

Discrete and Computational Geometry

Winter semester 2024/2025

Assignment 3

Problem 1: (4 Points)

Modify the Graham-Scan (Lecture 5, Algorithm 1) such that it also computes the convex hull of points that are not in *general position*, i.e.:

- For a given set of points in the plane we may assume that no three points lie on a common line.
- For points in the plane we may assume that no two points have the same x-coordinate or y-coordinate.

The modified algorithm should still run in $O(n \log(n))$ time and $O(n)$ space.

Problem 2: (4 Points)

For $n \geq 8$, describe a construction of an input instance of n points in \mathbb{R}^3 and a permutation of this input instance, where Algorithm 7.1 takes $\Omega(n^2)$ steps. Describe for which operations the algorithm needs to take this many steps.

Problem 3: (5+4 Points)

Consider Algorithm 7.1 from Lecture 7. The algorithm needs to find the cycle of horizon edges L that lie on the boundary of $Conflicts(i)$.

- (a) Design an algorithm for this step that takes $O(|Conflicts(i)|)$ time in the worst-case. Use the pointers of the DCEL. Show correctness and running time of your algorithm.
- (b) Show that the expected total time spent on computations for finding horizon edges during the course of the algorithm is in $O(n)$.