

# Artificial Life Summer 2025

## Evolutionary Algorithms 2

Master Computer Science [MA-INF 4201]

Mon 14:15 – 15:45, HSZ, HS-2

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Department of Computer Science, University of Bonn

# Last Lecture: Mon 19.5.2025

- Evolutionary Computation
- Historic Remarks
- Different Approaches
- Idea of Evolutionary Algorithms (EA)
- EA Steps
  - Individual, Genome, Fitness, Population
  - Parent selection
  - Inheritance
  - Mutation
  - Fitness evaluation
  - External selection

# Overview

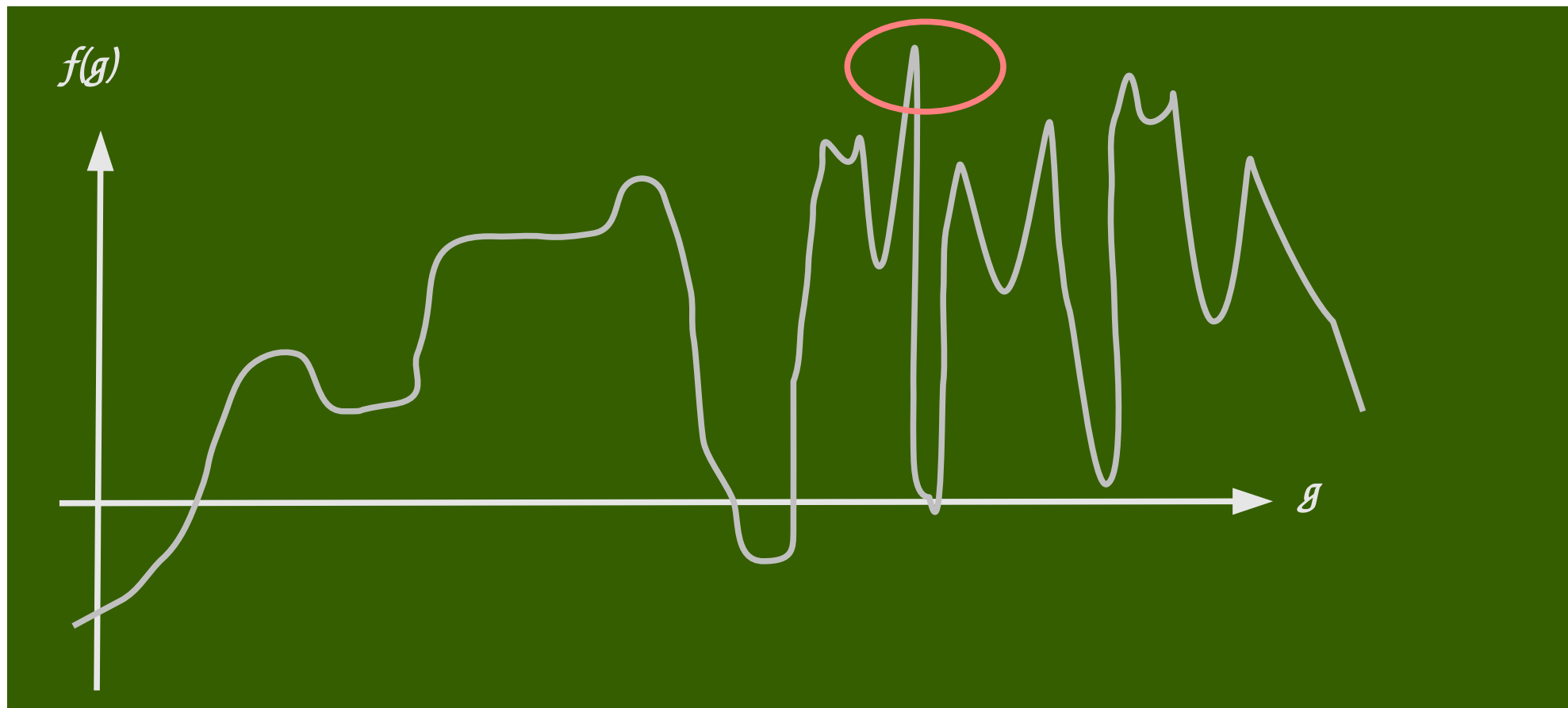
- EA Steps
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  - Initialization
- Strategy
- Performance Graph
- Genome Structure
- Examples

# Overview

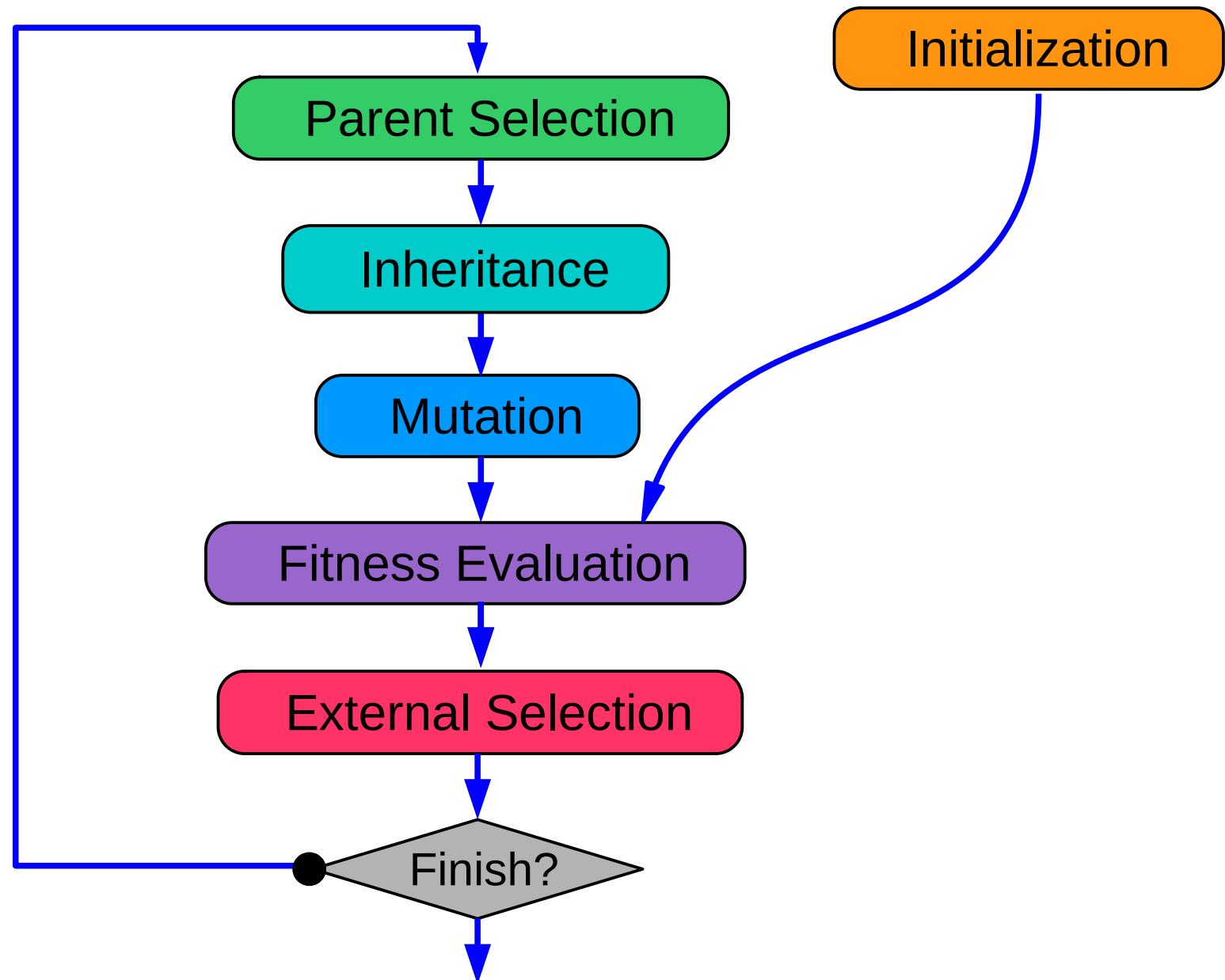
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# Optimization:

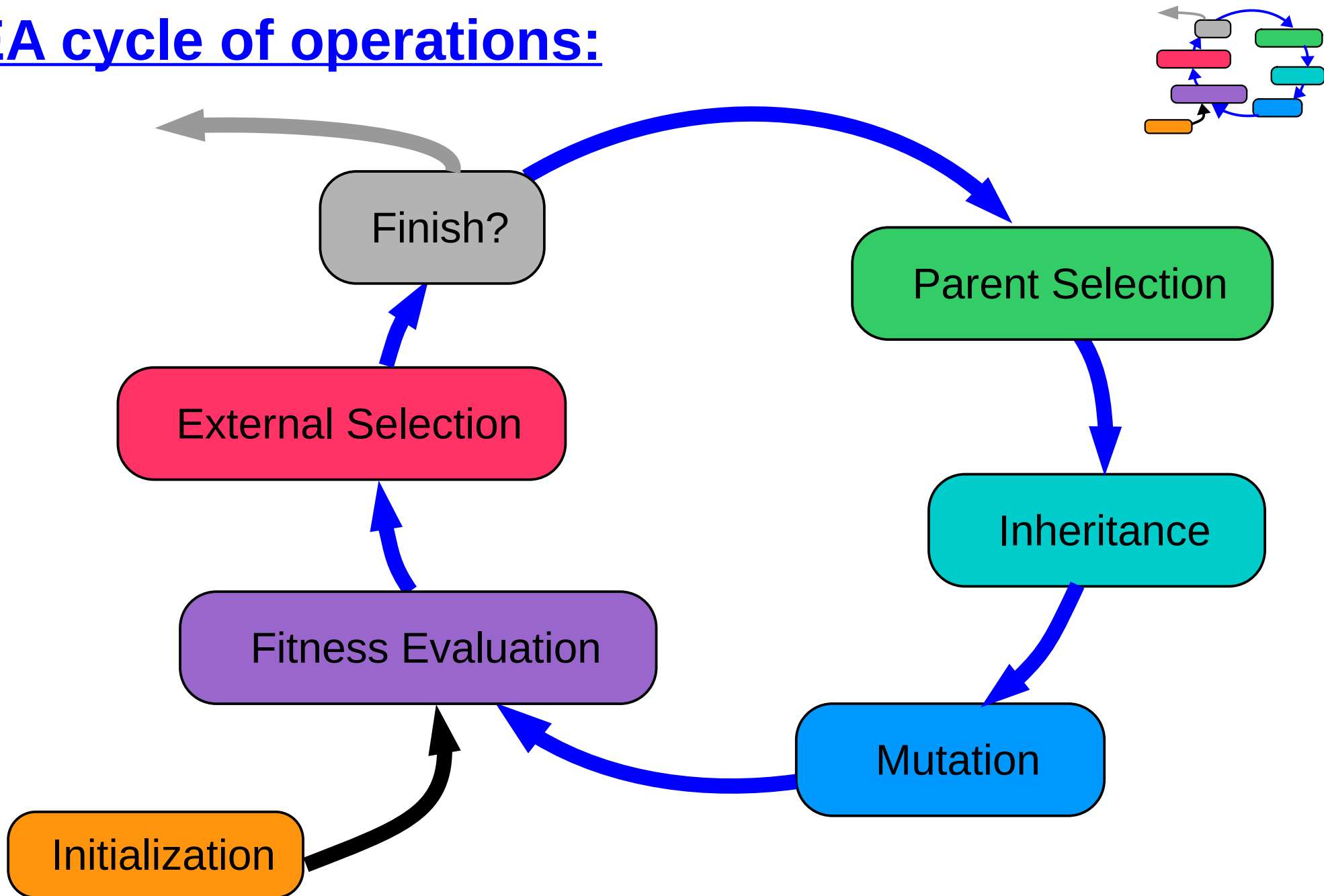
Draw a **nasty fitness function** on the blackboard:



## EA steps:



## EA cycle of operations:

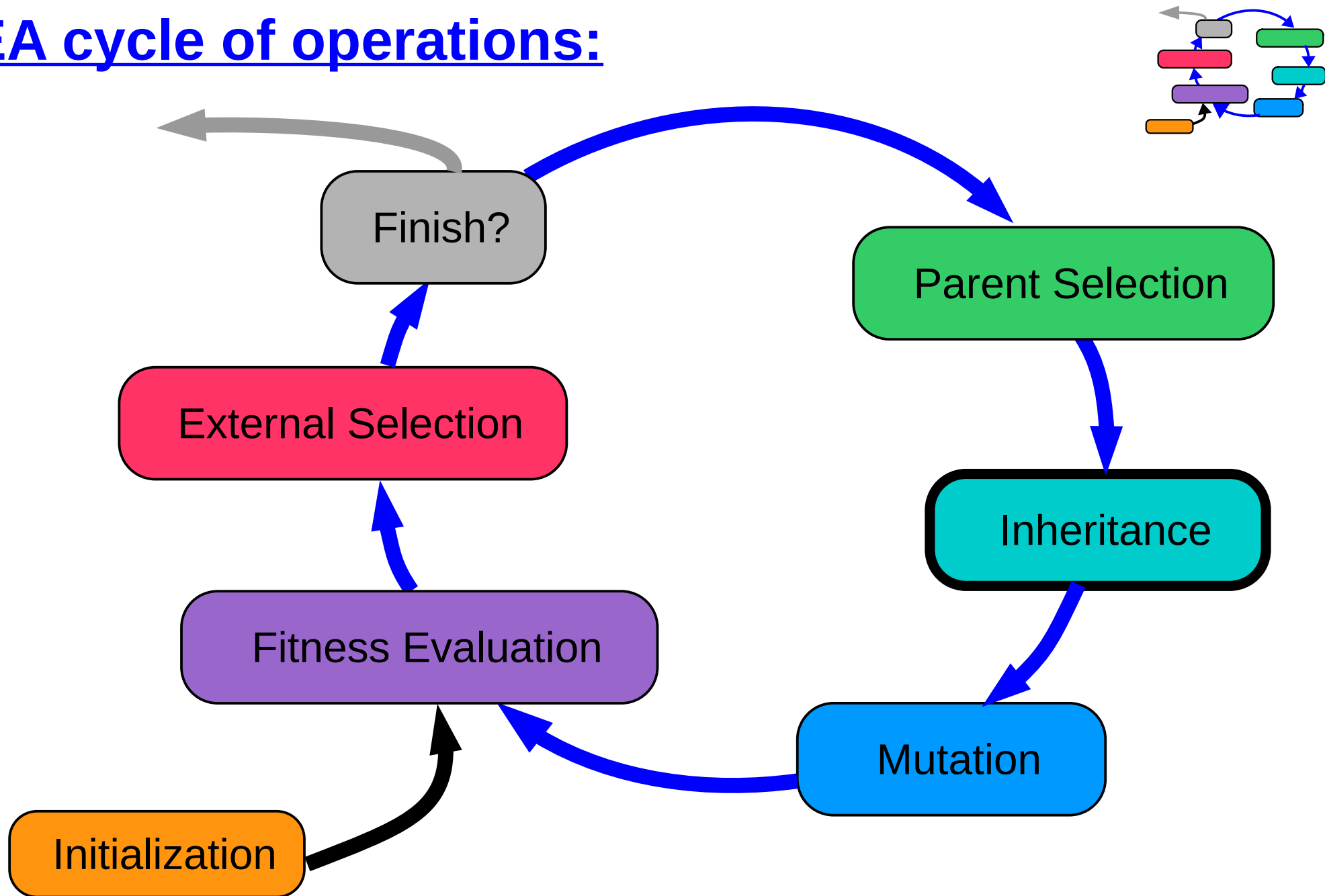


# Overview

- EA Steps
  - Individual, Genome, Fitness, Population
  - Parent selection
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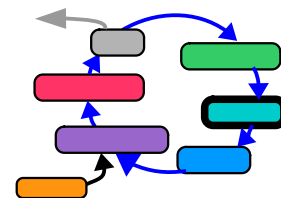


## EA cycle of operations:



## EA:

## Inheritance



**Inheritance** is the principle of transporting some of the information from the previous generation (parents), into the next generation, by generating the **offspring** out of the **parents genomes** .

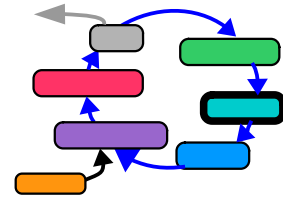
Thus, generating offspring means to create new individuals with new genomes.

**Inheritance** in the context of Evolutionary Algorithms means to use the genomes of those **parents** that have been selected.

There is a wide variety of possible ways to generate offspring.

## EA:

### Inheritance

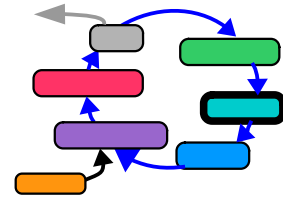


The most popular mechanism to implement the **inheritance** principle is to use  $k=2$  parents and to **combine** their genomes (mostly called recombination).

The most popular mechanism to implement **recombination** of 2 genomes is **cross over**:

### Inheritance:

- **recombination**
  - by **cross over**
  - (other recombination operators)
- (*other inheritance mechanism*)
  - (other inheritance operators)
  - ...

EA:**Inheritance**

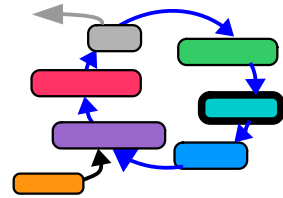
**Recombination** by **1-point cross over**:

Select two parents, **A** and **B** and **recombine** their genomes by taking one part from parent **A** and the other part from parent **B**

Choose a **random point C** where to split the genomes, and recombine the four remaining fractions to build new individuals, containing partial information from both parents.

EA:

Inheritance



Recombination by 1-point cross over:

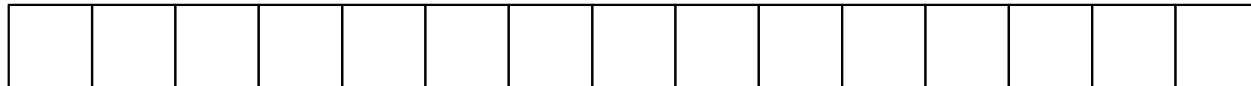
Parent A:

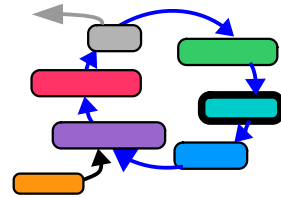
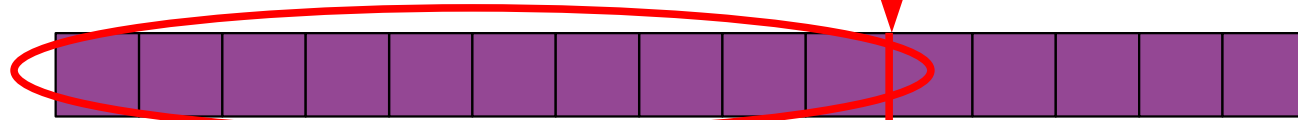


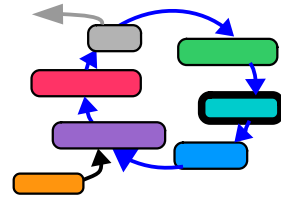
Parent B:

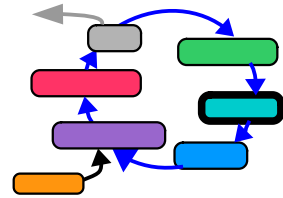


Child 1:

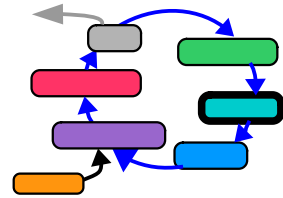


EA:**Inheritance****Recombination by 1-point cross-over:****cross-over point C****Parent A:****Parent B:****Child 1:**

EA:**Inheritance****Recombination by 1-point cross over:****cross over point C****Parent A:****Parent B:****Child 1:****Child 2:**

EA:**Inheritance****Recombination by 1-point cross over:****cross over point C****Parent A:****Parent B:****Child 1:****Child 2:**



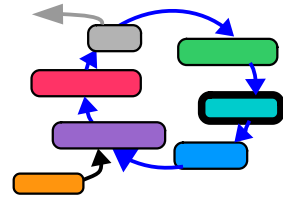
EA:**Inheritance**

**Recombination** by **1-point cross over**:

Parents **A**, **B**

$$\mathbf{A} = ( \mathbf{x}_a , \mathbf{y}_a )$$

$$\mathbf{B} = ( \mathbf{x}_b , \mathbf{y}_b )$$

EA:**Inheritance**

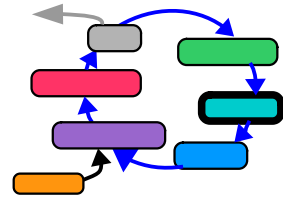
**Recombination by 1-point cross over:**

Parents **A**, **B**

$$\mathbf{A} = (x_a, y_a)$$

$$\mathbf{B} = (x_b, y_b)$$



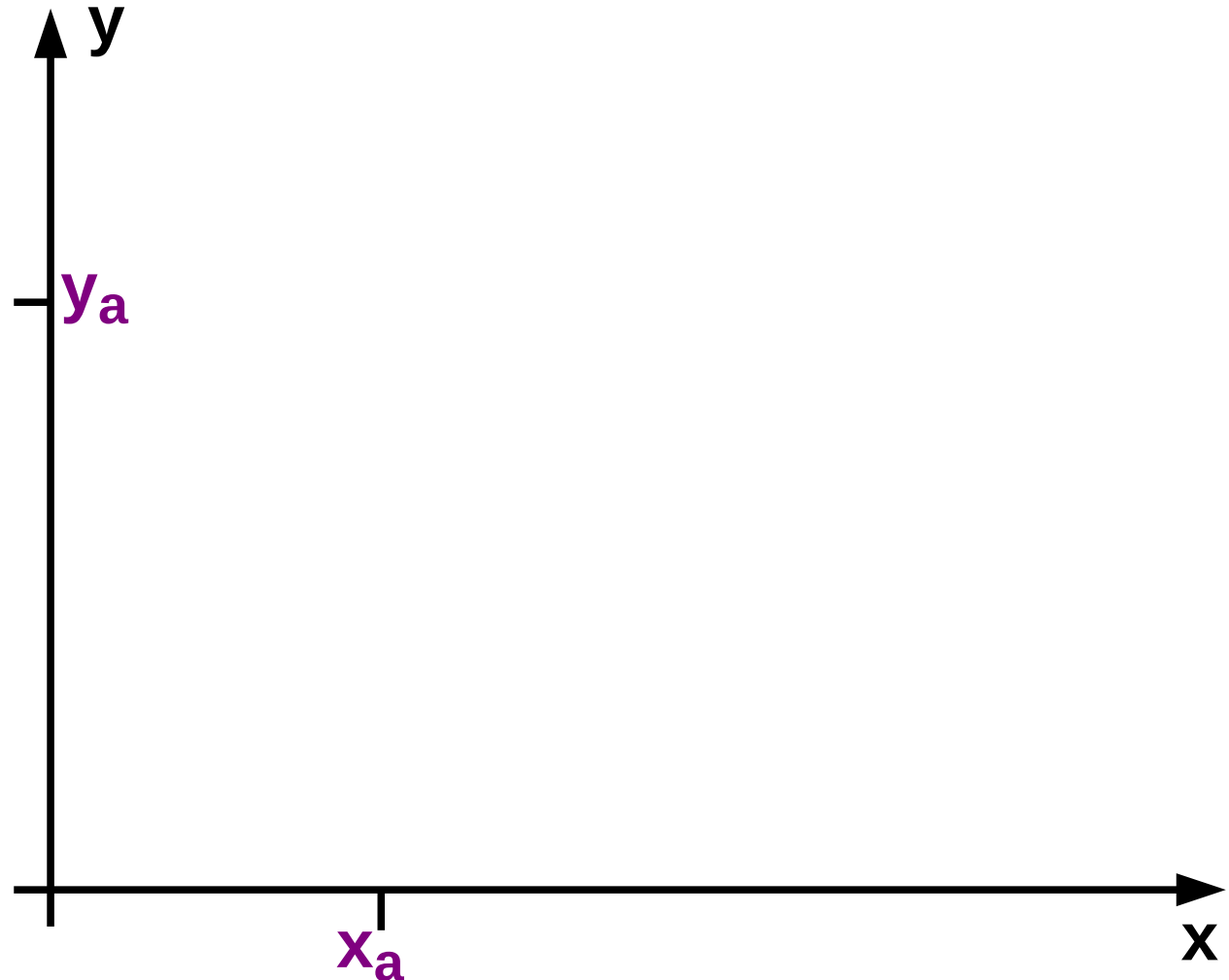
EA:**Inheritance**

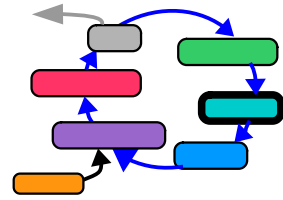
Recombination by 1-point cross over:

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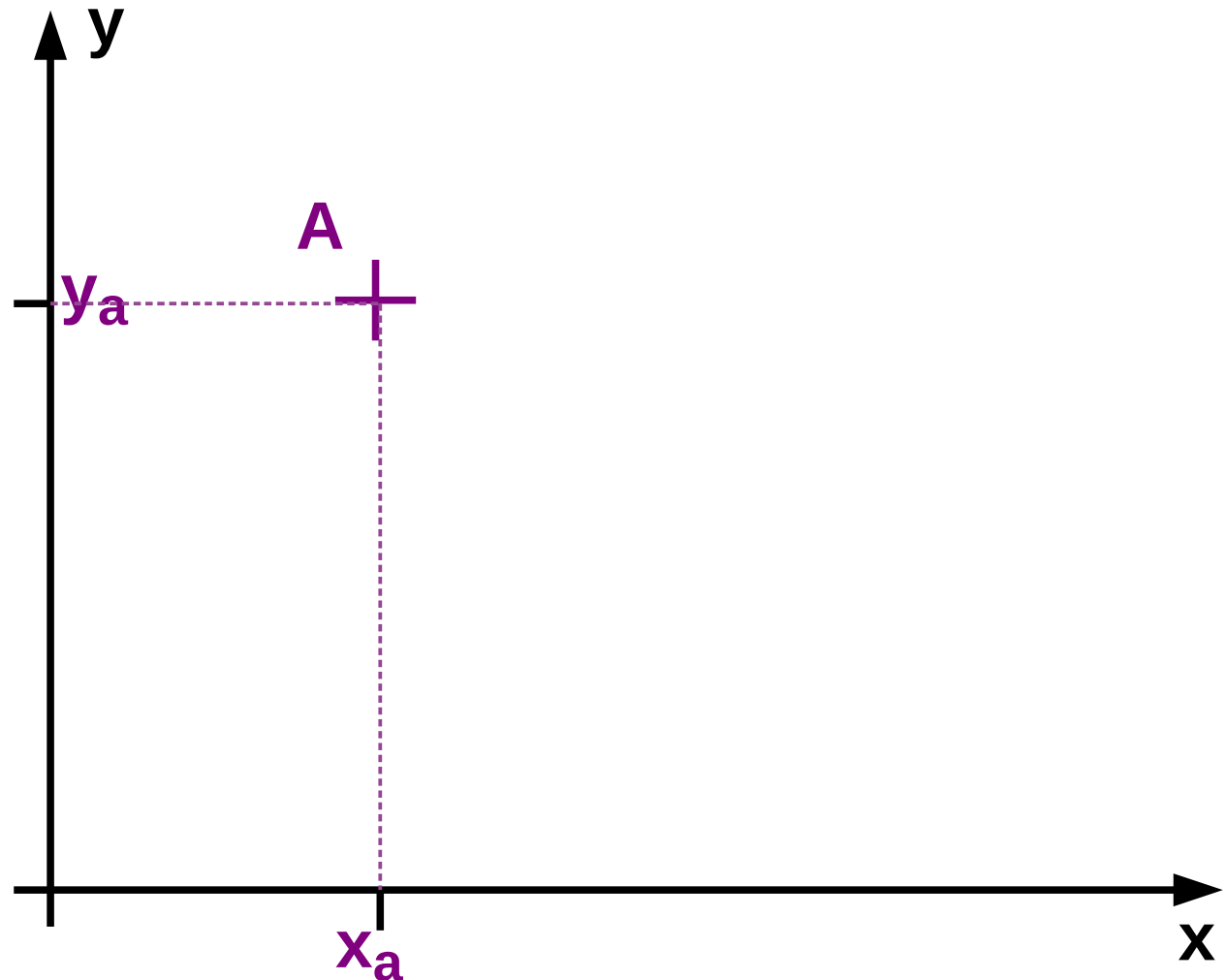
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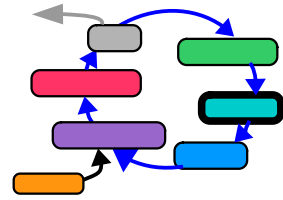
Recombination by 1-point cross over:

Parents **A**, **B**

$$\mathbf{A} = (x_a, y_a)$$

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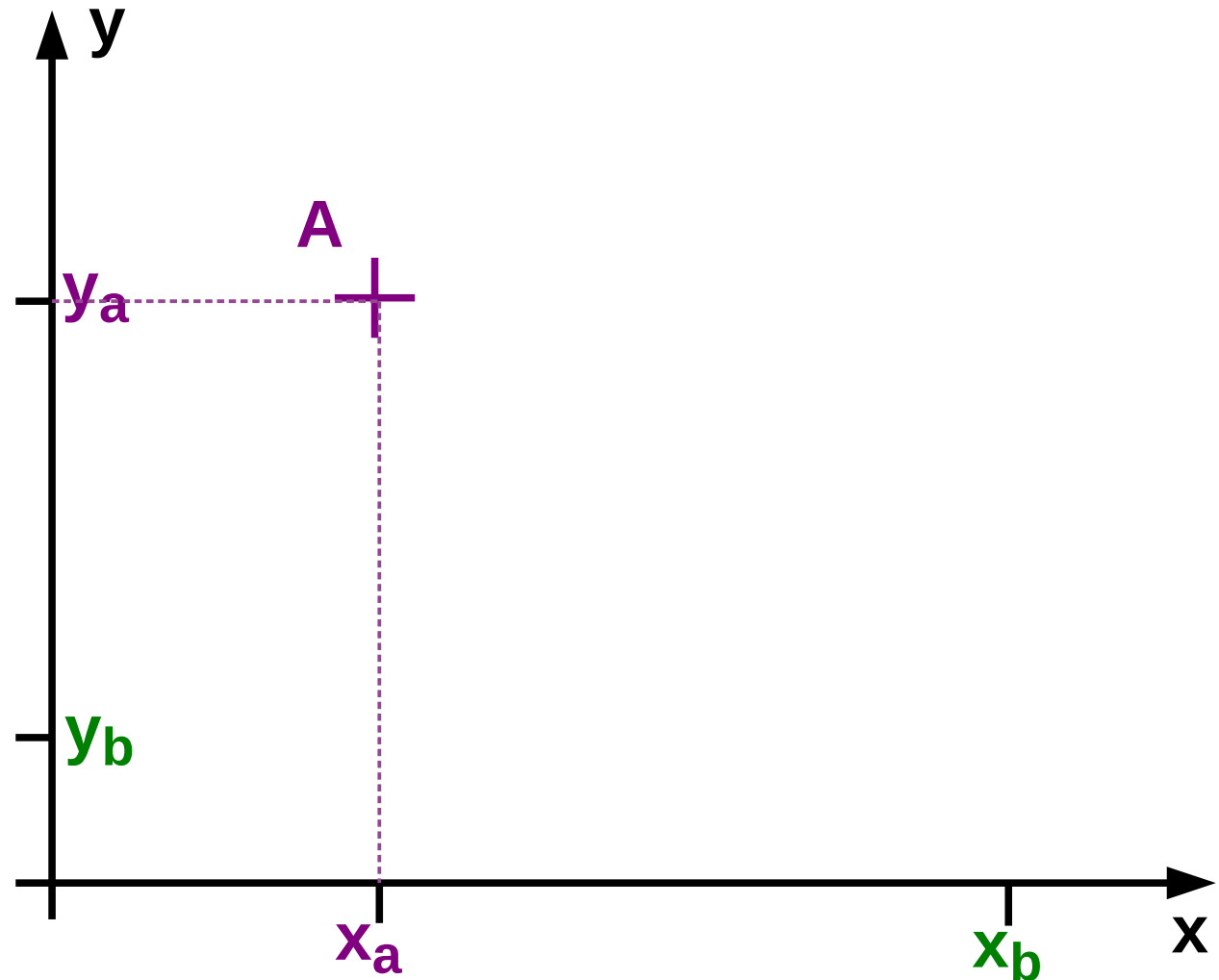
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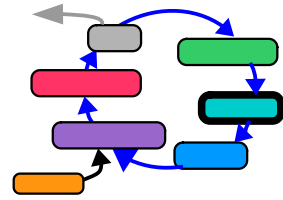
Recombination by 1-point cross over:

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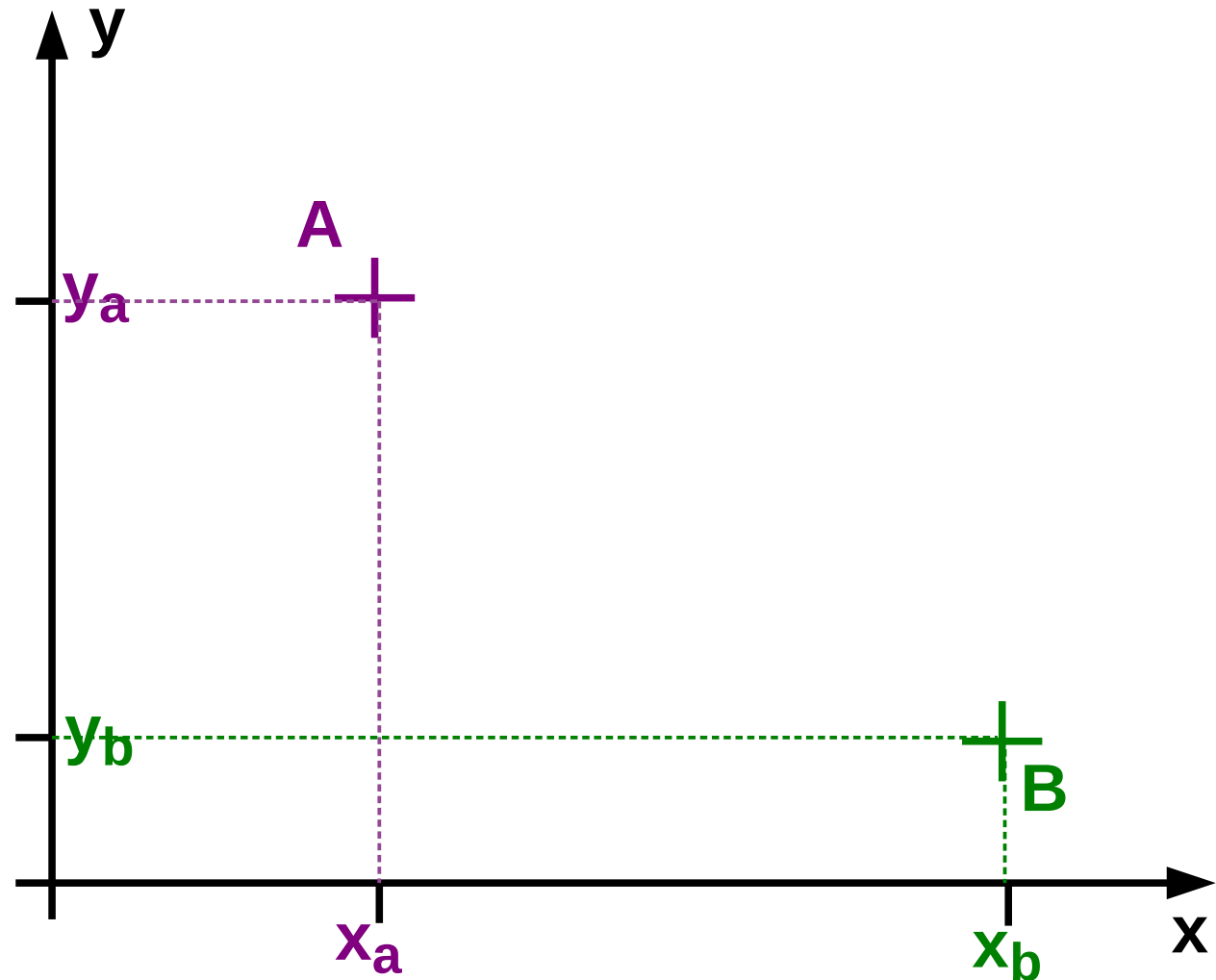
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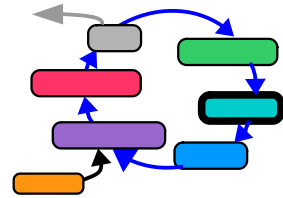
Recombination by 1-point cross over:

Parents **A**, **B**

$$\mathbf{A} = (x_a, y_a)$$

$$\mathbf{B} = (x_b, y_b)$$



EA:**Inheritance**

Recombination by 1-point cross over:

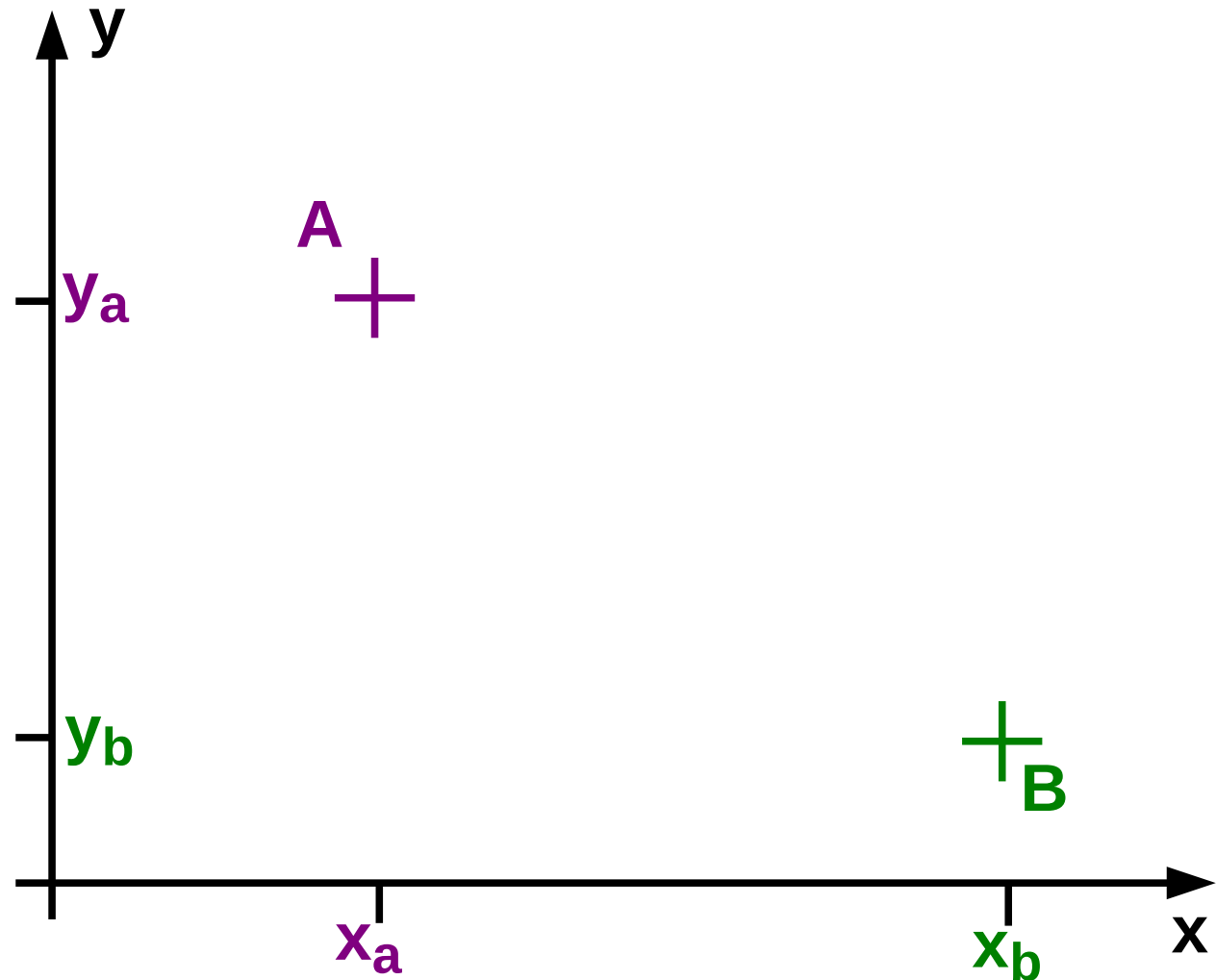
Parents **A**, **B**

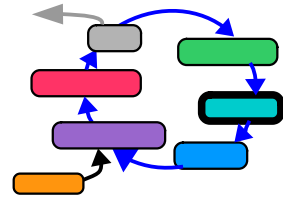
$$\mathbf{A} = (x_a, y_a)$$

$$= \left( \begin{array}{|c|c|} \hline x_a & y_a \\ \hline \end{array} \right)$$

$$\mathbf{B} = (x_b, y_b)$$

$$= \left( \begin{array}{|c|c|} \hline x_b & y_b \\ \hline \end{array} \right)$$



EA:**Inheritance****Recombination by 1-point cross over:****Parents A, B**

$$A = (x_a, y_a)$$

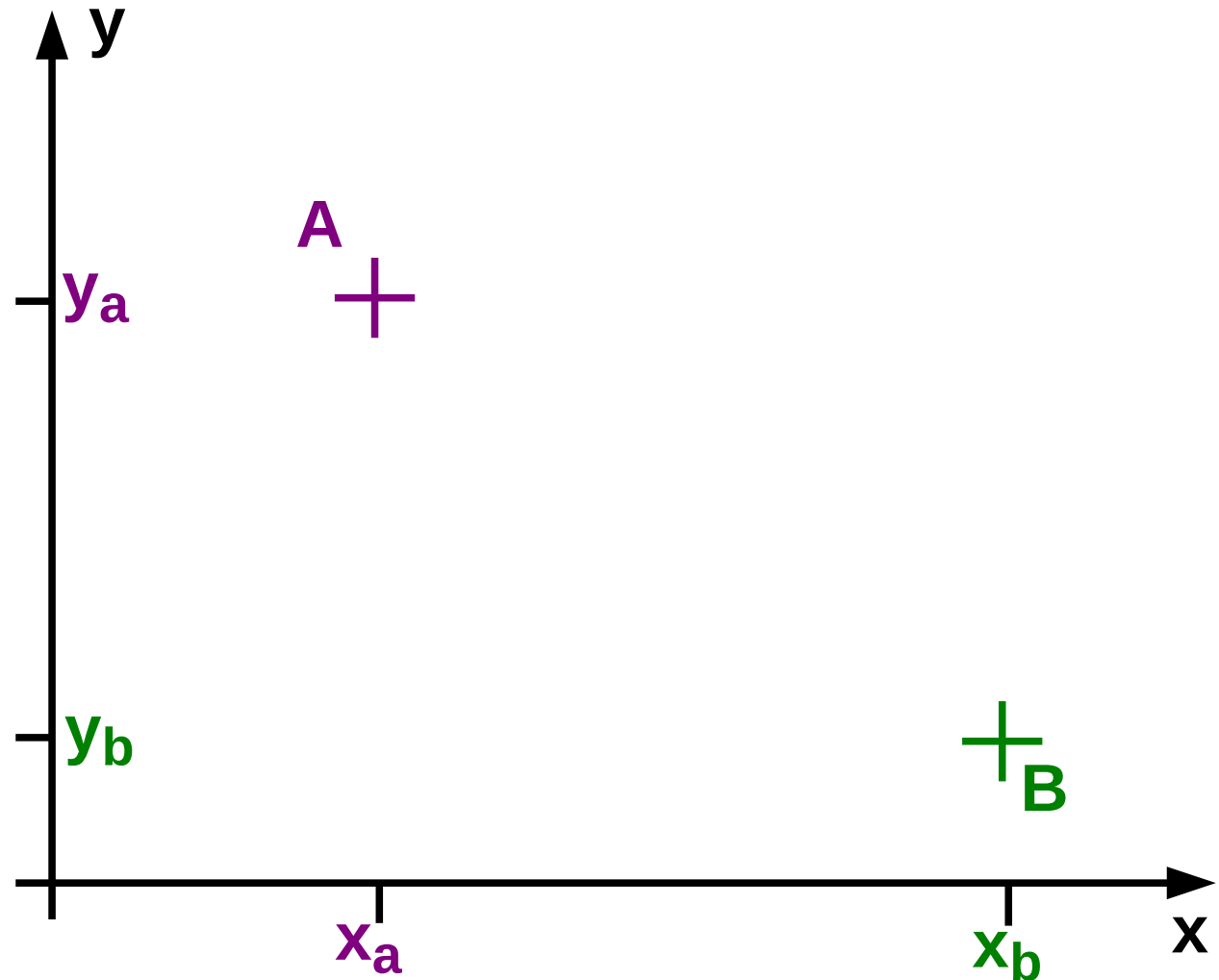
$$= \left( \begin{array}{|c|c|} \hline x_a & y_a \\ \hline \end{array} \right)$$

$$B = (x_b, y_b)$$

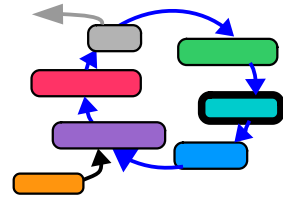
$$= \left( \begin{array}{|c|c|} \hline x_b & y_b \\ \hline \end{array} \right)$$

**Child C1**

$$C1 = ( \quad )$$





EA:**Inheritance****Recombination** by **1-point cross over**:Parents **A**, **B**

$$\mathbf{A} = (x_a, y_a)$$

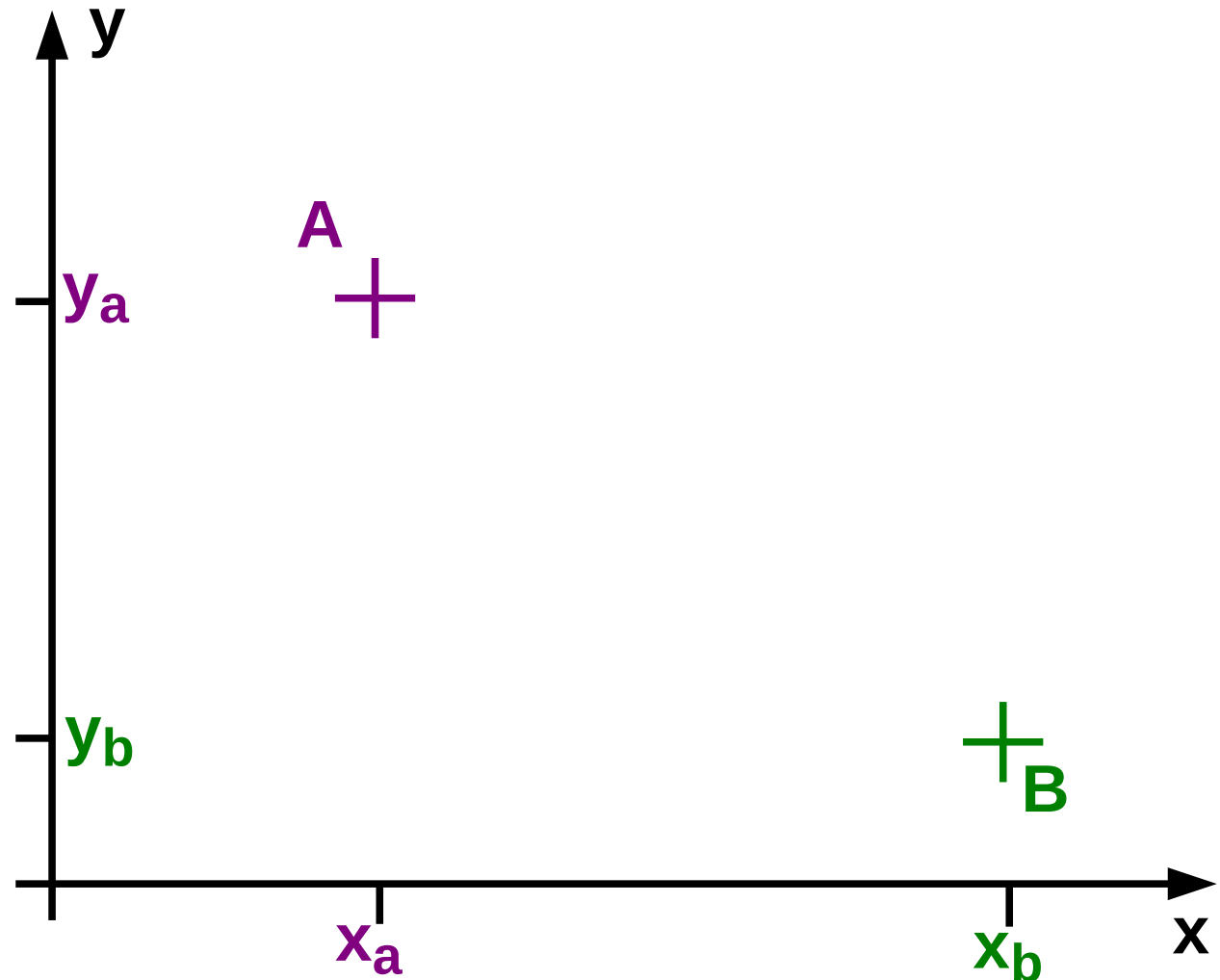
$$= \left( \begin{array}{|c|c|} \hline x_a & y_a \\ \hline \end{array} \right)$$

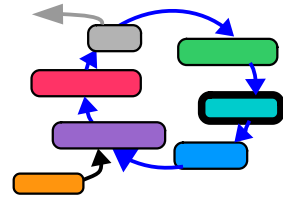
$$\mathbf{B} = (x_b, y_b)$$

$$= \left( \begin{array}{|c|c|} \hline x_b & y_b \\ \hline \end{array} \right)$$

Child **C1**

$$\mathbf{C1} = \left( \begin{array}{|c|c|} \hline x_a & y_b \\ \hline \end{array} \right)$$



EA:**Inheritance****Recombination by 1-point cross over:****Parents A, B**

$$A = (x_a, y_a)$$

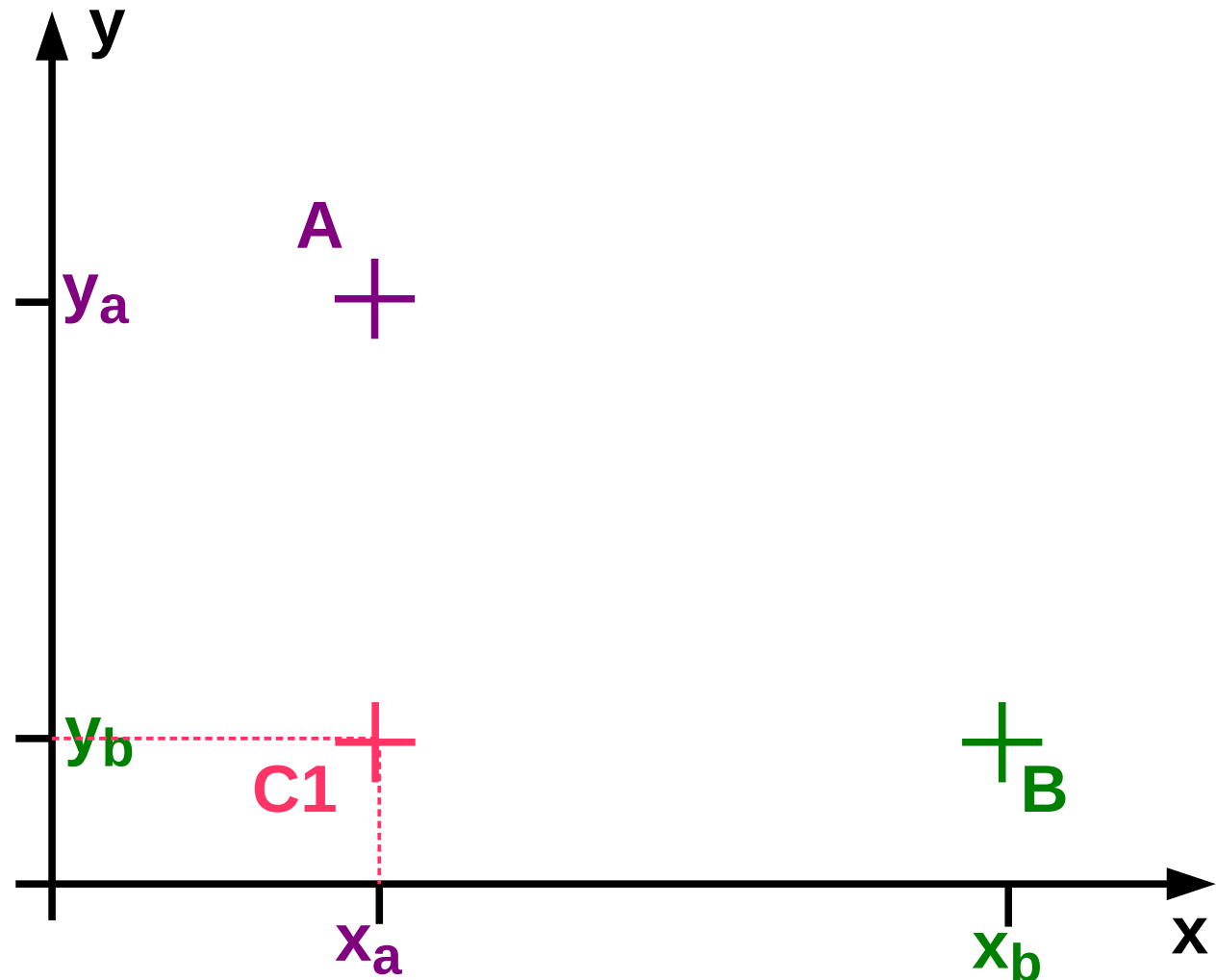
$$= \left( \begin{array}{|c|c|} \hline x_a & y_a \\ \hline \end{array} \right)$$

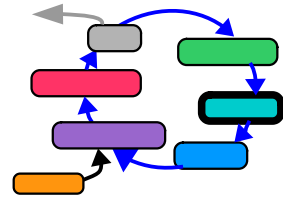
$$B = (x_b, y_b)$$

$$= \left( \begin{array}{|c|c|} \hline x_b & y_b \\ \hline \end{array} \right)$$

**Child C1**

$$C1 = \left( \begin{array}{|c|c|} \hline x_a & y_b \\ \hline \end{array} \right)$$



EA:**Inheritance****Recombination by 1-point cross over:****Parents A, B**

$$A = (x_a, y_a)$$

$$= \left( \begin{array}{|c|c|} \hline x_a & y_a \\ \hline \end{array} \right)$$

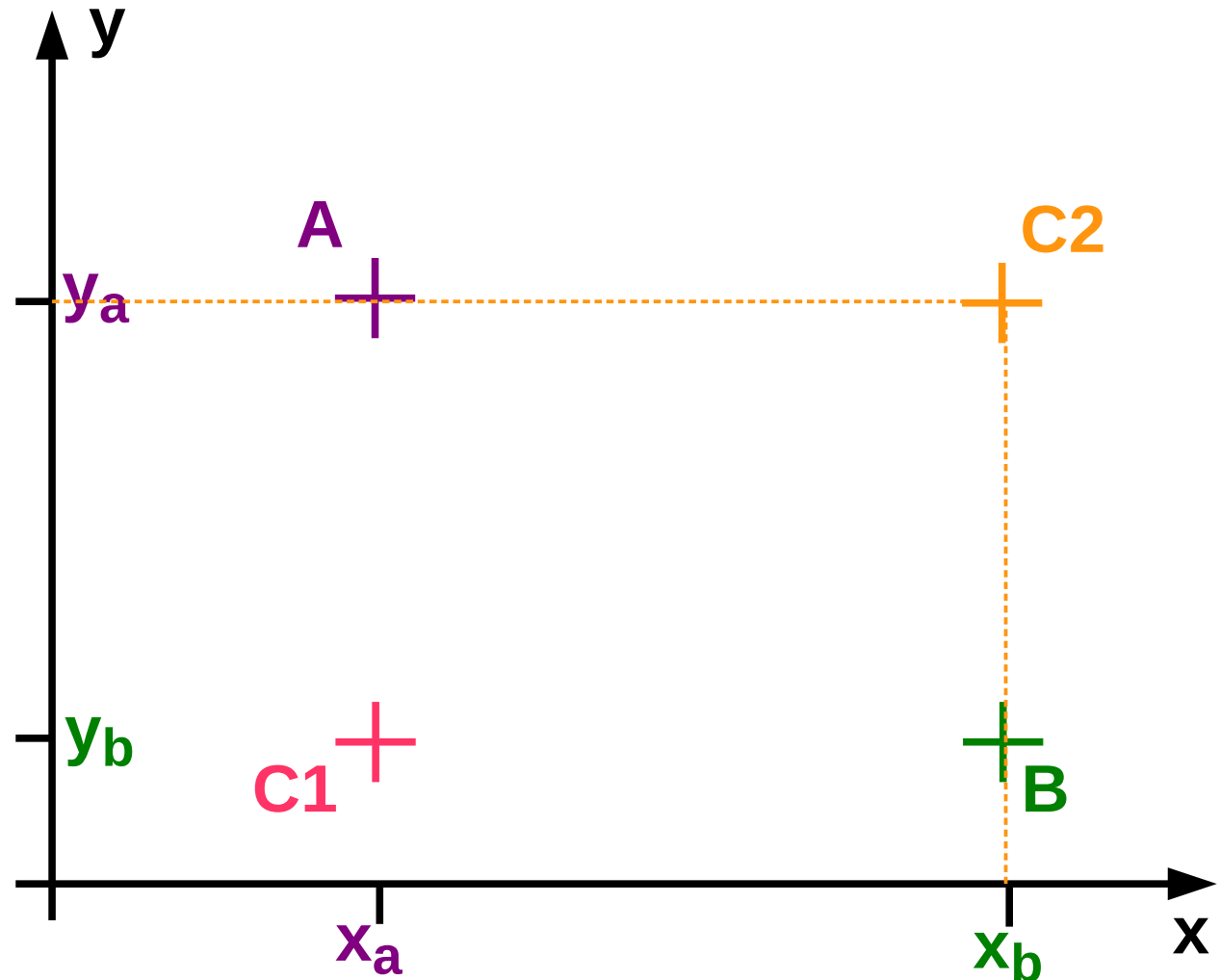
$$B = (x_b, y_b)$$

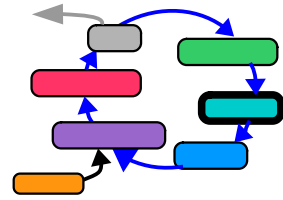
$$= \left( \begin{array}{|c|c|} \hline x_b & y_b \\ \hline \end{array} \right)$$

**Child C1, C2**

$$C1 = \left( \begin{array}{|c|c|} \hline x_a & y_b \\ \hline \end{array} \right)$$

$$C2 = \left( \begin{array}{|c|c|} \hline x_b & y_a \\ \hline \end{array} \right)$$



EA:**Inheritance**

Recombination by 1-point cross over:

Parents **A**, **B**

$$\mathbf{A} = (x_a, y_a)$$

$$= \left( \begin{array}{|c|c|} \hline x_a & y_a \\ \hline \end{array} \right)$$

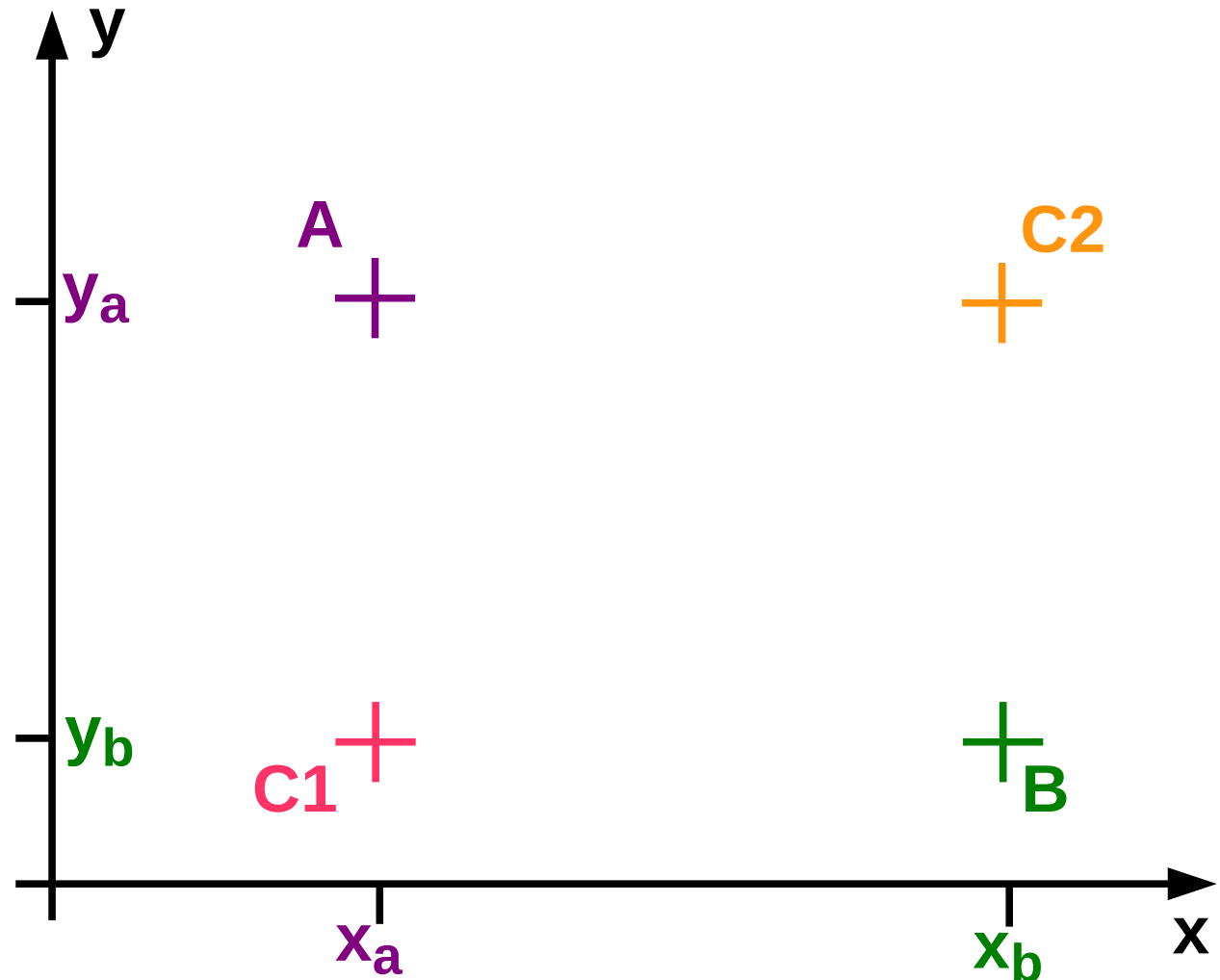
$$\mathbf{B} = (x_b, y_b)$$

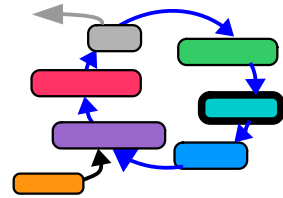
$$= \left( \begin{array}{|c|c|} \hline x_b & y_b \\ \hline \end{array} \right)$$

Child **C1**, **C2**

$$\mathbf{C1} = \left( \begin{array}{|c|c|} \hline x_a & y_b \\ \hline \end{array} \right)$$

$$\mathbf{C2} = \left( \begin{array}{|c|c|} \hline x_b & y_a \\ \hline \end{array} \right)$$



EA:**Inheritance**

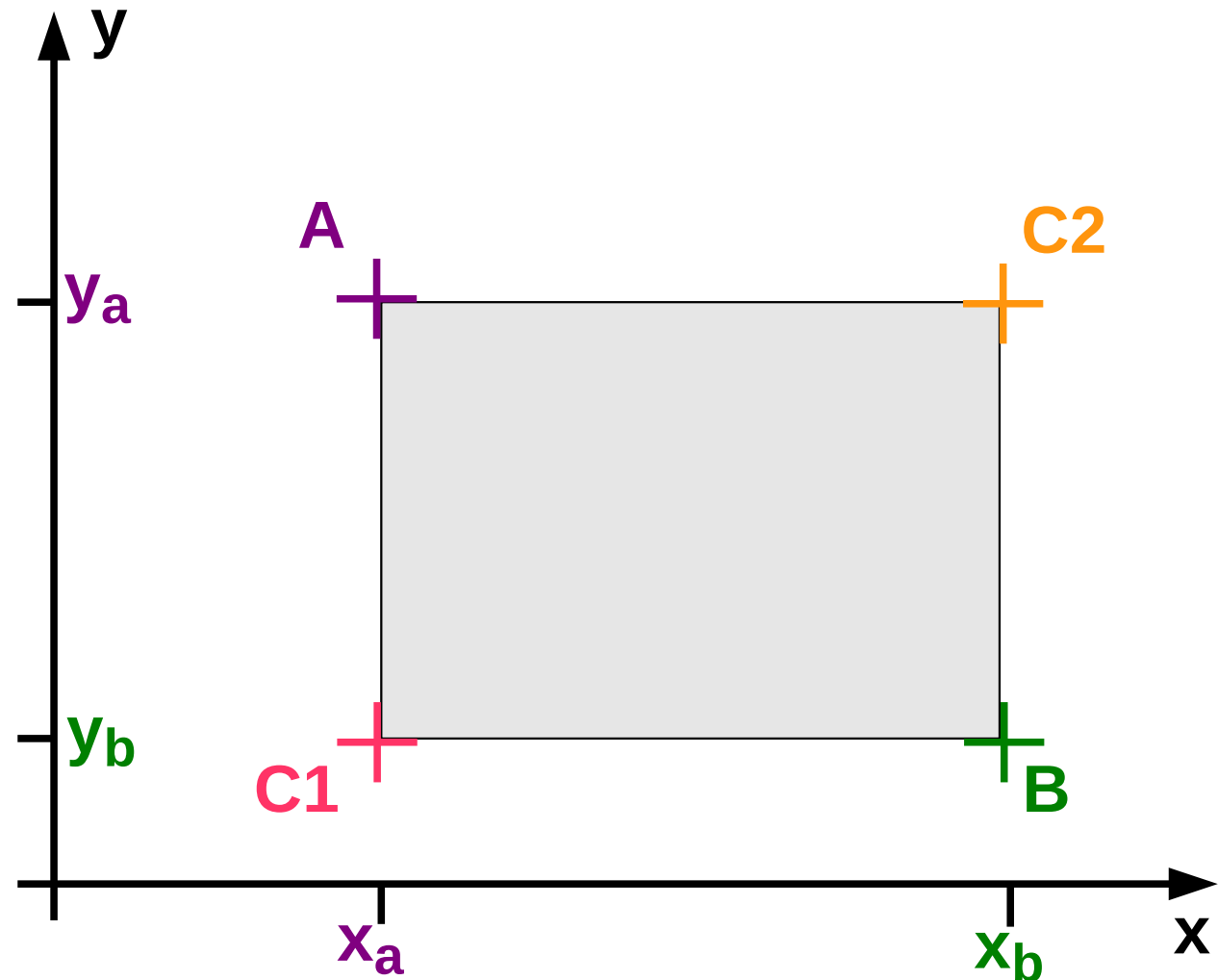
Recombination by 1-point cross over:

Parents **A**, **B**

$$\begin{aligned}
 \mathbf{A} &= (x_a, y_a) \\
 &= \left( \begin{array}{|c|c|} \hline x_a & y_a \\ \hline \end{array} \right) \\
 \mathbf{B} &= (x_b, y_b) \\
 &= \left( \begin{array}{|c|c|} \hline x_b & y_b \\ \hline \end{array} \right)
 \end{aligned}$$

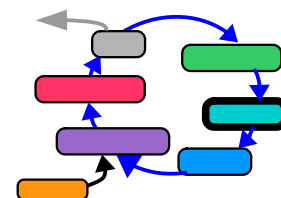
Child **C1**, **C2**

$$\begin{aligned}
 \mathbf{C1} &= \left( \begin{array}{|c|c|} \hline x_a & y_b \\ \hline \end{array} \right) \\
 \mathbf{C2} &= \left( \begin{array}{|c|c|} \hline x_b & y_a \\ \hline \end{array} \right)
 \end{aligned}$$



# EA:

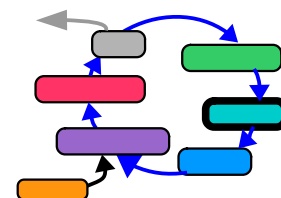
## Inheritance



**Inheritance** can involve  $k=(0), 1, 2, 3, \dots$  or even all  $\mu$  individuals from the parent generation.

Although, taking  $k=2$  parents is the most popular choice for implementing **inheritance**, the other possibilities are sensible, and are in use.

- **$k=0$  parents:** no inheritance, generate a novel genome
- **$k=1$  parent :** just copy the genome of the parent
- **$k=2$  parents:** recombination,  
e.g. cross-over, mean value, sum, ...
- **$k=3$  parents:** recombination, e.g. cyclic cross-over
- **$k=\mu$  parents:** recombination between all parents

EA:**Inheritance**

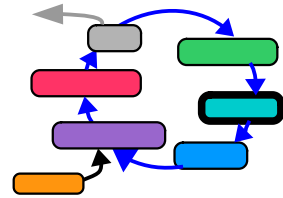
**$k=0$ , no inheritance**, generate a novel genome.

The decision to omit inheritance by generating novel genomes is possible, but not really a good idea for EAs.

Generating new individuals on a random basis, is a very strong implementation of **exploration**.

It can be seen as a special variant of **Random Search (RS)**, (with an extremely large hypersphere).

Typically in EAs, exploration is the role of the subsequent mutation step.

EA:**Inheritance**

**k=1, copy**, just copy the genome of the parent.

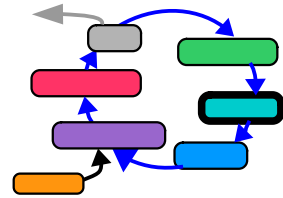
The decision to make just a **copy** of one of the parents for creating the offspring, is not a bad choice for the **inheritance** principle.

This is a very strong implementation of **exploitation**.

Just copying, bears the danger of losing the (necessary) diversity among all genomes within the population.

Only the subsequent EA step (mutation) that will potentially alter the resulting genome, can maintain the diversity to some extent (exploration).



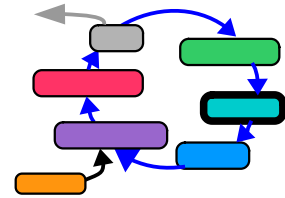
EA:**Inheritance**

**k=2, recombination**, combine the genomes of two parents.

The most popular mechanism to implement the **inheritance** principle is to use  $k=2$  parents and to **combine** their genomes (mostly called recombination).

## EA:

# Inheritance

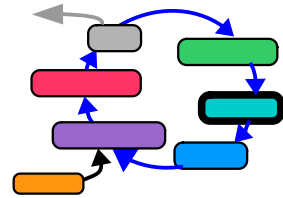


**k=2, recombination,** combine the genomes of two parents.

The most popular mechanism to implement the **inheritance** principle is to use  $k=2$  parents and to **combine** their genomes (mostly called recombination).

**k>2, recombination,** combine the genomes of more parents.

Most methods developed and proposed for recombining  $k=2$  parents apply as well for  **$k>2$  parents** with only small alterations.

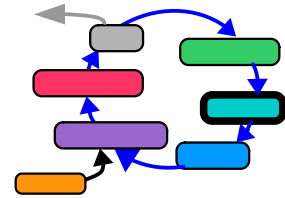
EA:**Inheritance**

**$k=2$  and  $k>2$ , recombination with  $k$  parents**

Several ways to **combine** (or **re-combine**) the genomes of the  $k$  parents have been proposed.

Some of them rely on a special structure of the genome.

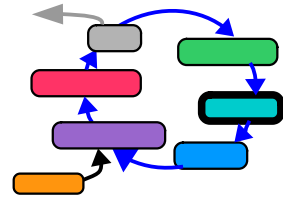
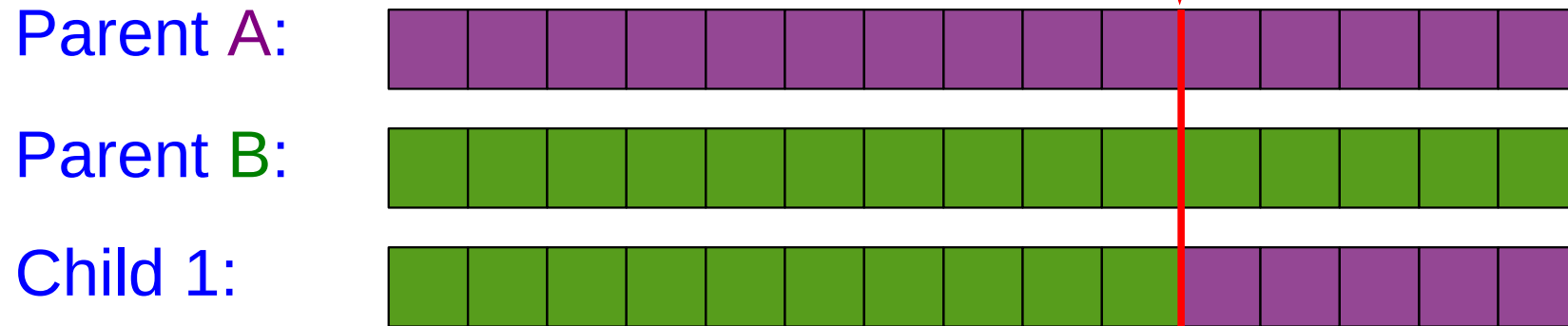
- **N-point cross over** (the most popular)
- **mean-values**, one component, all components
- **max, min, sum, difference** of values
- **union, intersection** of **sets** of items
- **concatenate** (strings)
- **...**

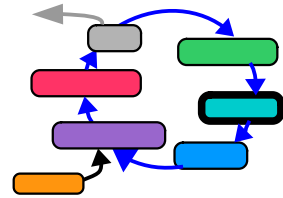
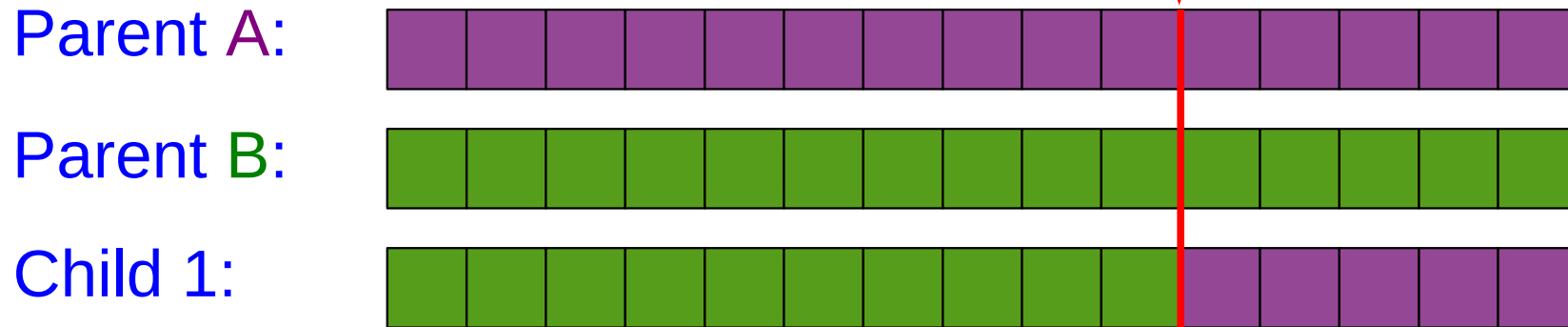
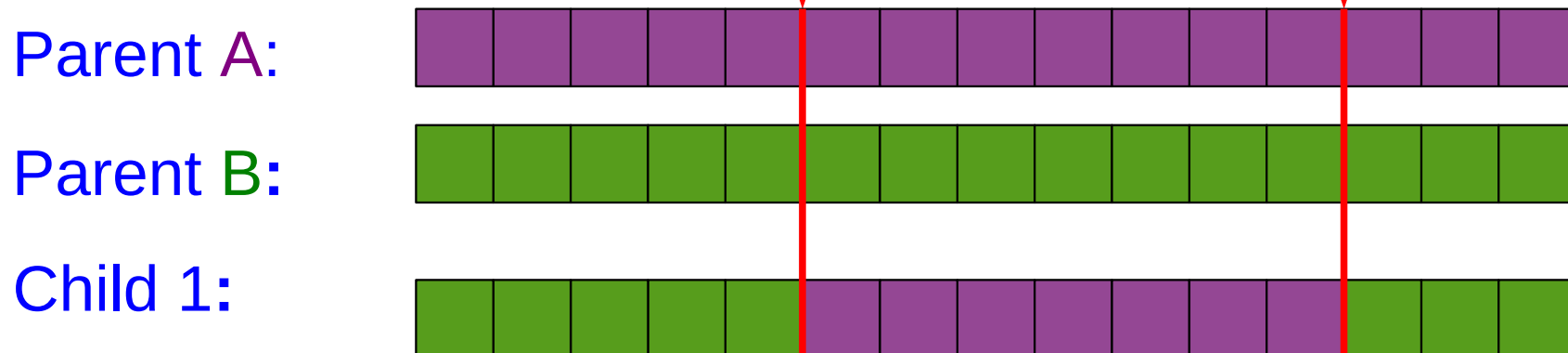
EA:**Inheritance**

**Recombination** by **N-point cross over**:

The **N-point cross over** combination operator, produces offspring that lie only on the corners of a hypercube.

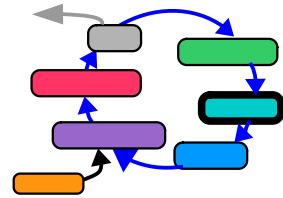
This is a reduced search space to look for solutions.

EA:**Inheritance****Recombination by 1-point cross over:**

EA:**Inheritance****Recombination by 1-point cross over:****Recombination by 2-point cross over:**

**EA:**

# Inheritance

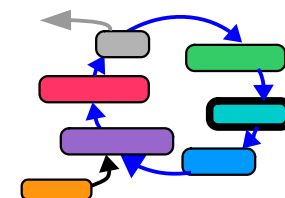


## k=2 and k>2, recombination with k parents

Several ways to **combine** (or **re-combine**) the genomes of the  $k$  parents have been proposed.

Some of them rely on a special structure of the genome.

- **N-point cross over** (the most popular)
- **mean-values**, one component, all components
- **max, min, sum, difference** of values
- **union, intersection** of **sets** of items
- **concatenate** (strings)
- **...**

EA:**Inheritance**

**Recombination** by point wise **mean values**:

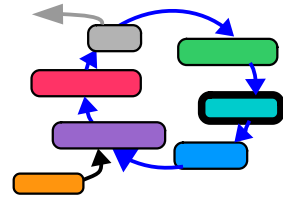
Select two parents, **A** and **B** and a random position **R** within the genome, and **recombine** the genomes by building the **mean value** between **corresponding parts** at position **R** of the genomes.

Pointwise numeric operations, like:

**mean value, sum, difference, maximum** or **minimum**

are only feasible, if the different parts of the genomes are comparable and numeric.



EA:**Inheritance****Recombination** by point wise mean valuesParent **A:**

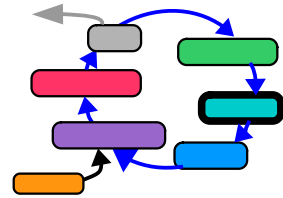
<b>0.333</b>	<b>144.6</b>	<b>42</b>	<b>92</b>	<b>4.75</b>	<b>5926</b>	<b>10716</b>
--------------	--------------	-----------	-----------	-------------	-------------	--------------

Parent **B:**

<b>0.123</b>	<b>1.40</b>	<b>40</b>	<b>42</b>	<b>1.532</b>	<b>5926</b>	<b>0</b>
--------------	-------------	-----------	-----------	--------------	-------------	----------

Child **C1:**

--	--	--	--	--	--	--

EA:**Inheritance****Recombination** by point wise mean valuesParent **A**:

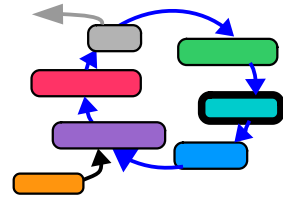
0.333	144.6	42	92	4.75	5926	10716
-------	-------	----	----	------	------	-------

Parent **B**:

0.123	1.40	40	42	1.532	5926	0
-------	------	----	----	-------	------	---

Child **C1**:

--	--	--	--	--	--	--

EA:**Inheritance****Recombination** by point wise mean valuesParent **A**:

0.333	144.6	42	92	4.75	5926	10716
-------	-------	----	----	------	------	-------

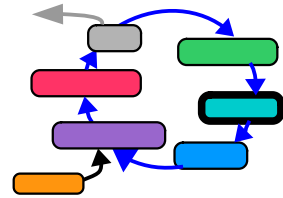
Parent **B**:

0.123	1.40	40	42	1.532	5926	0
-------	------	----	----	-------	------	---

$$\frac{1}{2} (4.75 + 1.532) = \frac{1}{2} (6.282) = 3.141$$

Child **C1**:

--	--	--	--	--	--	--

EA:**Inheritance****Recombination** by point wise mean valuesParent **A**:

0.333	144.6	42	92	4.75	5926	10716
-------	-------	----	----	------	------	-------

Parent **B**:

0.123	1.40	40	42	1.532	5926	0
-------	------	----	----	-------	------	---

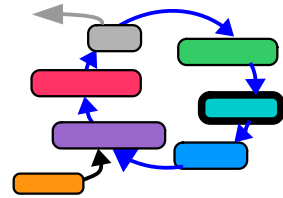
$$\frac{1}{2} (4.75 + 1.532) = \frac{1}{2} (6.282) = 3.141$$

Child **C1**:

				3.141		
--	--	--	--	-------	--	--

EA:

Inheritance



Recombination by point wise mean values

Parent A:

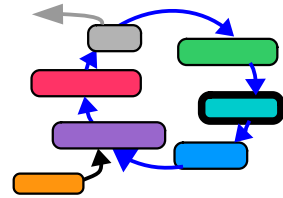
0.333	144.6	42	92	4.75	5926	10716
-------	-------	----	----	------	------	-------

Parent B:

0.123	1.40	40	42	1.532	5926	0
-------	------	----	----	-------	------	---

Child C1:

				3.141		
--	--	--	--	-------	--	--

EA:**Inheritance****Recombination by point wise mean values**Parent **A**:

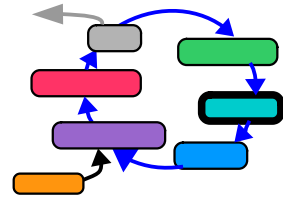
0.333	144.6	42	92	4.75	5926	10716
-------	-------	----	----	------	------	-------

Parent **B**:

0.123	1.40	40	42	1.532	5926	0
-------	------	----	----	-------	------	---

Child **C1**:

0.123	1.40	40	42	3.141	5926	0
-------	------	----	----	-------	------	---

EA:**Inheritance****Recombination by point wise mean values****Parent A:**

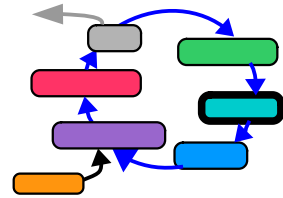
0.333	144.6	42	92	4.75	5926	10716
-------	-------	----	----	------	------	-------

**Parent B:**

0.123	1.40	40	42	1.532	5926	0
-------	------	----	----	-------	------	---

**Child C1:**

0.123	1.40	40	42	3.141	5926	0
-------	------	----	----	-------	------	---

EA:**Inheritance**

Recombination by point wise mean values

Parent **A**:

0.333	144.6	42	92	4.75	5926	10716
-------	-------	----	----	------	------	-------

Parent **B**:

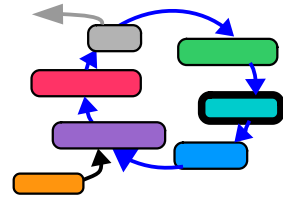
0.123	1.40	40	42	1.532	5926	0
-------	------	----	----	-------	------	---

mean value in one component **R**

Child **C1**:

0.123	1.40	40	42	3.141	5926	0
-------	------	----	----	-------	------	---



EA:**Inheritance****Recombination by point wise mean values**Parent **A**:

0.333	144.6	42	92	4.75	5926	10716
-------	-------	----	----	------	------	-------

Parent **B**:

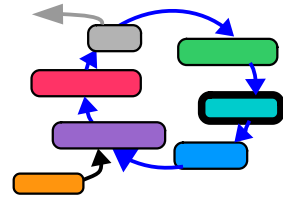
0.123	1.40	40	42	1.532	5926	0
-------	------	----	----	-------	------	---

mean value in one component **R**Child **C1**:

0.123	1.40	40	42	3.141	5926	0
-------	------	----	----	-------	------	---

mean value in **all** componentsChild **C2**:

--	--	--	--	--	--	--

EA:**Inheritance**

Recombination by point wise mean values

Parent **A**:

0.333	144.6	42	92	4.75	5926	10716
-------	-------	----	----	------	------	-------

Parent **B**:

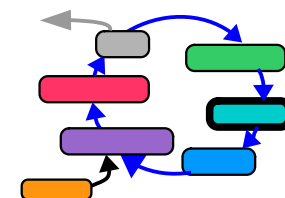
0.123	1.40	40	42	1.532	5926	0
-------	------	----	----	-------	------	---

mean value in one component **R**Child **C1**:

0.123	1.40	40	42	3.141	5926	0
-------	------	----	----	-------	------	---

mean value in **all** componentsChild **C2**:

0.228	73	41	67	3.141	5926	5358
-------	----	----	----	-------	------	------

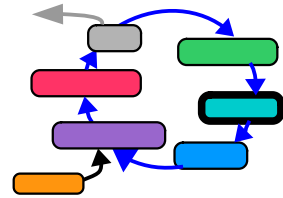
EA:**Inheritance**

**$k=2$  and  $k>2$ , recombination with  $k$  parents**

Several ways to **combine** (or **re-combine**) the genomes of the  $k$  parents have been proposed.  
Some of them rely on a special structure of the genome.

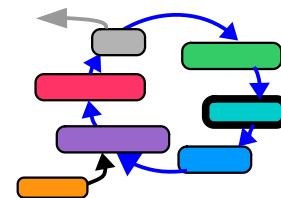
- **N-point cross over** (the most popular)
- **mean-values**, one component, all components
- **max, min, sum, difference** of values
- **union, intersection** of **sets** of items
- **concatenate** (strings)
- **...**

# Inheritance



## Recombination by union of sets

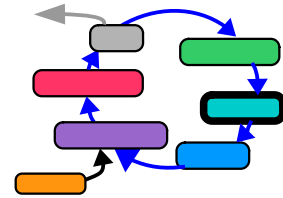
For genomes that are structured as a set of items, a valid and sensible **recombination** is to build the **union** of those sets.

EA:**Inheritance****Recombination** by **union of sets**

For genomes that are structured as a set of items, a valid and sensible **recombination** is to build the **union** of those sets.

**Parent A:**      { Soup, Pasta, Fish, White-Wine }

**Parent B:**      { Aperitif, White-Wine, Water, Cheese }

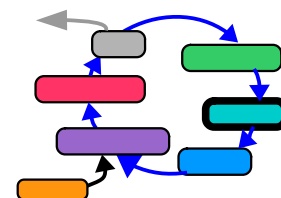
EA:**Inheritance****Recombination** by **union of sets**

For genomes that are structured as a set of items, a valid and sensible **recombination** is to build the **union** of those sets.

**Parent A:**      { Soup, Pasta, Fish, White-Wine }

**Parent B:**      { Aperitif, White-Wine, Water, Cheese }

**Child C4:**      { Soup, Pasta, Fish, White-Wine,  
                    Aperitif, Water, Cheese }

EA:**Inheritance**

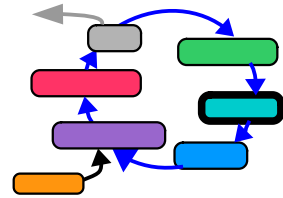
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- **mean-values**, one component, all components
- **max, min, sum, difference** of values
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- **concatenate** (strings)
- ...

EA:

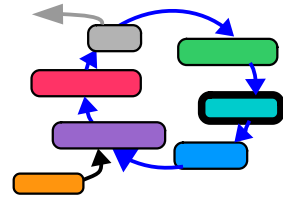
## Inheritance



## Recombination by concatenation

Genomes that are representing **strings**, can be **recombined** by a simple **concatenation** of their strings, just „**gluing**“ the genomes one after the other.  
In this case, the order is important.



EA:**Inheritance****Recombination by concatenation**

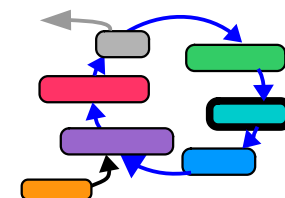
Genomes that are representing **strings**, can be **recombined** by a simple **concatenation** of their strings, just „**gluing**“ the genomes one after the other.  
In this case, the order is important.

**Parent A:** „**Throatwobbler**“

**Parent B:** „**Mangrove**“

**Child LY:** „**Throatwobbler Mangrove**“

*From: Monty Python's Flying Circus: Episode 22, Cosmetic surgery*

EA:**Inheritance**

**$k=2$  and  $k>2$ , recombination with  $k$  parents**

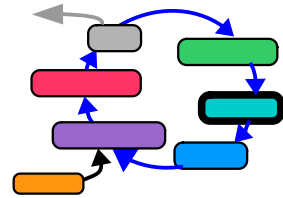
Several ways to **combine** (or **re-combine**) the genomes of the  $k$  parents have been proposed.

Some of them rely on a special structure of the genome.

- **N-point cross over** (the most popular)
- **mean-values**, one component, all components
- **max, min, sum, difference** of values
- **union, intersection** of **sets** of items
- **concatenate** (strings)
- **...**

## EA:

### Inheritance



Special forms of **genome coding** can require special **recombination operators**.

In some cases, the Genome is coded with a special semantics aligned with the application.

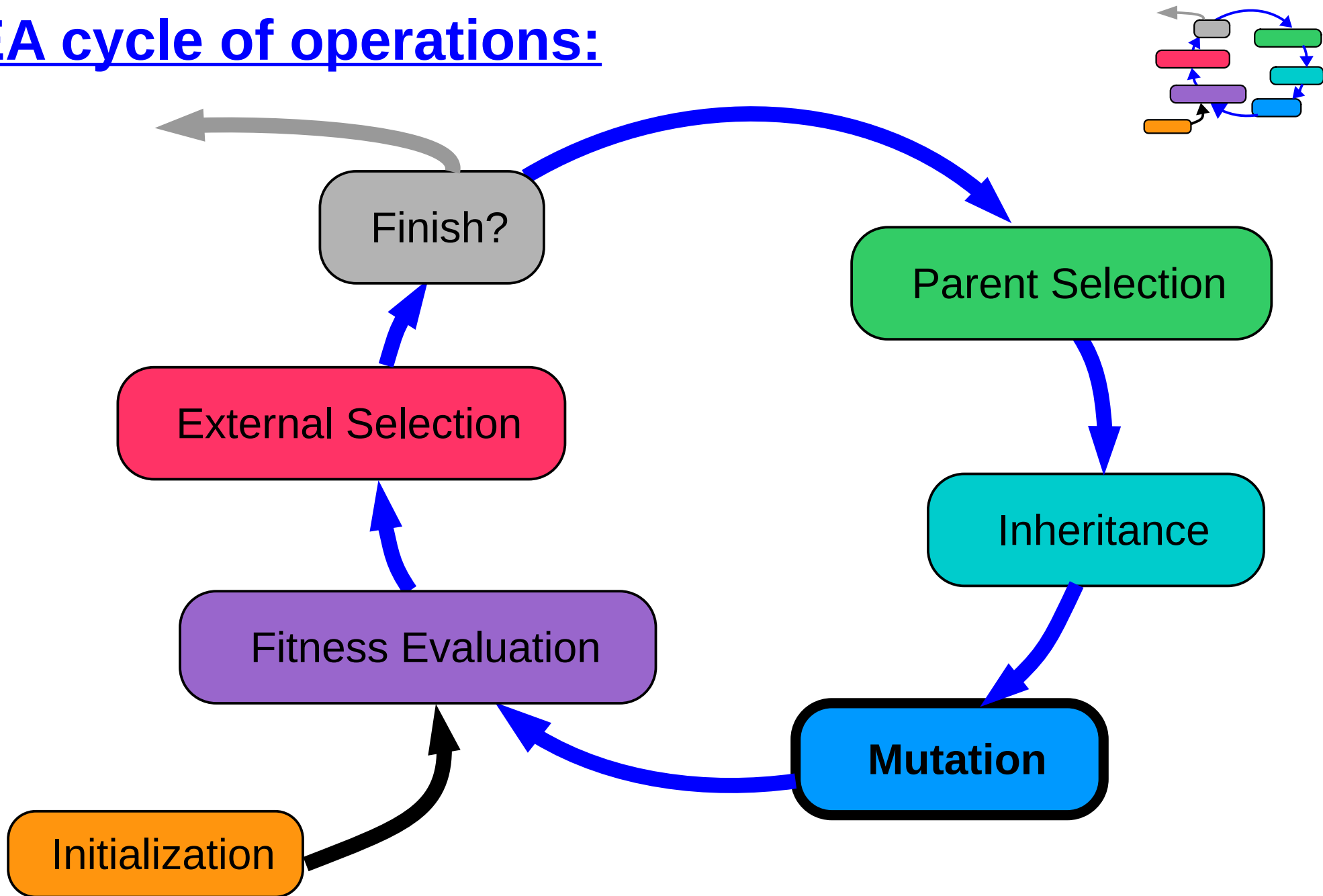
The **inheritance, recombination operator** **can** pay respect to this fact, and **can** therefore be structured in this way, but it **does not have to** .

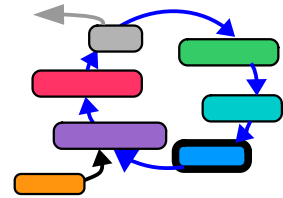
There are good arguments, to regard the genome as a complete context free binary vector.

# Overview

- EA Steps
  - Individual, Genome, Fitness, Population
  - Parent selection
  - Inheritance
  - **Mutation**
  - Fitness evaluation
  - External selection
  - Finish?
  - Initialization
- Strategy
- Performance Graph
- Genome Structure
- Examples

## EA cycle of operations:



EA:**Mutation**

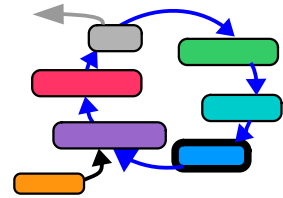
The job of the **mutation** step is to maintain the **diversity** of the genomes within the population:

Different positions in search space sample the fitness function at different points.

Changing the genome by **mutation** is implementing the **exploration** principle of **stochastic optimization**.

## EA:

### Mutation

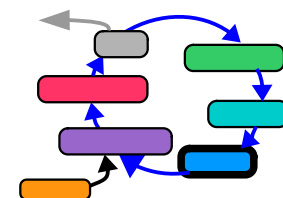


Depending on the different way the information is coded in the genome, the layout of the **mutation** step is to be designed differently.

Some common mutation operators are:

- **Binary genome**: a bit-flip
- **Vector of parameters**: change of value
- **Set of items**: replace a single item
- **String of characters**: replace a single character
- **Sequence**: change order within sequence
- **Other**: other method

Special codings of the genome will require that the **mutation** is paying respect to this coding.

EA:**Mutation**

The **mutation** for a binary genome is typically implemented as one or more **bit-flips**:

**Method 1:**

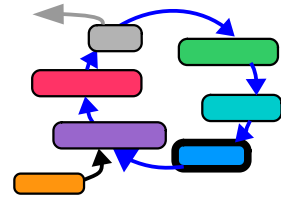
Choose a random position **r** in the genome and **flip** that **bit**. This yields a 100% chance that the genome is changed; the resulting Hamming distance is always exactly 1.

**Before:**



## EA:

### Mutation

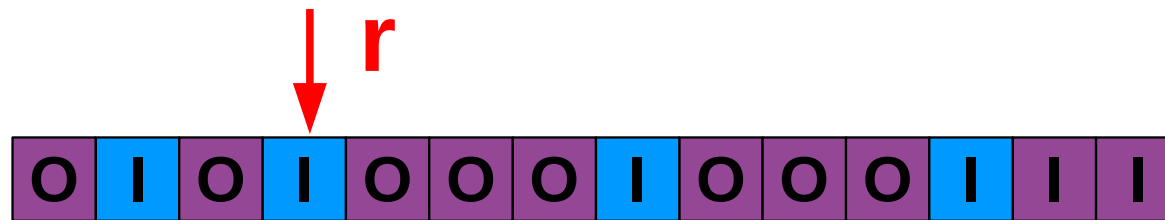


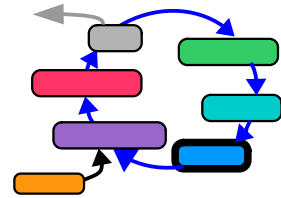
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**Before:**

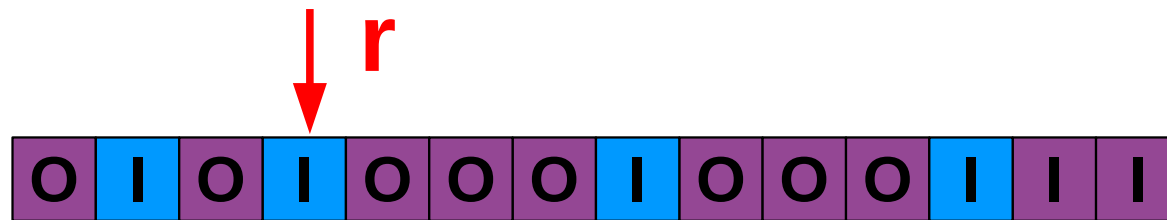
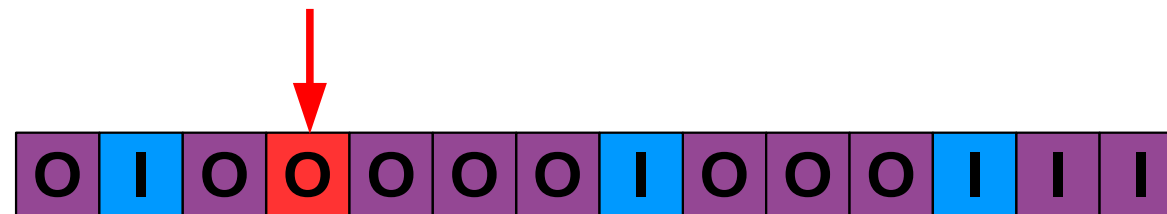


EA:**Mutation**

The **mutation** for a binary genome is typically implemented as one or more **bit-flips**:

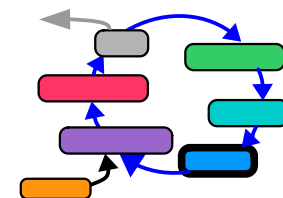
**Method 1:**

Choose a random position **r** in the genome and **flip** that **bit**. This yields a 100% chance that the genome is changed; the resulting Hamming distance is always exactly 1.

**Before:****After:**

## EA:

### Mutation



The **mutation** for a binary genome is typically implemented as one or more **bit-flips**:

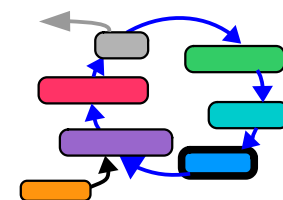
### Method 2:

Go through all positions in the genome, and **flip** each **bit** with a given **bit-flip**-probability  $\omega$ .

There is a chance, that all bits are flipped, and a chance that none is flipped.

**Before:**



EA:**Mutation**

The **mutation** for a binary genome is typically implemented as one or more **bit-flips**:

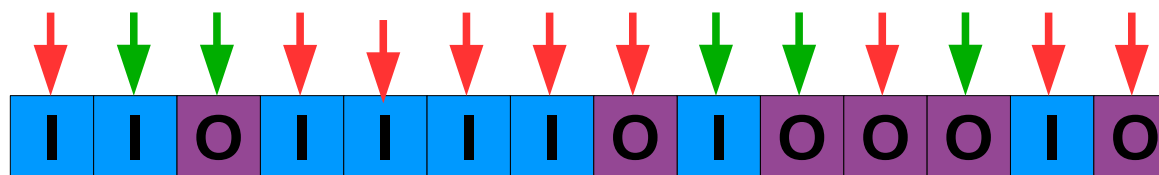
**Method 2:**

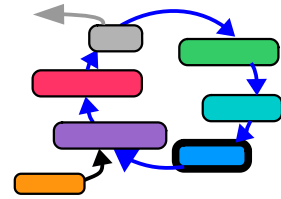
Go through all positions in the genome, and **flip** each **bit** with a given **bit-flip**-probability  $\omega$ .

There is a chance, that all bits are flipped, and a chance that none is flipped.

$$0.0 < \omega < 1.0$$

**Before:**



EA:**Mutation**

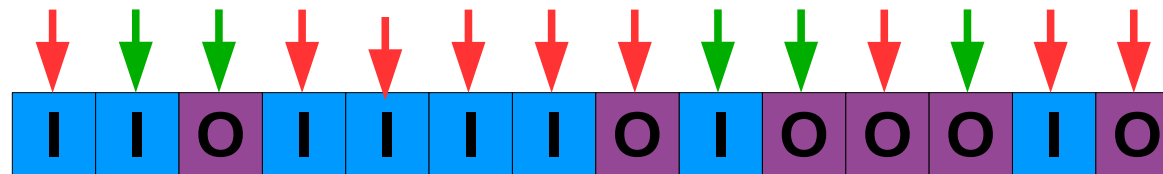
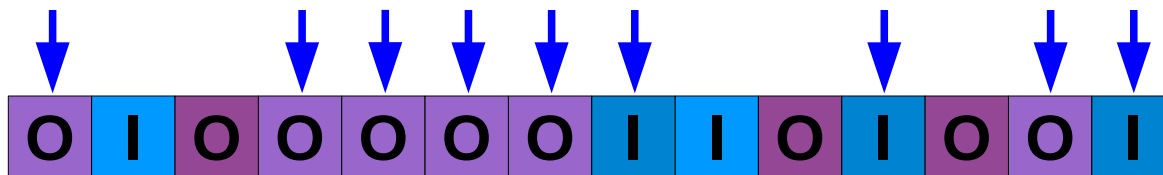
The **mutation** for a binary genome is typically implemented as one or more **bit-flips**:

**Method 2:**

Go through all positions in the genome, and **flip** each **bit** with a given **bit-flip-probability**  $\omega$ .

There is a chance, that all bits are flipped, and a chance that none is flipped.

$$0.0 < \omega < 1.0$$

**Before:****After:**

EA:

**Mutation**

&

**Inheritance**

It is possible, that the **inheritance** and the **mutation** process yield invalid genomes, which represent hypotheses **s** from outside the allowed search space **S**.

Those illegal genomes would stress the optimization process in an unwanted way.

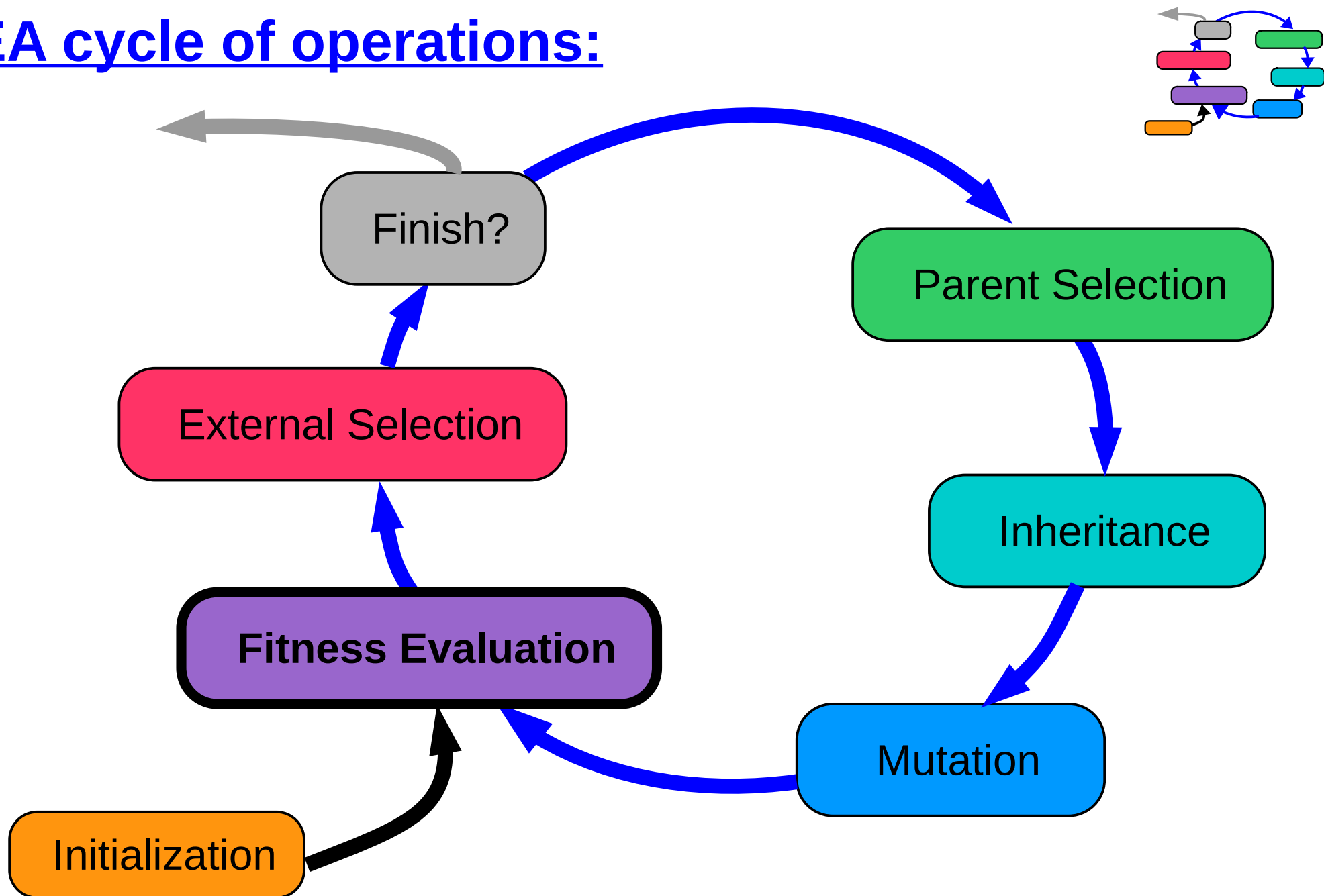
Therefore, it is a good advise, to take precautions against illegal genomes (if possible, and economic):

- shape the structure of the genome,
- shape the structure of the inheritance,
- shape the structure of the mutation.

# Overview

- EA Steps
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  - Inheritance
  - Mutation
  - **Fitness evaluation**
  - External selection
  - Finish?
  - Initialization
- Strategy
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- Examples

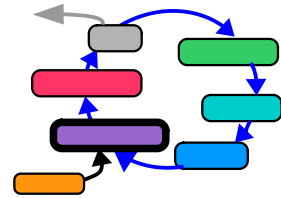
## EA cycle of operations:





EA:

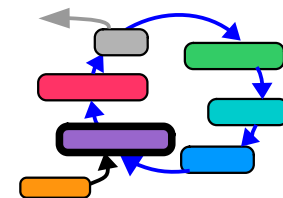
## Fitness Evaluation



The **fitness** of each individual is evaluated with respect to the **fitness-function** (most case identical to the objective function). For EAs, it is usual to have a **fitness** function that is to be maximized.

EA:

## Fitness Evaluation

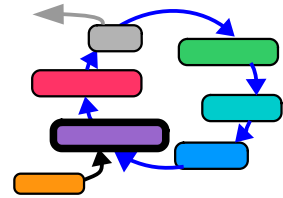


The **fitness** of each individual is evaluated with respect to the **fitness-function** (most case identical to the objective function). For EAs, it is usual to have a **fitness** function that is to be maximized.

If the **fitness evaluation** is expensive, it is a good idea to process only the new individuals/genomes from the **inheritance** step, including those individuals/genomes that have changed during **mutation**.

EA:

## Fitness Evaluation



The **fitness  $f(g_p)$**  of each individual  **$p$**  is evaluated with respect to the application using the given **fitness-function  $f(g)$** .

### Genome $g_p$

$p=P$

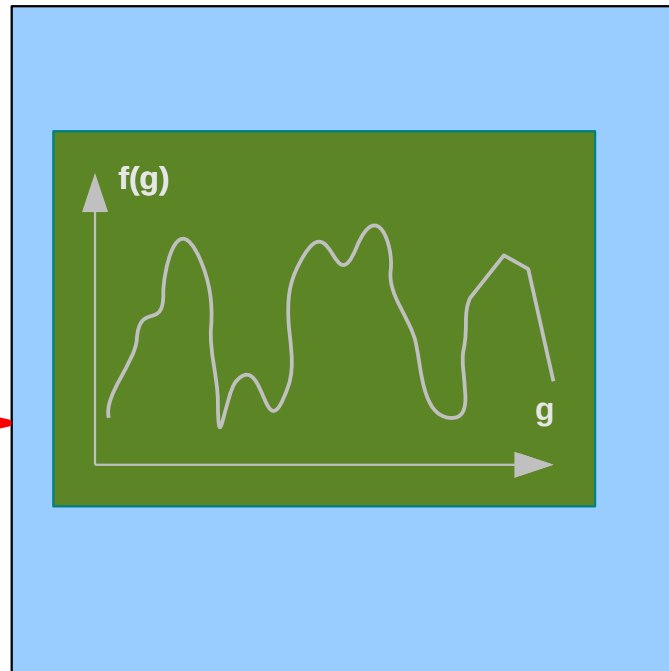
$p=5$

$p=4$

$p=3$

$p=2$

$p=1$



### Fitness Value $f(g_p)$

$f(g_p) =$

$f(g_4) =$

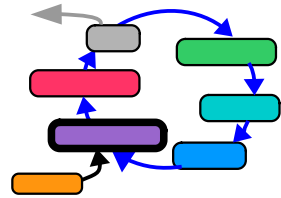
$f(g_3) = 0.228$

$f(g_2) = -171.0$

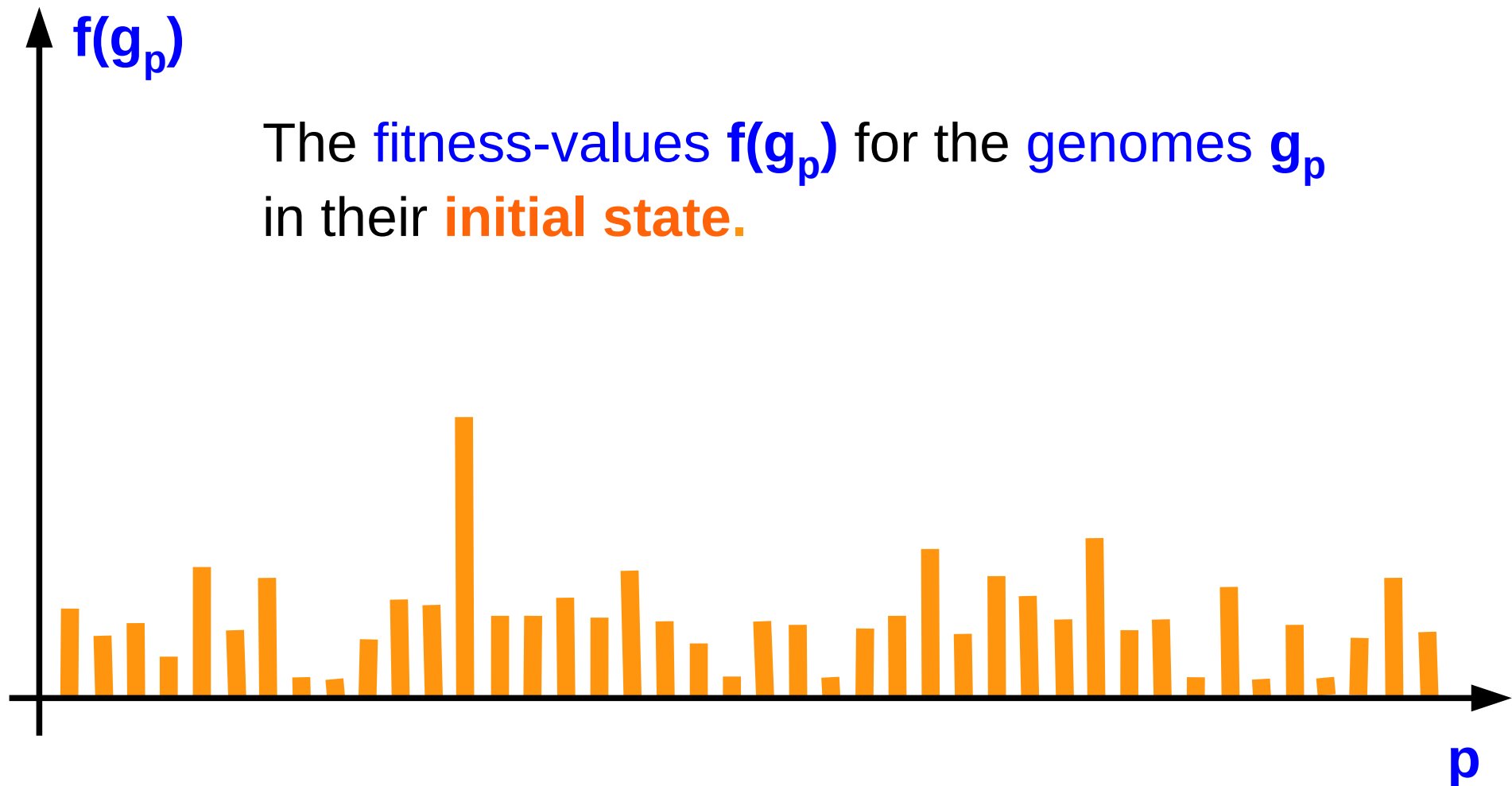
$f(g_1) = 0.5$

EA:

## Fitness Evaluation

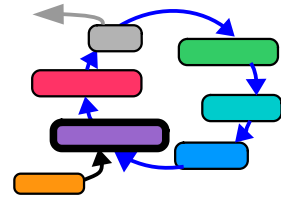


Distribution of **fitness values** over the population.

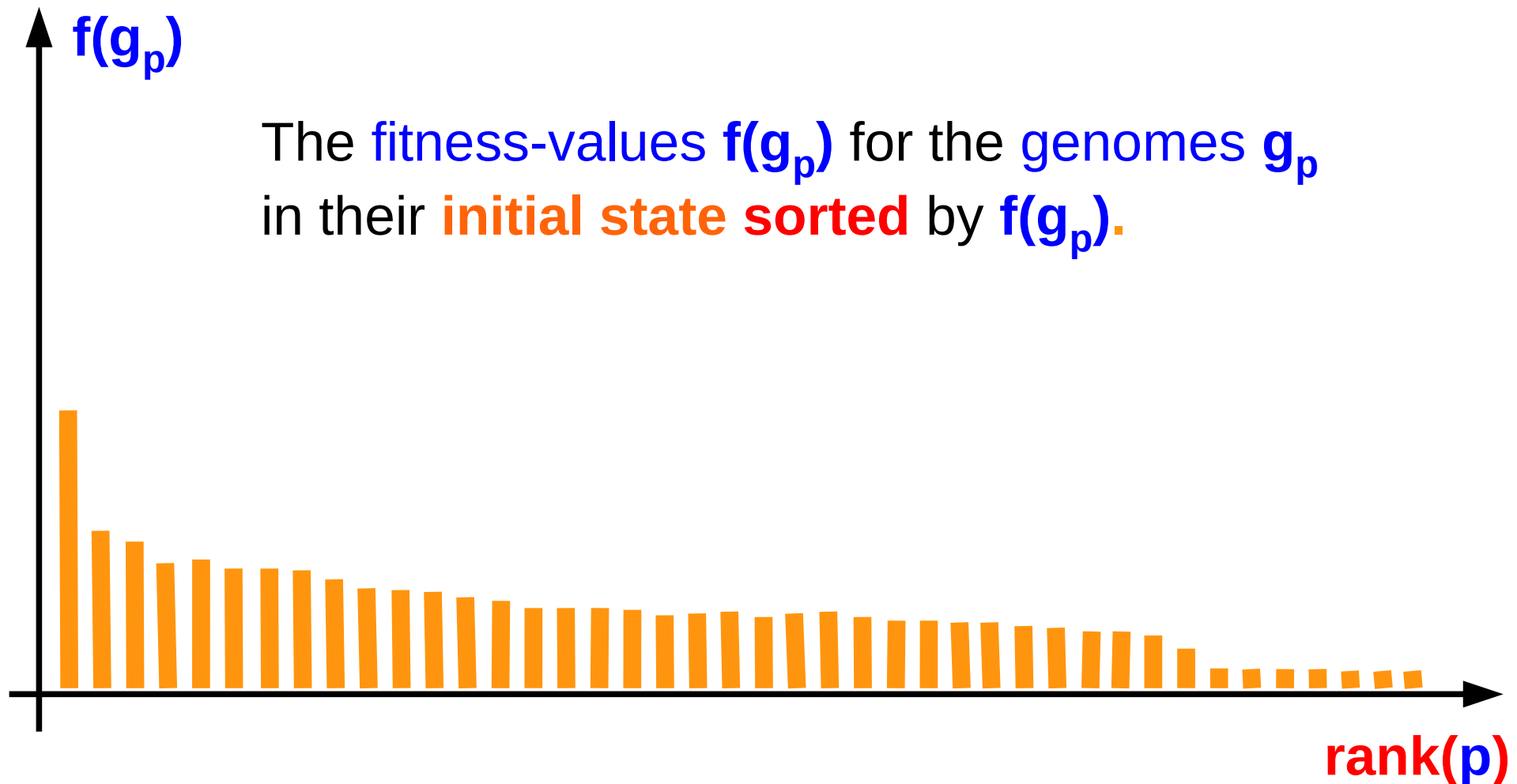


EA:

## Fitness Evaluation

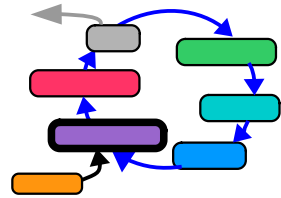


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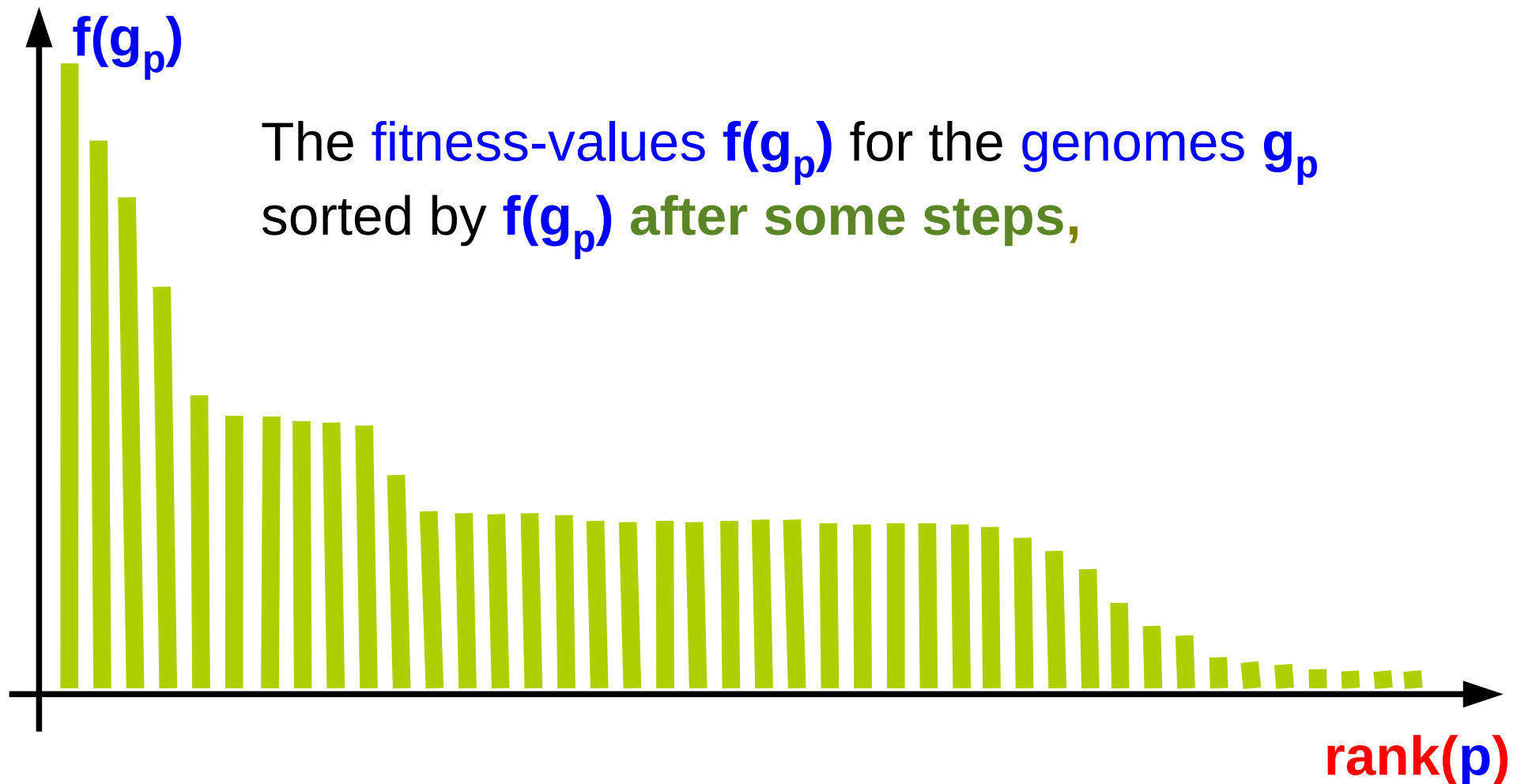


EA:

## Fitness Evaluation

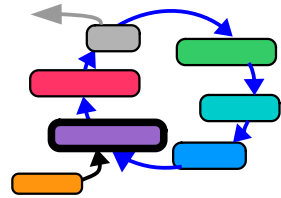


Distribution of **fitness values** over the population.

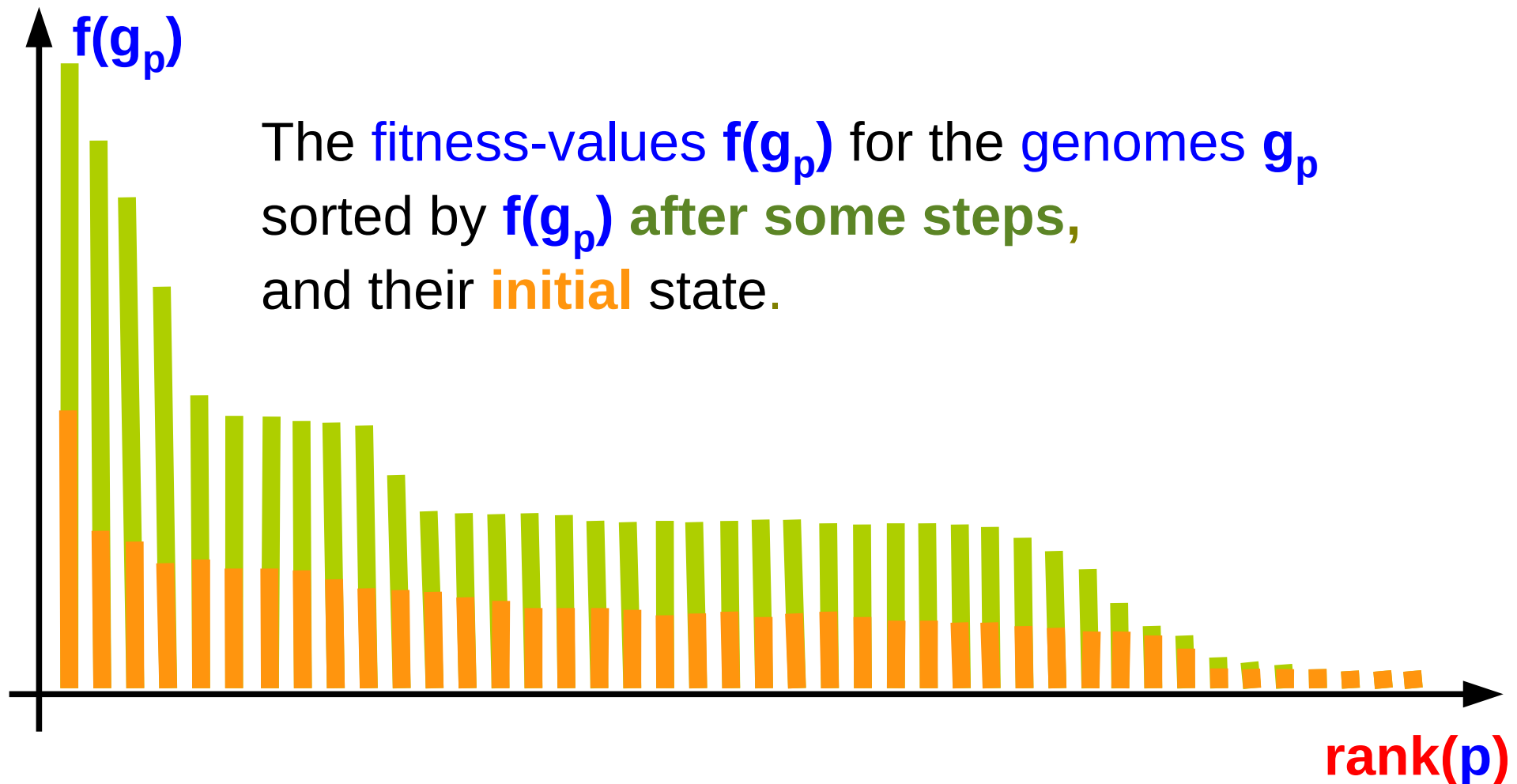


EA:

## Fitness Evaluation



Distribution of **fitness values** over the population.

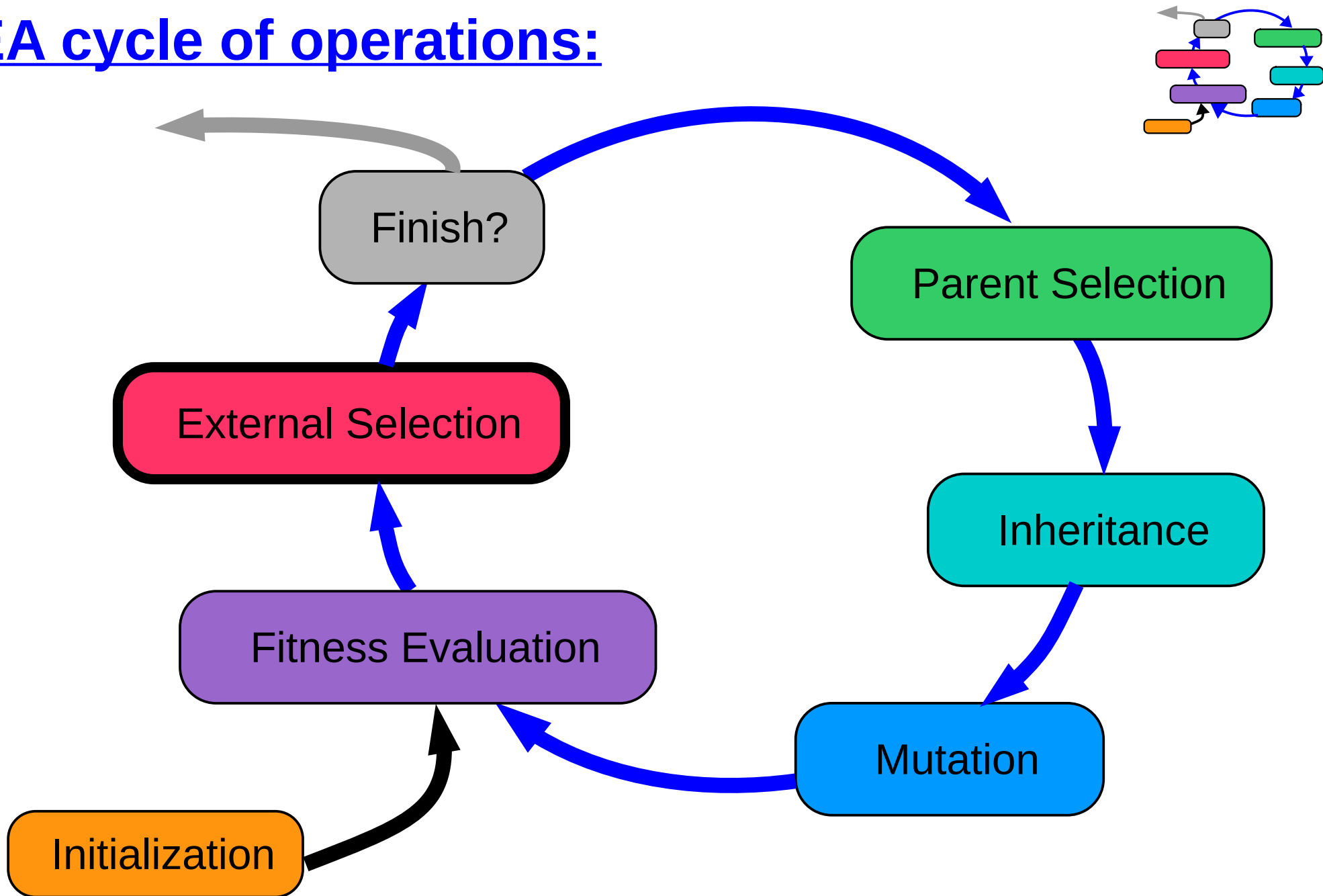


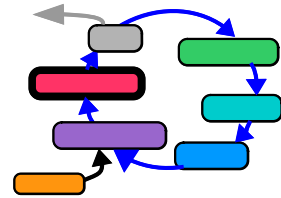
# Overview

- EA Steps
  - Individual, Genome, Fitness, Population
  - Parent selection
  - Inheritance
  - Mutation
  - Fitness evaluation
  - **External selection**
  - Finish?
  - Initialization
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## EA cycle of operations:

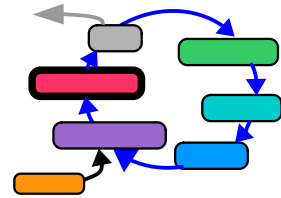


EA:**External Selection**

The purpose of the **selection process** is to keep the „Best“, and discard the „Losers“.

The **selection** is based on the achieved **fitness**, which is obtained by the **fitness function**, quasi from **external** to the EA.

The  $\mu$  individuals that survive the **external selection** process, will be the pool for the **parents** of the next generation. Later on, from these  $\mu$  parents the  $\lambda$  offspring will be generated through the inheritance process.

EA:**External Selection**

The two questions for the **external selection** are:

**How many ( $\mu$ ) of the individuals shall survive?**

and

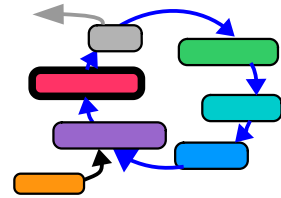
**Which are the ones to keep?**

A common way is to keep the population constant to **P** individuals.

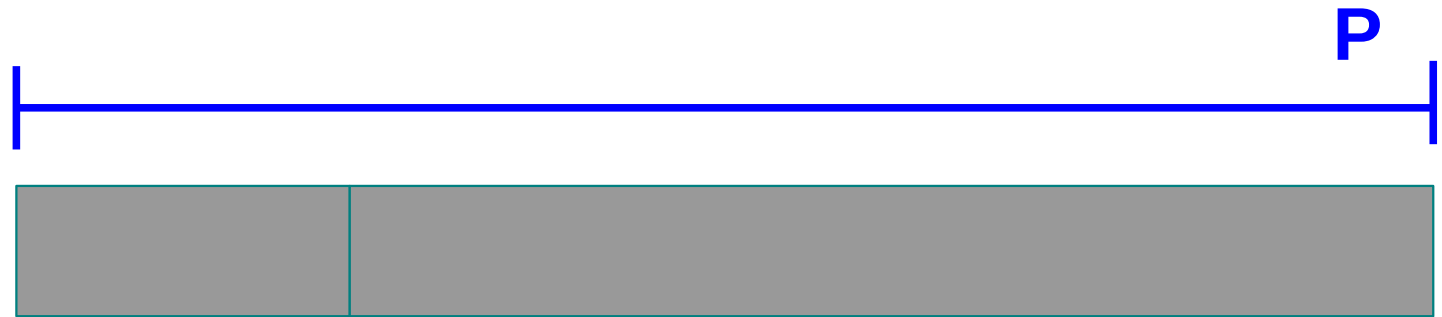
discard  $\lambda$  by external selection

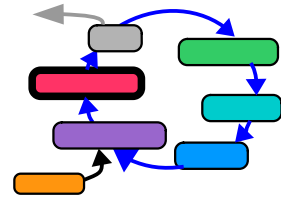
and keep  $\mu$  as pool for the parents,

then generate  $\lambda$  offspring for the next generation.

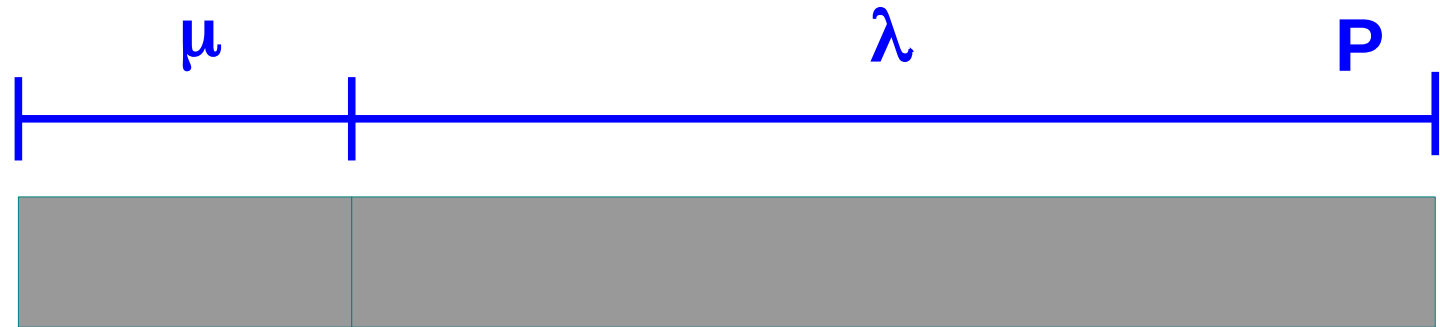
EA:**External Selection**

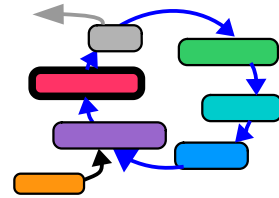
Have **P**, keep  $\mu$ , generate  $\lambda$  offspring



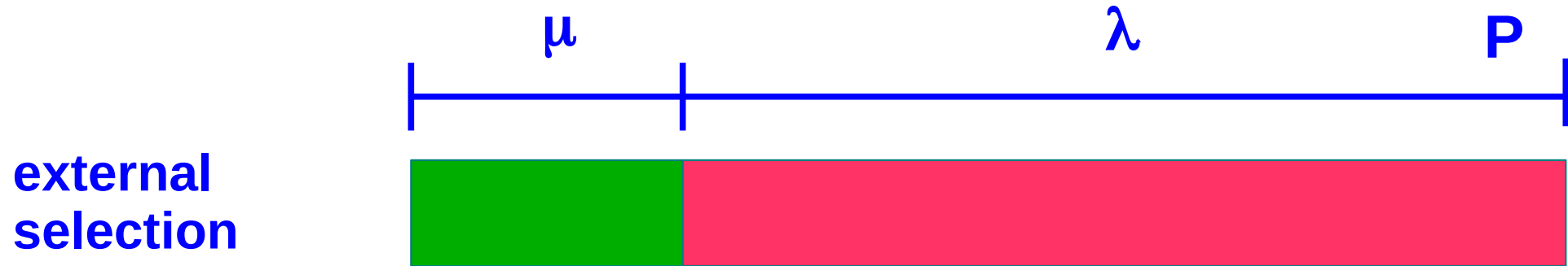
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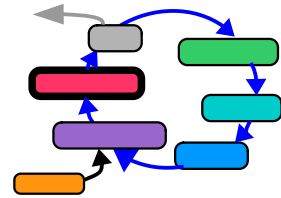
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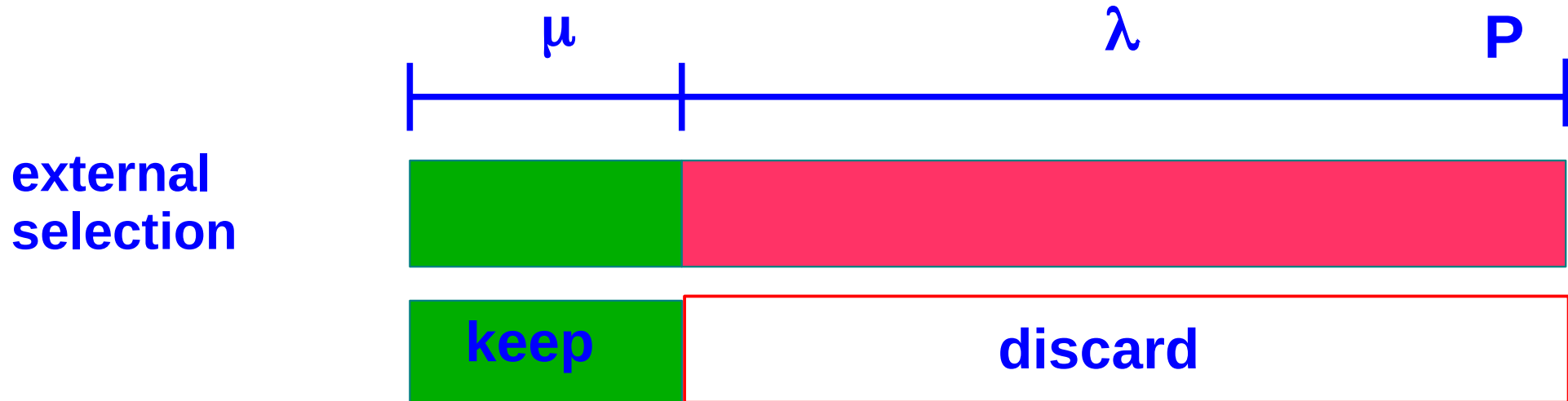
EA:**External Selection**

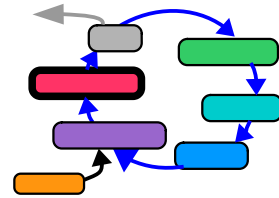
Have **P**, keep  $\mu$ , generate  $\lambda$  offspring



EA:**External Selection**

Have **P**, keep  $\mu$ , generate  $\lambda$  offspring



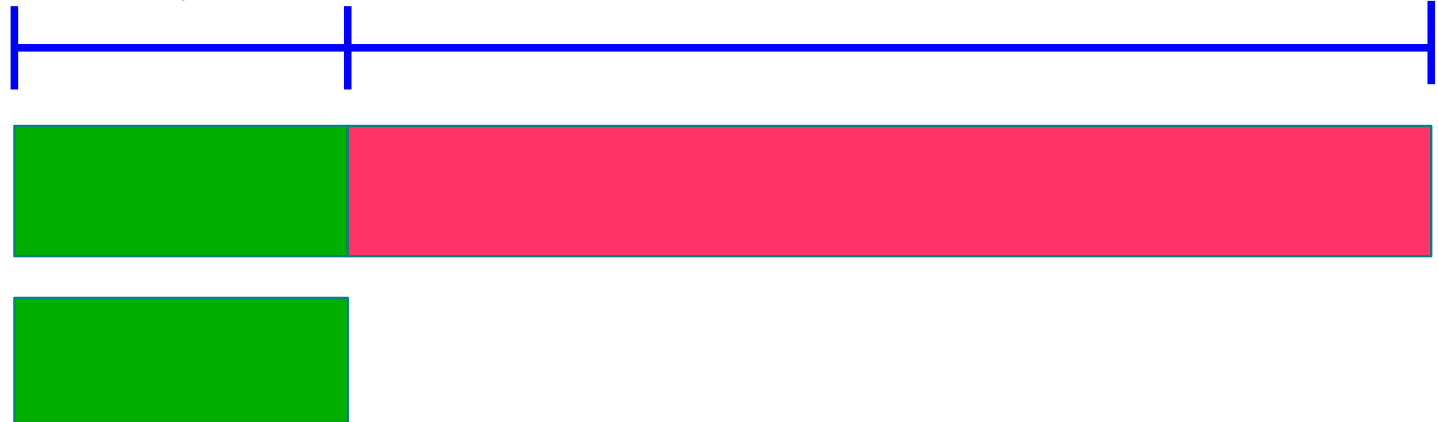
EA:**External Selection**

Have **P**, keep  $\mu$ , generate  $\lambda$  offspring

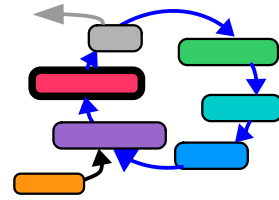
 $\mu$  $\lambda$ **P**

external  
selection

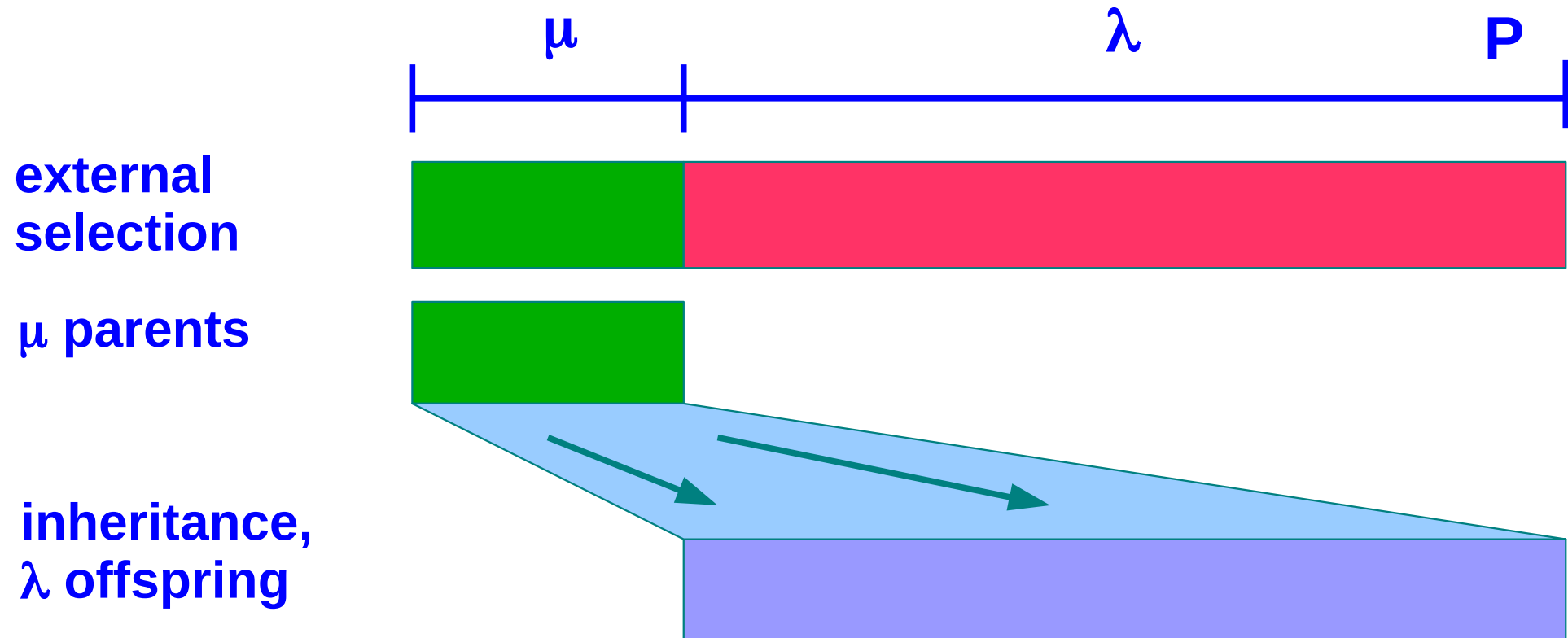
$\mu$  parents





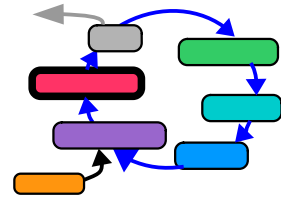
EA:**External Selection**

Have **P**, keep  $\mu$ , generate  $\lambda$  offspring

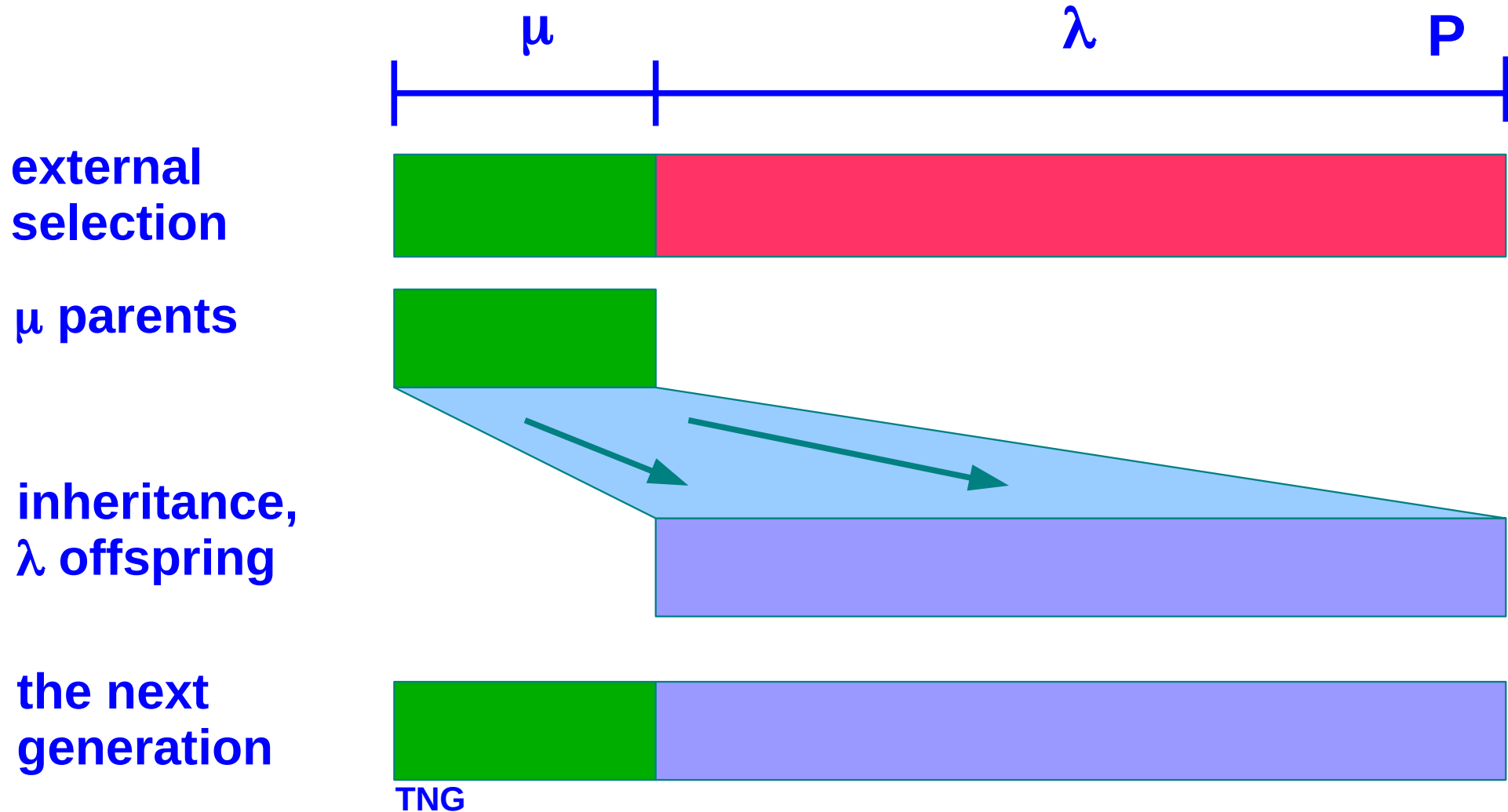


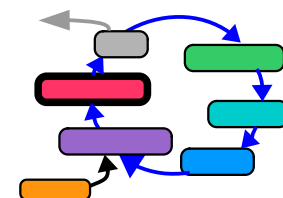
EA:

## External Selection



Have  $P$ , keep  $\mu$ , generate  $\lambda$  offspring



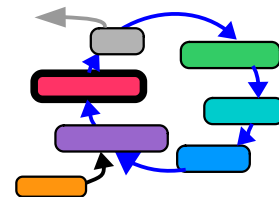
EA:**External Selection**

Common strategies for the **external selection** are:

- random choice
- schedule based, e.g. round robin
- fitness based elitism:
  - fitness proportional choice
  - rank proportional choice
- fitness based stochastic:
  - fitness proportionate, probabilistic choice
  - rank proportionate, probabilistic choice
- combinations of the above

## EA:

### External Selection



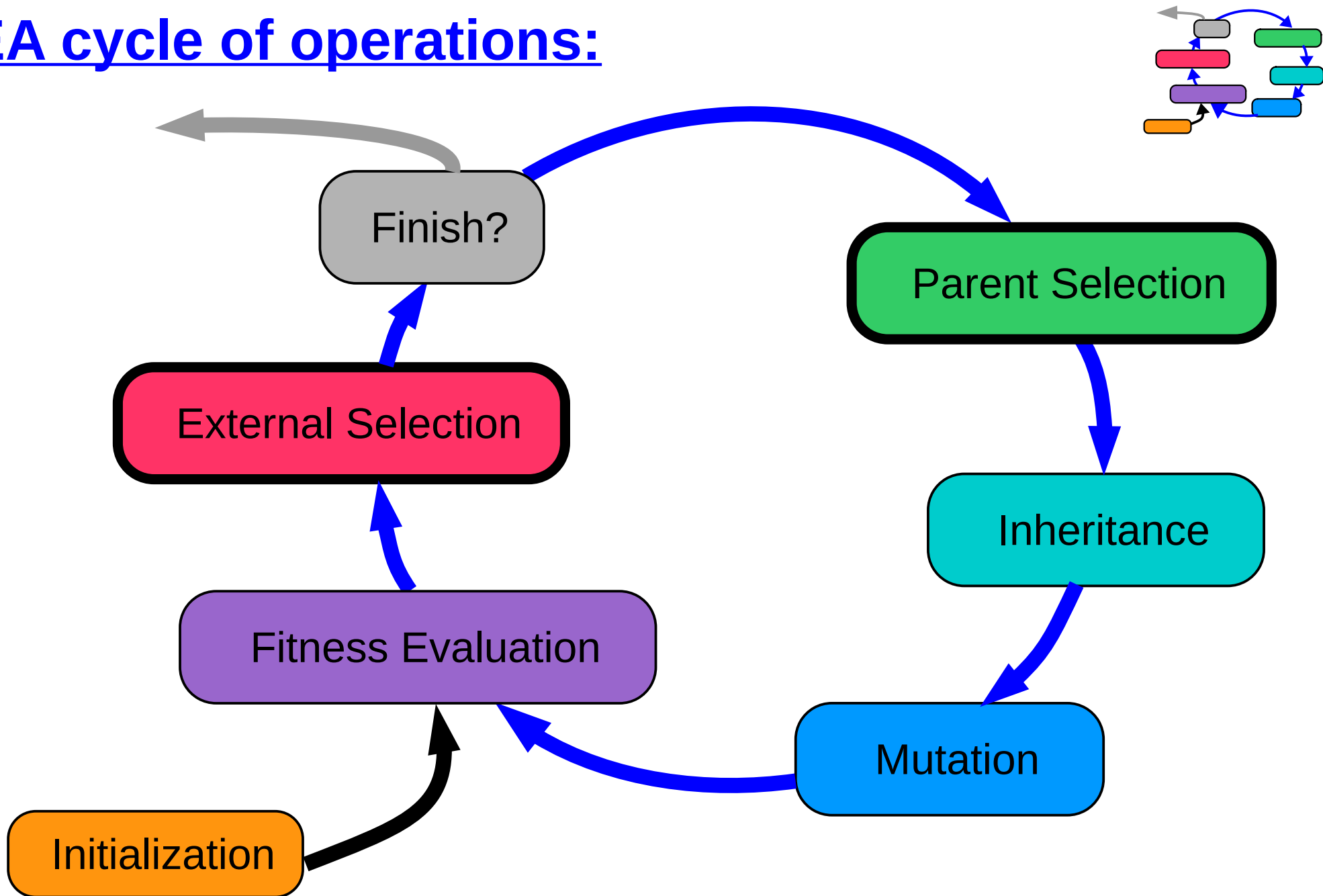
There are different ideas and principles behind the selection of the  $\mu$  possible parents from the population, with special pros and cons.

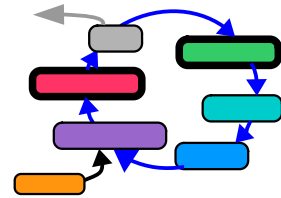
- **deterministic** or **stochastic** selection
- **fitness dependent** selection
- **fitness proportional** selection
- **rank-based** selection
- **tournament** selection
- **life-time based** selection
- **combinations** of the above + more

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## EA cycle of operations:



EA:**External Selection****Parent Selection**

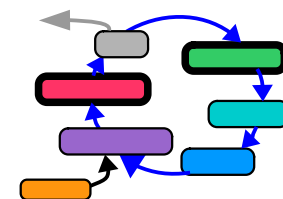
Selection in **E**volutionary **S**trategy **ES**-Systems:  
Following the initial idea of **E**volutionary **S**trategies  
there are two strategies for the selection process:

$$(\mu + \lambda) \quad \text{and} \quad (\mu, \lambda)$$

EA:

External Selection

Parent Selection

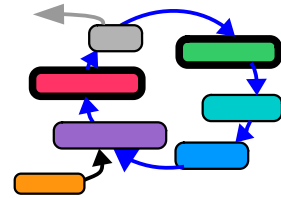


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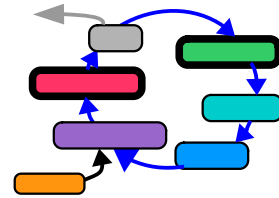
EA:**External Selection****Parent Selection**

( **1** + **1** ):  $\mu=1$  one parent,  $\lambda=1$  one child,  
inheritance by copying, only mutation  
rank based, deterministic external selection

EA:

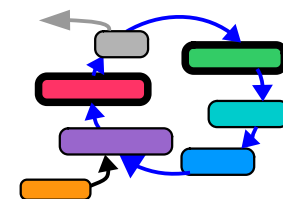
External Selection

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inheritance by copying, only mutation  
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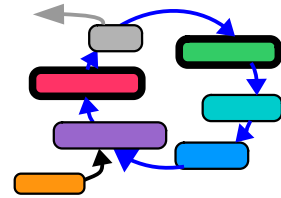
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(  $\mu + \lambda$  ):  $\mu$  parents,  $\lambda$  offspring, parents survive  
recombination, mutation, external selection.

EA:

External Selection

Parent Selection



(  $1 + 1$  ):  $\mu=1$  one parent,  $\lambda=1$  one child,  
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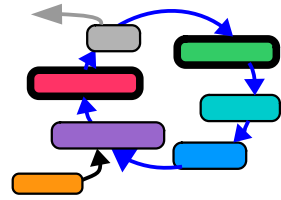
(  $\mu + \lambda$  ):  $\mu$  parents,  $\lambda$  offspring, parents survive  
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(  $\mu, \lambda$  ):  $\mu$  parents,  $\lambda$  offspring,  
recombination, mutation, external selection,  
parents are discarded.

EA:

External Selection

Parent Selection



Have  $P$ , keep  $\mu$ , generate  $\lambda$  offspring

$(\mu + \lambda)$

$\mu$

$\lambda$

$P$

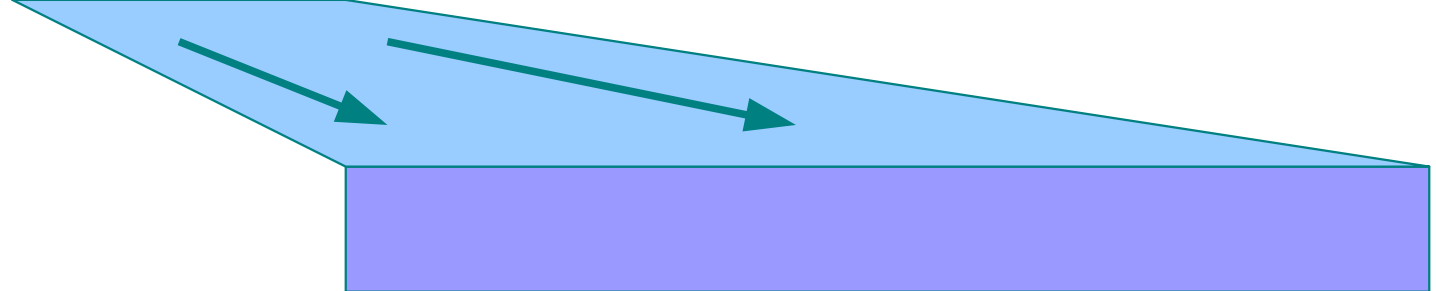
external  
selection



$\mu$  parents



inheritance,  
 $\lambda$  offspring



the next  
generation

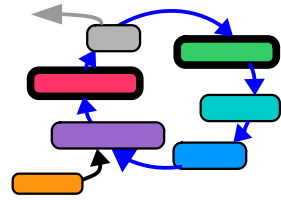


TNG

EA:

External Selection

Parent Selection



Have  $P$ , keep  $\mu$ , generate  $\lambda$  offspring

$(\mu, \lambda)$



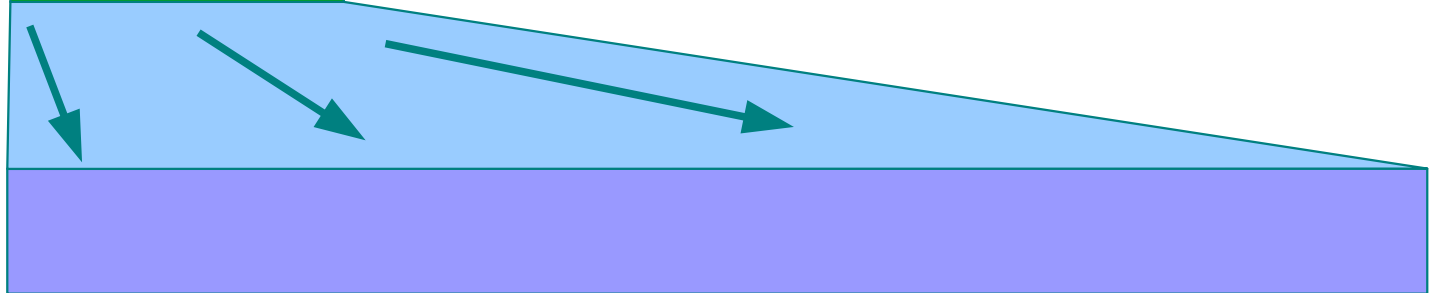
external  
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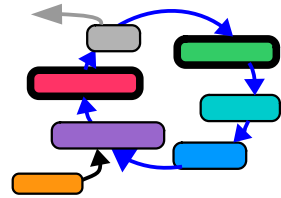


TNG

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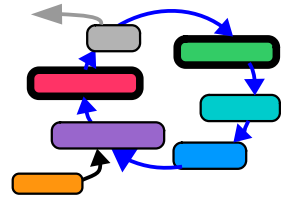
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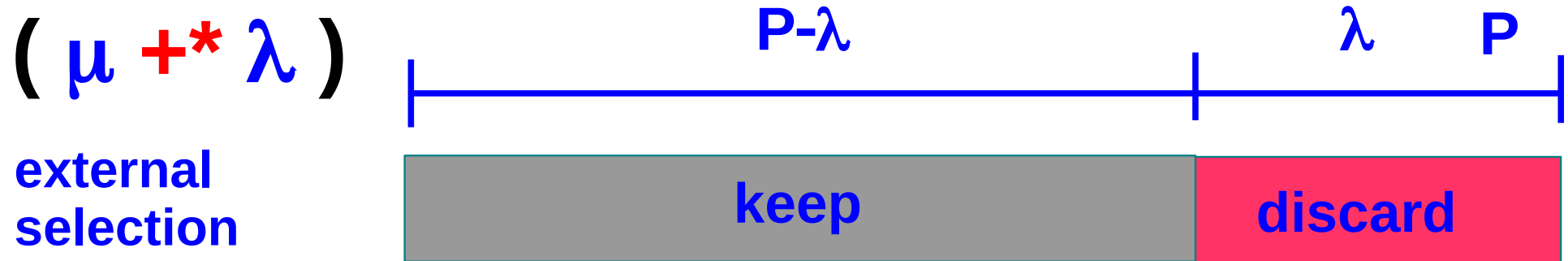
EA:

External Selection

Parent Selection



Have  $P$ , discard  $\lambda$ , select  $\mu$  parents, generate  $\lambda$  offspring

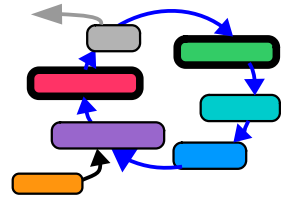




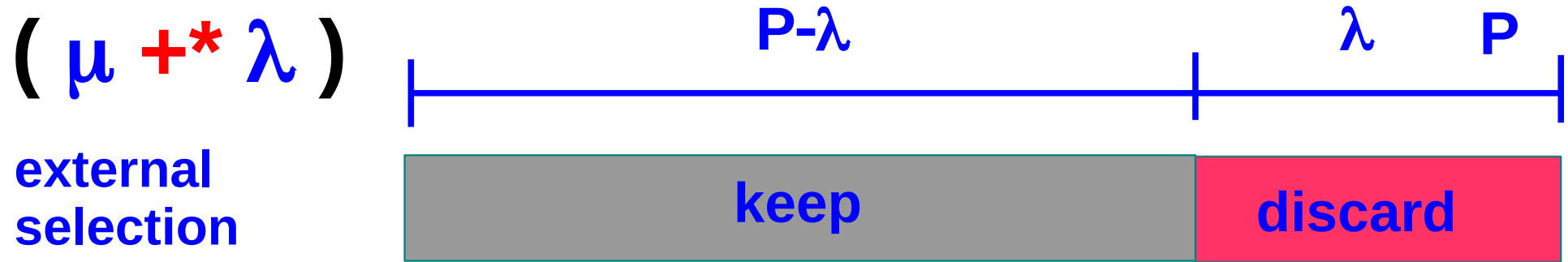
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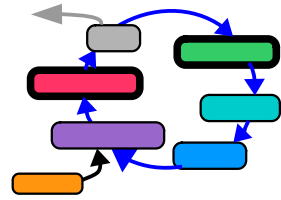


select  
 $\mu$  parents

EA:

External Selection

Parent Selection



Have  $P$ , discard  $\lambda$ , select  $\mu$  parents, generate  $\lambda$  offspring

$(\mu + \lambda)$

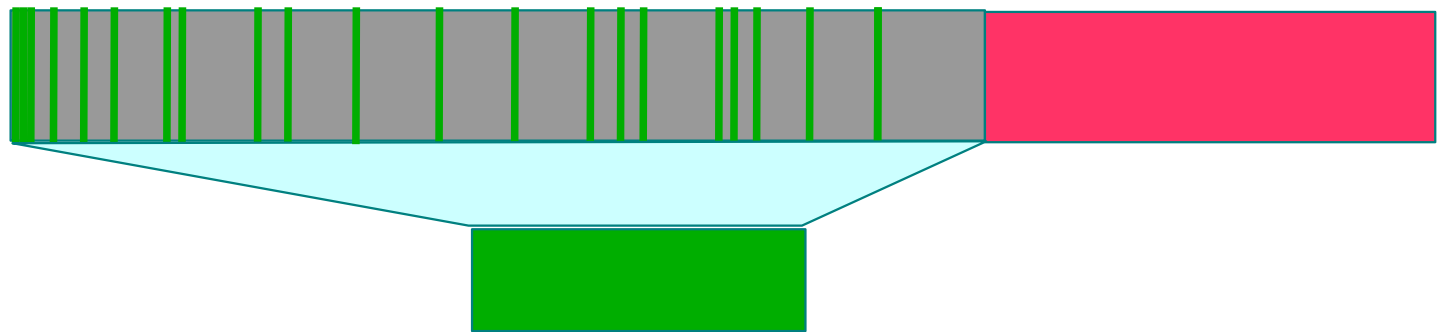
$P - \lambda$

$\lambda$

$P$

external  
selection

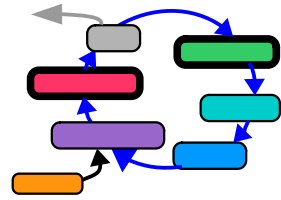
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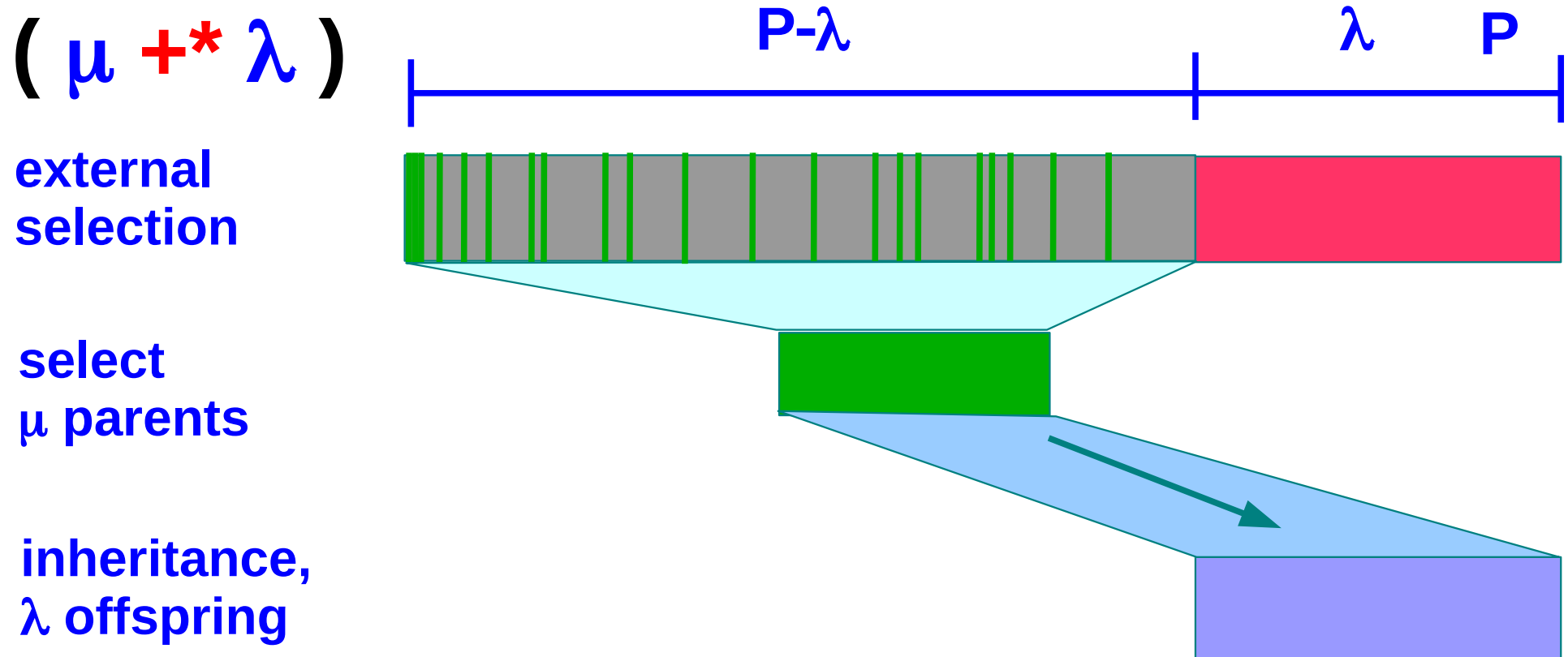
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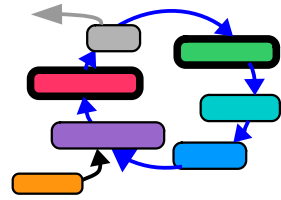
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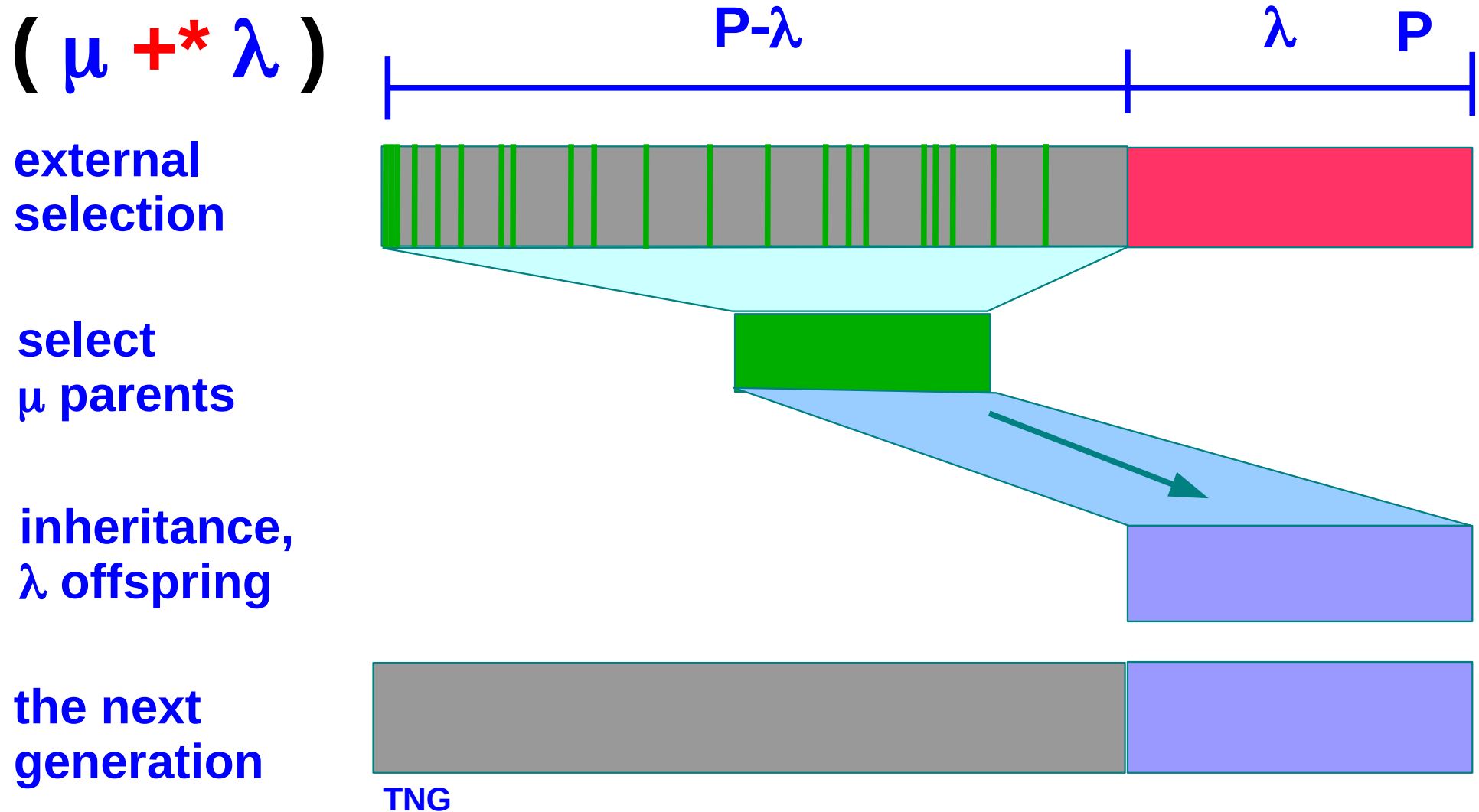
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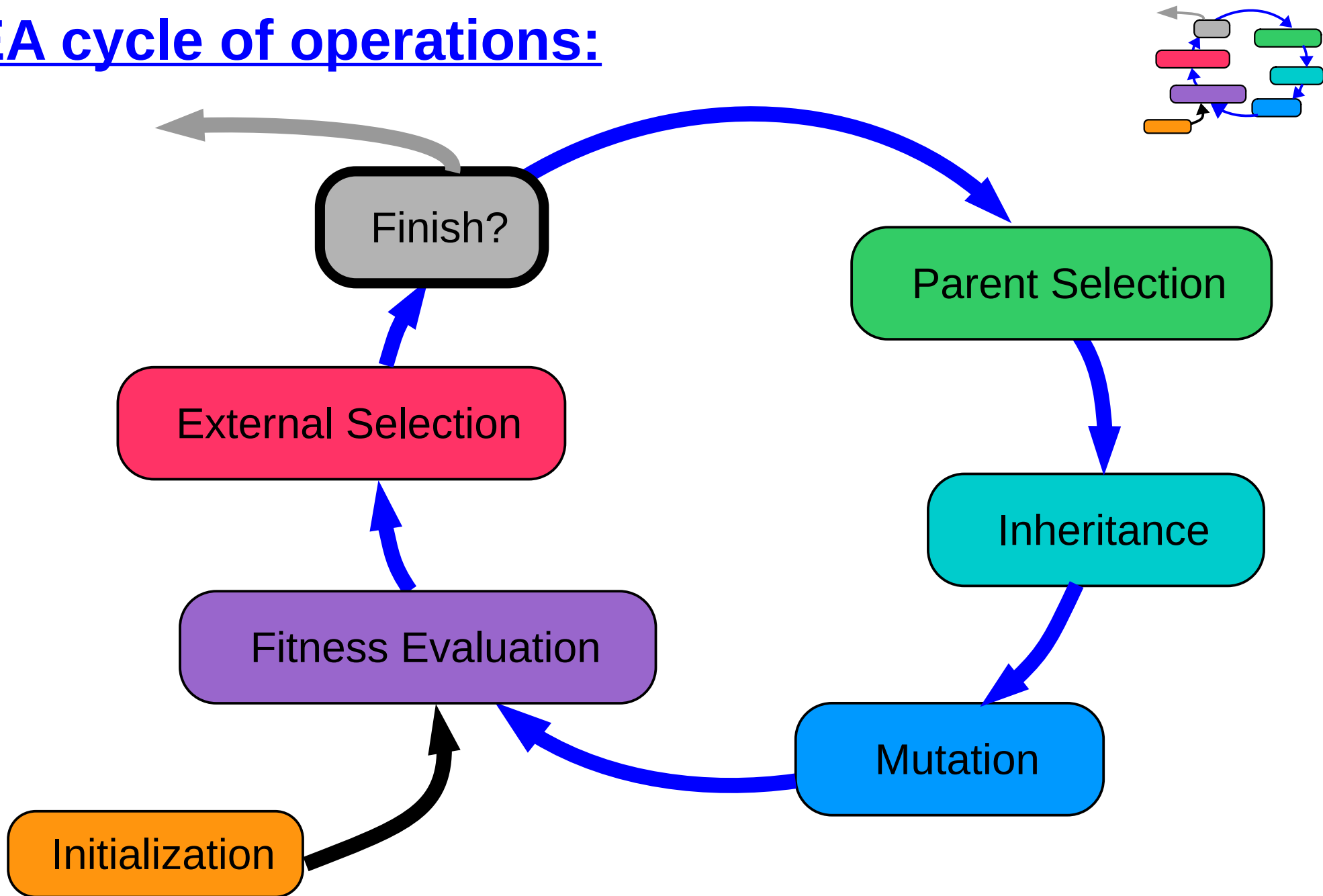
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# Overview

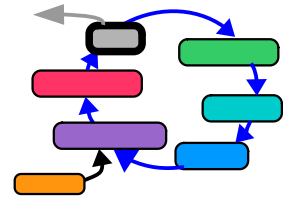
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## EA cycle of operations:



EA:

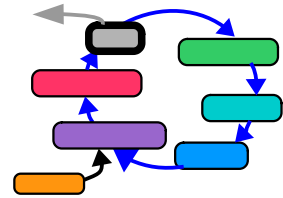
Finish?



There are several criteria to determine the **finishing** of the **EA** process:

EA:

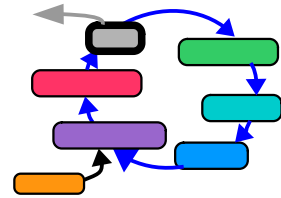
Finish?



There are several criteria to determine the **finishing** of the **EA** process:

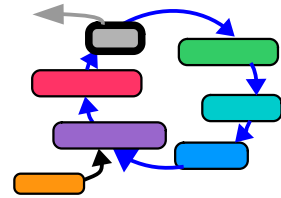
- **by performance** of best individual
- **by performance** of sub-population
- **by stagnation/development** of fitness improvement
- **by time**
- **by number** of generations
- ...



**EA:****Finish?**

There are several criteria to determine the **finishing** of the **EA** process:

- **by performance** of best individual
- **by performance** of sub-population
- **by stagnation/development** of fitness improvement
- **by time**
- **by number** of generations
- ...
- **by choice** of human operator

**EA:****Finish?**

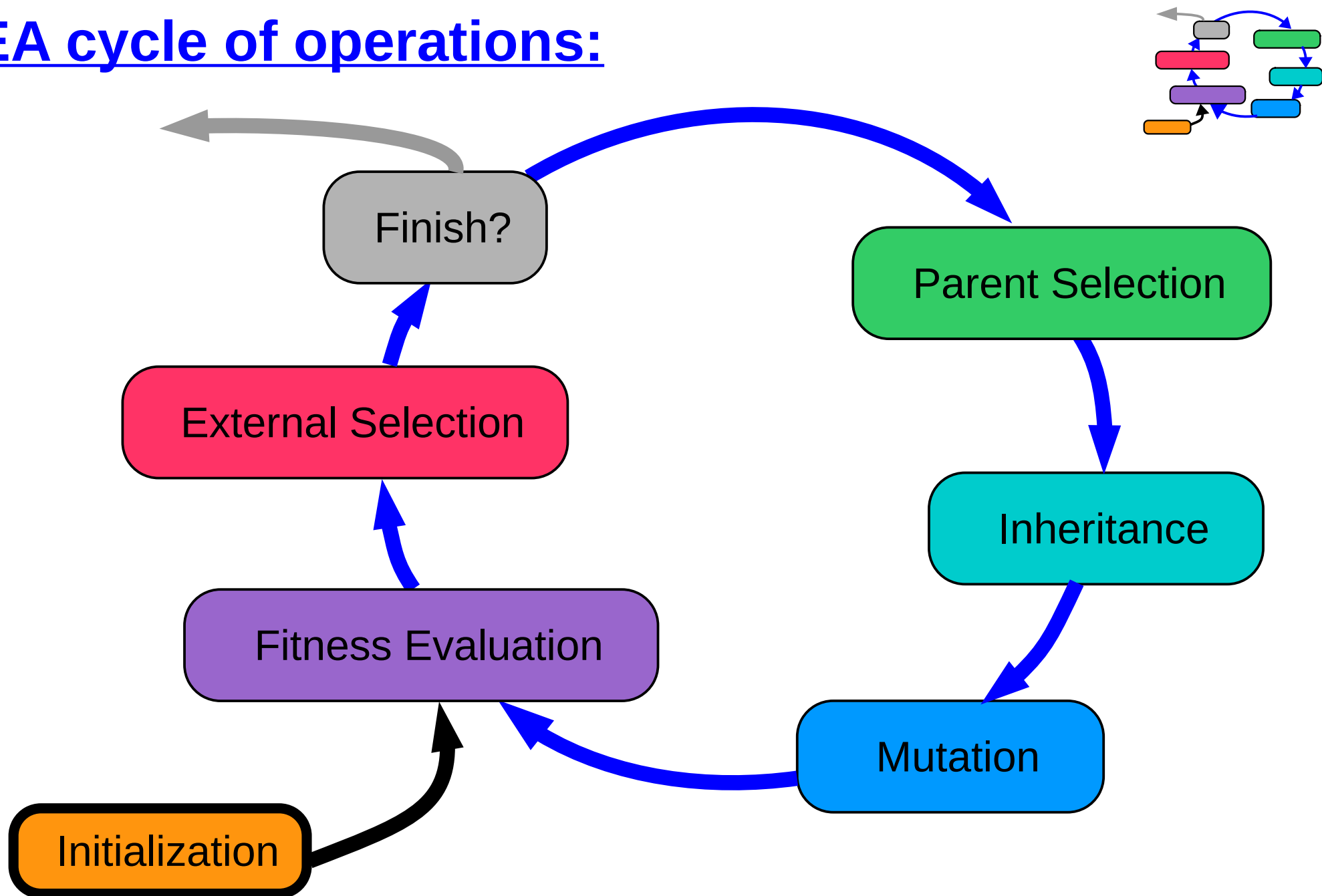
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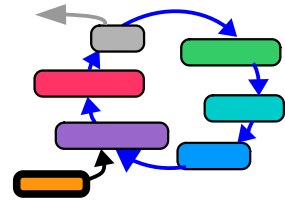
- **by performance** of best individual
- **by performance** of sub-population
- **by stagnation/development** of fitness improvement
- **by time**
- **by number** of generations
- ...
- **by choice** of human operator; **whatever that means.**

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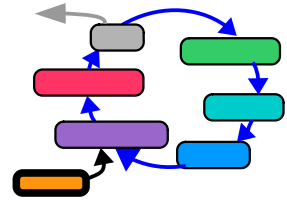
## EA cycle of operations:



EA:**Initialization**

The main principles driving the **initialization** of the genomes of the first population are:

# Initialization



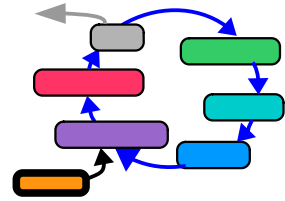
The main principles driving the **initialization** of the genomes of the first population are:

## Start as good as possible:

## Enough richness, enough diversity:

EA:

## Initialization

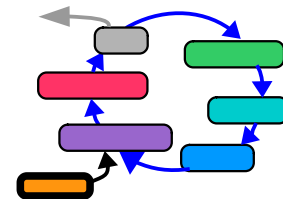


The main principles driving the **initialization** of the genomes of the first population are:

### Start as good as possible:

- Try to use all a priori knowledge available.
- Try to avoid illegal genomes (if possible).

### Enough richness, enough diversity:

EA:**Initialization**

The main principles driving the **initialization** of the genomes of the first population are:

**Start as good as possible:**

Try to use all a priori knowledge available.

Try to avoid illegal genomes (if possible).

**Enough richness, enough diversity:**

Sample as much from the fitness landscape as possible.

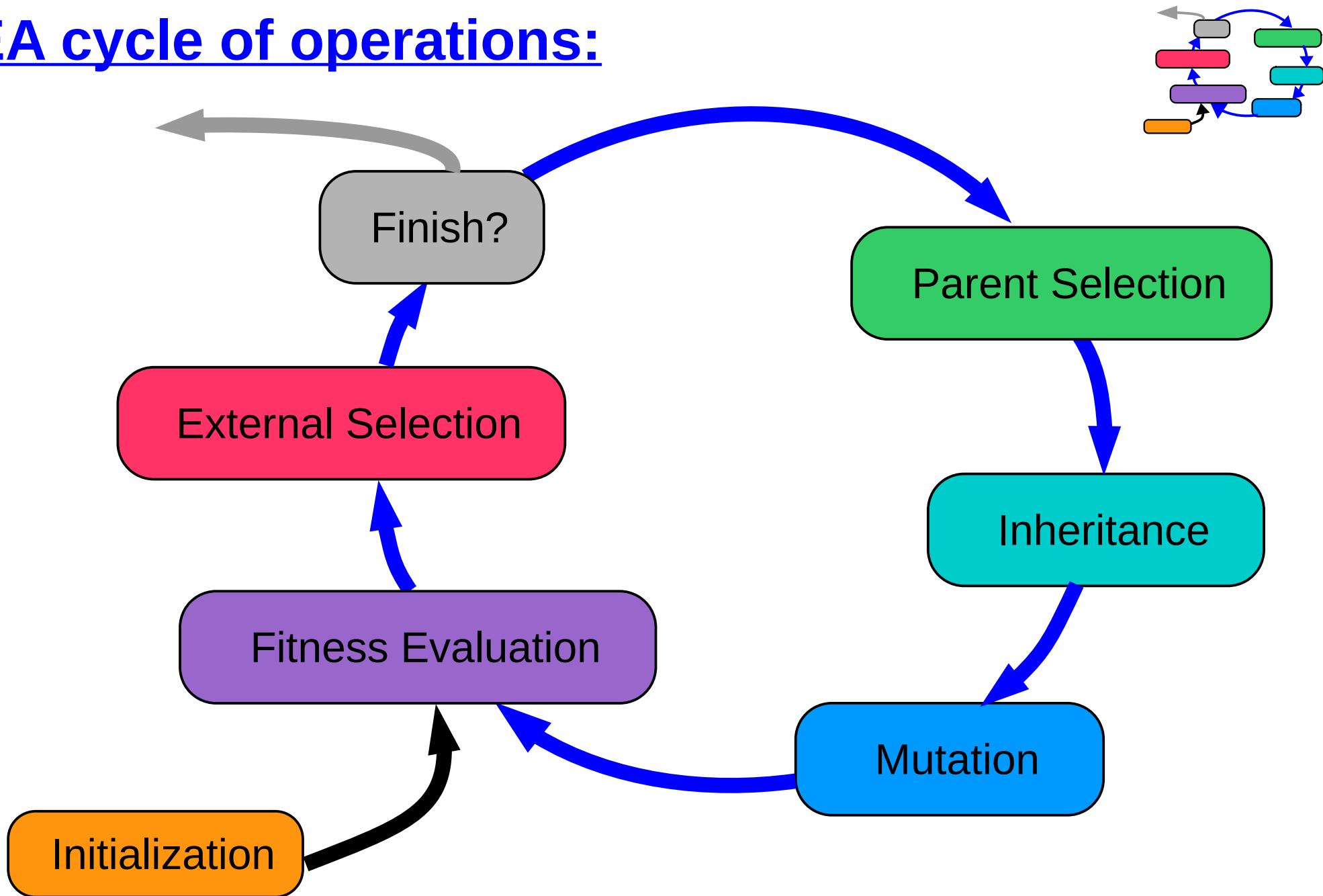
Try to cover the complete search space.



# Overview

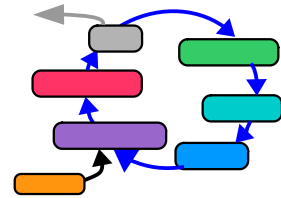
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## EA cycle of operations:



## EA:

### Performance Graph



The goal of the EA is to maximize the fitness.

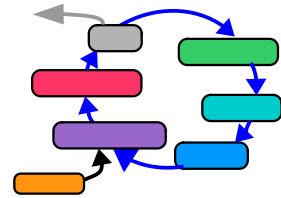
So, if the EA is working properly, somehow the fitness value within the complete population should rise with time.

How is the fitness of the best individual in the population changing over time?

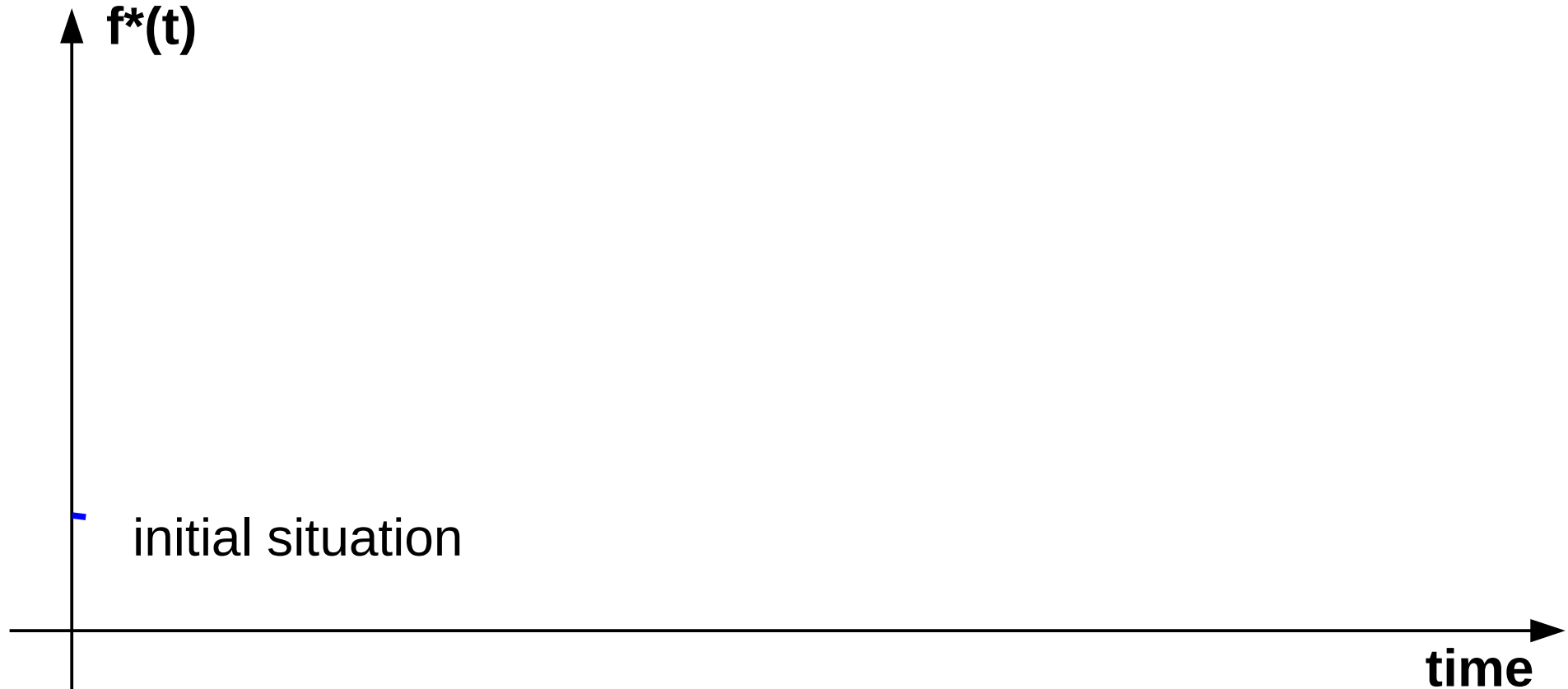
The performance graph is showing the development of the fitness  $f^*(t)$  of the best individual in each generation with respect to time.

EA:

## Performance Graph

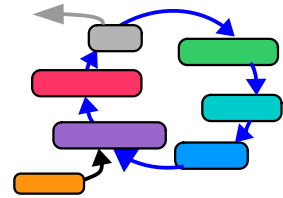


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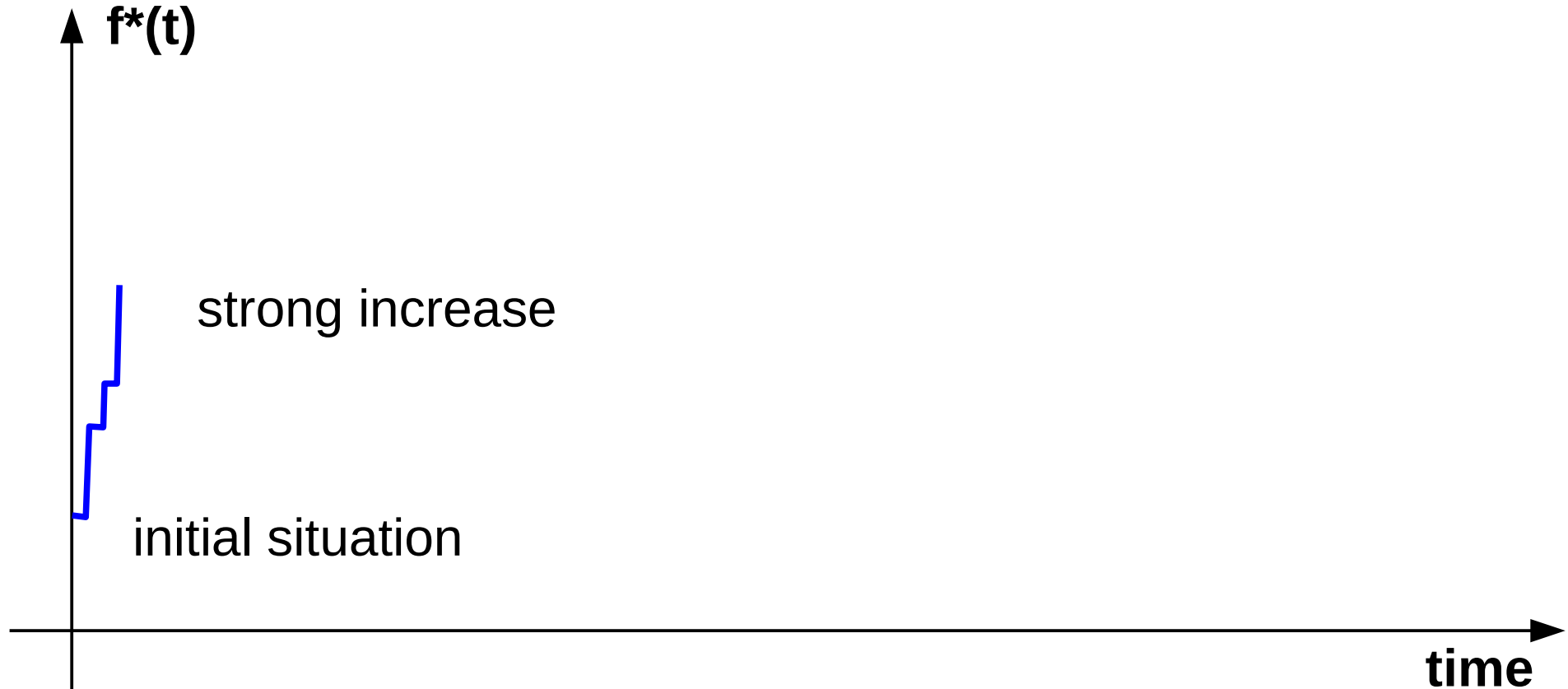


EA:

## Performance Graph

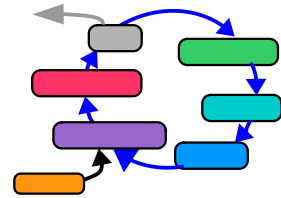


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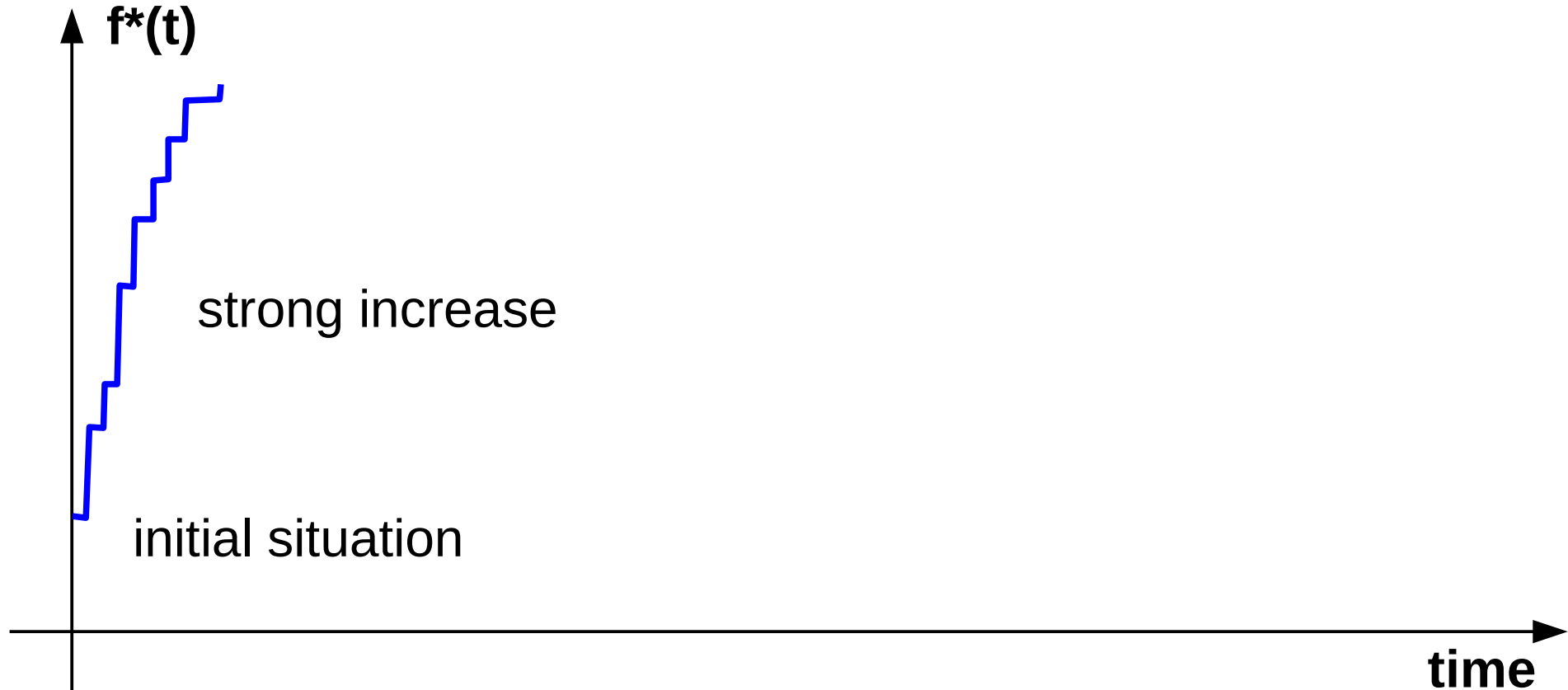


EA:

## Performance Graph

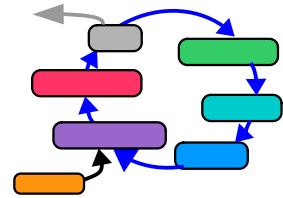


The performance graph is showing the development of the fitness  $f^*(t)$  of the best individual in each generation with respect to time.

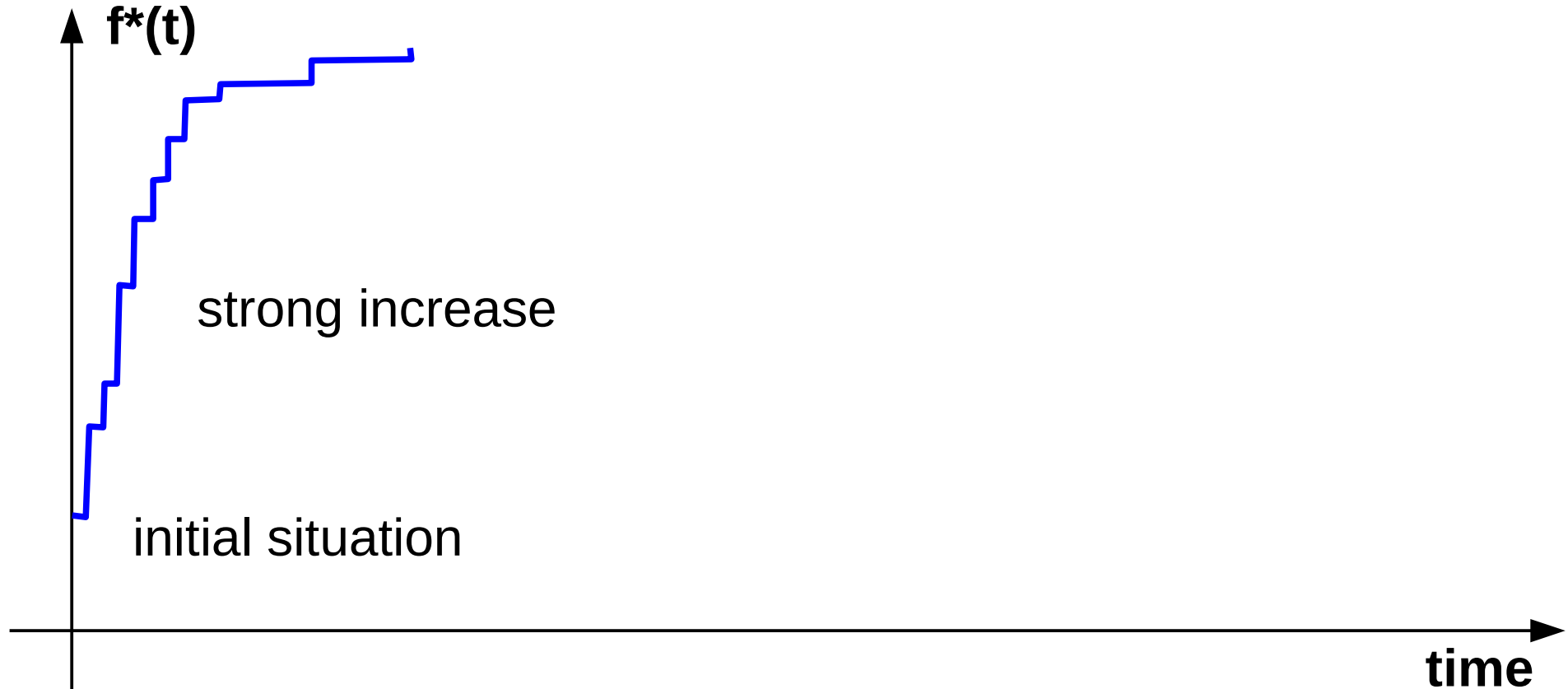


EA:

## Performance Graph

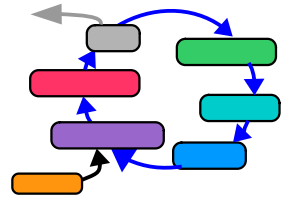


The performance graph is showing the development of the fitness  $f^*(t)$  of the best individual in each generation with respect to time.

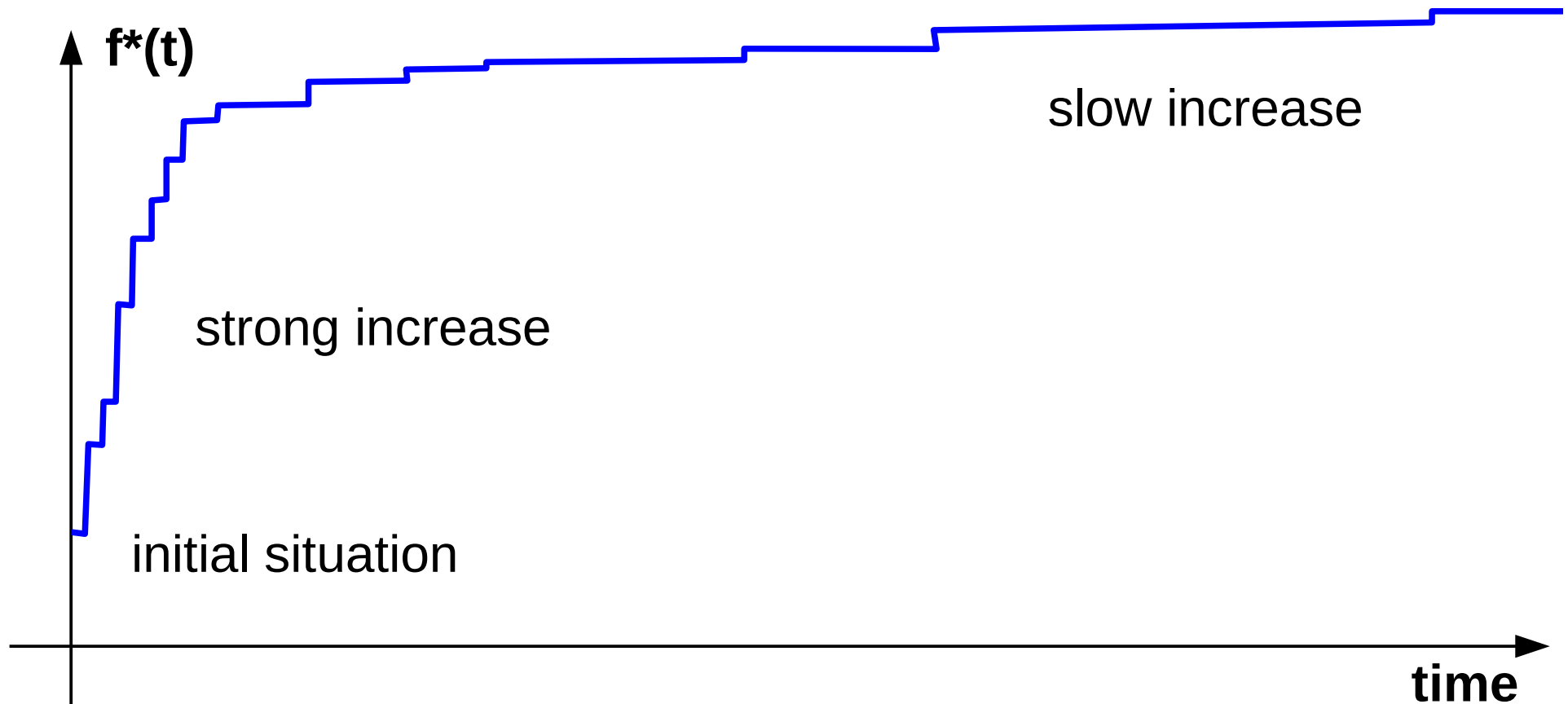


EA:

## Performance Graph



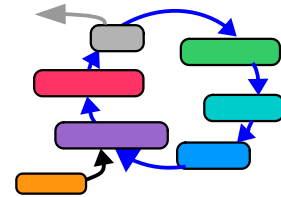
The performance graph is showing the development of the fitness  $f^*(t)$  of the best individual in each generation with respect to time.





## EA:

### Performance Graph

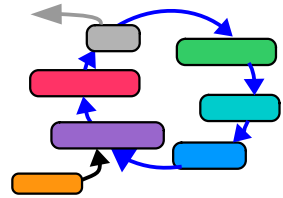


The performance graph is the most important tool to monitor the optimization process of a working evolutionary algorithm.

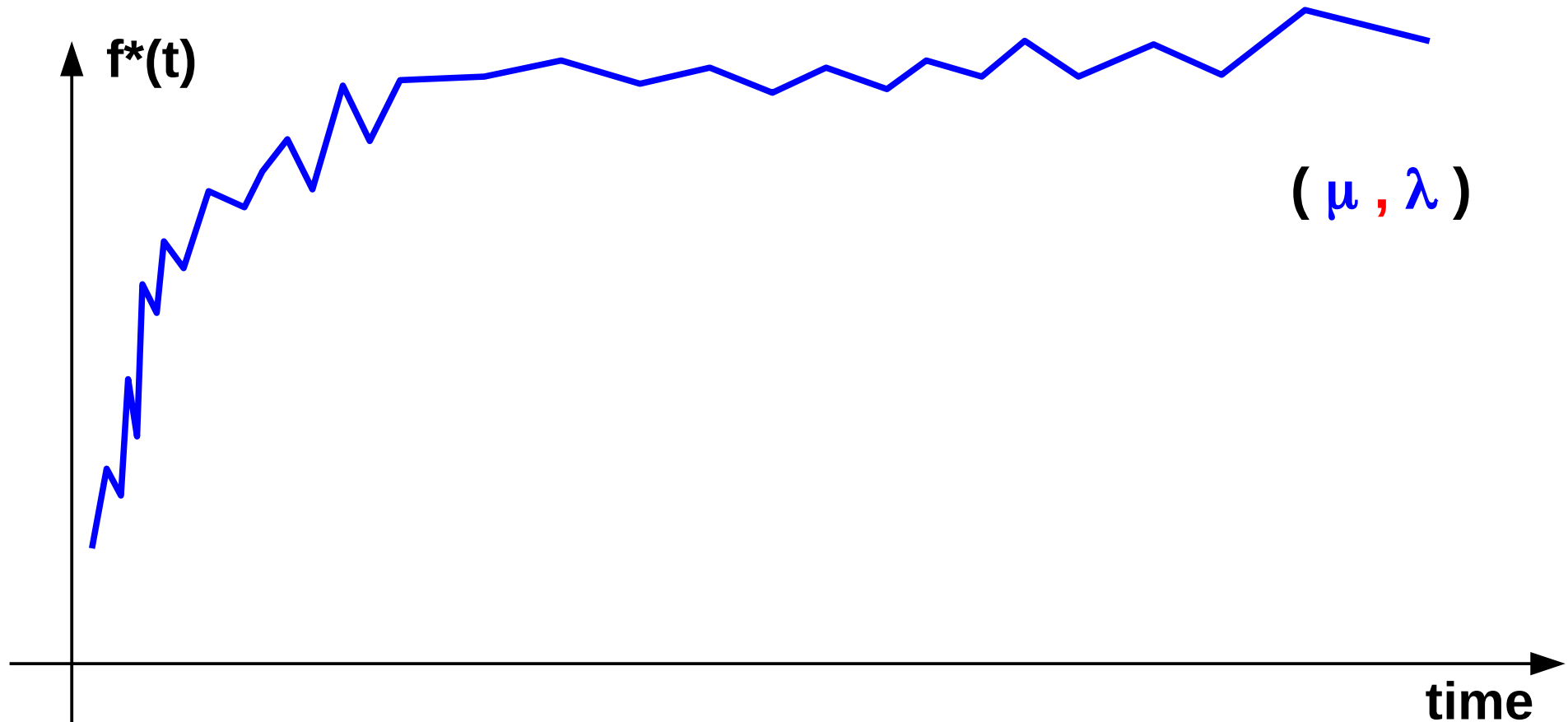
Depending on the chosen strategy, the performance graph shows different properties:

For a deterministic, rank dependent elitism strategy ( $\mu + \lambda$ ), the performance graph will increase monotonically.

For a probabilistic, non-elitism strategy, the performance graph can decrease, but should show an increase on the long run.

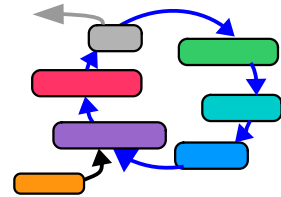
EA:**Performance Graph**

The performance graph is showing the development of the fitness  $f^*(t)$  of the best individual in each generation with respect to time.



## EA:

### Performance Graph



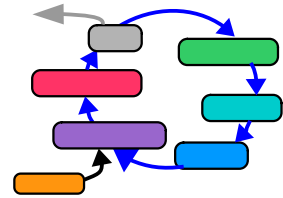
The performance graph is the most important tool to monitor the optimization process of a working evolutionary algorithm.

The performance graph can depict different aspects:

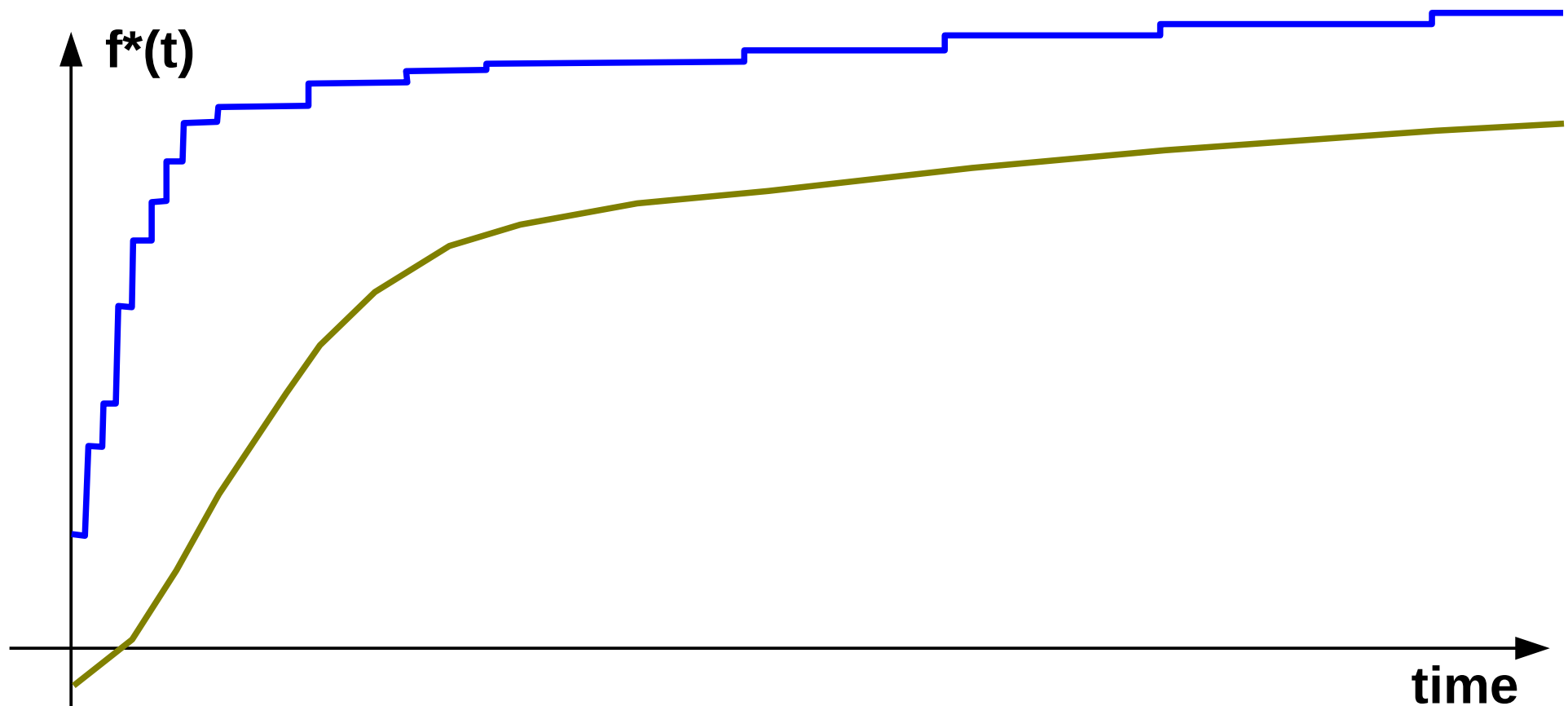
- the fitness  $f^*(t)$  of the best individual, (most important)
- the average fitness  $f_{av}(t)$  of the complete population
- the average and variance over the complete population
- the average fitness  $f_{avp}(t)$  of the parents
- all fitness values  $f_p(t)$  from all parents

EA:

## Performance Graph

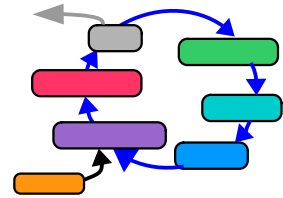


The performance graph is showing the development of the fitness  $f^*(t)$  of the best individual in each generation with respect to time.

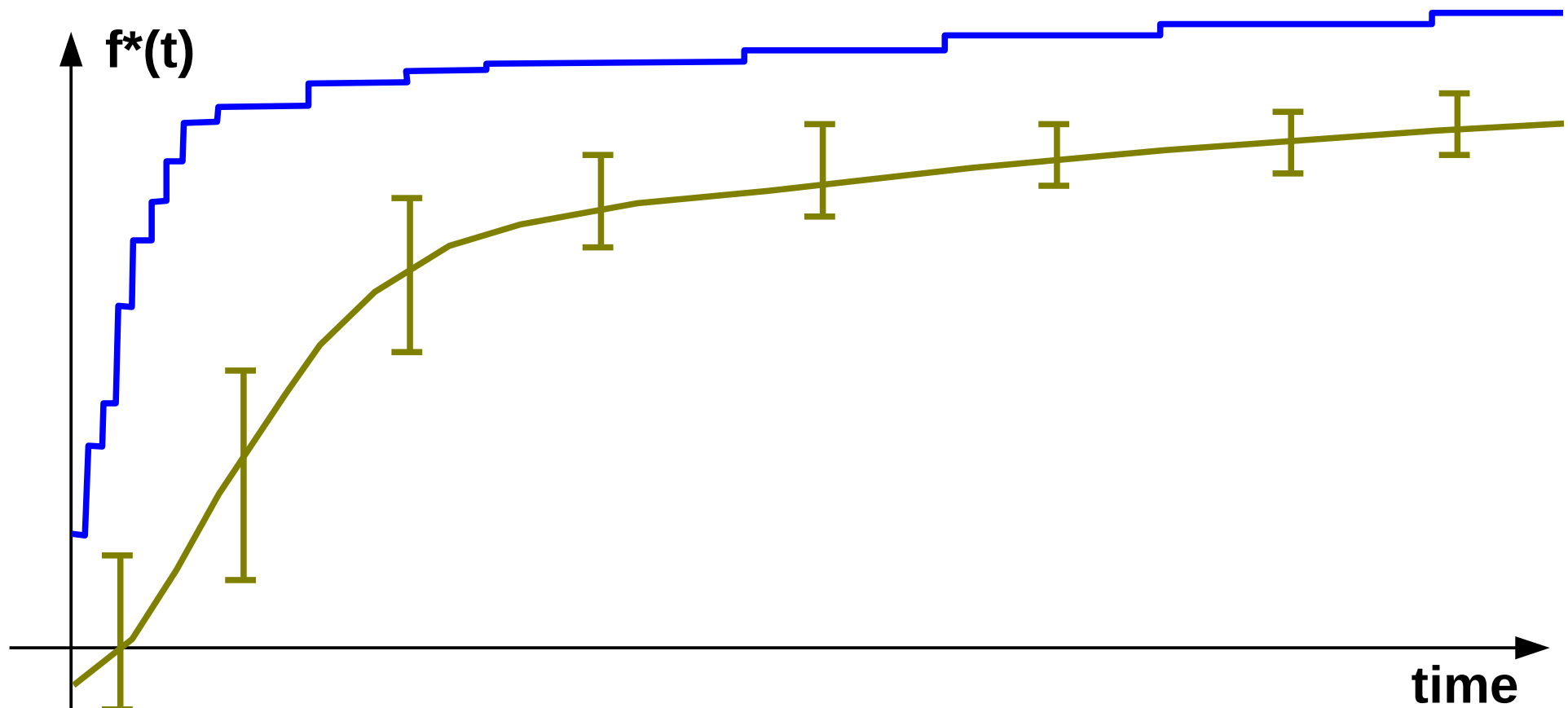


EA:

## Performance Graph



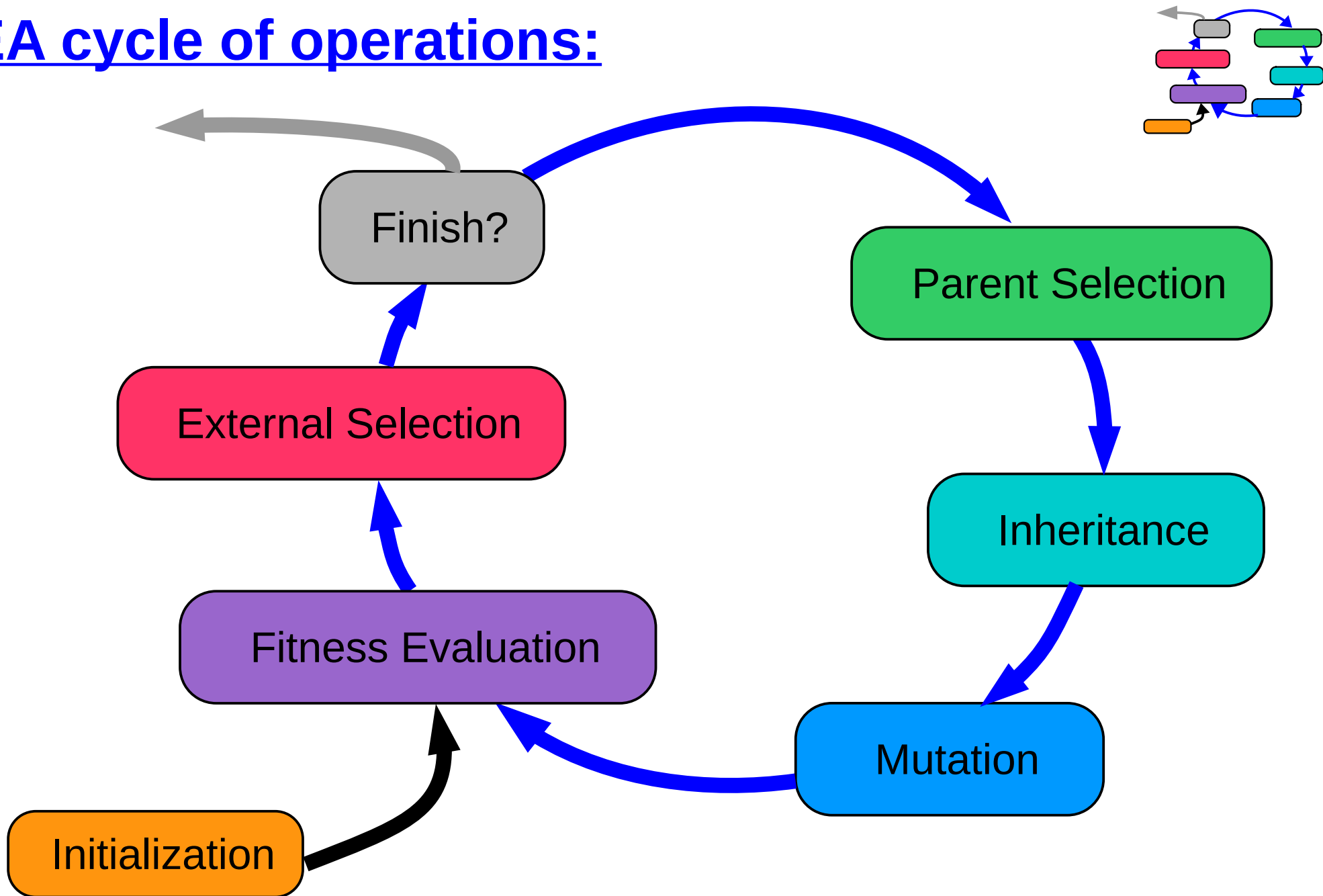
The performance graph is showing the development of the fitness  $f^*(t)$  of the best individual in each generation with respect to time.



# Overview

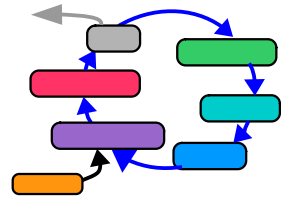
- EA Steps
  - Individual, Genome, Fitness, Population
  - Parent selection
  - Inheritance
  - Mutation
  - Fitness evaluation
  - External selection
  - Finish?
  - Initialization
- Strategy
- Performance Graph
- **Genome Structure**
- Examples

## EA cycle of operations:



EA:

## Genome Structure



**Naive structuring** of the genome:

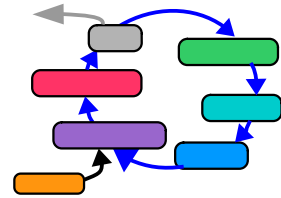
**“Normal” structuring** of the genome:

**Sophisticated structuring** of the genome:



EA:

## Genome Structure



**Naive structuring** of the genome:

**Pro:** easy to implement, only few knowledge necessary.

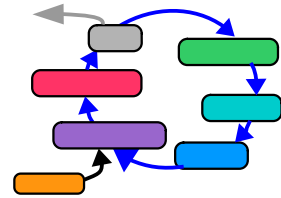
**Con:** large search space, a lot of local maxima possible, may be hard or impossible to find a good solution.

**“Normal” structuring** of the genome:

**Sophisticated structuring** of the genome:

## EA:

### Genome Structure



**Naive structuring** of the genome:

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**“Normal” structuring** of the genome:

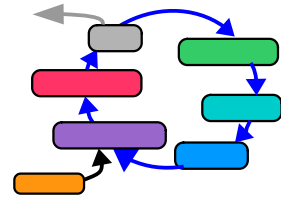
**Pro:** still easy to implement, some knowledge necessary, wide variety to implement inheritance and mutation.

**Con:** still a lot of bad or illegal genomes possible.

**Sophisticated structuring** of the genome:

## EA:

### Genome Structure



**Naive structuring** of the genome:

**Pro:** easy to implement, only few knowledge necessary.

**Con:** large search space, a lot of local maxima possible, may be hard or impossible to find a good solution.

**“Normal” structuring** of the genome:

**Pro:** still easy to implement, some knowledge necessary, wide variety to implement inheritance and mutation.

**Con:** still a lot of bad or illegal genomes possible.

**Sophisticated structuring** of the genome:

**Pro:** only legal, or good genomes are to be investigated.

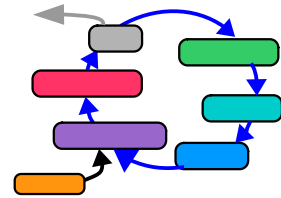
**Con:** profound knowledge about the process and of the kind of possible solutions is required, can become computational expensive to implement inheritance and mutation.

# Overview

- EA Steps
  - Individual, Genome, Fitness, Population
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- **Examples**

## EA:

### Example:



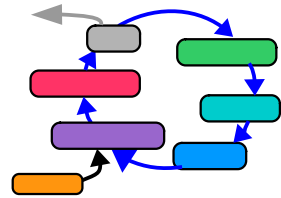
Implementing a task using an Evolutionary Algorithm, a series of structuring decision will be necessary.

*The sequence below is not common theory, but just my personal choice of doing it.*

- |                            |                                        |
|----------------------------|----------------------------------------|
| <b>Objective:</b>          | specify the objective                  |
| <b>Genome:</b>             | structure the genome                   |
| <b>Fitness Function:</b>   | define an appropriate fitness function |
| <b>Inheritance:</b>        | layout the inheritance process         |
| <b>Mutation:</b>           | layout the mutation process            |
| <b>Selection Strategy:</b> | specify the selection strategy         |

EA:

**Example: Fkt Maximum**



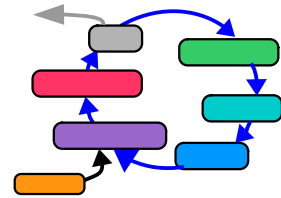
**Objective:**

**Genome:**

**Fitness Function:**

EA:

Example: Fkt Maximum



**Objective:**

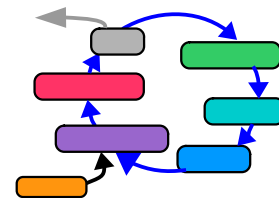
find the position  $X$ , where an objective function  $o(X)$  has its maximal value (or minimal value).

**Genome:**

**Fitness Function:**

## EA:

### Example: Fkt Maximum



### Objective:

find the position  $\mathbf{X}$ , where an objective function  $o(\mathbf{X})$  has its maximal value (or minimal value).

### Genome:

the N-dimensional vector  $\mathbf{X}$

$$\mathbf{g} = \mathbf{X} = \{x_1, x_2, \dots, x_i, \dots, x_N\}$$

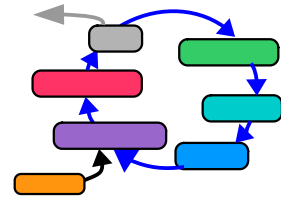
with N components  $x_1$  to  $x_N$

### Fitness Function:



## EA:

### Example: Fkt Maximum



### Objective:

find the position  $\mathbf{X}$ , where an objective function  $o(\mathbf{X})$  has its maximal value (or minimal value).

### Genome:

the N-dimensional vector  $\mathbf{X}$

$$\mathbf{g} = \mathbf{X} = \{x_1, x_2, \dots, x_i, \dots, x_N\}$$

with N components  $x_1$  to  $x_N$

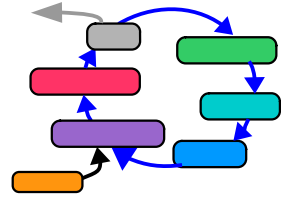
### Fitness Function:

fitness function  $\mathbf{f}(\mathbf{g})$  is identical to the objective function  $o(\mathbf{X})$

$$\mathbf{f}(\mathbf{g}) = o(\mathbf{X})$$

**EA:**

**Example: Fkt Maximum**



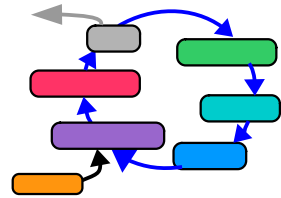
**Inheritance:**

**Mutation:**

**Selection Strategy:**

**EA:**

**Example: Fkt Maximum**



**Inheritance:**

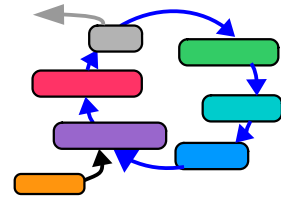
recombination with  $k=2$  parents, 1-point cross over

**Mutation:**

**Selection Strategy:**

**EA:**

**Example: Fkt Maximum**



**Inheritance:**

recombination with  $k=2$  parents, 1-point cross over

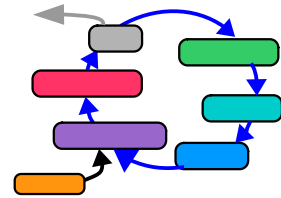
**Mutation:**

chose a random component  $x_i$ , and change that value to a complete new value.

**Selection Strategy:**

**EA:**

**Example: Fkt Maximum**



**Inheritance:**

recombination with  $k=2$  parents, 1-point cross over

**Mutation 1:**

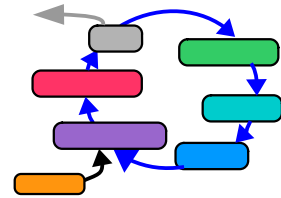
chose a random component  $x_i$ , and change that value to a complete new value.

**Mutation 2:**

**Selection Strategy:**

## EA:

### Example: Fkt Maximum



### Inheritance:

recombination with  $k=2$  parents, 1-point cross over

### Mutation 1:

choose a random component  $x_i$ , and change that value to a complete new value.

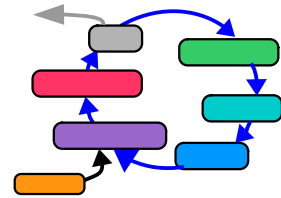
### Mutation 2:

choose one or two random components  $x_i$ ,  $x_j$  and add a small normally distributed value to them.

### Selection Strategy:

## EA:

### Example: Fkt Maximum



### Inheritance:

recombination with  $k=2$  parents, 1-point cross over

### Mutation 1:

chose a random component  $x_i$ , and change that value to a complete new value.

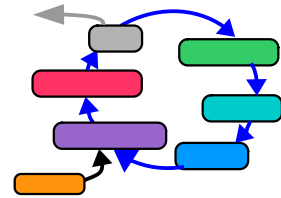
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EA:

Example: Fkt Maximum



### Inheritance:

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chose a random component  $x_i$ , and change that value to a complete new value.

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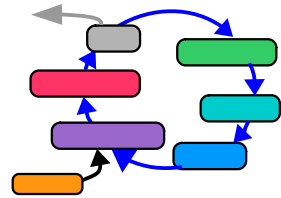
### Selection Strategy:

elitism,  $\mu$  parents, parents survive, no mutation for the parents  
deterministic, rank dependent, take the  $\mu$  best individuals  
( $\mu + \lambda$ ), with  $P=500$ ,  $\mu=100$ ,  $\lambda=400$ .



EA:

## Example: Fkt Maximum



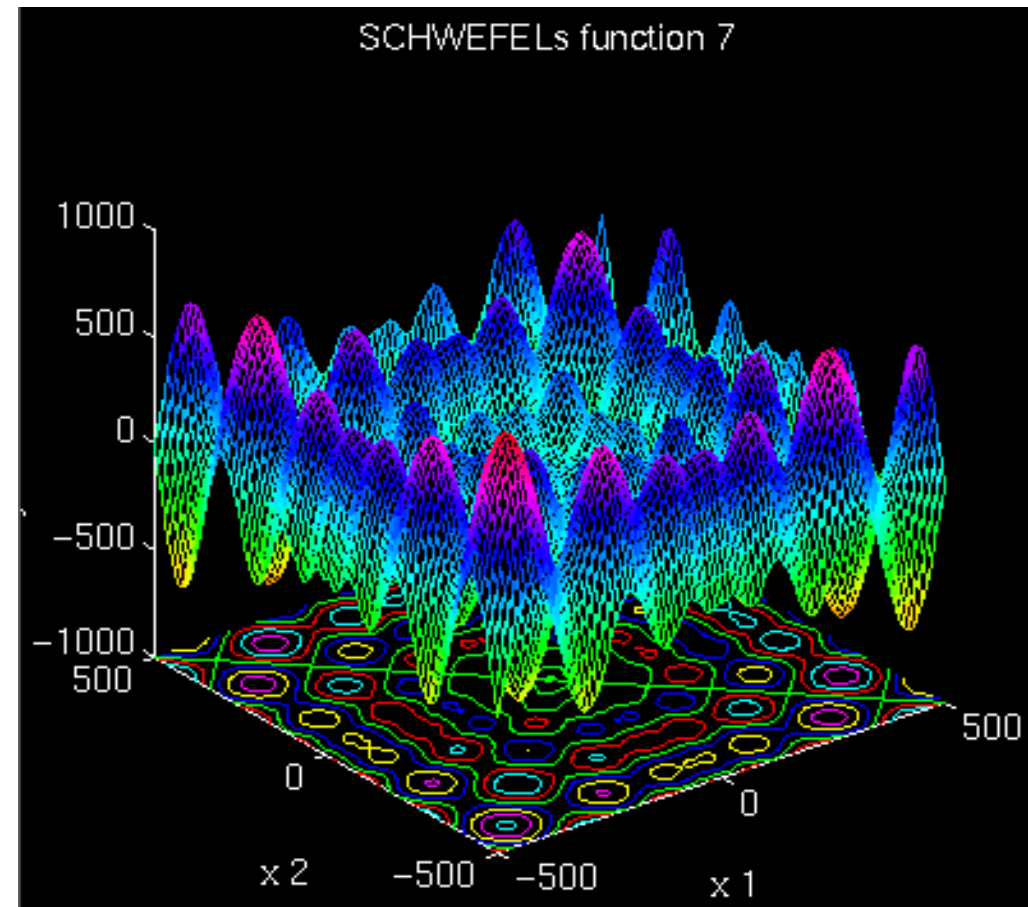
A typical **playground** to investigate the capabilities of optimization methods are **test functions** to minimize.

**Schwefel's Function (1981)**

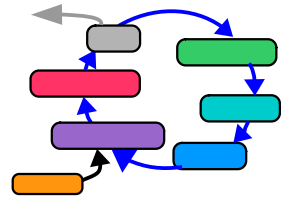
$$f(\mathbf{x}) = \sum_{j=1}^d -x_j \sin(\sqrt{\text{abs}(x_j)})$$

$$-500 < x_j < 500$$

Schwefel, H.-P.:  
Numerical optimization of computer models.  
Chichester: Wiley & Sons, 1981.

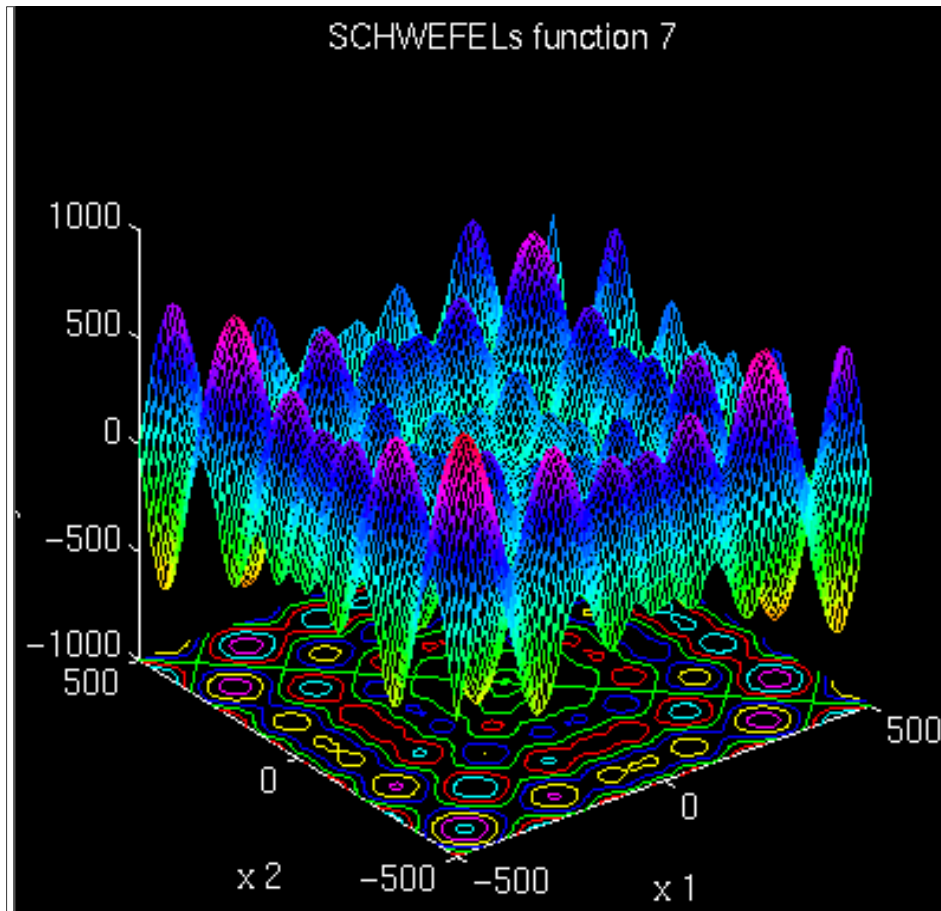


[http://www.tu-ilmenau.de/fakia/fileadmin/template/startIA/oek\\_systeme/Interess\\_adressen/pohlheim/GA\\_Toolbox/fcnindex.html](http://www.tu-ilmenau.de/fakia/fileadmin/template/startIA/oek_systeme/Interess_adressen/pohlheim/GA_Toolbox/fcnindex.html)

EA:**Example: Fkt Maximum**

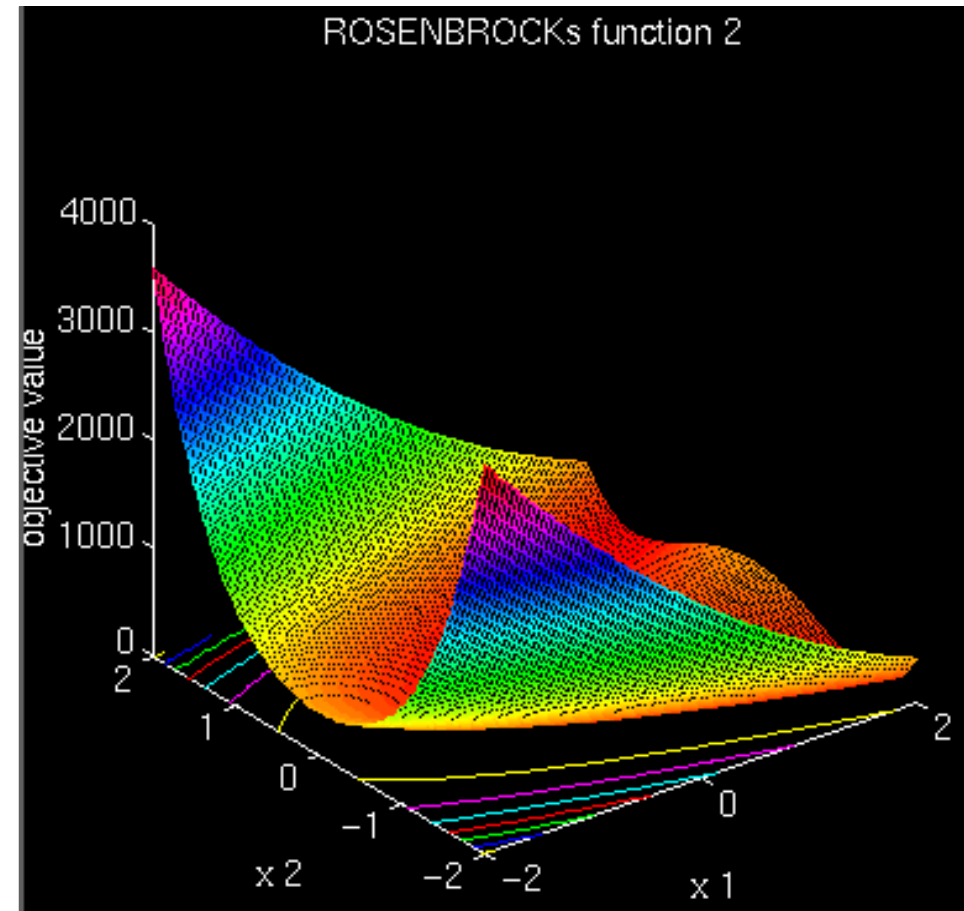
$$f(x) = \sum_{i=1:n} (-x(i) \cdot \sin(\sqrt{\text{abs}(x(i))})),$$

$$-500 \leq x(i) \leq 500.$$

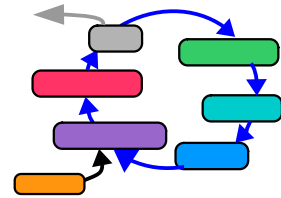


$$f(x) = \sum_{i=1:n-1} (100 \cdot (x(i+1) - x(i)^2)^2 + (1 - x(i))^2),$$

$$-2.048 \leq x(i) \leq 2.048.$$

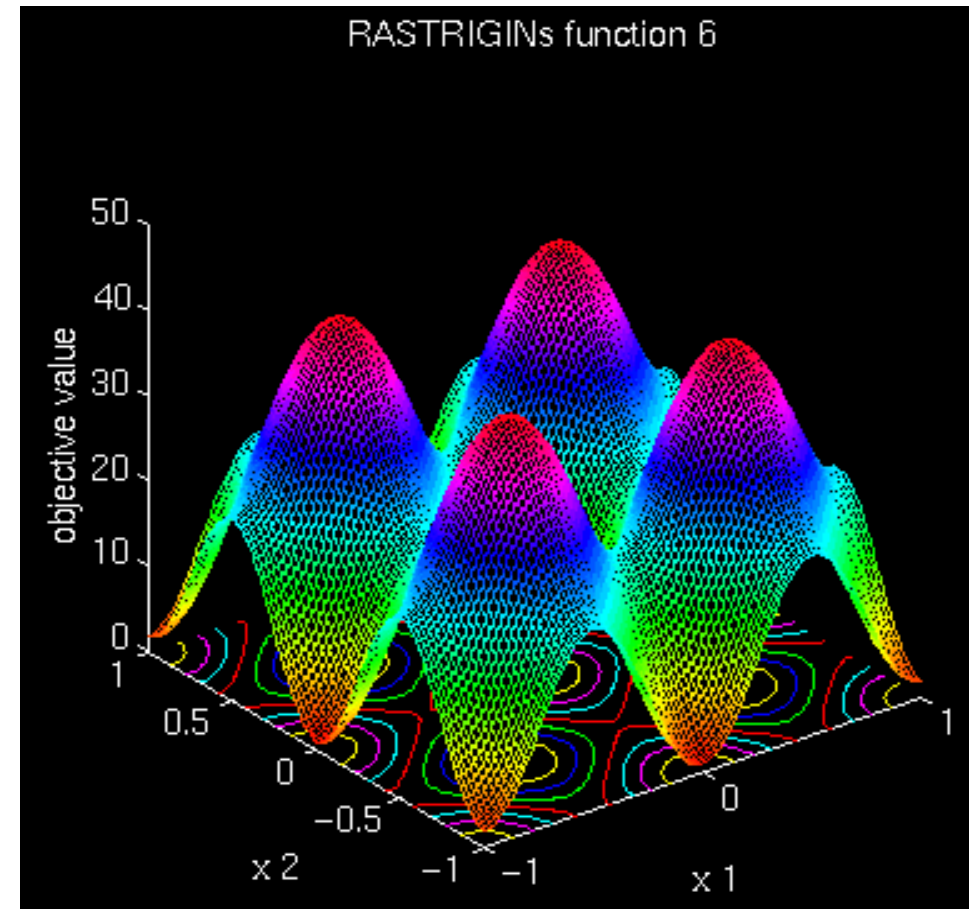
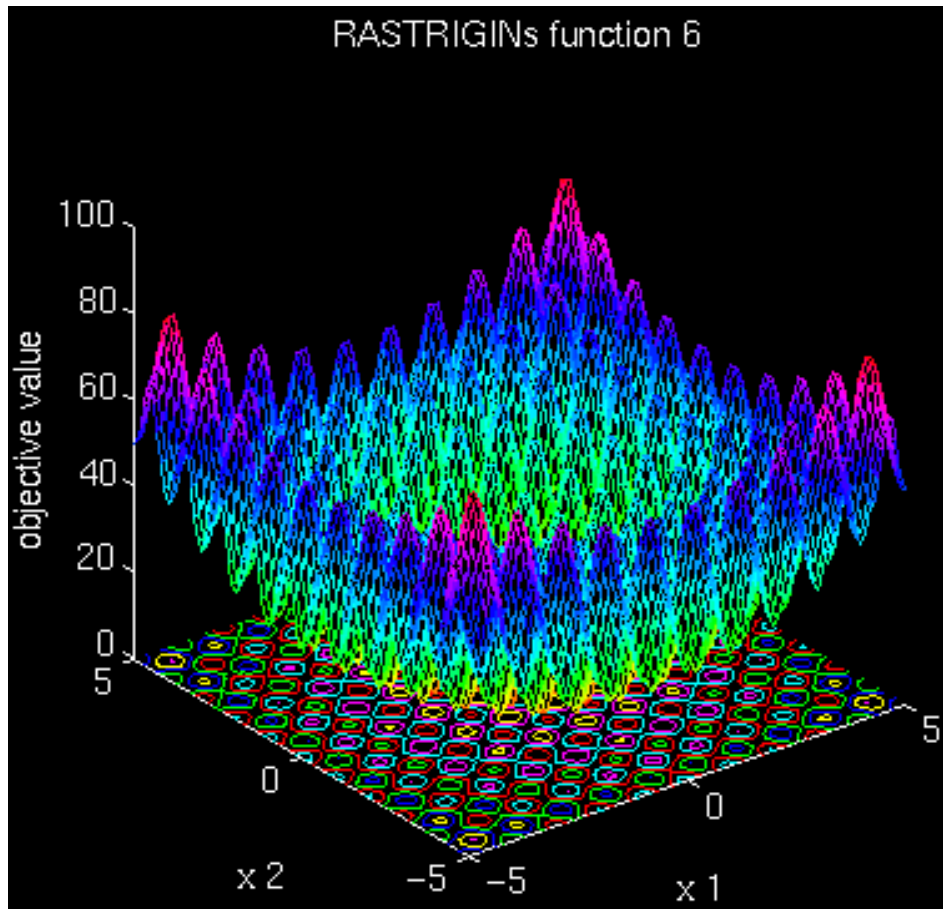


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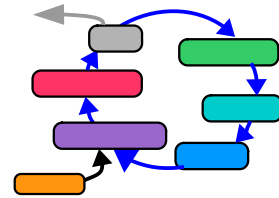
EA:**Example: Fkt Maximum**

$$f(x) = 10 \cdot n + \sum_{i=1:n} (x(i)^2 - 10 \cdot \cos(2 \cdot \pi \cdot x(i))),$$

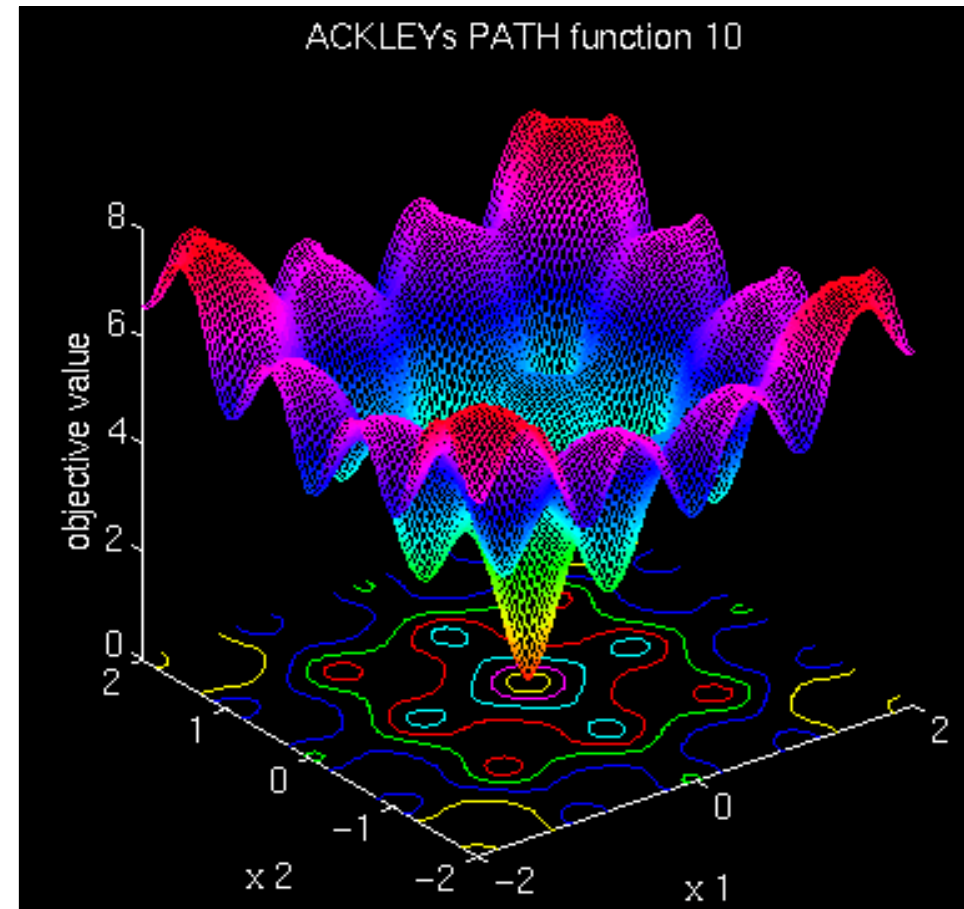
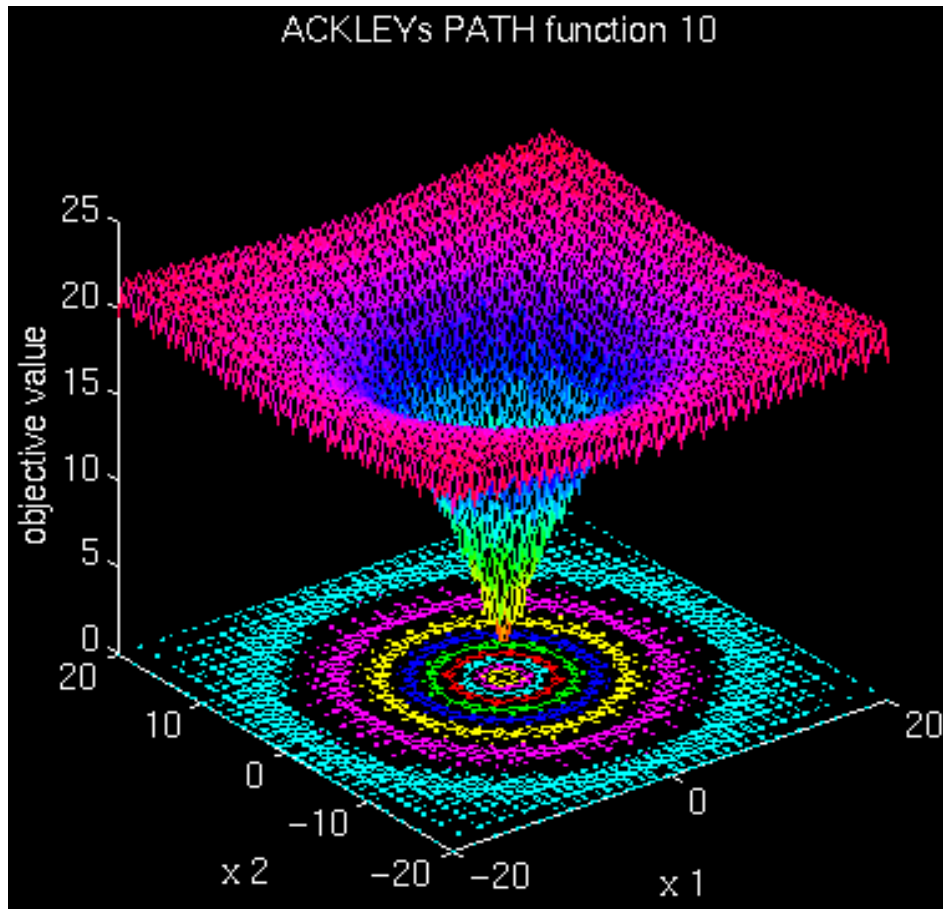
$$i=1:n; \quad -5.12 \leq x(i) \leq 5.12.$$



[http://www.tu-ilmenau.de/fakia/fileadmin/template/startIA/oek\\_systeme/Interess\\_adressen/pohlheim/GA\\_Toolbox/fcnindex.html](http://www.tu-ilmenau.de/fakia/fileadmin/template/startIA/oek_systeme/Interess_adressen/pohlheim/GA_Toolbox/fcnindex.html)

EA:**Example: Fkt Maximum**

$f(x) = -a \cdot \exp(-b \cdot \sqrt{1/n \cdot \sum(x(i)^2)}) - \exp(1/n \cdot \sum(\cos(c \cdot x(i)))) + a + \exp(1);$   
 $a=20; b=0.2; c=2 \cdot \pi; i=1:n;$

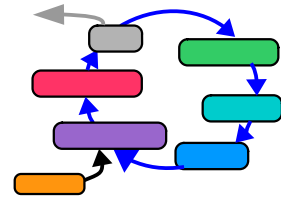


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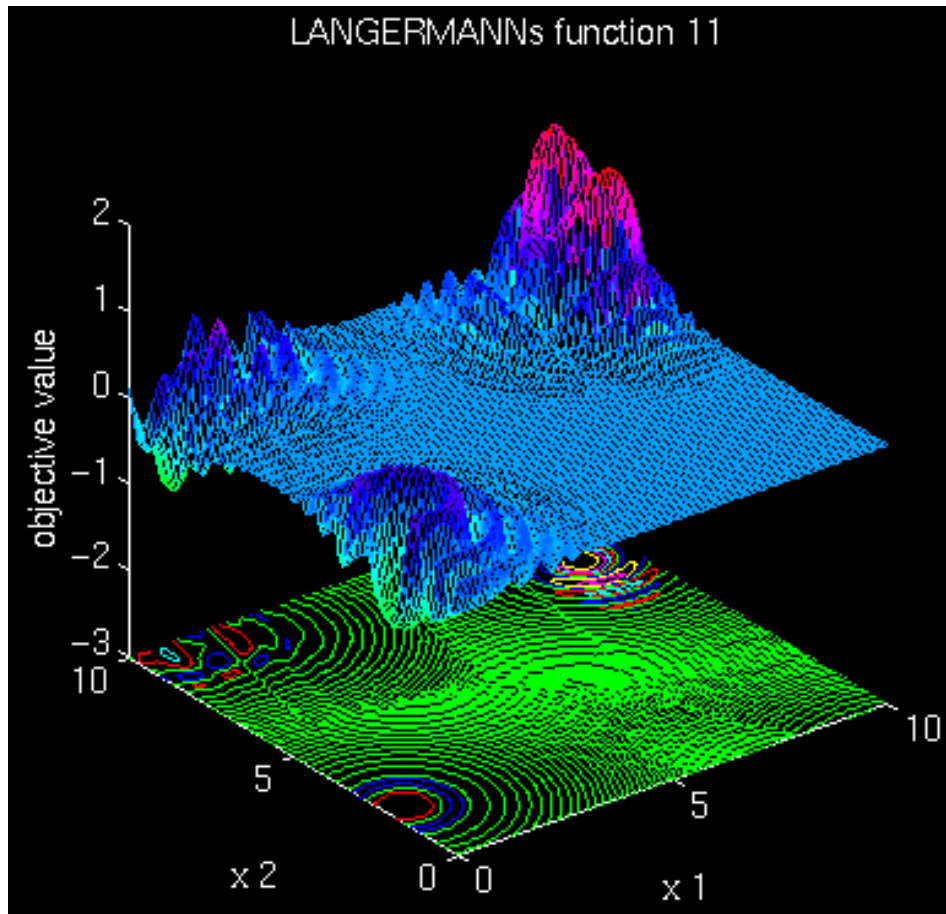
EA:

## Example: Fkt Maximum



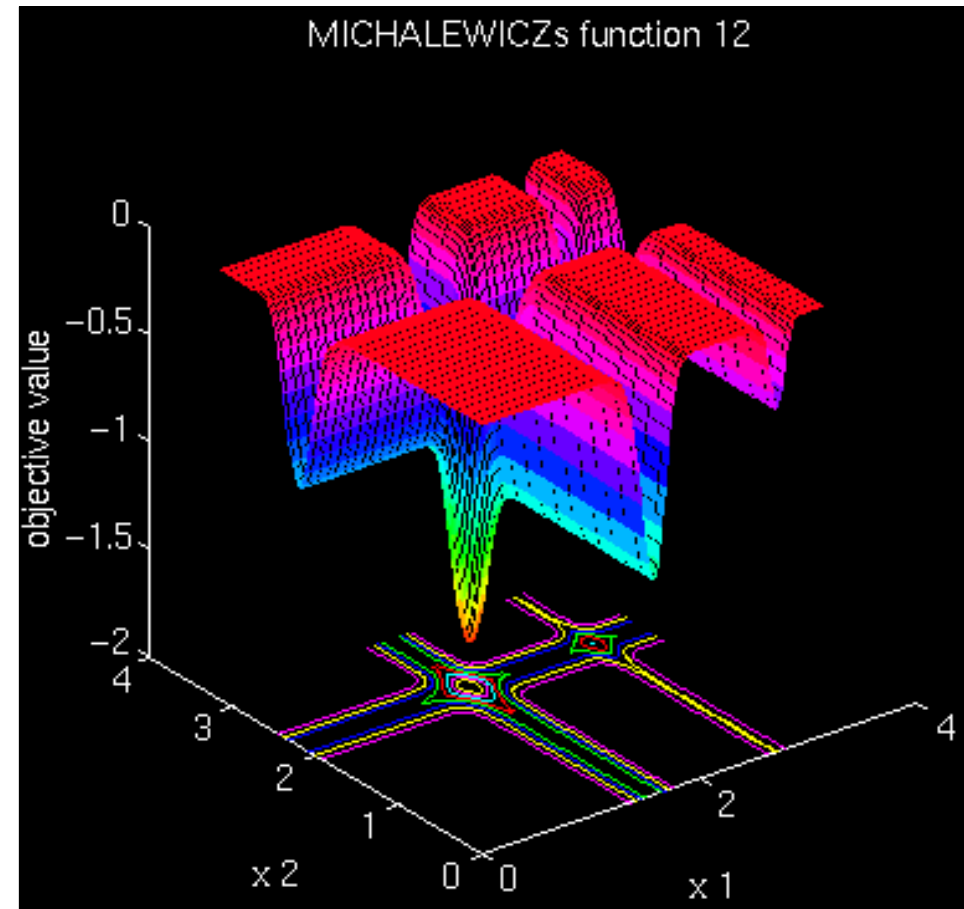
$$f(x) = -\sum_{i=1:m} (c(i) \cdot (\exp(-1/\pi \cdot \sum_{j=1:n} ((x-A(i,j))^2)) \cdot \cos(\pi \cdot \sum_{j=1:n} ((x-A(i,j))^2)))),$$

$i=1:m, m=5$

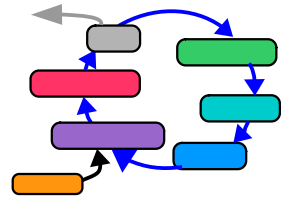


$$f(x) = -\sum_{i=1:n} (\sin(x(i)) \cdot (\sin(i \cdot x(i)^2/\pi))^{(2 \cdot m)}),$$

$i=1:n, m=10;$

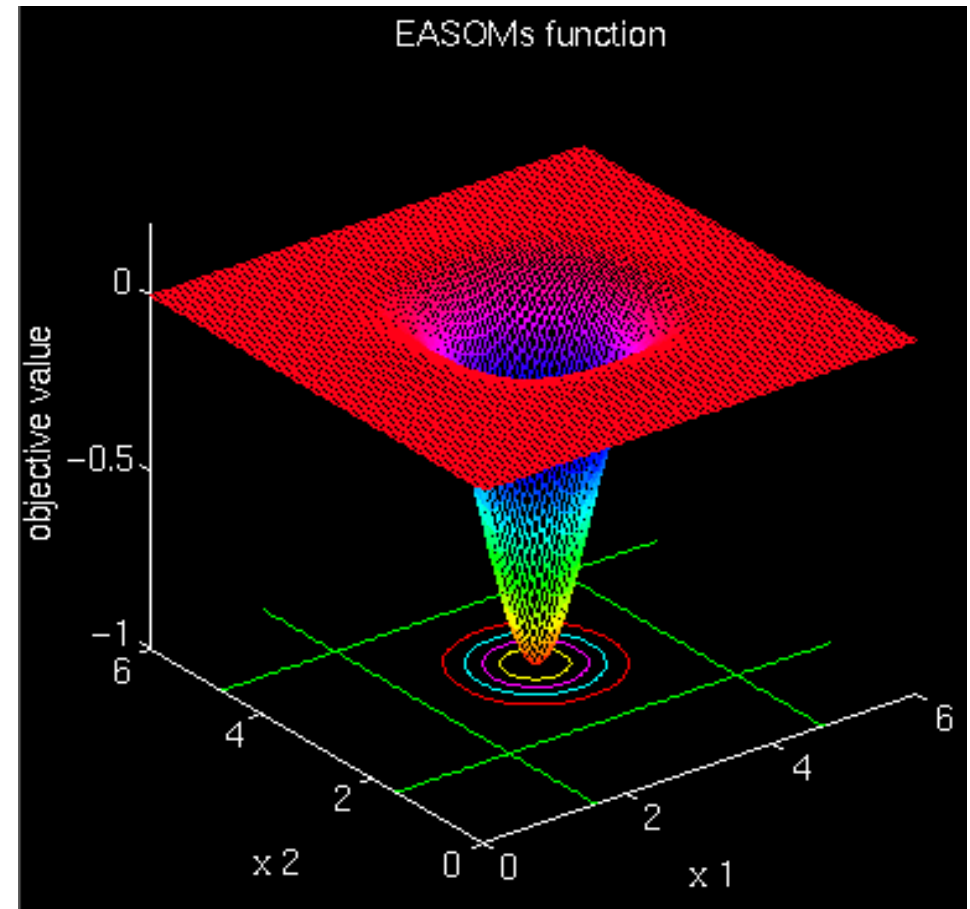
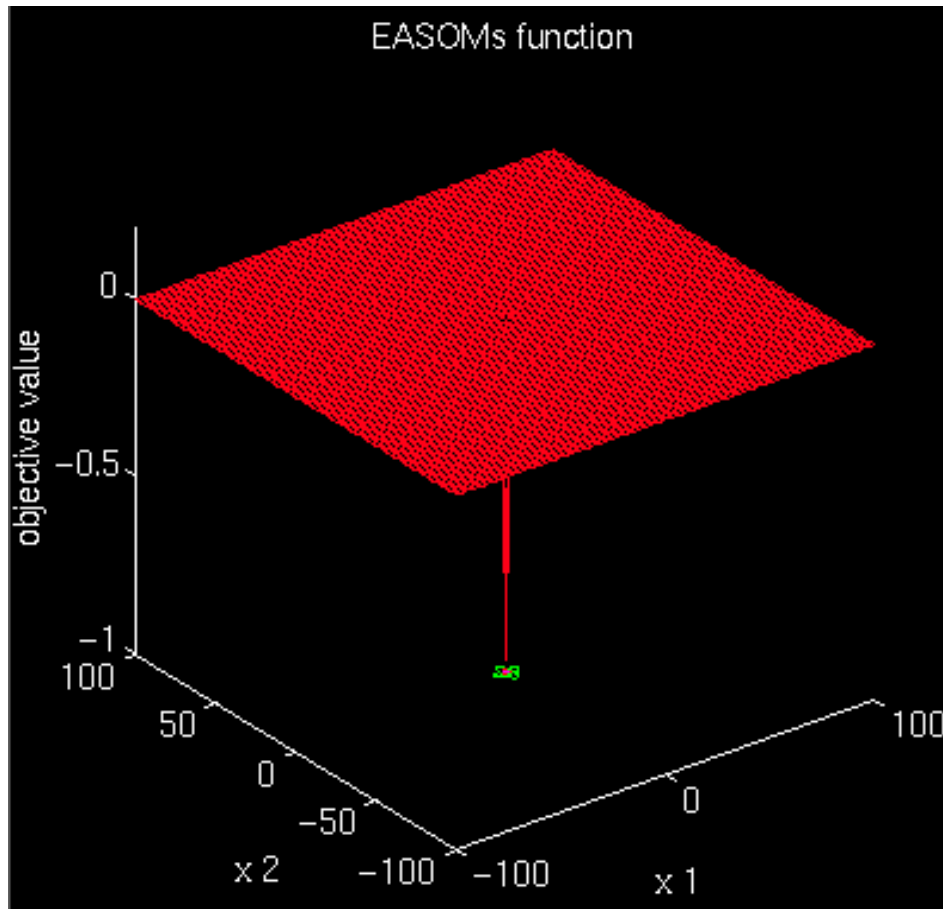


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EA:**Example: Fkt Maximum**

$$f(x_1, x_2) = -\cos(x_1) \cdot \cos(x_2) \cdot \exp(-((x_1 - \pi)^2 + (x_2 - \pi)^2));$$

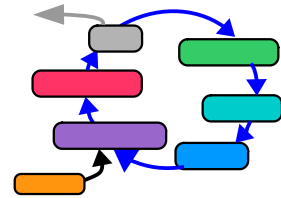
$$-100 \leq x(i) \leq 100, i=1:2.$$



[http://www.tu-ilmenau.de/fakia/fileadmin/template/startIA/oek\\_systeme/Interess\\_adressen/pohlheim/GA\\_Toolbox/fcnindex.html](http://www.tu-ilmenau.de/fakia/fileadmin/template/startIA/oek_systeme/Interess_adressen/pohlheim/GA_Toolbox/fcnindex.html)

## EA:

### Example: Fkt Maximum

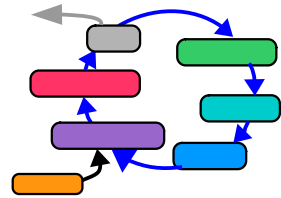


Below is a list of common **test functions (to minimize)** that are used to investigate the capabilities of optimization methods:

- Schwefel's Function
- Generalized Rosenbrock's Function
- Rastrigin's Function
- Ackley's (path) Function
- Langermann's Function
- Michalewicz's Function
- Easoms Function
- Griewangk's Function
- Bohachevsky's Function
- Watson's Function
- Colville's Function
- ...

EA:

Example: Sorting

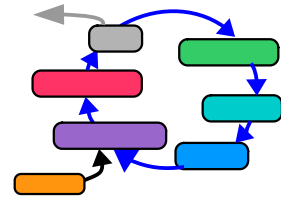


Objective:

Genome:

(easy) Fitness Function:



EA:**Example: Sorting****Objective:**

sort a set of numerical values  $x_i$  with respect to the square  $x_i^2$  of their value.

**Genome:**

an ordered set of values  $(x_i)$ ,  $\mathbf{g} = \{ x_1, x_2, \dots, x_i, \dots, x_{42}, \dots \}$

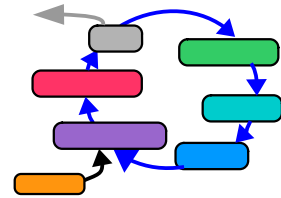
**(easy) Fitness Function:**

$f(\mathbf{g})$  is the accumulation of all comparisons of subsequent values add a **+1** if the order is correct, and a **-1** if the order is wrong.

$$f(\mathbf{g}) = \sum \text{sign}(x_i^2 - x_{i+1}^2)$$

## EA:

### Example: Sorting



### Easy Inheritance:

since a classical cross over will generate illegal genomes  
the  $k=1$ , copy operator with no recombination is fine.

### Mutation:

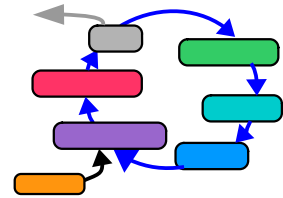
exchange 2 values within the sequence (genome),  
or even a complete subsequence,  
or invert the order of a subsequence within the genome.

### Selection Strategy:

elitism,  $\mu$  parents, parents survive, no mutation for the parents  
deterministic, rank dependent, take the  $\mu$  best individuals  
( $\mu + \lambda$ ), with  $P=500$ ,  $\mu=100$ ,  $\lambda=400$ .

EA:

Example: 42



## Objective:

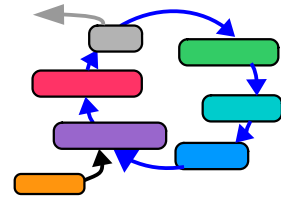
find the L-bit binary representation of the square root of 42;  
unfortunately, the math. function *sqrt* is not available.

## Genome:

## Fitness function:

EA:

Example: 42



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find the L-bit binary representation of the square root of 42; unfortunately, the math. function *sqrt* is not available.

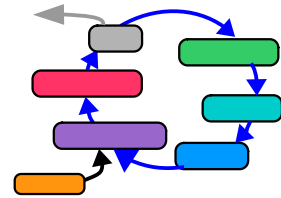
## Genome:

an L-bit binary vector **g**, in fix-point representation

## Fitness function:

EA:

Example: 42



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find the L-bit binary representation of the square root of 42;  
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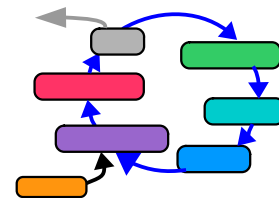
## Fitness function:

take the genome **g** as a binary, fix point number **x(g)**  
calculate the square of **x(g)**,  
compare the result to 42.0, the square of the objective number

$$f(g) = |42.0 - x(g)^2|$$

EA:

Example: 42



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find the L-bit binary representation of the square root of 42;  
unfortunately, the math. function *sqrt* is not available.

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an L-bit binary vector **g**, in fix-point representation

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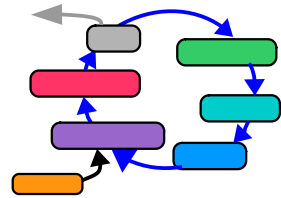
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calculate the square of x(g),  
compare the result to 42.0, the square of the objective number

$$f(g) = |42.0 - x(g)^2|$$

In fact, this is a *cost function*, that is to be minimized.

EA:

Example: 42

**Population:**

$P = 100$ , population size kept constant.

**Inheritance:**

recombination,  $k=2$  parents, 1-point cross over.

**Mutation:**

bit-flip, flip each bit of the genome with a probability  $\omega = 0.01$

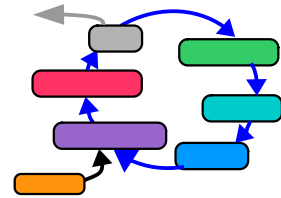
Caution: be aware of Hamming-cliffs.

**Selection Strategy:**

elitism,  $\mu$  parents, parents survive, no mutation for the parents  
 deterministic, rank dependent, take the  $\mu$  best individuals  
 ( $\mu + \lambda$ ), with  $\mu=20$ ,  $\lambda=80$ .

EA:

**Example: TSP**



**Objective:**

find the shortest route visiting each point from a set of given points (cities) exactly once (Traveling Salesman Problem).

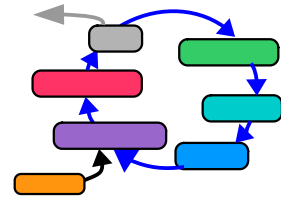
**Genome:**

**Fitness Function:**



## EA:

### Example: TSP



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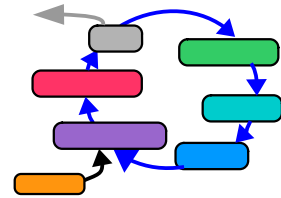
an ordered set of points (cities,  $C_i$ ), sequence of cities to be visited, containing each city exactly once.

$$g = \{ C_3, C_{17}, C_i, C_{42}, \dots \}$$

### Fitness Function:

## EA:

### Example: TSP



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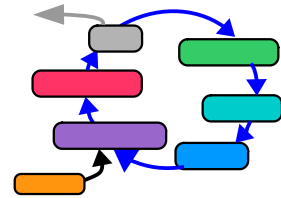
$$g = \{ C_3, C_{17}, C_i, C_{42}, \dots \}$$

### Fitness Function:

$f(g)$  is the sum of all distances  $d(C_i, C_{i+1})$  between subsequent cities in the list (genome). Of course all distances  $d(C_i, C_j)$  between the cities  $i$  and  $j$  is required; either by a distance matrix or by a calculation from the coordinates.

EA:

Example: TSP

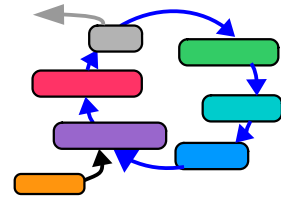


Easy Inheritance:

Mutation:

EA:

Example: TSP



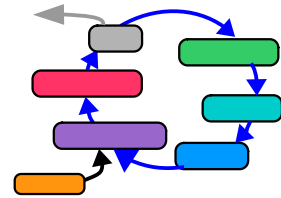
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## Mutation:

## EA:

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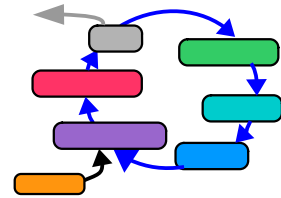


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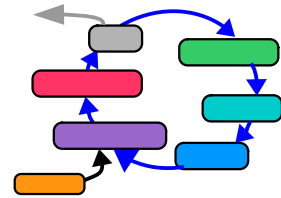
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**g** = { ... , London, Rio, Moscow, New York, Paris, Tokyo, LA, ... }

## EA:

### Example: TSP



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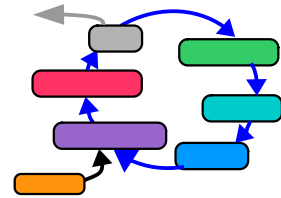
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$g' = \{ \dots, \text{London}, \text{Paris}, \text{Moscow}, \text{New York}, \text{Rio}, \text{Tokyo}, \text{LA}, \dots \}$

## EA:

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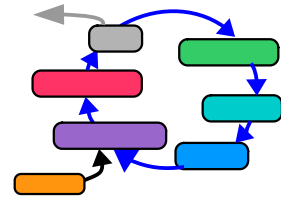
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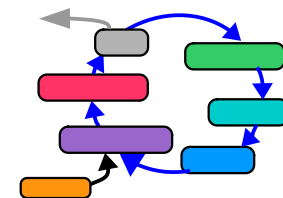
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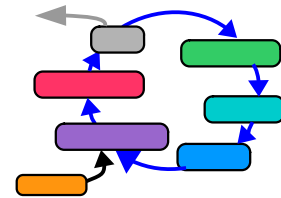
## Sophisticated Inheritance:

A more sophisticated inheritance operator would have to guarantee that the offspring still represent legal genomes,

- routes that visit each city exactly once - .

This may cause high computational effort, which might be larger than the potential benefit.

## Sophisticated Mutation:

EA:**Example: TSP**

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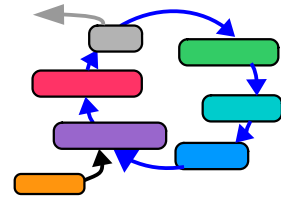
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## **Sophisticated Mutation:**

a more sophisticated mutation operator might take the fitness value  $f(g)$  of the genome  $g$  into account, to make only those changes to the genome that have a high probability for a fitness increase.

EA:**Example: TSP**

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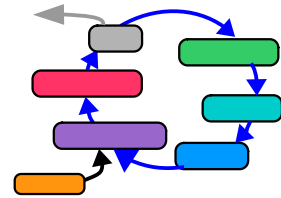
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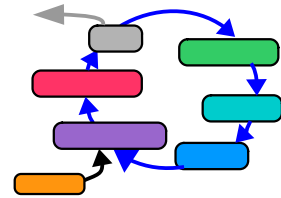
How do you know?

Either by theory (if available), or by a heuristics.

EA:

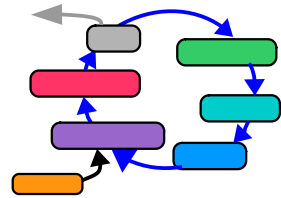
## Example: 8 Queens



EA:**Example: 8 Queens**

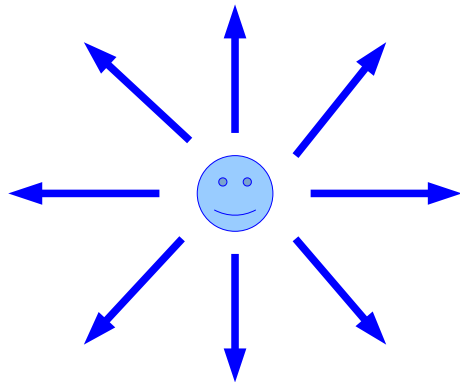
The 8 queens puzzle with an evolutionary algorithm:

The task of the 8 queens puzzle is to place 8 queens on a chess board (8x8) so that they can not reach each other.

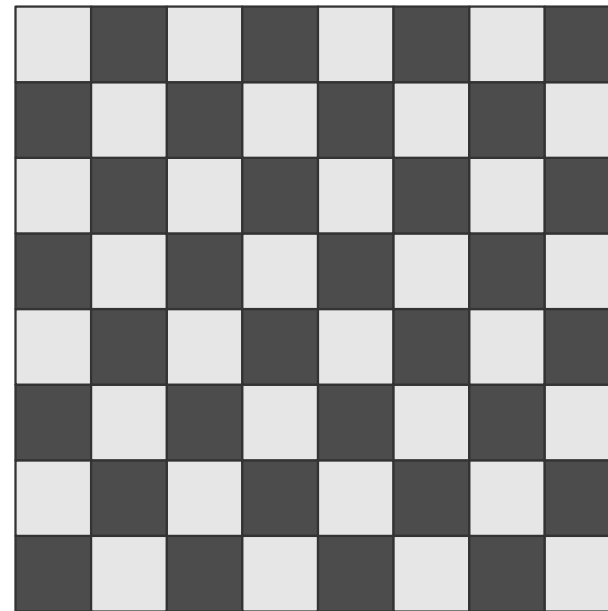
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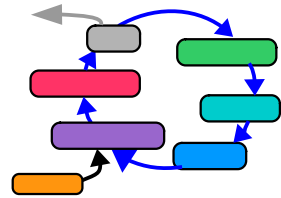
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allowed moves  
for a queen

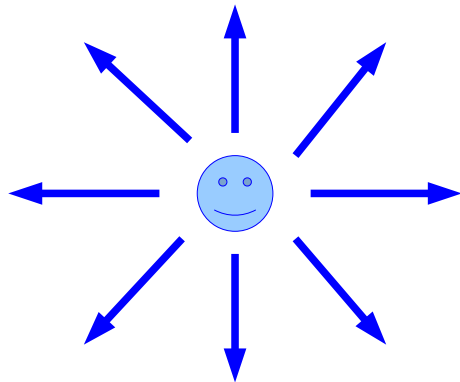


chess board with  
 $8 \times 8 = 64$  positions

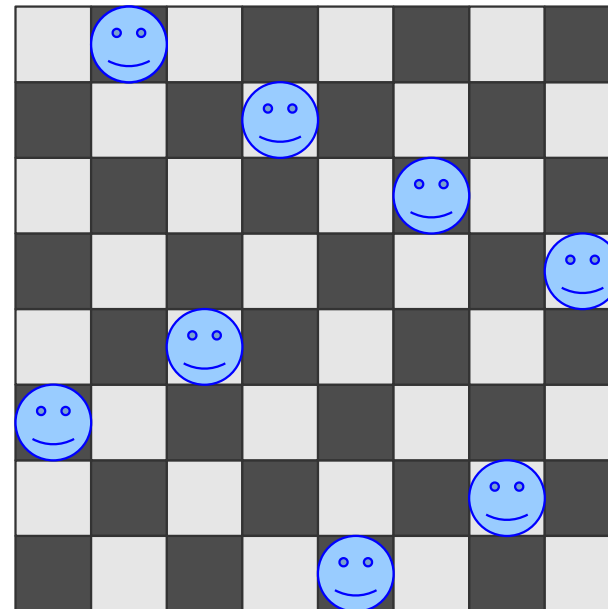
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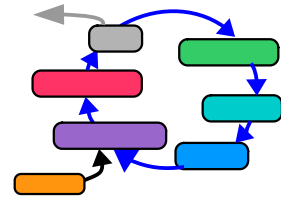
allowed moves  
for a queen



One of  
the **92**  
solutions

chess board with  
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EA:**Example: 8 Queens**

**Very naive** implementation of the genome:

a 64 bit binary vector; queen is 1, no queen is 0;  
more than 8 queens are possible.

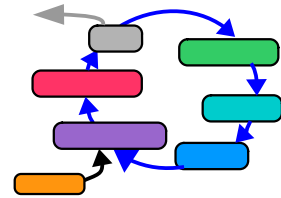
This very naive implementation is generating an extremely large search space with  $2^{64}$  possible genomes. This is beyond any computing power to be investigated in total.



**Semi naive** implementation of the genome:

a 64 bit binary vector; queen is 1, no queen is 0;  
exactly 8 queens == 8 bits set are possible.

Still the resulting search space is very large.

EA:**Example: 8 Queens**

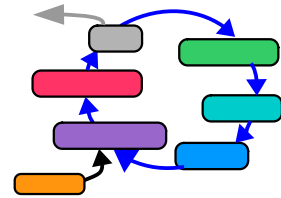
**Normal/Sophisticated** implementation of the genome:

8 rows of 8 bit binary vectors; queen is 1, no queen is 0;  
each row contains exactly one queen.

This is reducing the search space to  $8^8 = 16777216$   
possibilities, (which can be managed by brute-force).

EA:

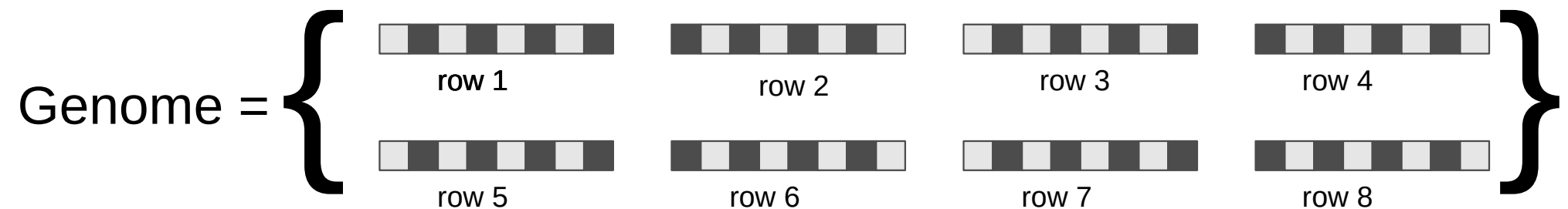
## Example: 8 Queens



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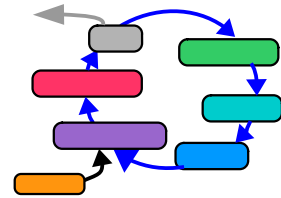
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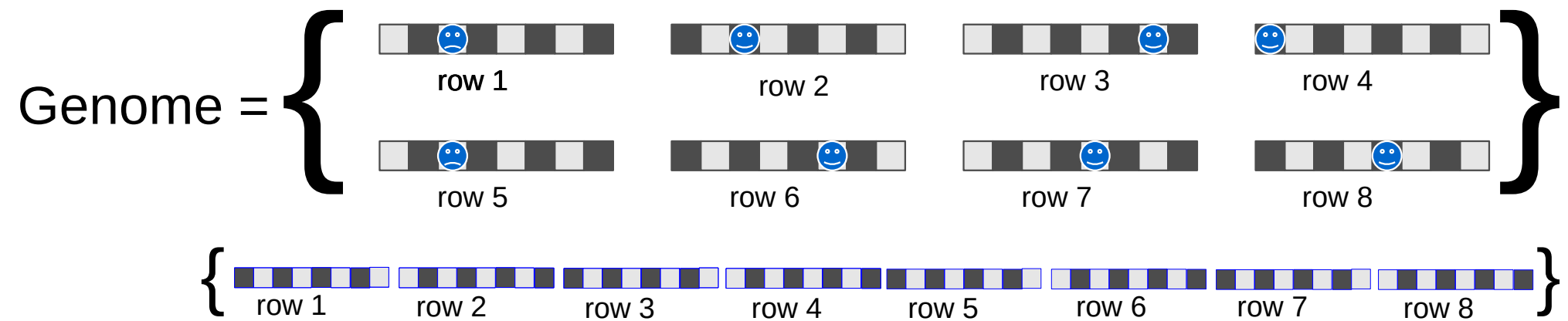
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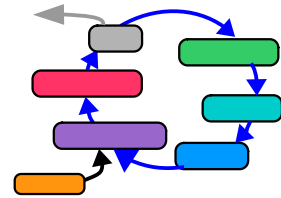


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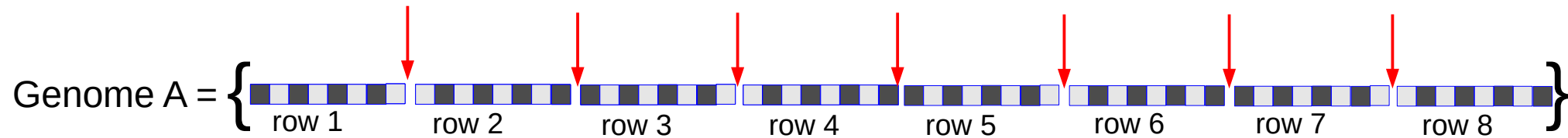
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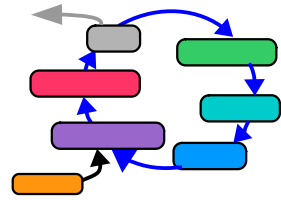
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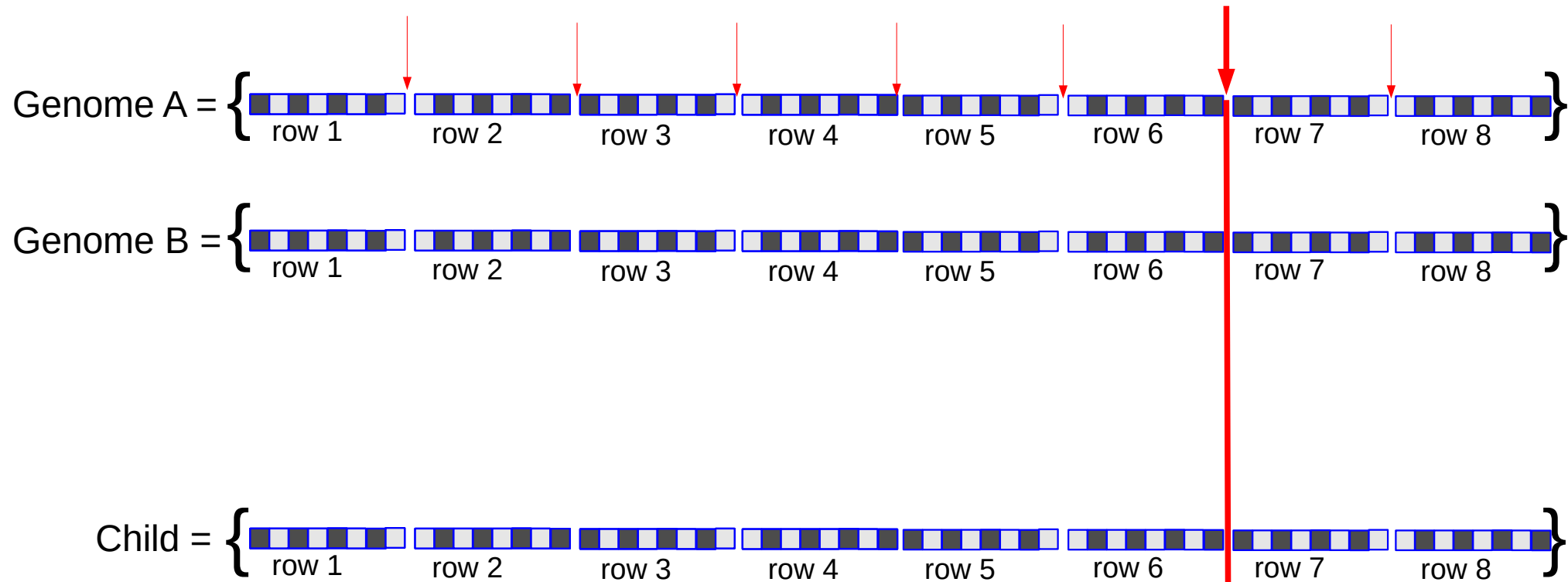
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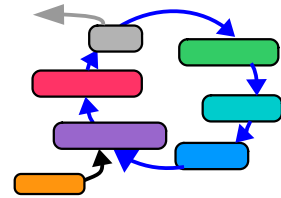
Inheritance, recombination, 1 point cross over  
cross-over points only between the rows




EA:**Example: 8 Queens**

Inheritance, recombination, 1 point cross over  
cross-over points only between the rows



EA:**Example: 8 Queens**

Mutation only the position of the queen **within** the row is altered

Genome A = {  }

Chose a random row **r** (1, ..., 8):  
and pick a new random position (1, ..., 8) for the queen 😞

before mutation

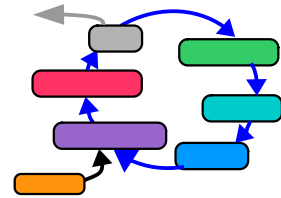


after mutation



EA:

## Example: 8 Queens



A more sophisticated implementation of the genome:

If you have an even more sophisticated idea for structuring the genome, don't forget to shape the inheritance and mutation operators and the fitness function accordingly.

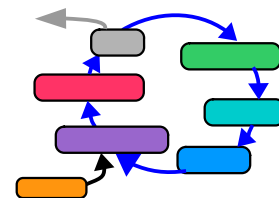
Go ahead,

feel free to do experiments



## EA:

### Example: 8 Queens



The objective, is to find a placement of the 8 queens so that they can't reach each other.

The fitness function for the EA should reflect this:

a large value of **f** if the placement is o.k. **I**

a small value of **f** if the placement is not o.k. **O**

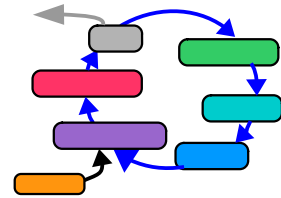
A fitness function that yields a binary value (**O**, **I**) is not a good idea for an evolutionary algorithm.

The resulting fitness surface is flat (**O**), with only a few isolated peaks (**I**).

The value of this kind of fitness function is not reflecting, that a genome can be close to a possible solution.

EA:

## Example: 8 Queens



The objective, is to find a placement of the 8 queens so that they can't reach each other.

An appropriate fitness function for an EA must be shaped accordingly;

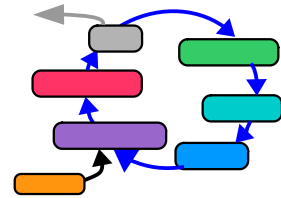
In addition, the fitness function  $f$  value should implement, that genomes  $g$ , close to an optimal solution should yield a larger fitness value  $f(g)$  than those far away.

The optimal would be, to have a fitness function  $f(g)$  that is proportional to the distance between the genome  $g$  to an optimal genome  $g^*$ .

Unfortunately this is not possible in general.

## EA:

### Example: 8 Queens



The objective, is to find a placement of the 8 queens so that they can't reach each other.

A proposal for a fitness function for the 8 queens problem:

Each possibility that one queen can attack another queen is counted as **-1**

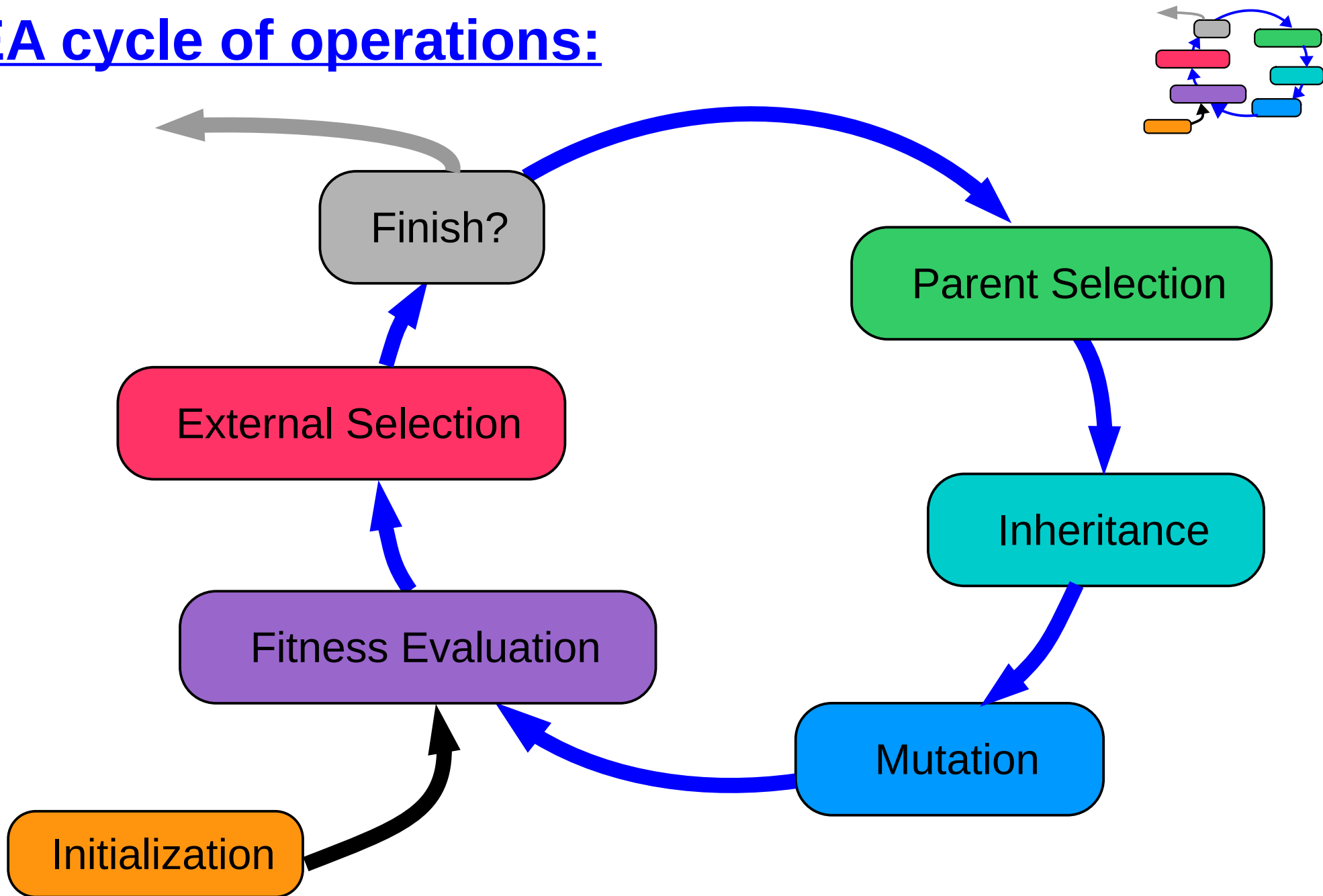
The fitness value  **$f(g)$**  is the sum of all attack possibilities.

This yields a graded response as required, with a maximal value  **$f(g^*) = 0.0$**  when no attack is possible.

### Caution:

This fitness function is only appropriate if the number of queens is fixed to 8 (*0 queens  $\Rightarrow$  0 attack possibilities*)

## EA cycle of operations:



# Overview

- EA Steps
  - Individual, Genome, Fitness, Population
  - Parent selection
  - Inheritance
  - Mutation
  - Fitness evaluation
  - External selection
  - Finish?
  - Initialization
- Strategy
- Performance Graph
- Genome Structure
- Examples

## Some important dates

Wed 14.5.25: **Dies Academicus** , special talks, no regular teaching

Thu 29.5.25 : **Ascension Day** , no lectures, no exercises, ...

Sun 8.6. - 9.6.25 : **Pentecost, Whitsun, Pfingsten**, Public holiday

Tue 10.6.25 – Fri 13.6.25 : **Excursion week**, no lectures, exercises, ...

Thu 19.6.25 : **Feast of Corpus Christi**, no lectures, exercises, ...

## Some important dates

**Thu 29.5.25 : Ascension Day** (Chr. Himmelfahrt),  
University is closed (building is closed, no exercises, lectures ...)  
shops are closed, restaurants are typically open.

Since Ascension Day is also known as Father's Day (Vatertag) or Men's Day (Männertag, Herrentag) in some parts of Germany, groups of male friends or male relatives spend a day together.

They often take part in an outdoor activity,  
such as a walk in the country or a horse-and-cart ride.  
There might be alcohol involved.

# Artificial Life Summer 2025

## Evolutionary Algorithms 2

Master Computer Science [MA-INF 4201]

Mon 14:15 – 15:45, HSZ, HS-2

Dr. Nils Goerke, Autonomous Intelligent Systems,  
Department of Computer Science, University of Bonn



# Artificial Life Summer 2025

## Evolutionary Algorithms 2

**Thank you for listening**

Dr. Nils Goerke, Autonomous Intelligent Systems,  
Department of Computer Science, University of Bonn