Autonomous Intelligent Systems,
Institute for Computer Science VI, University of Bonn

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Exercises for Artificial Life (MA-INF 4201), SS24

Exercises sheet 6, till: Mon 19. May, 2025

12.5.2025

Assignment 36 (2 Points)

Suppose a netto growth of a population of 1.5% per week.
How long will it take until this population has doubled its size?
Please derive a formula for the number of years necessary to reach the doubling.

Assignment 37 (2 Points)

What is a *Golden Spiral* ?
Please use your own words to explain it. Copying text from the internet is not sufficient.

Assignment 38 (1 Point)

Prove or disprove the following sentence in a formal way:
The Fibonnaci sequence is rising faster than the exponential function.

Assignment 39 (2 Points)

Derive a formula to calculate the limit β of the ratios of successive terms of the Fibonacci sequence analytically, and implement afew lines of code to check this limit experimentally.

$$\beta = \lim_{n\to\infty} \frac{F_{n+1}}{F_n} \qquad F_{n+2} = F_n + F_{n+1} \qquad F_0 = 0, \quad F_1 = 1$$

Assignment 40 (2 Points)

Derive a formula to determine the fixpoint x^* depending on a for the following iterated function (*logistic map*) : $x_{i+1} = a * x_i * (1 - x_i)$.
For a fixpoint of an iterated function the value is no longer changing, thus $x^*_{i+1} = x^*_i$.
Draw a diagram for the fixpoint $x^*(a)$ as a function of a with $0.0 < a < 4.0$.

Assignment 41 (4 Points)

Please determine the long-term behavior of the *logistic map* :
$x_{i+1} = a * x_i * (1 - x_i)$ for $a = 3.3$, for $a = 3.51$, and for $a = 3.75$
for the two starting conditions $x_{i=0} = 0.228734167$ and $\hat{x}_{i=0} = 0.228734168$.
Which class (Wolfram classification) of behavior is reached?
Draw at least one diagram that depicts the behavior.

The development of the difference between the values x_i and $\hat{x}_i$ is as well interesting, and helpful to understand the behavior.

Assignment 42 (2 Points)

Write down the Look-and-Say sequence up to step 6, starting with "42".

Programming Assignment: C (5 Points, due date **Mon 19.5.2025**)

Implement (in Python) a program to iterate a system of three nonlinear coupled differential equations using the Euler-Method (sometimes called forward Euler Method).

Set the initial states x_0, y_0, z_0 for timestep $t = 0$ (or iteration step $i = 0$) randomly between $0.0 < x_0, y_0, z_o < 1.0$. For each iteration step calculate the change of values $\Delta x_i, \Delta y_i, \Delta z_i$ according to the given differential equation and extrapolate to the next values for a small timestep Δt by taking the corresponding difference equations from the Euler-Method:
$x_{i+1} = x_i + \Delta x_i$
$y_{i+1} = y_i + \Delta y_i$
$z_{i+1} = z_i + \Delta z_i$

and

$\Delta x = \dot{x}(t) * \Delta t$
$\Delta y = \dot{y}(t) * \Delta t$
$\Delta z = \dot{z}(t) * \Delta t.$

You can chose between iterating the Lorenz-Attractor or the Rössler-Attractor, both for a minimum of 50000 steps with $\Delta t \leq 0.01$.
Depict the development of the values as three time series $x(i), y(i), z(i)$ and as a line in three dimensions.

Rössler Attractor: parameters are $a = 0.2, b = 0.2, c = 5.7$.
$\dot{x}(t) = -y - z$
$\dot{y}(t) = x + ay$
$\dot{z}(t) = b - zc + zx$

Lorenz-Attractor: parameters are $\rho = 28, \sigma = 10, \beta = \frac{8}{3}$.
$\dot{x}(t) = \sigma(y - x)$
$\dot{y}(t) = x(\rho - z) - y$
$\dot{z}(t) = xy - \beta z$