Image Histograms 2

Cyrill Stachniss

Summer term 2024 – Cyrill Stachniss

5 Minute Preparation for Today

https://www.youtube.com/watch?v=flI_Umo_VAU

Photogrammetry & Robotics Lab

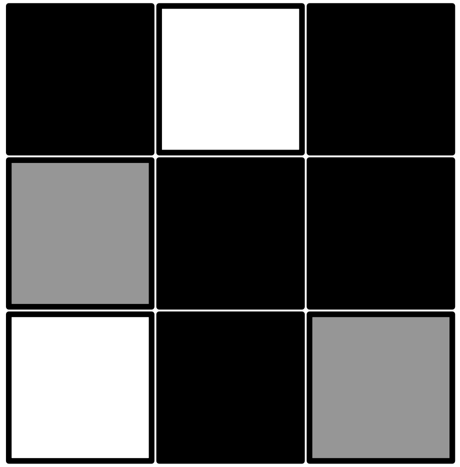
Histogram Transformations: Histogram Equalization & Noise Variance Equalization

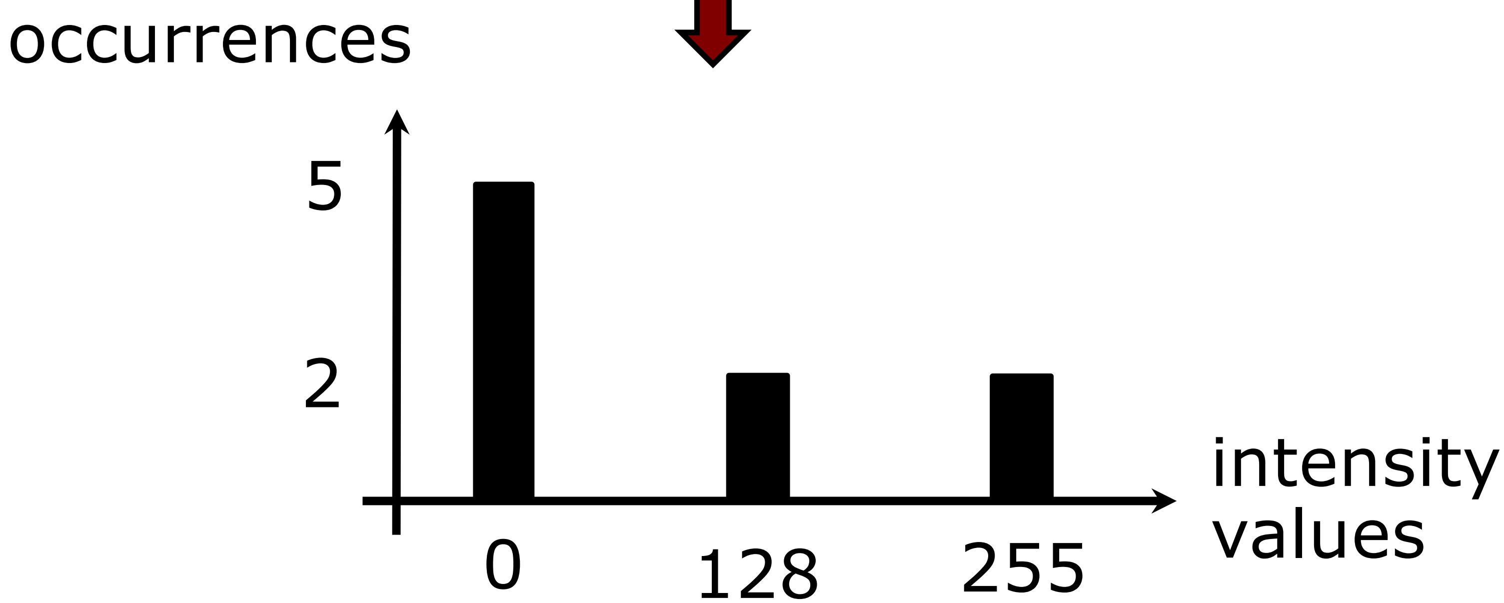
Cyrill Stachniss

The slides have been created by Cyrill Stachniss.

Image Histogram

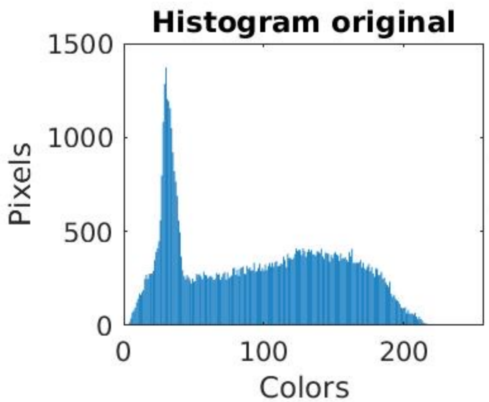
0	255	0
128	0	0
255	0	128





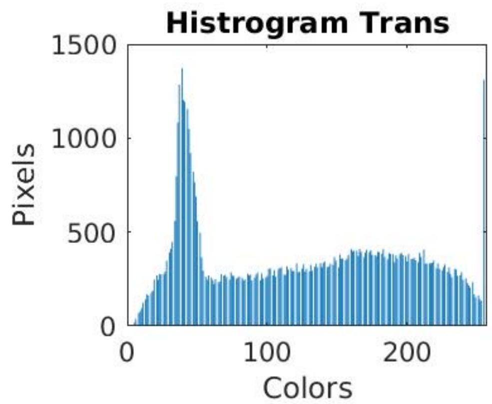
Brightness and Contrast

- A transformation that changes the mean, changes the image brightness
- Analogous for variance and contrast



increasing variance

increases contrast

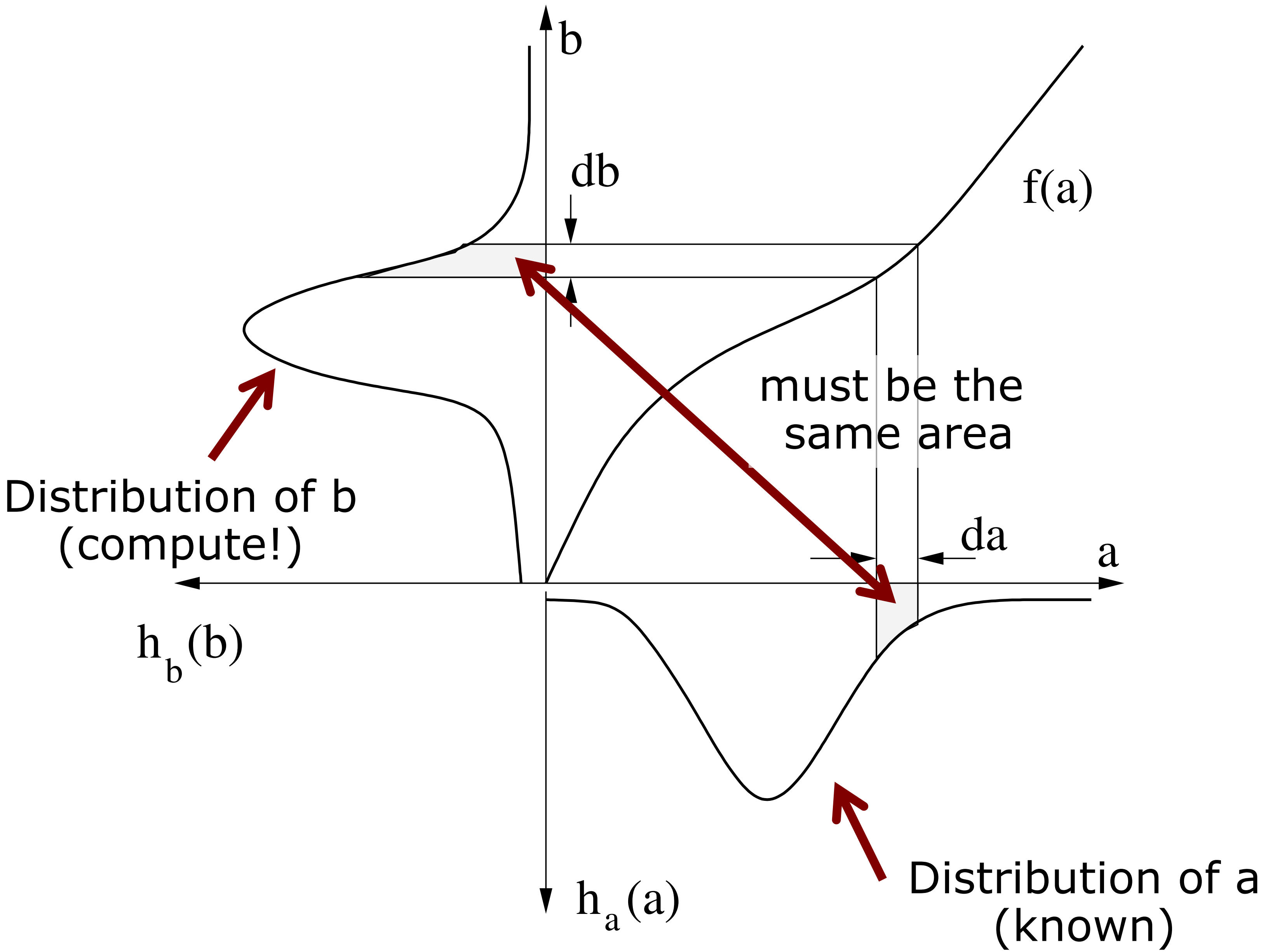


Can We Compute the Output Histogram Knowing the Transformation Function?

How Do Operators Affect the Distribution of Intensities?

- Monotonous function $b = f(a)$
- Histogram of the input image: $h_a(a)$
- Compute the histogram $h_b(b)$ of the output image given $h_a(a)$ and f
- Use gained knowledge to design f

Transformation of a PDF

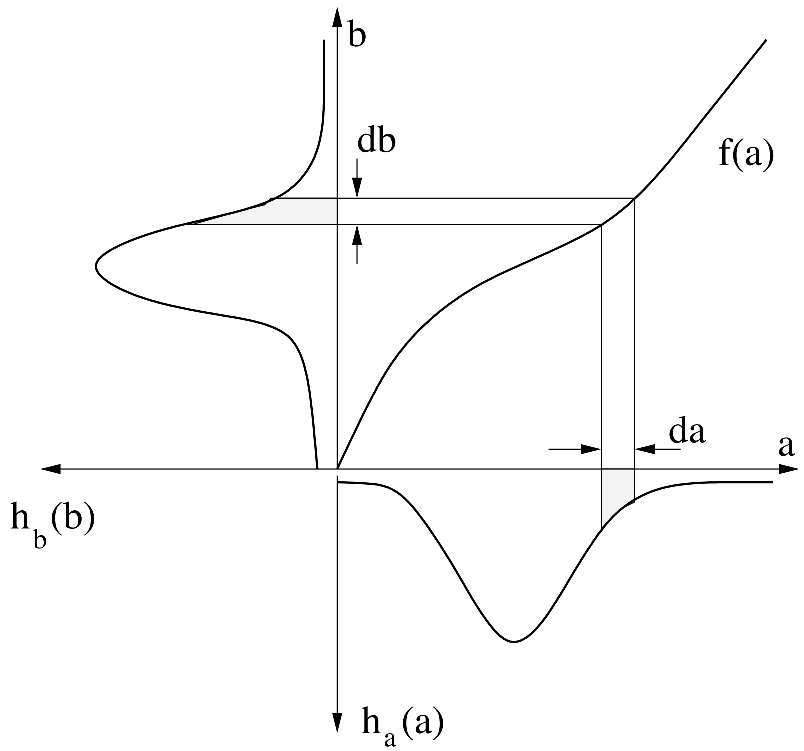


Effect of a Monotonous Point Operator on the PDF

- The gray "area" in the interval $[a, a + da]$ is mapped to the interval $[b, b + db]$
- So we have

$$h_b(b)\,db = h_a(a)\,da$$

("area stays the same")

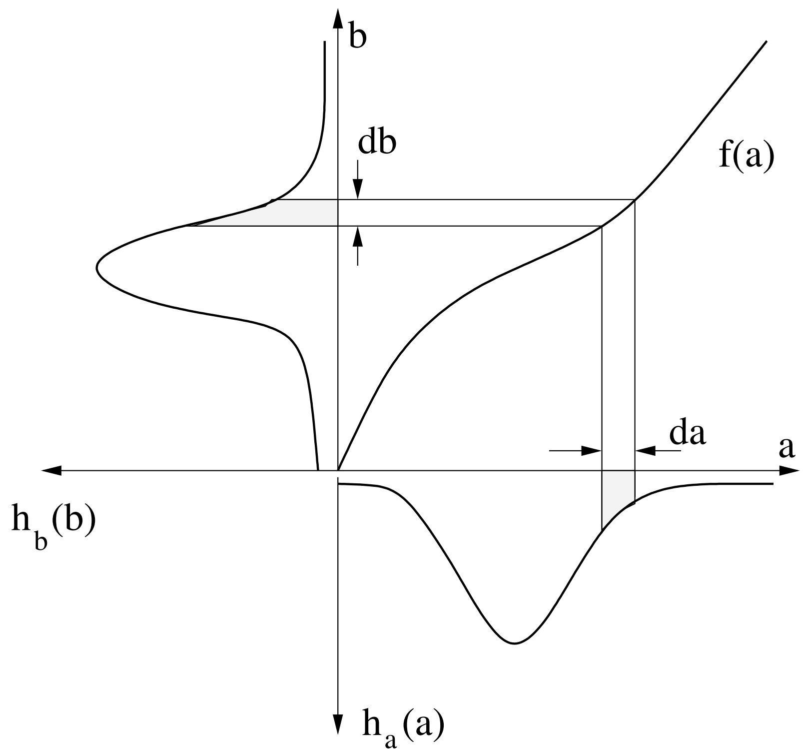


This leads to

we want to compute $h_b(b)$

$$h_b(b)\,db = h_a(a)\,da$$

$$h_b(b) = \frac{h_a(a)}{\left|\dfrac{db}{da}\right|}$$

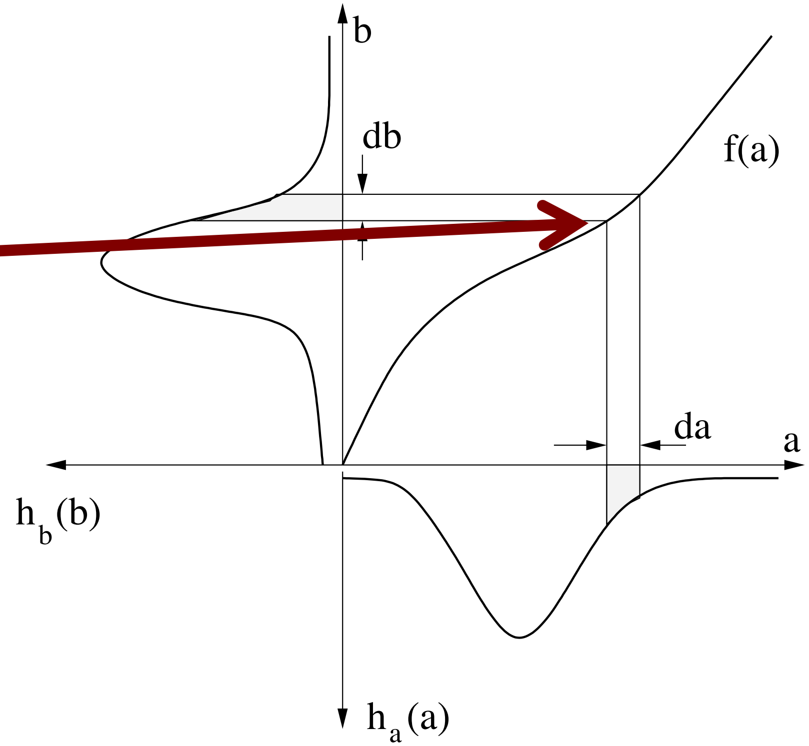


This leads to

$$h_b(b)\,db = h_a(a)\,da$$

$$h_b(b) = \frac{h_a(a)}{\left|\dfrac{db}{da}\right|}$$

derivative of f

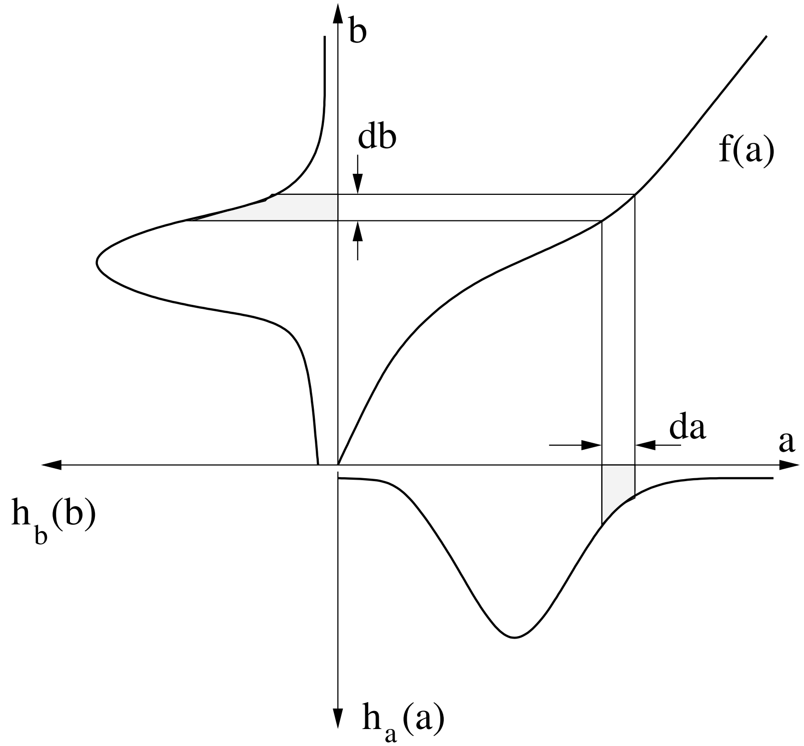


This leads to

first derivative

$$h_b(b)\,db = h_a(a)\,da$$

$$h_b(b) = \frac{h_a(a)}{\left|\dfrac{db}{da}\right|} = \frac{h_a(a)}{|f'(a)|}$$



This leads to

first derivative

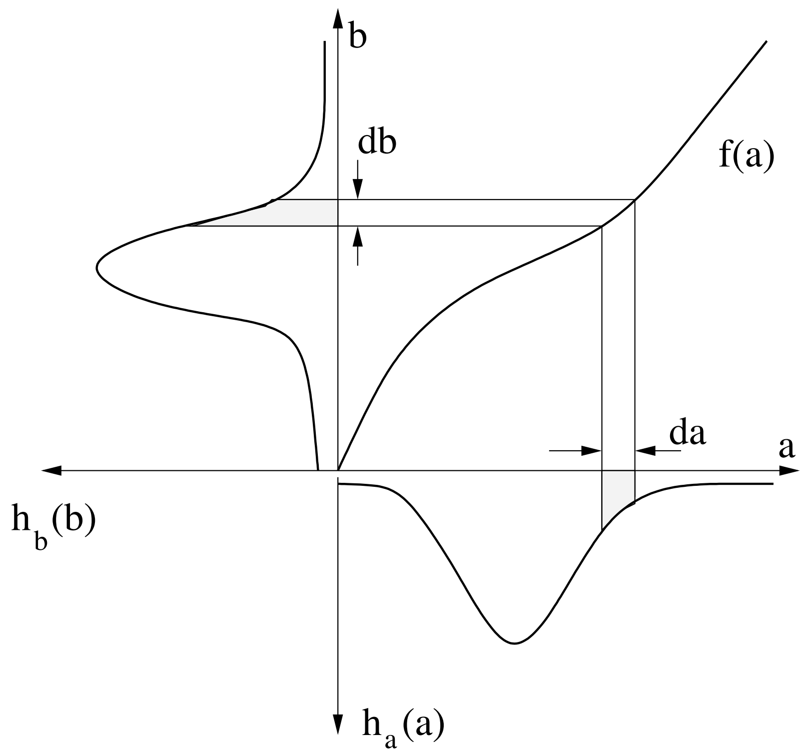
$$h_b(b)\, db = h_a(a)\, da$$

$$h_b(b) = \frac{h_a(a)}{\left|\dfrac{db}{da}\right|} = \frac{h_a(a)}{|f'(a)|}$$

function $h_b(b)$ should depend on b, not on a

$$b = f(a) \rightarrow a = f^{-1}(b)$$

$$\Rightarrow h_b(b) = \frac{h_a\left(f^{-1}(b)\right)}{\left|f'\left(f^{-1}(b)\right)\right|}$$



Example: Linear Function

- If we have the linear function

$$b = f(a) = k + ma$$

- with the first derivative

$$f'(a) = m$$

- and the inverse

$$a = f^{-1}(b) = \frac{b-k}{m}$$

- this yields $h_b(b) = \dfrac{h_a\left(f^{-1}(b)\right)}{\left|f'\left(f^{-1}(b)\right)\right|} = \dfrac{h_a\left(\dfrac{b-k}{m}\right)}{|m|}$

Example

- We can directly compute the output histogram given input and the (linear) transfer function
- Result

$$h_b(b) = \frac{h_a\left(\frac{b-k}{m}\right)}{|m|}$$

- This corresponds to a **shift by k** and a **scaling by 1/m**

Can We Design Transformations Such That the Resulting Image Has Certain Desired Properties?

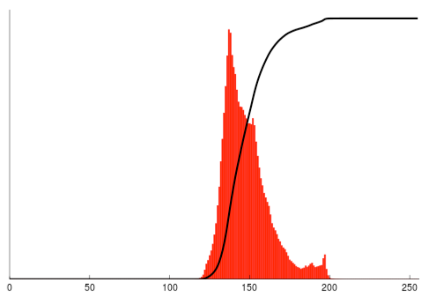
Histogram Equalization

Sometimes One Can Barely See Anything...

… but a Change in the Intensity Values Can Help

Histogram Equalization

How to transform the image so that all intensities are used equally often?



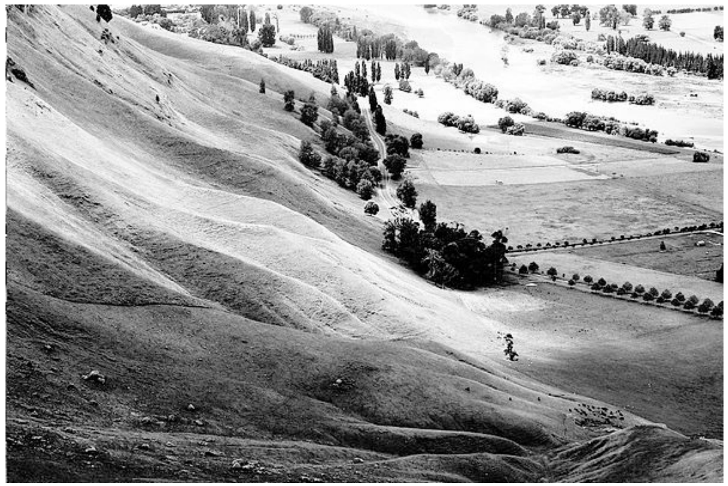
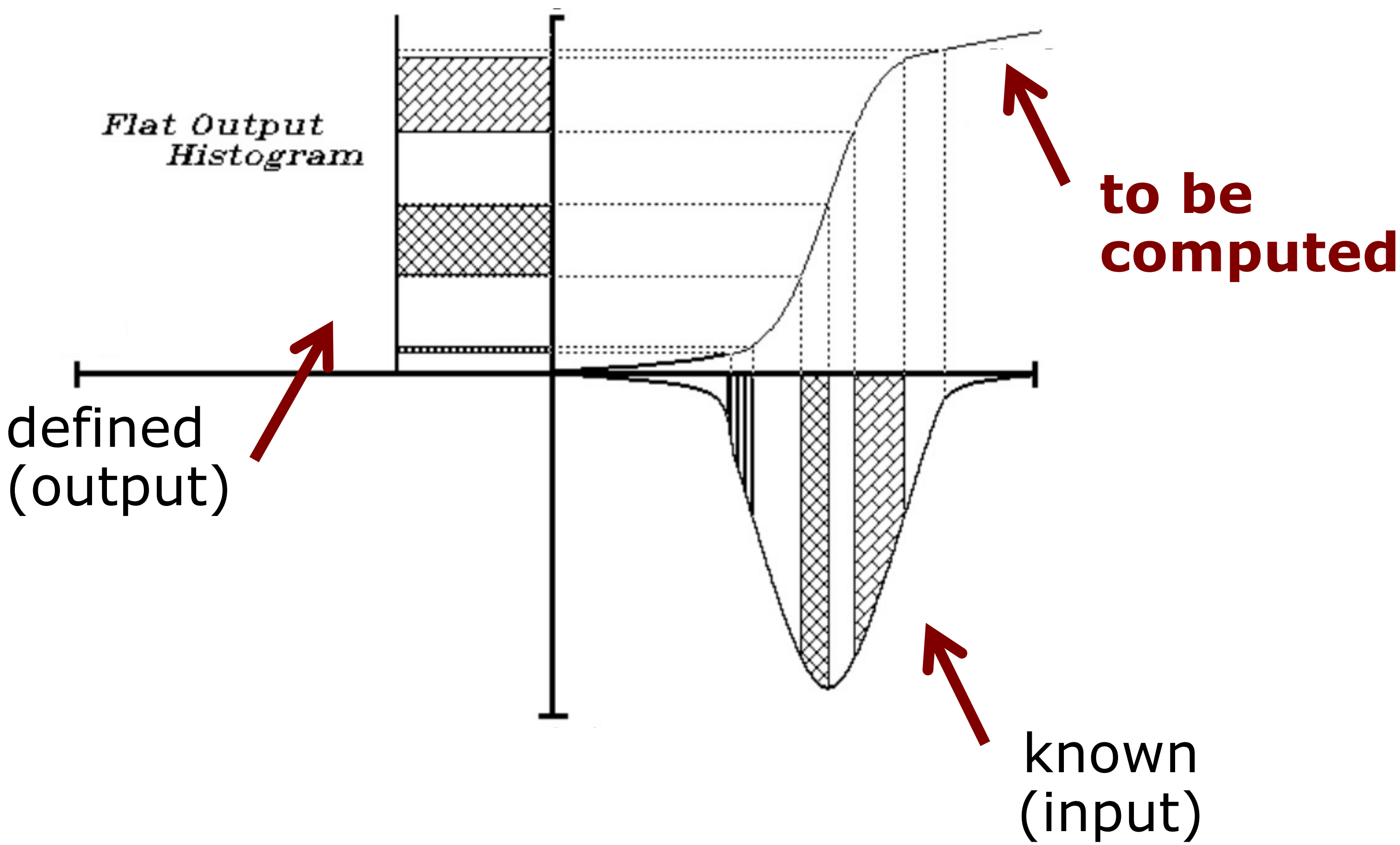


Image courtesy: Phillip Capper

Histogram Equalization

- **Goal: same number of values in every histogram bin** $h_b(b) = \text{const.}$



Histogram Equalization

- **Goal: same number of values in every histogram bin** $h_b(b) = \text{const.}$
- With a monotonically increasing f

$$h_b(b) = k \qquad\qquad h_b(b) = \frac{h_a(a)}{\frac{db}{da}}$$

$$\Rightarrow \quad k = \frac{h_a(a)}{\frac{db}{da}} \quad \Rightarrow \quad db = \frac{1}{k} h_a(a) da$$

Histogram Equalization

- How to solve $db = \frac{1}{k} h_a(a) da$?

Histogram Equalization

- How to solve $db = \frac{1}{k} h_a(a) da$?
- Solve via integration

$$\int db = \int \frac{1}{k} h_a(a) da$$

$$\Downarrow$$

$$b + C_1 = \frac{1}{k} \boxed{\int h_a(a) da}$$

Integral over the histogram of intensities $H(a)$

Histogram Equalization

- The equation

$$b + C_1 = \frac{1}{k} \int h_a(a) da$$

- simplifies to

$$b + C_1 = \frac{1}{k} H(a) + C_2$$

- with the cumulative histogram $H(a)$.
- Thus:

$$b = f(a) = \frac{1}{k} H(a) + C$$

Histogram Equalization

- The parameters k, C in

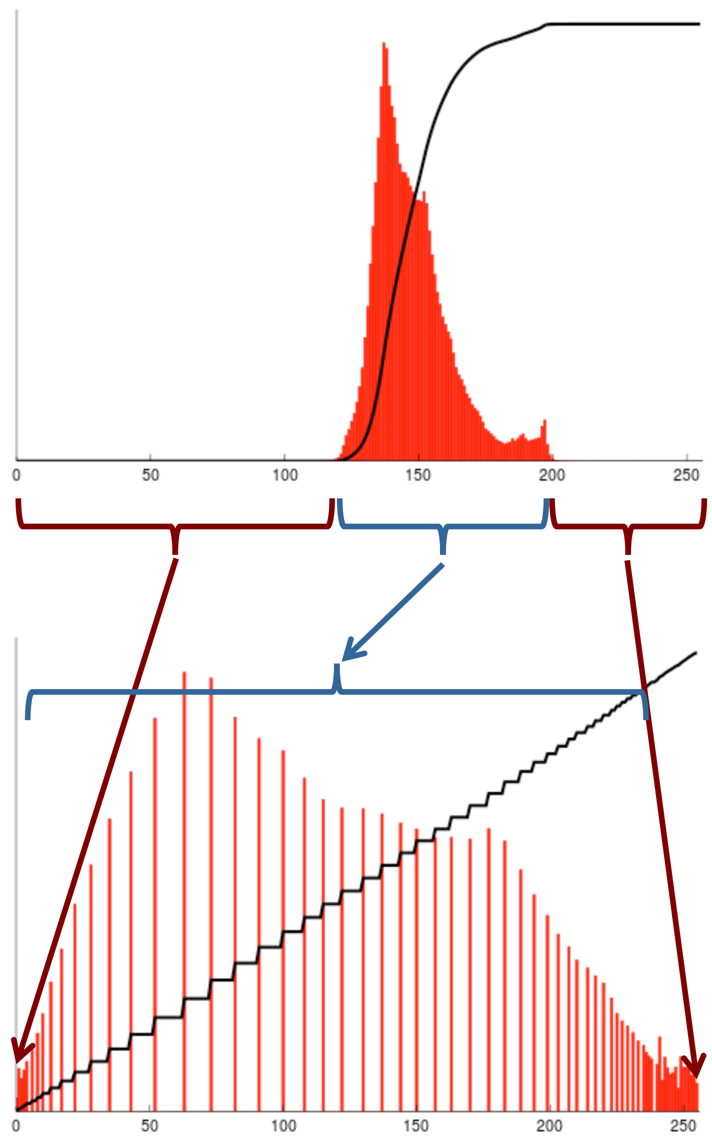
$$b = f(a) = \frac{1}{k} H(a) + C$$

- are typically chosen so that

$$f(0) = 0 \qquad f(255) = 255$$

- This maps the spectrum of intensity values to the full range [0, 255]

Mapping Illustration



$$f(0) = 0$$

$$f(255) = 255$$

Courtesy: Phillip Capper

Histogram Equalization

- Chose the parameters k, C so that

$$f(0) = 0 \qquad f(255) = 255$$

- From

$$\frac{1}{k} H(0) + C = 0 \qquad \frac{1}{k} \underbrace{H(255)}_{=N} + C = 255$$

- follows

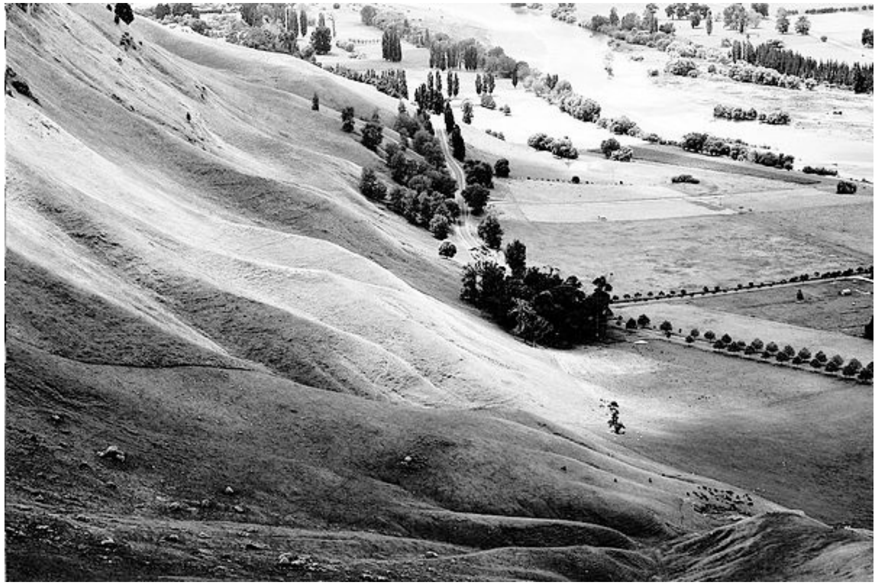
$$k = \frac{N - H(0)}{255} \qquad C = -H(0) \frac{255}{N - H(0)}$$

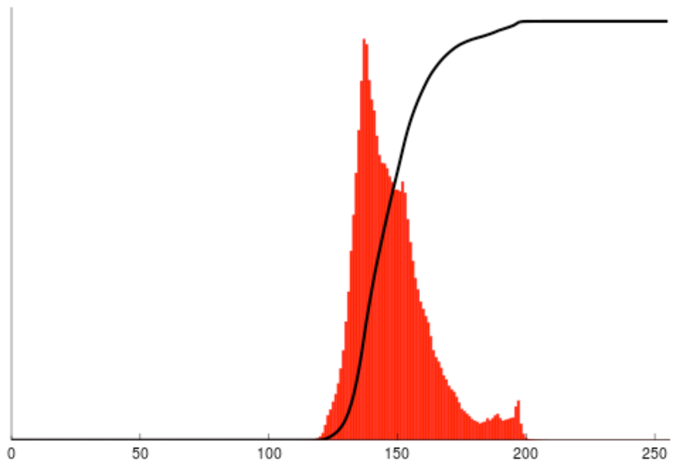
Histogram Equalization

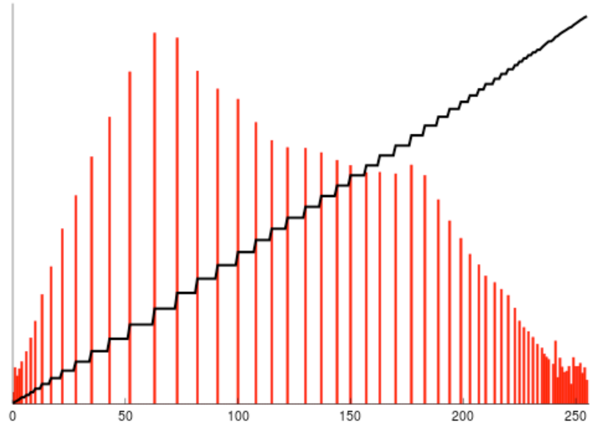
- This results in the point operator for histogram equalization

$$f(a) = \mathrm{round}\left(\frac{255\,(H(a) - H(0))}{N - H(0)}\right)$$

This Results in...

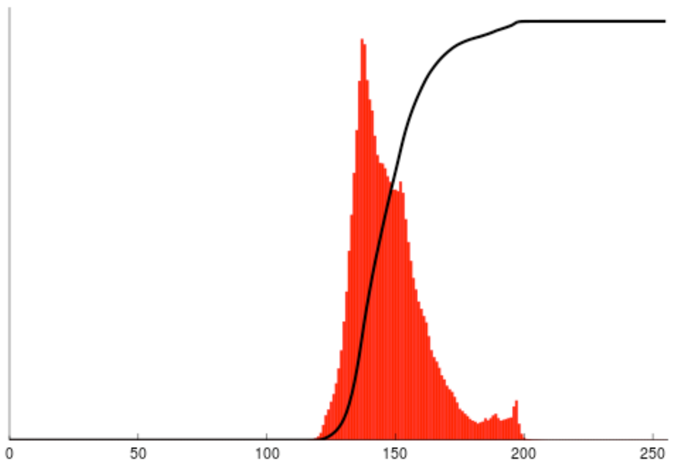


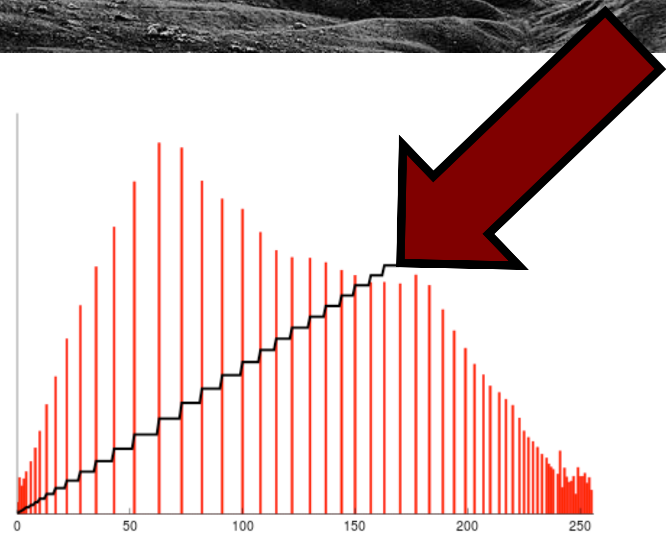




Courtesy: Phillip Capper

Uniform Distribution?

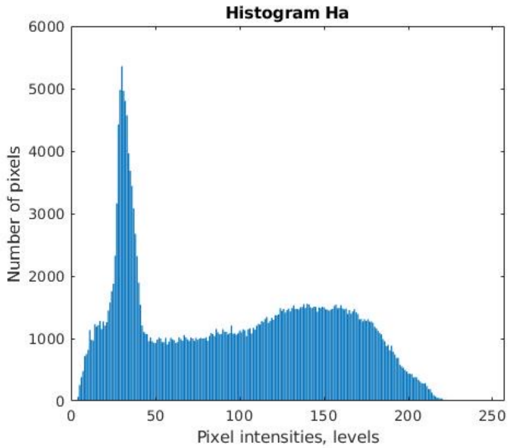




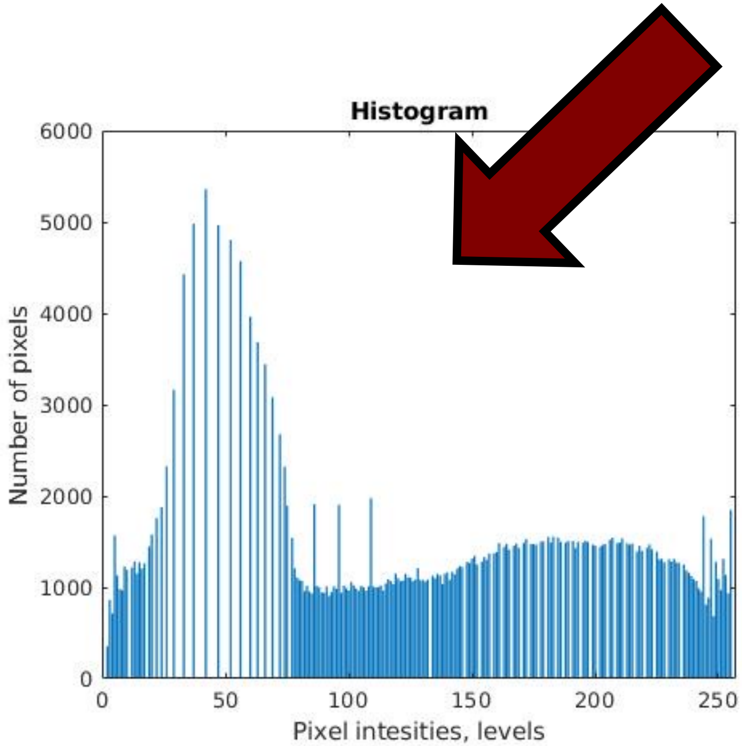
Courtesy: Phillip Capper

Uniform Distribution?

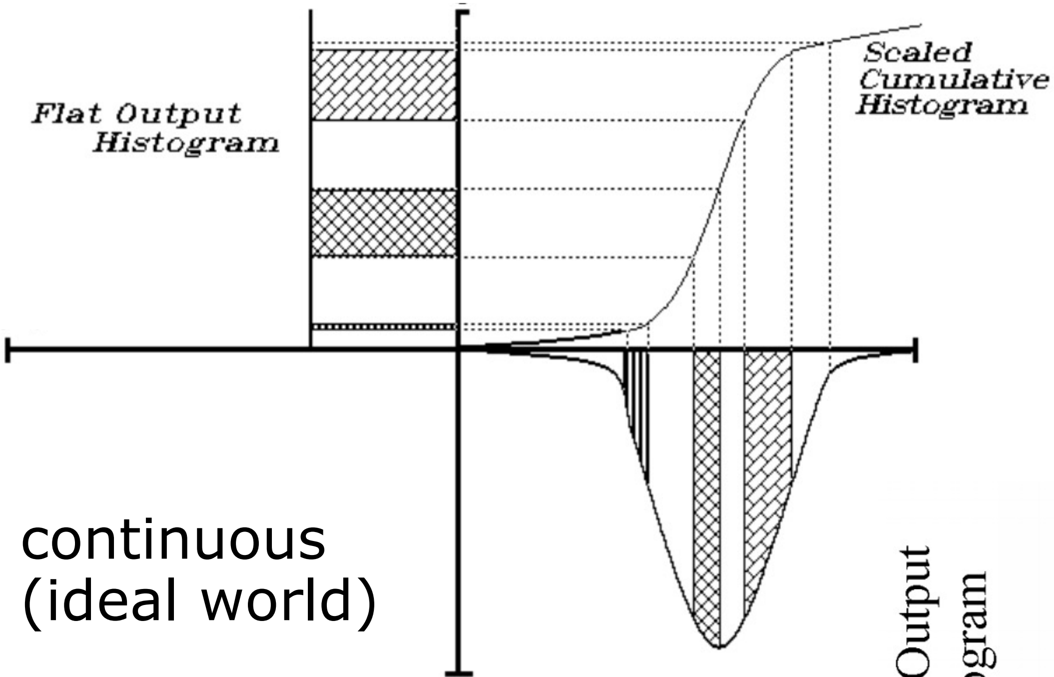
Original image



Equalized image

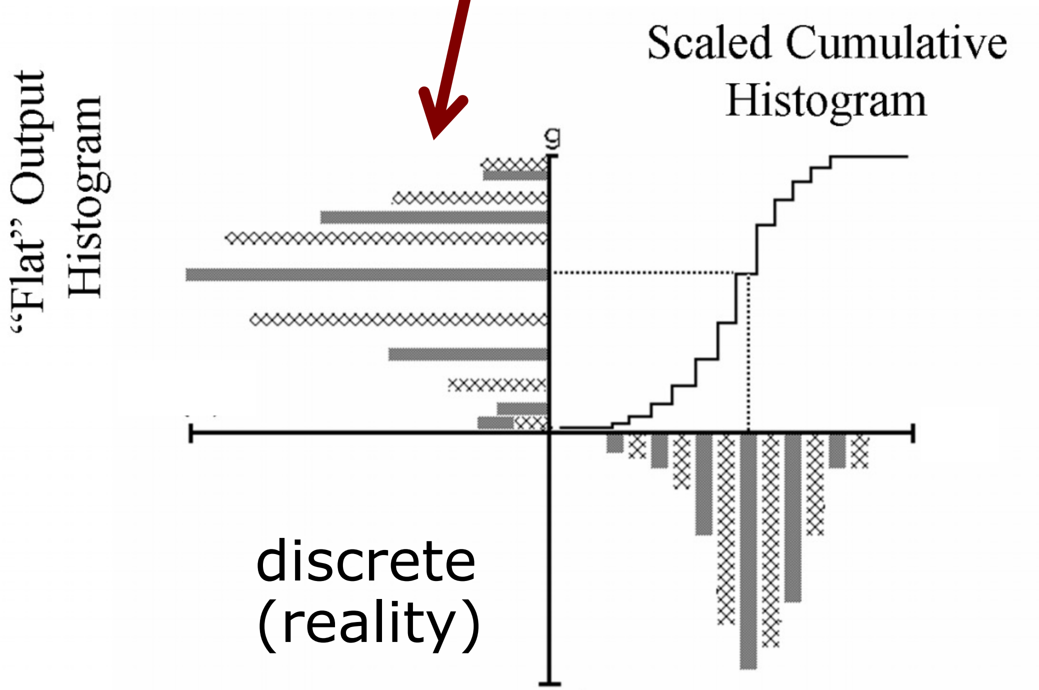


Continuous vs. Discrete Case



continuous
(ideal world)

As "flat" as it can get...

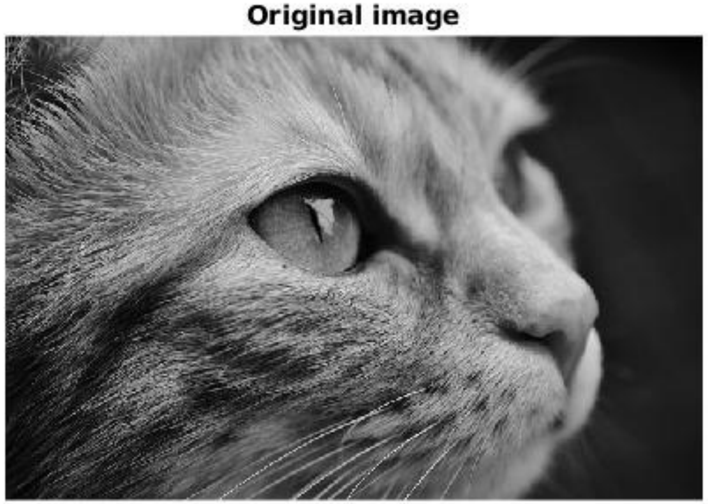


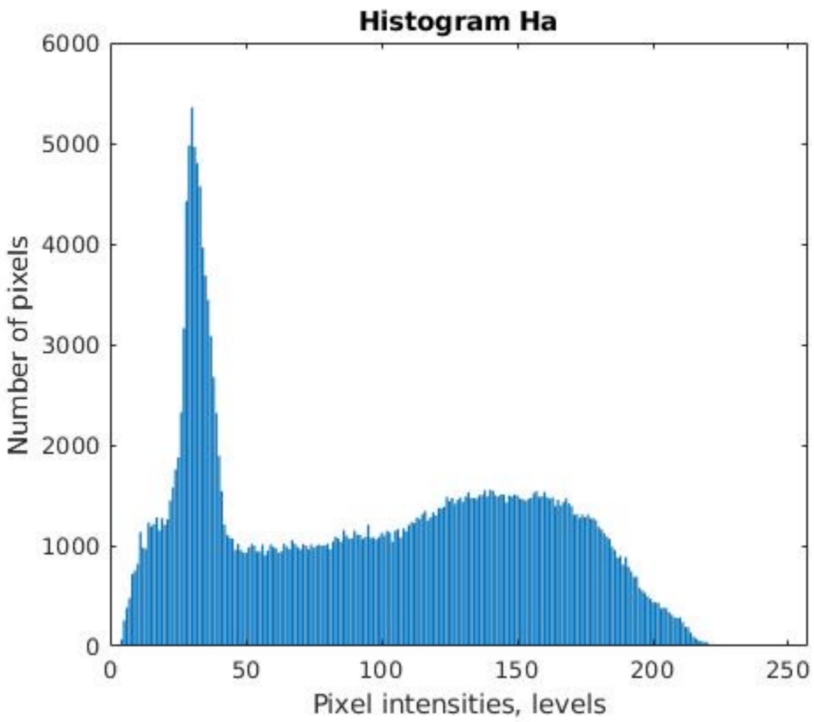
discrete
(reality)

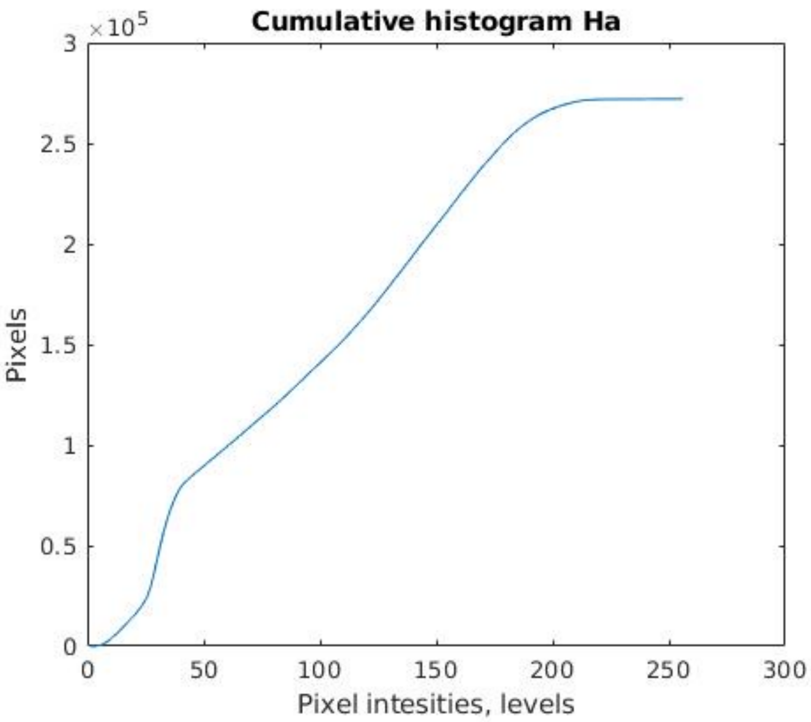
Effect of the Hist. Equalization

- Typically increases the contrast
- Areas of lower local contrast gain higher contrast
- Distributes the intensities over the histogram

Another Example

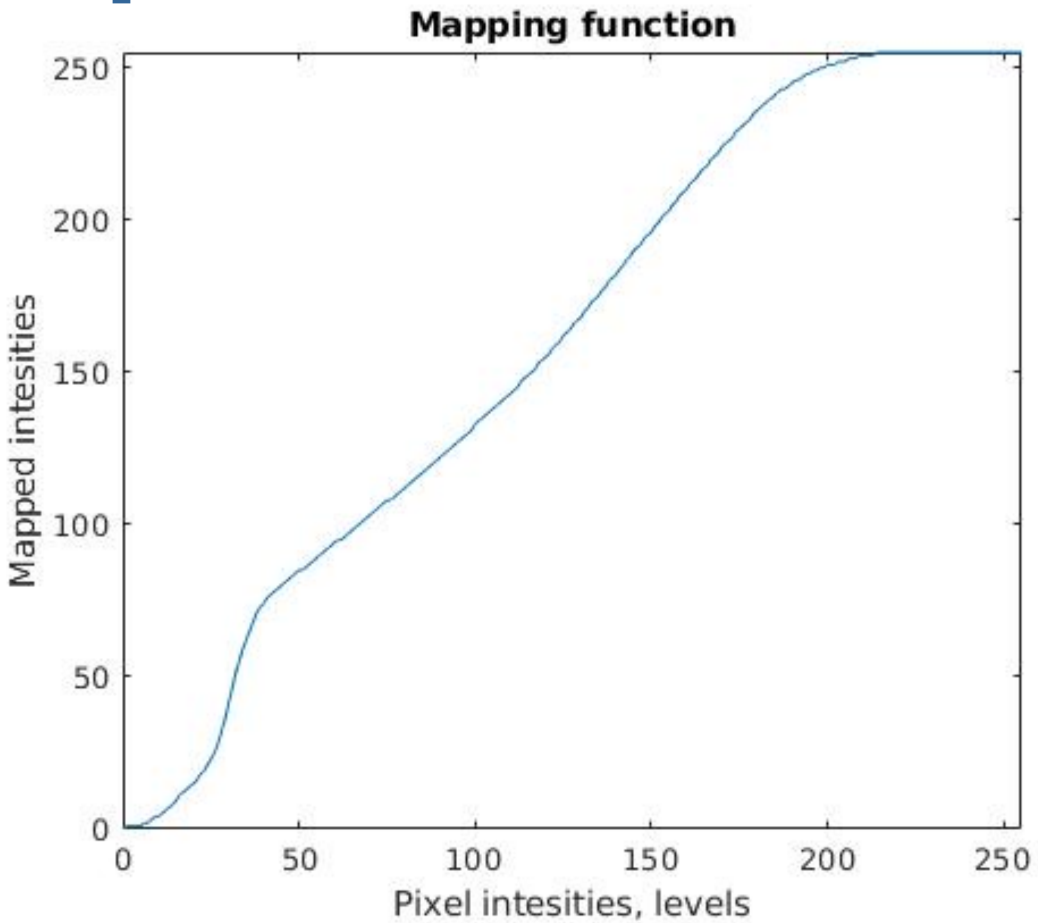






Another Example

Equalized image



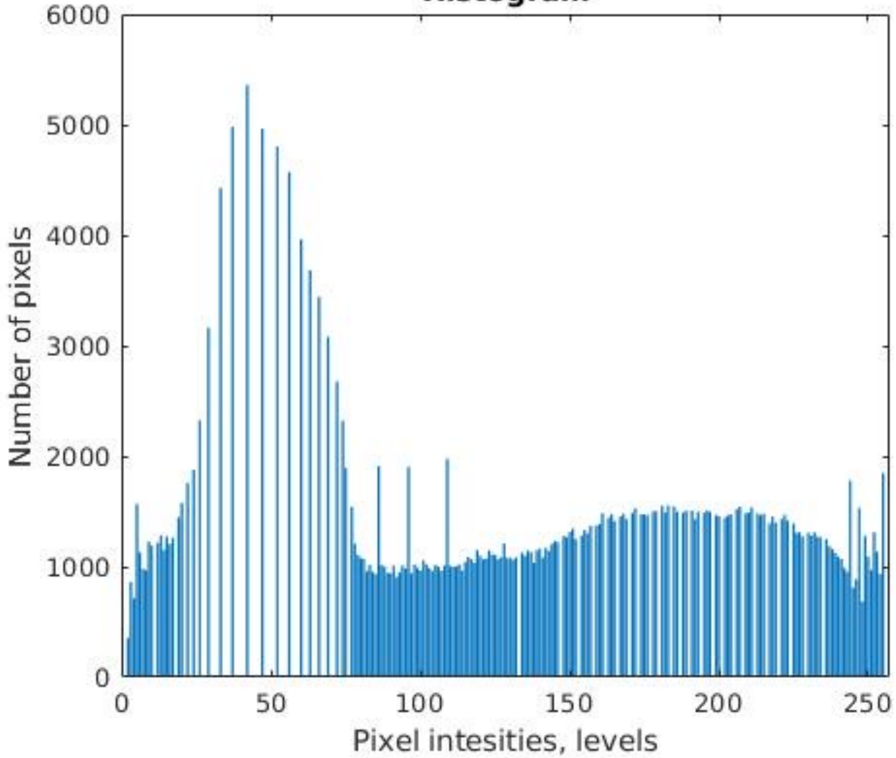
Original image

Another Example

Original image

Equalized image

Histogram



Variants of Histogram Equal.

- There exist variants of HE
- Adaptive HE (**AHE**) – performs HE in local patches, not the whole image
- AHE has issues in low contrast regions as it over-amplifies the corrections
- Contrast limited AHE (**CLAHE**) limits this over-amplifications in homogeneous regions

Noise Variance Equalization

Noise Variance Equalization

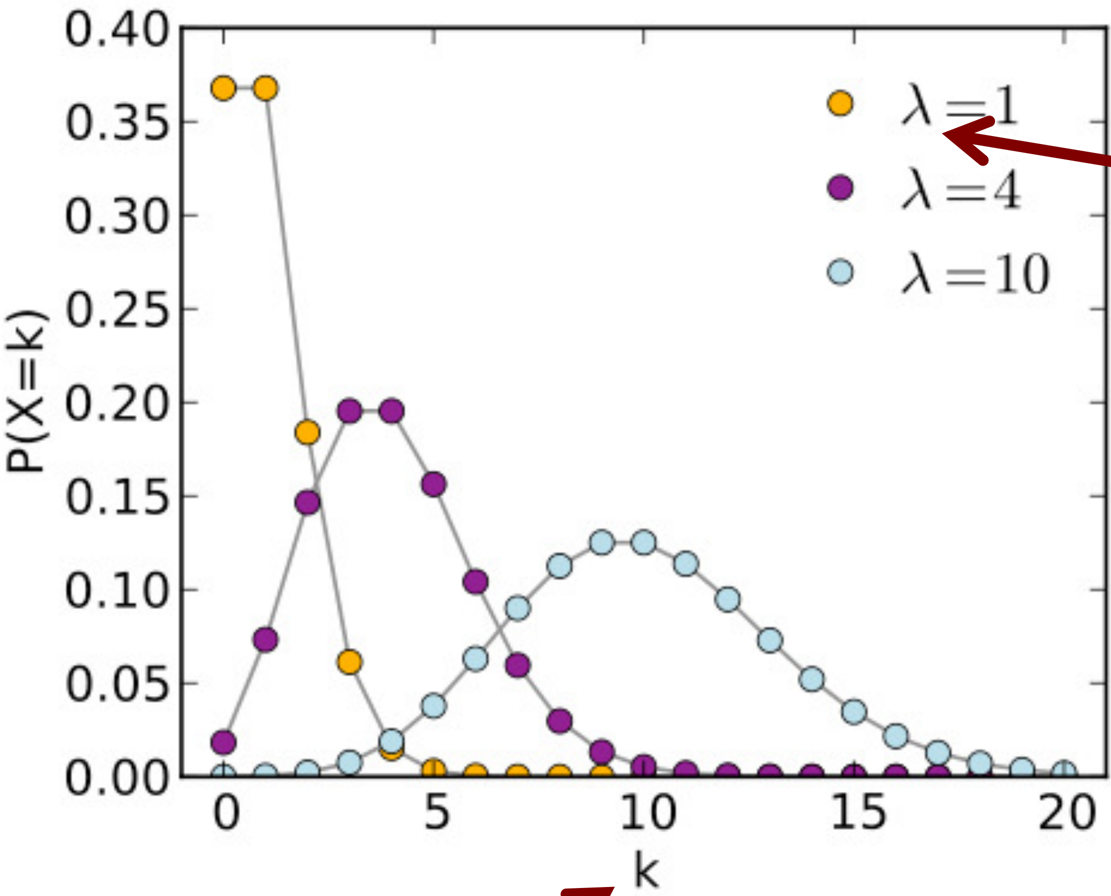
- Intensity measurements are noisy
- Sensor: variance of a measured intensity depends on the intensity itself
- **Goal:** adjust the **variance** of the intensities to a **fixed value**
- Useful for statistical analysis of images as all pixels have the same noise

Poisson Distribution

"The Poisson distribution is a **discrete** probability distribution that expresses the **probability of a given number k of events** occurring **in a fixed interval** of time and/or space if these events occur with a **known average rate** and **independently** of the time since the last event."

$$P(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

Poisson Distribution



parameter

number of occurrences

$$P(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

Image courtesy: Wikipedia 42

Distribution About the Number of Incoming Photons

- Poisson distribution for photon counts

$$P(k) = \frac{(\beta t)^k}{k!} e^{-\beta t}$$

- with
  - β : avg. number of incoming photons per second
  - t : exposure time
- Distribution about the number of photons reaching the sensor

Properties of the Poisson Distribution

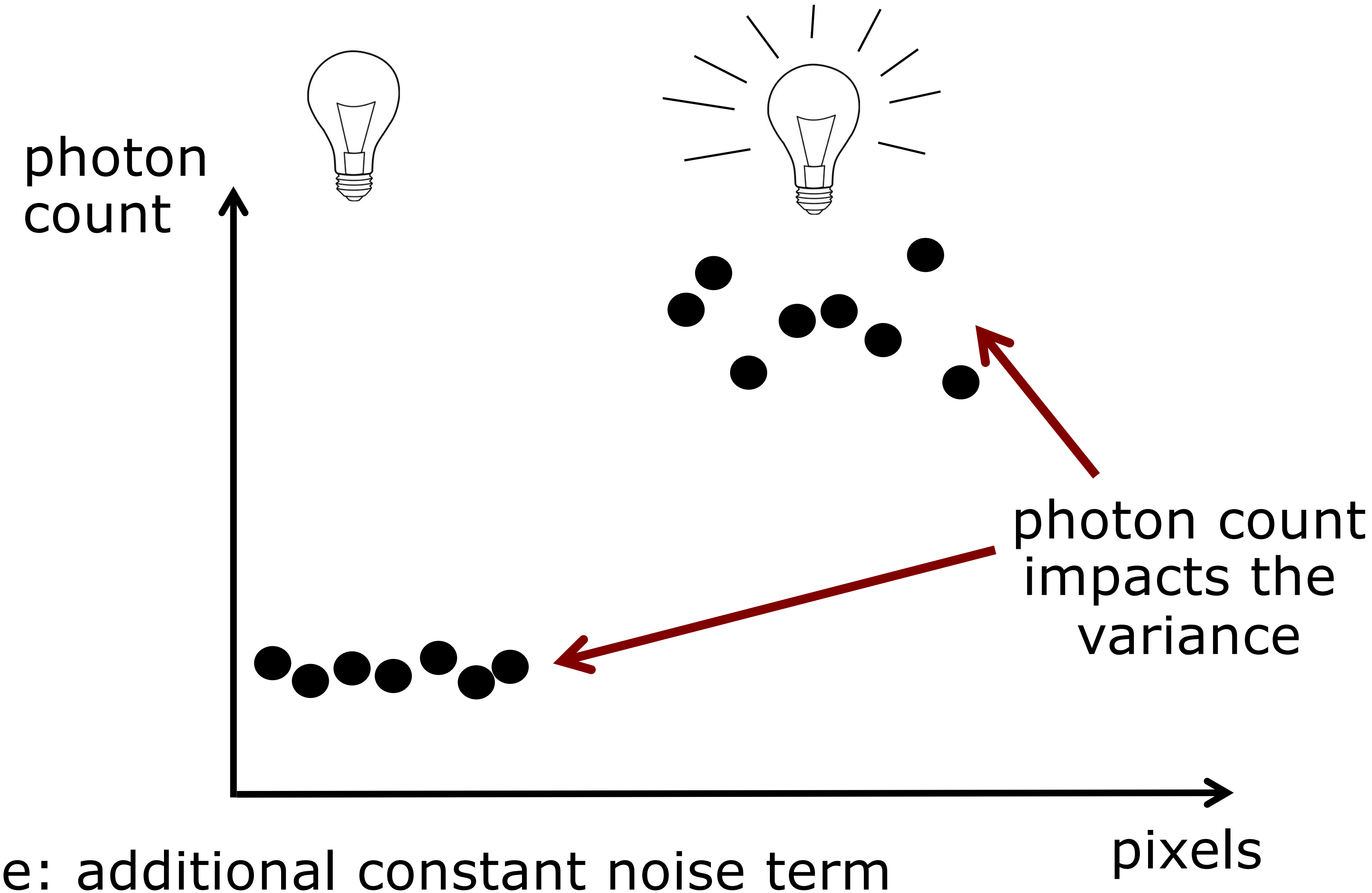
- Mean and variance are then

$$\mu_k = \beta t \qquad \sigma_k^2 = \beta t$$

- and thus a standard deviation of

$$\sigma_k = \sqrt{\beta t}$$

Variance Illustration



Note: additional constant noise term from electronics and quantization

Noise Variance Equalization

- Consider a proportional relationship between variance and intensity

  $$\sigma_a^2 = ma$$

- Goal: $\sigma_b = \sigma_0$ for all intensities

Noise Variance Equalization

- Consider a proportional relationship between variance and intensity

$$\sigma_a^2 = ma$$

- Goal: $\sigma_b = \sigma_0$ for all intensities
- Variance propagation yields

$$\sigma_b = \frac{db}{da}\sigma_a \overset{!}{=} \sigma_0 = \text{const.}$$

(given a monotonously increasing function)

Noise Variance Equalization

- We have

$$\sigma_b = \frac{db}{da}\sigma_a = \sigma_0 \qquad \sigma_a^2 = ma$$

- We can rewrite that as

$$db = \frac{\sigma_0}{\sigma_a}da \qquad \sigma_a = \sqrt{ma}$$

- Directly yields

$$db = \frac{\sigma_0}{\sqrt{m}}a^{-\frac{1}{2}}da$$

Noise Variance Equalization

- The equation

$$db = \frac{\sigma_0}{\sqrt{m}} a^{-\frac{1}{2}} da$$

- can be solved by integration to

$$b = f(a) = \frac{2\,\sigma_0}{\sqrt{m}} \sqrt{a} + C$$

- Again we want $\begin{array}{l} a = 0 \rightarrow b = 0 \\ a = 255 \rightarrow b = 255 \end{array}$

- Choose $C = 0$ to obtain $a = 0 \rightarrow b = 0$

Noise Variance Equalization

- Requiring $a = 255 \rightarrow b = 255$
- For the equation $b = \dfrac{2\,\sigma_0}{\sqrt{m}}\sqrt{a}$
- Directly leads to $255 = \dfrac{2\,\sigma_0}{\sqrt{m}}\sqrt{255}$

$$\frac{2\,\sigma_0}{\sqrt{m}} = \sqrt{255}$$

Noise Variance Equalization

- Requiring $a = 255 \rightarrow b = 255$

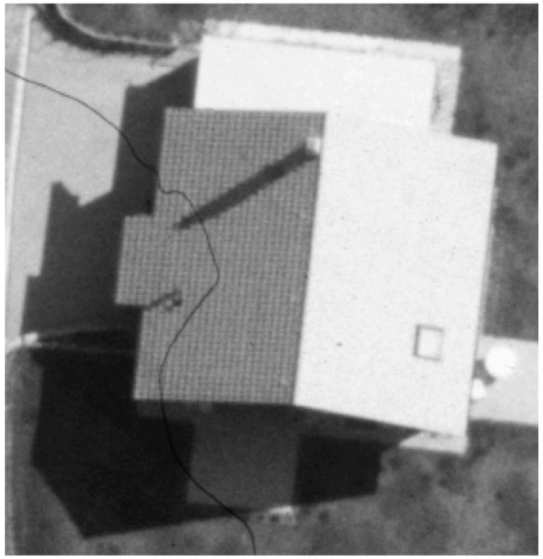
- Yields $\boxed{b = f(a) = \sqrt{255}\sqrt{a}}$

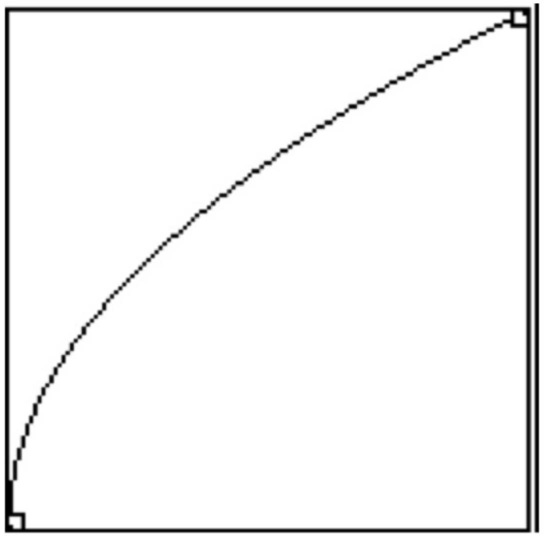
- Variance equalization yields a **square root function** with black stays black and white stays white

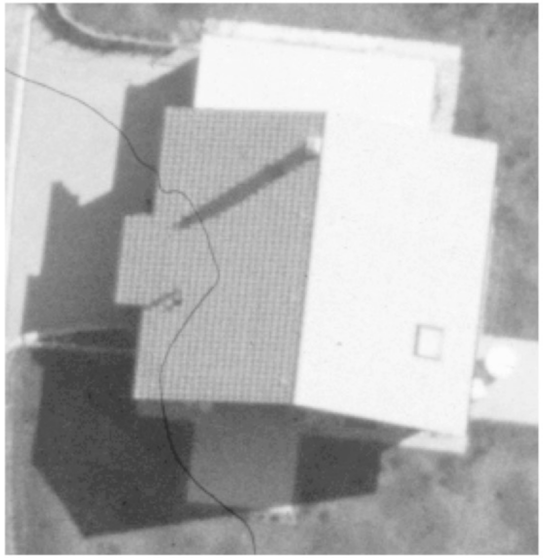
$$a = 0 \rightarrow b = 0 \qquad a = 255 \rightarrow b = 255$$

Noise Variance Equalization

- Result: $f(a) = \sqrt{255}\sqrt{a}$
- The square root stretches the dark areas and compresses bright areas







Summary

- **Image histograms** represent the image **intensity distributions**
- Point operators to manipulate images
- **Designing transformations** such that images have certain properties
- **Histogram equalization** to obtain a uniform distribution of intensities
- **Noise variance equalization:** All pixels intensities have the same noise

Slide Information

- The slides have been created by Cyrill Stachniss as part of the photogrammetry and robotics courses.
- **I tried to acknowledge all people from whom I used images or videos. In case I made a mistake or missed someone, please let me know.**
- The photogrammetry material heavily relies on the very well written lecture notes by Wolfgang Förstner and the Photogrammetric Computer Vision book by Förstner & Wrobel.
- Parts of the robotics material stems from the great Probabilistic Robotics book by Thrun, Burgard and Fox.
- If you are a university lecturer, feel free to use the course material. If you adapt the course material, please make sure that you keep the acknowledgements to others and please acknowledge me as well. To satisfy my own curiosity, please send me email notice if you use my slides.

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