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Discrete and Computational Geometry

Winter semester 2024/2025

Assignment 9

Problem 1: (3+4 Points)

Consider the example of 4 line segments in Figure 1 on the next page. Let $s_1, \dots, s_4$ be the permutation of line segments used in Algorithm 16.1.

a) Draw the current state of the trapezodial decomposition computed by the algorithm after each iteration of the for-loop.

b) Draw the search graph G computed by the algorithm after each iteration of the for-loop.

Problem 2: (7 Points)

Consider Algorithm 16.1 from Lecture 16. The algorithm needs to find the faces $f_2, \dots, f_k$ that properly intersect s_i in line 7. Show that the expected number of faces k that properly intersect s_i is in $O(1)$.
Hint: Use backwards analysis.

Problem 3: (6 Points)

Let $P \subset \mathbb{R}^2$ be a *star-shaped* polygon. That means there exists a kernel point $p \in P$ such that for every point $r \in P$ the line segment $\overline{pr}$ is contained in P. For an example of a *star-shaped* polygon see Figure 2 on the next page. Let $p_1, \dots, p_n \in \mathbb{R}^2$ be the vertices of the boundary of P in counterclockwise order. Design an algorithm that takes the ordered sequence $p_1, \dots, p_n$ and the kernel point p as input and computes a triangulation of P in $O(n)$ time. You may assume that each star shaped polygon with kernel point p and more than three vertices $p_1, \dots, p_n$ contains a vertex p_i such that the quadrilateral p, p_{i-1}, p_i, p_{i+1} is convex.

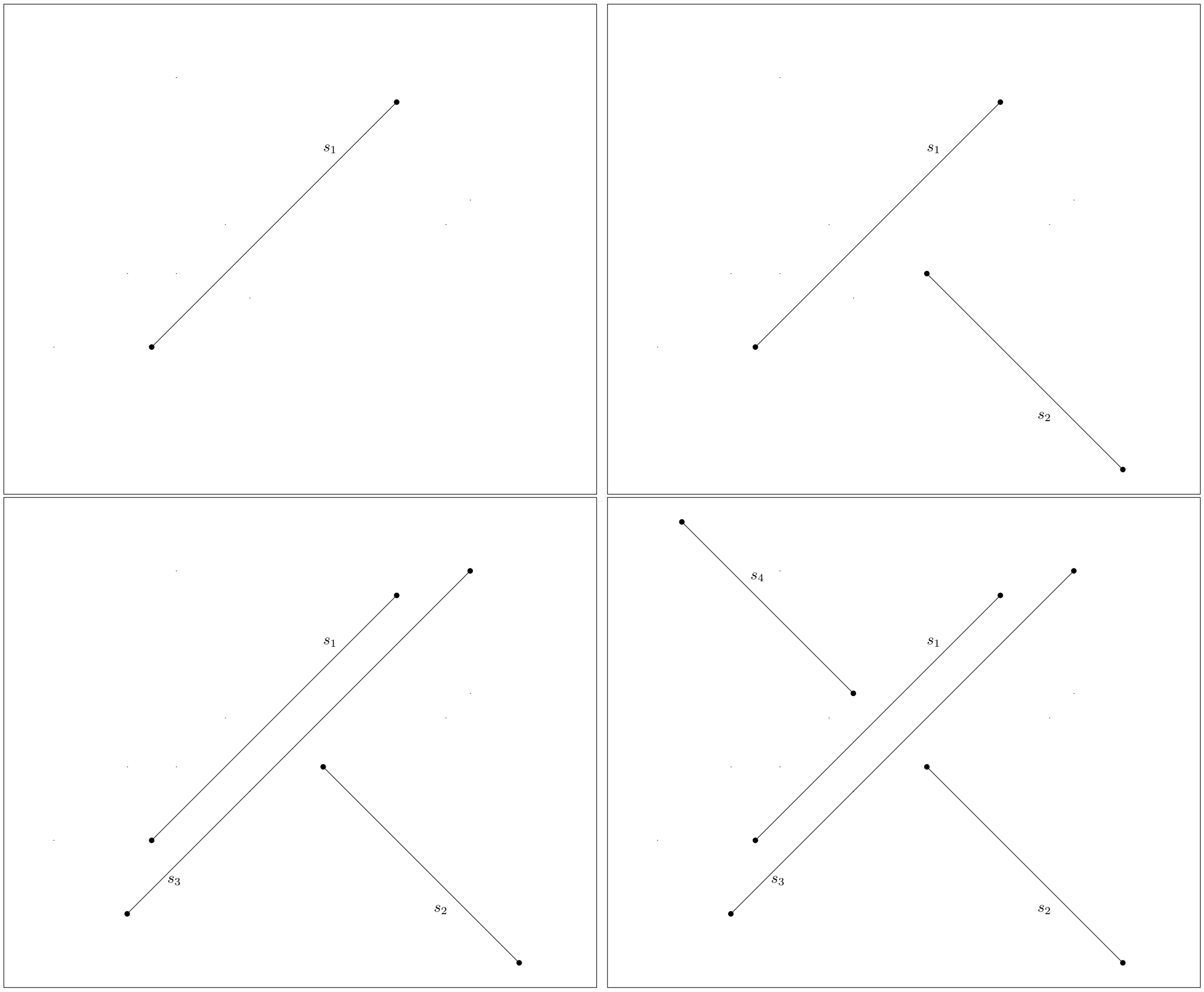

Figure 1: line segments

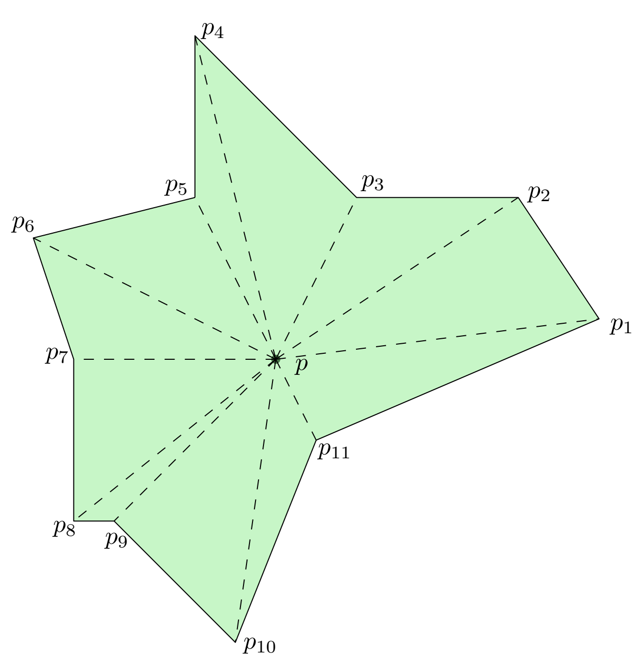

Figure 2: Example of a *star-shaped* polygon