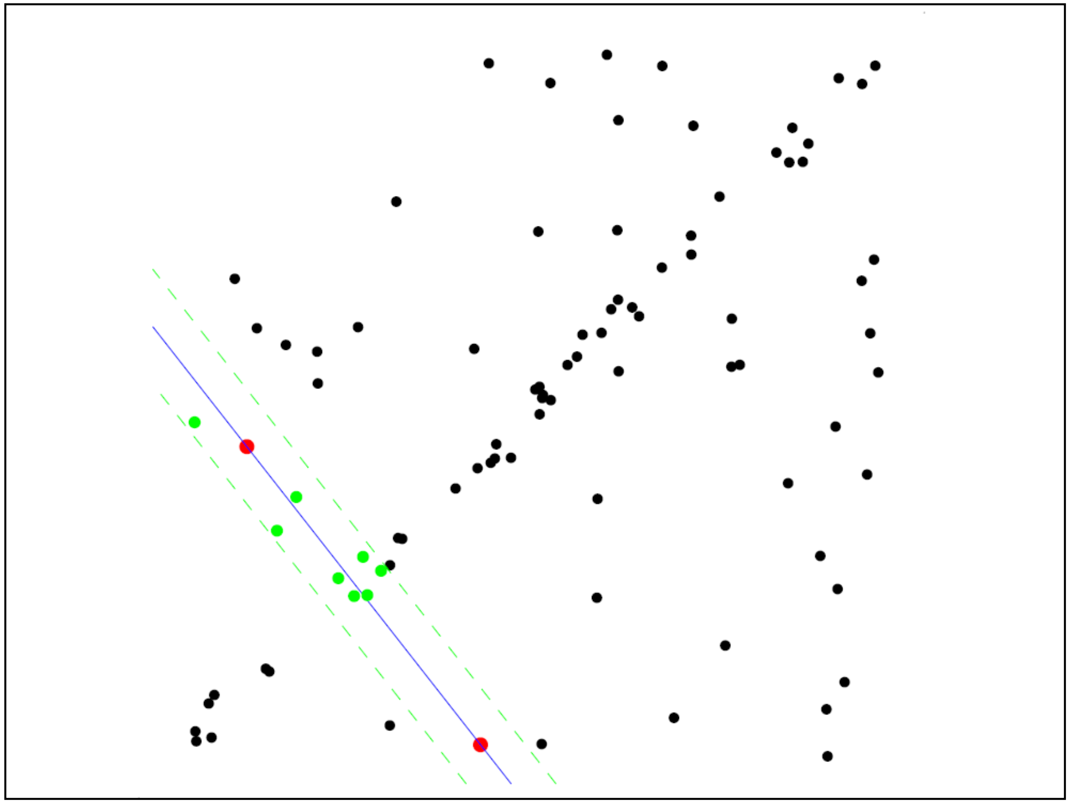

1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

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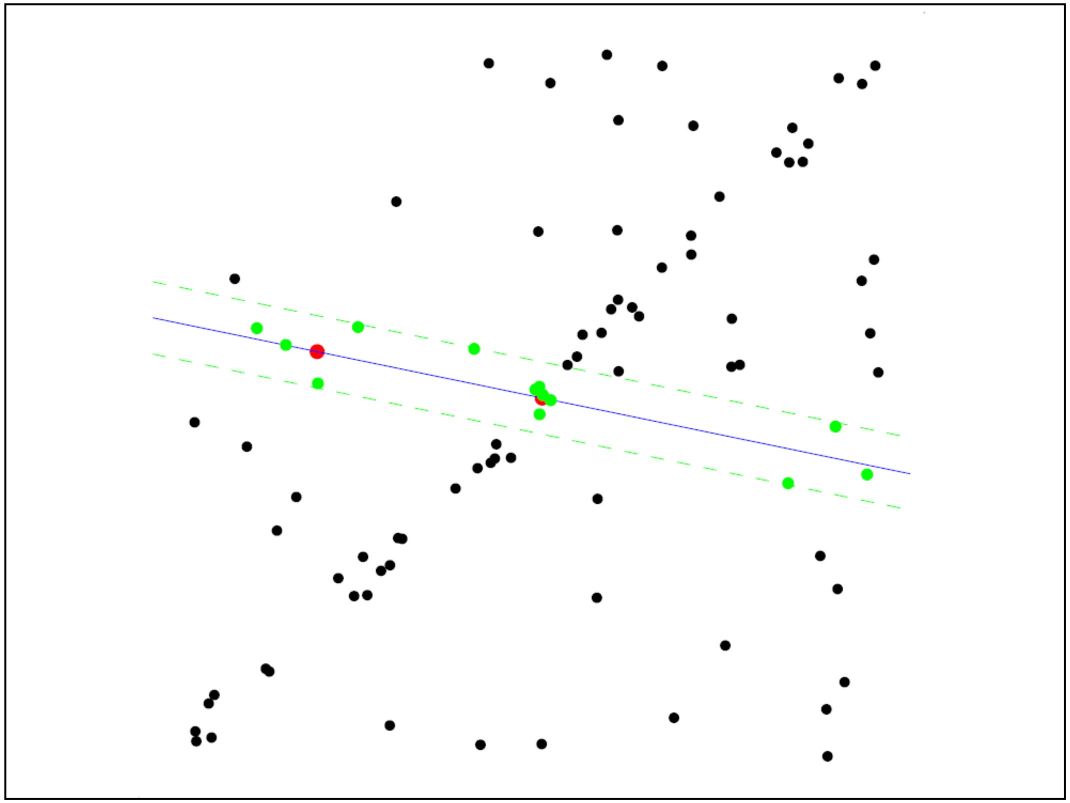
RANSAC for line fitting example



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Source: R. Raguram

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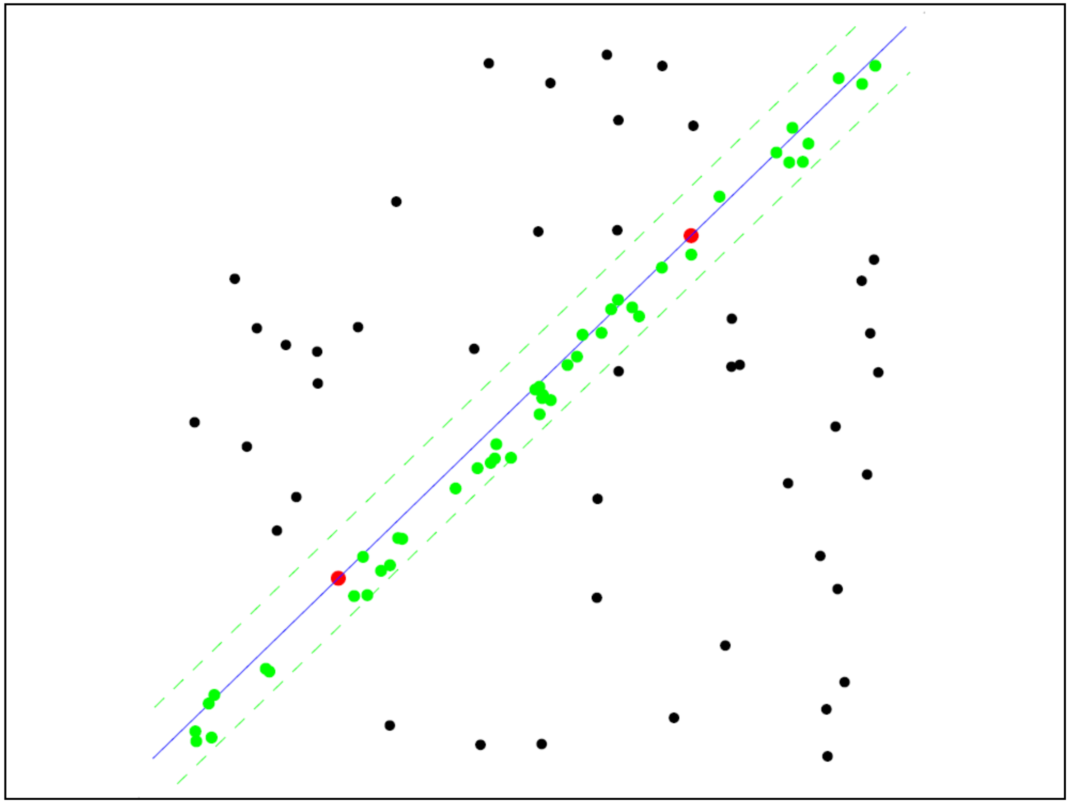
RANSAC for line fitting example

Uncontaminated sample



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4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

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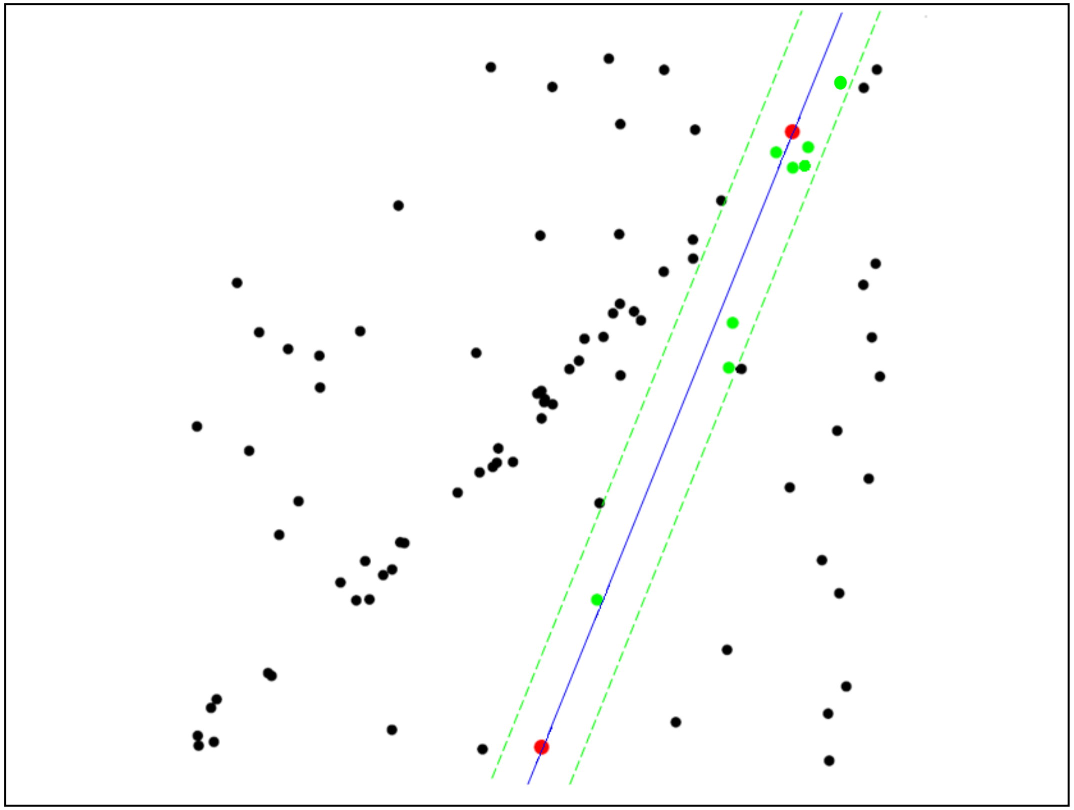
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
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5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

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RANSAC for line fitting

Repeat ***N*** times:

- Draw ***s*** points uniformly at random
- Fit line to these ***s*** points
- Find inliers to this line among the remaining points (i.e., points whose distance from the line is less than ***t***)
- If there are ***d*** or more inliers, accept the line and refit using all inliers

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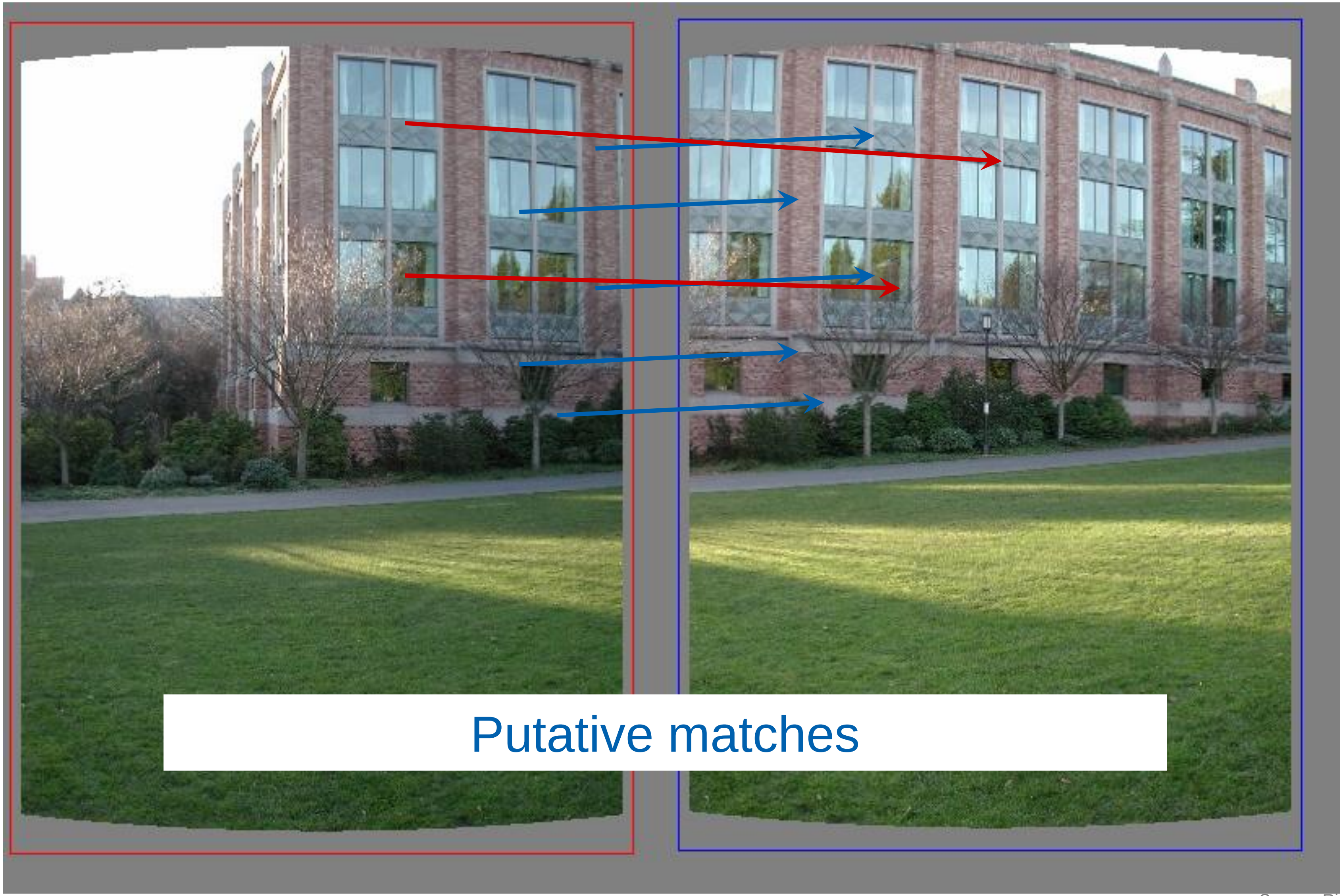
RANSAC example: Translation



Source: Rick Szeliski

RANSAC example: Translation

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RANSAC example: Translation

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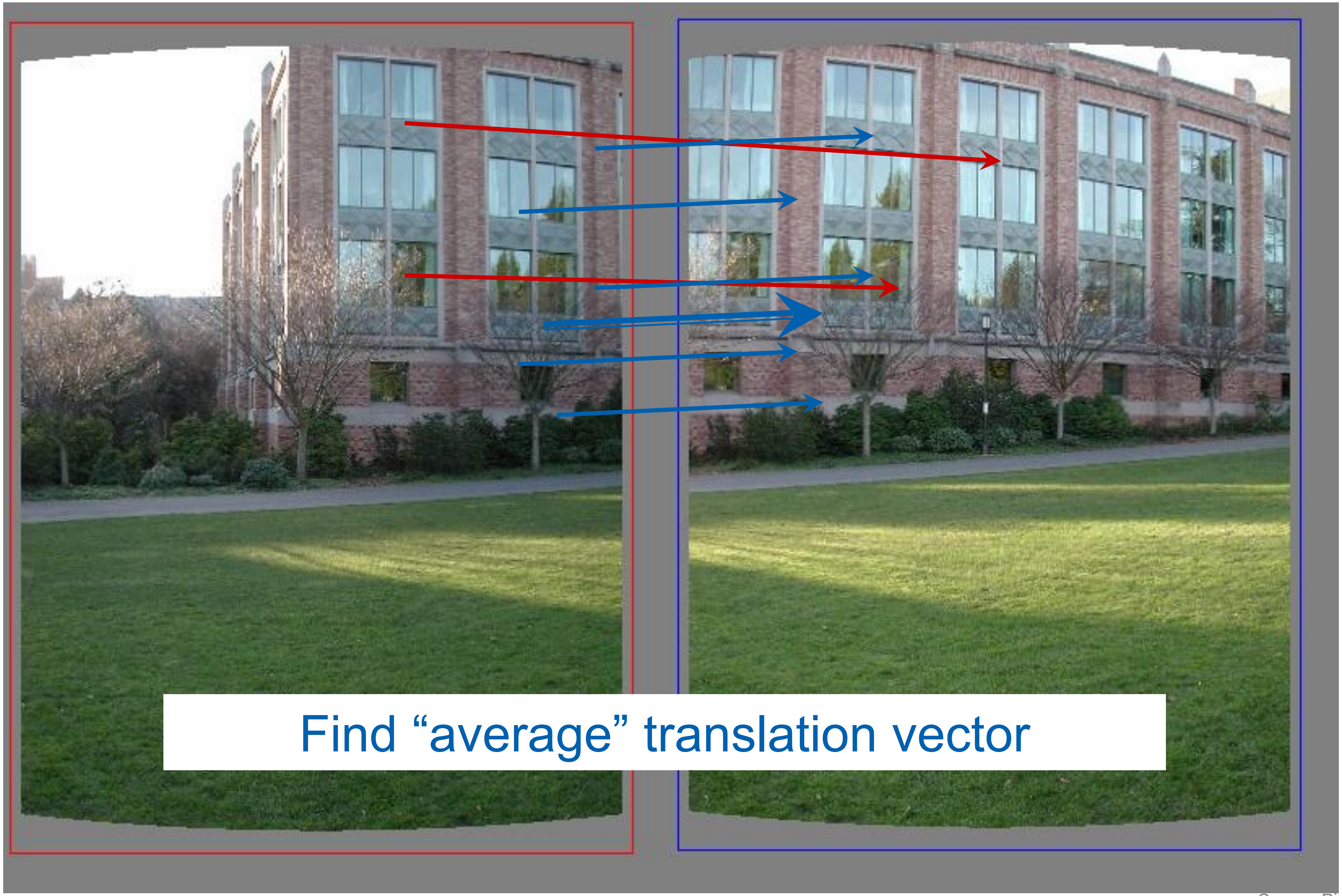
Source: Rick Szeliski

RANSAC example: Translation



Source: Rick Szeliski

How Many Samples?

On average

N	… number of point
I	… number of inliers
m	… size of the sample

P(good) = $$\frac{\binom{I}{m}}{\binom{N}{m}} = \prod_{j=0}^{m-1} \frac{I-j}{N-j}$$

mean time before the success

E(*k*) = 1 / P(good)

Ondřej Chum

How Many Samples?

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With confidence *p*

How large k?

...to hit at least one pair of points on the line l with probability <u>larger</u> than p (0.95)

Equivalently

...the probability of not hitting any pair of points on l is $\leq 1 - p$

Ondřej Chum

How Many Samples?

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With confidence *p*

N	… number of point
I	… number of inliers
m	… size of the sample

P(good) = $$\frac{\binom{I}{m}}{\binom{N}{m}} = \prod_{j=0}^{m-1} \frac{I-j}{N-j}$$

P(bad) = 1 – P(good)

P(bad *k* times) = $\left(1 - \text{P(good)}\right)^{k}$

Ondřej Chum

How Many Samples?

With confidence *p*

$$P(\text{bad } k \text{ times}) = \big(1 - P(\text{good})\big)^k \leq 1 - p$$

$$k \log \big(1 - P(\text{good})\big) \leq \log(1 - p)$$

$$k \geq \log(1 - p) \,/\, \log \big(1 - P(\text{good})\big)$$

$$P(\text{good}) = \frac{\binom{I}{m}}{\binom{N}{m}} = \prod_{j=0}^{m-1} \frac{I - j}{N - j}$$

Ondřej Chum

How Many Samples

I / *N* [%]

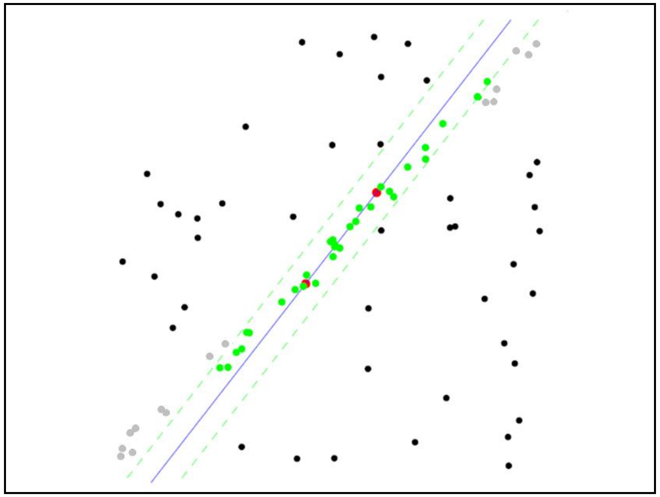
Size of the sample *m*

	15%	20%	30%	40%	50%	70%
2	132	73	32	17	10	4
4	5916	1871	368	116	46	11
7	$1.75 \cdot 10^6$	$2.34 \cdot 10^5$	$1.37 \cdot 10^4$	1827	382	35
8	$1.17 \cdot 10^7$	$1.17 \cdot 10^6$	$4.57 \cdot 10^4$	4570	765	50
12	$2.31 \cdot 10^{10}$	$7.31 \cdot 10^8$	$5.64 \cdot 10^6$	$1.79 \cdot 10^5$	$1.23 \cdot 10^4$	215
18	$2.08 \cdot 10^{15}$	$1.14 \cdot 10^{13}$	$7.73 \cdot 10^9$	$4.36 \cdot 10^7$	$7.85 \cdot 10^5$	1838
30	∞	∞	$1.35 \cdot 10^{16}$	$2.60 \cdot 10^{12}$	$3.22 \cdot 10^9$	$1.33 \cdot 10^5$
40	∞	∞	∞	$2.70 \cdot 10^{16}$	$3.29 \cdot 10^{12}$	$4.71 \cdot 10^6$

Ondřej Chum

RANSAC pros and cons

- Pros
  - Simple and general
  - Applicable to many different problems
  - Often works well in practice
- Cons
  - Doesn't work well for low inlier ratios (too many iterations, or can fail completely)
  - Can't always get a good initialization of the model based on the minimum number of samples



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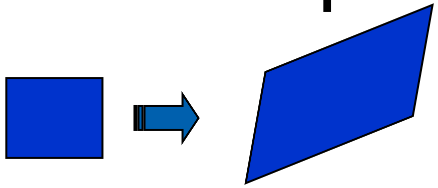
Recall: 2D Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Affine transformations are combinations of …
  - Linear transformations, and
  - Translations

- Parallel lines remain parallel



Source: K. Grauman

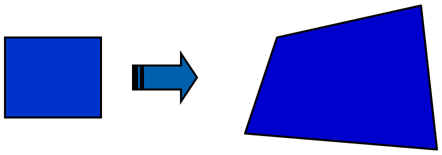
Projective Transformations

Projective transformations:

- Affine transformations, and
- Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Parallel lines do not necessarily remain parallel



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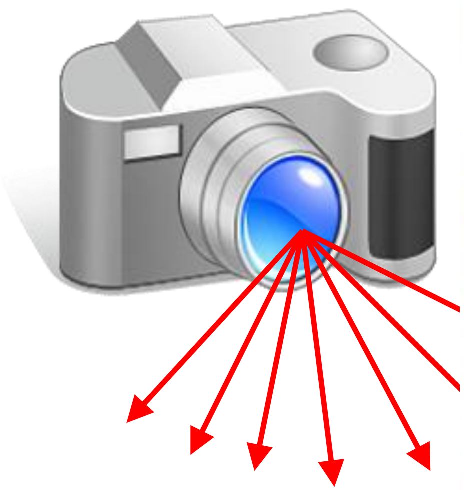
Mosaics



. . .

image from S. Seitz

Obtain a wider angle view by combining multiple images.

Kristen Grauman

How to stitch together a panorama (a.k.a. mosaic)?

Basic Procedure

- Take a sequence of images from the same position
  - Rotate the camera about its optical center
- Compute transformation between second image and first
- Transform the second image to overlap with the first
- Blend the two together to create a mosaic
- (If there are more images, repeat)

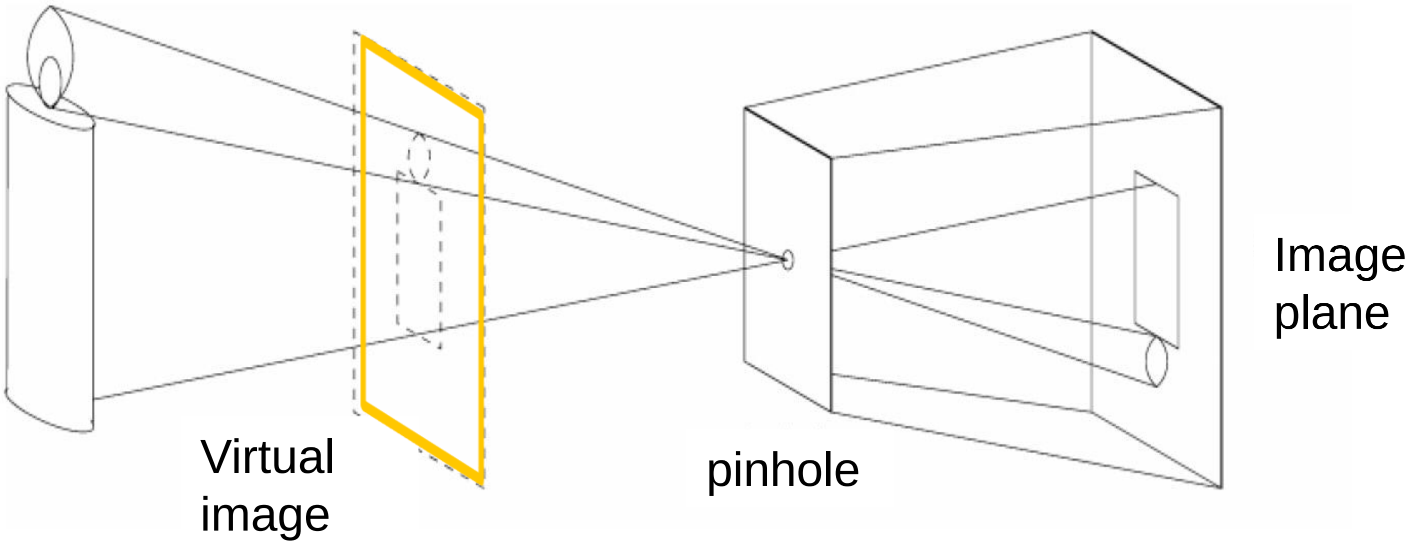
…but **wait**, why should this work at all?

- What about the 3D geometry of the scene?
- Why aren't we using it?

Source: Steve Seitz

Pinhole camera

Pinhole camera is a simple model to approximate imaging process, perspective **projection**.

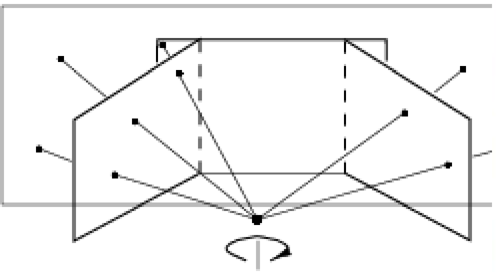


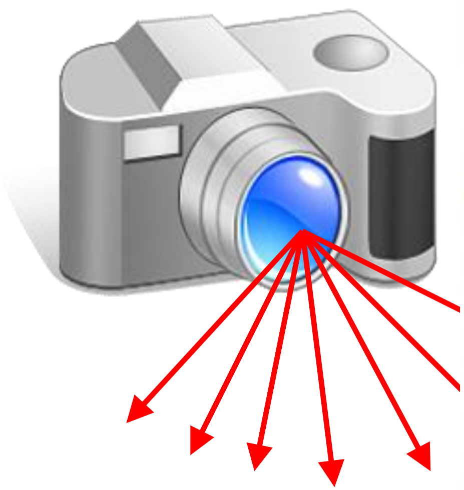
If we treat pinhole as a point, only one ray from any given point can enter the camera.

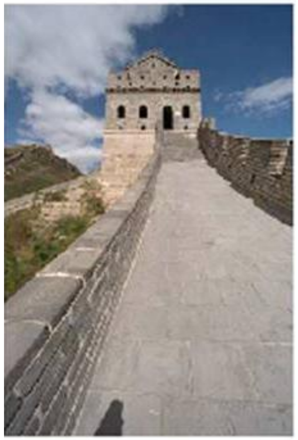
Fig from Forsyth and Ponce

Kristen Grauman

Mosaics







. . .

image from S. Seitz

Obtain a wider angle view by combining multiple images.

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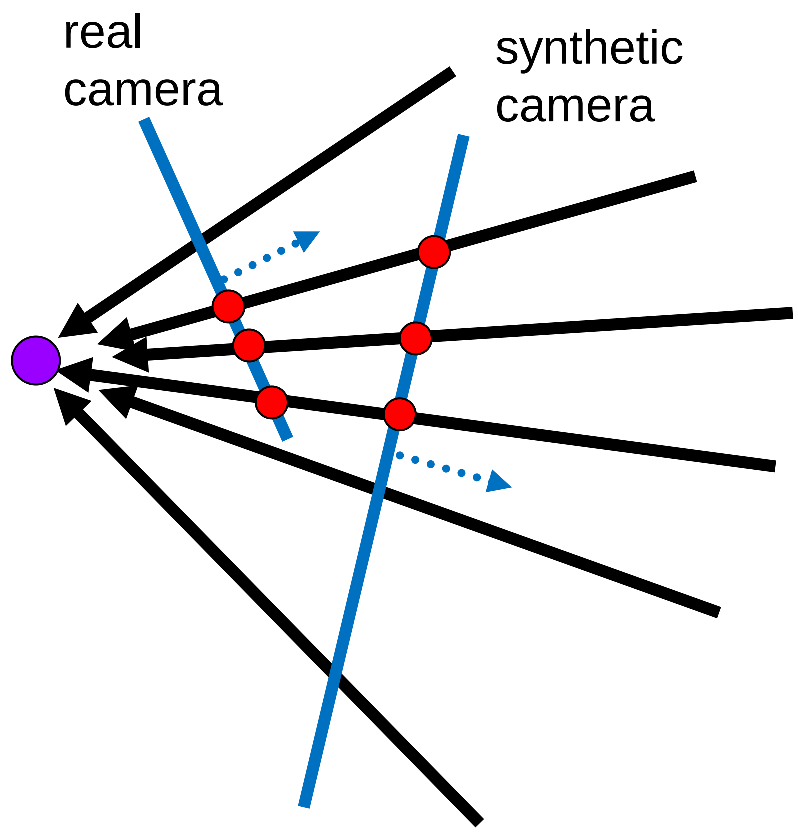
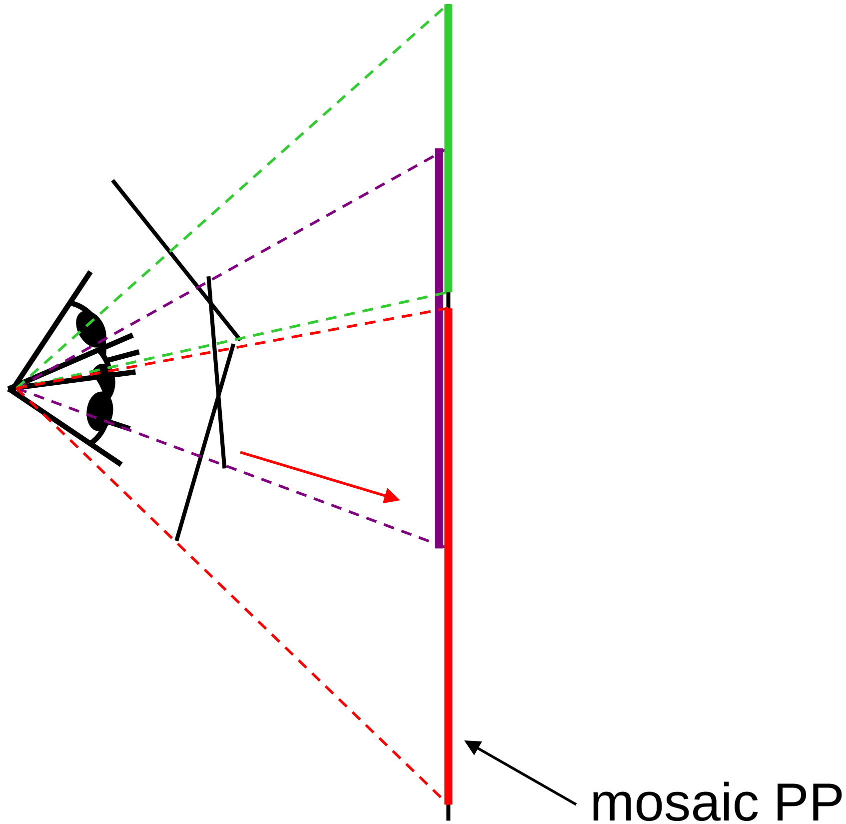
Mosaics: generating synthetic views



Source: Alyosha Efros

Image reprojection



The mosaic has a natural interpretation in 3D

- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*

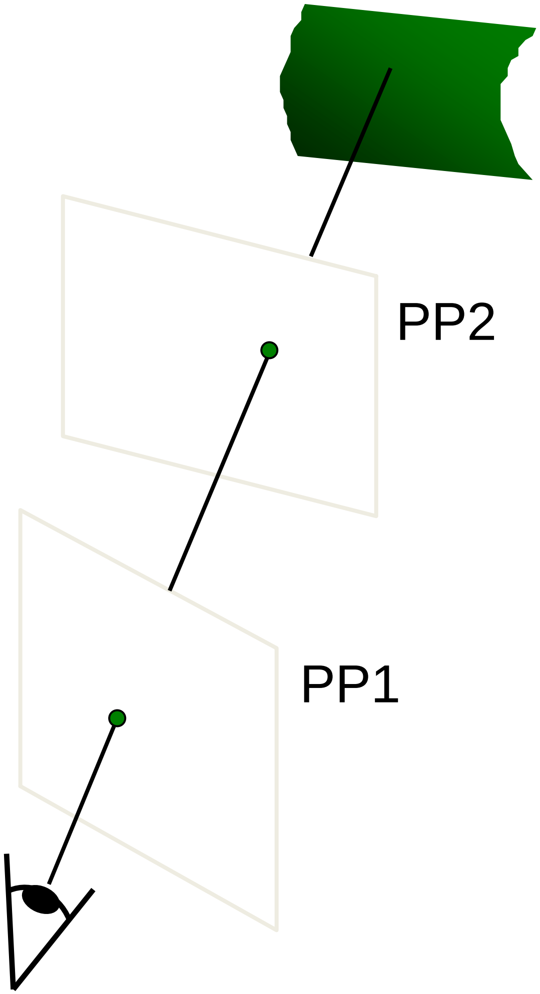
Source: Steve Seitz

Image reprojection

- How to relate two images from the same camera center? How to map a pixel from PP1 to PP2?

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2

- Rather than thinking of this as a 3D reprojection, think of it as a 2D image warp from one image to another.



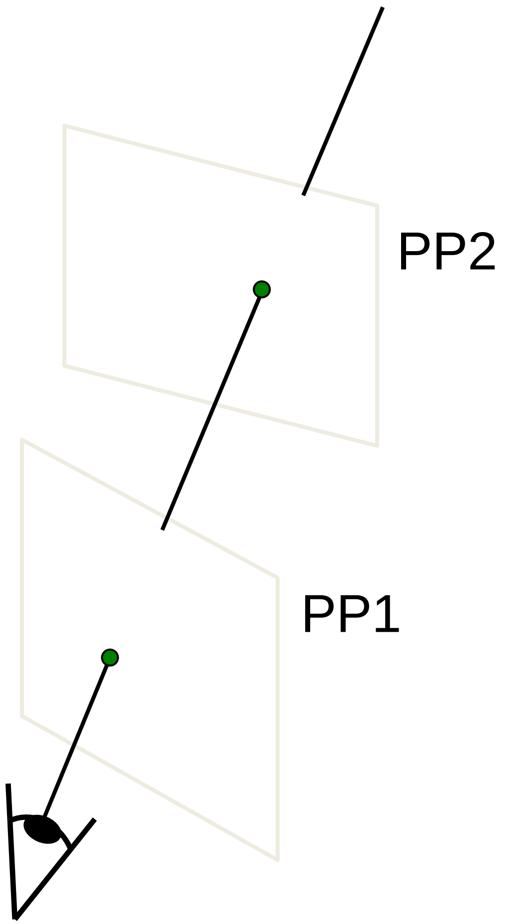
Source: Alyosha Efros

Image reprojection: Homography

A projective transform is a mapping between any two PPs with the same center of projection

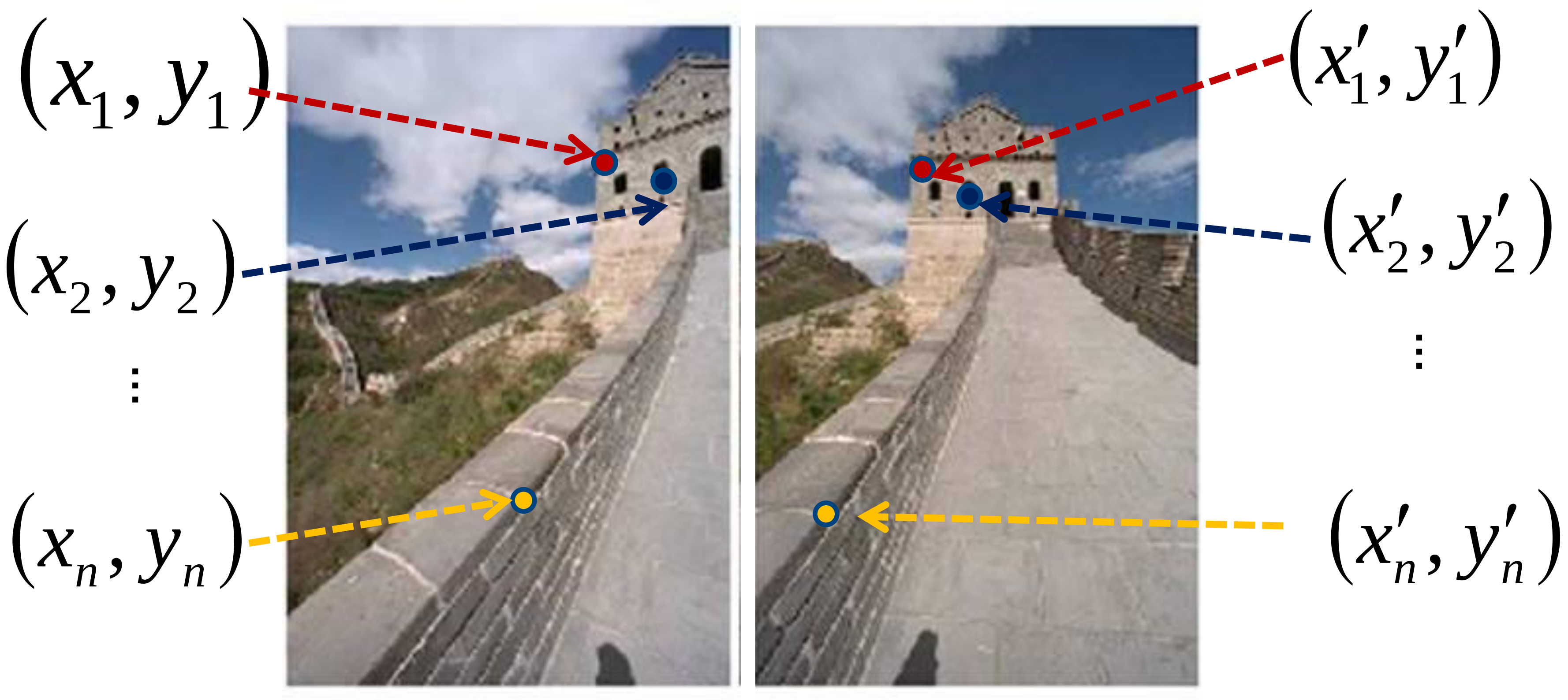
Called **Homography**

$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{\mathbf{p'}} = \underbrace{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\mathbf{p}}$$



Source: Alyosha Efros

Homography



To **compute** the homography given pairs of corresponding points in the images, we need to set up an equation where the parameters of **H** are the unknowns…

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Solving for homographies

$$\begin{bmatrix} \mathrm{w}x_i' \\ \mathrm{w}y_i' \\ \mathrm{w} \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Solving for homographies

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ & & & & & \vdots & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

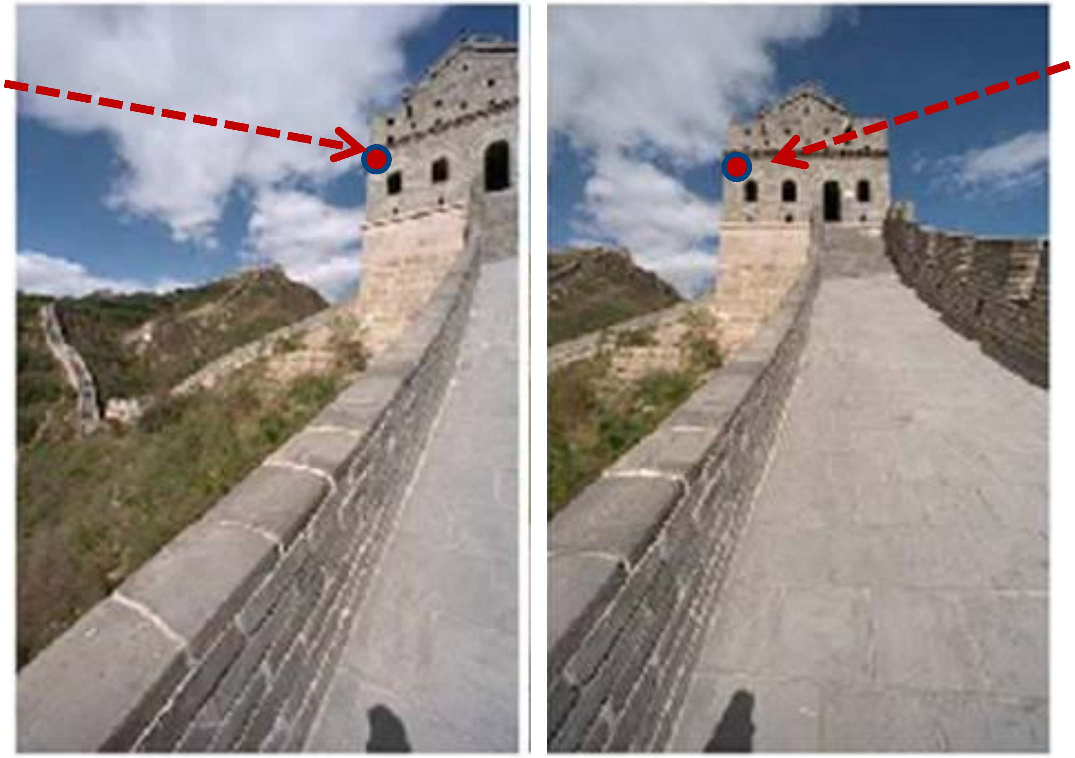
$$\underset{2n \times 9}{\mathrm{A}} \qquad \underset{9}{\mathrm{h}} \qquad \underset{2n}{\mathbf{0}}$$

Defines a least squares problem: $\quad \text{minimize } \|\mathbf{A}\mathbf{h} - \mathbf{0}\|^2$

- Since h is only defined up to scale, solve for unit vector $\hat{h}$
- Solution: $\hat{h}$ = eigenvector of A^TA with smallest eigenvalue

Homography

$$\left(x, y\right)$$



$$\left({}^{wx'}\!/_{w}, {}^{wy'}\!/_{w}\right) = \left(x', y'\right)$$

To **apply** a given homography **H**

- Compute **p' = Hp** (regular matrix multiply)
- Convert **p'** from homogeneous to image coordinates

$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{\mathbf{p'}} = \underbrace{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\mathbf{p}}$$

Kristen Grauman

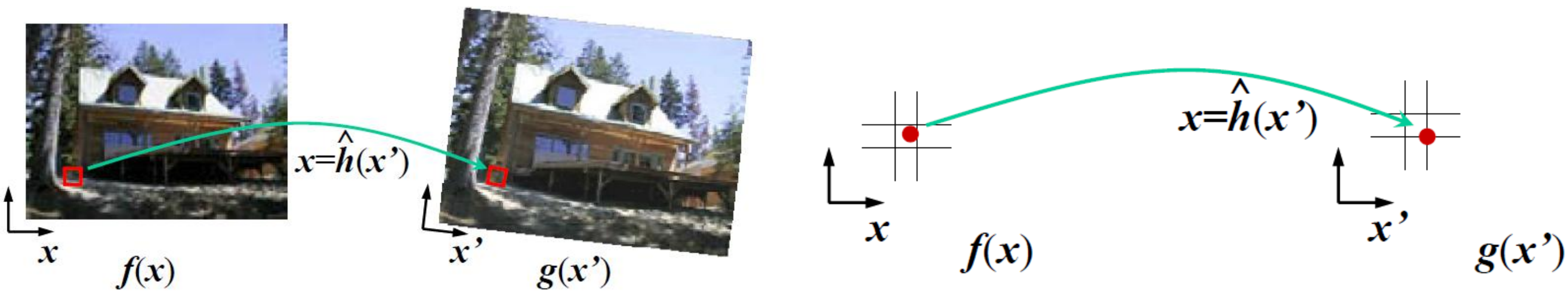
RANSAC for estimating homography

- RANSAC loop:
- 1. Select four feature pairs (at random)
- 2. Compute homography H (exact)
- 3. Compute *inliers* where $SSD(p_i', \mathbf{H}p_i) < \varepsilon$
- 4. Keep largest set of inliers
- 5. Re-compute least-squares H estimate on all of the inliers

Slide credit: Steve Seitz

Recall: Warping

- GPU/OpenGL rendering
- Inverse map with interpolation:



procedure $inverseWarp(f, \boldsymbol{h}, \textbf{out } g)$:

For every pixel $\boldsymbol{x}'$ in $g(\boldsymbol{x}')$

1. Compute the source location $\boldsymbol{x} = \hat{\boldsymbol{h}}(\boldsymbol{x}')$
2. Resample $f(\boldsymbol{x})$ at location $\boldsymbol{x}$ and copy to $g(\boldsymbol{x}')$

Recap: How to stitch together a panorama (a.k.a. mosaic)?

Basic Procedure

- Take a sequence of images from the same position
  - Rotate the camera about its optical center
- Compute transformation (homography) between second image and first using corresponding points.
- Transform the second image to overlap with the first.
- Blend the two together to create a mosaic.
- (If there are more images, repeat)

Source: Steve Seitz

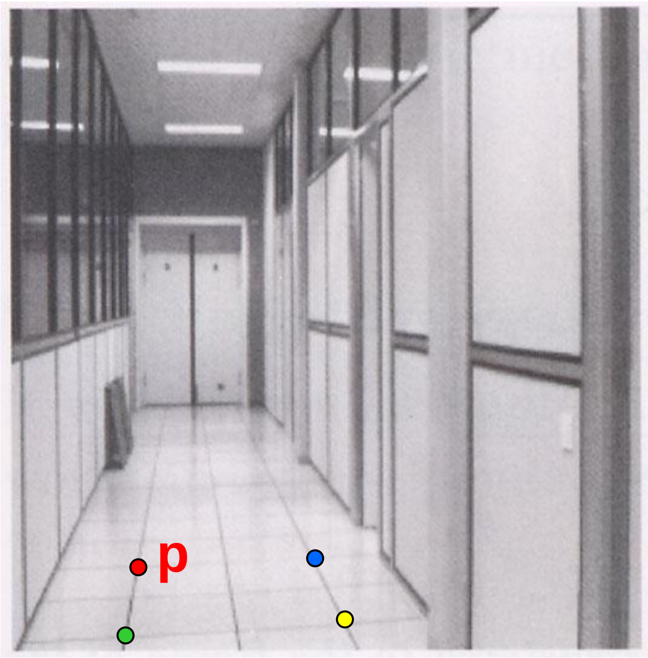
Image warping with homographies

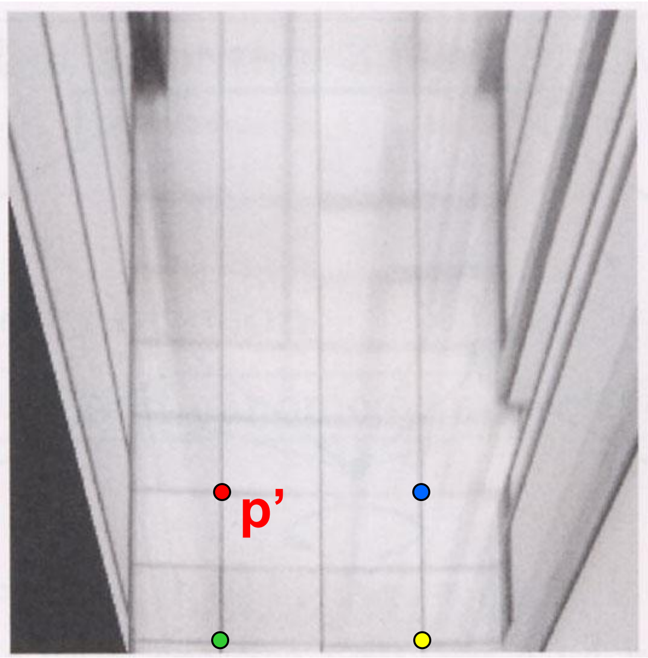
image plane in front

image plane below

Source: Steve Seitz

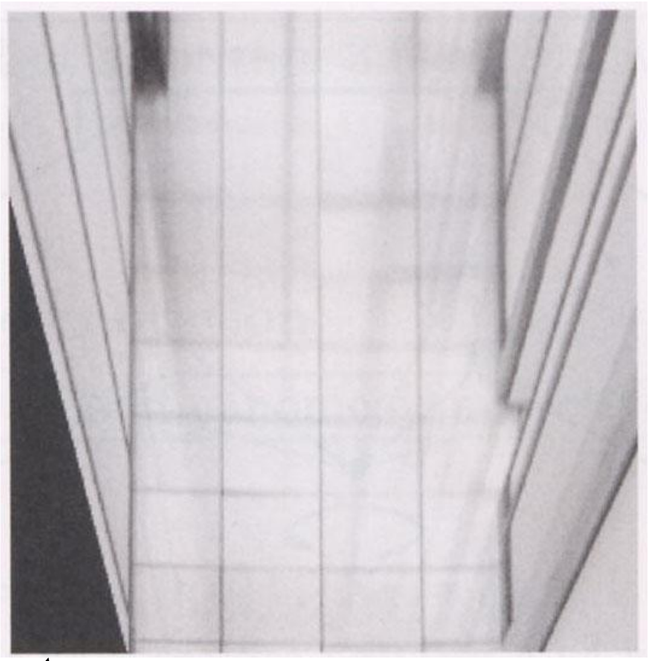
Image rectification





Source: Steve Seitz

Image warping with homographies



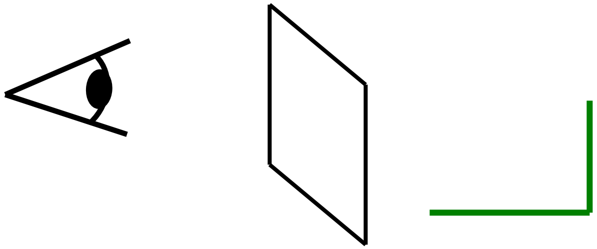


image plane in front

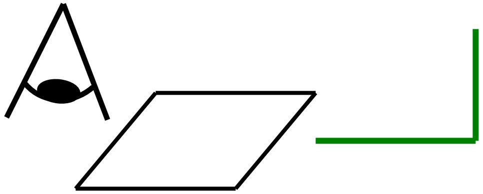
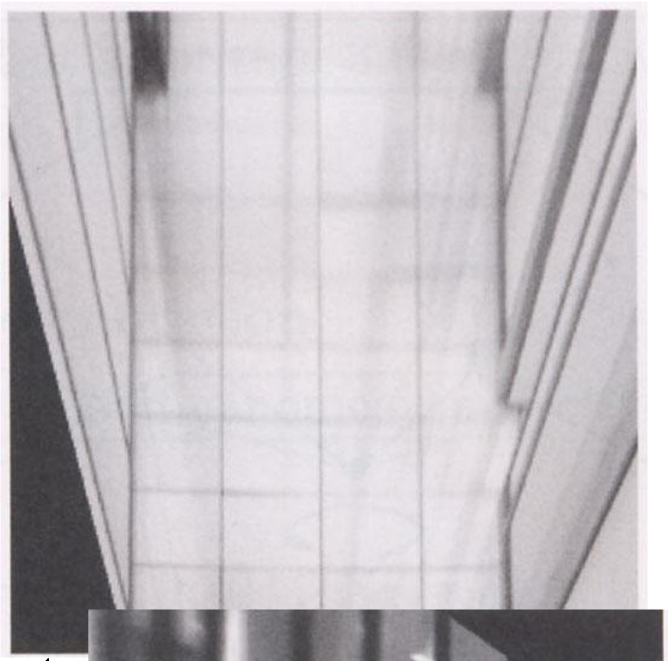


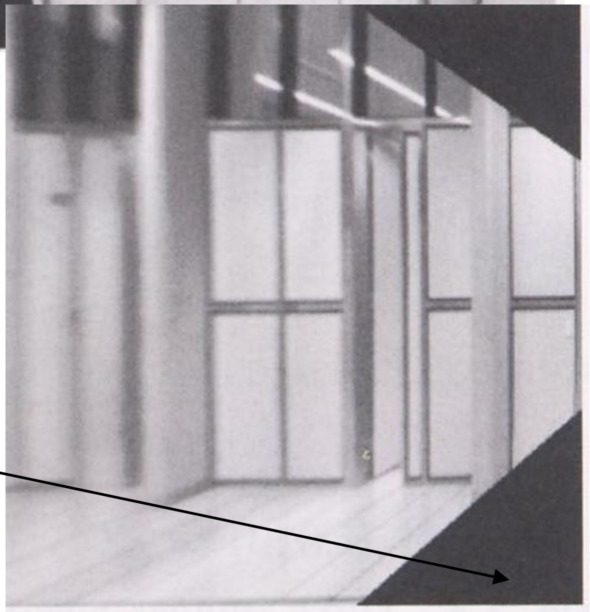
image plane below

black area
where no pixel
maps to

Source: Steve Seitz

Image warping with homographies





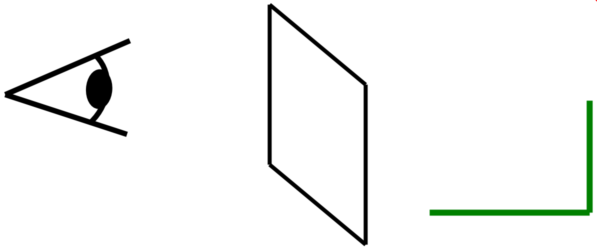


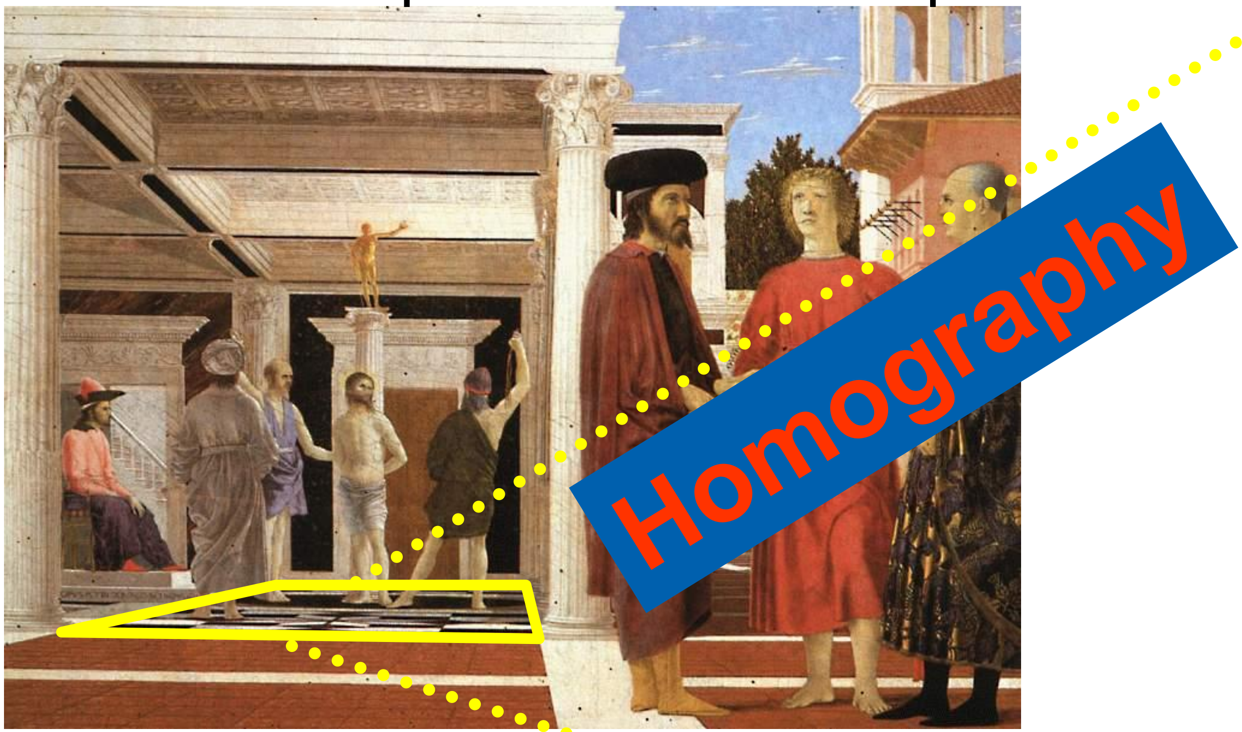
image plane in front

black area
where no pixel
maps to

Source: Steve Seitz

Analysing patterns and shapes

What is the shape of the b/w floor pattern?



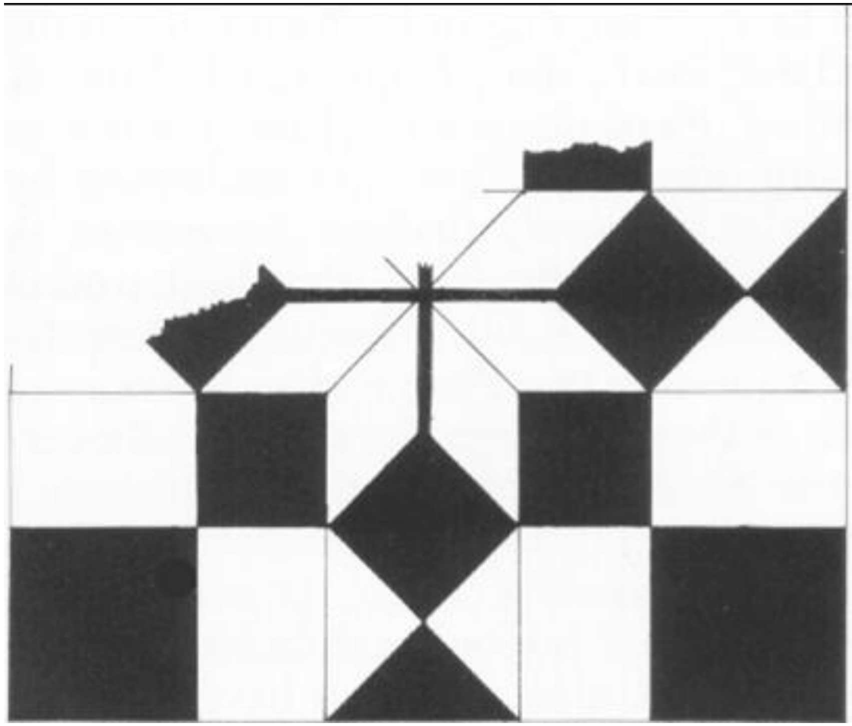
The floor (enlarged)

Automatically rectified floor

Slide from Antonio Criminisi

Analysing patterns and shapes

Automatic rectification



From Martin Kemp *The Science of Art*
(manual reconstruction)

Slide from Antonio Criminisi

Analysing patterns and shapes

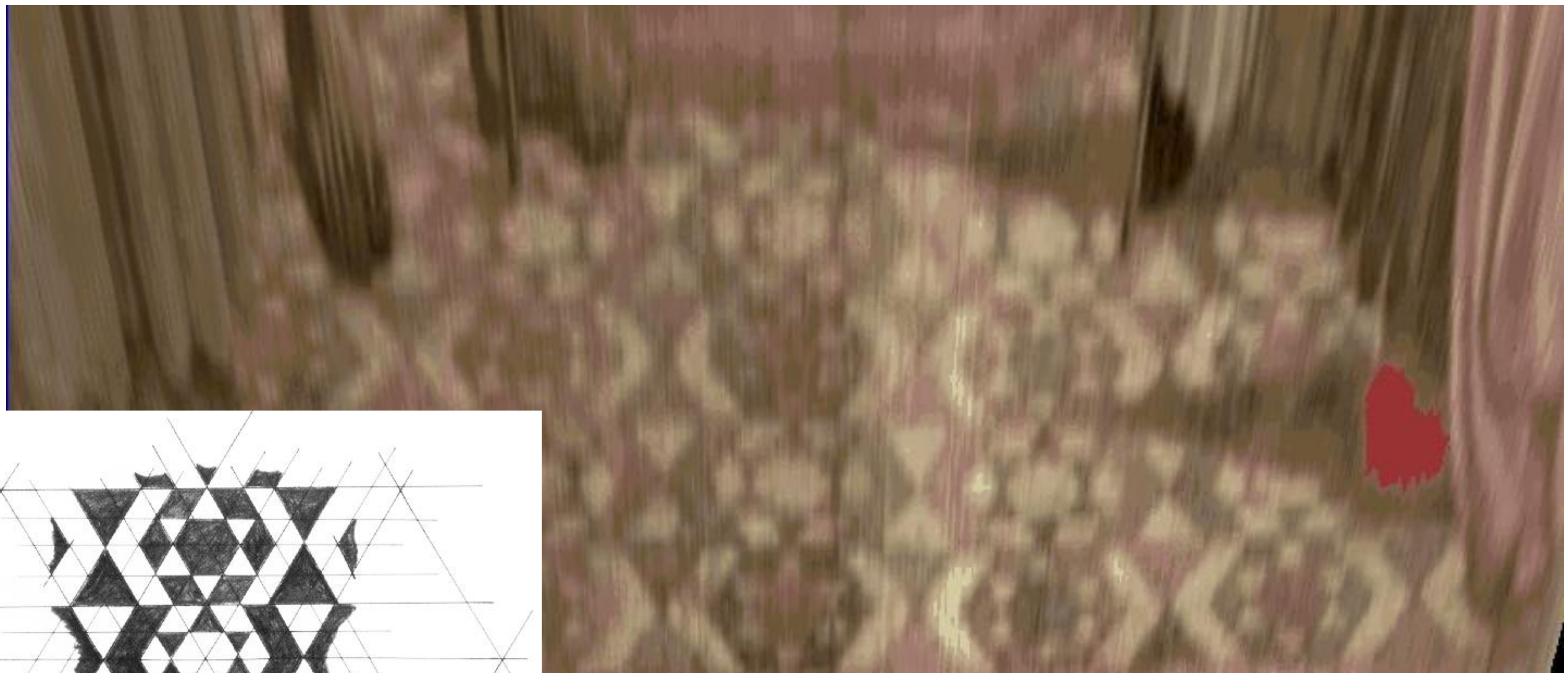
St. Lucy Altarpiece, D. Veneziano

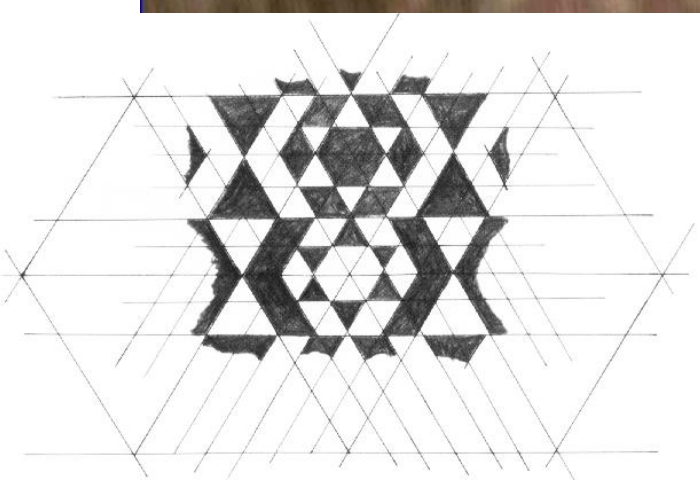
Automatically rectified floor

Slide from Criminisi

Analysing patterns and shapes

Automatic
rectification



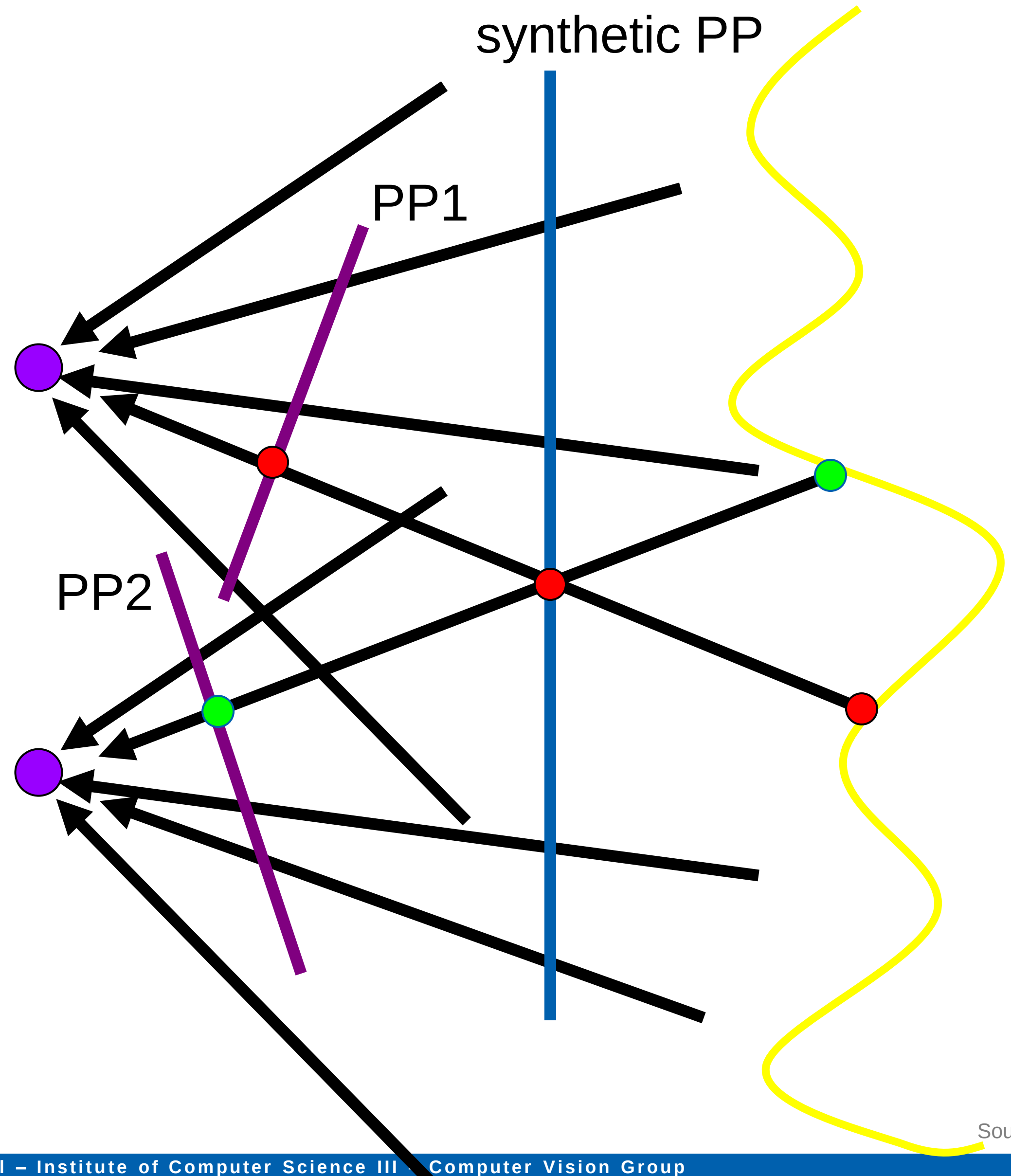


From Martin Kemp, *The Science of Art*
(manual reconstruction)

Slide from Criminisi

Changing camera center

Does it still work?



Source: Alyosha Efros

Recall: same camera center

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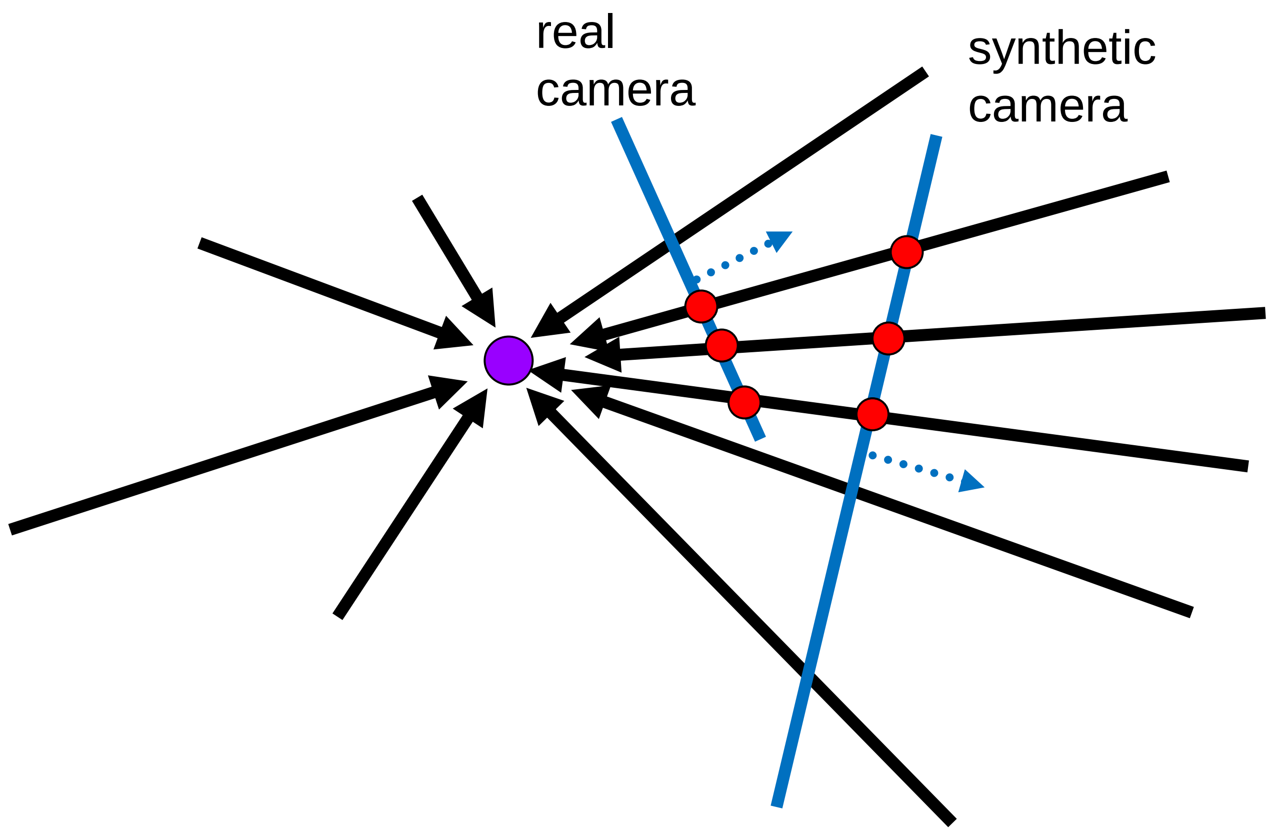
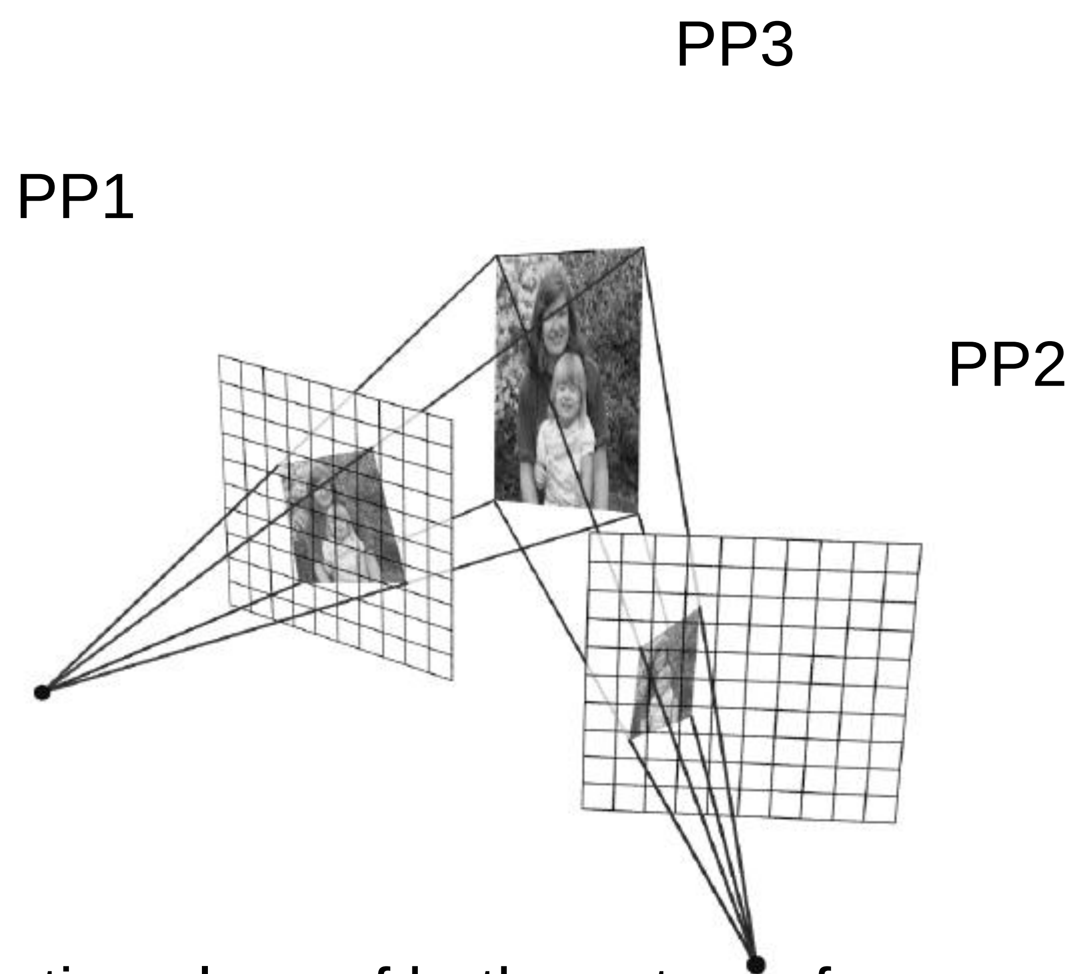


Can generate synthetic camera view
as long as it has **the same center of projection**.

Source: Alyosha Efros

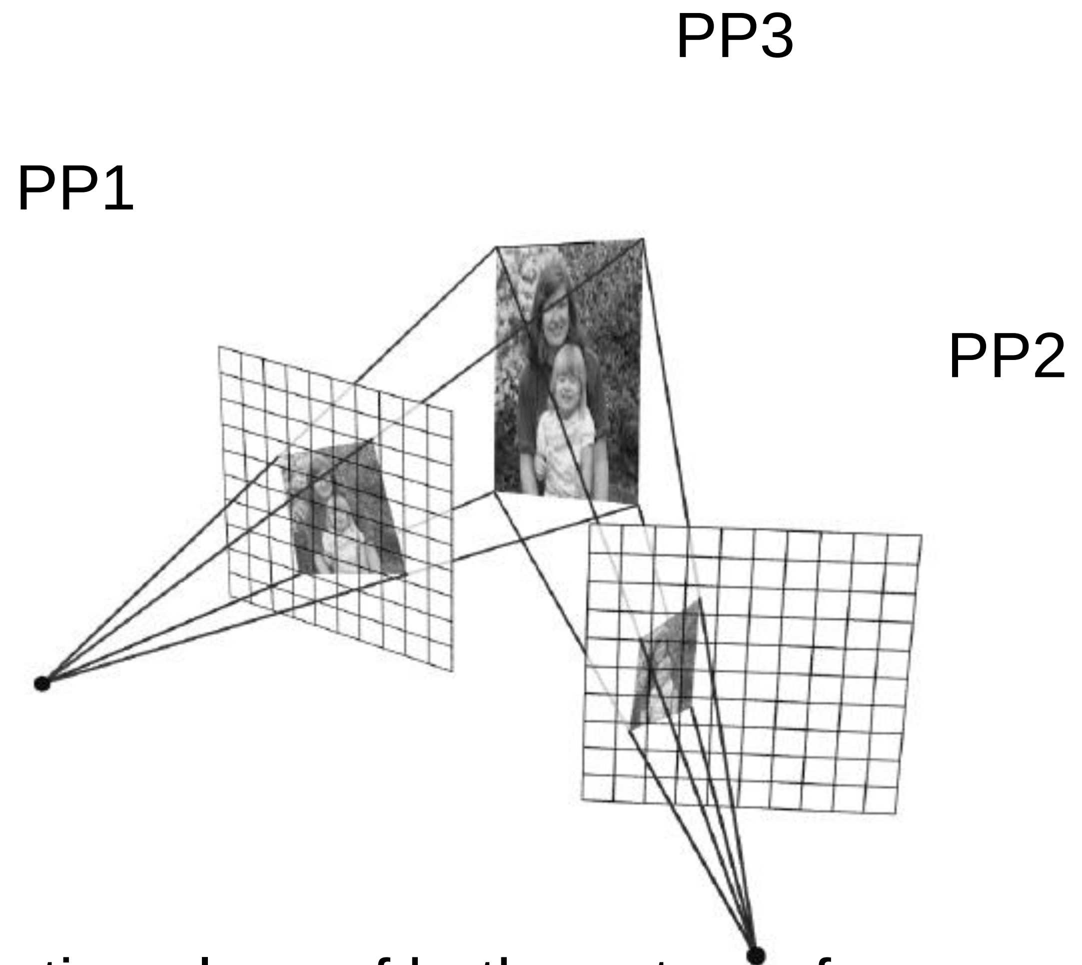
…or planar scene



- PP3 is a projection plane of both centers of projection, so we are OK!

Source: Alyosha Efros

…or planar scene (or far away)



- PP3 is a projection plane of both centers of projection, so we are OK!
- This is how big aerial photographs are made

Source: Alyosha Efros


Map
Satellite
Hybrid
200 ft
100 m
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Kristen Grauman

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