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May 7, 2025
Due date: May 14, 2025

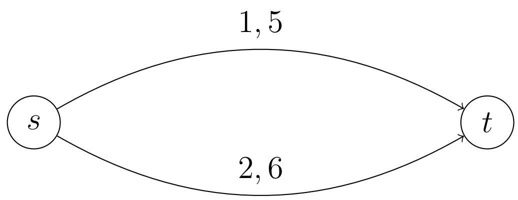
Algorithmic Game Theory

Summer Term 2025

Exercise Set 4

If you would like to present one of the solutions in class, please also send an email to rlehming@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated on a first-come-first-served basis, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1:
Consider the following symmetric network congestion game with two players:



(a) What is the Price of Anarchy and the Price of Stability of pure Nash equilibria?

(b) What is the Price of Anarchy and the Price of Stability of mixed Nash equilibria?

Hint: First of all, determine all mixed Nash equilibria. You might start with a sentence like "Let σ be a mixed Nash equilibrium with $\sigma_1 = (p_1, 1-p_1)$, $\sigma_2 = (p_2, 1-p_2)$" and subsequently derive properties of p_1 and p_2.

Exercise 2:
Consider a (λ, μ)-smooth game with N players and let $s^{(1)}, \dots, s^{(T)}$ be a sequence of states such that the external regret of every player is at most $R^{(T)}$. Moreover, let s^* denote a state that minimizes the social cost. We want to upper bound the average social cost of the sequence of states. To this end, show the following bound

$$\frac{1}{T}\sum_{t=1}^{T} SC(s^{(t)}) \leq \frac{N \cdot R^{(T)}}{(1-\mu)T} + \frac{\lambda}{1-\mu} SC(s^*)\,.$$

Hint: In this setting, the external regret for player i is the difference between the cost they have incurred and the cost they would have incurred with the best fixed strategy in hindsight.

Exercise 3:
A *fair cost-sharing game* is a congestion game such that for all resources $r \in R$ the delay function can be modeled as $d_r(x) = c_r/x$ for a constant c_r.
Show that fair cost sharing games with n players are $(n, 0)$-smooth.