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Discrete and Computational Geometry

Winter semester 2024/2025

Assignment 4

Note: Please acknowledge sources.

Problem 1: (8 Points)

The *Gabriel Graph (GG)* of a set of points in the plane is defined as follows: Two points p and q are connected by an edge of the Gabriel graph if and only if the disc with diameter pq does not contain any other point of P.

1. Prove that the Delaunay triangulation of P contains the Gabriel Graph.
2. Prove that p and q are connected by an edge of the Gabriel Graph of P if and only if the Delaunay edge between p and q intersects its dual Voronoi edge.
3. Give an $O(n \log n)$ time algorithm to compute the Gabriel graph using the previous parts of this question (here, $|P| = n$). (Assume, of course, that you are provided an $O(n \log n)$ time algorithm for Delaunay triangulation from the lecture notes)

Problem 2: (8 Points)

A *Euclidean Minimum Spanning Tree (EMST)* of a set of points in the plane is a tree of minimum total edge length connecting all the points. Not surprisingly, these have many applications.

1. Prove that the set of edges of a Delaunay triangulation of a point set P contain the EMST for P.
2. Using part 1. of this question, give an $O(n \log n)$ algorithm for EMST on the plane (here, $|P| = n$). (Assume, of course, that you are provided an $O(n \log n)$ time algorithm for Delaunay triangulation from the lecture notes)
3. Prove that $EMST \subseteq GG$. GG is the Gabriel Graph from the previous question.

Problem 3: (9 Points)

The farthest point Voronoi diagram of point set $P = p_1, \ldots, p_n$, divides the plane into cells in which the same point of P is the farthest. More precisely, define the farthest region for p_i to be $\{x \in R^2 : d(x, p_i) \geq d(x, p_j), \forall p_j \neq p_i\}$.

1. Prove that the farthest region for p_i is convex.
2. Characterize the sites with non-empty farthest Voronoi regions. Can the regions be bounded? (Give proofs!)
3. Sketch a lower bound for constructing the farthest-point Voronoi diagram (this is closely related to what you have already understood in the lectures).