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Discrete and Computational Geometry

Winter semester 2024/2025

Assignment 8

Cite your sources!

Problem 1: (7 Points)
Consider n lines in the plane in general position (their arrangement is simple). Call a vertex v of their arrangement an *extreme* if one of its defining lines has a positive slope and the other one has a negative slope.

a) Prove that there are at most $O((k+1)^2)$ extremes of level at most k, for $k = 0, \ldots, n$. Imitate the proof of Clarkson's theorem on levels.

b) Show that the bound in a) cannot be improved in general.

Problem 2: (6 Points)
Show that the total number of edges of level at most k in an arrangement of n hyperplanes in $\mathbb{R}^2$ is at most $O(n(k+1))$.
HINT: Again, modify the proof by Clarkson from Lecture 14.

Problem 3: (7 Points)
Let $D_1, \ldots, D_n$ be circular disks in the plane.
Assume that the number of intersections of the boundary circles that are not contained in the interior of any of the disks is in $O(n)$.
Show that the number of intersections of their boundary circles that are contained in the interior of at most k disks, for $k = 1, \ldots, n$, is bounded by $O(n(k+1))$.