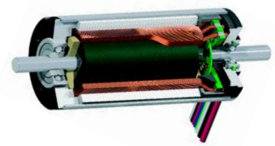
Grundlagen der Robotik

4. Kinematik

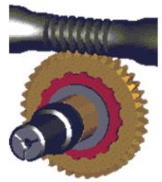
Prof. Sven Behnke

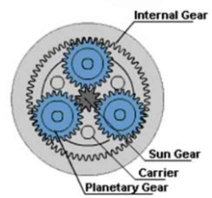
Letzte Vorlesung

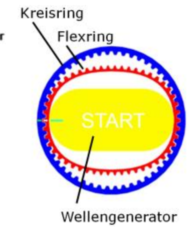
- Aktuatoren
  - Elektromotoren

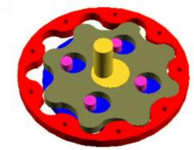


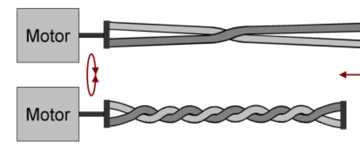
  - Getriebe







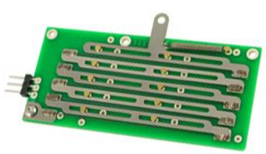


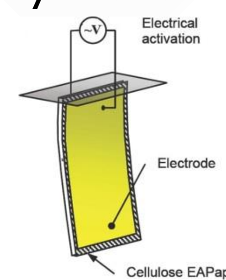


  - Servos

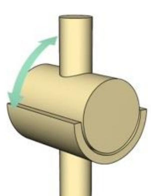
  - Pneumatik, Hydraulik

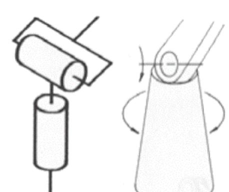
  - Piezo-Aktuatoren, Gedächtnismetalle, Elektroaktive Polymere





- Gelenke



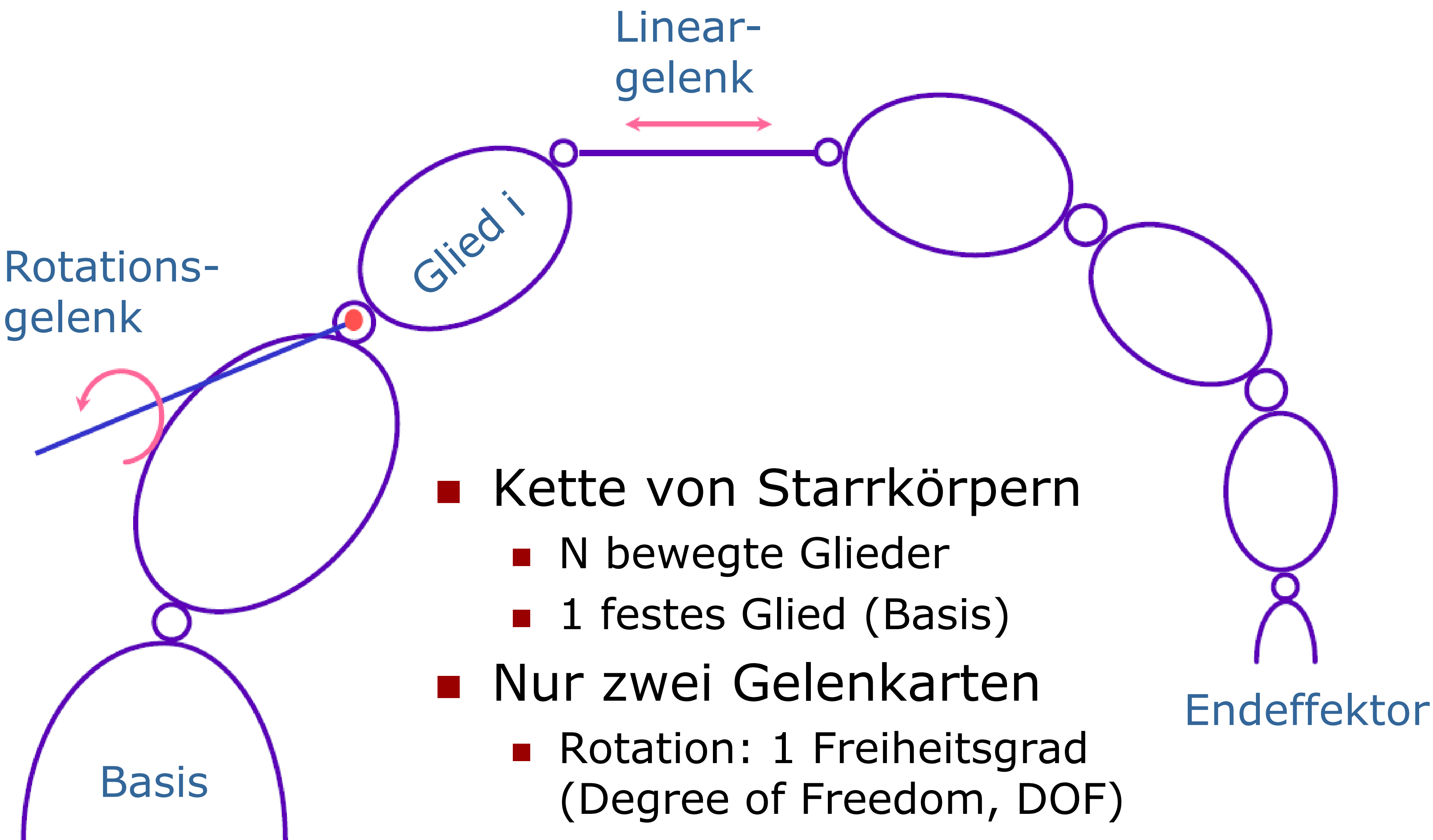


360° Roll –
180° Pitch

Kinematik

- Vorwärts
  - Gegeben: Gelenkwinkel, Robotermodell
  - Gesucht: Endeffektorpose
- Invers
  - Gegeben: Endeffektorpose, Robotermodell
  - Gesucht: Gelenkwinkel
- Nötig: Mathematisches Modell der Roboterbewegung

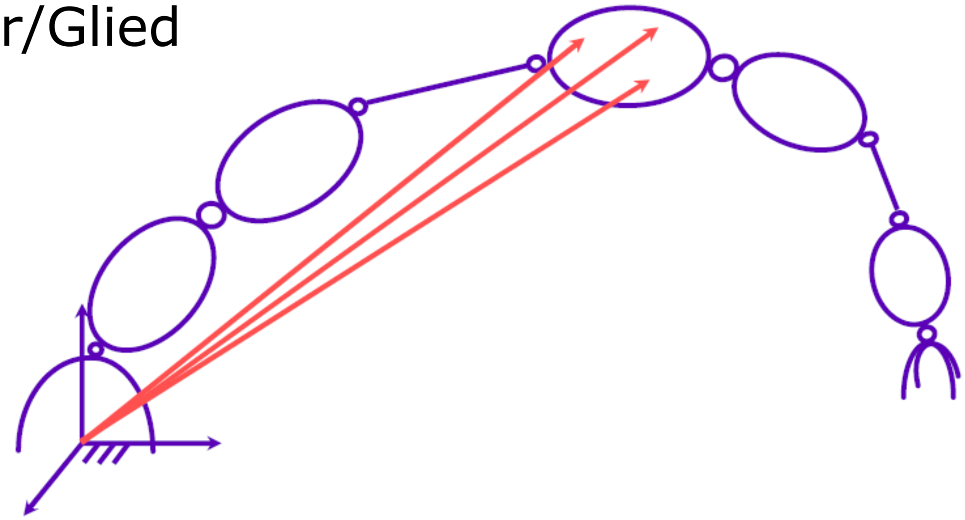
Manipulator als kinematische Kette von Gelenken



- Kette von Starrkörpern
  - N bewegte Glieder
  - 1 festes Glied (Basis)
- Nur zwei Gelenkarten
  - Rotation: 1 Freiheitsgrad (Degree of Freedom, DOF)
  - Linear: 1 DOF

Konfigurationsparameter

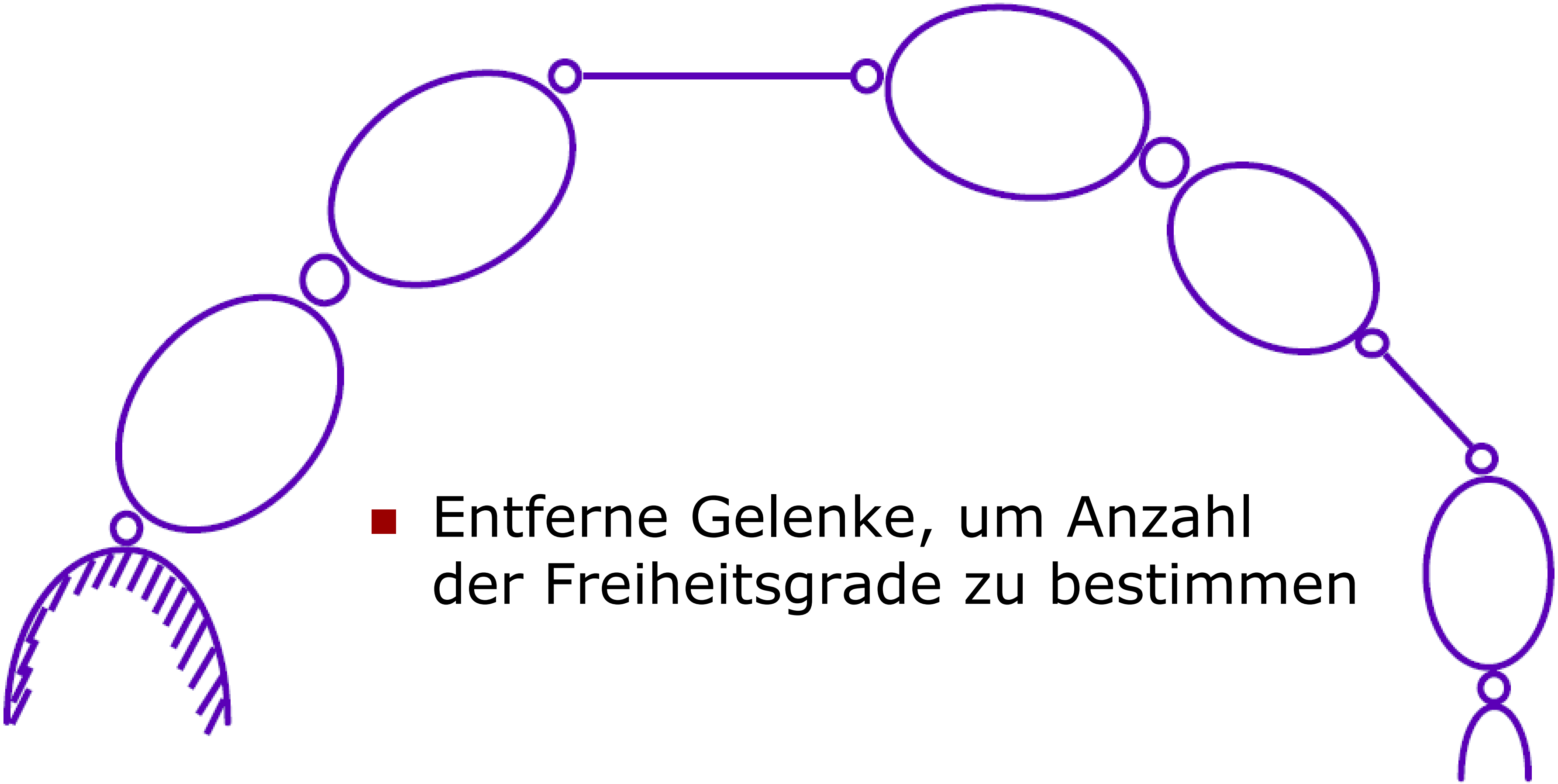
- **Konfigurationsparameter**: Menge von Positionspara-metern, welche die Konfiguration des Systems beschreiben
- Beispiel: Drei 3D-Punkte, um Lage eines Glied im Raum zu bestimmen: 9 Parameter/Glied



- **Generalisierte Koordinaten**:
  Eine Menge unabhängiger Konfigurationsparameter

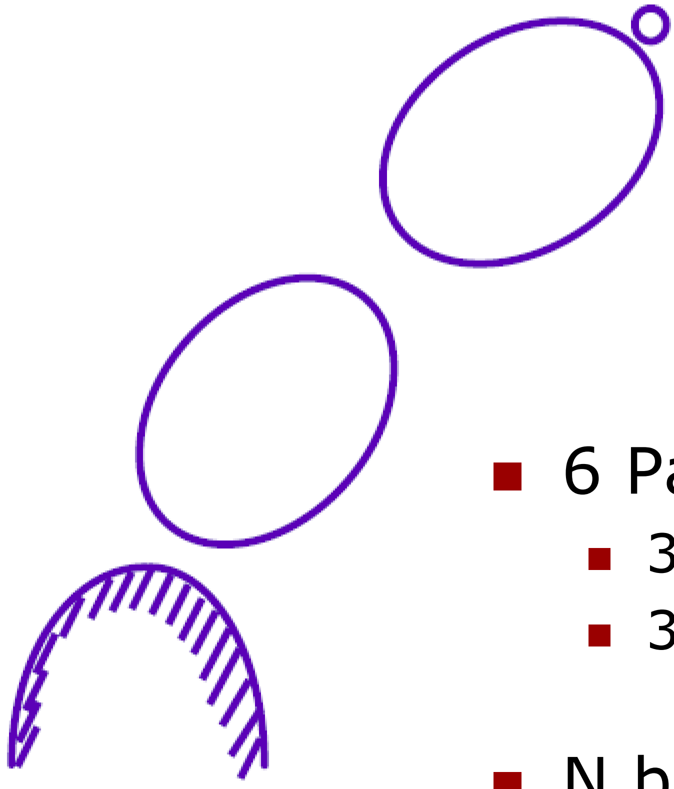
- **Freiheitsgrade** (Degrees of Freedom, DoF):
  Anzahl generalisierter Koordinaten

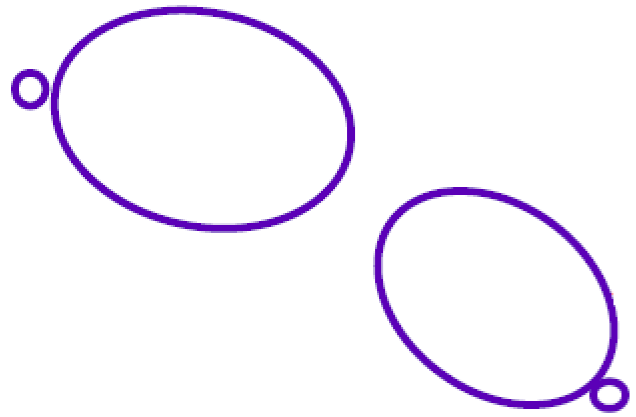
Generalisierte Koordinaten



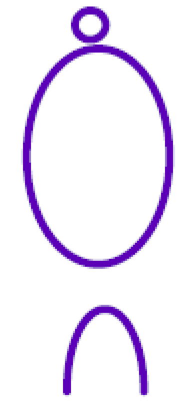
- Entferne Gelenke, um Anzahl der Freiheitsgrade zu bestimmen

Generalisierte Koordinaten

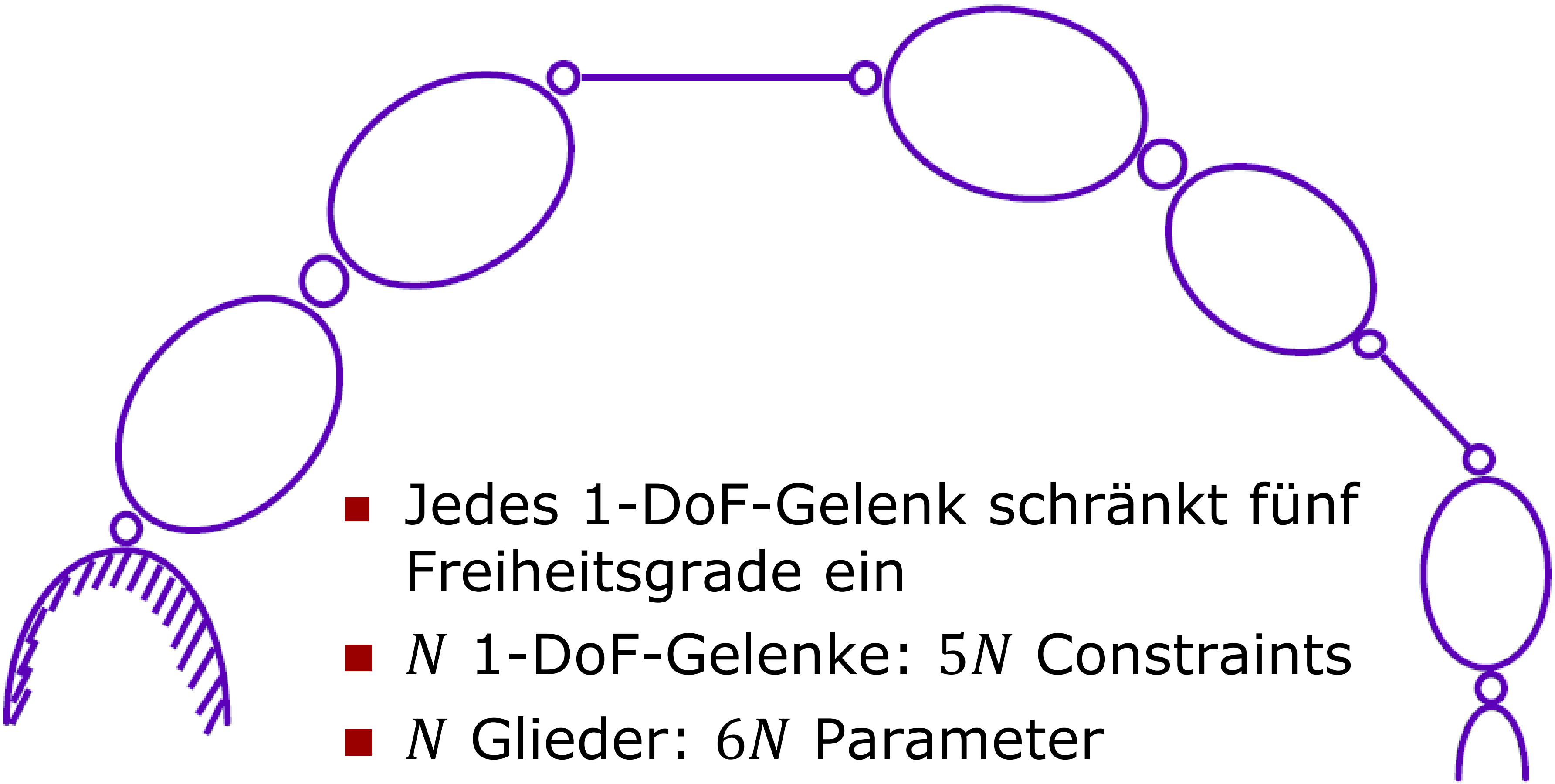




- 6 Parameter pro Glied
  - 3 Positionskoordinaten
  - 3 Orientierungen
- N bewegte Glieder
  => 6N Parameter



Generalisierte Koordinaten

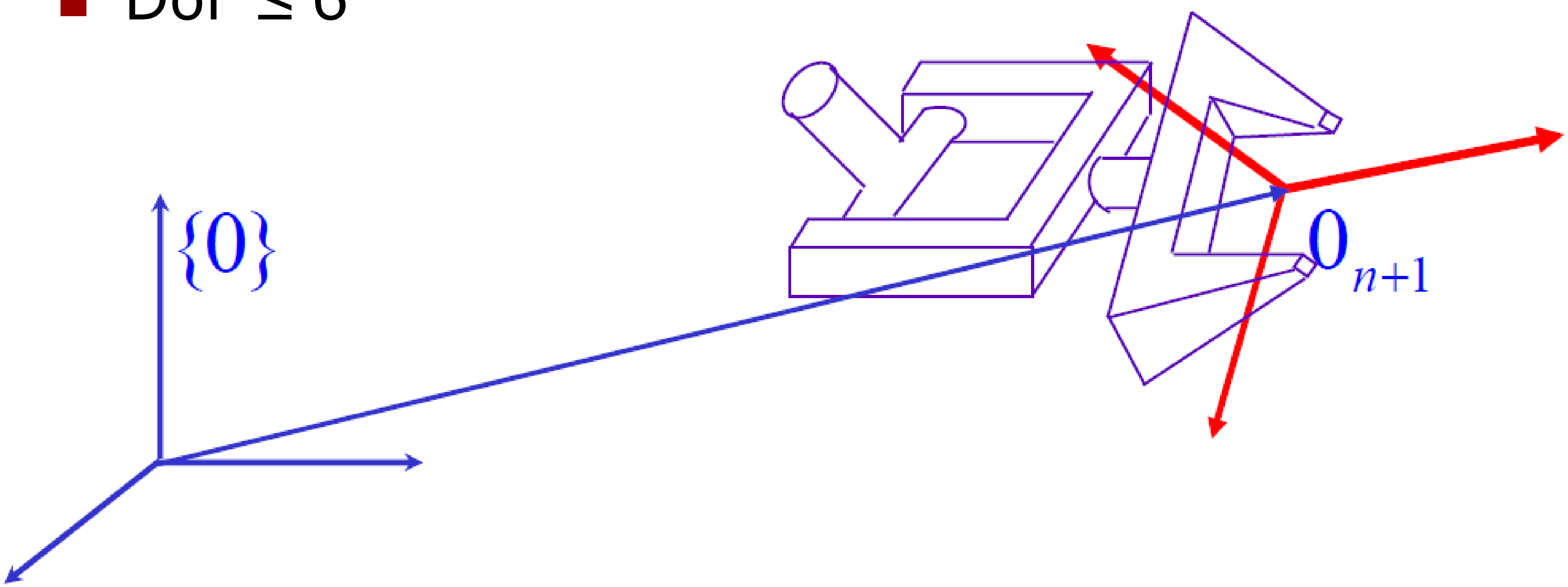


- Jedes 1-DoF-Gelenk schränkt fünf Freiheitsgrade ein
- N 1-DoF-Gelenke: $5N$ Constraints
- N Glieder: $6N$ Parameter

=> N Freiheitsgrade (DoF)

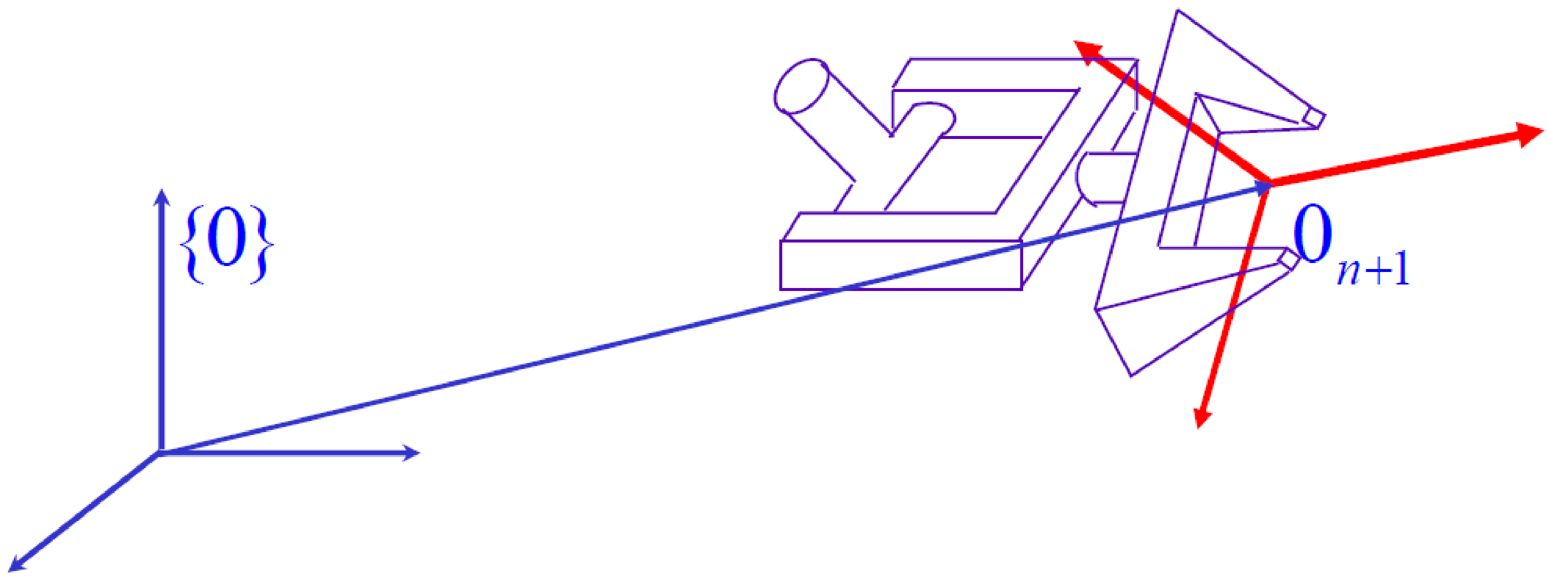
Endeffektor-Konfigurationsparameter

- Eine Menge von m Parametern $(x_1, x_2, \ldots, x_m)$, welche die Endeffektorposition und –orientierung vollständig beschreiben
- DoF ≤ 6



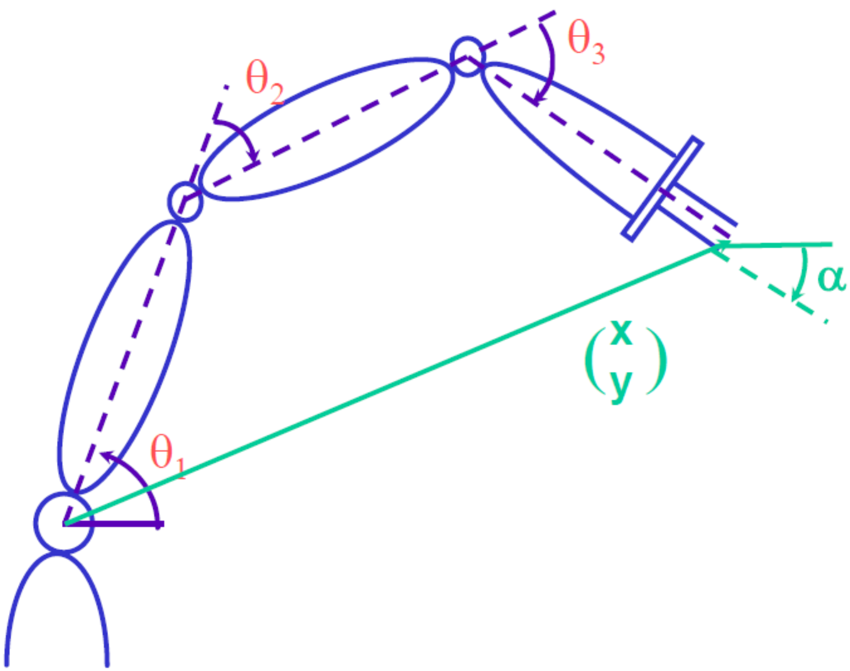
Arbeits-Koordinaten

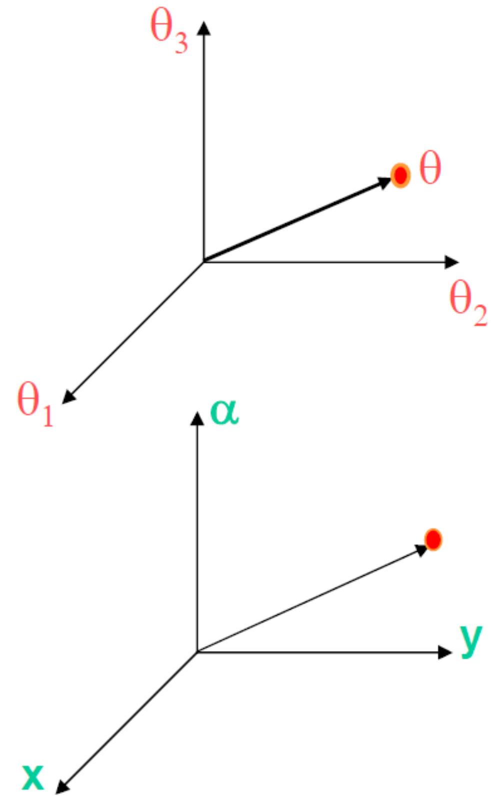
- Eine Menge von M **unabhängigen** Parameter $(x_1, x_2, \ldots, x_M)$, welche die Endeffektorposition und –orientierung vollständig beschreiben
- *M: Freiheitsgrade des Endeffektors*



Gelenkraum vs. Arbeitsraum

- Gelenkstellungen -> Konfigurationsraum





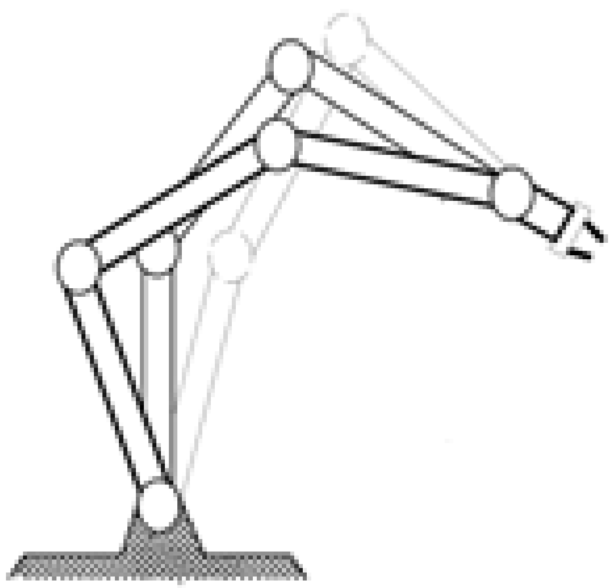
- Arbeits-Koordinaten -> Arbeitsraum

Redundanz

- Ein Manipulator wird redundant genannt, wenn die Anzahl der Gelenke N größer ist als die Anzahl der Freiheitsgrade M des Endeffektors

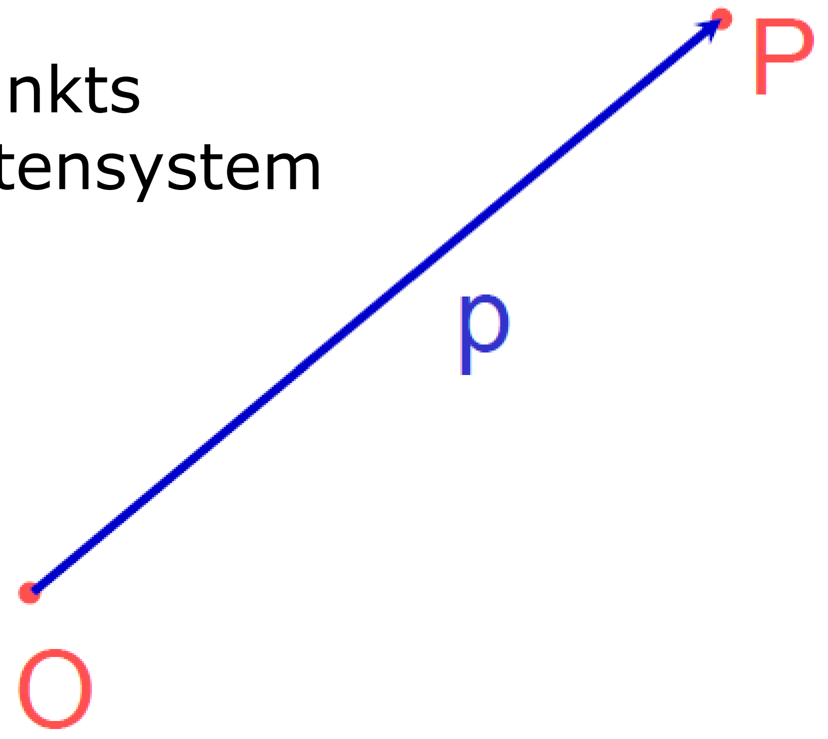
$$N > M$$

- Redundante Freiheitsgrade:
  $N - M$ *(Dimensionalität des Nullraums)*



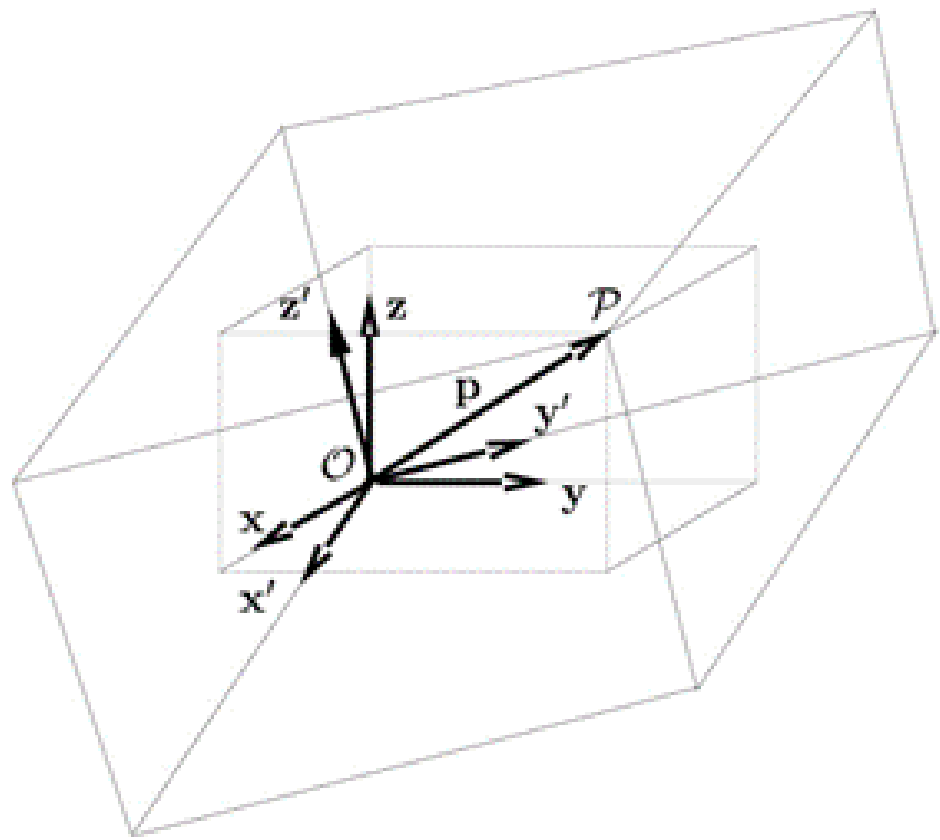
Position eines Punkts

- Bezüglich eines festen Ursprungs O wird die Position eines Punkts P durch den Vektor $\overrightarrow{OP}$ beschrieben, Kurzschreibweise: $\boldsymbol{p}$
- Beschreibung des Punkts hängt vom Koordinatensystem ab



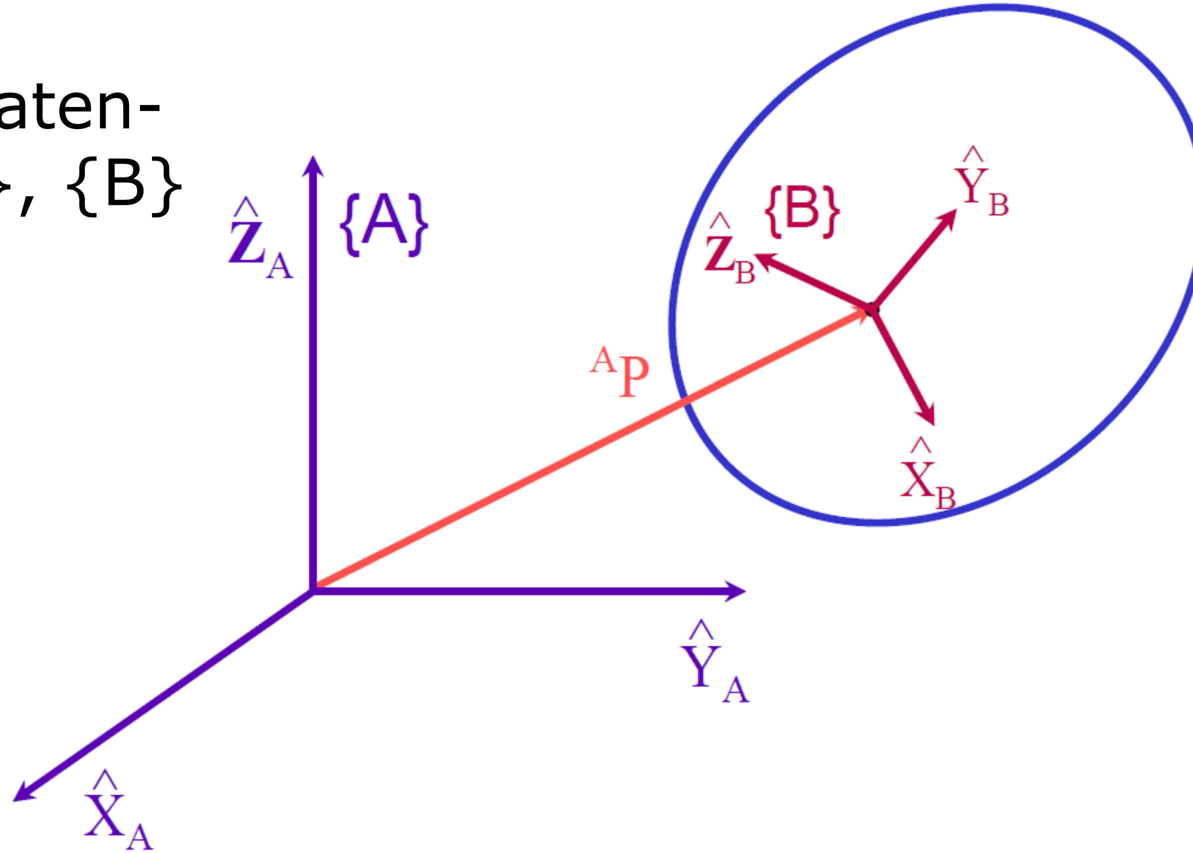
Koordinatensysteme

- Je nach Rotation des Koordinatensystems ist der Punkt P durch andere Koordinaten (x, y, z) beschrieben



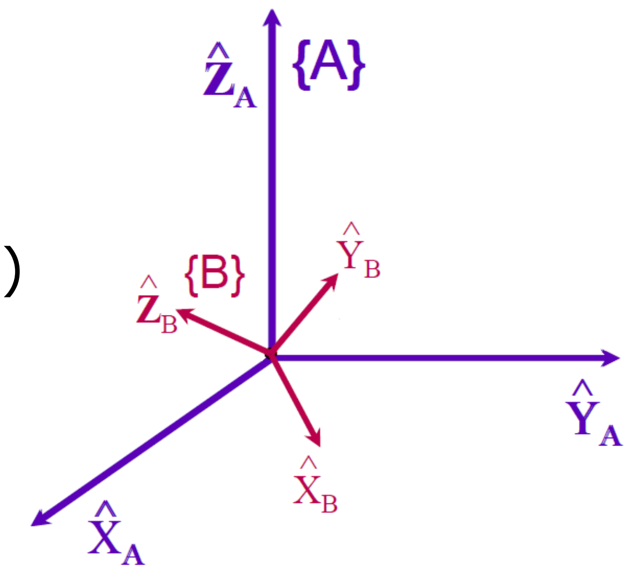
Konfiguration von Starrkörpern

- Zwei Koordinaten-systeme: {A}, {B}



- Position ${}^A P$ des Ursprungs von {B} in {A} beschreibt Translation
- Orientierungen $\left\{ {}^A\hat{X}_B, {}^A\hat{Y}_B, {}^A\hat{Z}_B, \right\}$ der Achsen des Systems {B} im System {A} beschreiben Rotation

Rotationsmatrix



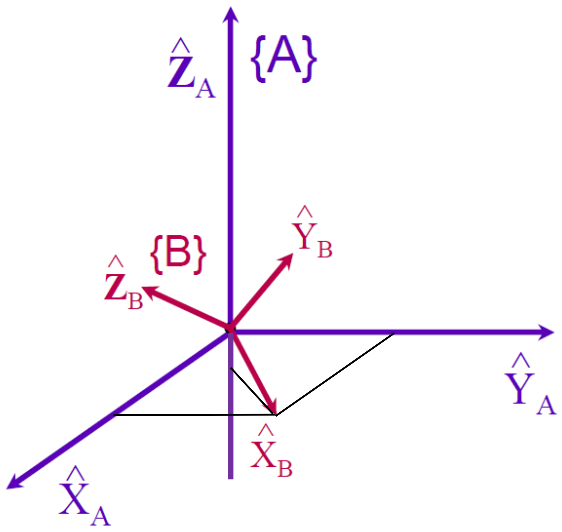
- Zunächst keine Verschiebung (${}^{A}P = 0$)
- Rotationsmatrix: $${}^{A}_{B}R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$
- Transformation von Koordinatensystem {B} in Koordinatensystem {A}: $${}^{A}\hat{X}_B = {}^{A}_{B}R \; {}^{B}\hat{X}_B$$
- Betrachte die Achsen in {B}:

$${}^{A}\hat{X}_B = {}^{A}_{B}R \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad {}^{A}\hat{Y}_B = {}^{A}_{B}R \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad {}^{A}\hat{Z}_B = {}^{A}_{B}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Jeweils eine Spalte von ${}^{A}_{B}R$ wird ausgewählt

- Rotationsmatrix hat als Spalten die Beschreibung der Achsen von {B} in Frame A $${}^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{X}_B & {}^{A}\hat{Y}_B & {}^{A}\hat{Z}_B \end{bmatrix}$$

Rotationsmatrix II



- Rotationsmatrix: $${}^{A}_{B}R = \left[{}^{A}\hat{X}_B \quad {}^{A}\hat{Y}_B \quad {}^{A}\hat{Z}_B \right]$$

- Berechnung als Skalarprodukt der Achsen von B mit den Achsen von A:

$${}^{A}\hat{X}_B = \begin{bmatrix} \hat{X}_B . \hat{X}_A \\ \hat{X}_B . \hat{Y}_A \\ \hat{X}_B . \hat{Z}_A \end{bmatrix}$$

- Rotationsmatrix hat Zeilen, die Achsen von A in B beschreiben

$${}^{A}_{B}R = \begin{bmatrix} \hat{X}_B . \hat{X}_A & \hat{Y}_B . \hat{X}_A & \hat{Z}_B . \hat{X}_A \\ \hat{X}_B . \hat{Y}_A & \hat{Y}_B . \hat{Y}_A & \hat{Z}_B . \hat{Y}_A \\ \hat{X}_B . \hat{Z}_A & \hat{Y}_B . \hat{Z}_A & \hat{Z}_B . \hat{Z}_A \end{bmatrix} \begin{array}{l} {}^{B}X_A^{T} \\ \\ \\ \end{array}$$

Inverse Rotationsmatrix

- Betrachte Rotationsmatrizen beider Richtungen:

$$ {}^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{X}_{B} & {}^{A}\hat{Y}_{B} & {}^{A}\hat{Z}_{B} \end{bmatrix} = \begin{bmatrix} {}^{B}\hat{X}_{A}^{T} \\ {}^{B}\hat{Y}_{A}^{T} \\ {}^{B}\hat{Z}_{A}^{T} \end{bmatrix} = \begin{bmatrix} {}^{B}\hat{X}_{A} & {}^{B}\hat{Y}_{A} & {}^{B}\hat{Z}_{A} \end{bmatrix}^{T} = {}^{B}_{A}R^{T} $$

- Diese sind Transponierte voneinander:

$$ {}^{A}_{B}R = {}^{B}_{A}R^{T} $$

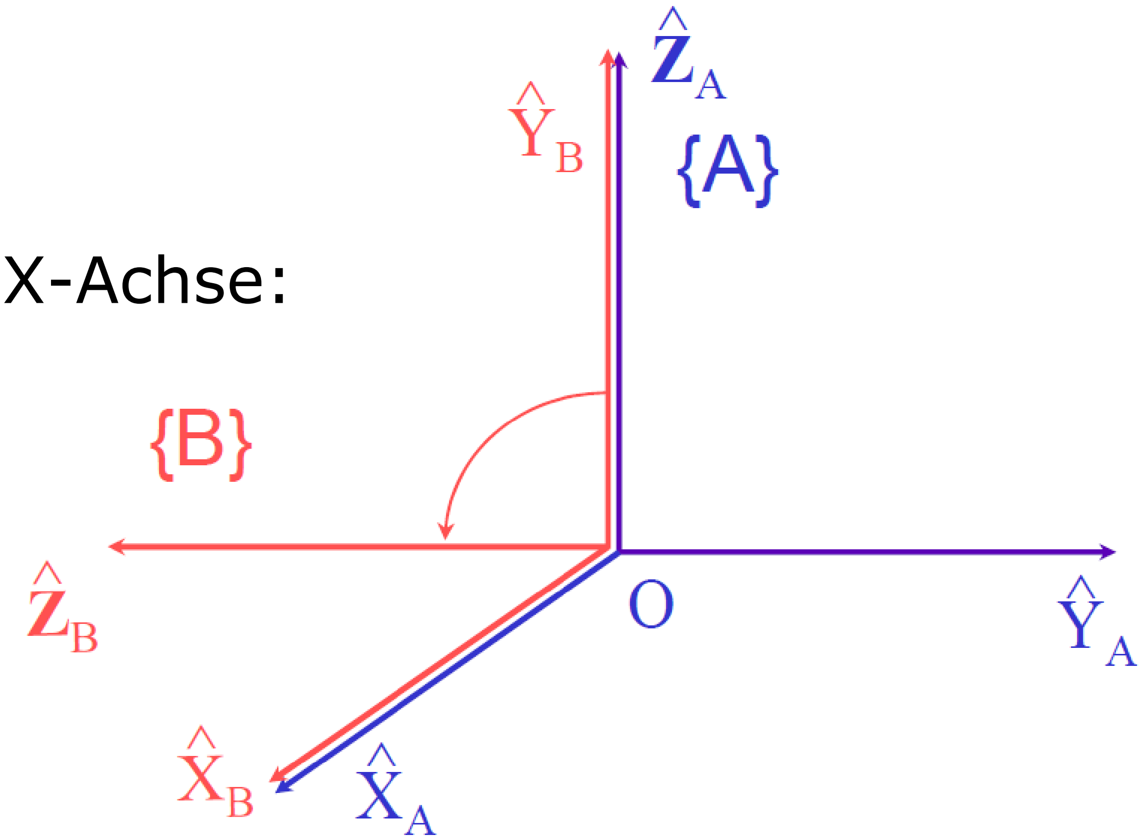
- Inverse Rotationsmatrix: $\;{}^{A}_{B}R^{-1} = {}^{B}_{A}R = {}^{A}_{B}R^{T}$

$$ \boxed{{}^{A}_{B}R^{-1} = {}^{A}_{B}R^{T}} $$

- Die Rotationsmatrizen sind orthonormal.

Beispiel

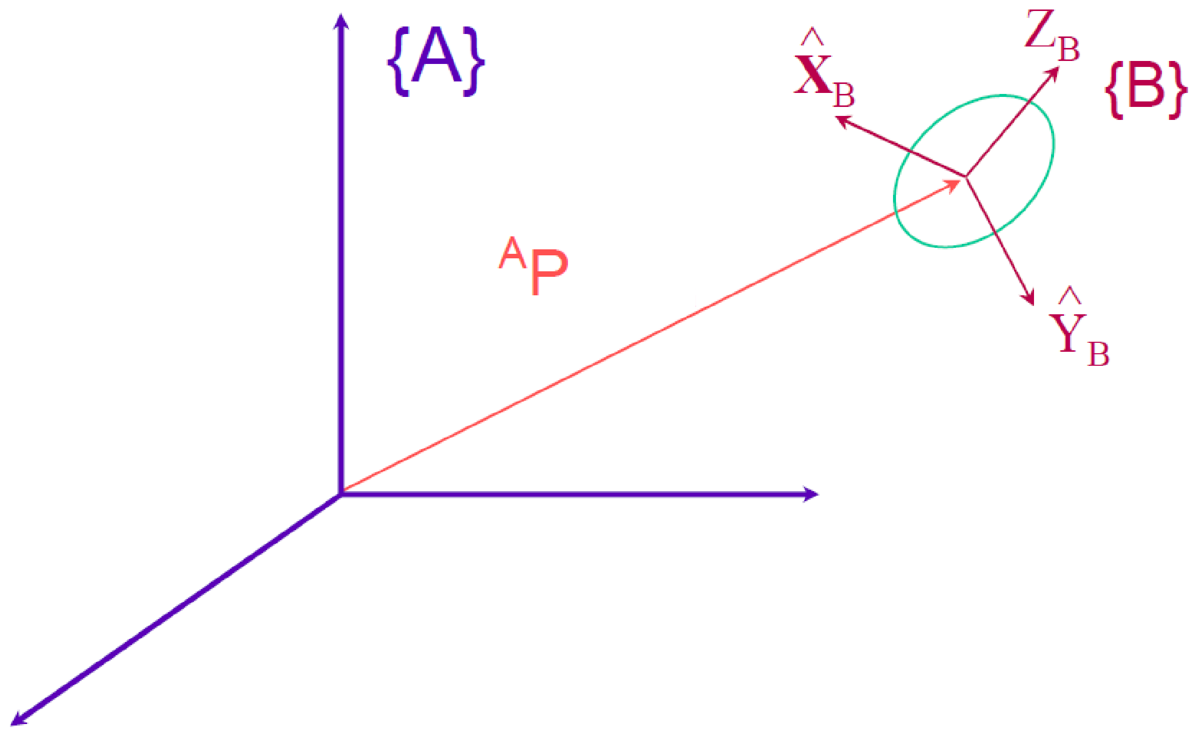
- 90° Rotation um X-Achse:



- Rotationsmatrix:

$$
\begin{array}{cc}
{}^{A}_{B}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} & \begin{array}{l} \leftarrow {}^{B}\hat{X}_A^T \\ \leftarrow {}^{B}\hat{Y}_A^T \\ \leftarrow {}^{B}\hat{Z}_A^T \end{array} \\
\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ {}^{A}\hat{X}_B & {}^{A}\hat{Y}_B & {}^{A}\hat{Z}_B \end{array} &
\end{array}
$$

Beschreibung eines Koordinatensystems



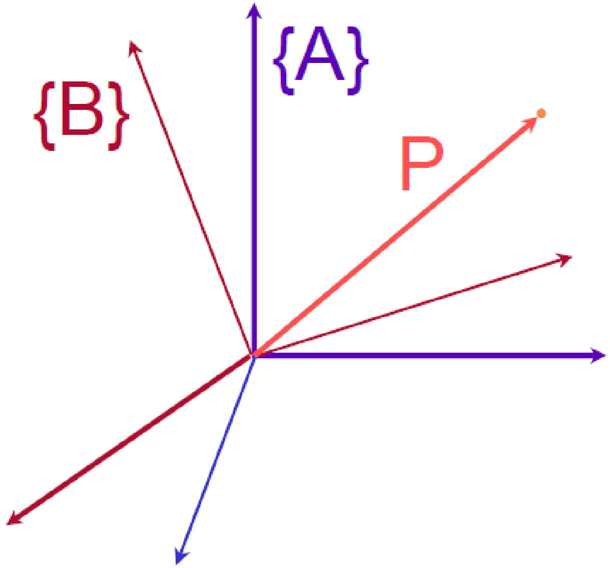
- Rotation und Translation bezüglich A:

Frame {B}: ${}^{A}\hat{X}_{B}, {}^{A}\hat{Y}_{B}, {}^{A}\hat{Z}_{B}, {}^{A}P$

$$\{B\} = \left\{ {}^{A}_{B}R \quad {}^{A}P \right\}$$

Mapping: Wechsel des Koordinatensystems

- Nur Rotation:

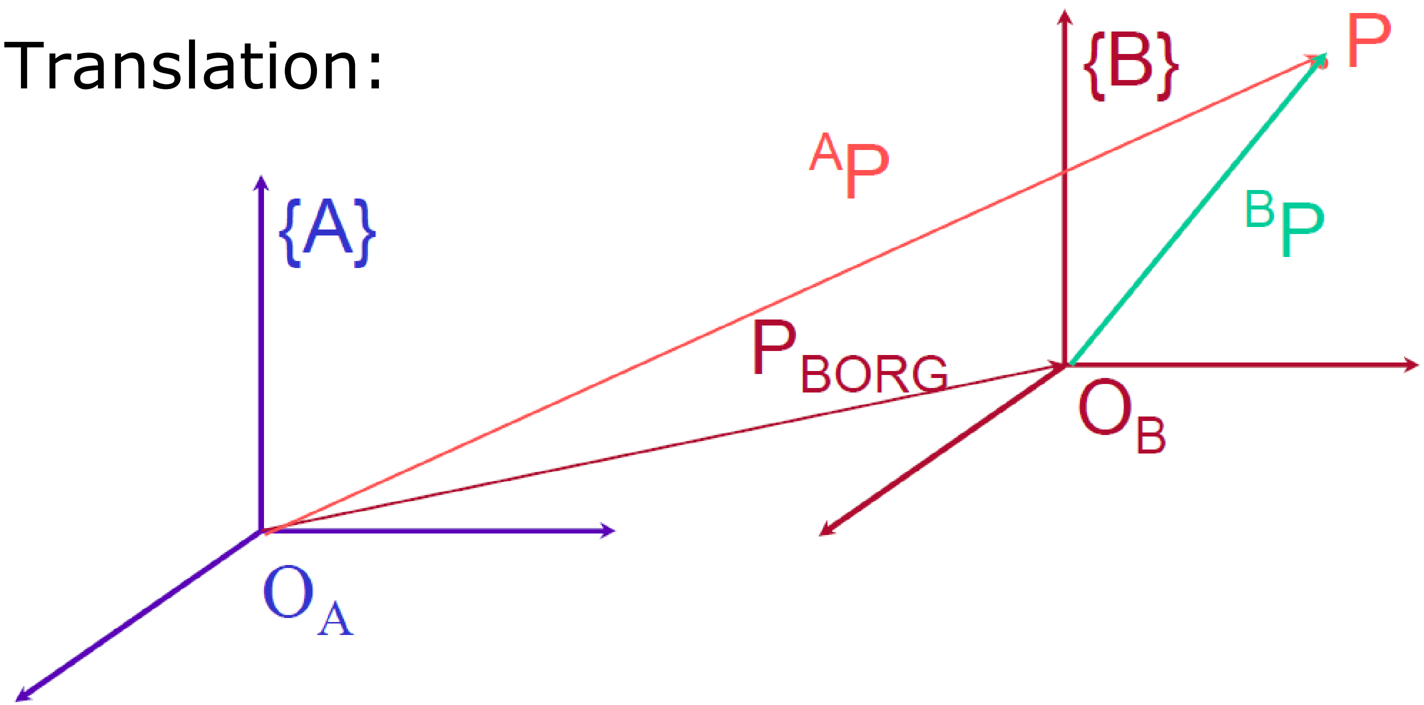


- Wenn P in {B} gegeben ist: ${}^{B}P$

$$ {}^{A}P = \begin{pmatrix} {}^{B}\hat{X}_{A} \cdot {}^{B}P \\ {}^{B}\hat{Y}_{A} \cdot {}^{B}P \\ {}^{B}\hat{Z}_{A} \cdot {}^{B}P \end{pmatrix} = \begin{pmatrix} {}^{B}\hat{X}_{A}^{T} \\ {}^{B}\hat{Y}_{A}^{T} \\ {}^{B}\hat{Z}_{A}^{T} \end{pmatrix} \cdot {}^{B}P \quad \Rightarrow \quad {}^{A}P = {}^{A}_{B}R \; {}^{B}P $$

Mapping: Translation

- Nur Translation:



- Wechsel der Beschreibung eines Punkts P:

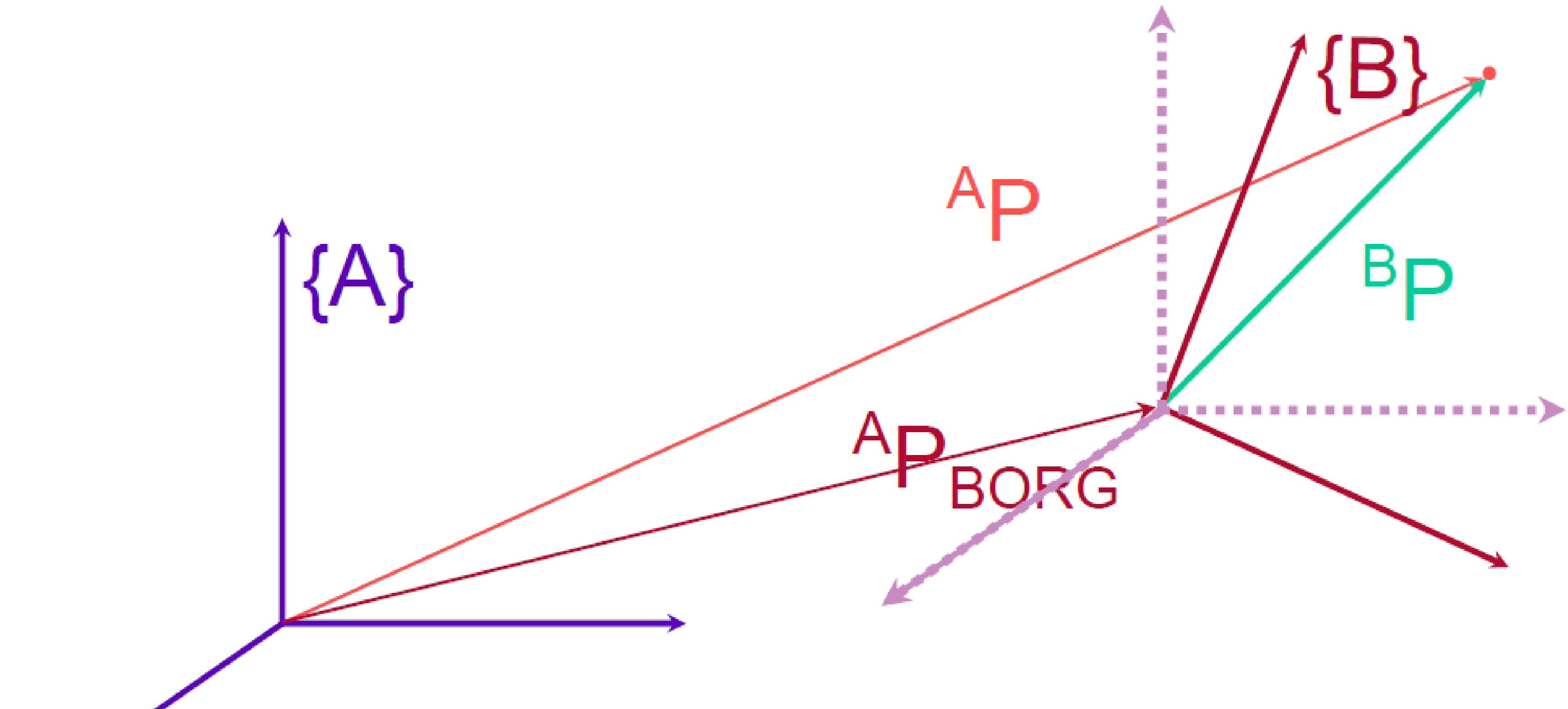
$$\overrightarrow{O_BP} \Rightarrow \overrightarrow{O_AP}$$

$$P_{O_B} \Rightarrow P_{O_A}$$

$$\boxed{{}^AP = {}^BP + {}^AP_{BORG}}$$

Allgemeine Abbildung

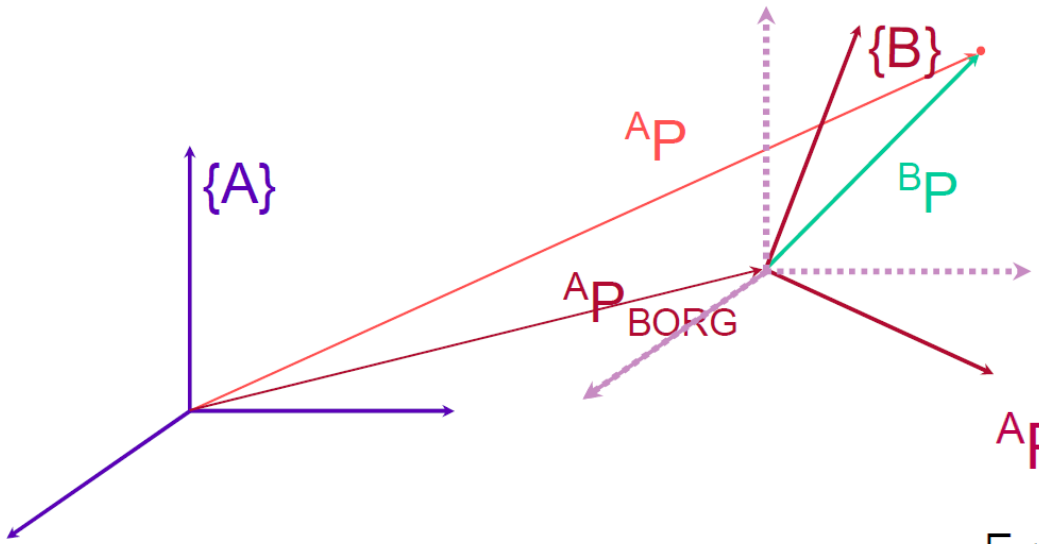
- Rotation und Translation



$$^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG}$$

Homogene Transformation

- Problem: Aneinanderreihung allgemeiner Transformationen wird unschön



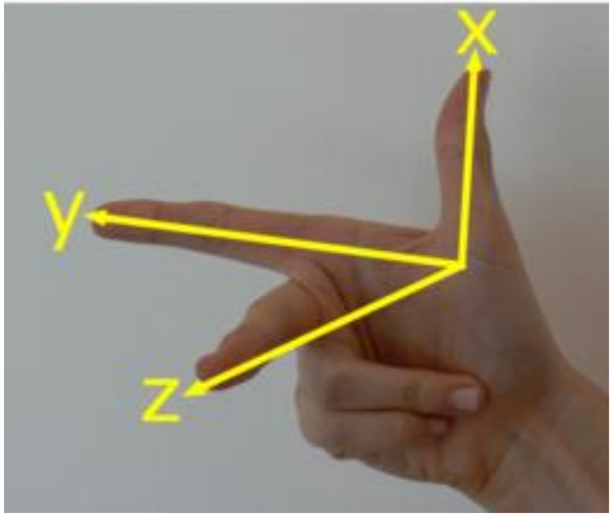
$$^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG}$$

- Idee: Homogene Koordinaten $\left[\begin{array}{c} {}^{A}P \\ \hline 1 \end{array}\right] = \left[\begin{array}{ccc|c} & {}^{A}_{B}R & & {}^{A}P_{Borg} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} {}^{B}P \\ \hline 1 \end{array}\right]$

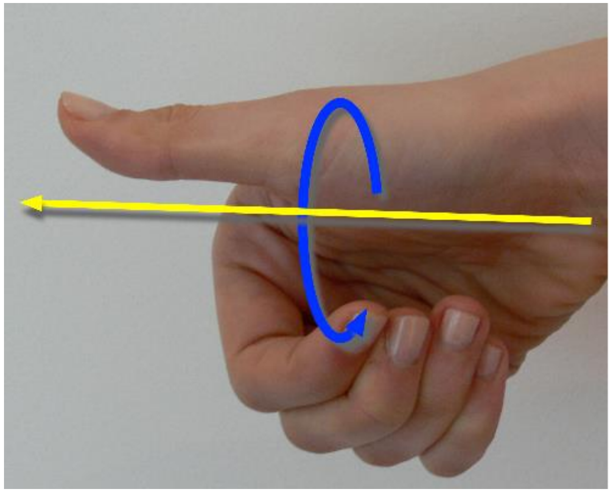
- Transformation durch Matrix: $\underset{(4x1)}{{}^{A}P} = \underset{(4x4)}{{}^{A}_{B}T}\ \underset{(4x1)}{{}^{B}P}$

Rechte-Hand-Regeln

- Rechtshändiges Koordinatensystem (Finger zeigen Koordinatenachsen)



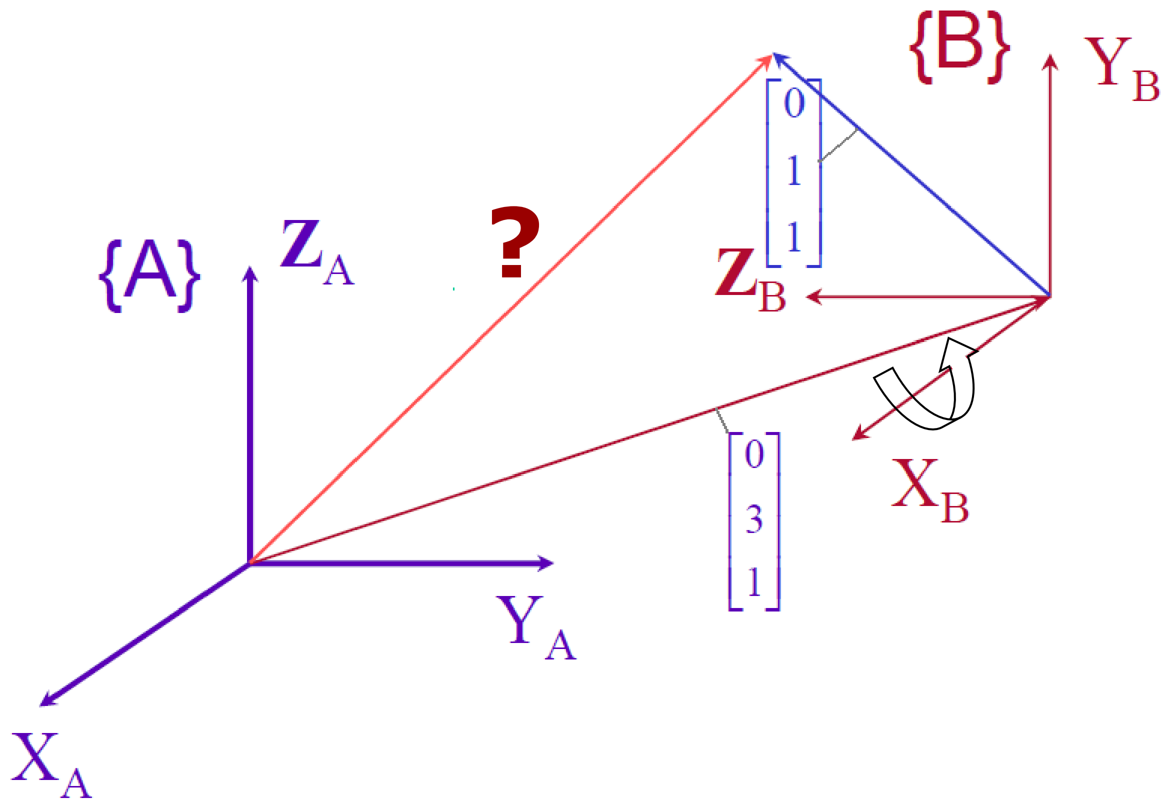
- Rotation um einen Vektor (Vier Finger zeigen positive Drehrichtung)



[Bildquelle: Corke RVC]

Beispiel

- 90° Rotation um X und Translation [0 3 1]



- Homogene Transformation

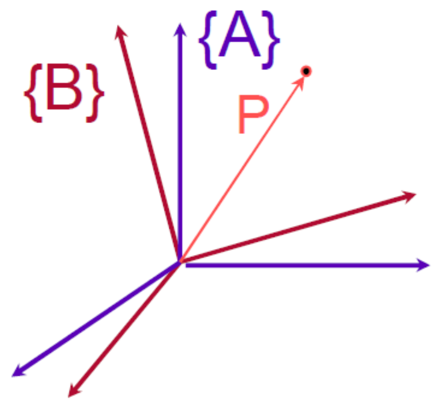
$$
{}^{A}_{B}T = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad {}^{B}P = \left[\begin{array}{c} 0 \\ 1 \\ 1 \\ \hline 1 \end{array}\right]
$$

$$
{}^{A}P = {}^{A}_{B}T\ {}^{B}P
$$

$$
\Rightarrow \quad {}^{A}P = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}
$$

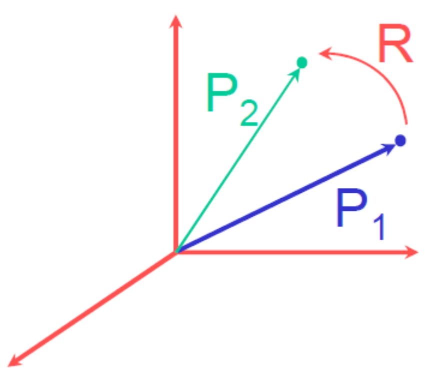
Rotation: Mapping vs. Operator

- **Mapping**: Wechsel des Koordinatensystems



$$ {}^{A}P = {}^{A}_{B}R \; {}^{B}P $$

- **Operator**: Bewegung der Punkte
  (im selben Koordinatensystem)



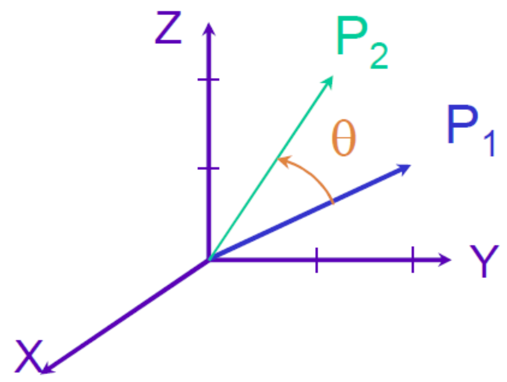
$$ R\colon\; P_1 \longrightarrow P_2 $$

$$ \boxed{P_2 = R \; P_1} $$

Rotation um eine Achse

- Rotation um Achse K mit Winkel θ:

$$R_K(\theta): \quad P_1 \longrightarrow P_2 \qquad\qquad P_2 = R_K(\theta)\, P_1$$



- Beispiel: Rotation um X-Achse

$$R_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$P_2 = R_X(\theta) P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8 & -0.6 \\ 0 & 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

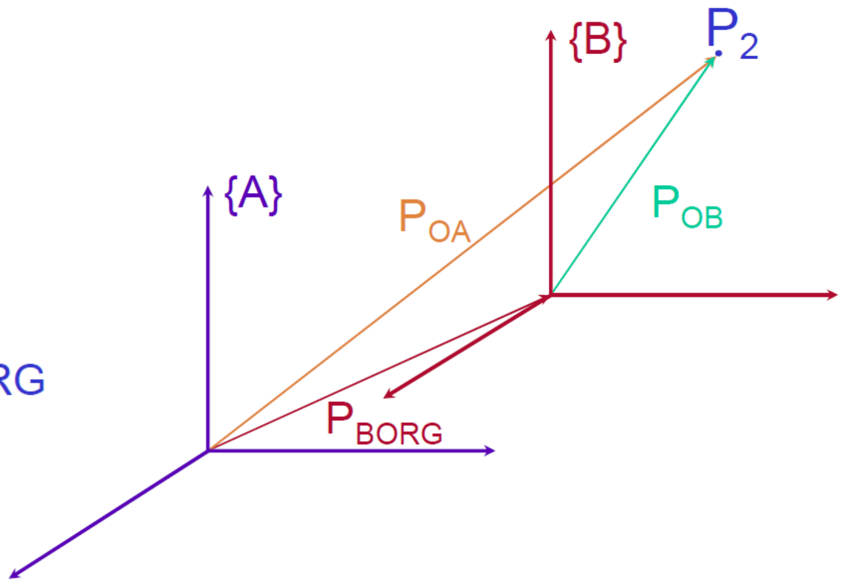
Θ=36,87°

Translation: Mapping vs. Operator

- Mapping:

$$P_{BORG} : \quad P_{OB} \longrightarrow P_{OA}$$

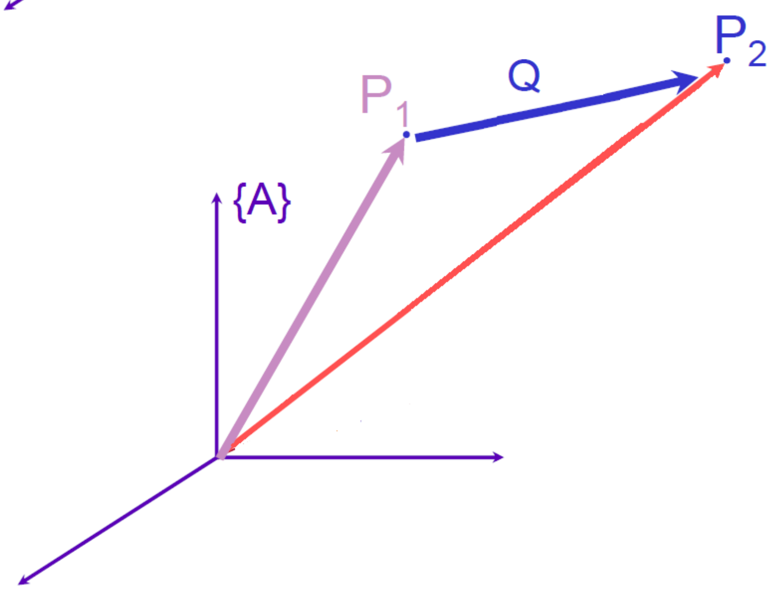
$$P_{OA} = P_{OB} + P_{BORG}$$



- Operator:

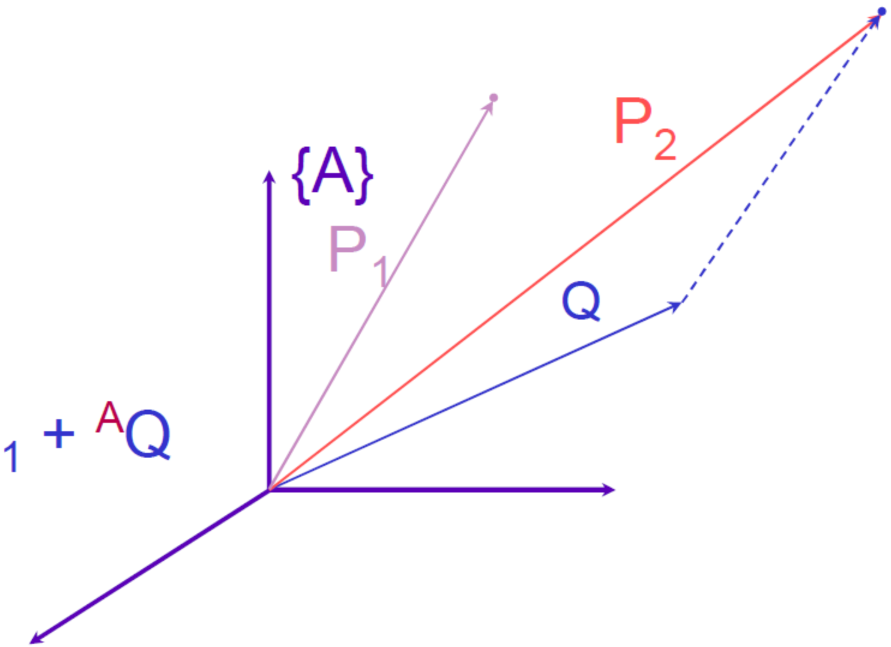
$$Q : \quad P_1 \longrightarrow P_2$$

$$P_2 = P_1 + Q$$



Translationsoperator

- Operator:



$$^{A}P_2 = {}^{A}P_1 + {}^{A}Q$$

- Homogene Transformation:

$$D_Q = \begin{bmatrix} 1 & 0 & 0 & q_x \\ 0 & 1 & 0 & q_y \\ 0 & 0 & 1 & q_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow {}^{A}P_2 = {}^{A}D_Q {}^{A}P_1$$

Allgemeiner Operator

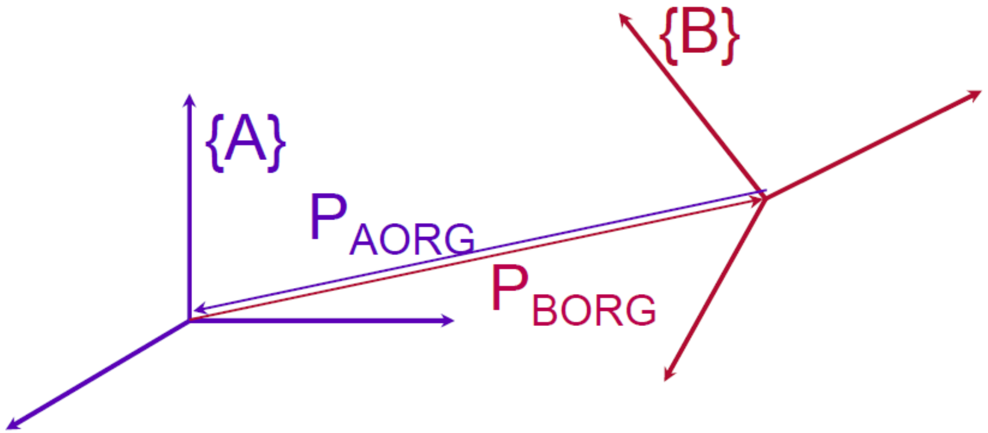
- In homogener Form

$$P_2 = \underbrace{\left(\begin{array}{c|c} R_K(\theta) & Q \\ \hline 0\ 0\ 0 & 1 \end{array}\right)} P_1$$

- Transformationsmatrix T:

$$\boxed{P_2 = T P_1}$$

Inverse Transformation



- Hin-Transformation:

$$ {}^{A}_{B}T = \begin{bmatrix} & {}^{A}_{B}R & & {}^{A}P_{Borg} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

- Rücktransformation:

$R^{-1} = R^{T}$, aber $T^{-1} \neq T^{T}$

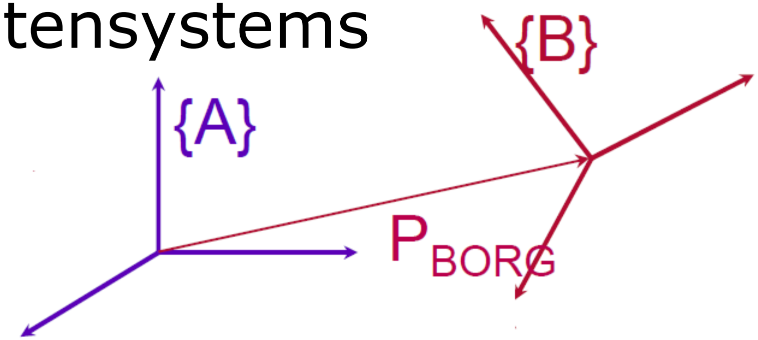
$$ {}^{A}_{B}T^{-1} = {}^{B}_{A}T = \begin{bmatrix} & {}^{A}_{B}R^{T} & & -{}^{A}_{B}R^{T} \cdot {}^{A}P_{Borg} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

${}^{B}P_{AORG}$

Interpretation Homogener Transformationen

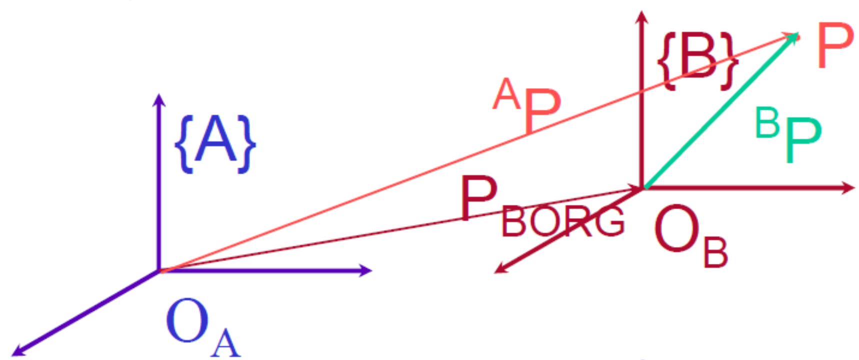
- Beschreibung des Koordinatensystems

$$\{B\} = \left\{ {}^{A}_{B}R \quad {}^{A}P_{Borg} \right\}$$



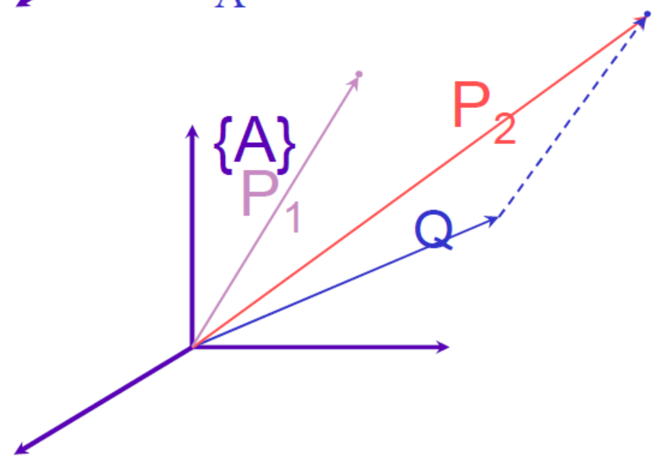
- Mapping von B nach A

$${}^{A}_{B}T : \quad {}^{B}P \rightarrow {}^{A}P$$

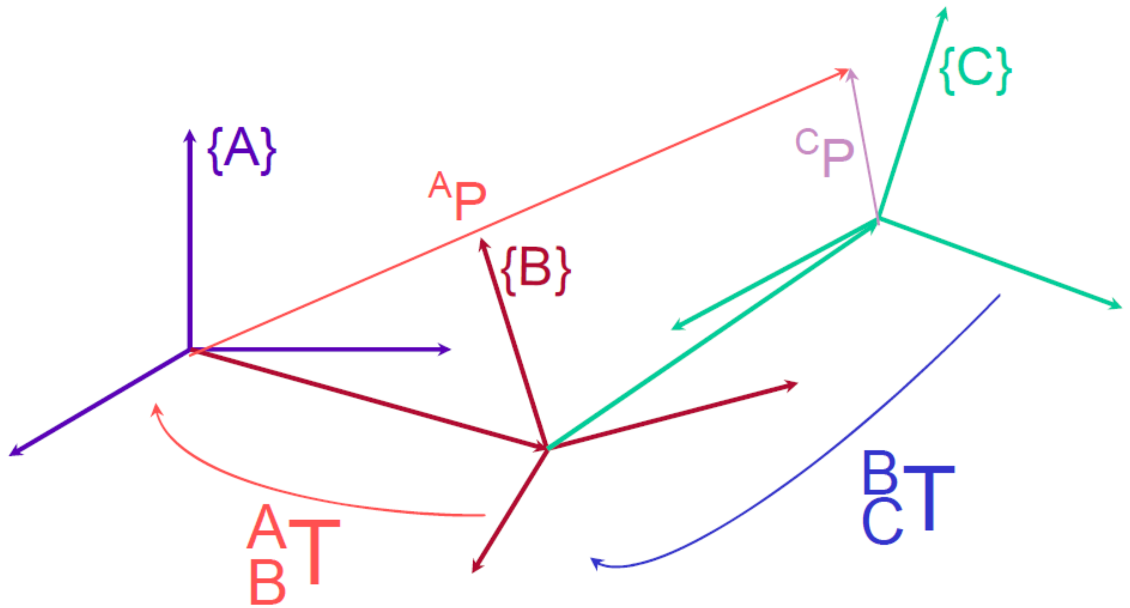


- Operator

$$T : \quad P_1 \rightarrow P_2$$



Transformationskette



$$^{A}P = {}^{A}_{B}T\ {}^{B}P \qquad {}^{B}P = {}^{B}_{C}T\ {}^{C}P$$

$$^{A}P = {}^{A}_{B}T\ {}^{B}_{C}T\ {}^{C}P$$

- Multipliziere Transformationsmatrizen

$$\Rightarrow \quad {}^{A}_{C}T = {}^{A}_{B}T\ {}^{B}_{C}T$$

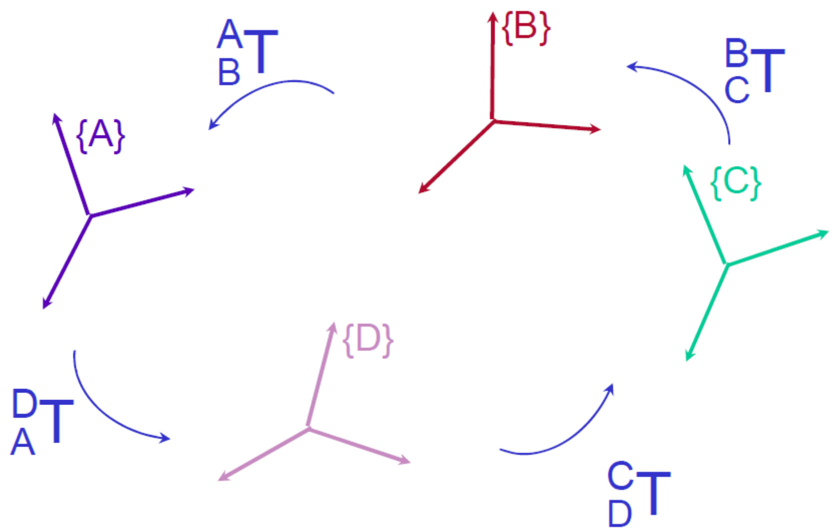
Resultierende Transformation

$$ {}^{A}_{C}T = {}^{A}_{B}T \, {}^{B}_{C}T $$

$$ {}^{A}_{C}T = \left[\begin{array}{ccc|c} & {}^{A}_{B}R \, {}^{B}_{C}R & & {}^{A}_{B}R \, {}^{B}P_{Corg} + {}^{A}P_{Borg} \\ \hline 0 & 0 & 0 & 1 \end{array} \right] $$

Transformationsgleichung

- Zyklische Transformationskette:



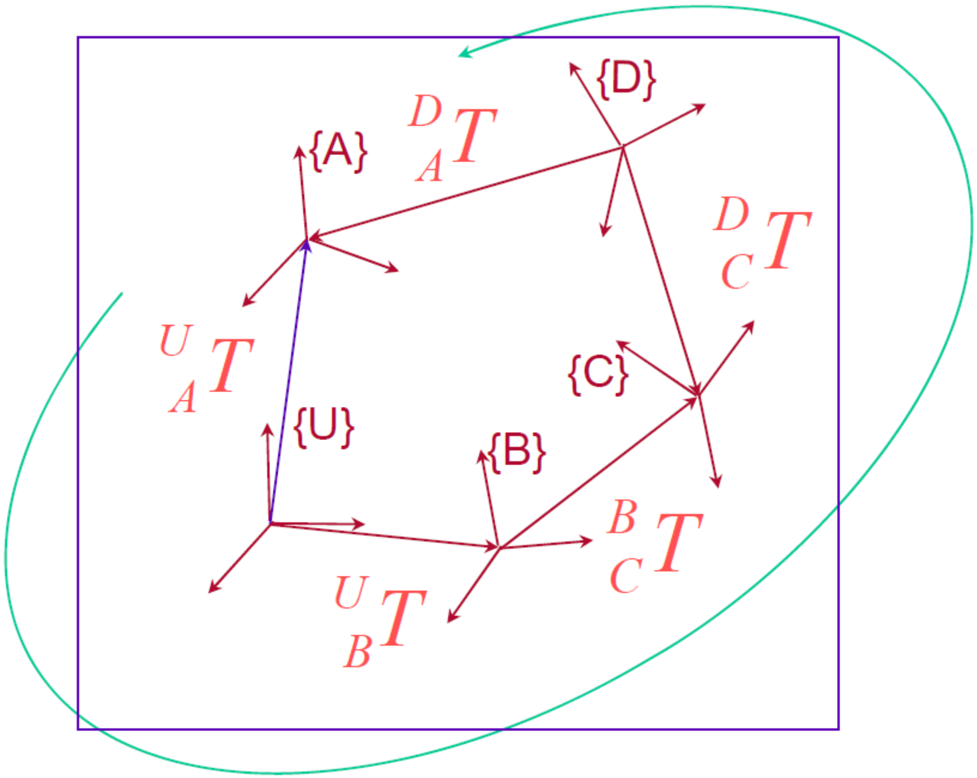
- Produkt ist Identität: $\boxed{{}^{A}_{B}T\,{}^{B}_{C}T\,{}^{C}_{D}T\,{}^{D}_{A}T = I}$

- Wir können jede fehlende Transformation berechnen:

$$\Rightarrow \quad {}^{B}_{A}T = {}^{B}_{C}T\,{}^{C}_{D}T\,{}^{D}_{A}T$$

Unterschiedliche Richtungen

- Lege Richtung fest:



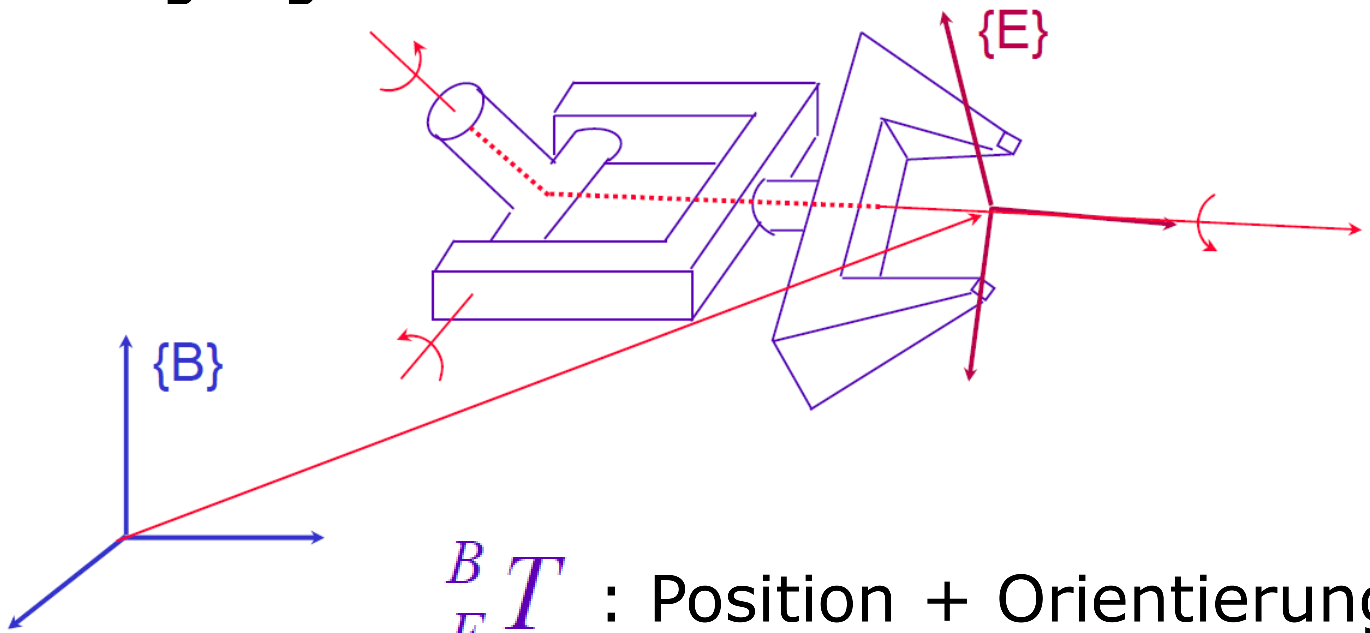
- Multipliziere mit inverser Transformation, wenn Richtung falsch:

$$ {}^{D}_{A}T^{-1} \cdot {}^{D}_{C}T \cdot {}^{B}_{C}T^{-1} \cdot {}^{U}_{B}T^{-1} \cdot {}^{U}_{A}T \equiv I $$

- Berechne Unbekannte: ${}^{U}_{A}T = {}^{U}_{B}T \cdot {}^{B}_{C}T \cdot {}^{D}_{C}T^{-1} \cdot {}^{D}_{A}T$

Endeffektor-Konfiguration

- Bewegung relativ zu Basis



$${}^{B}_{E}T$$: Position + Orientierung

- Endeffektor-Konfigurationsparameter:

$$X = \begin{bmatrix} X_P \\ X_R \end{bmatrix}$$

Position
Orientierung

Positionsrepräsentationen

- Kartesisch

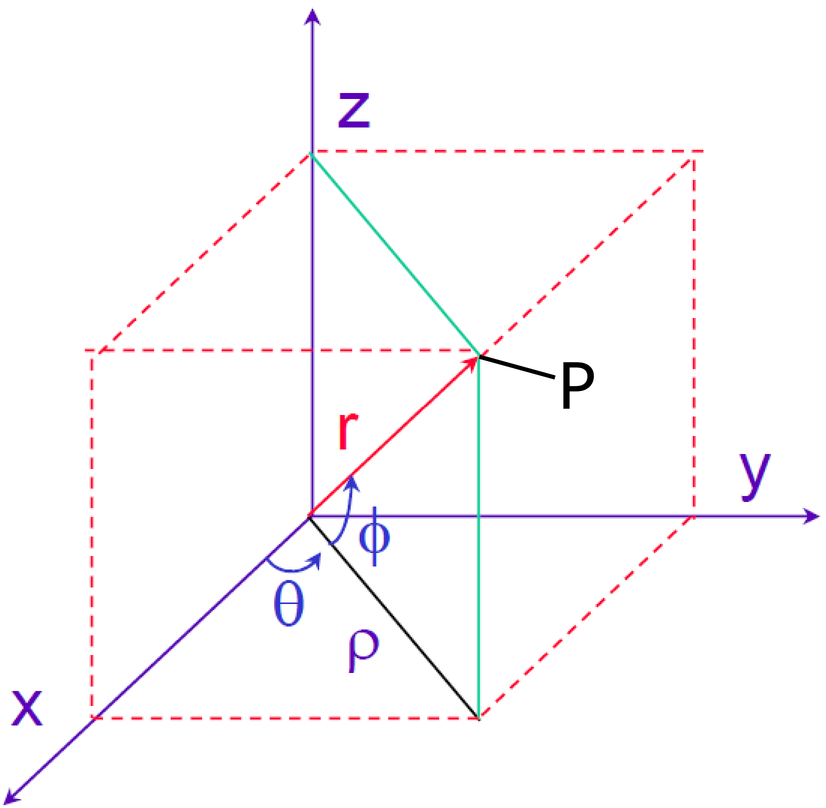
  (x, y, z)

- Zylindrisch

  (ρ, θ, z)

- Sphärisch

  (r, θ, ϕ)



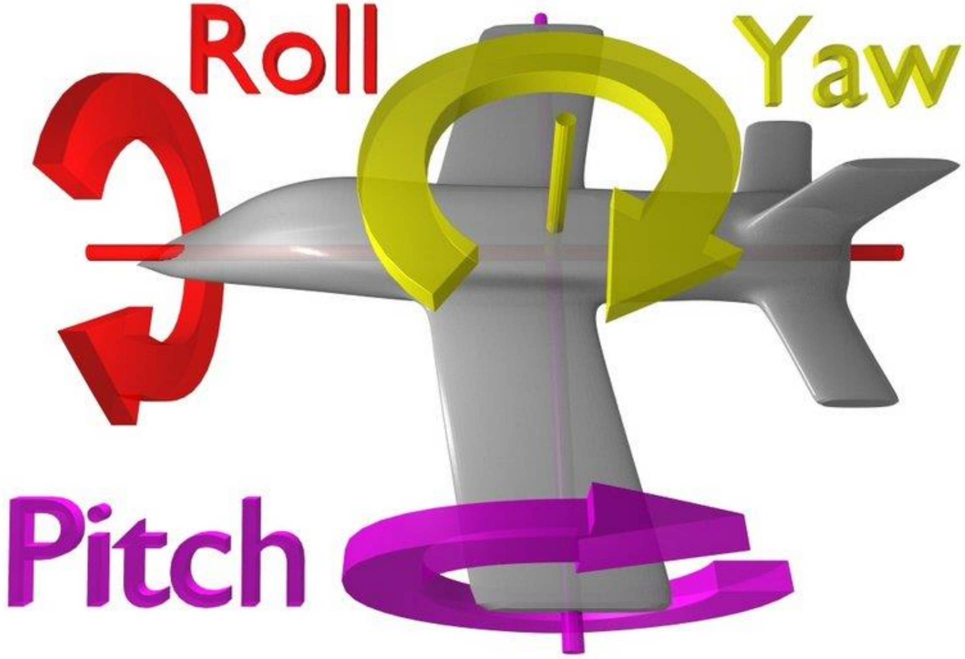
Jeweils drei unabhängige Parameter

Beschreibung von Rotationen

- Z.B. Orientierung eines Flugzeugs

Rollen

Gieren



Nicken

Rotationsrepräsentationen

- Rotationsmatrix:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{bmatrix}$$

- Cosinuswerte der Richtungen zu den Achsen:

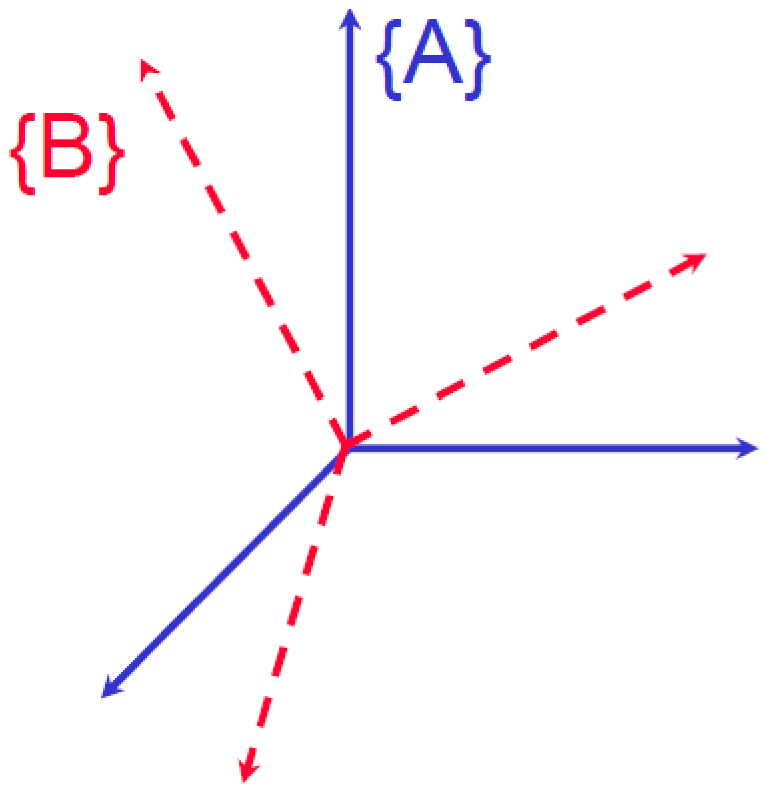
$$x_r = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \end{bmatrix}_{(9x1)}$$

- Sechs Nebenbedingungen:
  - Einheitslänge: $|\mathbf{r}_1| = |\mathbf{r}_2| = |\mathbf{r}_3| = 1$
  - Orthogonalität: $\mathbf{r}_1 . \mathbf{r}_2 = \mathbf{r}_1 . \mathbf{r}_3 = \mathbf{r}_2 . \mathbf{r}_3 = 0$

=> Drei Freiheitsgrade

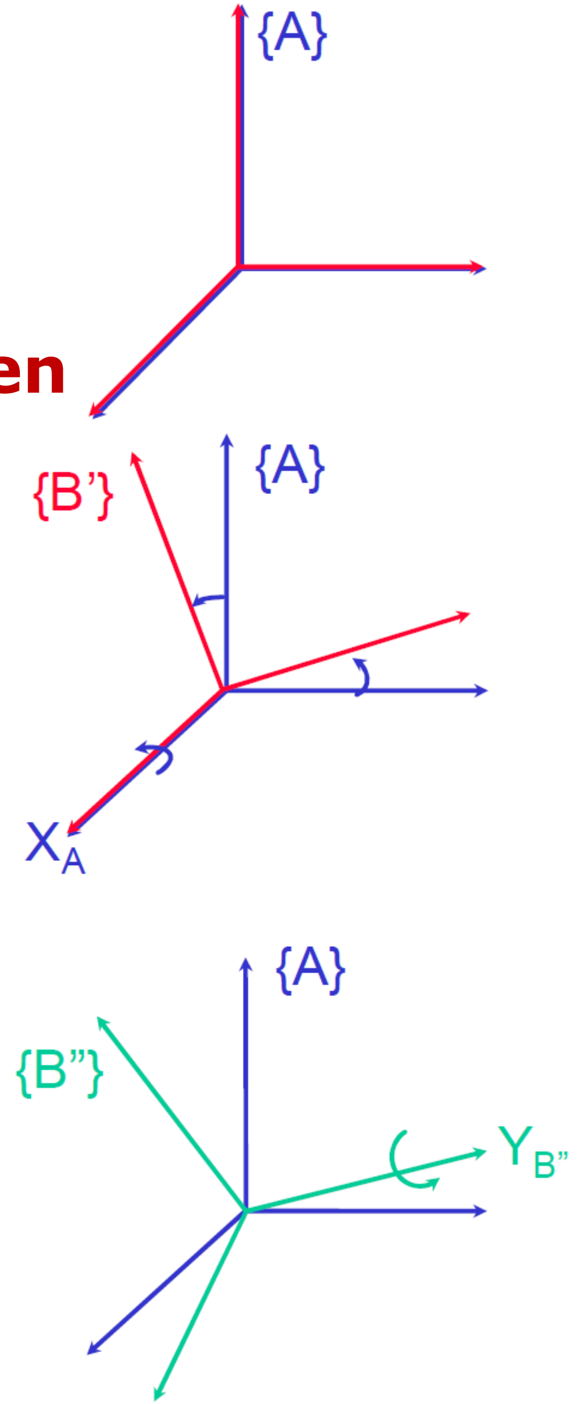
Repräsentation mit drei Winkeln

- Es gibt viele Möglichkeiten, die Reihenfolge von Elementar-Rotationen zu kombinieren.
- Die Rotationsachse kann fest oder beweglich sein.

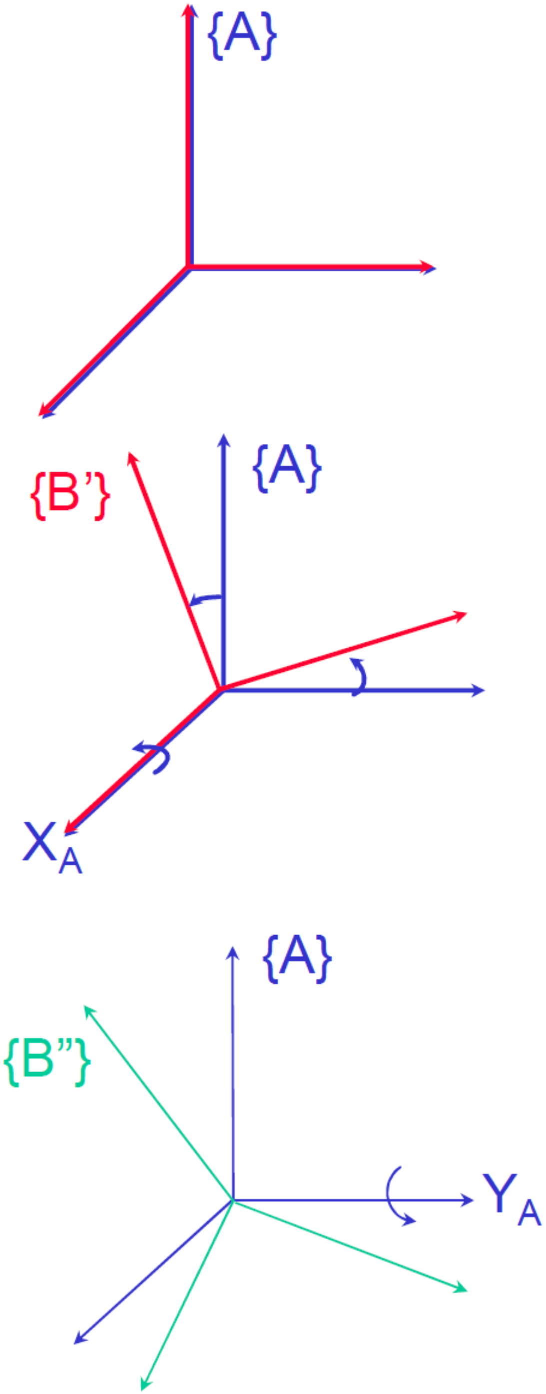


Euler-Winkel

- **Bewegliche Rotationsachsen**
- Z.B. X-Y-Z, d.h.
  zuerst Rotation um
  X-Achse
- Anschließend Rotation
  um **rotierte** Y-Achse
- Dann Rotation um
  resultierende Z-Achse
- 12 Varianten, abhängig von
  Reihenfolge der Rotationen



Absolute Winkel



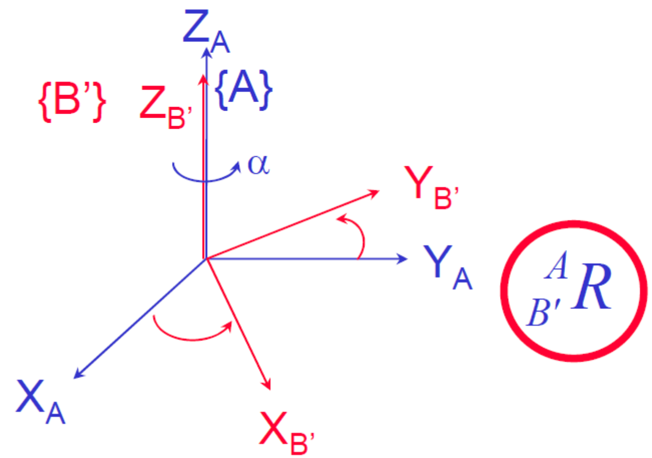
- **Feste Rotationsachsen**
- Z.B. X-Y-Z, d.h.
  zuerst Rotation um
  X-Achse
- Anschließend Rotation
  um **ursprüngliche** Y-Achse
- Dann Rotation um
  ursprüngliche Z-Achse
- 12 Varianten, abhängig von
  Reihenfolge der Rotationen

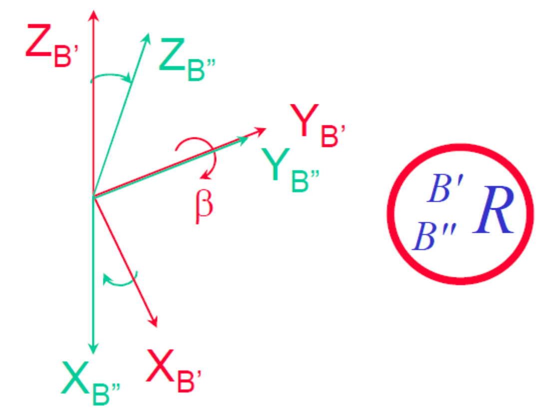
(Z-Y-X)-Eulerwinkel

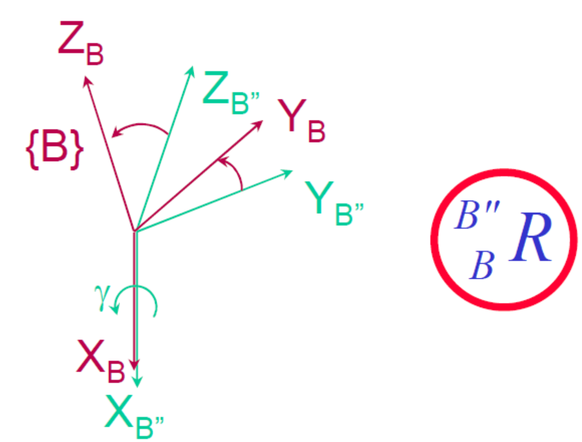
- Rotation um Z-Achse mit Winkel α
- Rotation um resultierende Y-Achse mit Winkel β
- Rotation um resultierende X-Achse mit Winkel γ
- Gesamtrotation

$$ {}^{A}_{B}R = {}^{A}_{B'}R \,.\, {}^{B'}_{B''}R \,.\, {}^{B''}_{B}R $$

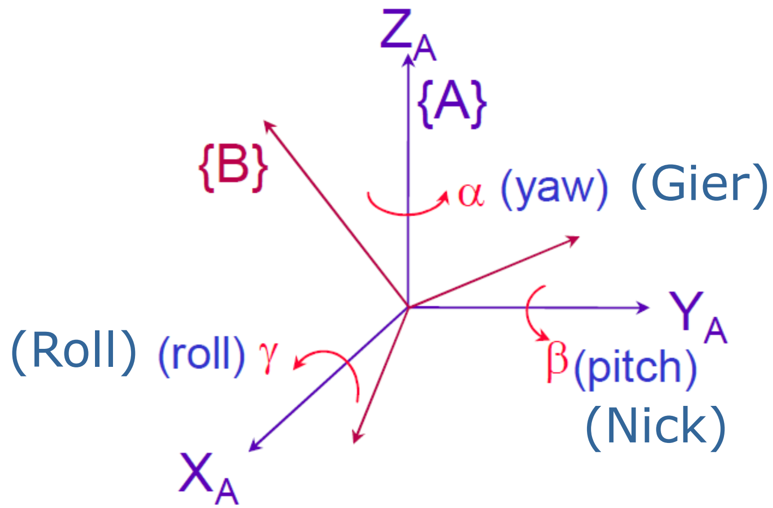
$$ \boxed{{}^{A}_{B}R = R_Z(\alpha).R_Y(\beta).R_X(\gamma)} $$







(X-Y-Z)-Absolutwinkel



- Drei Rotationen um die ursprünglichen Achsen

$$R_X(\gamma): \; v \rightarrow R_X(\gamma).v$$

$$R_Y(\beta): \; (R_X(\gamma).v) \rightarrow R_Y(\beta).(R_X(\gamma).v)$$

$$R_Z(\alpha): \; (R_Y(\beta).R_X(\gamma).v) \rightarrow R_Z(\alpha).(R_Y(\beta).R_X(\gamma).v)$$

- Gesamtrotation

$$\boxed{{}_B^A R = {}_B^A R_{XYZ}(\gamma,\beta,\alpha) = R_Z(\alpha).R_Y(\beta).R_X(\gamma)}$$

(Z-Y-X)-Eulerwinkel

- Multiplikation elementarer Rotationsmatrizen

$$ {}^{A}_{B}R = R_Z(\alpha).R_{Y'}(\beta).R_{X''}(\gamma) $$

$$ = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix} $$

- Gesamtrotation

$$ {}^{A}_{B}R = {}^{A}_{B}R_{Z\,Y'X''}(\alpha,\beta,\gamma) = \begin{bmatrix} c\alpha.c\beta & X & X \\ s\alpha.c\beta & X & X \\ -s\beta & c\beta.s\gamma & c\beta.c\gamma \end{bmatrix} $$

- Wie kann man die Winkel aus der Matrix ablesen?

(Z-Y-Z) - Eulerwinkel

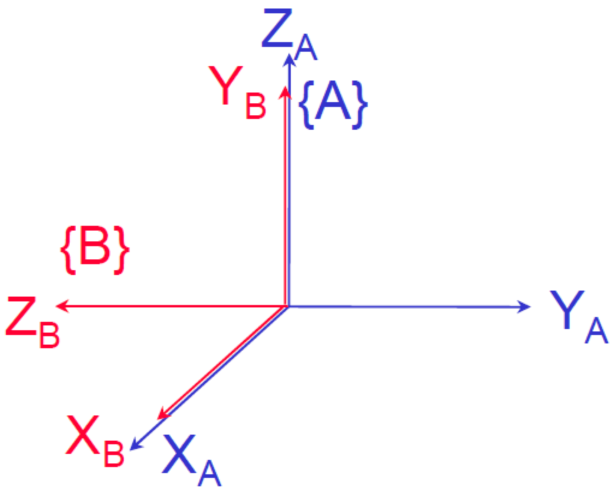
- Multiplikation elementarer Rotationen

$$ {}^{A}_{B}R = R_Z(\alpha).R_{Y'}(\beta).R_{Z''}(\gamma) $$

- Gesamtrotation

$$ {}^{A}_{B}R = {}^{A}_{B}R_{Z\,Y'Z''}(\alpha,\beta,\gamma) = \begin{bmatrix} X & X & c\alpha.s\beta \\ X & X & s\alpha.s\beta \\ -s\beta.c\gamma & s\beta.s\gamma & c\beta \end{bmatrix} $$

Beispiel: (Z-Y-X)-Eulerwinkel



$R_{Z\,Y'X''}(\alpha,\beta,\gamma)$:

$$\alpha = 0$$
$$\beta = 0$$
$$\gamma = 90^o$$

Absolutwinkel vs. Eulerwinkel

- (X-Y-Z) – Absolutwinkel

$$R_{XYZ}(\gamma,\beta,\alpha) = R_Z(\alpha).R_Y(\beta).R_X(\gamma)$$

- (Z-Y-X) – Eulerwinkel

$$R_{Z\,Y'X''}(\alpha,\beta,\gamma) = R_Z(\alpha).R_{Y'}(\beta).R_{X''}(\gamma)$$

- Identität

$$\boxed{R_{Z\,Y'X''}(\alpha,\beta,\gamma) = R_{XYZ}(\gamma,\beta,\alpha)}$$