

Algorithmic Game Theory

Summer Term 2025

Exercise Set 2

If you would like to present one of the solutions in class, please also send an email to rlehming@uni-bonn.de containing the **task** which you would like to present and in **which of the tutorials** you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be allocated on a first-come-first-served basis, so sending this email earlier than Tuesday evening is highly recommended.

Exercise 1:

Consider the bimatrix game *Battle of the Sexes* given in Example 3.3 of the third lecture.

- a) Compute a mixed Nash equilibrium by choosing probabilities for one player that will make the other player indifferent between his pure strategies (see Example 3.11).
- b) Determine the probabilities of the mixed Nash equilibrium graphically by plotting the players' expected costs.

Exercise 2:

We define a strategy $s_i \in S_i$ of a normal-form cost-minimization game to be *strictly dominated*, if there exists a strategy s'_i such that $c_i(s'_i, s_{-i}) < c_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$. Prove that for all mixed Nash equilibria σ , there is no player $i \in N$ with a mixed strategy σ_i such that $\sigma_{i,s_i} > 0$ for a strictly dominated strategy $s_i \in S_i$.

Exercise 3:

Have a look at the proof of Nash's Theorem (4.3) in which normal-form payoff-maximization games are considered. Let $N = \{1, \dots, n\}$ and $S_i = \{1, \dots, m_i\}$ for all $i \in N$. The set of mixed states X can be considered as a subset of \mathbb{R}^m with $m = \sum_{i=1}^n m_i$.

Show that X is convex and compact.