Thomas Kesselheim
Rebecca Lehming

April 23, 2025
Due date: May 6, 2025

Algorithmic Game Theory

Summer Term 2025

Exercise Set 3

If you would like to present one of the solutions in class, please also send an email to rlehming@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated on a first-come-first-served basis, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1:
Consider the local search problem *Positive Not-All-Equal kSat* (Pos-NAE-kSAT) which is defined the following way:

Instances: Propositional logic formula with n binary variables $x_1, \ldots, x_n$ that is described by m clauses $c_1, \ldots, c_m$. Each clause c_i has a weight $w_i \in \mathbb{N}$ and consists of exactly k literals, which are all positive (i.e., the formula does not contain any negated variable $\overline{x}_i$).

Feasible solutions: Any variable assignment $s \in \{0, 1\}^n$

Objective function: Sum of weights of clauses c_i in which not all literals are mapped to the same value.

Neighbourhood: Assignments s and s' are *neighbouring* if they differ in the assignment of a single variable.

You can assume that Pos-NAE-kSAT is in PLS. Now:

(a) Show that Pos-NAE-2SAT $\leq_{PLS}$ MaxCut

(b) Show that Pos-NAE-3SAT $\leq_{PLS}$ Pos-NAE-2SAT

Exercise 2:
We define a Congestion Game to be *symmetric*, if $\Sigma_1 = \ldots = \Sigma_n$. Let $PNE_{\text{Cong. Game}}$ and $PNE_{\text{Sym. Cong. Game}}$ be the local search problems in PLS of finding a pure Nash equilibrium of a general or symmetric Congestion Games, respectively.

Show: $PNE_{\text{Cong. Game}} \leq_{PLS} PNE_{\text{Sym. Cong. Game}}$.

Hint: Add an auxiliary resource for each player with a suitable delay function.

The following exercises require knowledge of lectures 6 and 7.

Exercise 3:
We want to derive properties of the sets of correlated and coarse correlated equilibria.

(a) Show that the set of correlated equilibria of a cost-minimization game Γ is convex, i.e. for two correlated equilibria p, p' and $\lambda \in [0,1]$, also $\lambda p + (1-\lambda)p'$ is a correlated equilibrium.

(b) Show that every correlated equilibrium is also a coarse correlated equilibrium.

Exercise 4:
Consider the following regret-minimization-algorithm.

Greedy

- Set $p_1^1 = 1$ and $p_j^1 = 0$ for all $j \neq 1$.
- In each round $t = 1, \ldots, T$:

  Let $L_{min}^t = \min_{i \in N} L_i^t$ for $L_i^t = \sum_{t' \leq t} \ell_i^{(t')}$ and $S^t = \{i \in N \mid L_i^t = L_{min}^t\}$.
  Set $p_i^{t+1} = 1$ for $i = \min S^t$ and $p_j^{t+1} = 0$ otherwise.

You can assume that $\ell_i^{(t)} \in \{0,1\}$ for all i and t.

(a) Show that the costs of Greedy are at most $N \cdot L_{min}^T + (N-1)$.

(b) State a scheme for an example such that the stated upper bound of (a) is tight for an infinite number of values T.