Juergen Gall

Optical Flow

MA-INF 2201 - Computer Vision

WS24/25

Visual motion

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Uses of motion

- Estimating 3D structure
- Segmenting objects based on motion cues
- Learning and tracking dynamical models
- Recognizing events and activities

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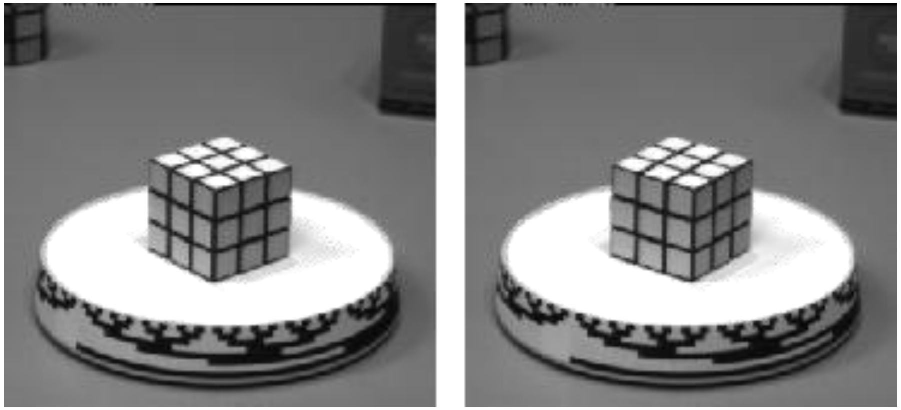
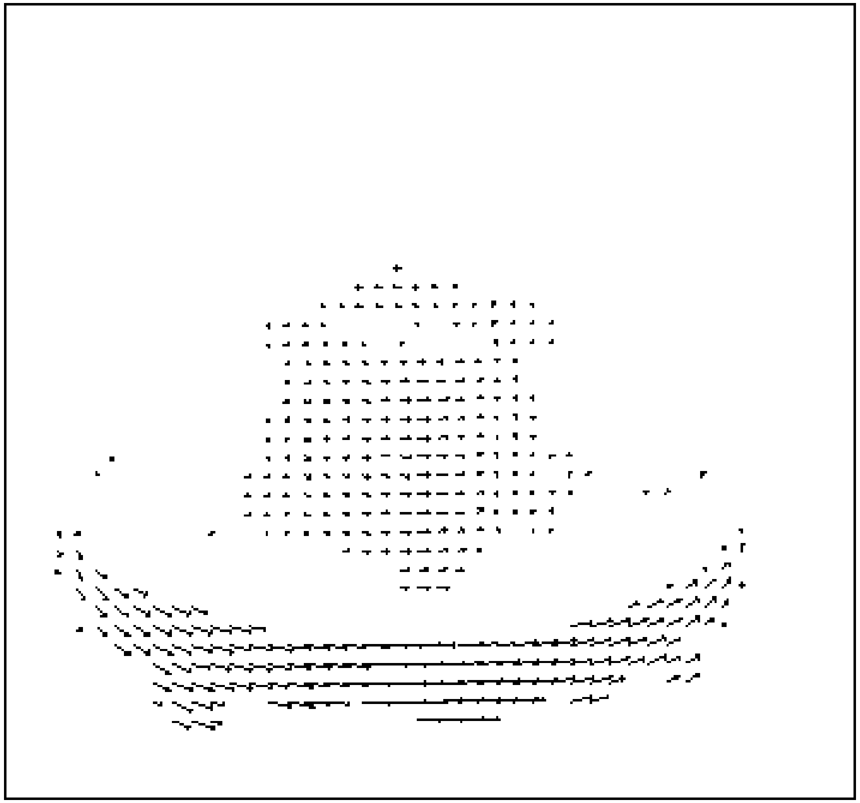
Motion field

- The motion field is the projection of the 3D scene motion into the image





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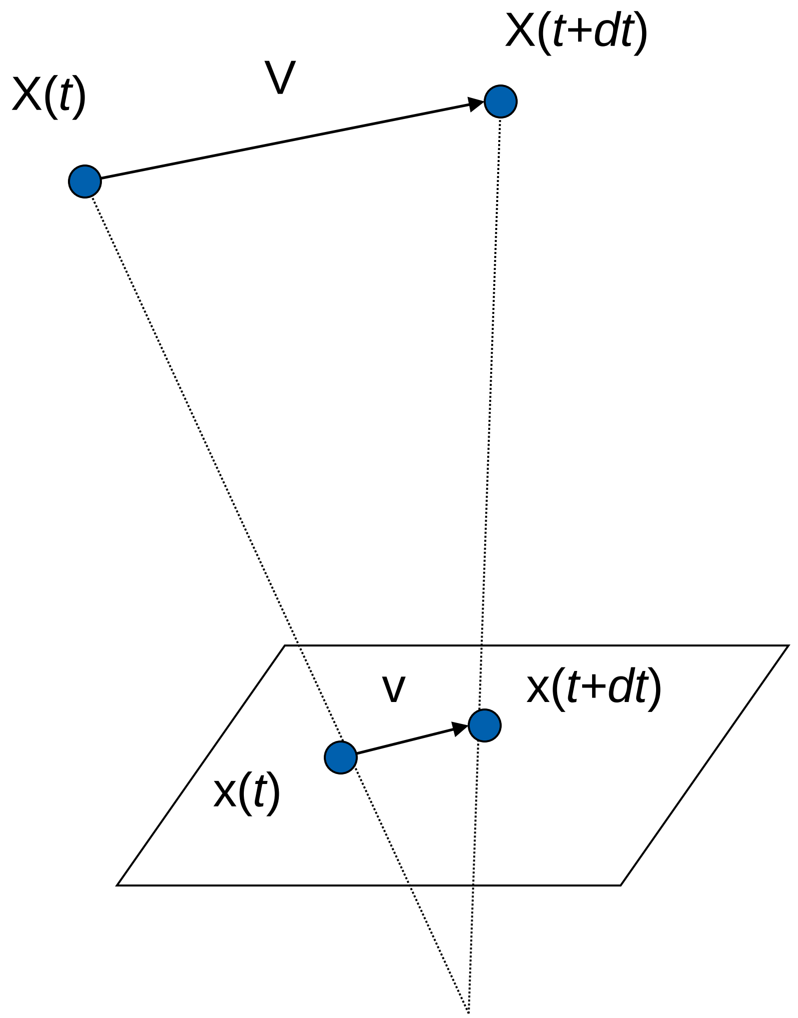
Motion field and optical flow

- $\mathbf{X}(t)$ is a moving 3D point
- Velocity of scene point: $\mathbf{V} = d\mathbf{X}/dt$
- $\mathbf{x}(t) = (x(t),y(t))$ is the projection of **X** in the image
- Apparent velocity **v** in the image: given by components $v_x = dx/dt$ and $v_y = dy/dt$
- These components are known as the *motion field* of the image



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Optical flow

- Definition: optical flow is the *apparent* motion of brightness patterns in the image
- Ideally, optical flow would be the same as the motion field
- Have to be careful: apparent motion can be caused by lighting changes without any actual motion

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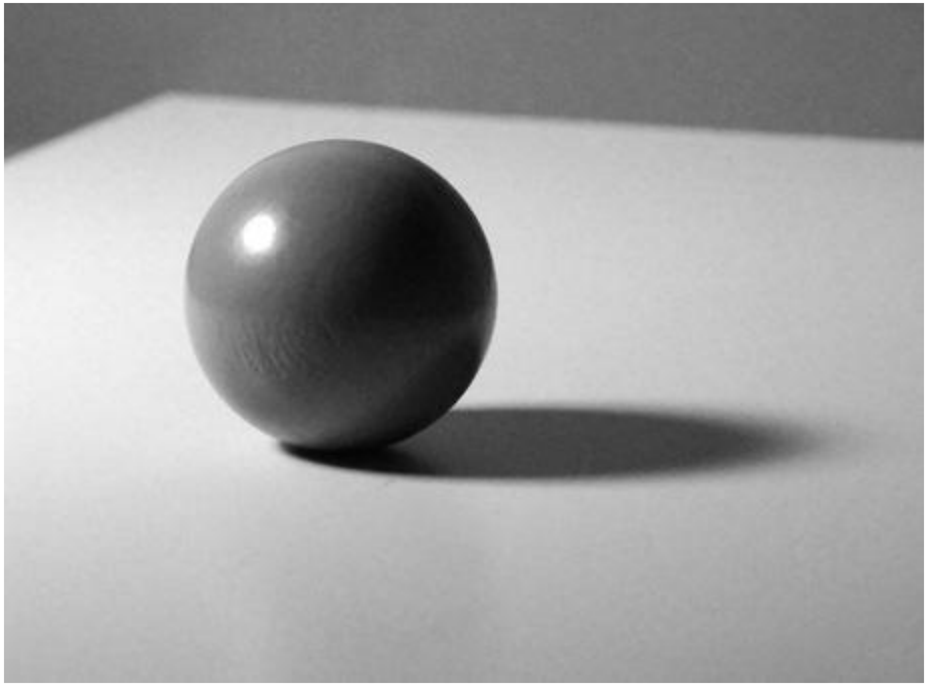
Optical flow

- Have to be careful: apparent motion can be caused by lighting changes without any actual motion



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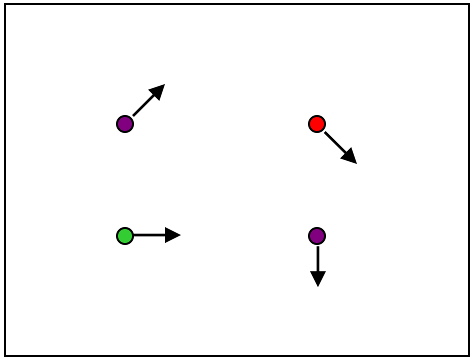
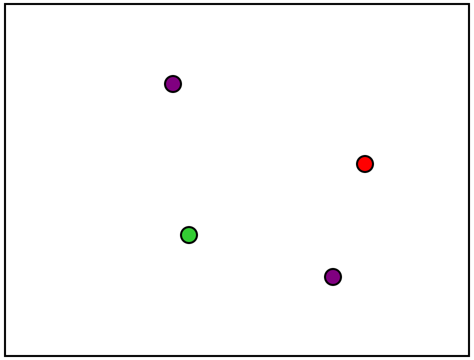
Estimating optical flow



$I(x,y,t-1)$



$I(x,y,t)$

- Given two subsequent frames, estimate the apparent motion field $u(x,y)$ and $v(x,y)$ between them

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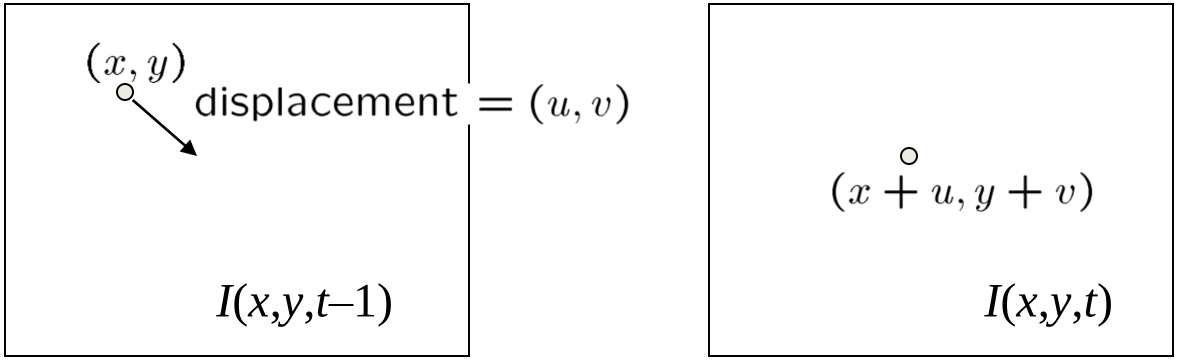
The brightness constancy constraint



- Brightness Constancy Equation:

$$I(x, y, t-1) = I(x + u(x, y), y + v(x, y), t)$$

Linearizing the right side using Taylor expansion:

$$I(x, y, t-1) \approx I(x, y, t) + I_x \cdot u(x, y) + I_y \cdot v(x, y)$$

Hence, $I_x \cdot u + I_y \cdot v + I_t \approx 0$ or $\nabla I \cdot (u, v) + I_t \approx 0$

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The brightness constancy constraint

$$I_x \cdot u + I_y \cdot v + I_t = 0$$

- How many equations and unknowns per pixel?
  – One equation, two unknowns

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The brightness constancy constraint

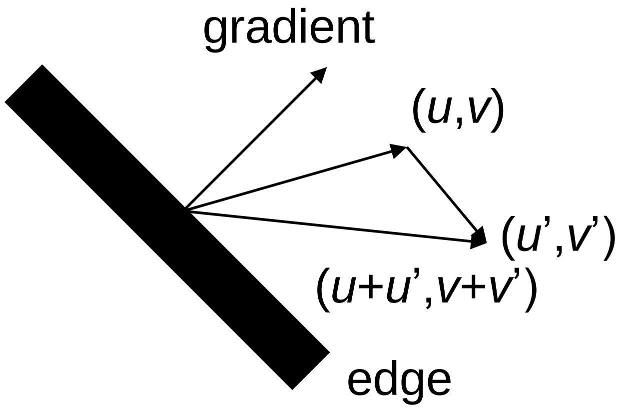
- Intuitively, what does this constraint mean?

$$\nabla I \cdot (u, v) + I_t = 0$$

- The component of the flow perpendicular to the gradient (i.e., parallel to the edge) is unknown

- If (u, v) satisfies the equation, so does (u+u', v+v') if

$$\nabla I \cdot (u', v') = 0$$



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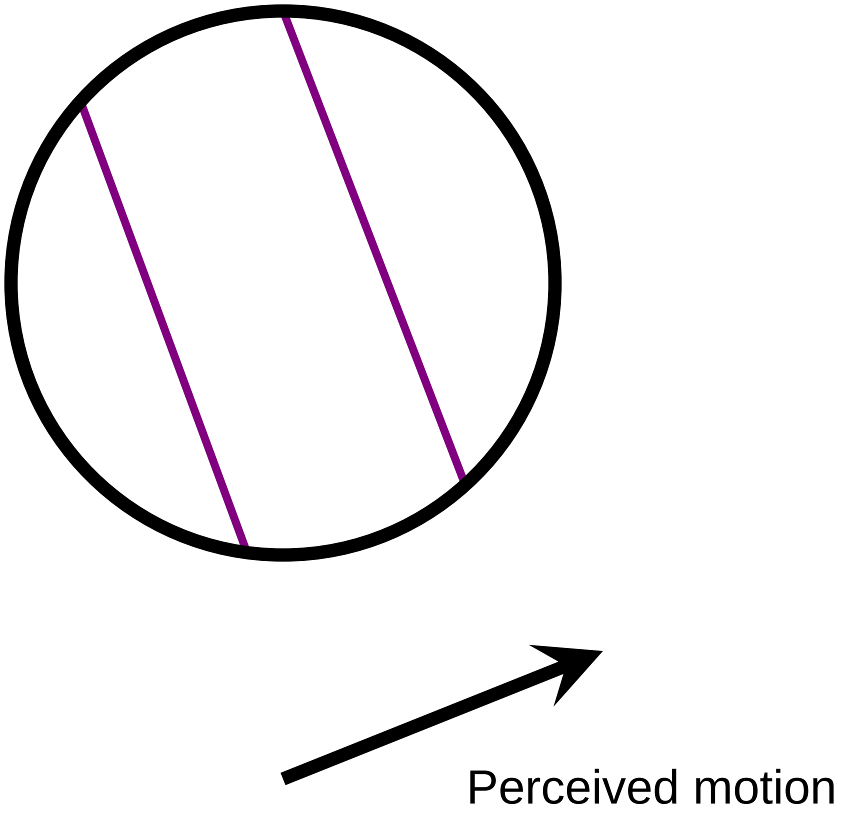
The aperture problem



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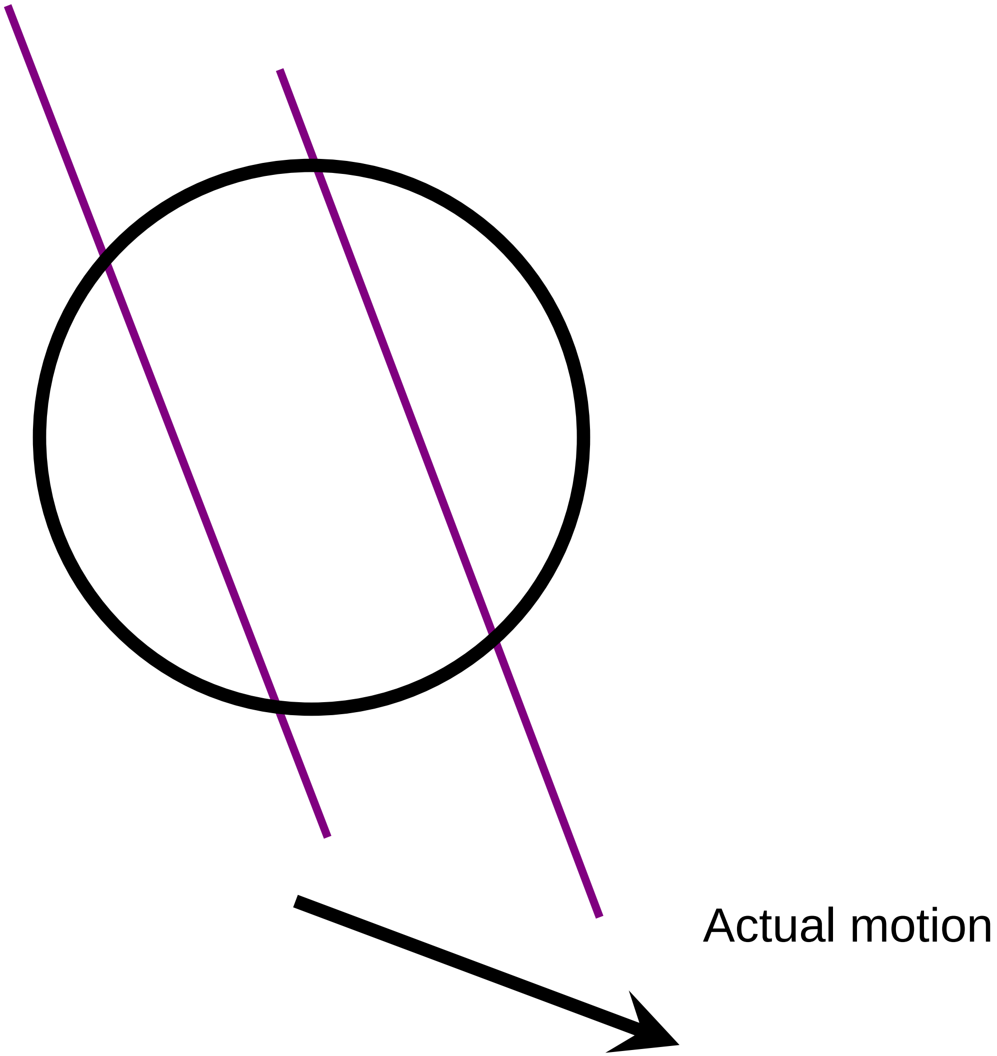
The aperture problem



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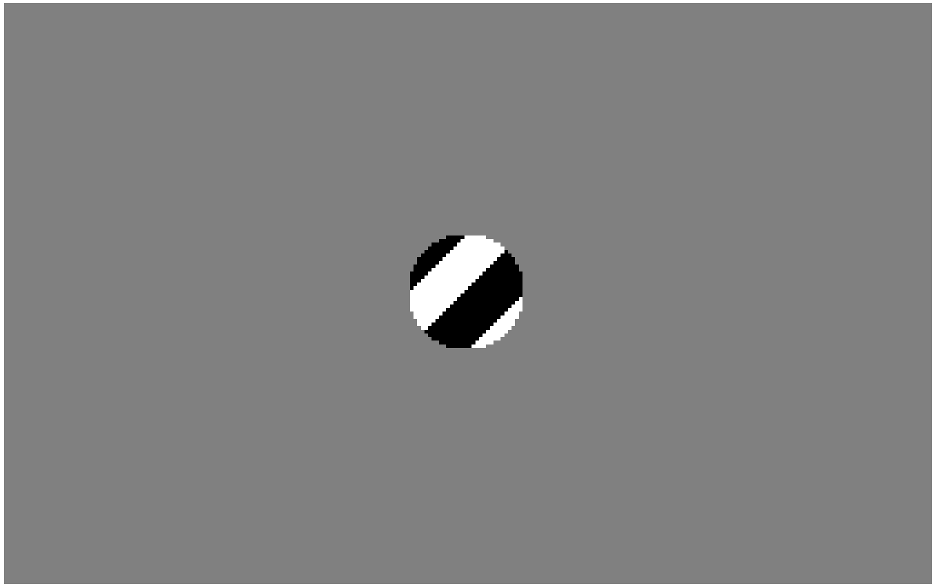
The barber pole illusion



http://en.wikipedia.org/wiki/Barberpole_illusion

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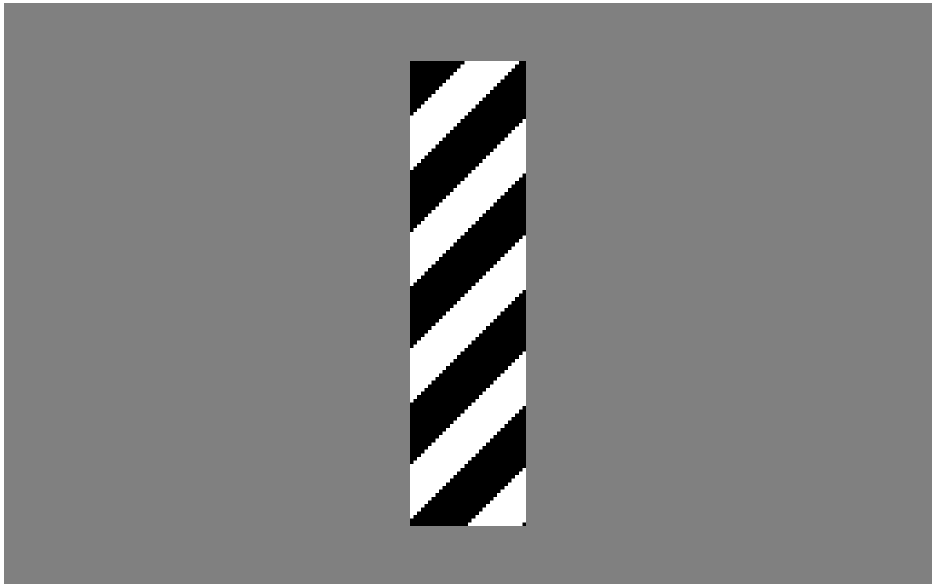
The barber pole illusion



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The barber pole illusion

http://en.wikipedia.org/wiki/Barberpole_illusion

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Solving the aperture problem

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- How to get more equations for a pixel?
- **Spatial coherence constraint:** pretend the pixel's neighbors have the same (u,v)
  - If we use a 5x5 window, that gives us 25 equations per pixel

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \;\; v]$$

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix}$$

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

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Solving the aperture problem

- Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix} \qquad \underset{25\text{x}2}{A} \; \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b}$$

- Is this system always solvable?
- Only if rank of A is 2
- Not the case for edges

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

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Conditions for solvability

Bad case

Good case

Lucas-Kanade flow

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- Linear least squares problem

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix} \qquad \underset{25\text{x}2}{A} \ \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b}$$

Solution given by $(A^T A)\ d = A^T b$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

The summations are over all pixels in the window

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

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Lucas-Kanade flow

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- Recall the Harris corner detector: $M = A^T A$ is the *second moment matrix*

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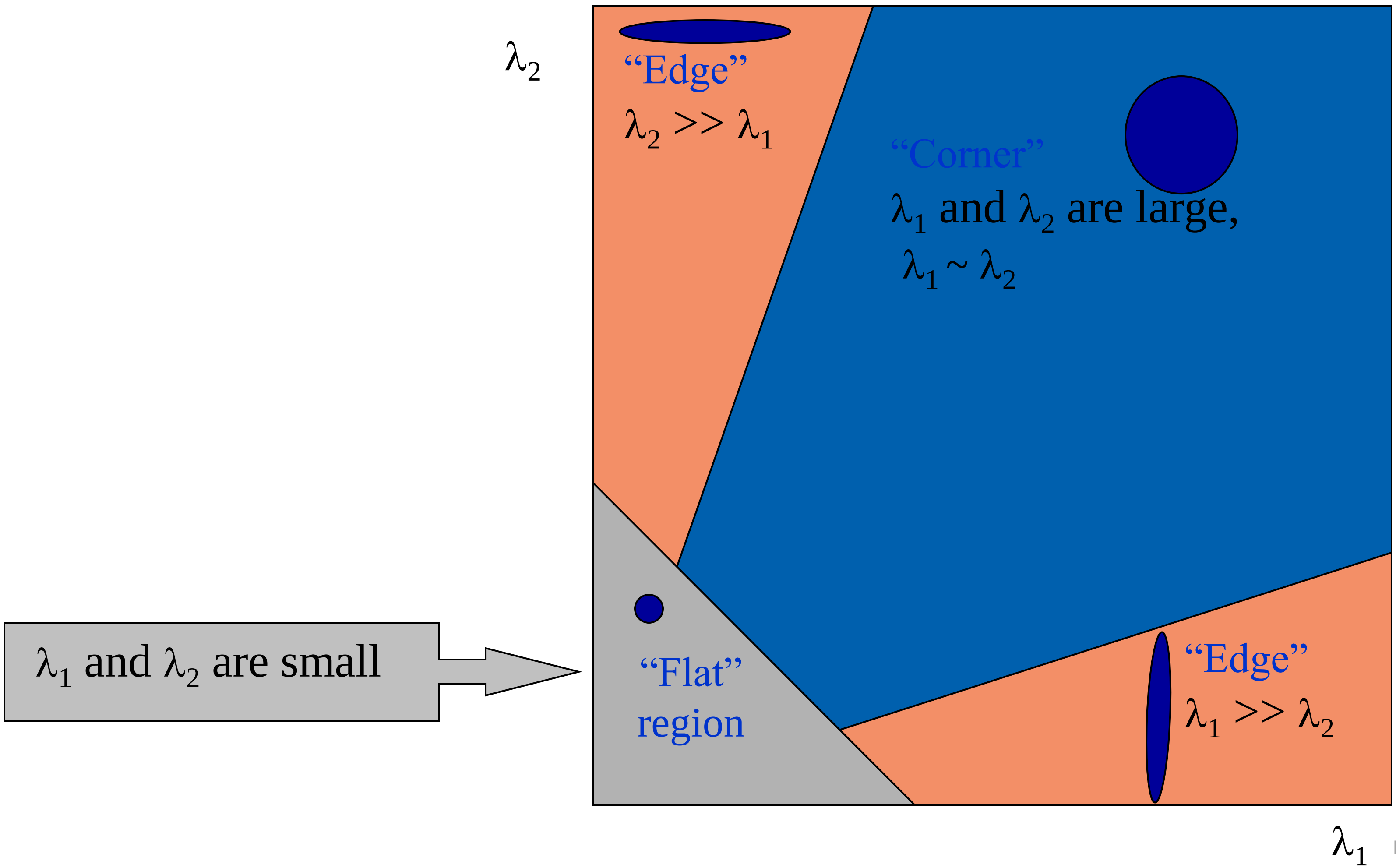
Interpreting the eigenvalues

Classification of image points using eigenvalues of the second moment matrix:



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Uniform region

– gradients have small magnitude
– small λ_1, small λ_2
– system is ill-conditioned

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Edge

– gradients have one dominant direction

– large λ_1, small λ_2

– system is ill-conditioned

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High-texture or corner region

– gradients have different directions, large magnitudes
– large λ_1, large λ_2
– system is well-conditioned

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Errors in Lucas-Kanade

- The motion is large (larger than a pixel)
  - Iterative refinement
  - Coarse-to-fine estimation
  - Exhaustive neighborhood search (feature matching)
- A point does not move like its neighbors
  - Motion segmentation
- Brightness constancy does not hold
  - Exhaustive neighborhood search with normalized correlation

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Feature tracking

- So far, we have only considered optical flow estimation in a pair of images
- If we have more than two images, we can compute the optical flow from each frame to the next
- Given a point in the first image, we can in principle reconstruct its path by simply “following the arrows”

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Shi-Tomasi feature tracker

- Find good features using eigenvalues of second-moment matrix
  - Key idea: “good” features to track are the ones whose motion can be estimated reliably
- From frame to frame, track with Lucas-Kanade
  - This amounts to assuming a translation model for frame-to-frame feature movement
- Check consistency of tracks by *affine* registration to the first observed instance of the feature
  - Affine model is more accurate for larger displacements
  - Comparing to the first frame helps to minimize drift

J. Shi and C. Tomasi. Good Features to Track. CVPR 1994.

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Tracking example

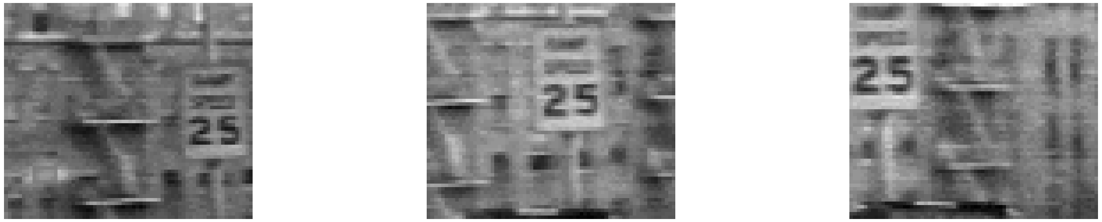

Figure 1: Three frame details from Woody Allen's *Manhattan*. The details are from the 1st, 11th, and 21st frames of a subsequence from the movie.

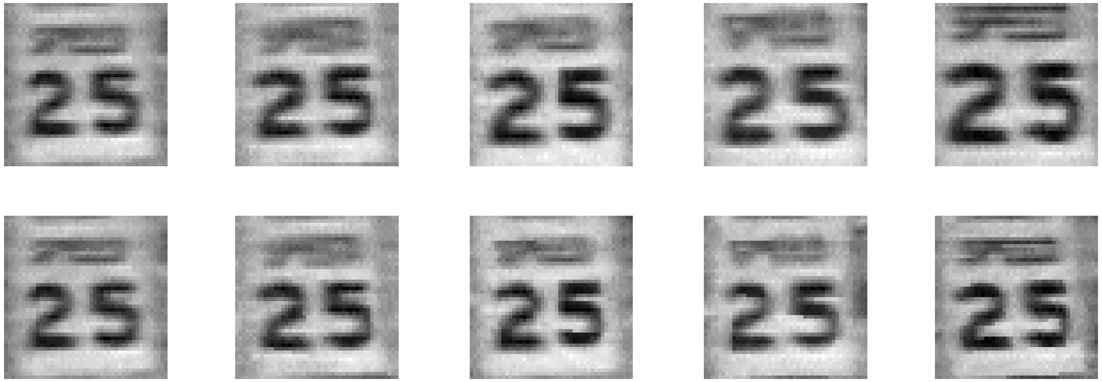

Figure 2: The traffic sign windows from frames 1,6,11,16,21 as tracked (top), and warped by the computed deformation matrices (bottom).

J. Shi and C. Tomasi. Good Features to Track. CVPR 1994.

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Dense motion estimation

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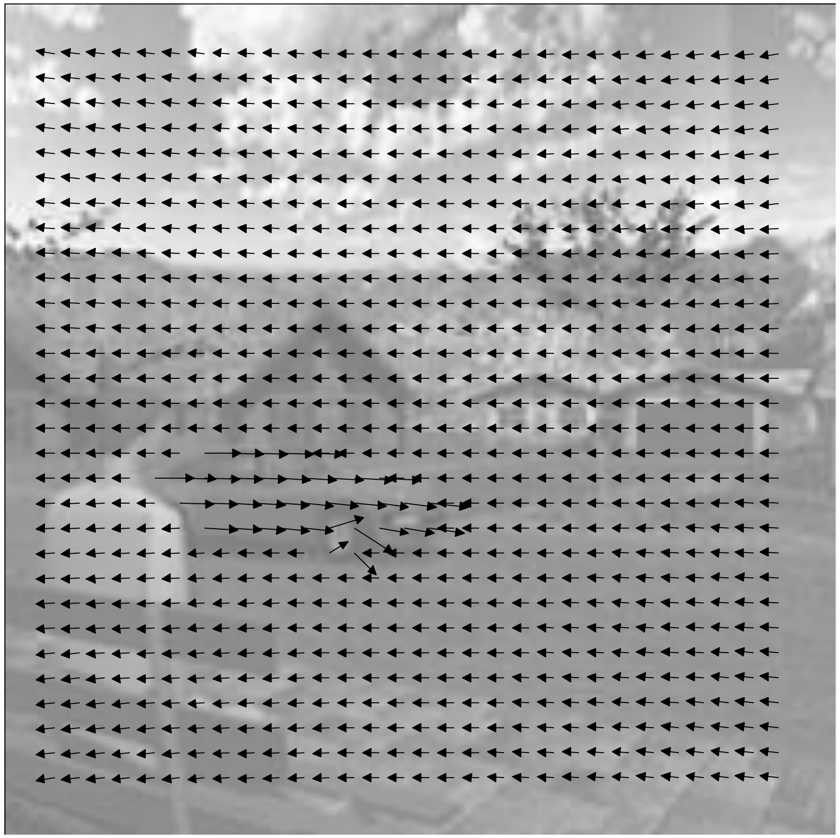
- For each pixel in an image, optical flow determines the corresponding pixel in the next image
- Can be cast as a variational problem

Image sequence $I(x, y, t)$



Optical flow field $(u, v)(x, y, t)$.

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Horn-Schunck model

- Most basic model → good to start with

- Assumption I:
  Gray value constancy (optic flow constraint)

  $I_x u + I_y v + I_t = 0$ reminder: subscripts denote partial derivatives

  Linearized version of

  $$I(x+u, y+v, t+1) - I(x, y, t) = 0$$

- Assumption II:
  Smoothness of optic flow field
  $|\nabla u|^2 + |\nabla v|^2 \rightarrow \min$

- Combined in an energy functional:

$$E(u, v) = \int_\Omega (I_x u + I_y v + I_t)^2 + \alpha(|\nabla u|^2 + |\nabla v|^2)\, dx dy$$

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Analysis of Horn-Schunck model

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$$E(u,v) = \int_\Omega (I_x u + I_y v + I_t)^2 + \alpha(|\nabla u|^2 + |\nabla v|^2)\, dx dy$$

- First term penalizes deviations from optic flow constraints
- Second term penalizes deviations from smoothness
- Parameter α steers relative importance of assumptions

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Theoretical analysis

- Both terms of the energy are convex in (u, v)

$$E(u,v) = \int_\Omega (I_x u + I_y v + I_t)^2 + \alpha(|\nabla u|^2 + |\nabla v|^2)\, dx dy$$

- Linear combination of two convex functionals is again convex

- There exists one unique minimum (Schnörr 1994)

- Further properties (Schnörr 1994):
  – Solution depends continuously on the input data
  – Smoothness: $u, v \in H^2(\Omega)$ (twice differentiable)

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Euler-Lagrange equations

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$$E(u,v) = \int_\Omega (I_x u + I_y v + I_t)^2 + \alpha(|\nabla u|^2 + |\nabla v|^2)\, dx dy$$

- Necessary conditions for a minimum: Euler-Lagrange equations

$$\frac{d}{d\epsilon} E(u(x) + \epsilon h(x), v(x))\Big|_{\epsilon=0} = 0 \qquad \forall h(x)$$

$$\frac{d}{d\epsilon} E(u(x), v(x) + \epsilon h(x))\Big|_{\epsilon=0} = 0 \qquad \forall h(x)$$

- This leads to:

$$(I_x u + I_y v + I_t) I_x - \alpha \Delta u = 0$$

$$(I_x u + I_y v + I_t) I_y - \alpha \Delta v = 0$$

- Discretization needed to solve for (u, v)

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Spatial discretization

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- Euler-Lagrange equation (for u):

$$(I_x u + I_y v + I_t) I_x - \alpha \Delta u = 0$$

- Approximate Laplacian by

0	1	0
1	-4	1
0	1	0

$$\Delta u = u_{xx} + u_{yy}$$

$$\approx \frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{\boxed{i,j}}}{h^2}$$

Pixel indices

- Grid size usually set to $h = 1$

- Relevant in multi-grid schemes

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Gradient descent

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- Gradient descent $$-\frac{dE}{du} = \Delta u - \frac{1}{\alpha}(I_x u + I_y v + I_t) I_x$$
- Most simple iterative scheme (explicit scheme)

$$u_{i,j}^{0} = 0$$

Iteration index

$$u_{i,j}^{k+1} = u_{i,j}^{\boxed{k}} + \tau \left(\Delta u_{i,j}^{k} - \frac{1}{\alpha}(I_x u_{i,j}^{k} + I_y v_{i,j}^{k} + I_t) I_x \right)$$

- Right hand side contains only known values
  → easy to evaluate
- Stable for step sizes $\tau \leq \frac{1}{4}$
- Drawbacks:
  – Terribly slow
  – Updates eventually become too small (number precision)

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Linear system of equations

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- Euler-Lagrange equations

$$(I_x u + I_y v + I_t) I_x - \alpha \Delta u = 0$$

$$(I_x u + I_y v + I_t) I_y - \alpha \Delta v = 0$$

- Discretized version

$$\frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}}{h^2} - \frac{1}{\alpha}(I_x u_{i,j} + I_y v_{i,j} + I_t) I_x = 0$$

$$\frac{v_{i+1,j} + v_{i-1,j} + v_{i,j+1} + v_{i,j-1} - 4v_{i,j}}{h^2} - \frac{1}{\alpha}(I_x u_{i,j} + I_y v_{i,j} + I_t) I_y = 0$$

- Linear in (u, v)

Concatenation of u and v

- Can be written in matrix-vector notation $A^h w = b$

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Linear system of equations

- Sparse linear system

$$A^h = \left(\begin{array}{ccccc|ccccc} d &&&&& d &&&& \\ & d &&&&& d &&& \\ && d &&&&& d && \\ &&& d &&&&& d & \\ &&&& d &&&&& d \\ \hline d &&&&& d &&&& \\ & d &&&&& d &&& \\ && d &&&&& d && \\ &&& d &&&&& d & \\ &&&& d &&&&& d \end{array}\right) + \left(\begin{array}{ccccc|ccccc} s & s & & s & & & & & & \\ s & s & s & & s & & & & & \\ & s & s & s & & & & & & \\ s & & s & s & s & & & & & \\ & s & & s & s & & & & & \\ \hline & & & & & s & s & & s & \\ & & & & & s & s & s & & s \\ & & & & & & s & s & s & \\ & & & & & s & & s & s & s \\ & & & & & & s & & s & s \end{array}\right)$$

- Blocks due to coupling of u and v
- Data term contributes only to block main diagonals
- Smoothness term has four additional block off-diagonals (coupling of neighboring pixels)

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Solving the linear system

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- Gauß-elimination
  Not recommendable for sparse matrices (very slow)
- Iterative methods: Jacobi method (quite slow)
- Faster: Gauß-Seidel method
- Even faster: Successive over-relaxation (SOR)
- Other approaches:
  - Multi-grid methods
  - Conjugate gradient (CG) with a preconditioner (good for GPU implementation)
  - Primal-dual methods (require another numerical approach)

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SOR for Horn-Schunck method

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- Split the matrix into the diagonal part D, the lower triangle L and the upper triangle U

$$Aw = b \quad \Leftrightarrow \quad Dw = b - Lw - Uw$$

- Update scheme:

$$u_i^{k+1} = (1-\omega)\, u_i^k + \omega \frac{\sum_{j \in \mathcal{N}^-(i)} u_j^{k+1} + \sum_{j \in \mathcal{N}^+(i)} u_j^k - \frac{1}{\alpha}((I_x I_y)_i v_i^k + (I_x I_t)_i)}{\sum_{j \in \mathcal{N}^-(i) \cup \mathcal{N}^+(i)} 1 + \frac{1}{\alpha}(I_x^2)_i}$$

- Relaxation parameter $\omega \in (0, 2)$

- Gauß-Seidel method for $\omega = 1$

- Optimal ω (fastest convergence) depends on the problem
  Horn-Schunck method: $\omega = 1.98$

[Thomas Brox. From Pixels to Regions: Partial Differential Equations in Image Analysis. 2005]

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Computation speed with various solvers

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- Results from Bruhn et al. 2005, CPU

	Iterations	Frames/sec	Speedup
Gauß-Seidel	21931	0.026	1
SOR	286	1.381	54
Cascadic GS	237	1.667	65
Cascadic SOR	25	8.125	316
Full multi-grid	1	18.229	708

- Speedup of multi-grid techniques much smaller for extended models

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Average angular error

- Established measure of how accurate an optic flow method is

- Computes the angle between the correct $(u_c, v_c)^\top$ and estimated $(u, v)^\top$ flow vector

$$\text{AAE} := \frac{1}{n}\sum_{i=1}^{n} \arccos\left(\frac{(u_c)_i u_i + (v_c)_i v_i + 1}{\sqrt{((u_c)_i^2 + (v_c)_i^2 + 1)(u_i^2 + v_i^2 + 1)}}\right)$$

- Alternative: average endpoint error:

$$\text{AEE} := \frac{1}{n}\sum_{i=1}^{n} \left(((u_c)_i - u_i)^2 + ((v_c)_i - v_i)^2\right)$$

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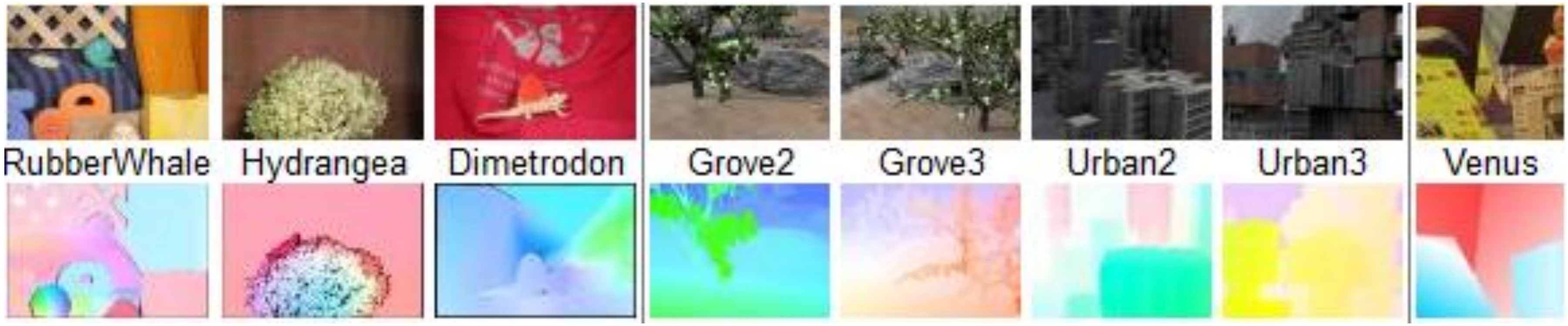
Middlebury flow benchmark

Popular optical flow benchmark
http://vision.middlebury.edu/flow/
Poor correlation with real-world problems

8 test sequences with ground truth



8 training sequences with ground truth

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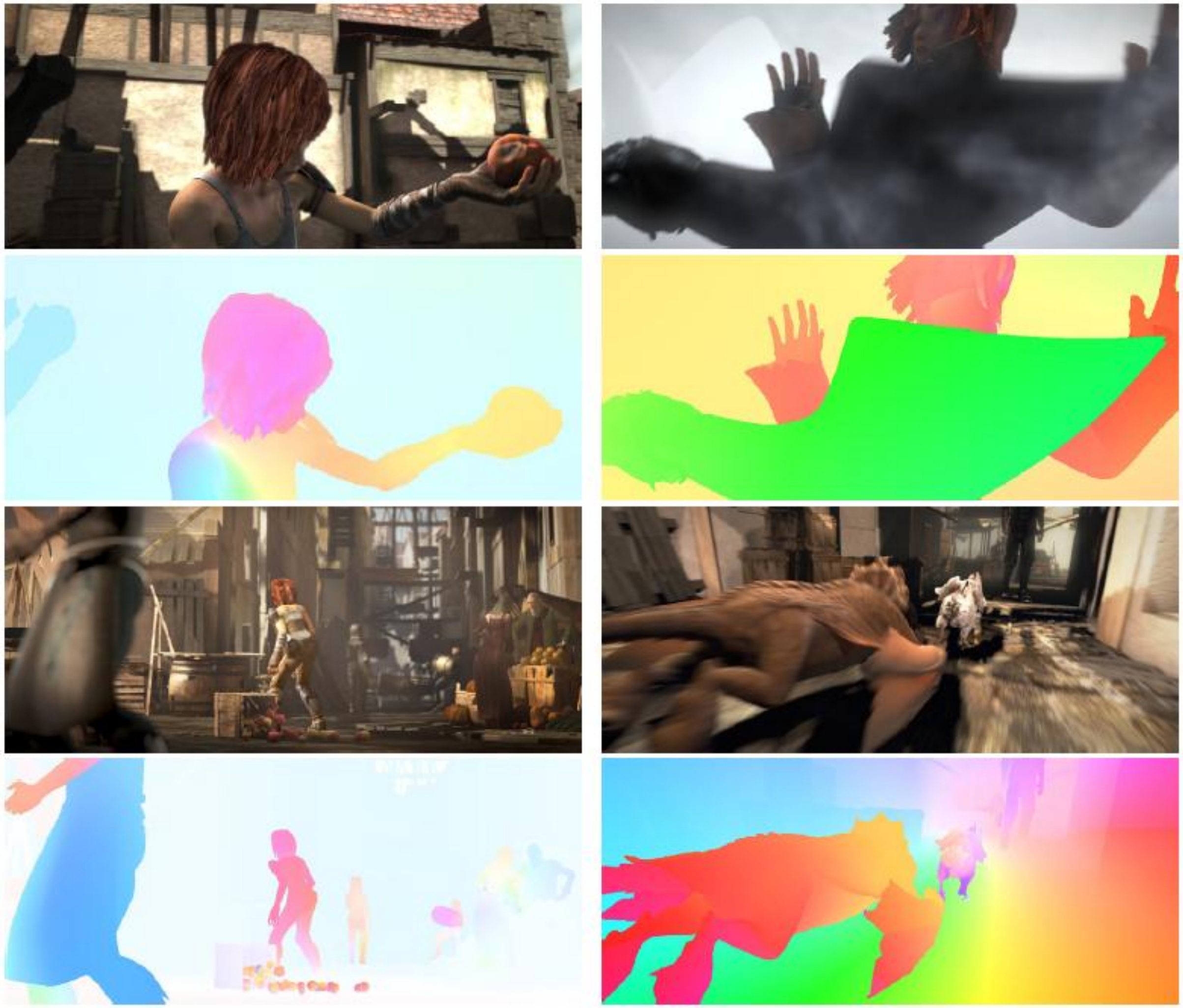
MPI-Sintel Optical Flow Dataset

Ground-truth from open-source movie Sintel (Butler et al. ECCV 2012)



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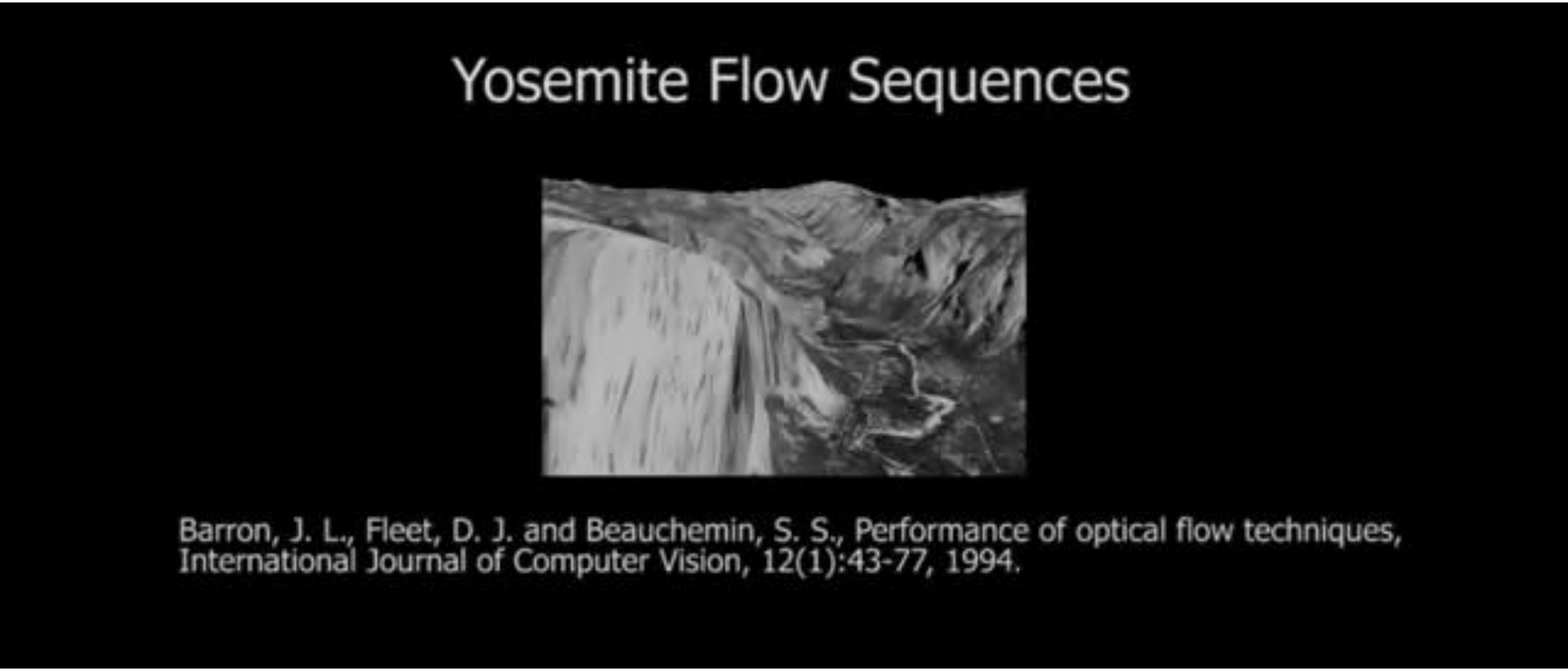
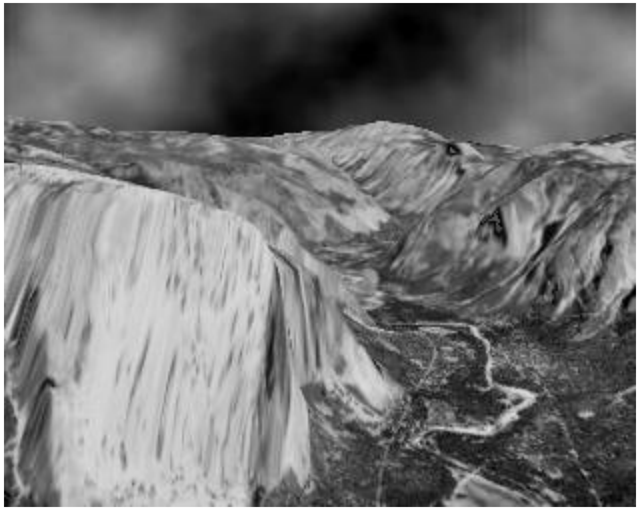
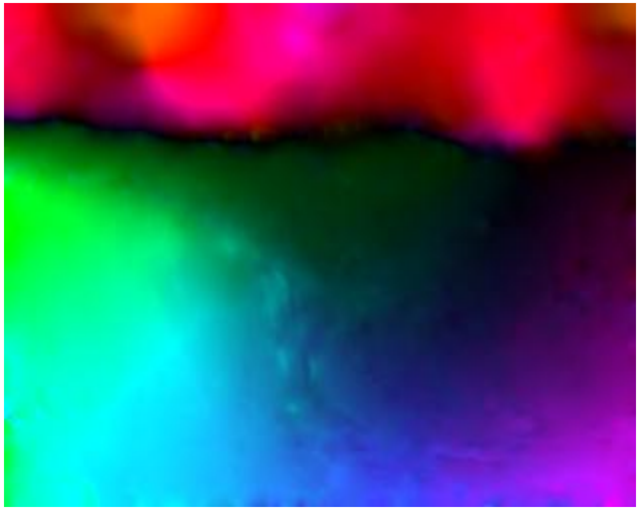
Optical Flow Dataset



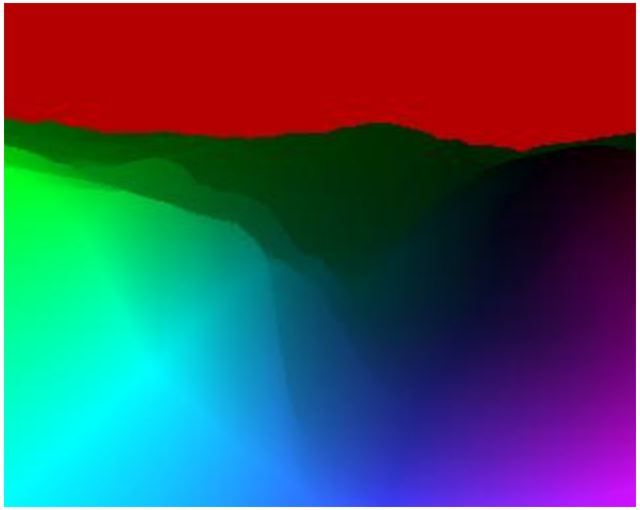
Results with Horn-Schunck method



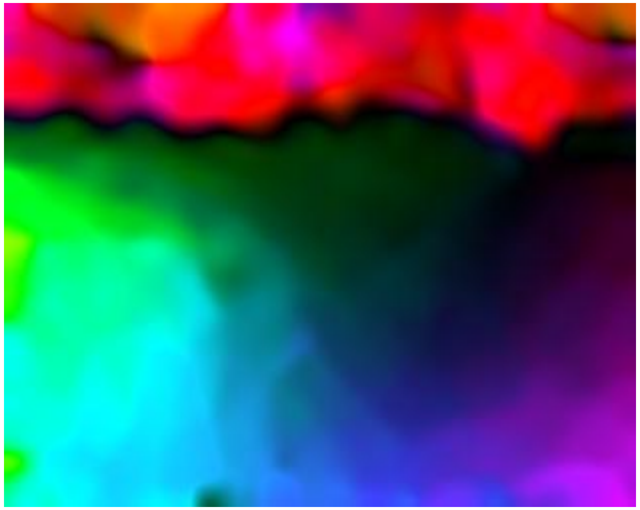
Input sequence



Horn-Schunck (7.17°)



Ground truth



Lucas-Kanade (8.78°)

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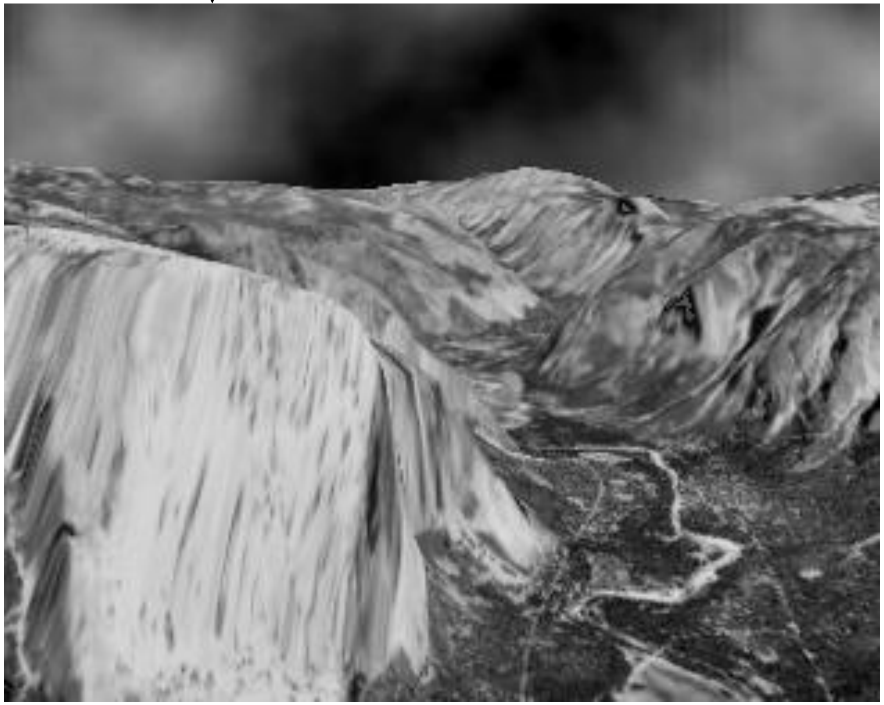
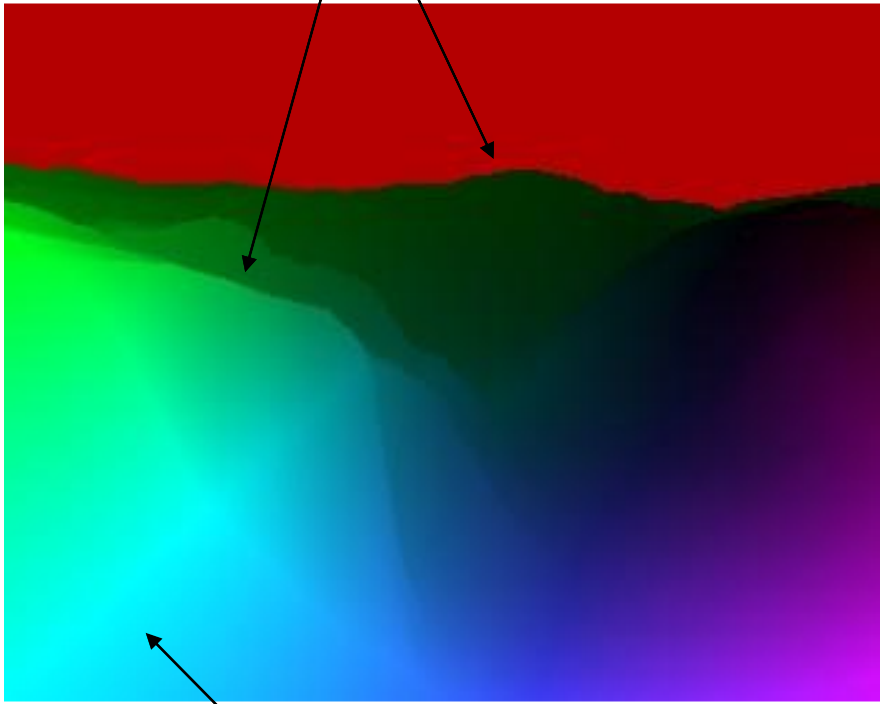
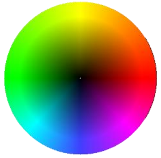
Yosemite test sequence

Illumination changes

Motion discontinuities





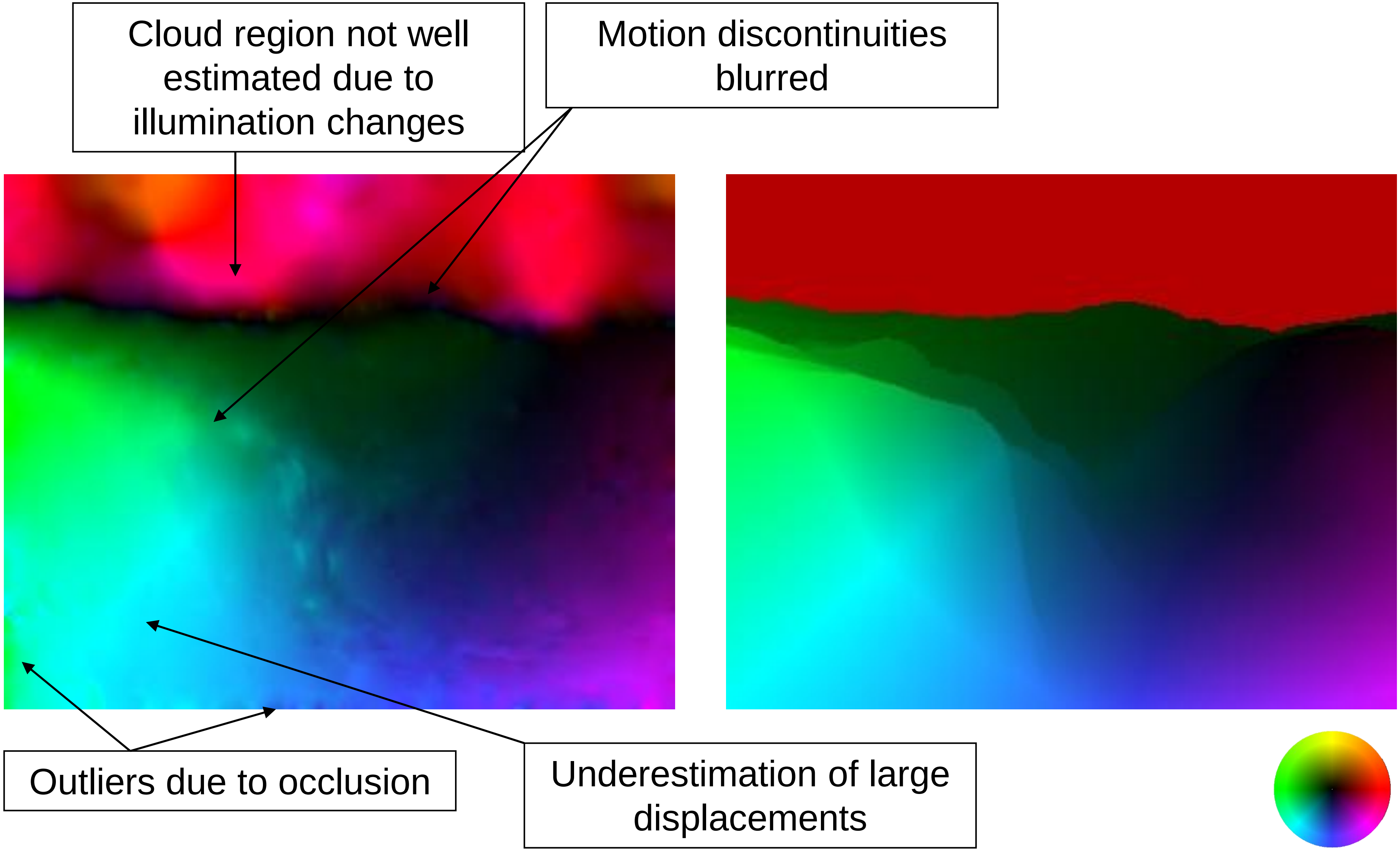


Occlusion

“Large” displacements

Thomas Brox

Limitations of Horn-Schunck method



Thomas Brox

Motion discontinuities

Occlusions

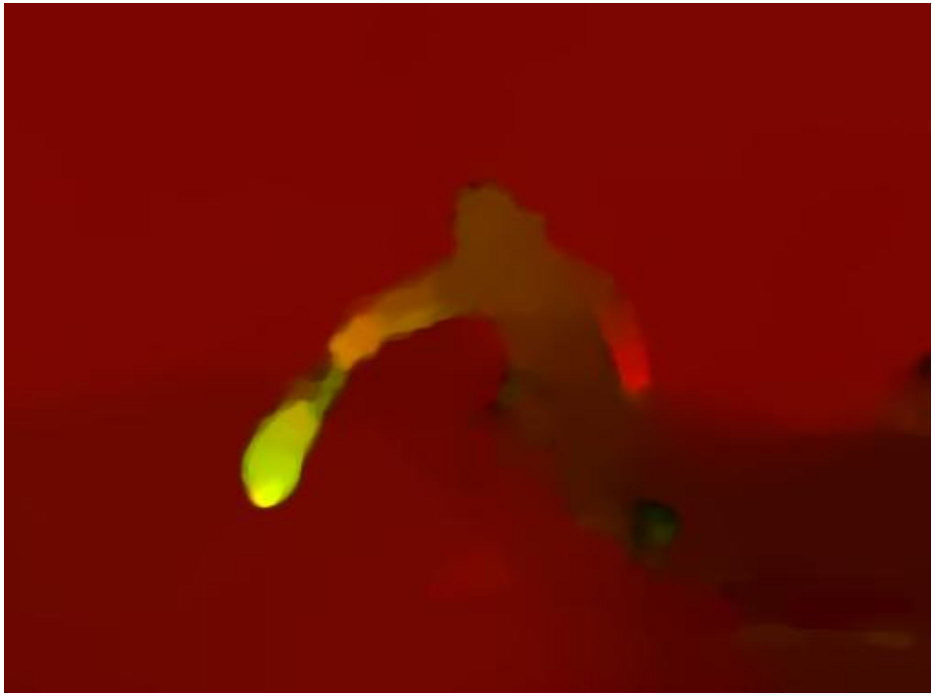
Illumination changes

Large displacements

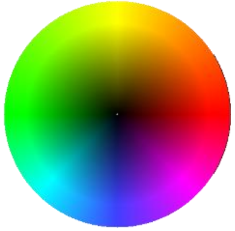
Really large displacements

Thomas Brox

Motion discontinuities



- Different, independently moving objects cause discontinuities in the correct flow field
- Usually we do not know the object boundaries



Thomas Brox

Motion discontinuities and the smoothness assumption

- Fact: smoothness assumption not satisfied everywhere
- Quadratic penalizer → Gaussian error distribution

$$E(u,v) = \int_\Omega (I_x u + I_y v + I_t)^2 + \alpha(|\nabla u|^2 + |\nabla v|^2)\, dx dy$$

- Too much influence given to outliers → blurring

Thomas Brox

Non-quadratic smoothness term

Applying a robust function:

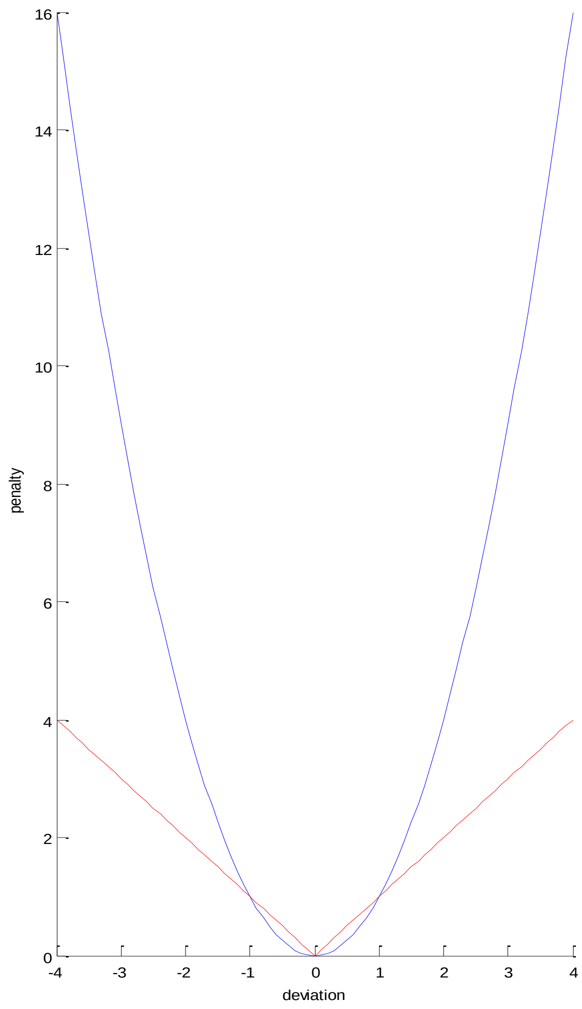
$$E_S(u,v) = \int_\Omega \Psi(|\nabla u|^2 + |\nabla v|^2)\, dxdy$$

For example:

Total variation (TV)

$$\Psi(s^2) = \sqrt{s^2 + \epsilon^2}, \quad \epsilon = 0.001$$

Advantage: still convex
→ global optimum!



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Fixed-point iteration

Lagrange equations for regularizer

$$-\text{div}\left(\Psi'(|\nabla u|^2 + |\nabla v|^2)\nabla u\right) = 0$$

$$-\text{div}\left(\Psi'(|\nabla u|^2 + |\nabla v|^2)\nabla v\right) = 0$$

TV penalizer depends on the unknown flow

$$\Psi'(s^2) = \frac{1}{\sqrt{s^2 + \epsilon^2}}$$

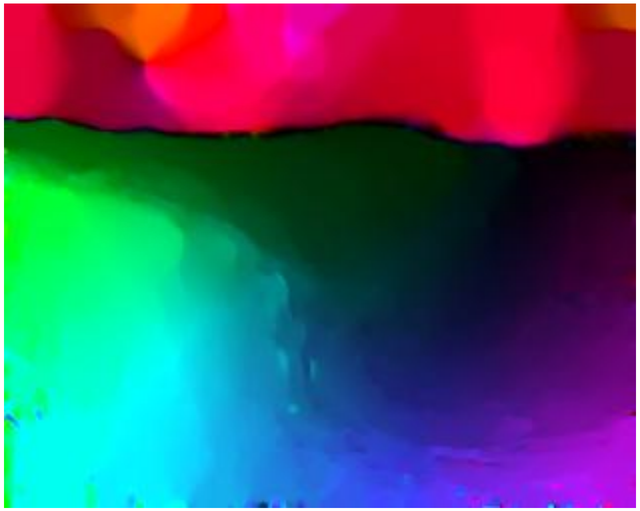
Solution:

- Keep Ψ' fixed for an (initial) solution (u, v) (fixed point)
- Solve resulting linear system to obtain a new fixed point
- Update Ψ', iterate

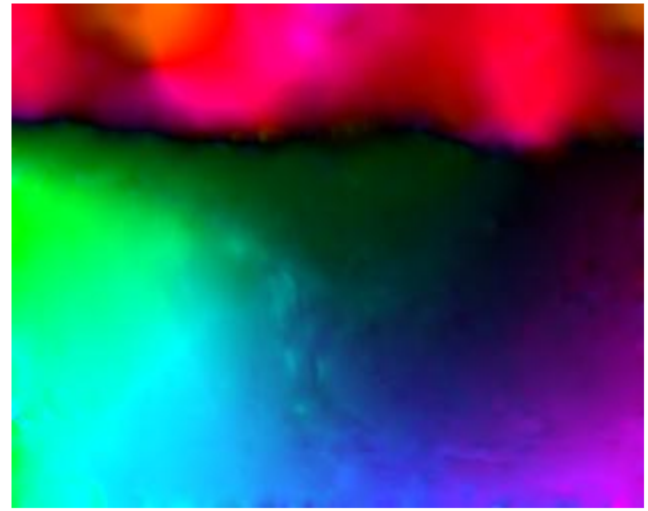
[Brox et al. **High Accuracy Optical Flow Estimation Based on a Theory for Warping**. ECCV 2004]

Thomas Brox

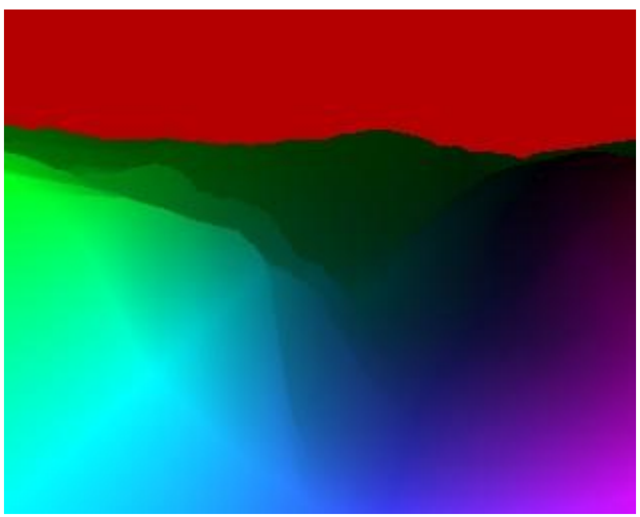
Result: improvements at motion discontinuities



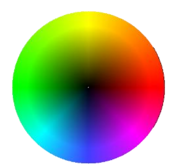
Non-quadratic smoothness term (6.36°)



Horn-Schunck (7.17°)



Ground truth



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Occlusions

Some pixels of frame 1 no longer visible in frame 2

Pixels matched to most similar pixels in frame 2
→ erroneous motion vectors

Thomas Brox

Dealing with occlusions and non-Gaussian noise

Robust function applied to the data term:
(Black-Anandan 1996, Mémin-Pérez 1998)

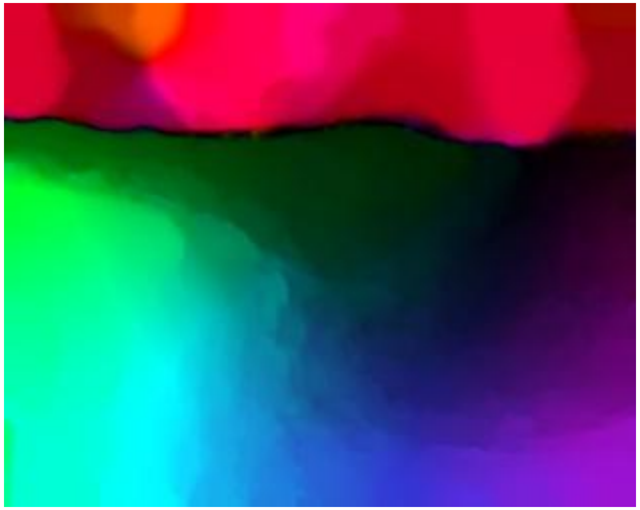
$$E(u,v) = \int_{\Omega} \Psi\left((I_x u + I_y v + I_t)^2\right) + \alpha \Psi\left(|\nabla u|^2 + |\nabla v|^2\right) \, dx dy$$

Interpretation: less importance given to pixels with high matching cost → mainly occlusions

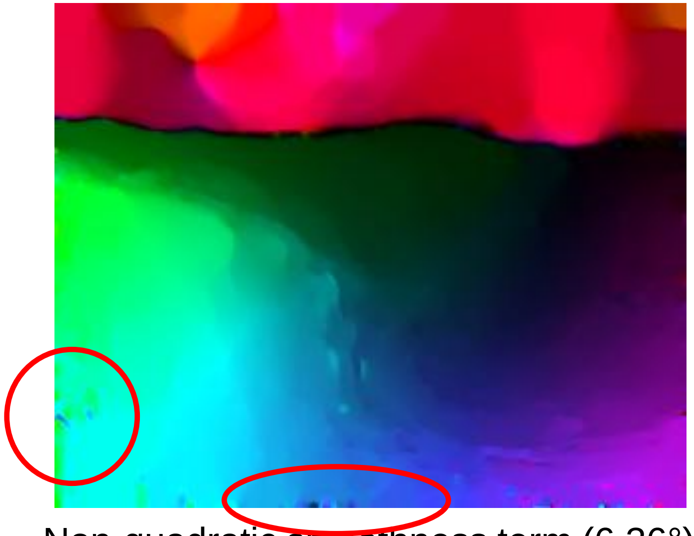
Nonlinear system can again be solved with fixed point iterations

Thomas Brox

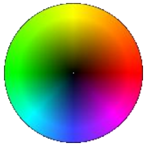
Result: Better treatment of occlusions

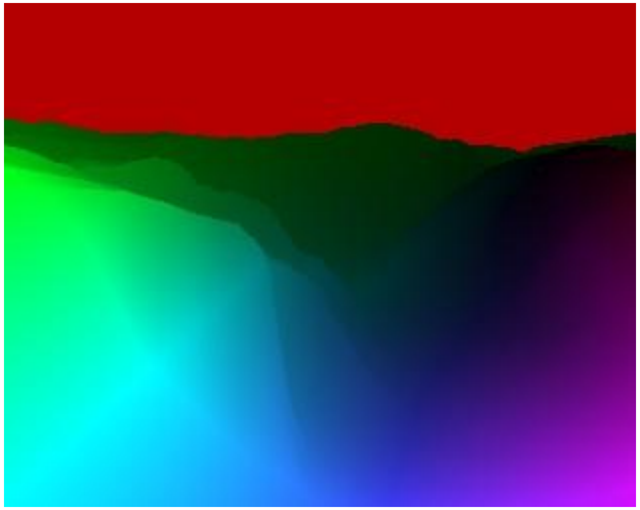


Both terms non-quadratic (5.97°)

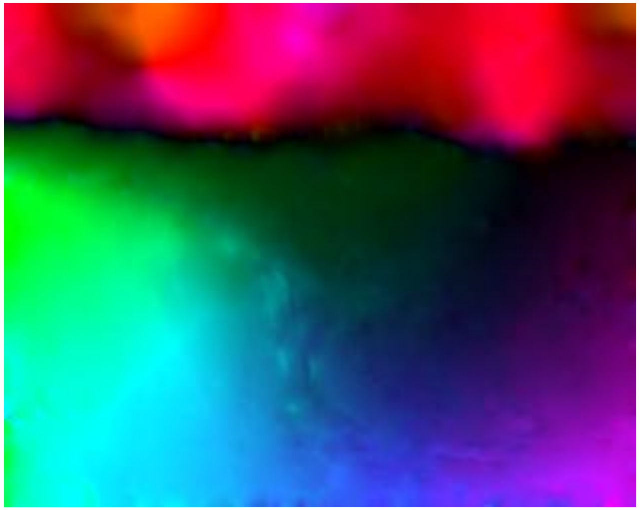


Non-quadratic smoothness term (6.36°)





Ground truth



Horn-Schunck (7.17°)

Thomas Brox

Illumination changes

Typically caused by:

- shadows
- light source flickering
- self-adaptive cameras
- different viewing angles and non-Lambertian surfaces

“Schefflera” from Middlebury benchmark

Two frames from a Miss Marple movie

Thomas Brox

Matching the gradient

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- Brightness constancy assumption:

$$I(x+u, y+v, t+1) - I(x, y, t) = 0$$

- Gradient constancy assumption
(Uras et al. 1988, Schnörr 1994, Brox et al. 2004)

$$\nabla I(x+u, y+v, t+1) - \nabla I(x, y, t) = 0$$

- Two important positive effects:
  1. More information: two more equations
  2. Invariant to additive brightness changes

- Negative effects:
  1. Not rotation invariant
  2. More sensitive to noise

Thomas Brox

Combination of features

- Linearization:

$$I_{xx}u + I_{xy}v + I_{xt} = 0$$

$$I_{xy}u + I_{yy}v + I_{yt} = 0$$

- Combine with gray value constancy assumption:

$$E(u,v) = \int_\Omega \Psi\left((I_x u + I_y v + I_t)^2\right) dxdy$$

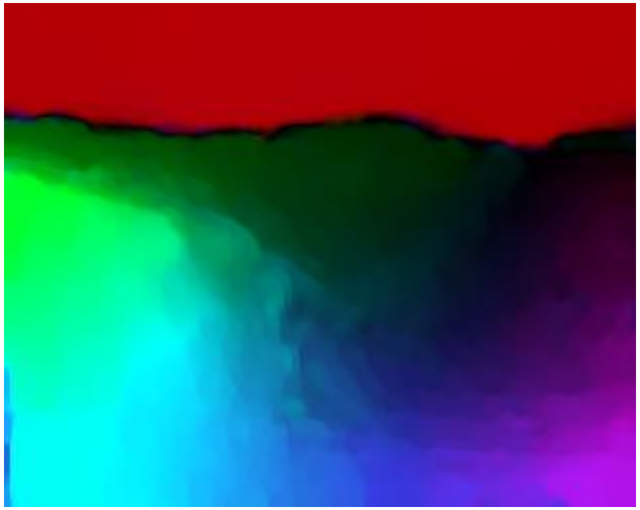
$$+\gamma \int_\Omega \Psi\left((I_{xx}u + I_{xy}v + I_{xt})^2 + (I_{xy}u + I_{yy}v + I_{yt})^2\right) dxdy$$

$$+ \alpha E_{\text{Smooth}}$$

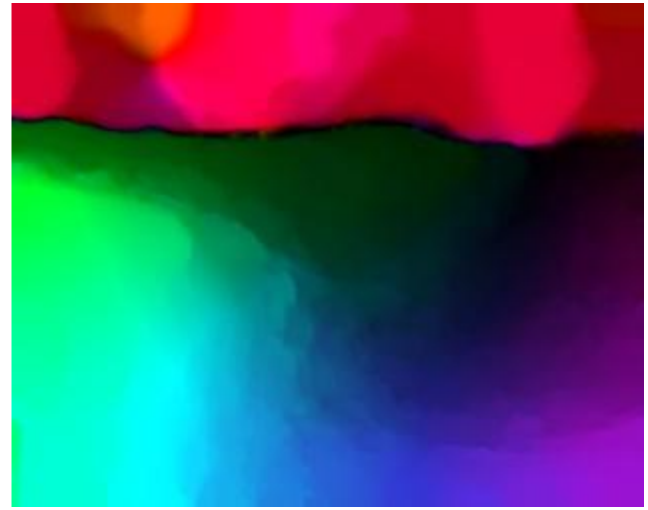
- Separate robust penalizer Ψ for each feature (Bruhn-Weickert 2005)

Thomas Brox

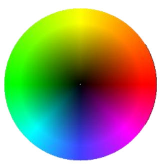
Result: Illumination changes no longer a problem

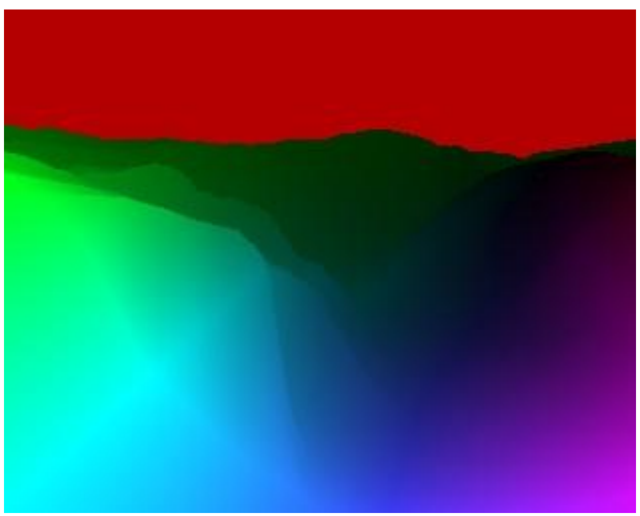


Gradient constancy (3.5°)

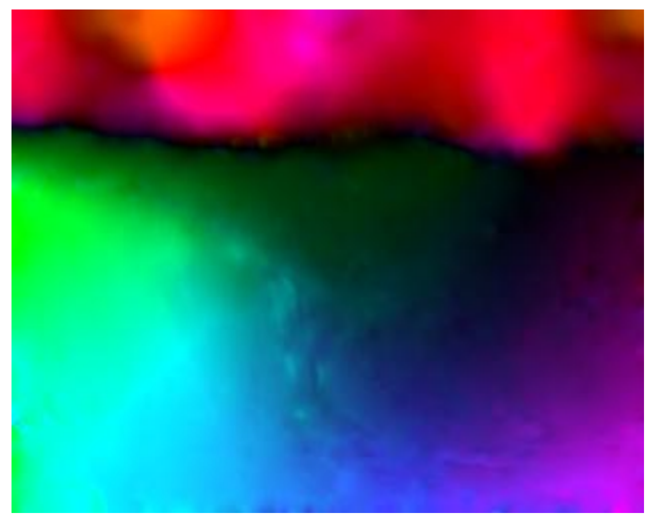


Both terms non-quadratic (5.97°)





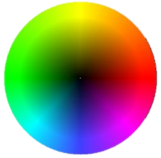
Ground truth

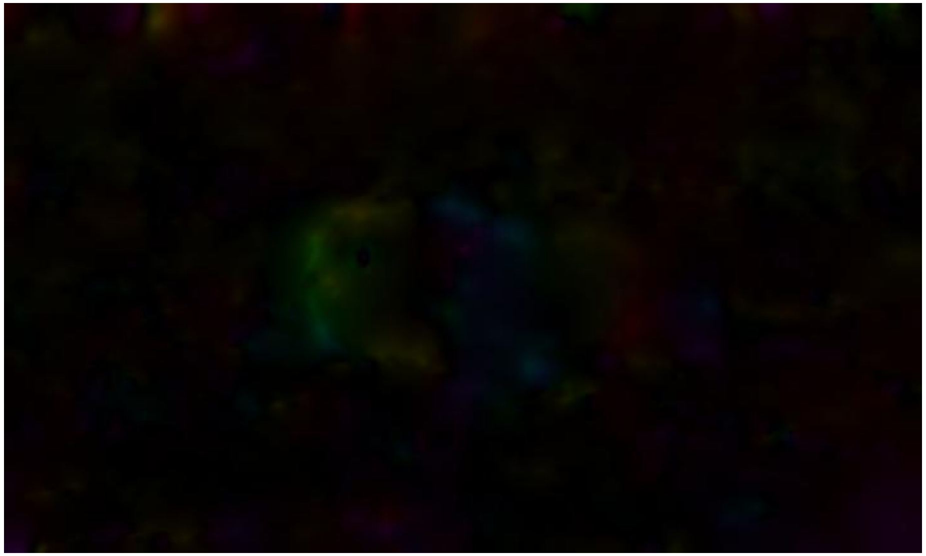


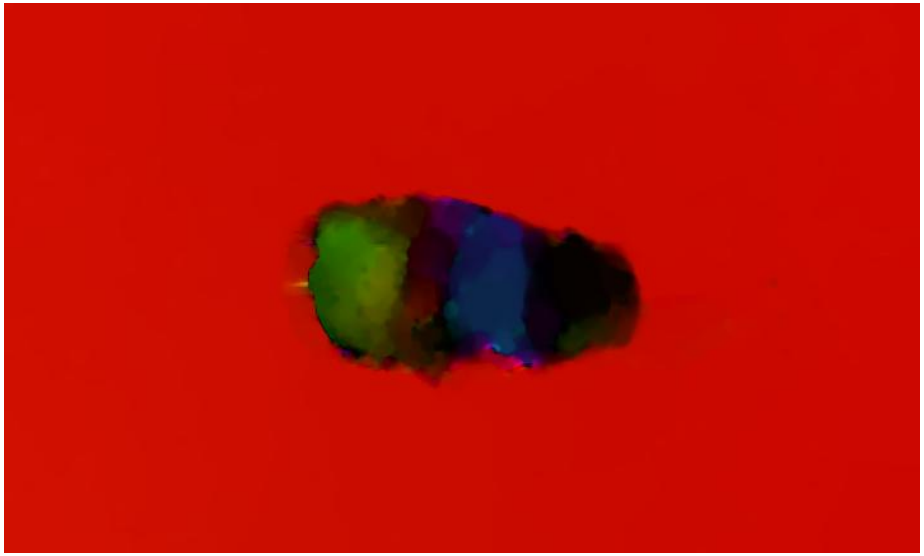
Horn-Schunck (7.17°)

Thomas Brox

Large displacements







Horn&Schunck

Brox et al. 2004

Thomas Brox

Linearization and large displacements

- Constancy assumptions have been linearized

$$I(x+u, y+v, t+1) - I(x, y, t) = 0 \quad \rightarrow \quad I_x u + I_y v + I_t = 0$$

- Assumes that the image is a linear gradient

- Locally (between two pixels), this is true
  → Linearization only valid for very small displacements (subpixel displacements)

- If images are quite smooth, approximation is good enough for larger displacements

- But typical images are not smooth at all

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Non-linearized constancy assumption

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- Energy functional:

$$E(u,v) = \int_{\Omega} \Psi\left((I(\mathbf{x}+\mathbf{w}) - I(\mathbf{x}))^2\right) + \alpha\Psi\left(|\nabla u|^2 + |\nabla v|^2\right) \mathbf{dx}$$

$$\mathbf{x} := (x, y, t) \qquad \mathbf{w} := (u, v, 1)$$

- Correct description of the matching criterion (no approximation)

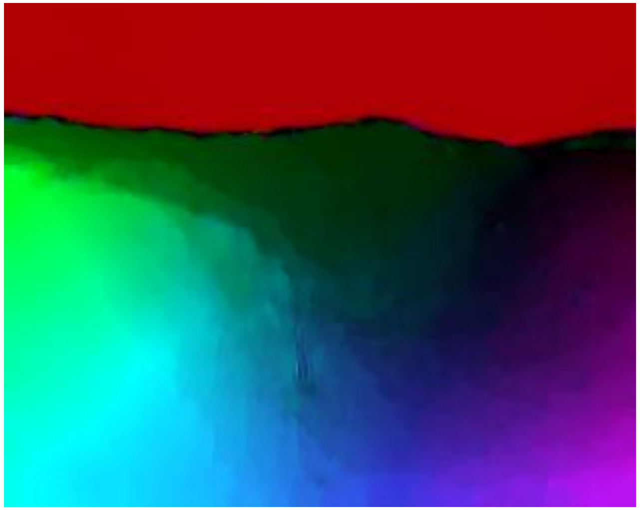
- Problem:
  - Not convex in $\mathbf{w}$ → multiple local minima

- One can work with the non-linearized assumptions and iteratively estimate an increment using the Gauss-Newton method

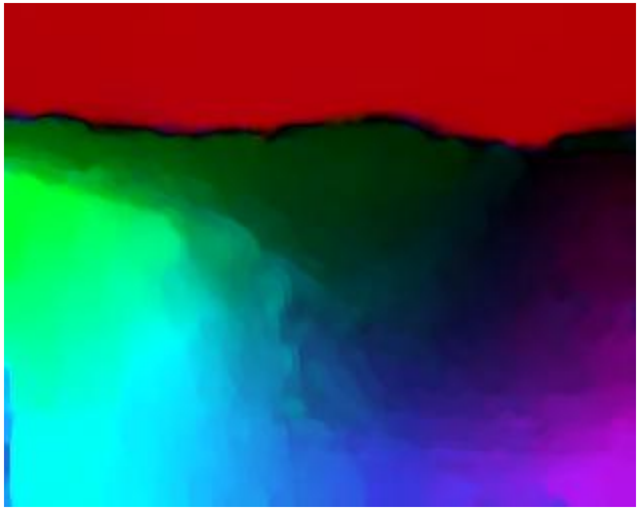
- Computing the increment requires computation of warped images

Thomas Brox

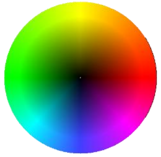
Result: higher accuracy, large displacements possible

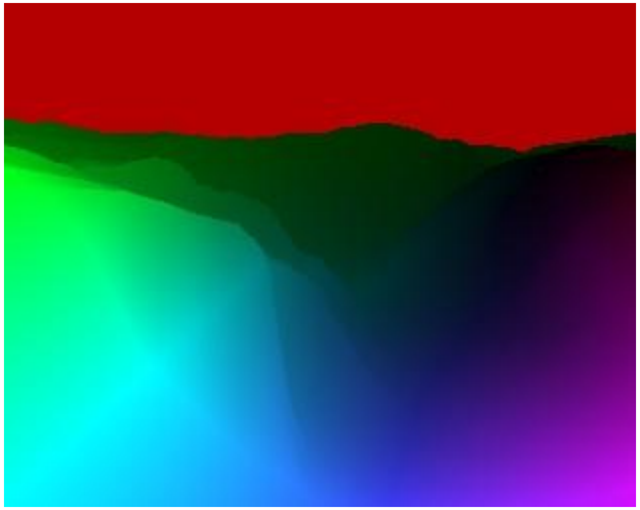


Non-linearized constancy (2.44°)

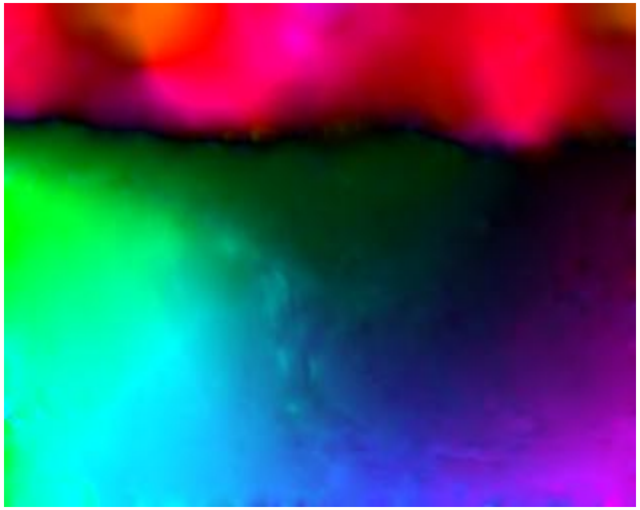


Gradient constancy (3.5°)





Ground truth



Horn-Schunck (7.17°)

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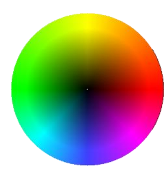
Other results: zoom into a scene



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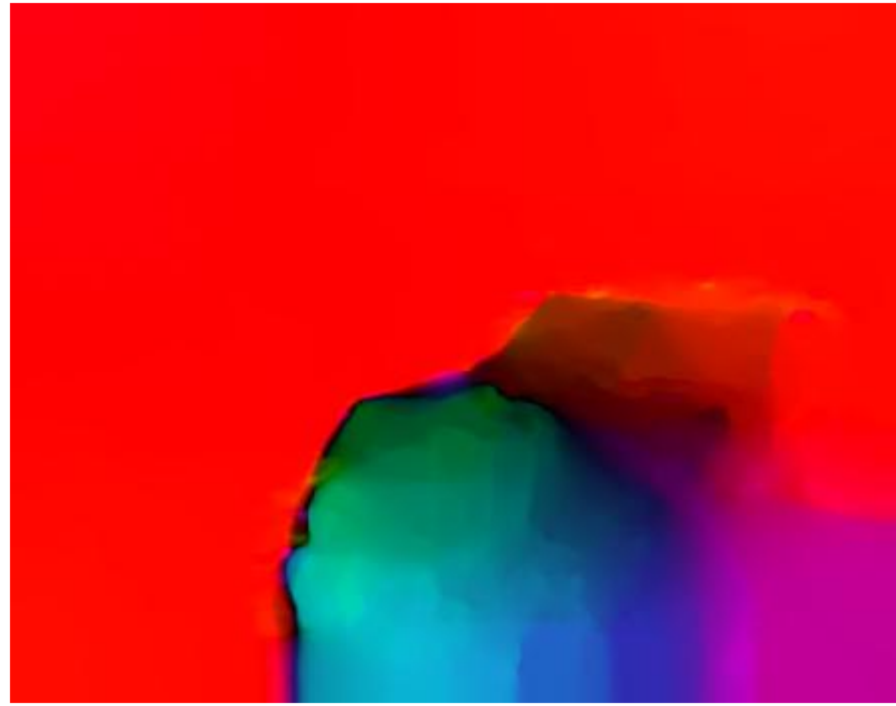
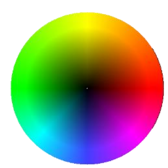
Car in a movie



from “Miss Marple: A Pocket Full of Rye”



Thomas Brox

SIFT flow (Liu et al. 2008)

- Label transfer from training to test data

Training image

Training annotation

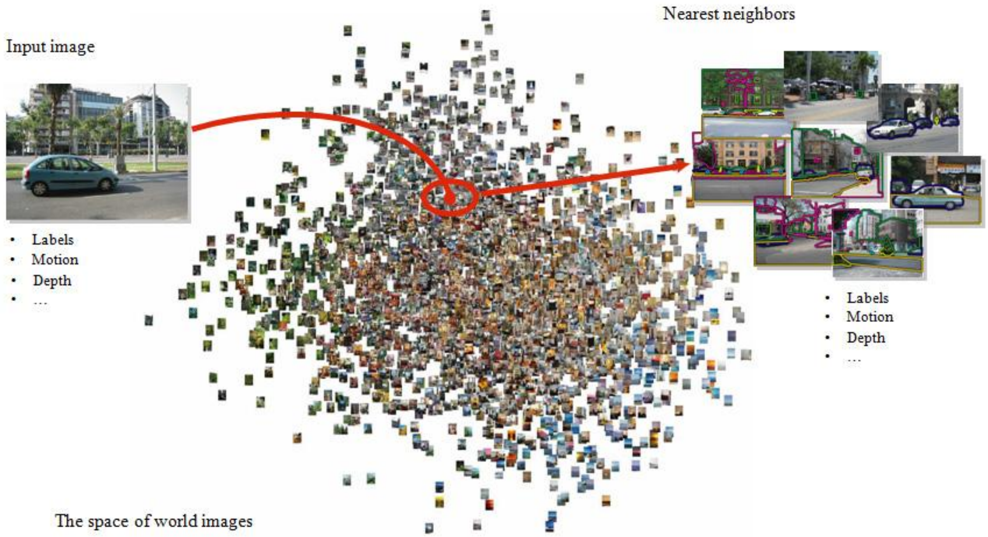
Test image

Warped annotation

- Requires flow from test image to training image

- Only feasible if the two images are sufficiently similar

- Search most similar training image in a database using global image descriptors



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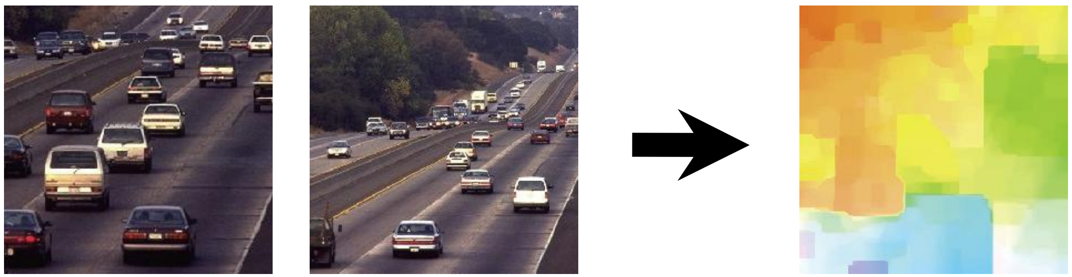
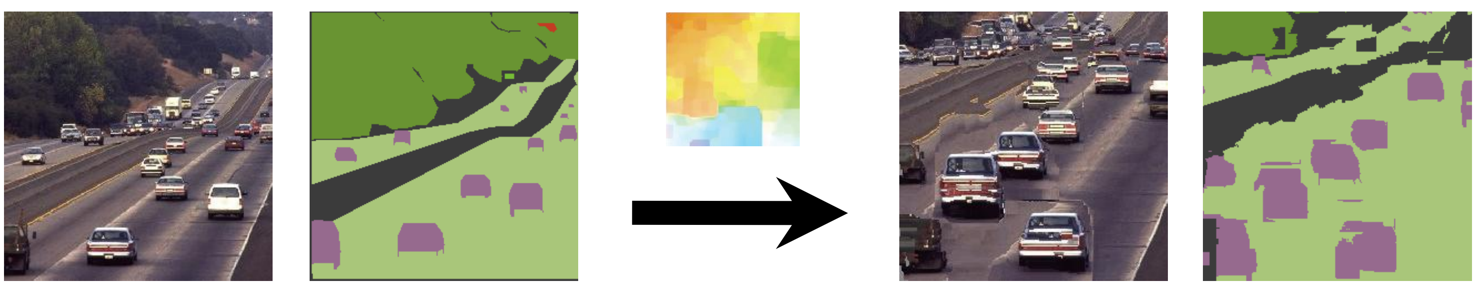
SIFT flow

- Compute SIFT descriptors densely in both images
- Apply combinatorial method (Shekhovtsov et al. Efficient MRF Deformation Model for Non-Rigid Image Matching. CVIU 2008) to estimate the flow between both images



- Warping the annotation of the training image yields the transferred annotation of the test image



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More results

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Image 1

Image 2

SIFT flow

Warping

Image 1

Image 2

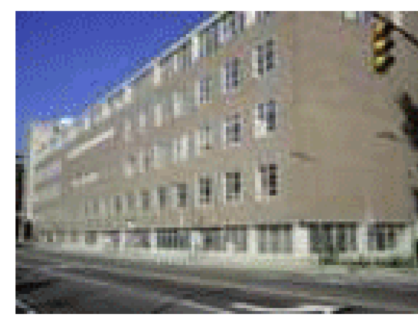
SIFT flow

Warping

Image 1

Image 2

SIFT flow



Warping

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Incorporating correspondences from descriptor matching

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$$
\begin{aligned}
E(\mathbf{w}(\mathbf{x})) = & \int \Psi\left(|I_2(\mathbf{x}+\mathbf{w}(\mathbf{x})) - I_1(\mathbf{x})|^2\right) d\mathbf{x} \\
& + \gamma \int \Psi\left(|\nabla I_2(\mathbf{x}+\mathbf{w}(\mathbf{x})) - \nabla I_1(\mathbf{x})|^2\right) d\mathbf{x} \\
& + \alpha \int \Psi\left(|\nabla u(\mathbf{x})|^2 + |\nabla v(\mathbf{x})|^2\right) d\mathbf{x}
\end{aligned}
$$

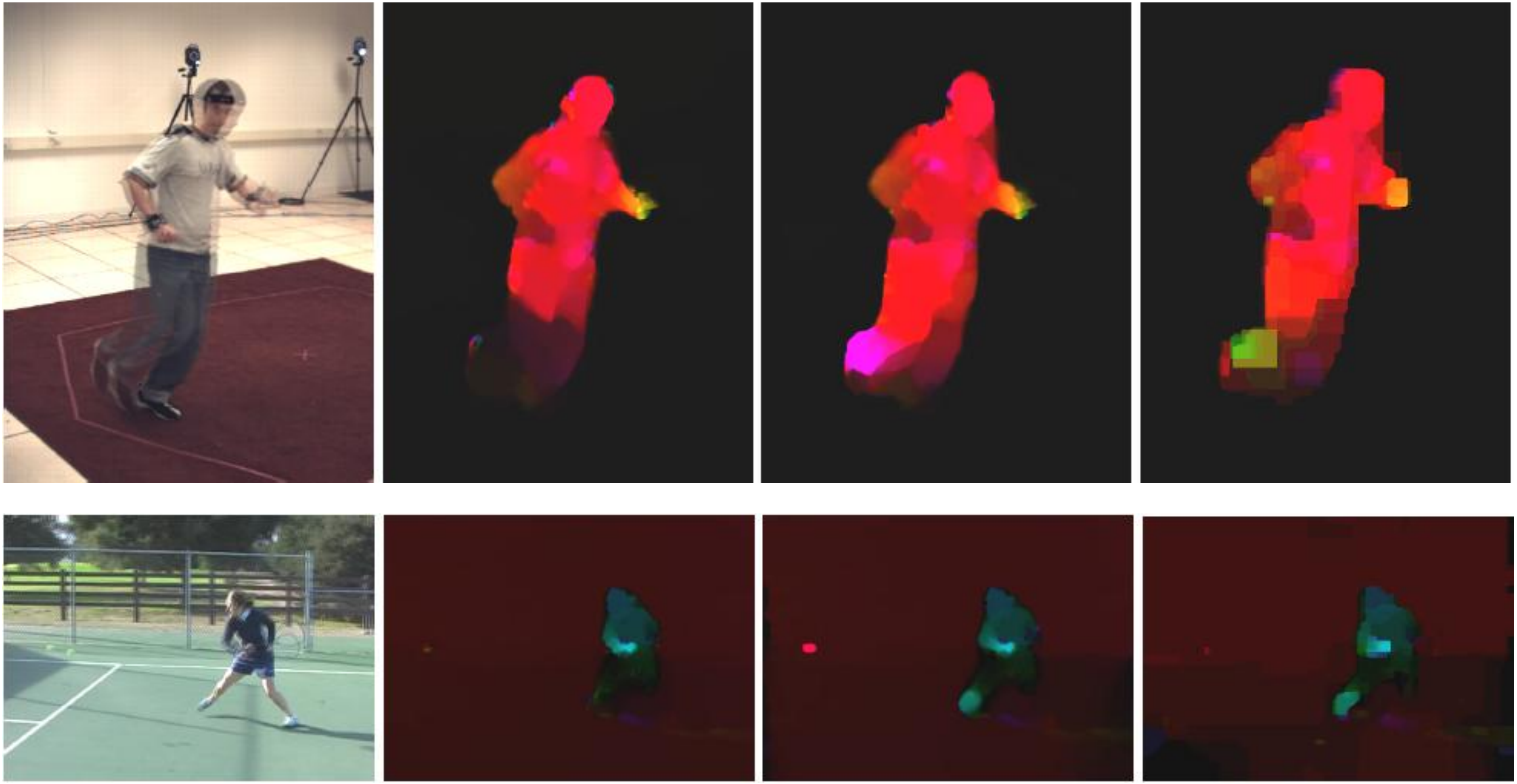
point correspondences from HOG descriptors

$$
+\beta \int \underbrace{\rho(\mathbf{x})}_{\text{Matching score}} \underbrace{\Psi}_{\text{Robust function}}\Big(\underbrace{(u(\mathbf{x}) - u'(\mathbf{x}))^2 + (v(\mathbf{x}) - v'(\mathbf{x}))^2}_{\text{Flow = correspondence vector}}\Big) d\mathbf{x}
$$

[T. Brox and J. Malik. **Large displacement optical flow: descriptor matching in variational motion estimation**. PAMI 2011]

Thomas Brox

Comparison to SIFT flow



Input images (overlaid) | Classical coarse-to-fine method | LDOF | SIFT flow

Thomas Brox

How do we find object regions?

Video analysis/motion segmentation

- Motion cues can be very clear when objects move
- Several indications in biological vision (studies with infants and healed blind people)

Thomas Brox

Motion segmentation

- Goal: separate regions by their motion

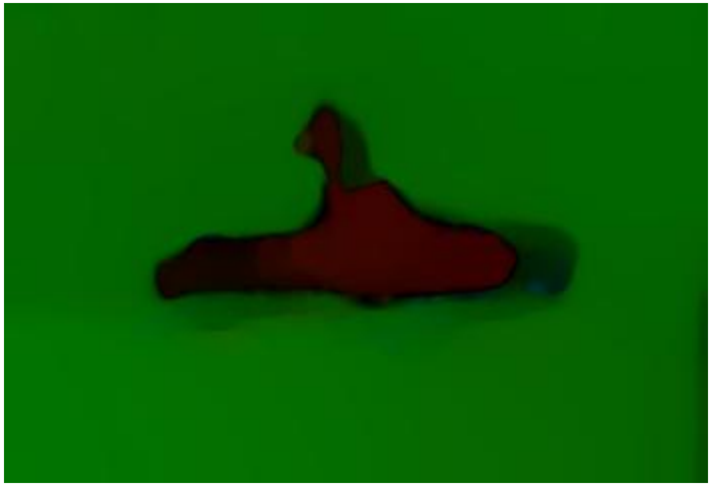
- Typical chicken-egg problem:
  - A motion field is necessary for estimating the partitioning
  - A partitioning helps estimate the motion field

- Standard approach (layered motion: Wang-Adelson 1994):
  1. First compute optical flow
  2. Then compute segmentation based on the optical flow

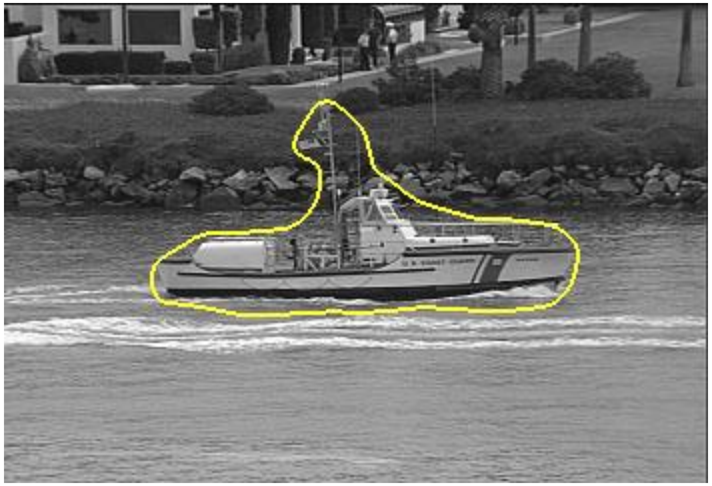
Thomas Brox

Example

Input sequence



Estimated flow field



Flow segmentation

Thomas Brox

Simultaneous flow estimation and segmentation

- So far: two-step approach
  - Decide on the optic flow
  - Use result for segmentation
- Problem: wrong decisions in the first step cannot be corrected → optical flow estimation cannot benefit from segmentation
- Solution: state joint problem in one energy functional
- Optimize in both variables simultaneously

Thomas Brox

Motion competition

- Joint energy functional (Cremers-Soatto 2005):

$$E(u_i, v_i, C) = \sum_{i=1}^{N} \int_{\Omega_i} (I_x u_i + I_y v_i + I_z)^2 \,\mathbf{dx} + \nu |C|$$

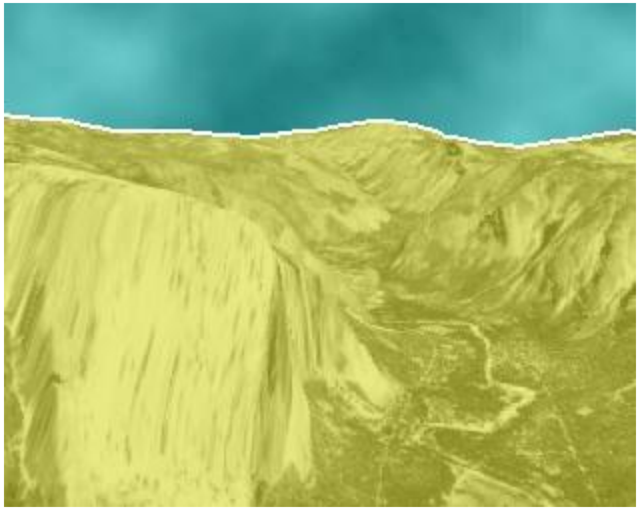
- Remarks:
  – Partitioning of the image domain into N disjoint regions
  – Simple parametric (constant) motion field in each region Ω_i
  – Length constraint prefers smooth contours

- Alternating optimization:
  – Estimate motion field in the initial regions (Lucas-Kanade method restricted to the region's domain)
  – Evolve the contour according to the motion field (e.g. with level sets)
  – Iterate

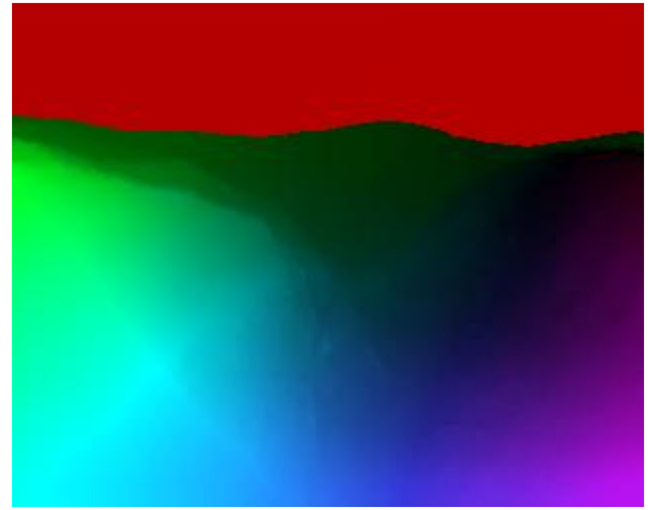
- Segmentation and motion estimation are coupled and benefit from each other

Thomas Brox

Two easy examples



Segmentation



Optical flow

Segmentation

Optical flow

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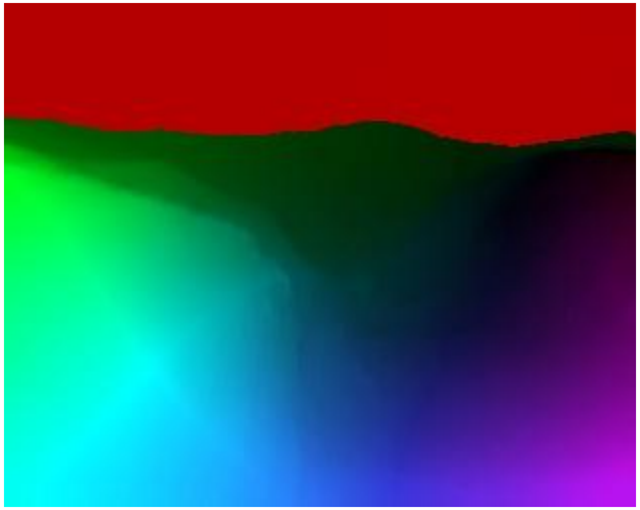
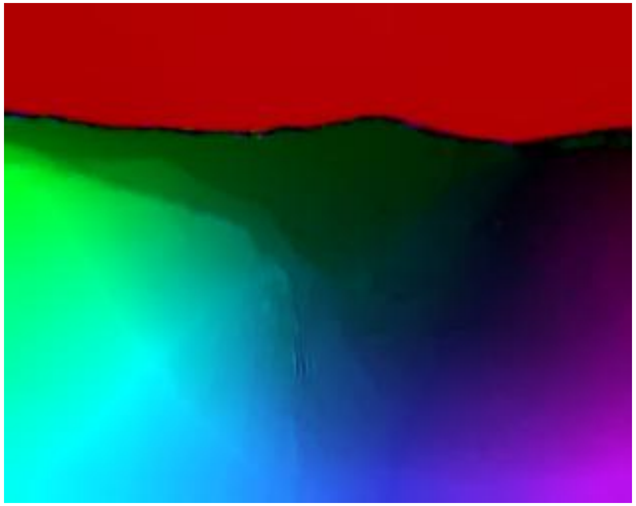
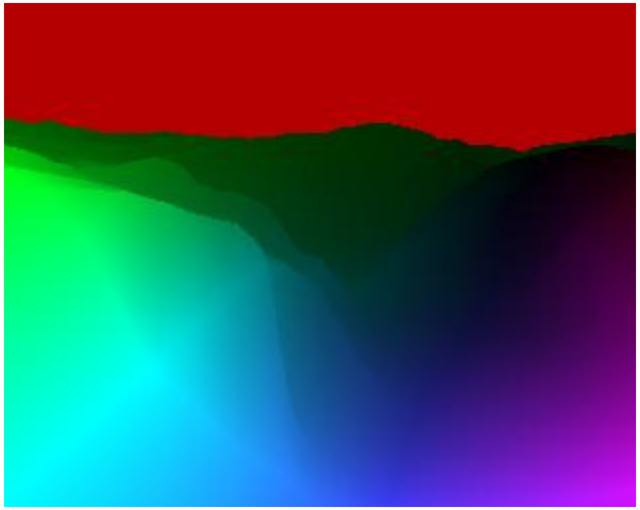
Accurate motion fields



With segmentation (1.22°)



Without segmentation (1.78°)



Correct flow field

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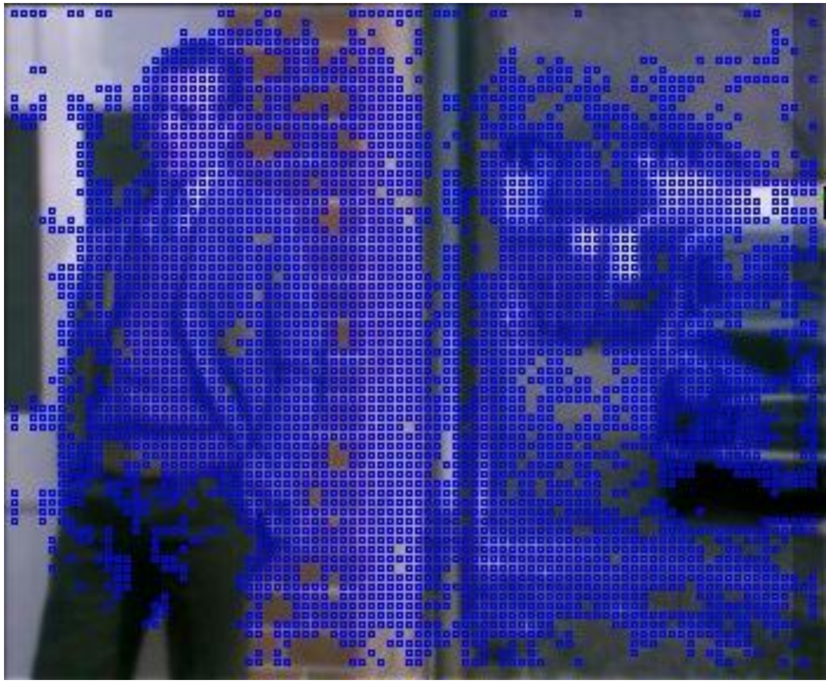
Motion segmentation based on point trajectories

- Two-frame optical flow can only separate objects with different motion in these two particular frames
- Idea: make use of the temporal context
  → point trajectories over multiple frames
- Trajectories obtained via point tracking
- Clustering based on this information can separate objects even at times where they do not move



Trajectories obtained with tracker from Sundaram et al. 2010

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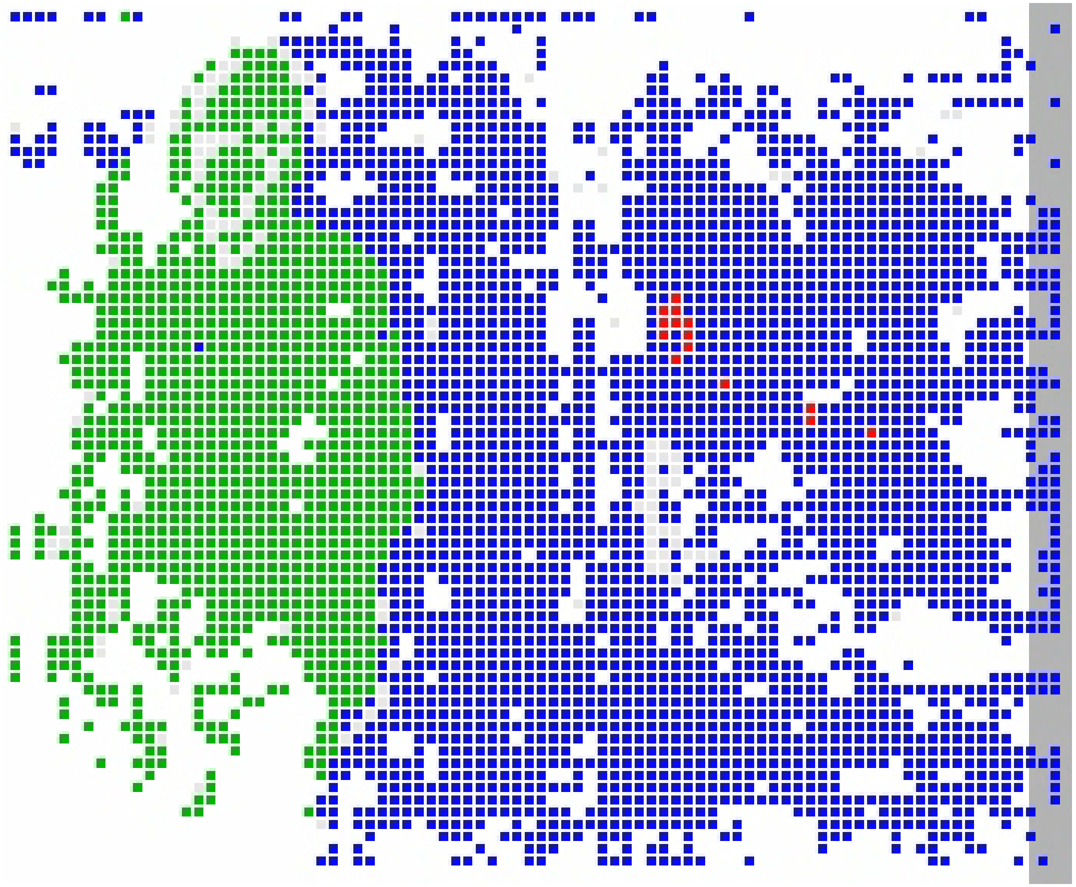
Clustering of point trajectories

Point trajectories → Single graph, pairwise distances → Spectral clustering → Clustered trajectories



[P. Ochs. **Segmentation of moving objects by long term video analysis**. PAMI 2014]

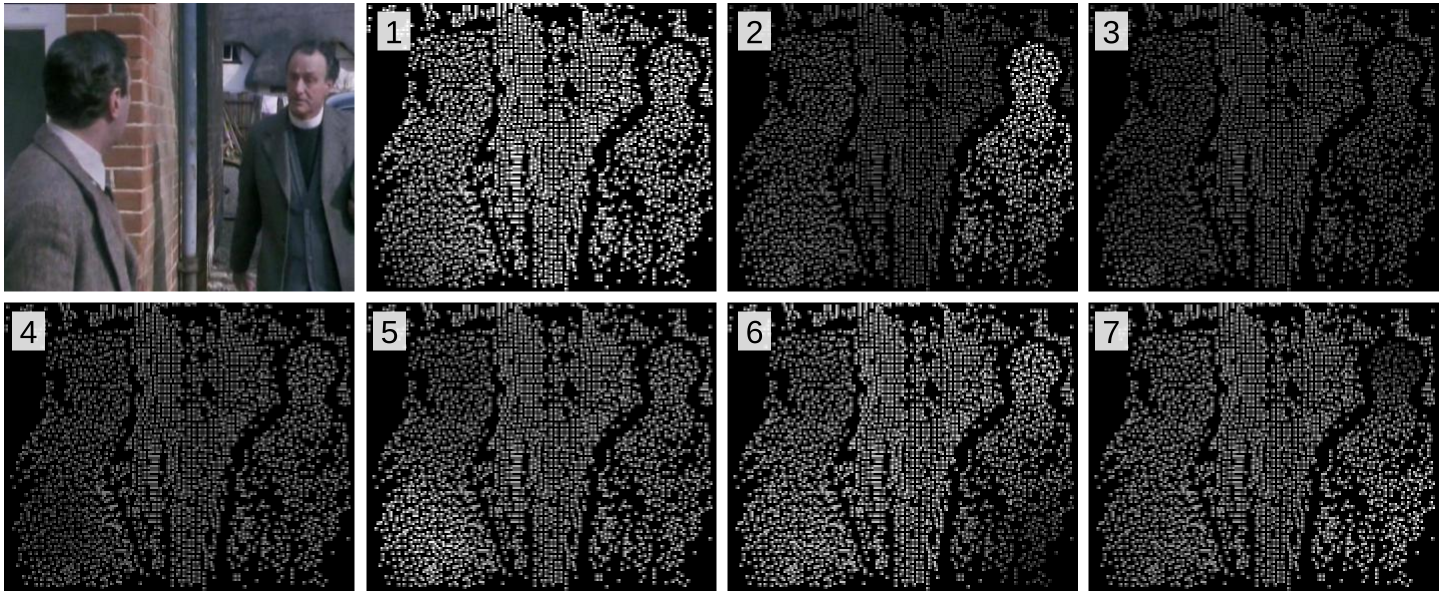
Thomas Brox

Recall: Spectral clustering

1. Compute Laplacian eigenmap

$$V^\top \Lambda V = D^{-1/2}(D - W)D^{-1/2}$$

2. Consider all eigenvectors with eigenvalue $\lambda < 0.04$

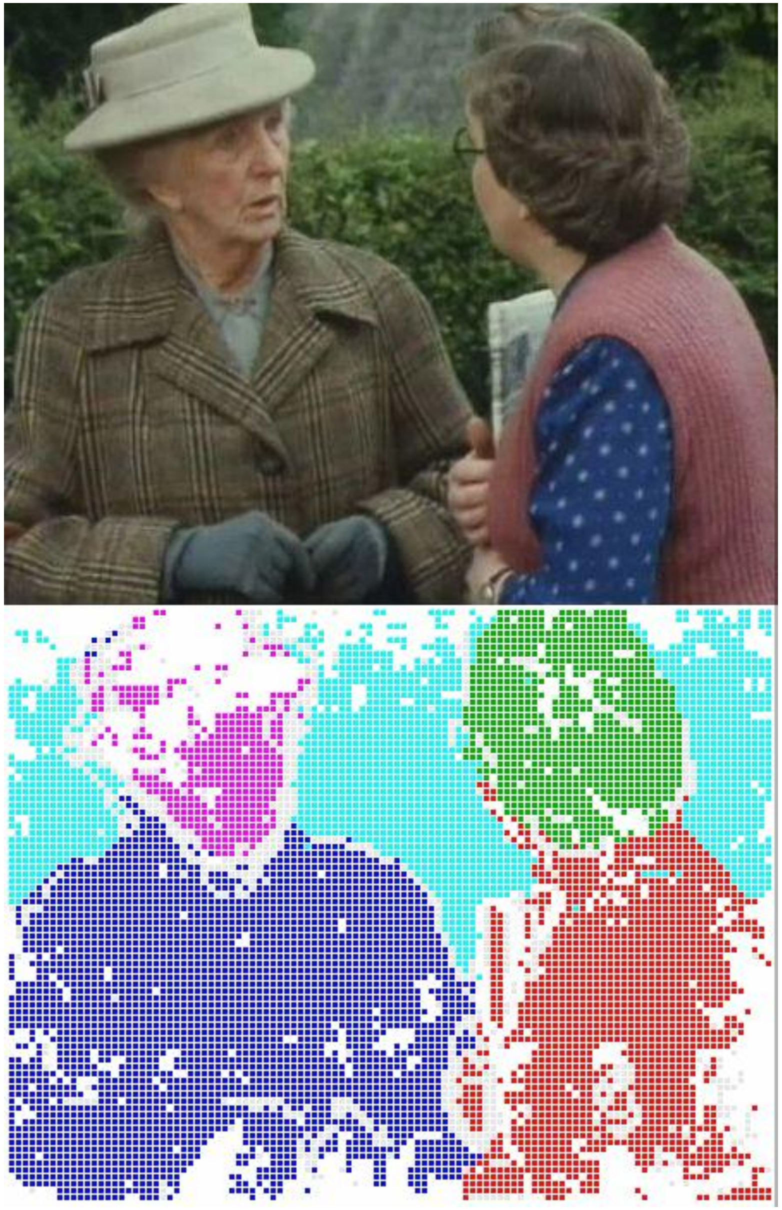


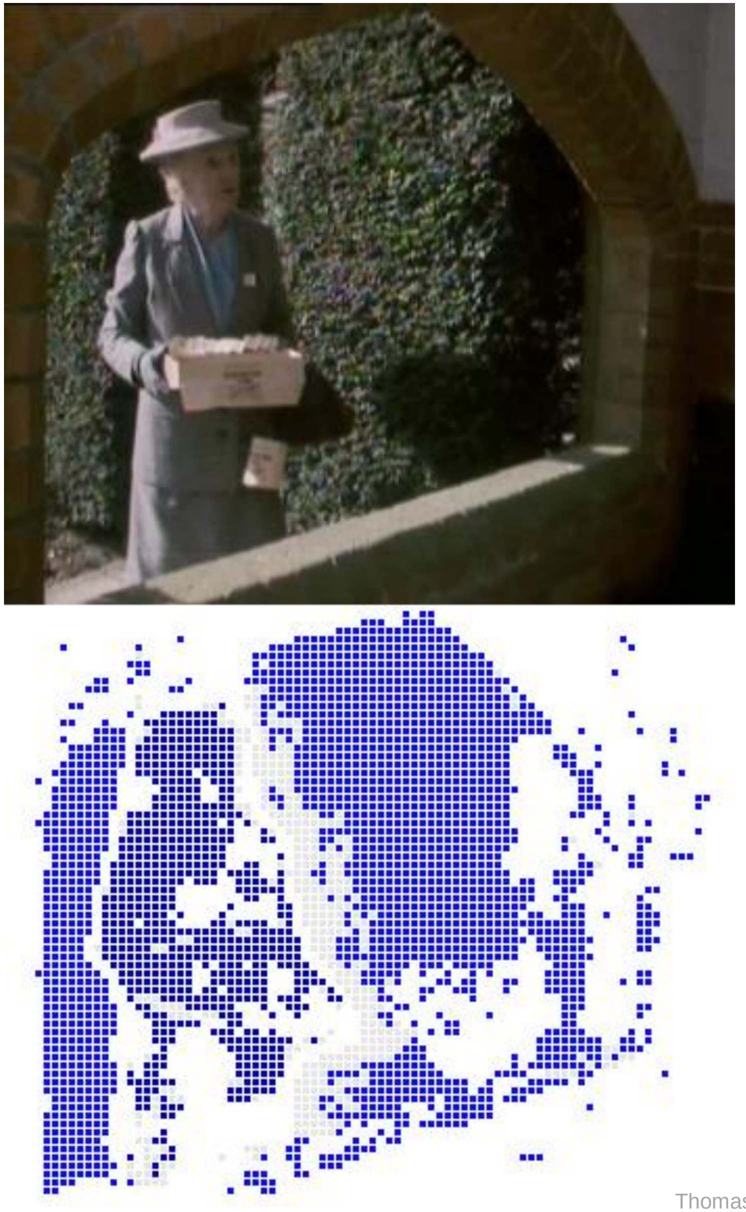
…

3. Run multi-region segmentation on eigenvectors

Thomas Brox

Results on movie shots





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