Thomas Kesselheim
Rebecca Lehming

May 14, 2025
Due date: May 21, 2025

Algorithmic Game Theory

Summer Term 2025

Exercise Set 5

If you would like to present one of the solutions in class, please also send an email to rlehming@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated on a first-come-first-served basis, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1:
We call s an ϵ-approximation to a pure Nash equilibrium if $c_i(s) \leq (1+\epsilon)c_i(s'_i, s_{-i})$ for all i and s'_i.

(a) Consider a (λ, μ)-smooth cost-minimization game and let and $0 < \epsilon < \frac{1}{\mu} - 1$. Prove that the PoA of ϵ-approximations to pure Nash equilibria is at most $\frac{(1+\epsilon)\lambda}{1-(1+\epsilon)\mu}$.

(b) Can you state a similar result for more general equilibrium concepts?

Exercise 2:
Consider a second-price auction with a fixed value profile $(v_i)_{i \in N}$. Since the value profile is fixed, we get a normal-form utility-maximization game.

(a) Show that there exists a pure Nash equilibrium in the defined game.

(b) Now, consider a game in which only two players participate and $v_1 \gg v_2$ holds. Prove that even in this setting there exists a pure Nash equilibrium such that bidder 2 wins.

Exercise 3:
We consider an auction of k identical items. Each bidder can acquire at most one of the items. If bidder i gets one of the items, she has a value of v_i. Otherwise, that is, if she does not get an item, she has a value of 0.

(a) State a generalization of the second-price auction and prove that it is truthful (the second-price auction covers the case of $k = 1$). Follow steps in the spirit of Lecture 10.

(b) Now, consider a mechanism which sequentially performs k second-price auctions. That is, initially each bidder reports one bid. Then, in each auction, one item is sold among the remaining players using their initial bids. Show that truthful bidding does not necessarily lead to a pure Nash equilibrium even in the special case of three players and $k = 2$.