UNIVERSITÄT BONN

Chapter 3:
Magnetic Resonance Imaging

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3.1 Introduction to MRI

Magnetic Resonance Imaging (MRI)

- Developed in 1970s
  - 1976: First MR scan of a human body part
- Based on the magnetic resonance of atomic nuclei (mostly hydrogen)
- Requires a strong static magnetic field, and high-frequency electromagnetic pulses
- Typical image resolution:
  Several mm^3

Discovery of NMR and MRI

Nuclear spin manipulation through external fields:

- Static magnetic field: $B_0 \cdot \boldsymbol{e}_z$ $\Rightarrow$ spin polarization
- Radiofreq. field: $B_1(t) \cdot \boldsymbol{e}_{xy}$ $\Rightarrow$ resonance signal
- Gradient field: $(\boldsymbol{G}(t) \cdot \boldsymbol{r}) \cdot \boldsymbol{e}_z$ $\Rightarrow$ spatial frequencies

➔ **Imaging!**

Felix Bloch
(1905-1983)

Edward Purcell

1952 Nobel Prize in Physics

Paul Lauterbur and Sir Peter Mansfield

2003 Nobel Prize in Medicine

Computed Tomography (CT/CAT)

- Introduced in 1968
- Nobel Prize in 1979
- Based on measuring X-ray absorption from many different directions
  - Algorithmic reconstruction of a slice image
  - Multiple slices combined into a 3D volume
    - Often acquired simultaneously
- Can achieve sub-mm voxel size
  - e.g., inspect fine fractures

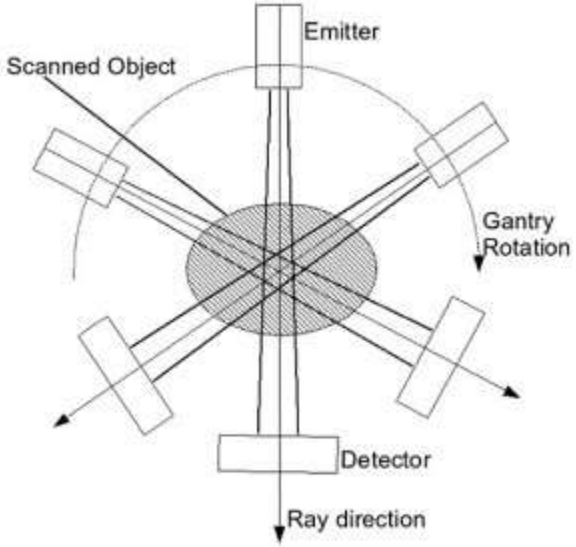


Image from Siemens Medical

MRI vs. CT: Applications

- CT ideally suited for **skeletal structures**
- MRI provides a larger variety of contrasts for **soft tissue** (e.g., brain)
- Compared to MRI, CT is **more quantitative**
  – There are ways to make MRI quantitative; some of them will be treated in this lecture
- CT **cheaper** and **more widely available**

CT scan

MRI scan

MRI vs. CT: Safety

- X-ray radiation in CT is **ionizing** and potentially harmful to biological tissue
  - Rules on allowable dose
- MRI has **no known harmful effects,** safe to use on healthy volunteers
  - Footnote: Beware of implants, heating of certain tattoos,

…and office chairs

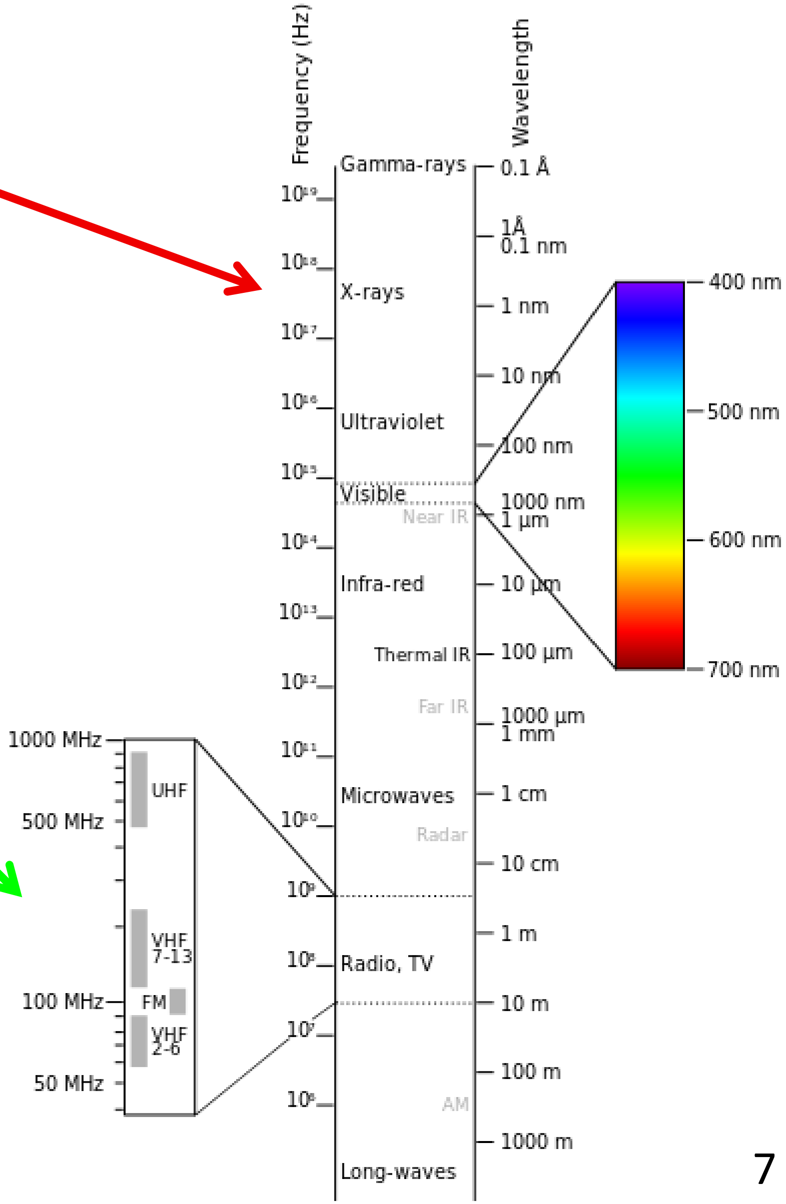
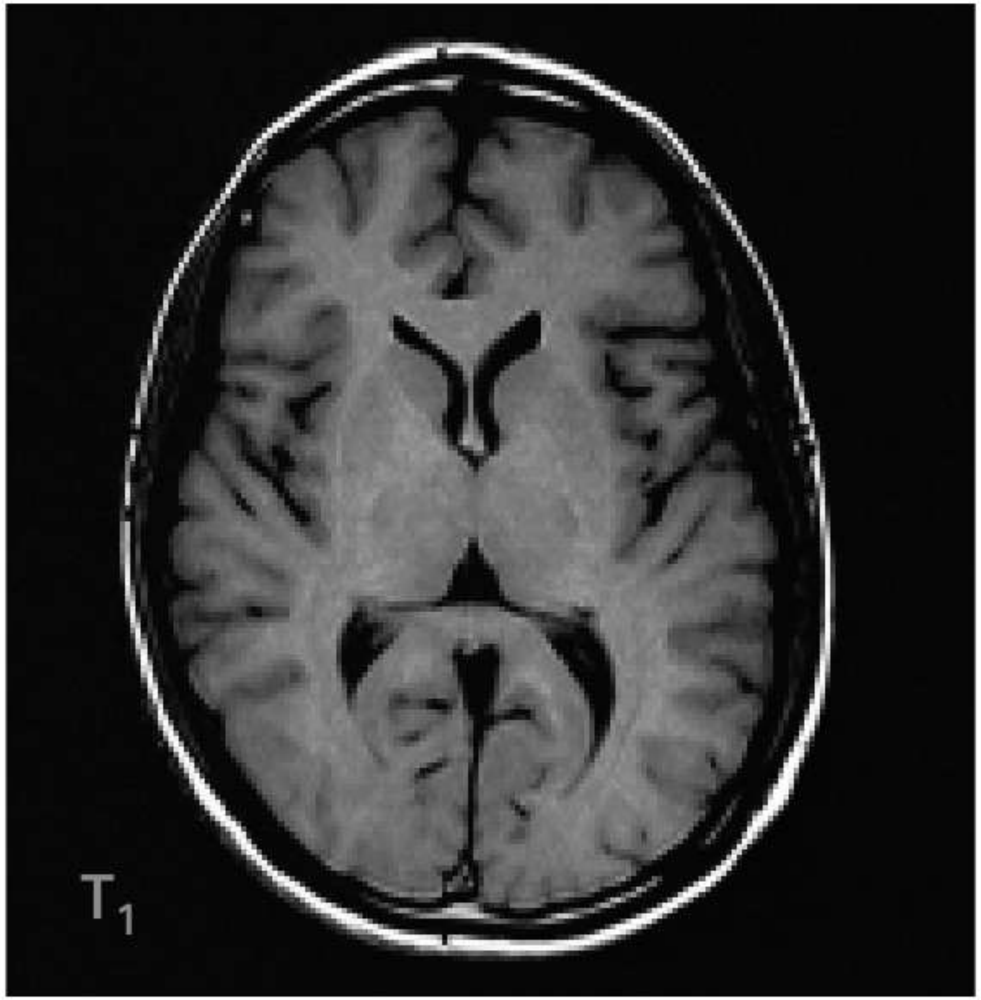
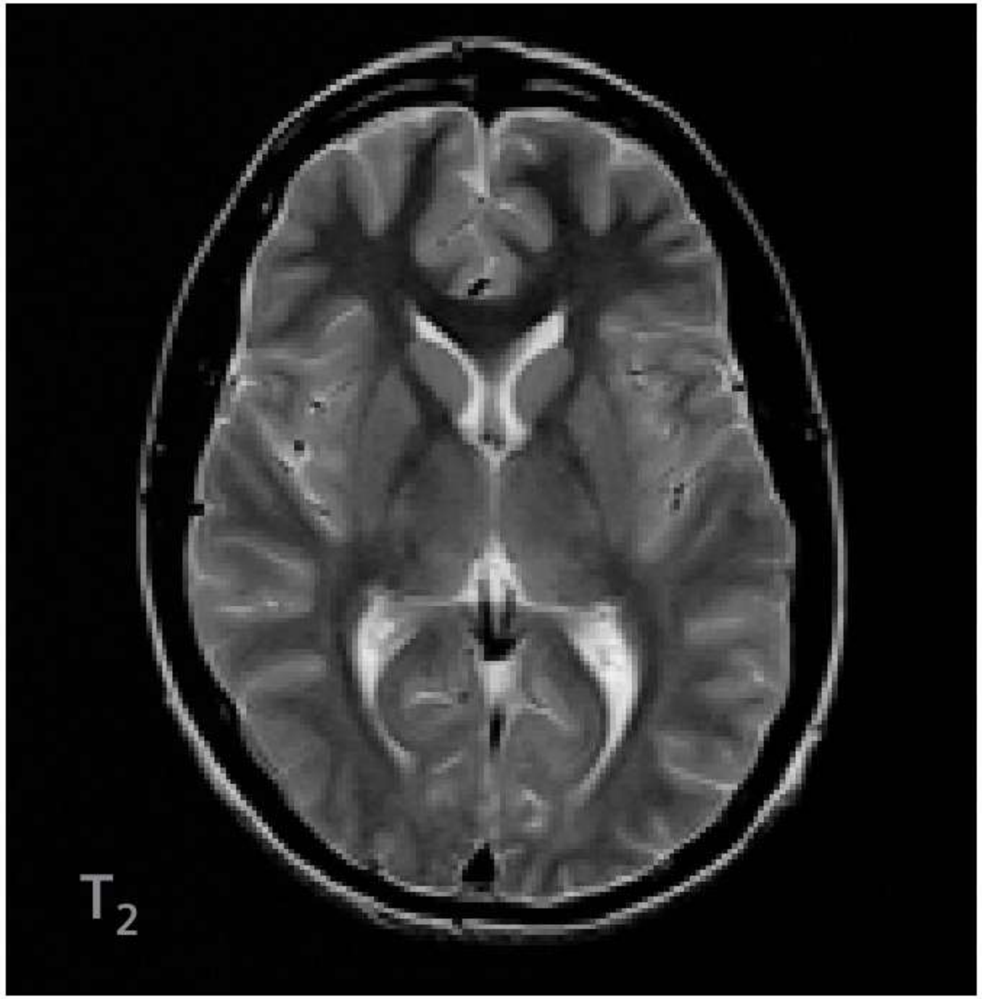


Image Sources: Victor Blacus / www.impactednurse.com

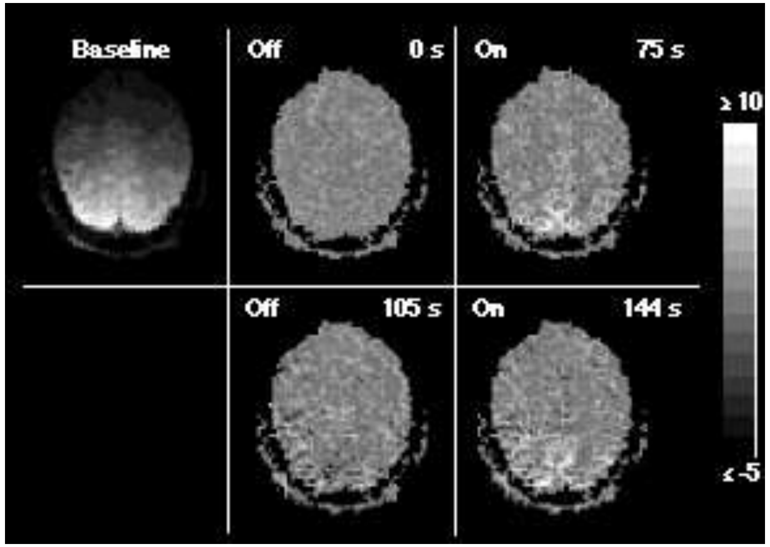
Examples: Multi-Contrast (T1/T2-weighting)

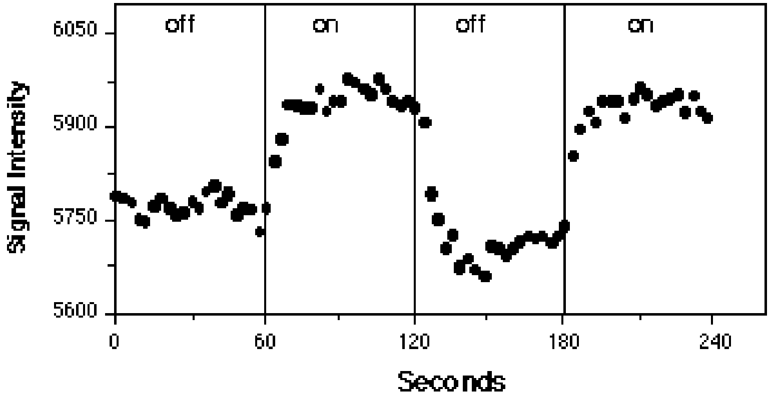




Functional MRI

- Functional MRI takes repeated images in quick succession
- The level of blood oxygenation leads to a slight variation in MR contrast
- „Watch the brain work“

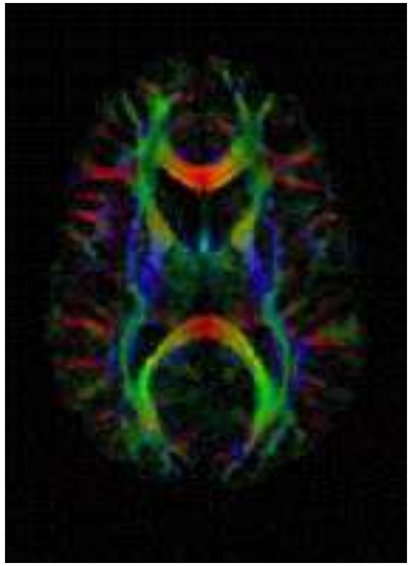


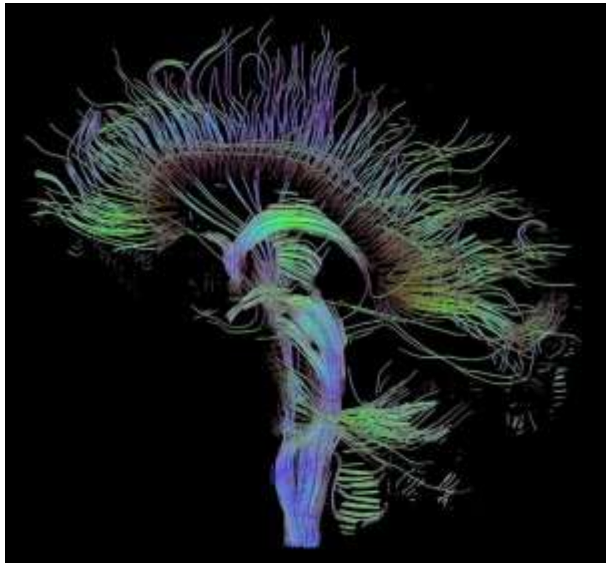


Images from Kwong et al., 1992

Diffusion MRI

- Developed in mid-1980s, much refinement since then
- MR signal attenuated by heat motion (Brownian motion) of imaged nuclei
- Spatial resolution: 1-10 mm^3
  - Probe of much smaller microstructure due to microscale interactions between molecules and tissue





Summary

- **Advantages of MRI** as an imaging modality:
  - **No ionizing radiation**
  - **Contrast agents** only required for specific use cases
  - **Well-suited for soft tissue** such as the brain
  - Fast enough for **functional imaging**
  - Flexible sequences offer **variety of contrasts**
    - different physical properties (T_1, T_2, diffusion, etc.)
  - Can be made **quantitative**
    - E.g., measuring T_1 as opposed to having a T_1-weighted image in arbitrary units, in which brightness also depends on other factors
  - **Flexible orientation** of slices

3.2 Nuclear Magnetic Resonance

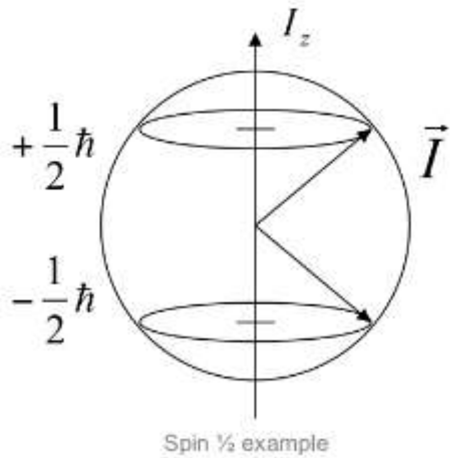
Partly based on slides by Prof. Tony Stöcker

Key Questions of this Section

- How does the **signal** arise that an MR scanner measures?
- Why do we need such a **strong magnetic field**?
- Why is it called **“Magnetic Resonance”**?
- How does MR differentiate between different **types of soft tissue**?

Quantum Mechanical Spin

- Human body consists of ≈70% $H_2 0$ 
- Hydrogen nuclei (protons) have *nuclear spin,* a quantum mechanical property that is somewhat similar to classical **angular momentum** (e.g., rotating ball)
  - – But it is **intrinsic,** cannot speed up or slow down
  - – Protons have spin ½, $I_z = \pm\frac{1}{2}\hbar$
  - – Individual spins are in a *superposition* of parallel and antiparallel states with a certain phase



h = Planck constant ≈ 6.626068·10^{-34} Js

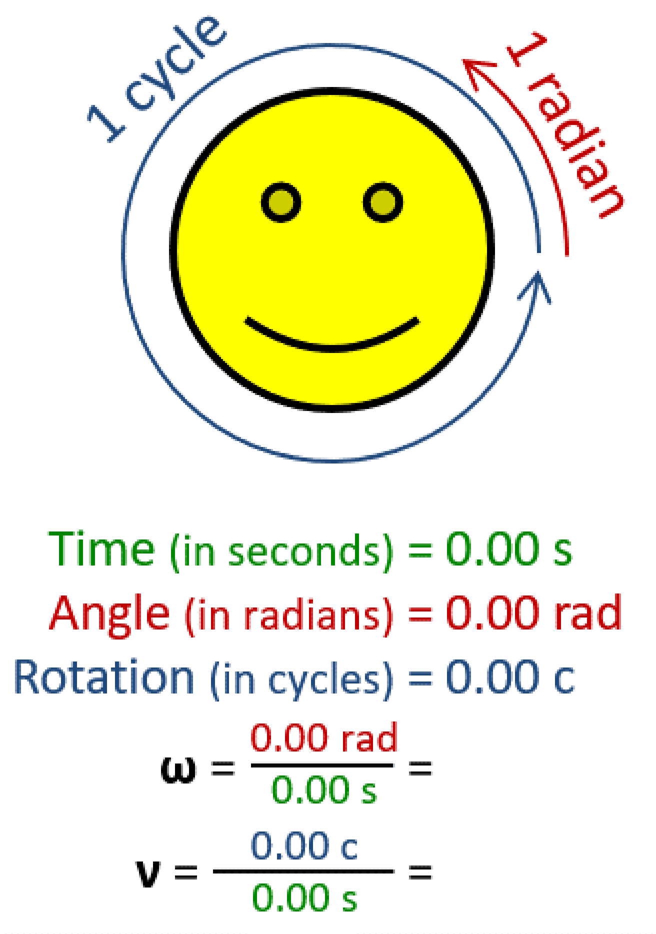
$\hbar = \frac{h}{2\pi}$ "reduced Planck constant"

Frequency vs. Angular Frequency

- **Rotational frequency** ν refers to the number of cycles per second (unit: Hz)
- **Angular frequency** ω refers to the change in angle (in radians) per second

$$\omega = 2\pi\nu$$

- Fourier basis with respect to
  - Rotational frequency: $e^{2\pi i\nu t}$
  - Angular frequency: $e^{i\omega t}$

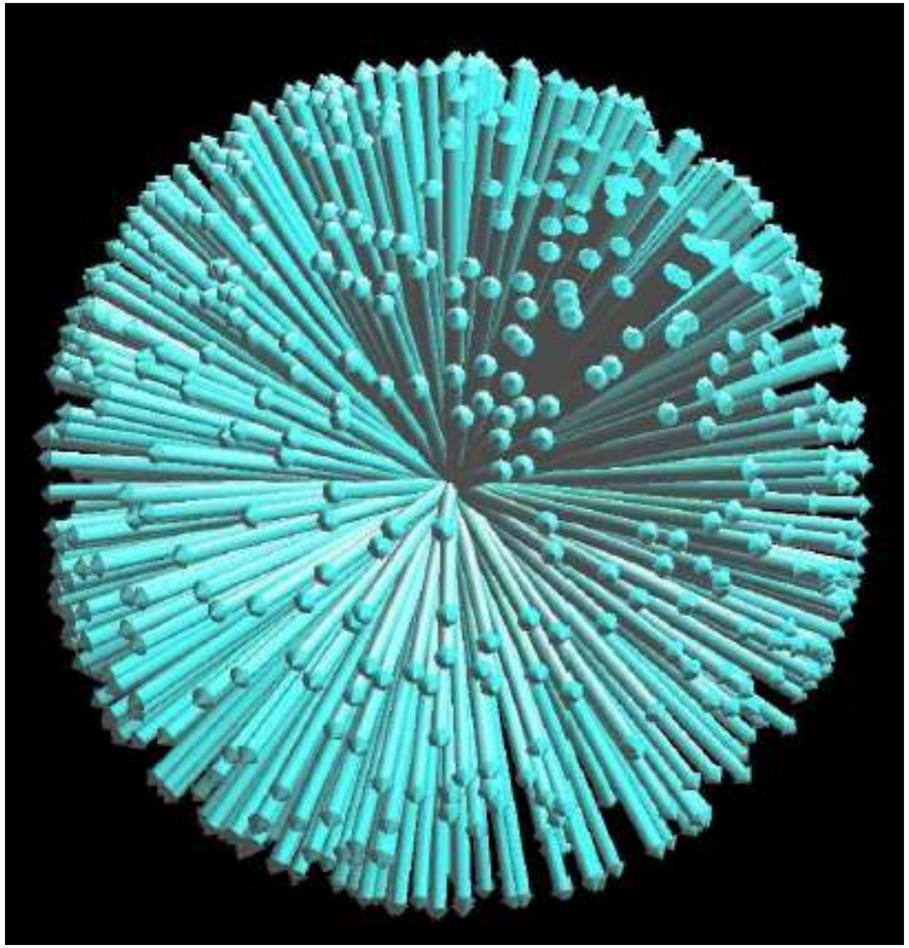


Animation: Steve Byrnes via Wikipedia

Nuclear Magnetic Moment

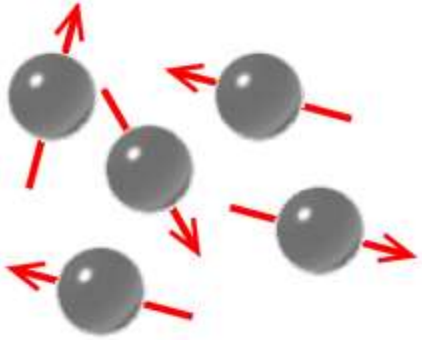
- Every nuclear spin comes with a **nuclear magnetic moment** $\boldsymbol{\mu} = \gamma \boldsymbol{I}$
  - γ = nucleus-specific gyromagnetic ratio
  - For hydrogen, γ = $267.513 \cdot 10^6$ rad/(s·T).

Spins at $B_0 = 0$



Hanson 2008

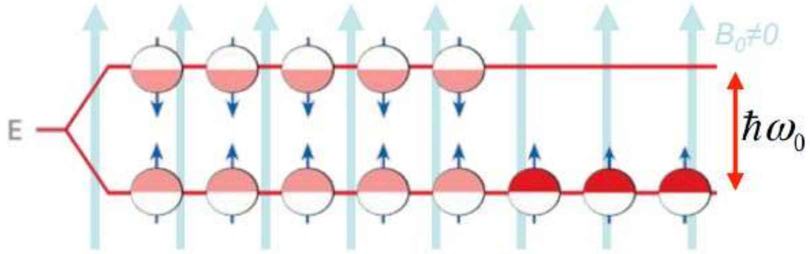
- Spin states can be represented as vectors whose
  - Polar angle indicates parallel / anti-parallel alignment
  - Azimuth angle indicates phase
- Without external magnetic field
  - Spins have random orientation
  - Vector sum over spins is zero
  - No net magnetisation



Nuclear Zeeman Effect

- When exposed to a static external magnetic field $\mathbf{B}_0$, the z-components of the nuclear magnetic moments align with $\mathbf{B}_0$.
  - For our purposes, the direction of $\mathbf{B}_0$ defines the z-axis.
- Two possible orientations of the individual magnetic moments: parallel and anti-parallel with energies

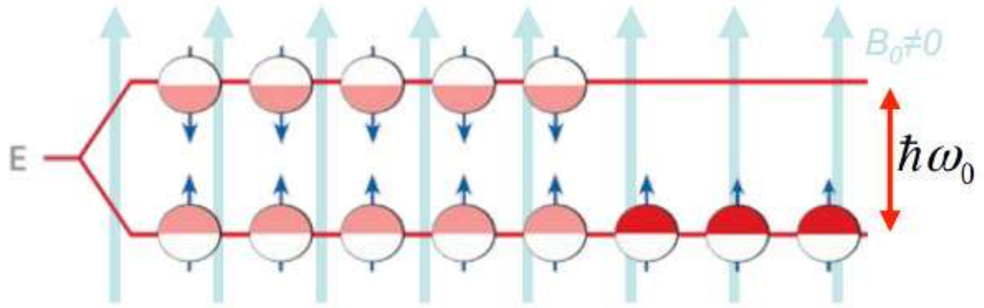
$$E_{\uparrow,\downarrow} = \mu_{\uparrow,\downarrow} B_0 = \mp \frac{1}{2} \hbar \gamma B_0$$



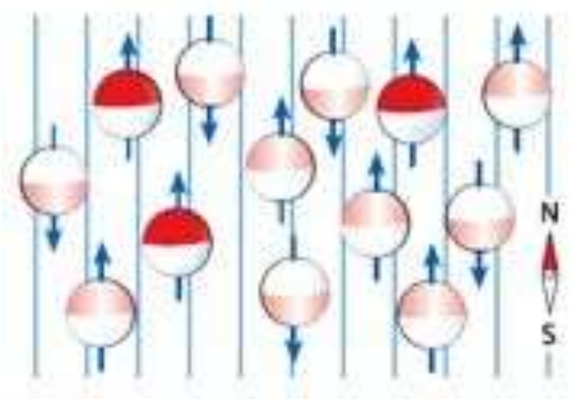
Zeeman splitting:

$$\Delta E = \hbar \gamma B_0$$

Equilibrium Magnetization

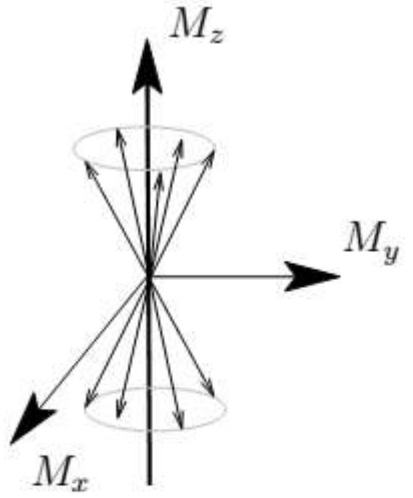


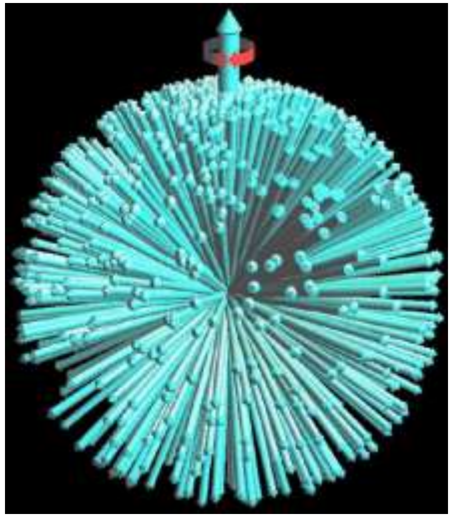
- At equilibrium, there is a tiny excess of spins in the lower energy state
  - Boltzmann statistics $\frac{n_\downarrow}{n_\uparrow} = e^{-\frac{\Delta E}{k_B T}}$
  - If you plug in the Boltzmann constant $k_B \approx 1.381 \cdot 10^{-23} \mathrm{J/K}$, body temperature T, and ΔE at clinical magnetic field strength B_0, you get a number very close to one
- The number of spins per voxel is large enough so that ensemble average produces a **macroscopic magnetization** $M_0 \sim B_0$



Spins in a Static Field: Larmor Precession

- Since the axis of rotation does not align with $\boldsymbol{B_0}$, precession around the direction of $\boldsymbol{B_0}$ takes place
  - Larmor frequency: $\omega_0 = \gamma B_0$
  - Spin distribution skewed slightly towards magnetic north pole, phases are random
  - Macroscopic magnetization is static and aligned with $\boldsymbol{B_0}$

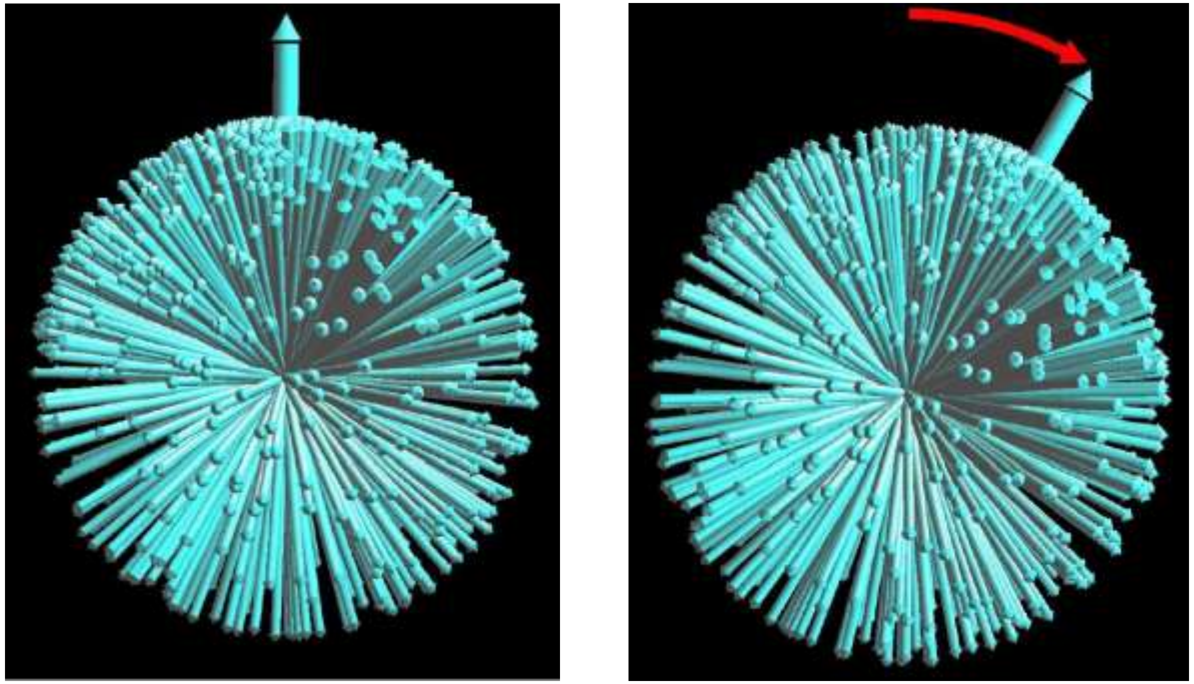




[Hanson 2008]

Nuclear Magnetic Resonance

- **Nuclear Magnetic Resonance:**
  - Electromagnetic wave $\boldsymbol{B_1}$ perpendicular to $\boldsymbol{B_0}$ at ω_0 **(RF pulse)** *coherently rotates* the spin distribution, deflects the macroscopic magnetization
  - Net magnetization starts to precess with ω_0

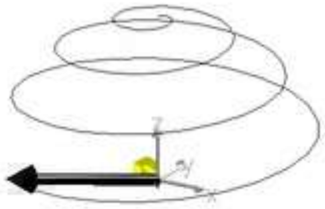


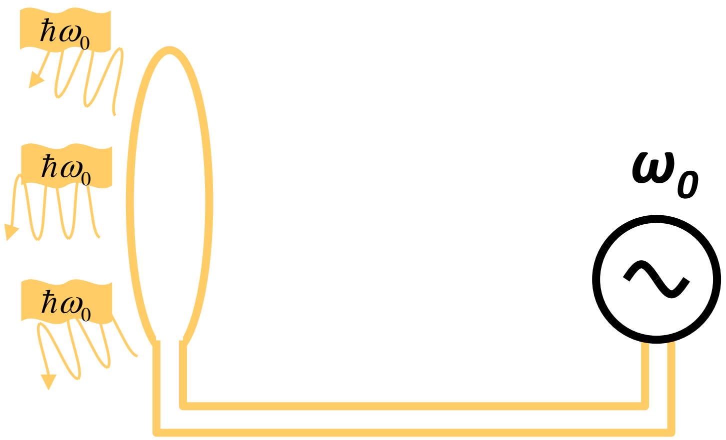
[Hanson 2008]

Excitation / Magnetic Resonance

➔Apply a **resonant radiofrequency (RF) pulse** to excite all spins, i.e. excite the macroscopic magnetisation from its equilibrium.

➔**Resonance** ↔ frequency of RF pulse = Larmor frequency $\boldsymbol{\omega_0}$.

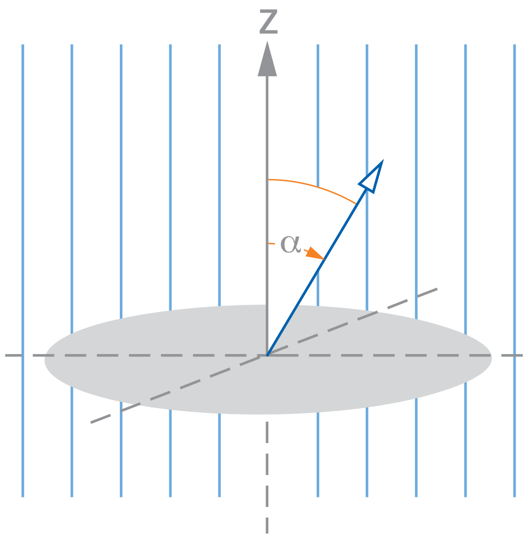




Excitation Flip Angle

- The **flip angle** by which we are deflecting the macroscopic magnetization from the z axis depends on the strength B_1 and duration τ of the transverse on-resonance RF field

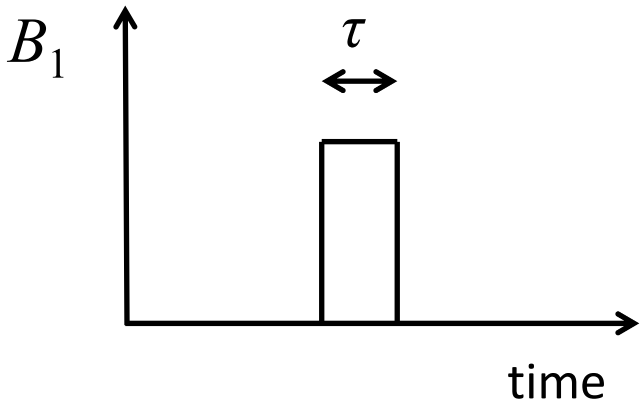
$$\alpha = \gamma \int_{-\tau/2}^{\tau/2} B_1(t)dt$$



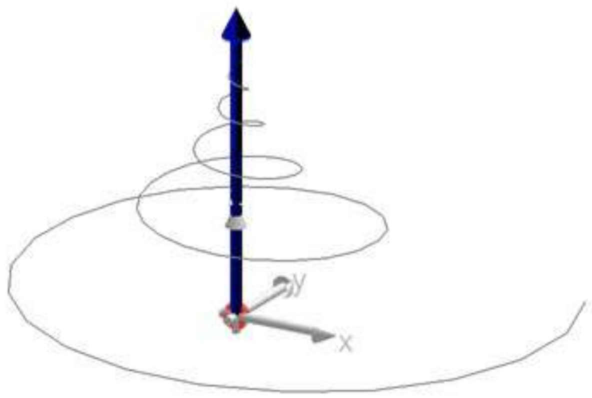
Reminder: $\gamma = 267.513 \cdot 10^6$ rad/(s·T)

- With constant amplitude and phase („rectangular pulse“):

$$\alpha = \gamma B_1 \tau$$



Relaxation



- Following an excitation, the magnetization returns to its equilibrium state
- Longitudinal magnetization regains its equilibrium value M_0
- Transverse magnetisation decays to zero
- Both happen exponentially, transverse decay is faster

Longitudinal Relaxation

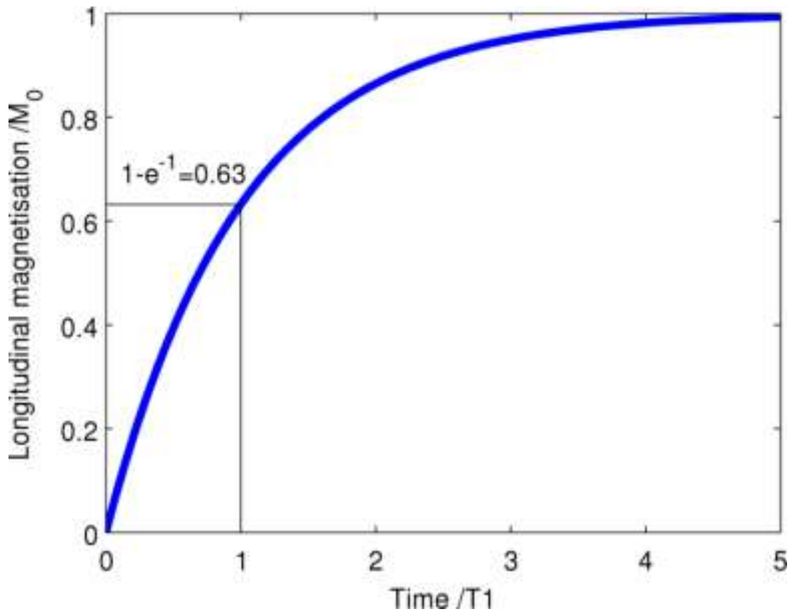
- Longitudinal relaxation due to exchange of energy with the surrounding tissue (lattice)
- Time evolution is given by

$$M_z(t) = M_0 + (M_z(0) - M_0)e^{-\frac{t}{T_1}}$$

- Assuming a 90° excitation pulse at t=0; $M_z(0)=0$:

$$M_z = M_0(1 - e^{-\frac{t}{T_1}})$$

M_0 = longitudinal equilibrium magnetization
T_1 = spin-lattice relaxation time constant

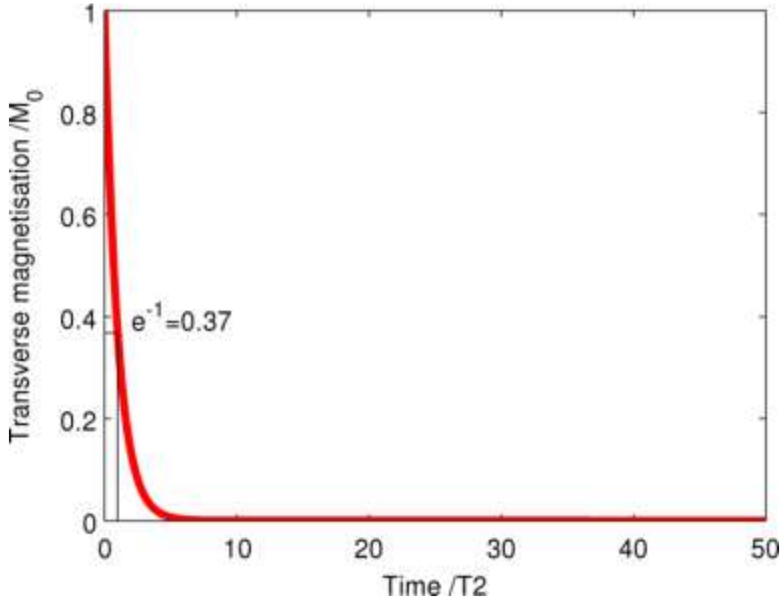


Transverse Relaxation

- Transient and random local field fluctuations occur on atomic level through spin-spin interactions
- Leads to *irreversible* loss of phase coherence between the spins
- Assuming a 90° pulse at t=0 and $M_{xy}(0)=M_0$ the solution is

$$M_{xy} = M_0 e^{-\frac{t}{T_2}}$$

T_2 is the spin-spin relaxation time constant.



Typical T1 and T2 Times in Tissue

- T1 times (in ms):

	0.2 T	1.0 T	3T
Fat		240	≈400
Muscle	370	730	≈1200
White Matter	388	680	≈900
Gray Matter	492	809	≈1500
CSF	1400	2500	≈4000

- T2 times (in ms):

Fat	84
Muscle	47
White Matter	92
Gray Matter	101
CSF	1400

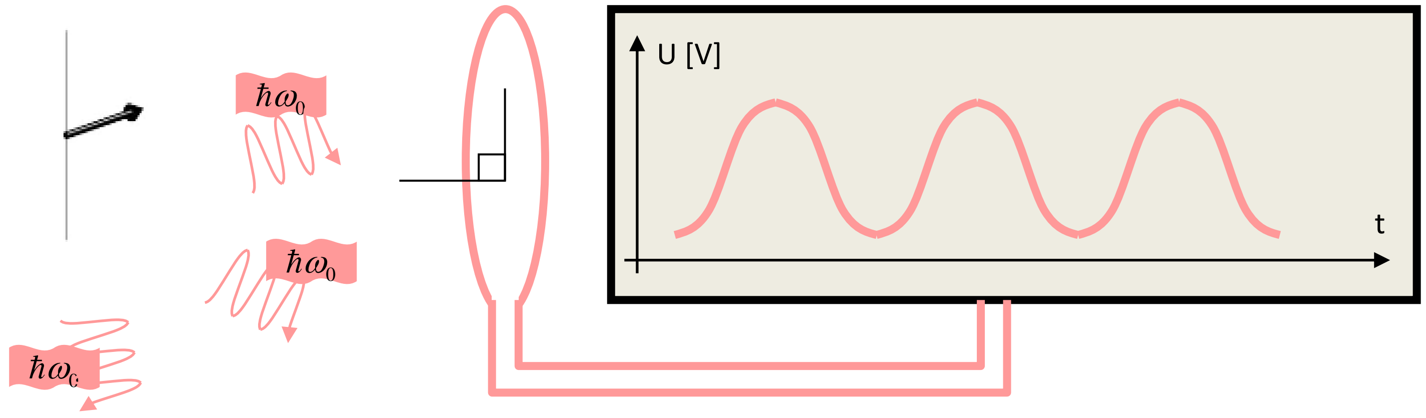
Note: Measurements show high variability. (Bojorquez et al., What are normal relaxation times of tissues at 3T? MRI 2017)

MR Signal Reception

- Faraday's law: "A changing magnetic flux induces a voltage in a conductive loop."

(cf. bicycle dynamo)

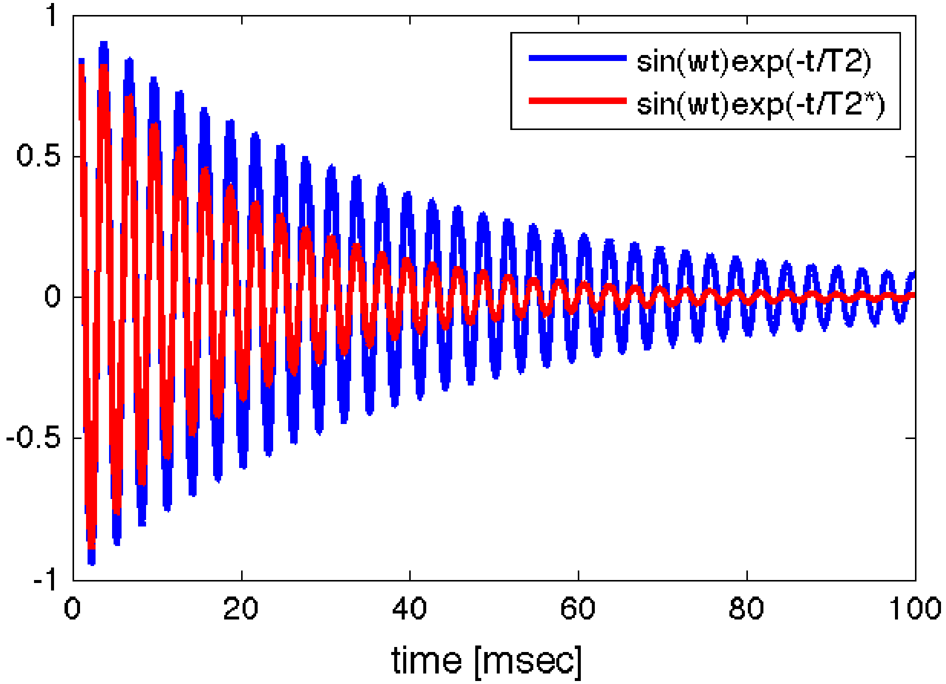
- Maximize the area of the loop by orienting it perpendicular to the transverse plane



Free Induction Decay (FID) and T_2^* (effective transverse relaxation)

- Spin-spin interaction lead to irreversible loss of phase coherence between the spins (T_2-decay)
- *B_0 inhomogeneities and susceptibility effects cause additional shortening with time constant T_2'*
- **Resulting decay constant: T_2***

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$



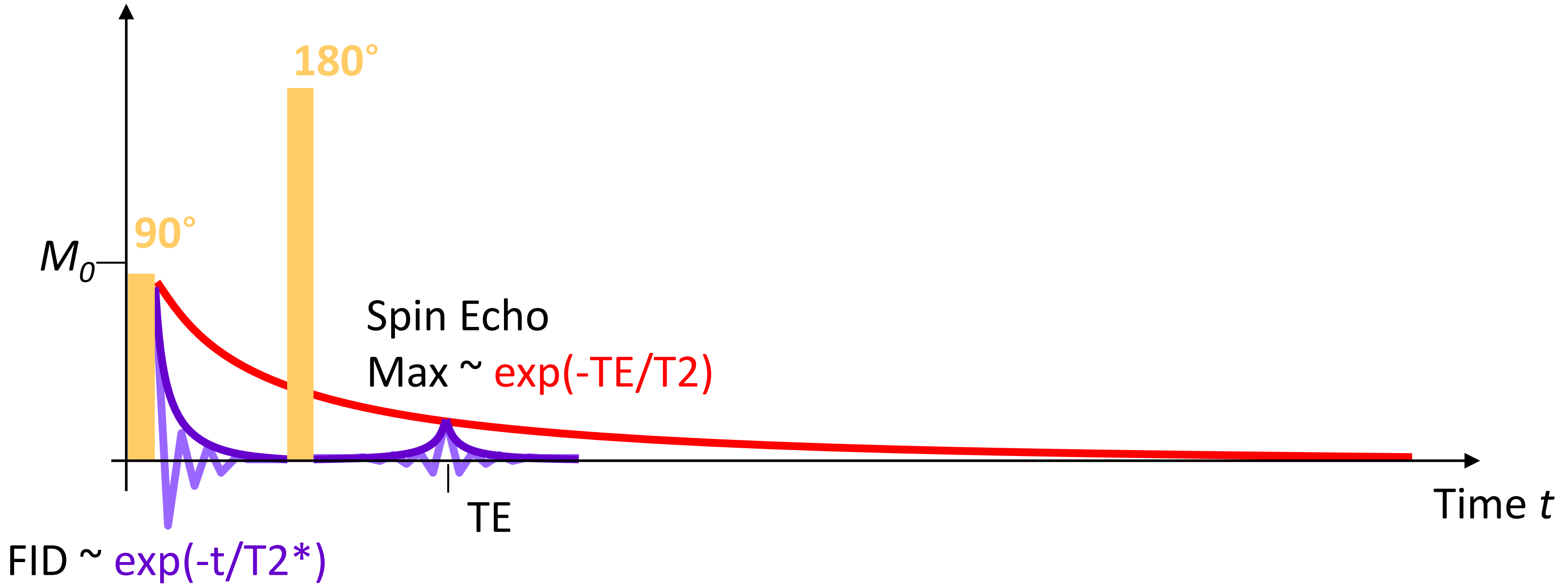
Key Questions of this Section

- How does the **signal** arise that an MR scanner measures?
- Why do we need such a **strong magnetic field**?
- Why is it called **“Magnetic Resonance”**?
- How does MR differentiate between different **types of soft tissue**?

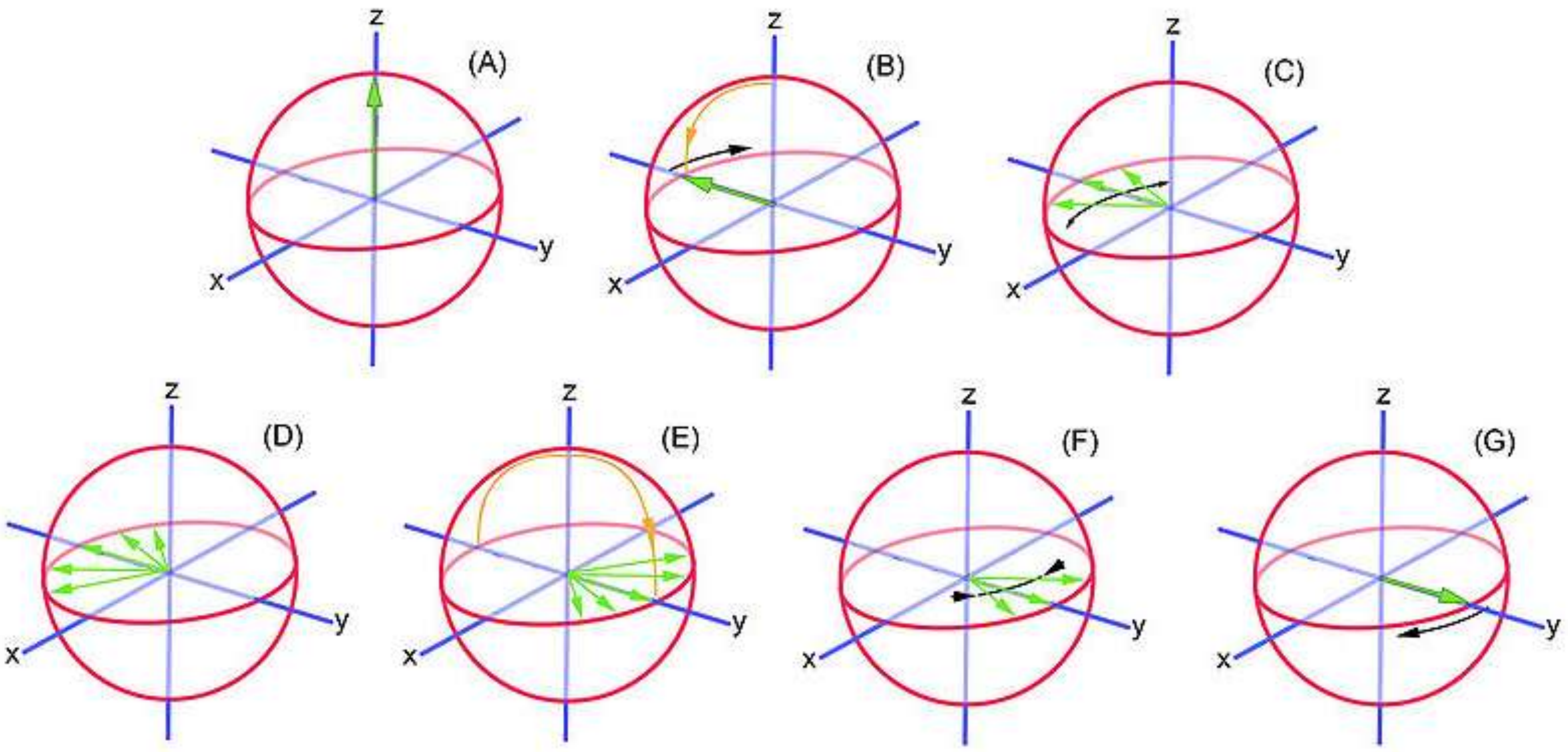
- What is the basic setup of an **echo measurement**?
- What are the main **safety issues** related to MRI?

Spin Echo (SE)

- Signal losses due to B_0 inhomogeneities is reversible!
- Recall a ***spin echo*** by inversion of the spins using a 180° pulse.
- The individual spin's phase evolution due to the B_0 induced *frequency dispersion* (T_2') is "rewound". But T_2 *decay is irreversible*.

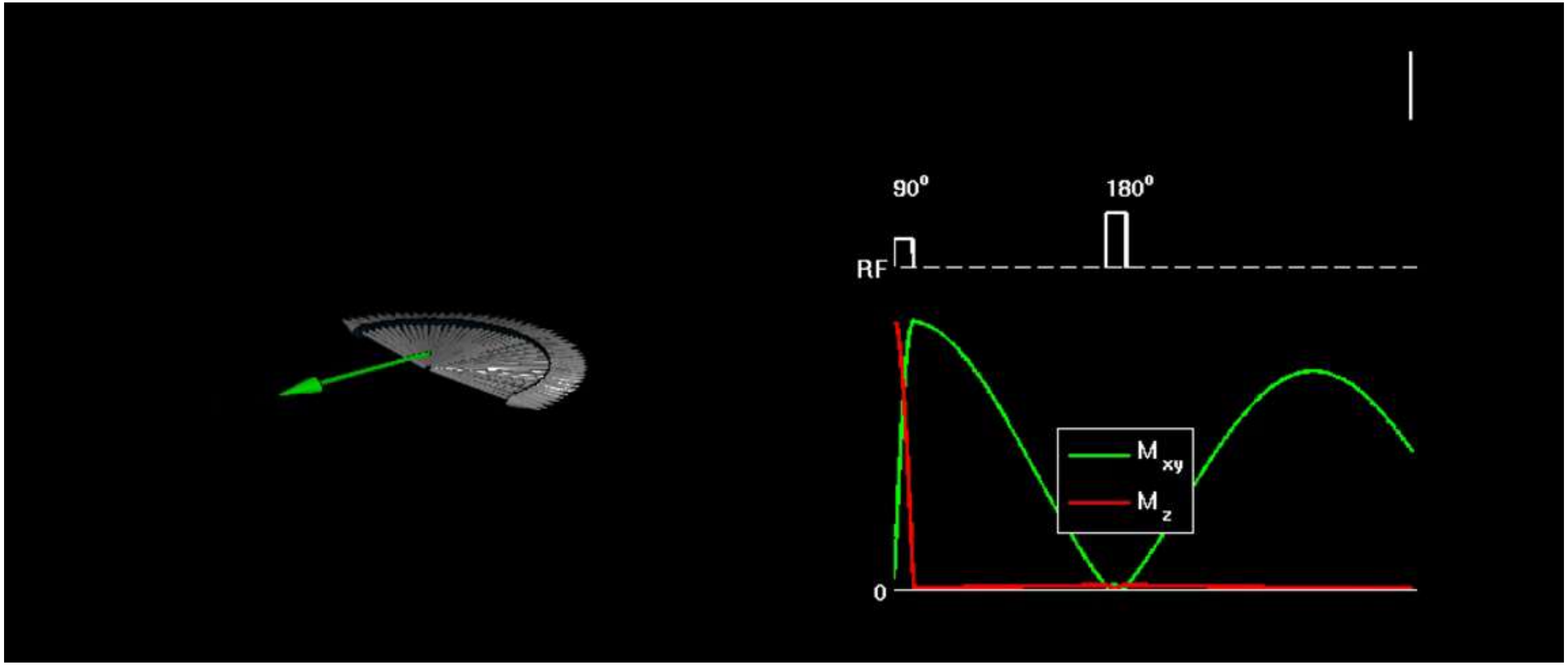


Spin Echo

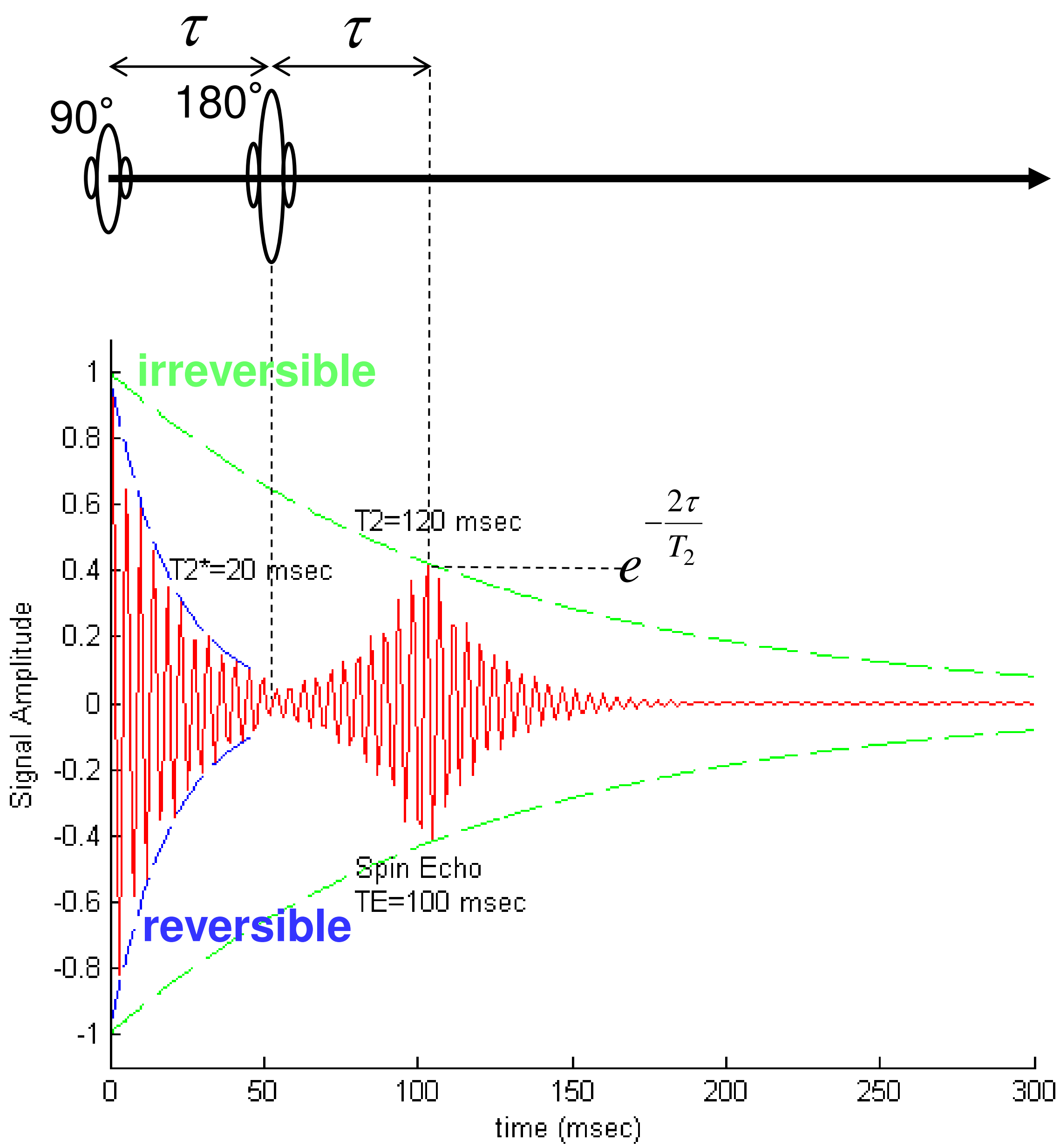


A) Equilibrium
B) 90 degree RF pulse (excitation)
C)-D) de-phasing of spins
E) 180 degree RF pulse (refocusing)
F)-G) re-phasing of spins => Spin Echo!

Illustration: Spin Echo



Spin Echo



τ
τ
90°
180°
irreversible
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
T2=120 msec
$e^{-\frac{2\tau}{T_2}}$
T2*=20 msec
Spin Echo
TE=100 msec
reversible
Signal Amplitude
0
50
100
150
200
250
300
time (msec)

Gradient Recalled Echo (GRE)

- Recall a ***gradient echo*** by *dephasing* and subsequent *rephasing* in the opposite direction.
- Much faster and less RF than spin echo.
- But: the individual spin's phase evolution due to the B_0 induced *frequency dispersion (T_2*)* is *NOT* "rewound".

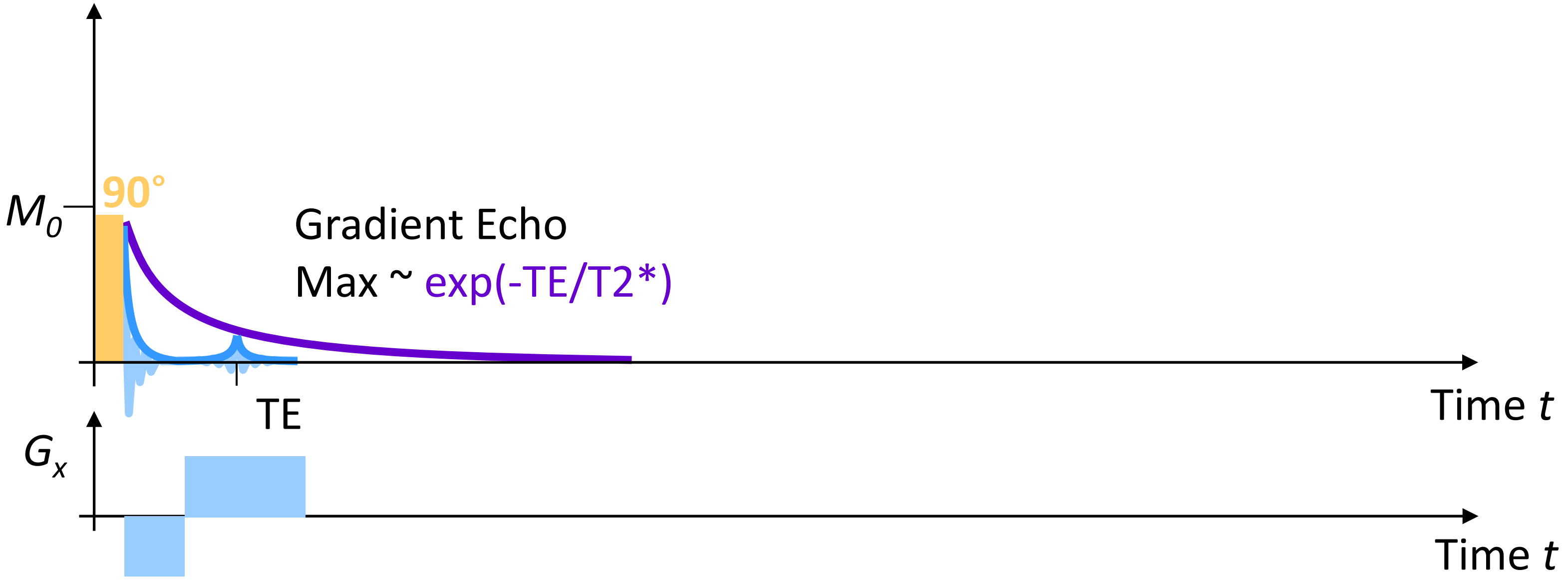
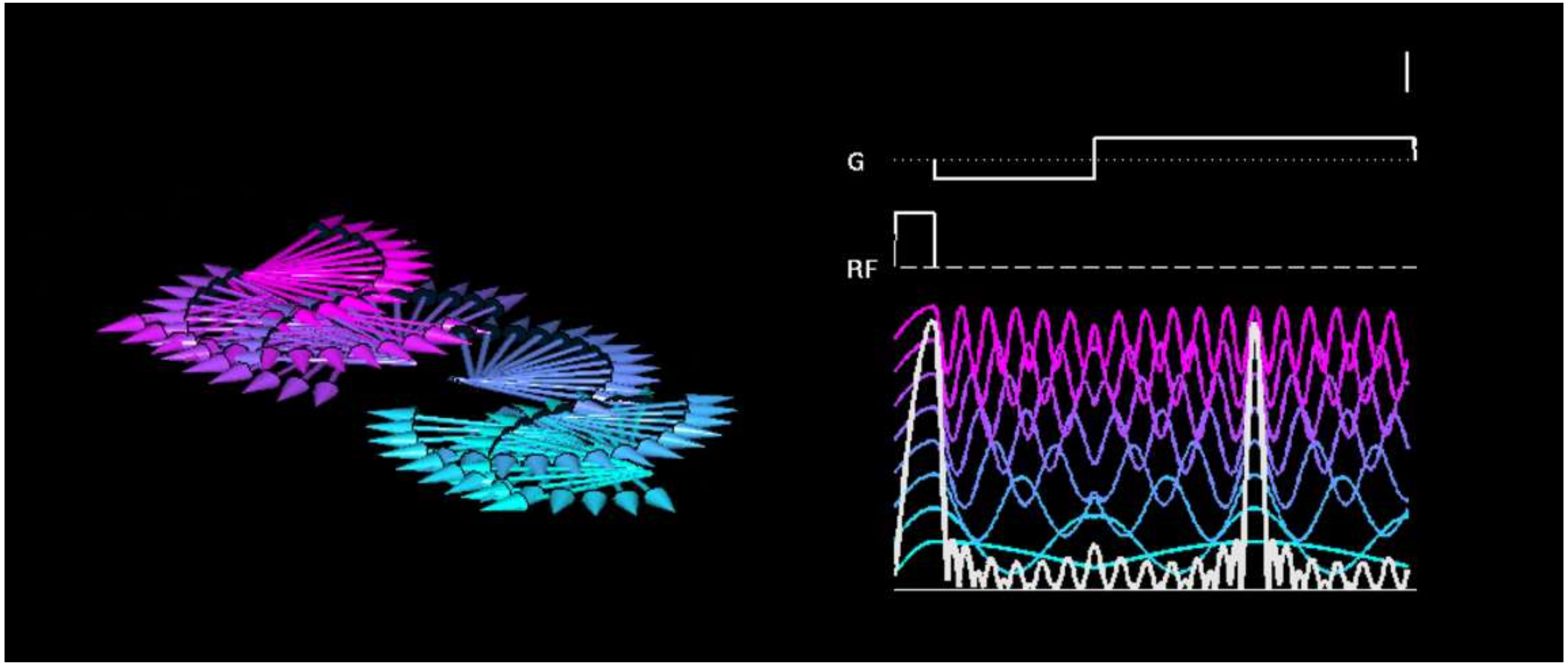


Illustration: Gradient Echo



Basic Safety Issues – The Main Field

- Permanently cooled-down (liquid Helium, ~ 4K) superconductor produces the high, static magnetic field: B_0 ~ 1 – 3 Tesla (1T = 10000 Gauss).
  - Earth's magnetic field on its surface ~ 0.5 Gauss.

➔Huge force on ferromagnetic material growing rapidly when approaching the magnet.

➔Objects like a normal fire extinguisher, tools, keys etc. can cause immense (live-threatening) damage.

➔Credit cards, cell phones, pacemakers will not work anymore.

➔Currents are induced in conducting material by changing magnetic flux, e.g. in organs when entering the scanner too quickly (from 0 to B_0). Can cause dizziness.

Basic Safety Issues – Field Gradients

- Imaging gradients are ramped up and down repetitively during an imaging sequence.
- Max. Amplitude ~ 40 mT/m.

➔Slew rate is limited to max. 200 mT/(m ms) in order not to stimulate the patient's peripheral nerves (muscle contraction in arms, back, …)

➔Patients should not cross their hands in order not to create a conducting loop through their body.

➔Again, ferro-magnetic parts are not allowed, they would move and cause live-threatening damage.

Basic Safety Issues – RF fields

- RF fields can lead to hot spots at metallic edges, e.g. cables in contact with the skin or MR-unsafe implants.
  - Heating of make-up / tattoos with magnetic properties

- Higher B_0 fields require higher frequencies, wavelength of the RF fields approaches the extent of, e.g., the head.
  - High amplitudes due to standing-wave effects.
  - Amplitude and duration of RF must be controlled due to SAR (Specific Absorption Rate) constraints.

Summary

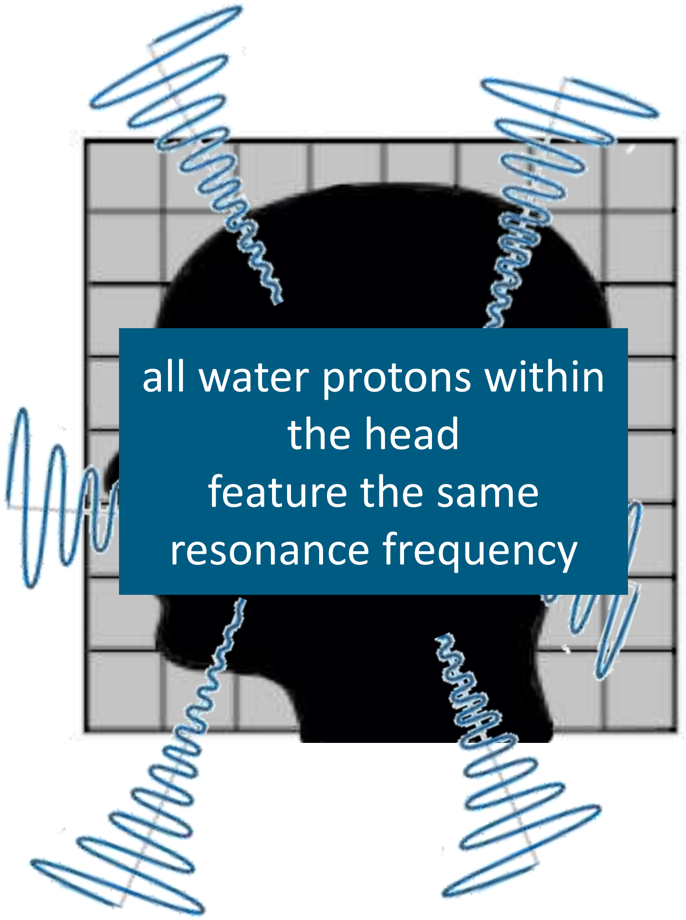
- MR places proton-spins in high B_0
  - Results in macroscopic magnetization (along B_0 in equilibrium).
- Spins excitable using *resonant RF* pulses.
  - Flips macroscopic magnetization vector
  - Rotating transverse part induces a current
- *Relaxation* described by T_1, T_2, T_2*.
- MRI acquires echo-signals, usually *Spin echoes* or *Gradient recalled echoes.*

3.3 Magnetic Resonance Imaging

Partly based on slides by Prof. Tony Stöcker

Imaging ?

- So far, the MR signal is the sum of all nuclear magnetic resonances within the entire sample.
- Since signals are not localized spatially, we cannot use them to create an image.

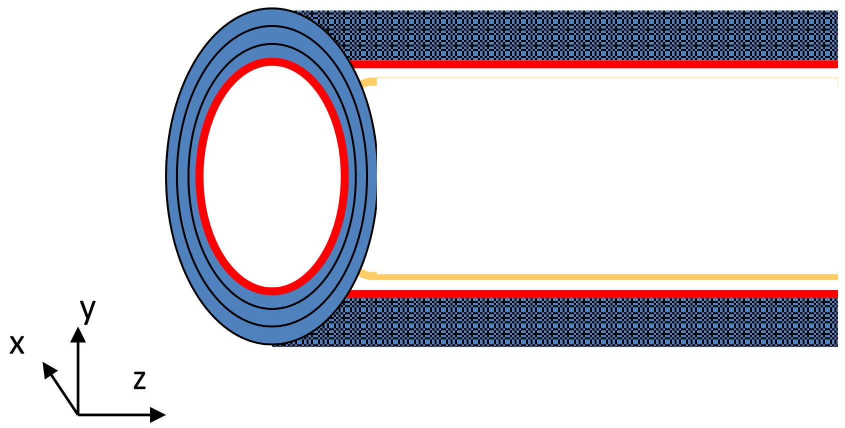


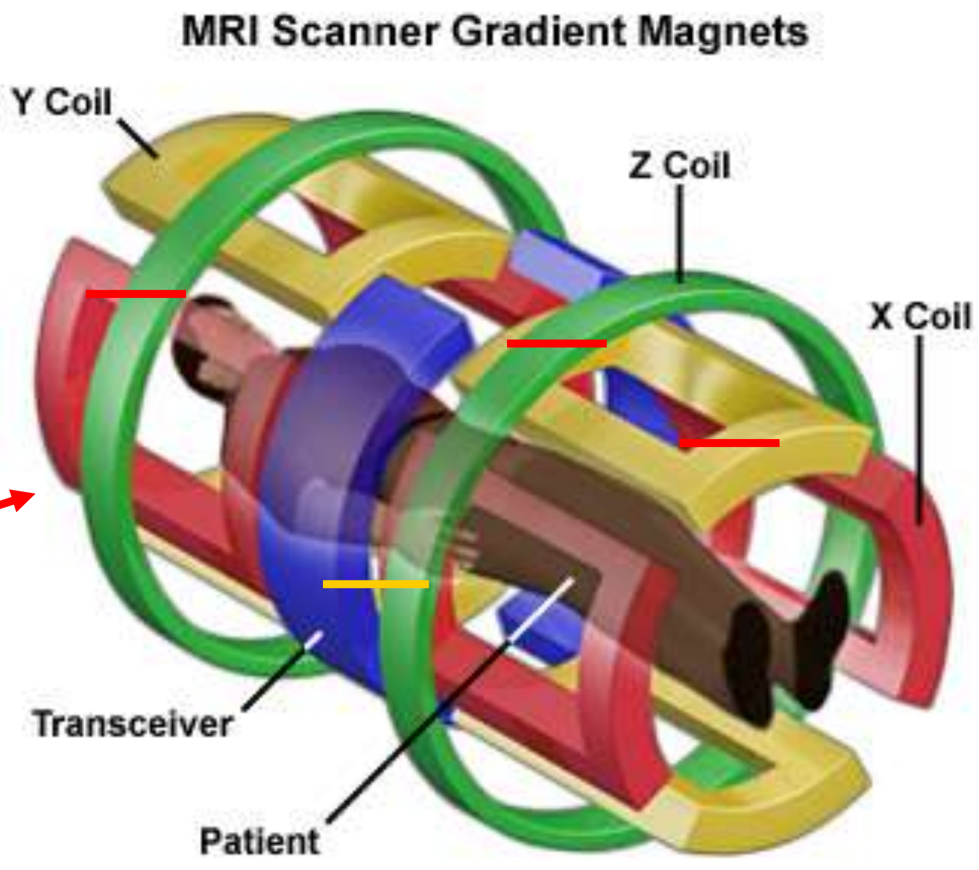
Source: Siemens, Magnete, Spins und Resonanzen

MR Scanner Components (Simplified)

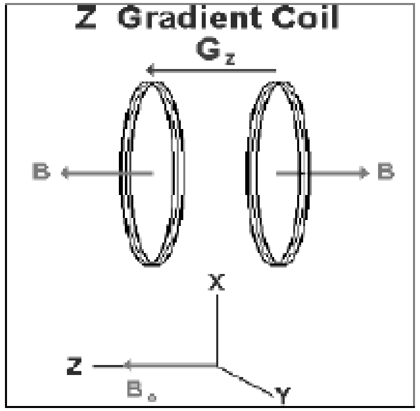
Three types of magnetic fields:

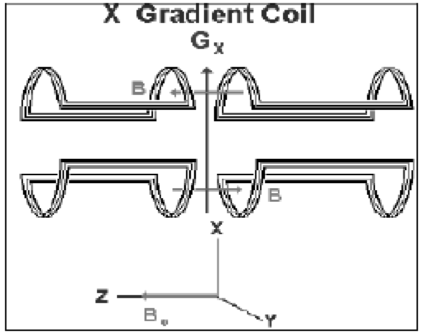
1. Static field ($B_0 \parallel z$)
2. **Dynamic "gradients"** ($G_i = dB_z/dr_i$)
3. Radio-frequency ($B_1 \perp z$)

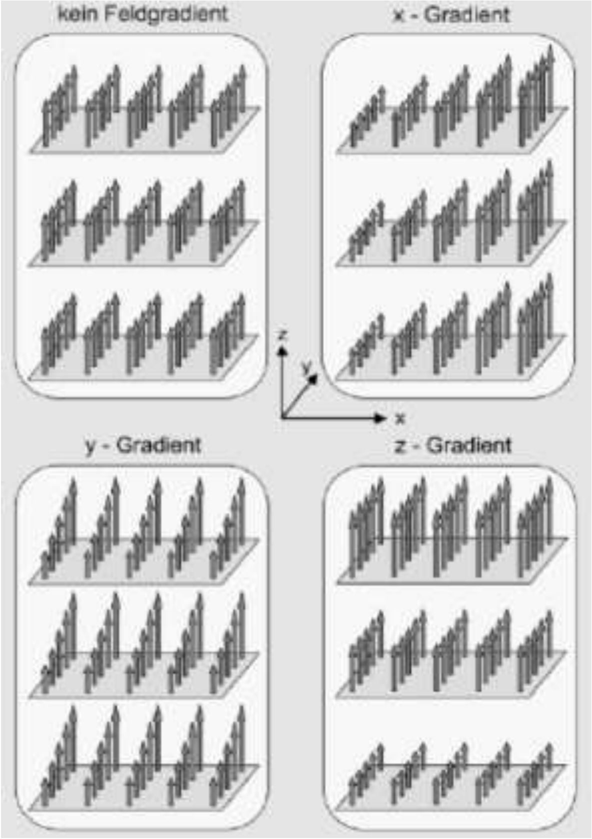




Gradient Coils: Linear Field Variation







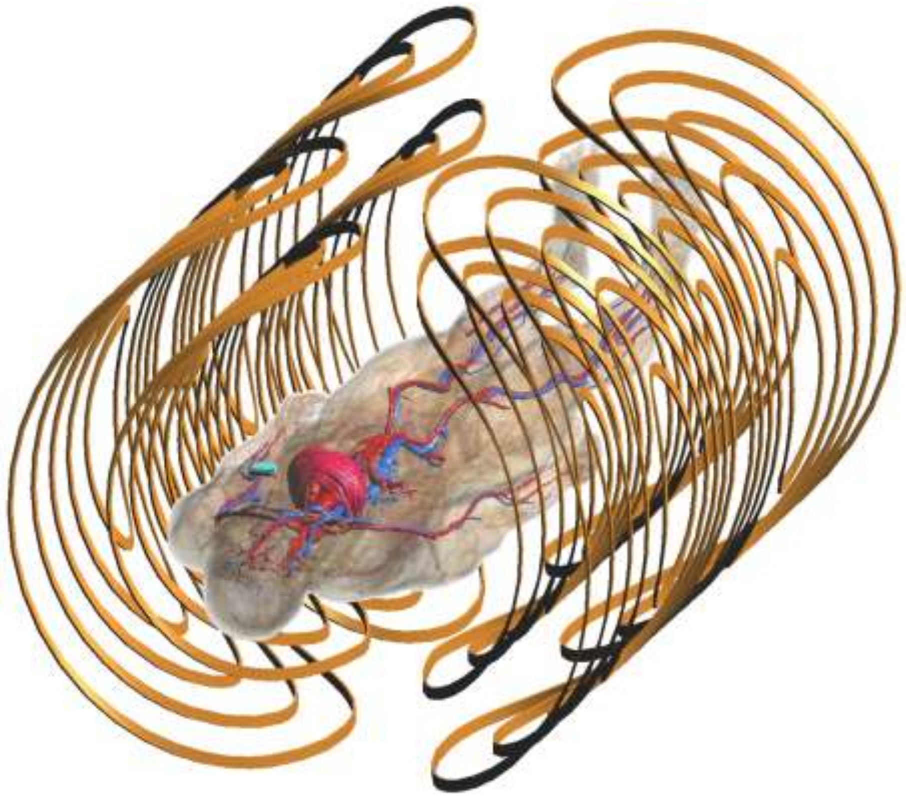
$$\Rightarrow \mathbf{B}(t) = (B_0 + \mathbf{G}(t) \cdot \mathbf{r})\mathbf{e}_z$$

Some Facts About Gradient Coils

- typical range: 1-40 $\times$ 10^{-3} T/m

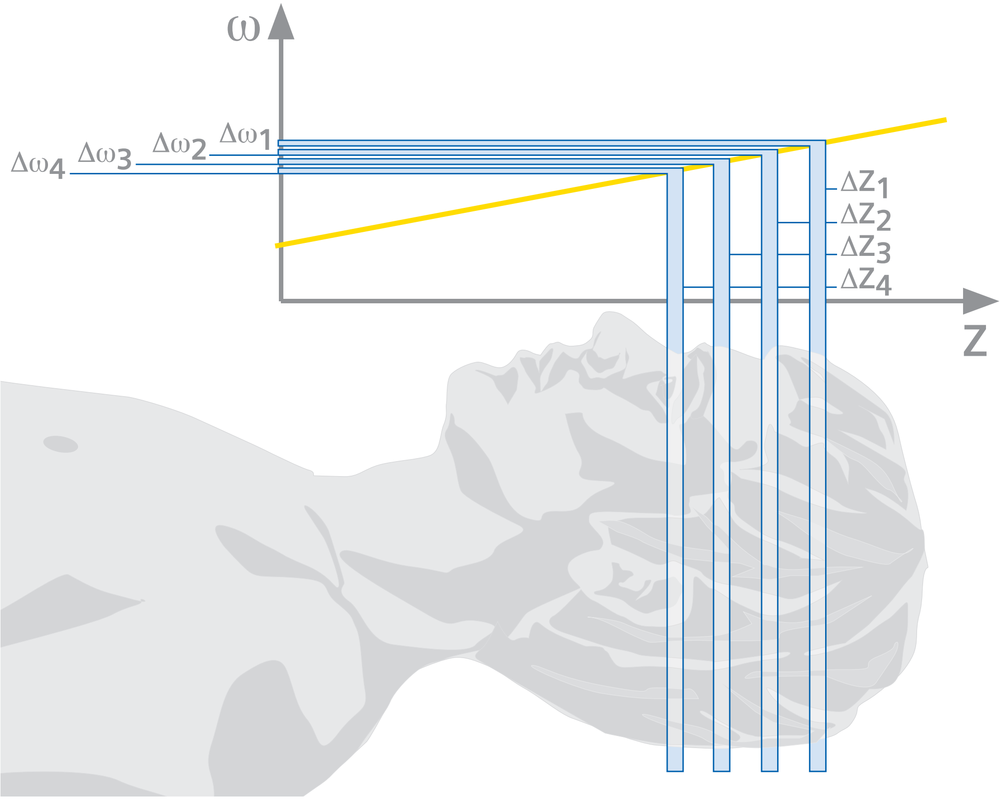
  e.g.: x=30 cm, B_0=1T, G=10 mT/m

  ➔ B = 0.9985 T … 1.0015 T
- rise time: 200-600 µs
- linearity: 40-60 cm
- power: e.g. 500 A at 2000 V in short time, power in MW range
- need liquid cooling
- “make the noise”



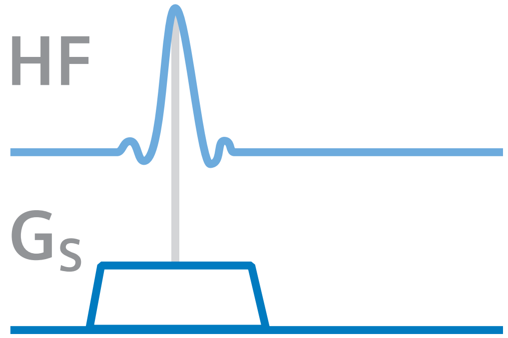
Slice Selection - Motivation

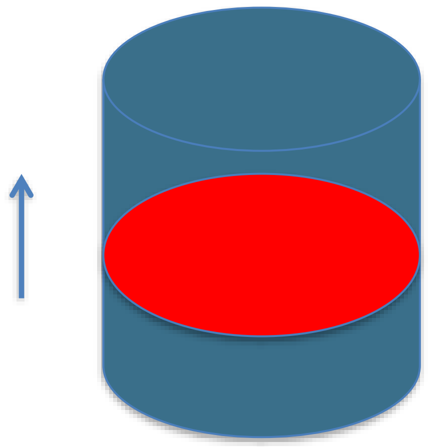
- Magnetic Resonance occurs only when RF pulse matches Larmor frequency
- Larmor frequency proportional to field strength



Slice Selection

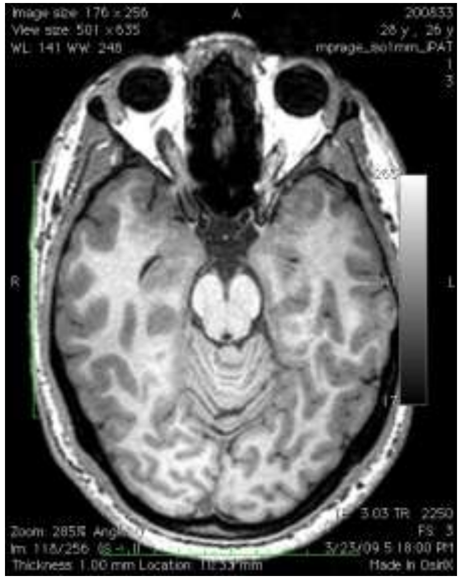
- Simultaneous application of gradients and RF pulses
- Slices can be created in arbitrary orientations (*double oblique*) by simultaneous application of several gradients





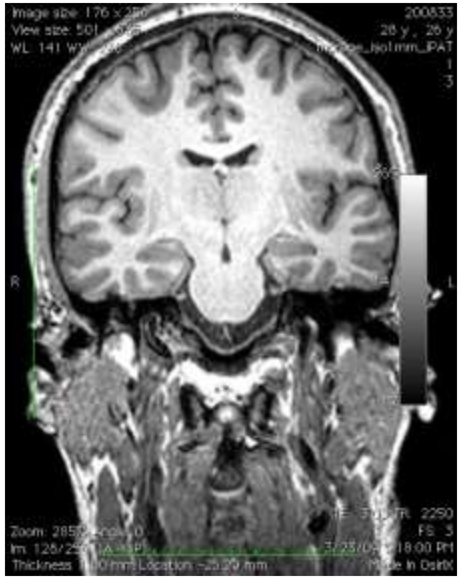
Examples of Oblique Slices

- Achieved through superposition of physical gradients G_x, G_y, G_z



axial slice

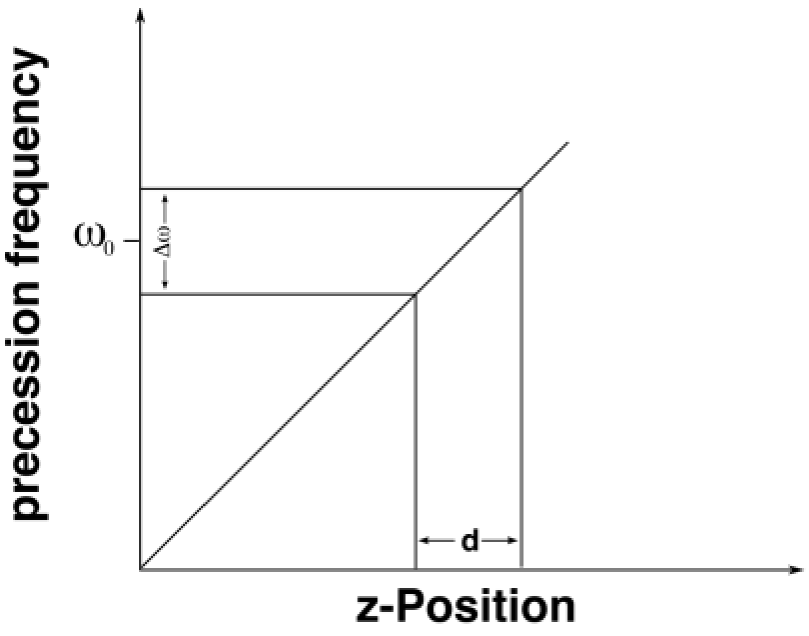
sagittal slice



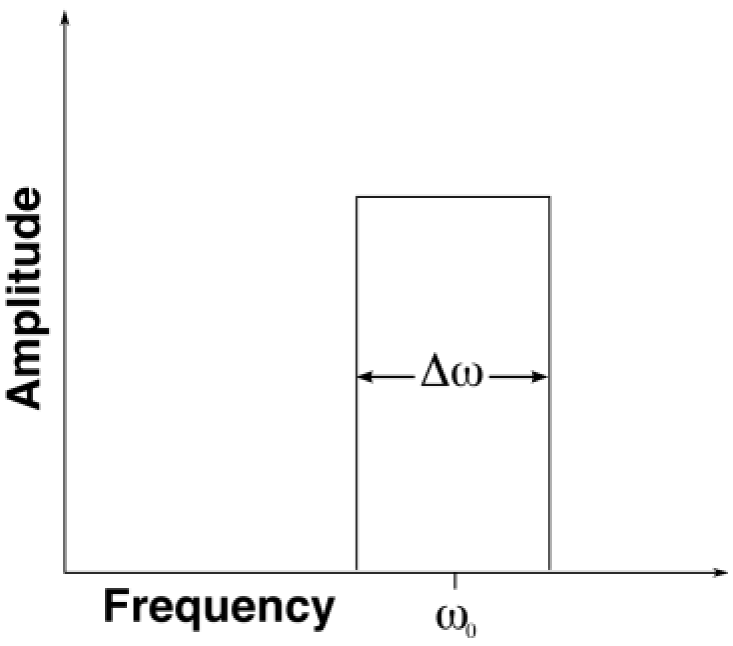
coronal slice

Slice Selection – Pictorial Description

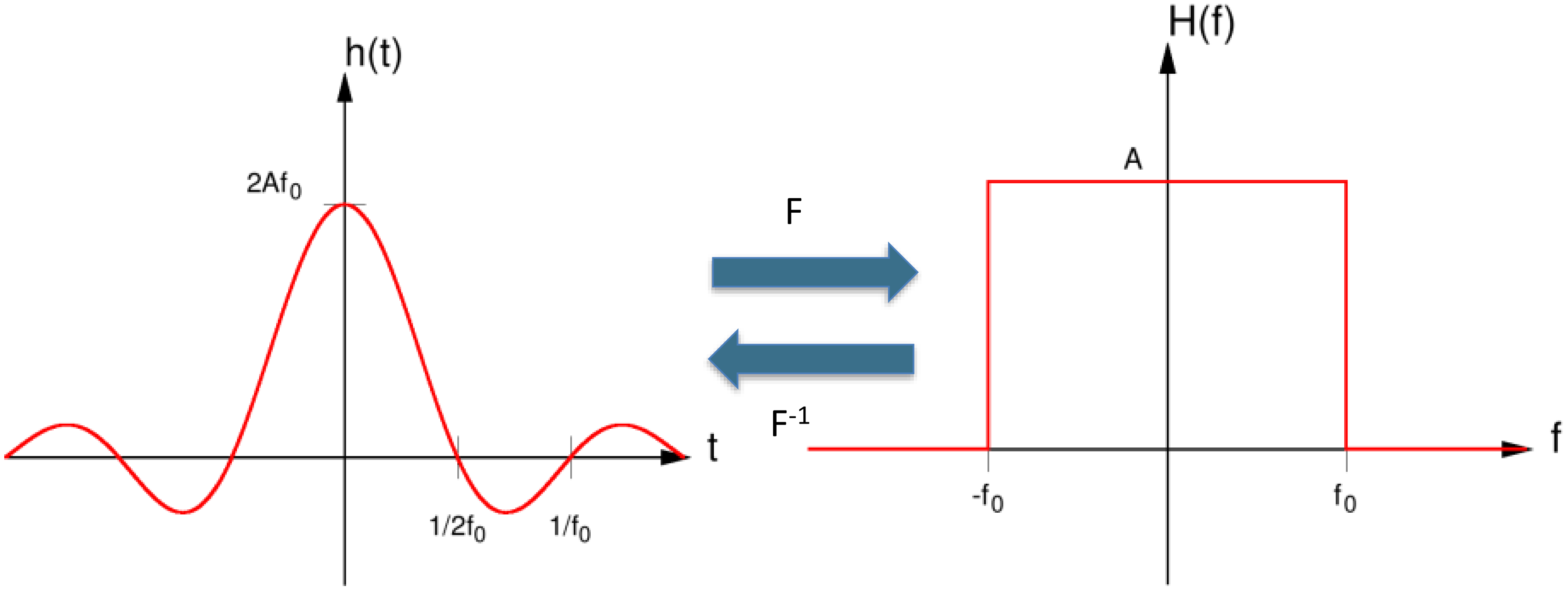
Gradient in z direction



Desired excitation



Exciting a rectangular slice profile



RF pulse

Frequency response

Slice Selection – Slice Thickness

- The RF bandwidth Δf determines the *range of frequencies* contained in a pulse
- The slice thickness is determined by the frequency band of the excitation pulse (as a function of gradient amplitude)

$$\Delta z = \frac{2\pi \Delta f}{\gamma G_z}$$

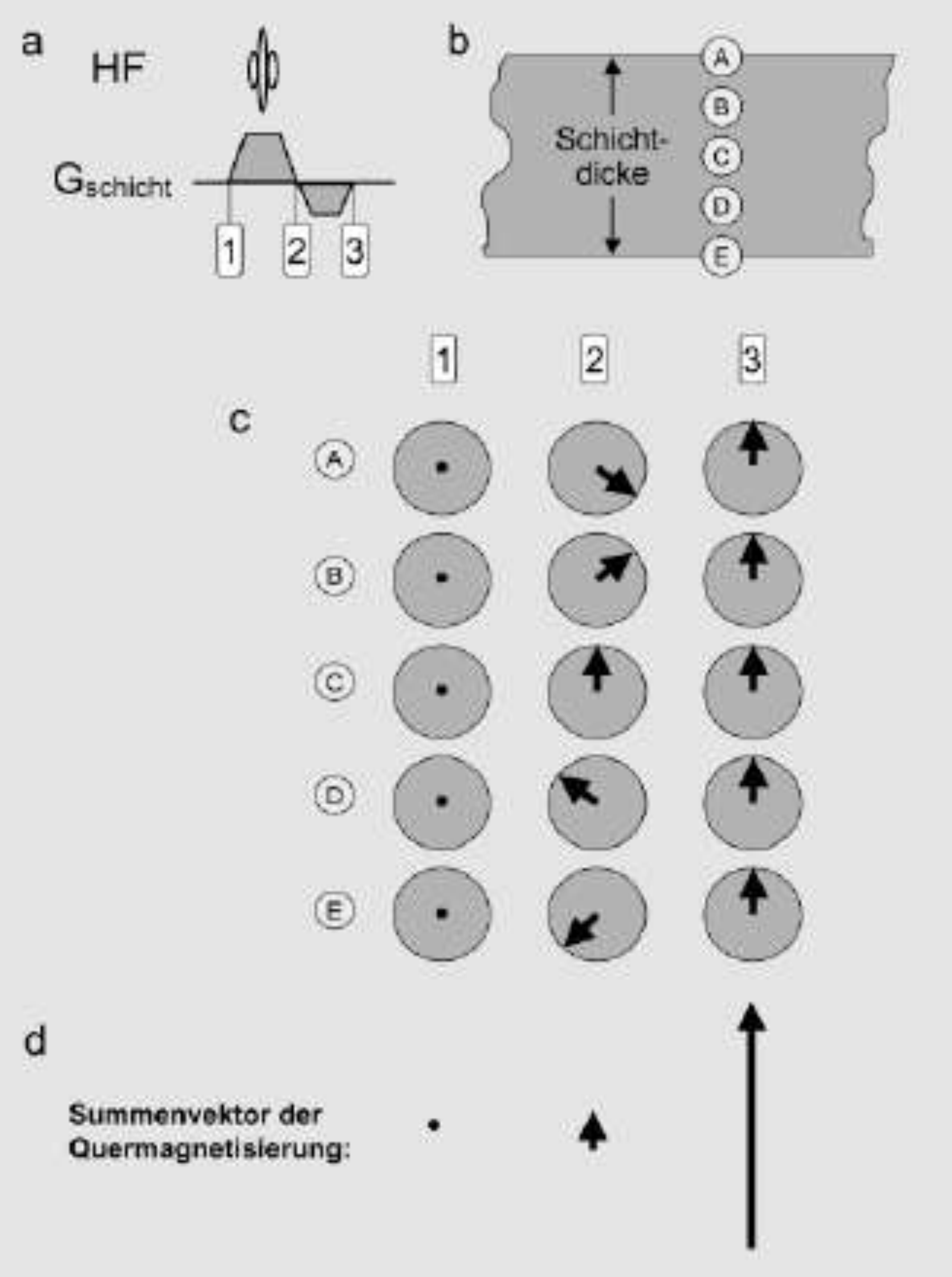
- Practical RF pulses do not have a rectangular freq. response; *Full-Width-Half-Maximum (FWHM)* is usually defined as bandwidth

Slice Refocusing

- Different Larmor frequencies within slice lead to phase dispersion by factor

$$\exp\left(\frac{i\gamma G_z \tau}{2}\right)$$

  - τ = duration of RF pulse
- Removed by application of opposite *z*-gradient (“refocusing lobe” or “slice rewinder”)

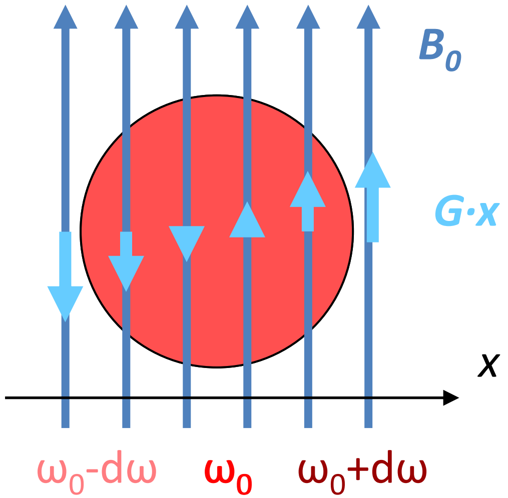


Spatial Encoding

- The basic idea of MRI:

Make the precessional frequency a function of space!

- The "spectrum" then reflects spatial distribution.
- Linear field gradients of the B-field in z-direction, e.g. $G_x = dB_z/dx$

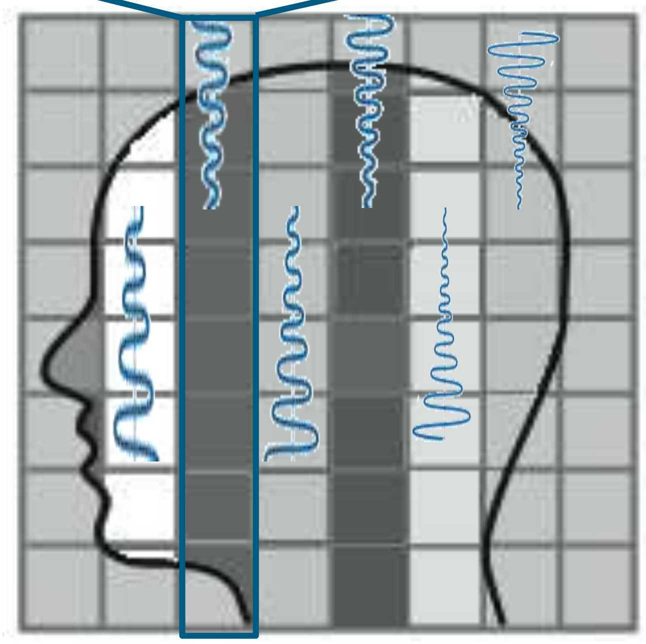


Frequency Encoding

- Frequency encoding: apply gradient during data acquisition
  - "Readout gradient"

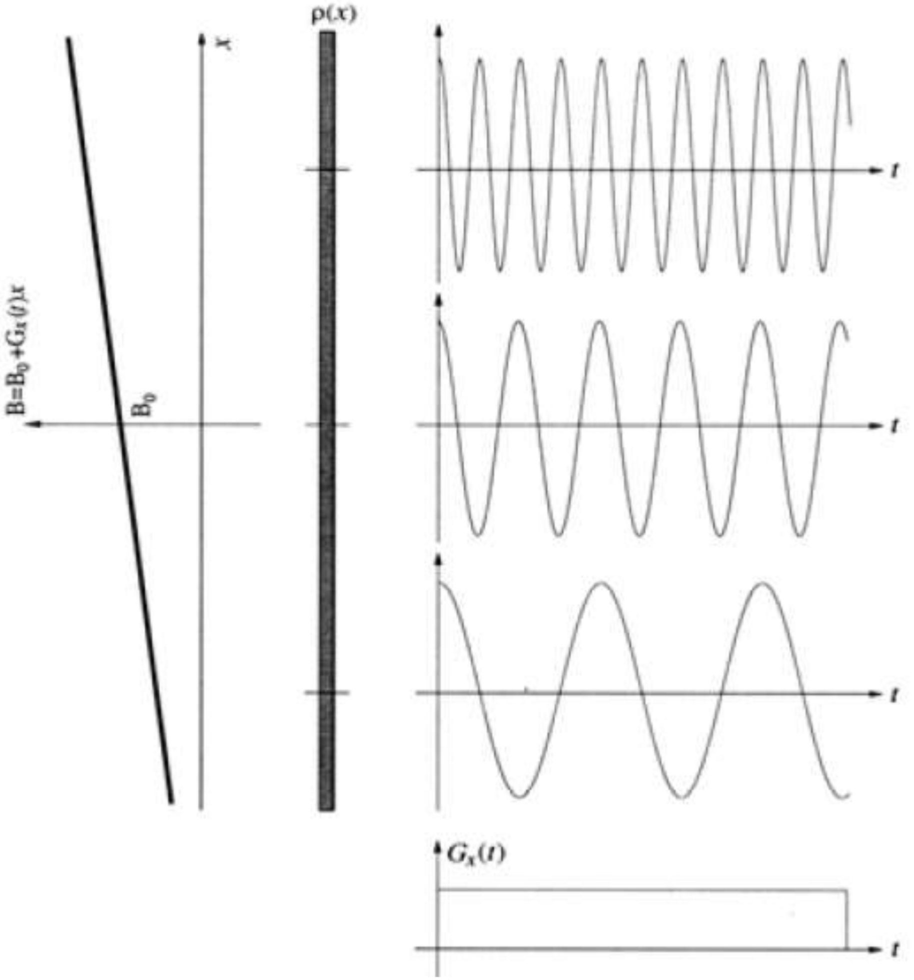
$$\Delta\omega = \gamma G_x x$$

$$M_\perp^n(t) = M_0 e^{-\frac{t}{T_2^*}} e^{-i(\omega_0 + \Delta\omega^n)t}$$



Source: Siemens, Magnete, Spins und Resonanzen

Frequency Encoding

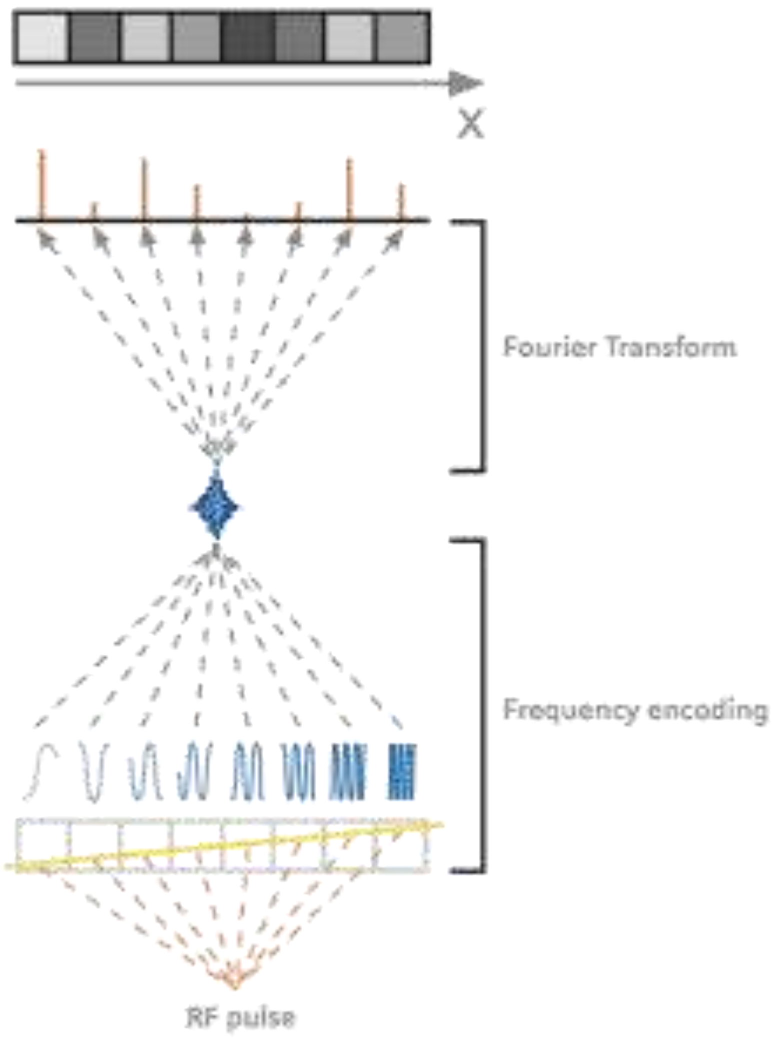


Larmor frequency is a linear function of the spatial coordinate:

$$\omega(x) = \gamma(B_0 + G_x \cdot x)$$

Frequency Encoding

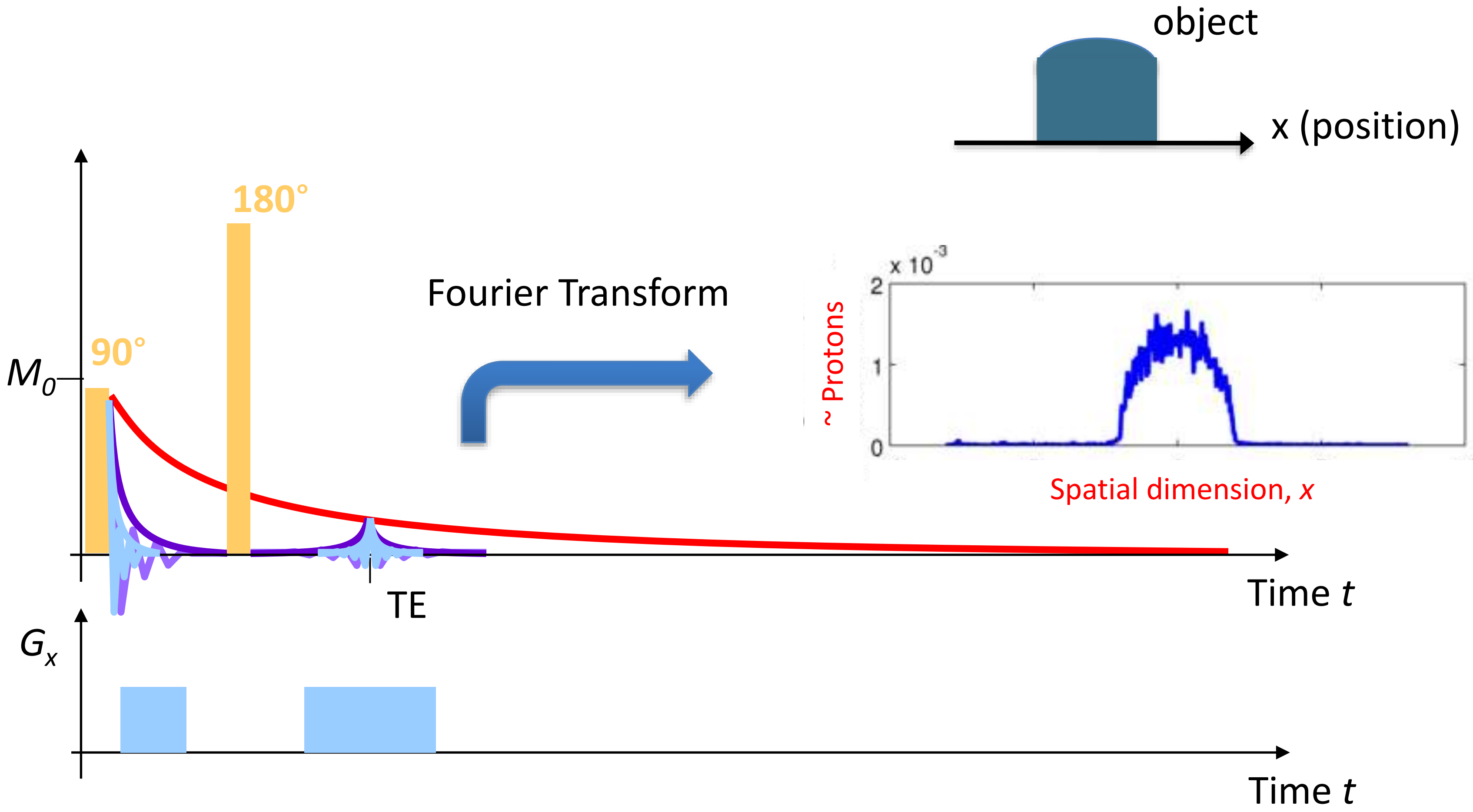
- The Fourier Transformation allows us to transform that frequency information back to spatial location



Source: Siemens, Magnete, Spins und Resonanzen

Spin Echo with Frequency Encoding

Take the spin echo and introduce gradients along *x* => 1D image !



Frequency Encoding: Sampling

- MR signal S(t) is digitized by using an “analog to digital converter” ADC and a discrete timing interval Δt in a total acquisition window t_{aq}
- For the Fourier-analysis of the MR signal there are in total N = t_{aq}/Δt measured data points: S(Δt), S(2Δt), S(3Δt), … , S(NΔt)
- Sampling theorem links Field of View (FOV; maximum object diameter) to sampling interval Δt:

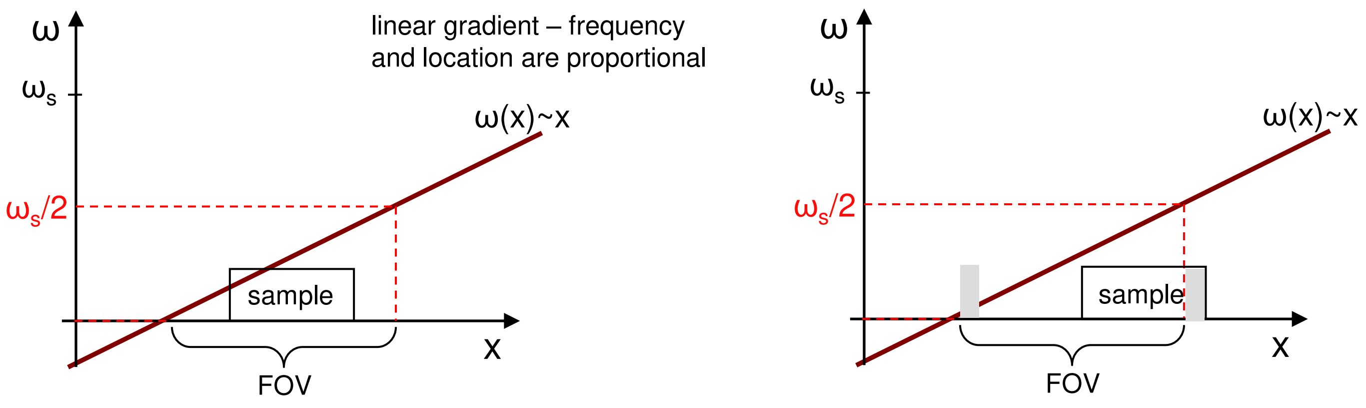
$$f_{\max} = \frac{\gamma G_x}{2\pi}\frac{\mathrm{FOV}}{2} < \frac{1}{2\Delta t} \Rightarrow \mathrm{FOV} = \frac{2\pi}{\gamma G_x \Delta t}$$

- Spatial resolution Δx results as Δx = FOV / N
- *Example*: with N = 256, Δt = 30 μs, Gx = 1.566 mT/m the spatial resolution in x-direction (pixel resolution in x) is:

Δx = 1.953 mm and X = N Δx = 50 cm (= field-of-view FOV)

Aliasing in MRI

- When using frequency encoding, **aliasing manifests as a fold-over** of objects outside the FOV to incorrect locations.



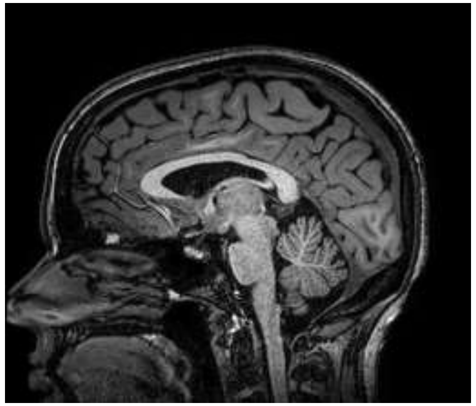
$$\omega_{max} < \frac{\omega_s}{2}$$

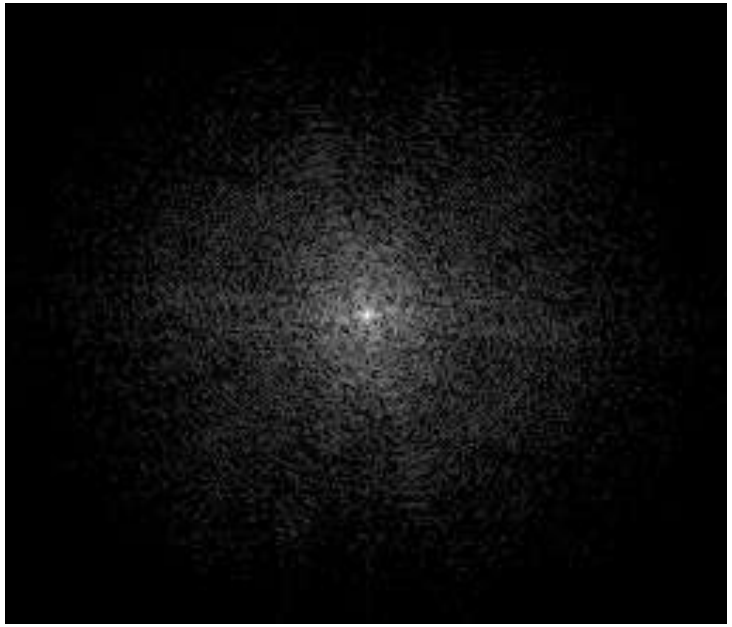
no aliasing

$$\omega_{max} > \frac{\omega_s}{2}$$

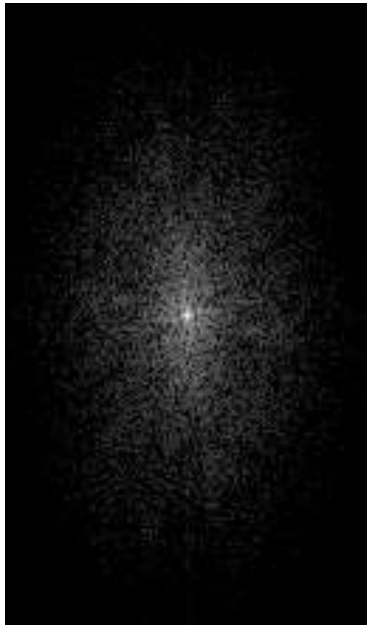
aliasing

Aliasing - Illustration

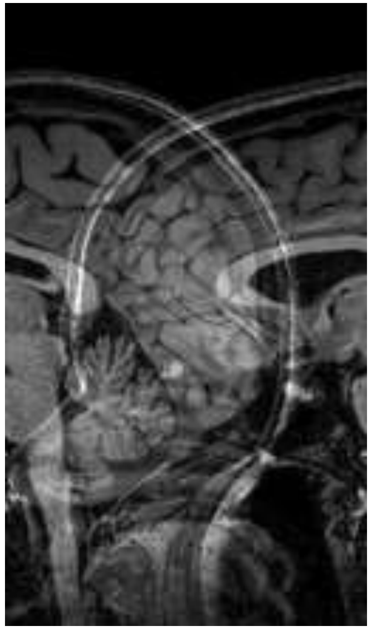




Original k space representation



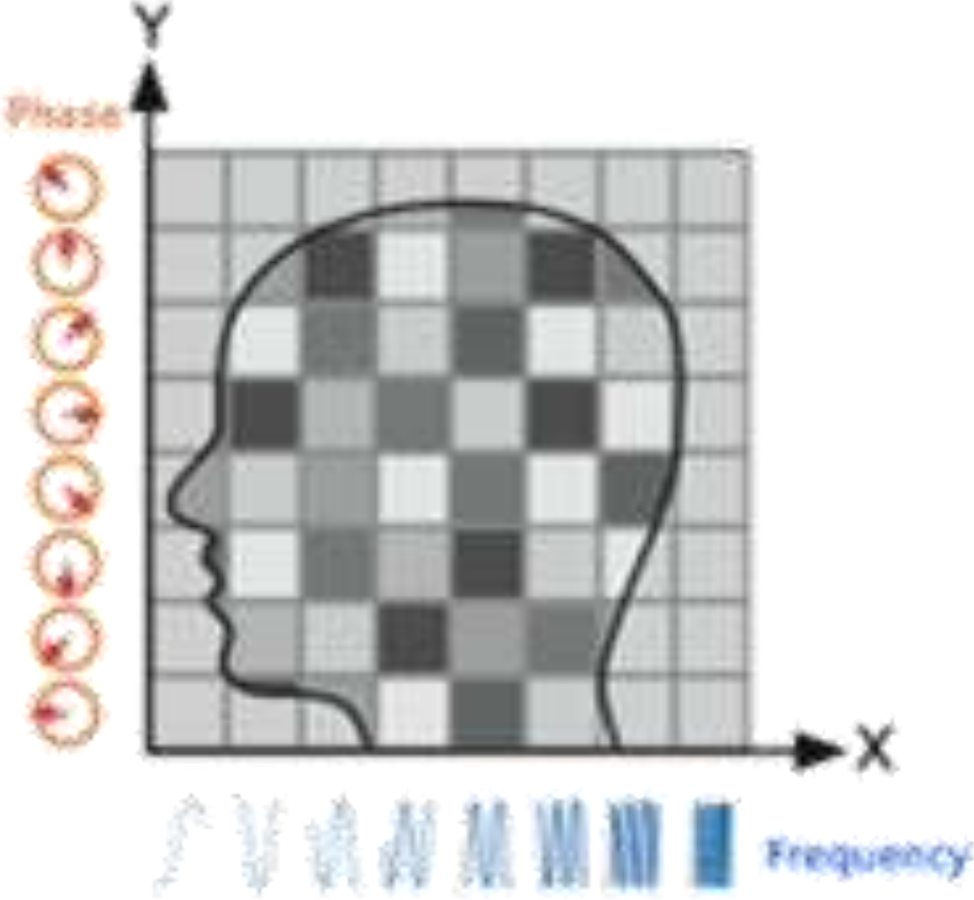
Remove every other column



Aliased image

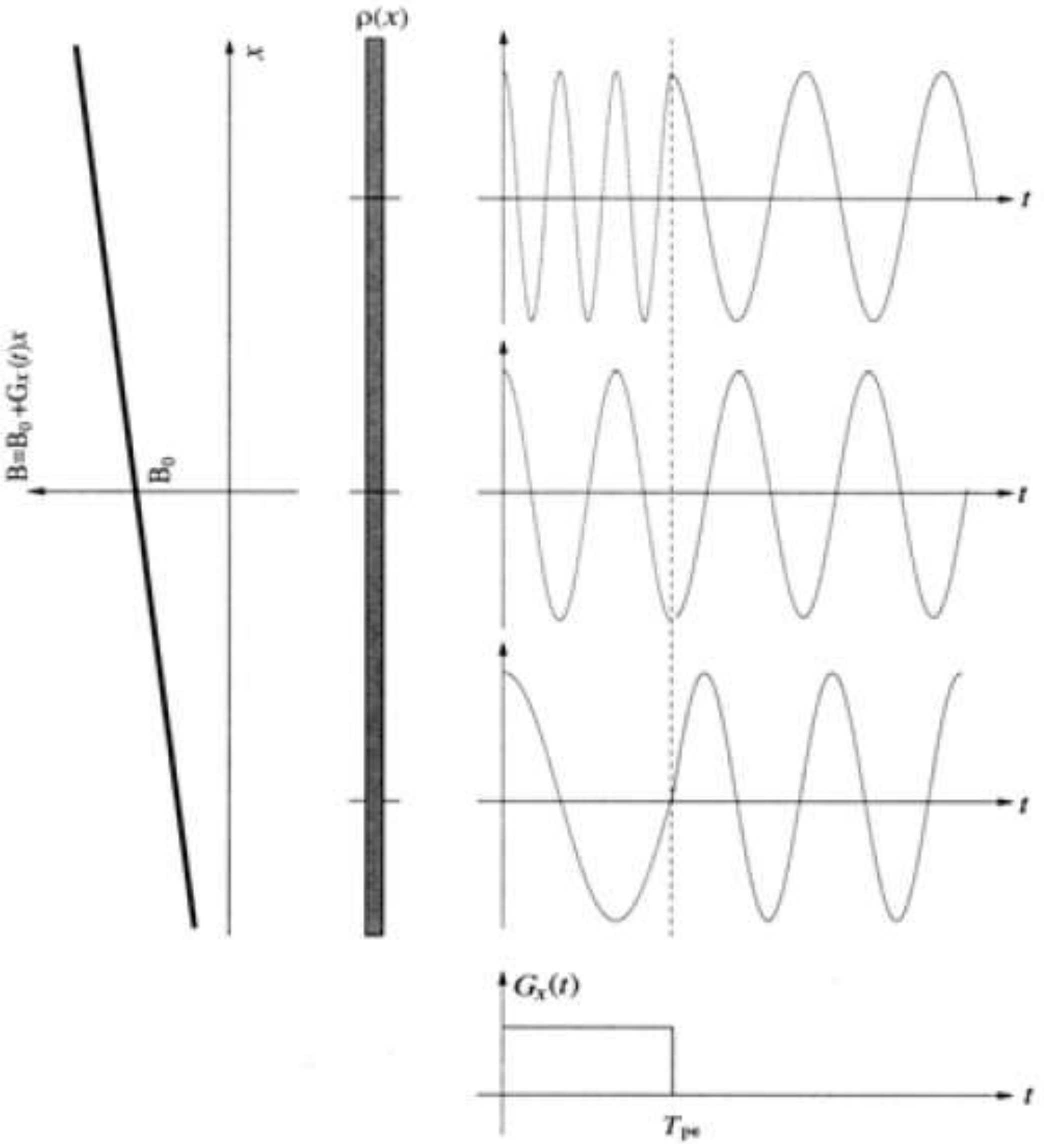
Phase Encoding

- Frequency encoding is applied to one spatial dimension, e.g. the x-axis. How to encode the y-axis?
- Switch on the y-gradient for a short time *between excitation and readout,* in order to modulate the spins’ phase in y direction.
- *Repetition of the process with linearly varying phase* again encodes a frequency!



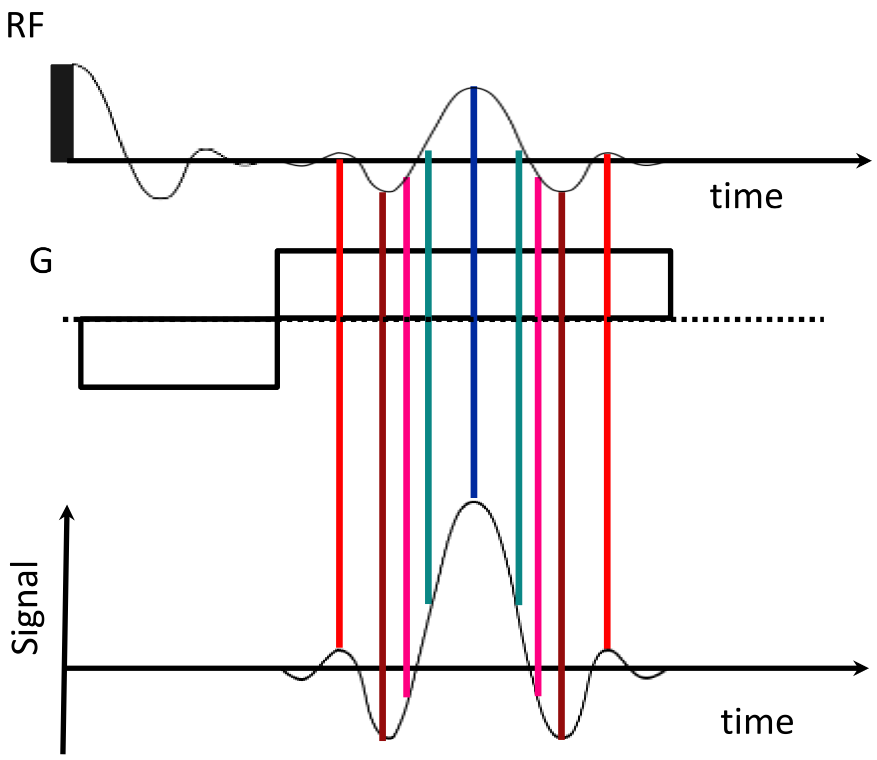
Source: Siemens, Magnete, Spins und Resonanzen

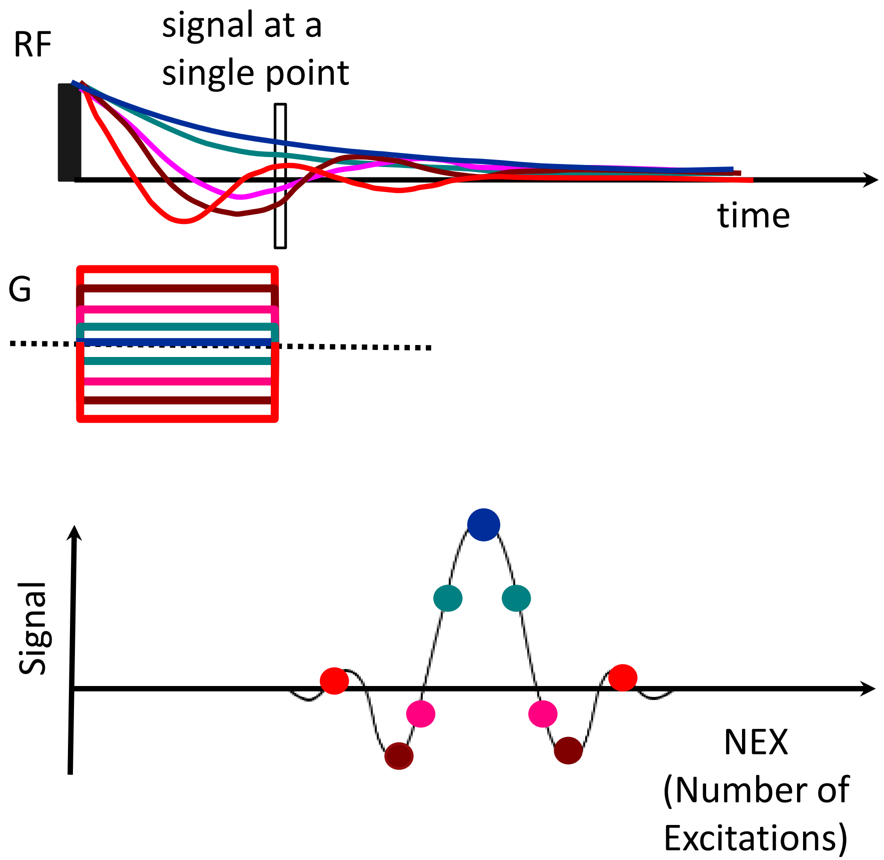
Phase Encoding



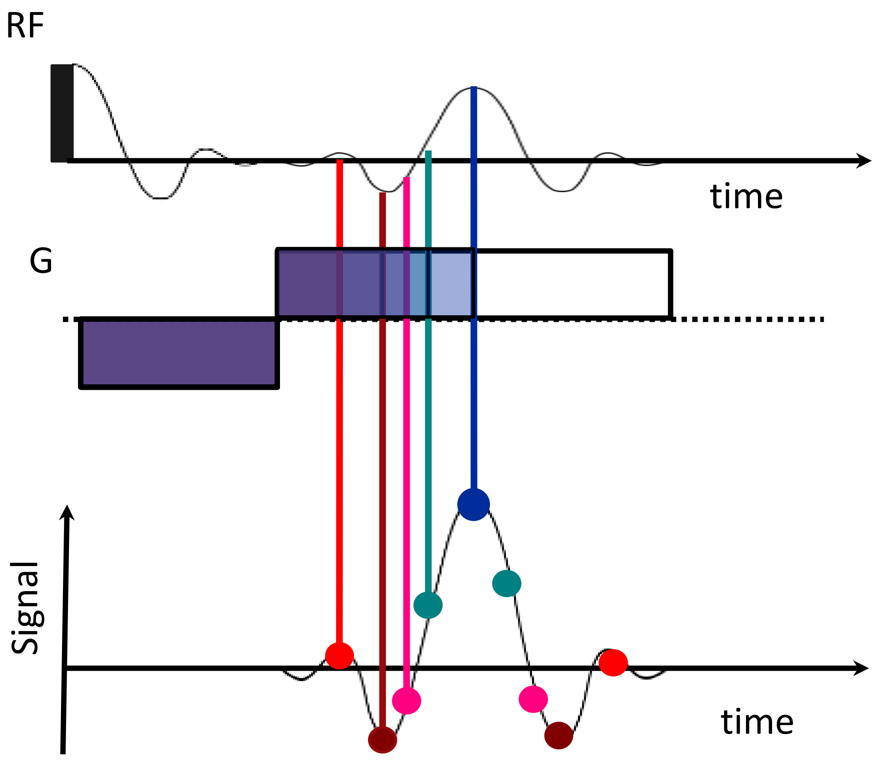
- Phase encoding stamps an initial phase angle onto the excited spins
- After switching off the phase encoding gradient, precession continues at a fixed frequency $\omega_{0,}$ but with different phase
- Phase information of an activated MR-signal is linearly dependent on spatial coordinate

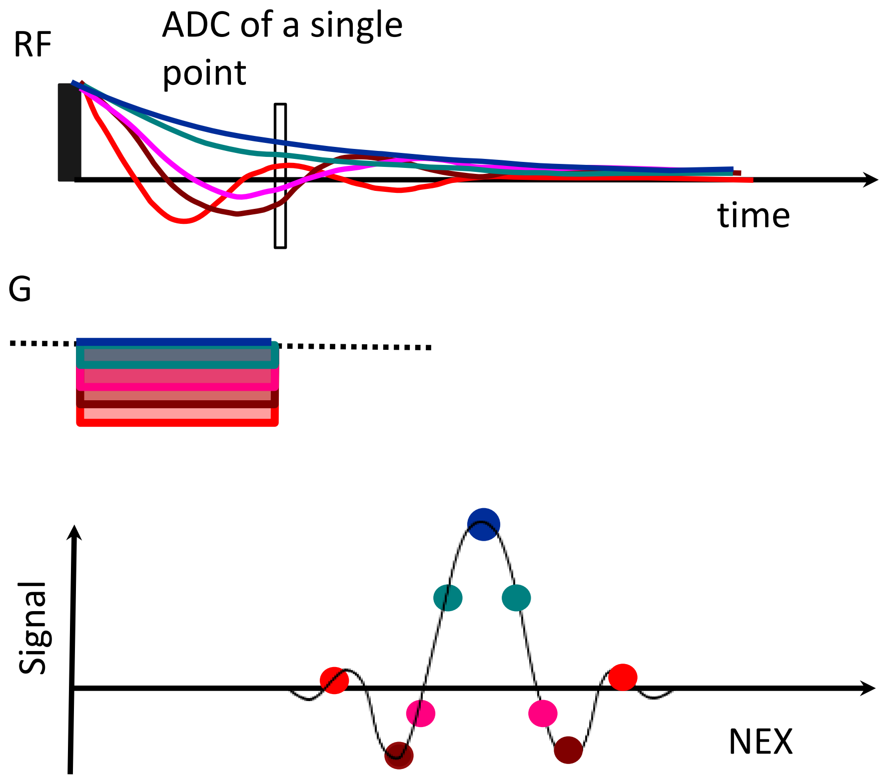
Frequency Encoding vs. Phase Encoding





Frequency Encoding vs. Phase Encoding





Signal Equation – Slice Selection

- The overall signal after a 90° pulse is given as:

neglected!

$$s(t) = \iiint\limits_{xyz} M_0(x,y,z) \exp\left(\cancel{-\frac{t}{T_2^*}}\right) \exp(-i\omega_0 t)\, dx\, dy\, dz$$

equilibrium magnetisation ~ proton density

spin-spin relaxation

precession of the spins

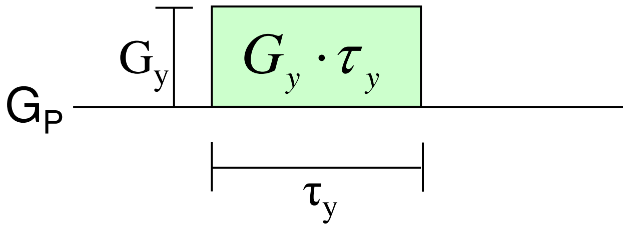
- 2D integral after slice selection:

$$s(t) = \iint\limits_{xy} m(x,y) \exp(-i\omega_0 t)\, dx\, dy$$

with $m(x,y) = \int_{z_0 - \frac{\Delta z}{2}}^{z_0 + \frac{\Delta z}{2}} M_0(x,y,z)\,dz$

Signal Equation – Phase Encoding

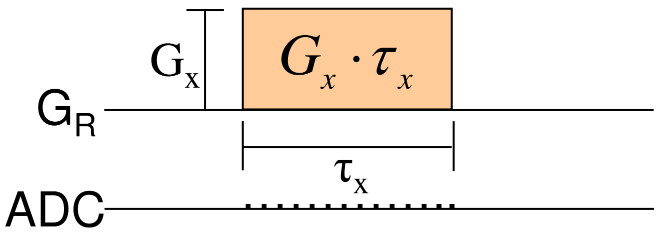
If we apply a linear gradient in y-direction between excitation and readout, it changes the signal equation as follows:



$$s(G_y, \tau_y, t) = \iint_{xy} m(x, y) \exp(-i\omega_0 t) \exp(-i\gamma G_y \tau_y y)\, dx dy$$

Signal Equation – Frequency Encoding

Similarly, a frequency encoding gradient in x-direction while readout changes the signal equation:



$$s\left(G_x, G_y, \tau_x, \tau_y, t\right) =$$

$$\iint\limits_{xy} m(x, y) \exp(-i\omega_0 t) \exp(-i\gamma G_x \tau_x x) \exp\left(-i\gamma G_y \tau_y y\right) dxdy$$

Signal Equation - Demodulation

$$\iint_{xy} m(x,y)\, \exp(-i\omega_0 t)\, \exp(-i\gamma G_x \tau_x x)\, \exp\left(-i\gamma G_y \tau_y y\right) dxdy$$

$\Vert$

demodulation in frequency by ω_0

(change from laboratory to rotating frame)

$\Downarrow$

$$\iint_{xy} m(x,y)\, \cancel{\exp(-i\omega_0 t)}\, \exp(-i\gamma G_x \tau_x x)\, \exp\left(-i\gamma G_y \tau_y y\right) dxdy$$

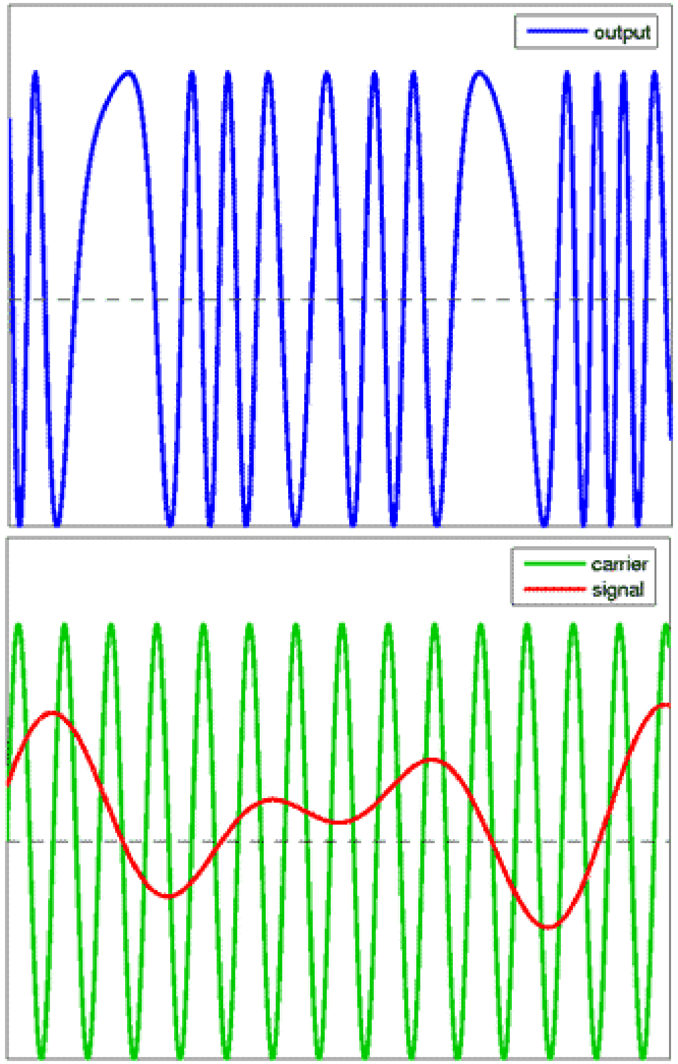


Figure from: http://en.wikipedia.org/wiki/Phase_modulation

Signal Equation – Fourier Interpretation

Thus, the demodulated signal from the slice selectively excited sample which was exposed to linear gradients in x- and y-direction is:

$$s\left(G_x, G_y, \tau_x, \tau_y\right) = \iint_{xy} m(x, y) \exp(-i\gamma G_x \tau_x x) \exp\left(-i\gamma G_y \tau_y y\right) dxdy$$

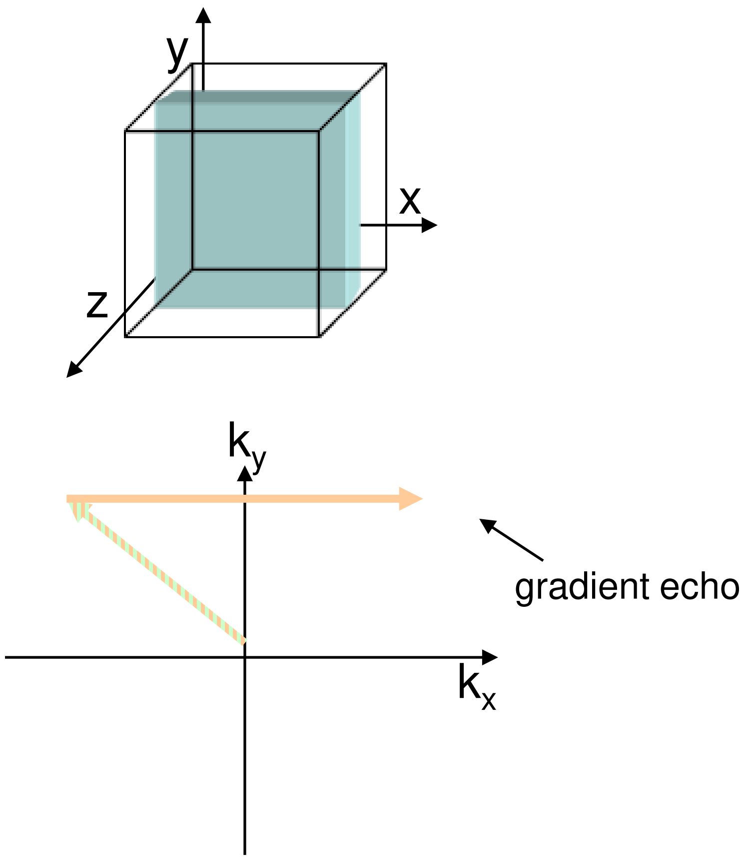
Our signal is the Fourier transform of the magnetisation!

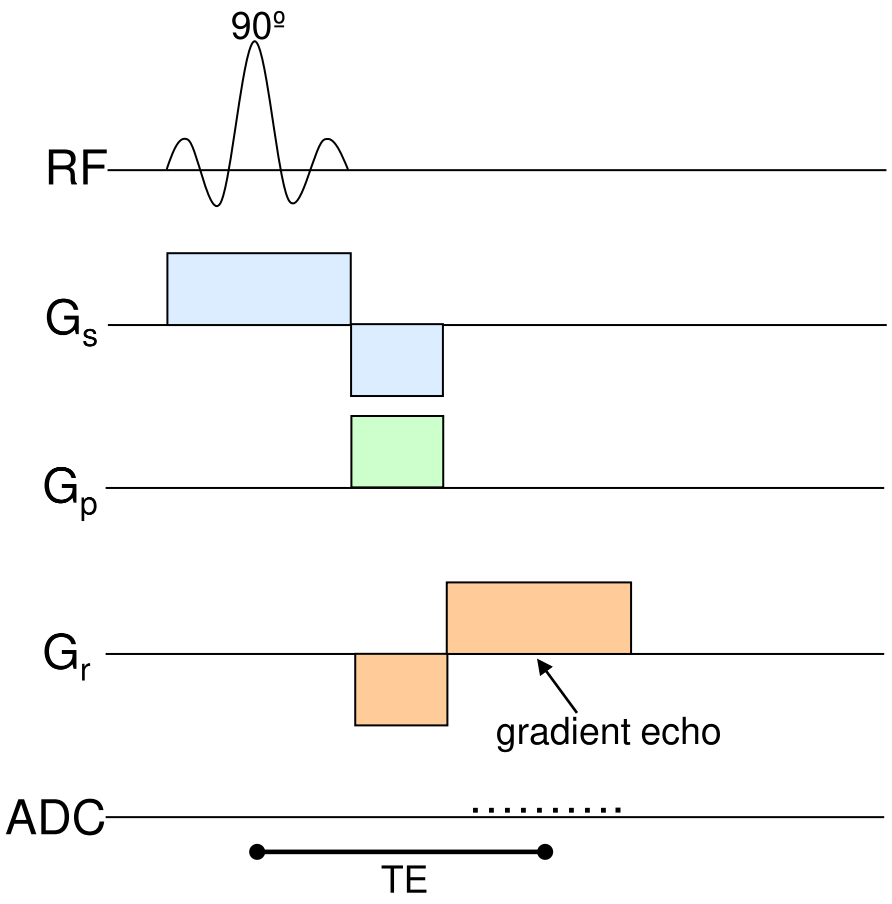
Definition: $k_x(G_x, \tau_x) = \frac{\gamma}{2\pi} G_x \tau_x, \qquad k_y\left(G_y, \tau_y\right) = \frac{\gamma}{2\pi} G_y \tau_y$

$$\Downarrow$$

$$\boxed{s\left(k_x, k_y\right) = \iint_{xy} m(x, y) \exp(-i2\pi k_x x) \exp\left(-i2\pi k_y y\right) dxdy}$$

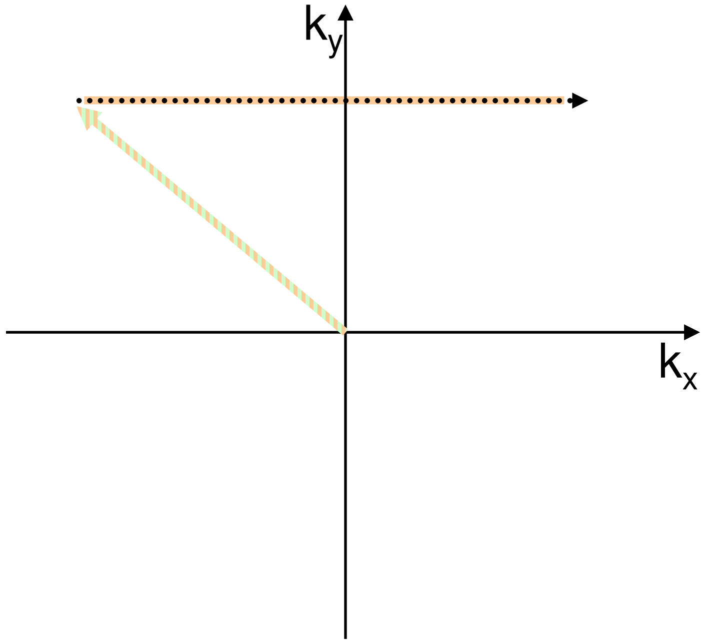
Gradient Echo MRI

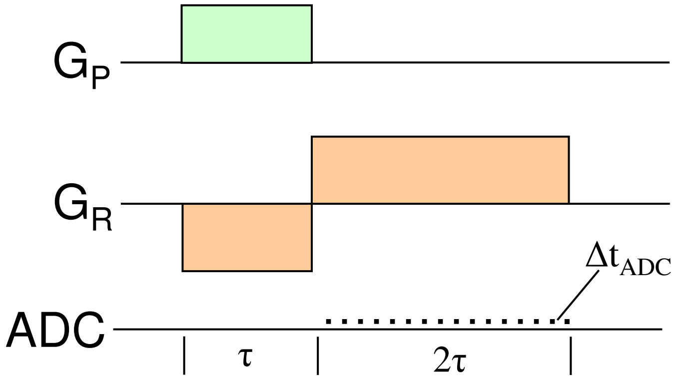




k-space: Sampling

The combination of phase and frequency encoding we just learned about acquires **one line** of *k*-space*:*



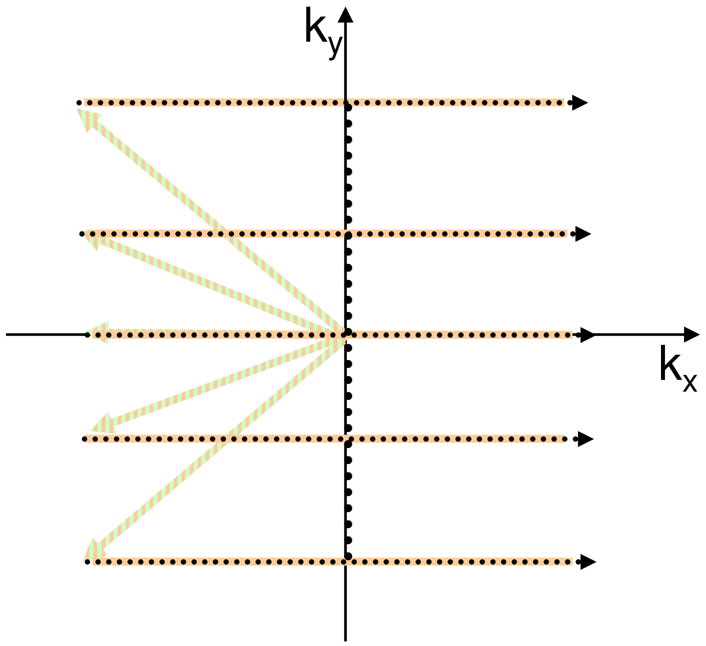


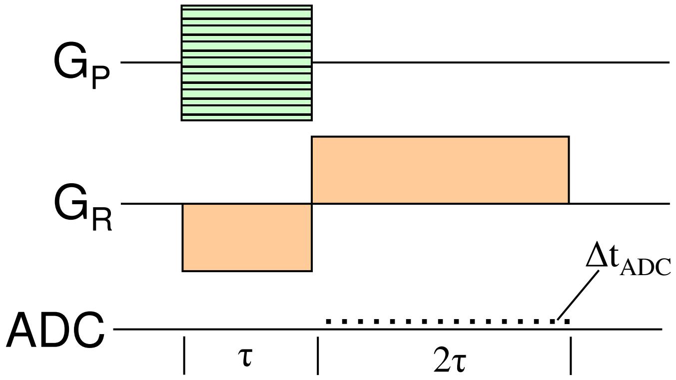
$$k_x = \frac{\gamma}{2\pi}\left(-G_x\tau + G_x\{0, \Delta t_{ADC}, 2\Delta t_{ADC}, \ldots, 2\tau\}\right)$$

$$k_y = \frac{\gamma}{2\pi}G_y\tau = const.$$

k-space: Sampling

The sequence is repeated with different y-gradient amplitudes to acquire **a whole** 2-dimensional *k*-space.



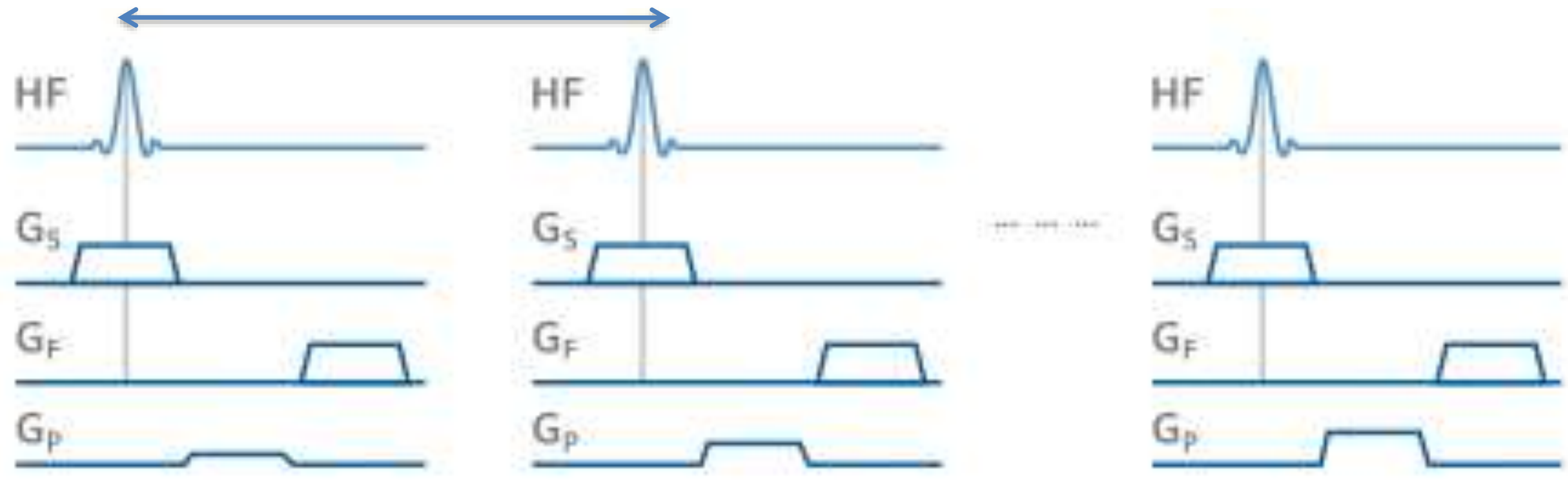


$$k_x = \frac{\gamma}{2\pi}\left(-G_x\tau + G_x\{0, \Delta t_{ADC}, 2\Delta t_{ADC}, \ldots, 2\tau\}\right)$$

$$k_y = \frac{\gamma}{2\pi}\{G_{y,\max}, \ldots, G_{y,\min}\}\tau$$

Timing for Phase and Frequency Encoding

TR (repetition time)



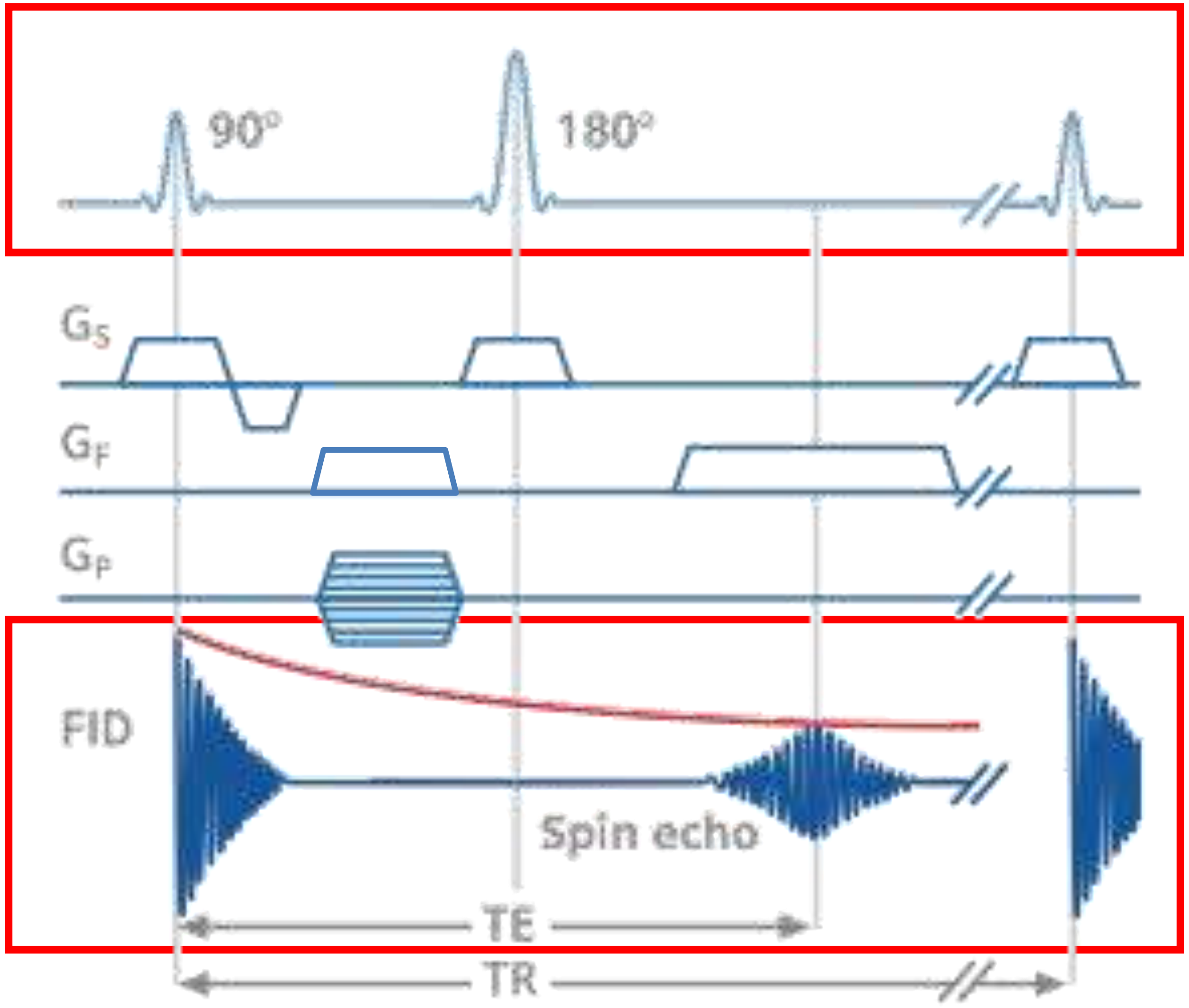
PE step 1 PE step 2 … PE step N

Wait for T1 relaxation

The total acquisition time is given by the repetition time, TR, times the number of phase encode steps!

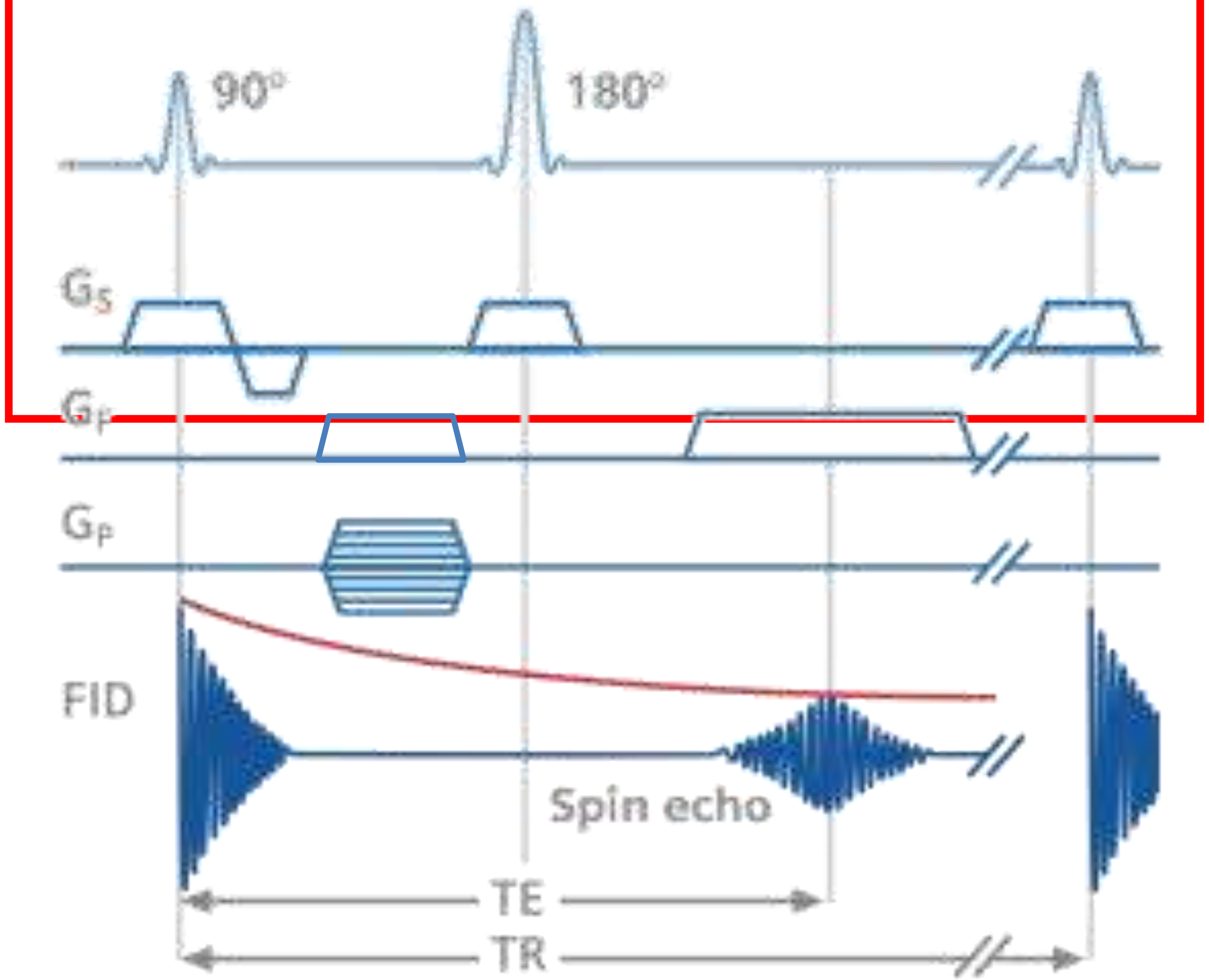
Source: Siemens, Magnete, Spins und Resonanzen

Spin Echo Sequence: Echo Generation



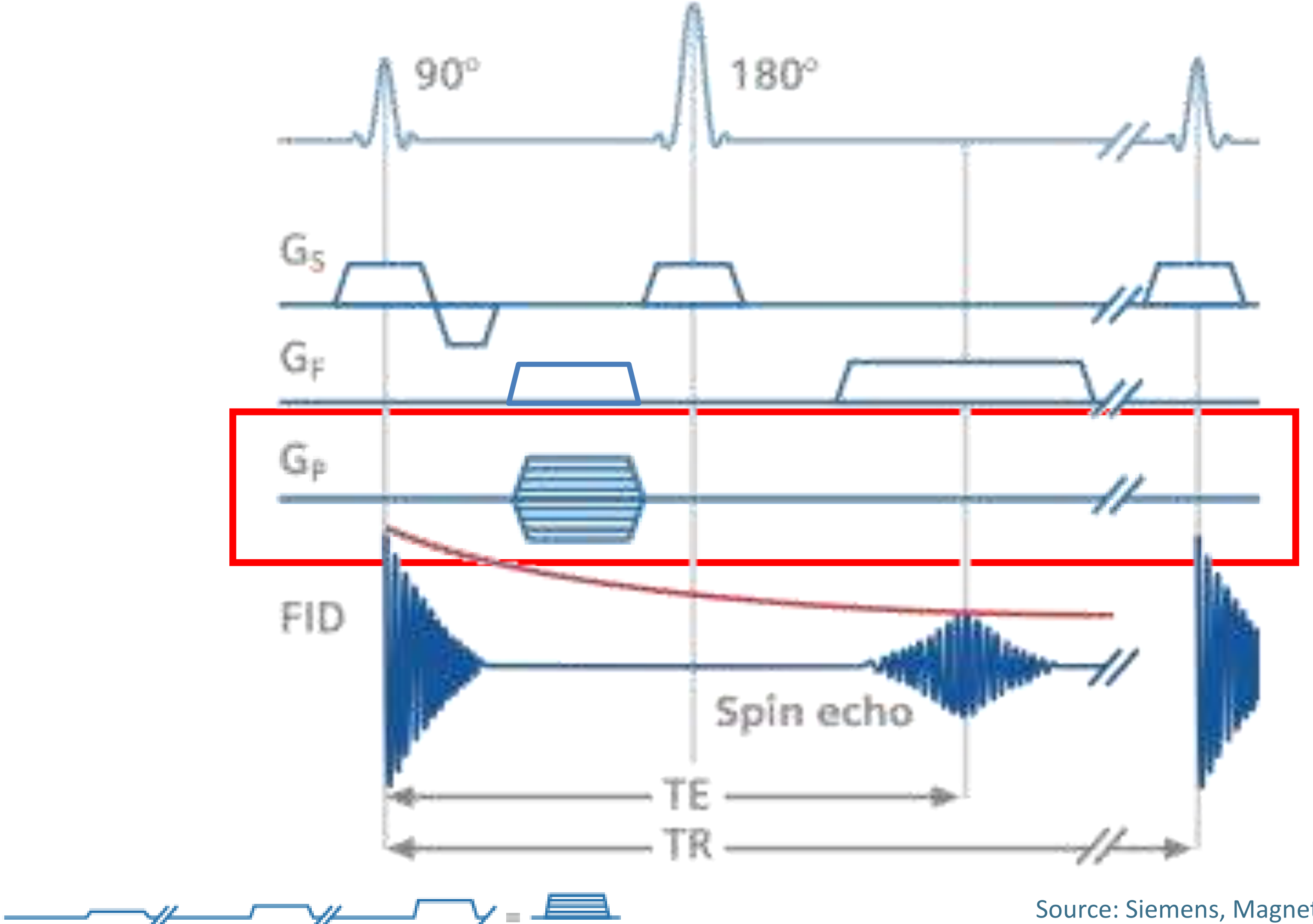
Source: Siemens, Magnete, Spins und Resonanzen

Spin Echo Sequence: Slice Selection



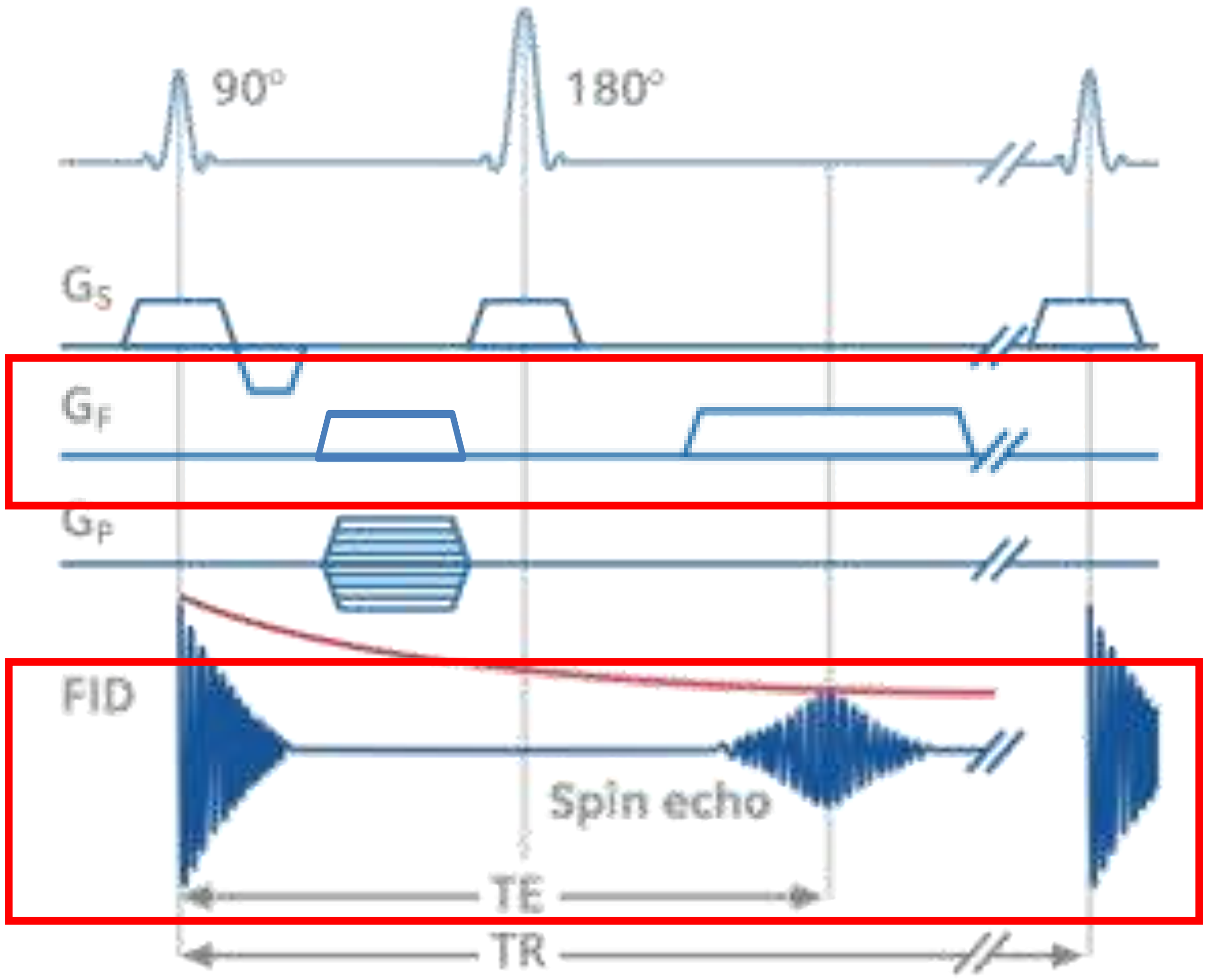
Source: Siemens, Magnete, Spins und Resonanzen

Spin Echo Sequence: Phase Encoding



Source: Siemens, Magnete, Spins und Resonanzen

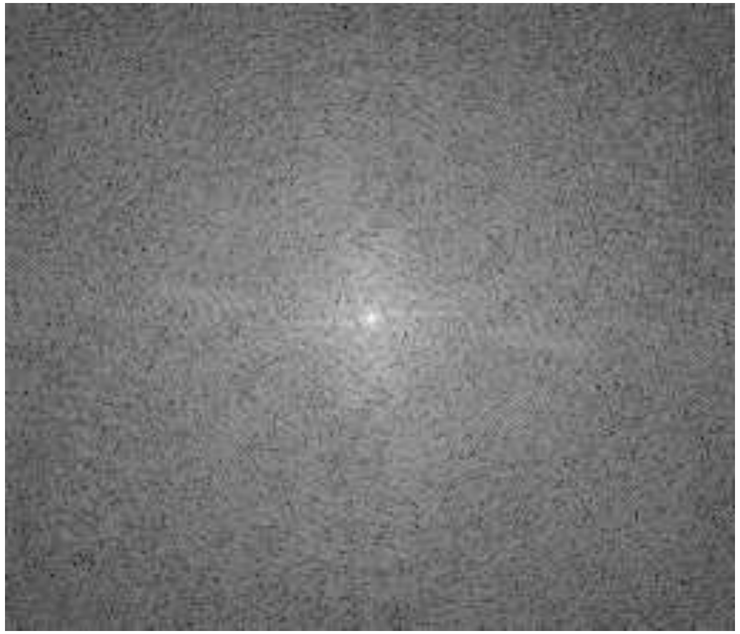
Spin Echo Sequence: Frequency Encoding



Source: Siemens, Magnete, Spins und Resonanzen

Image Reconstruction

Application of inverse 2D Fourier Transform:



k-space

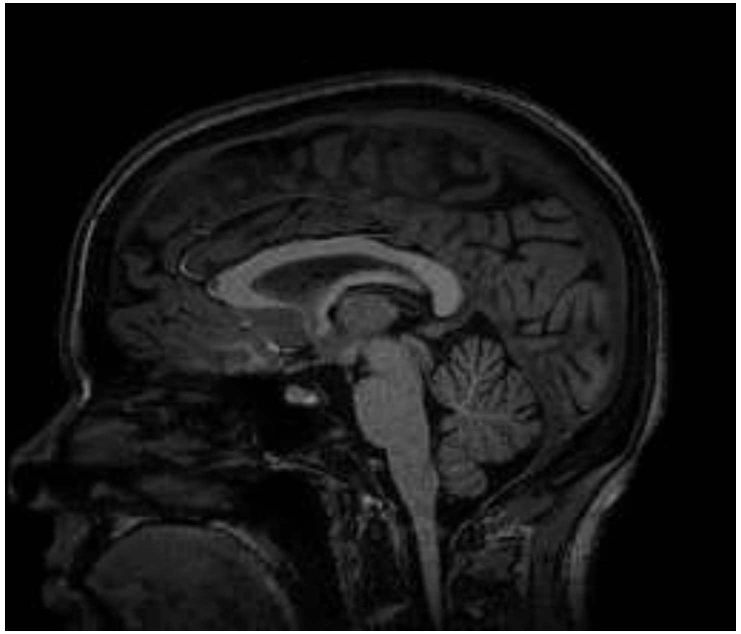
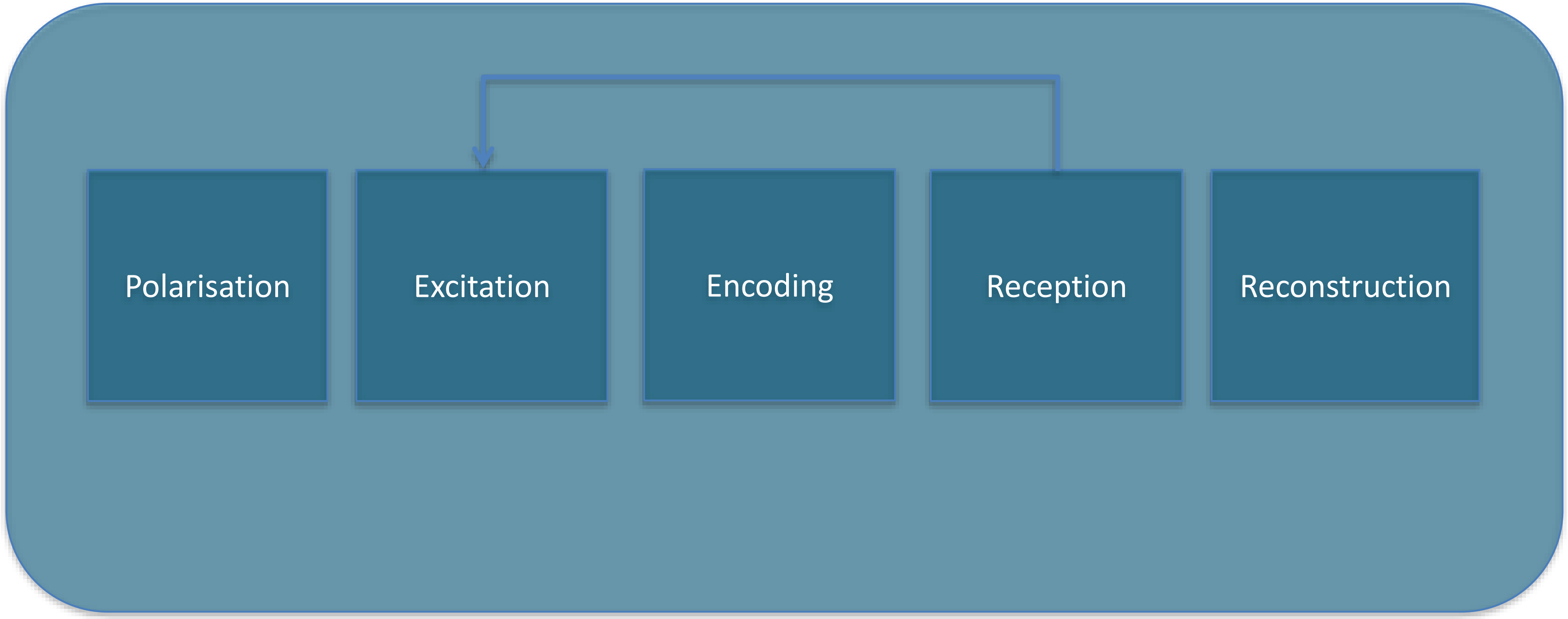


Image space

In theory, it should be enough to acquire half of *k*-space.

In practice, full sampling improves SNR.

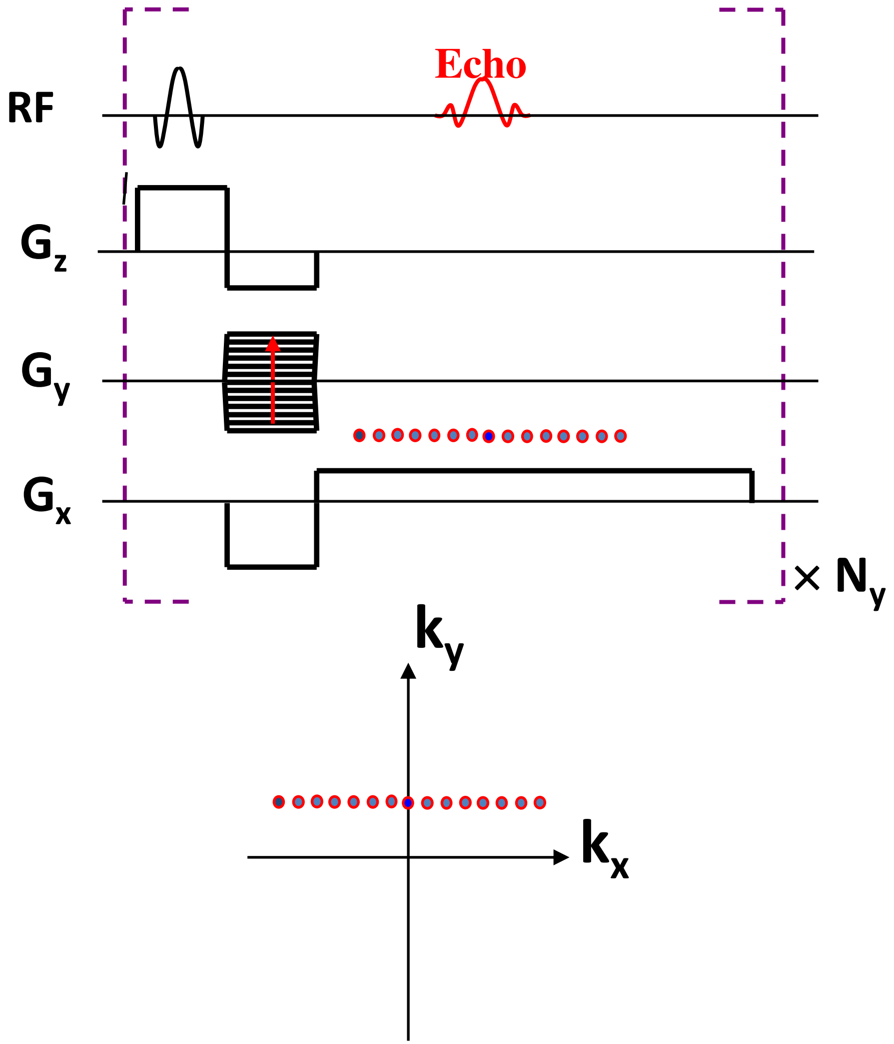
The Whole Picture



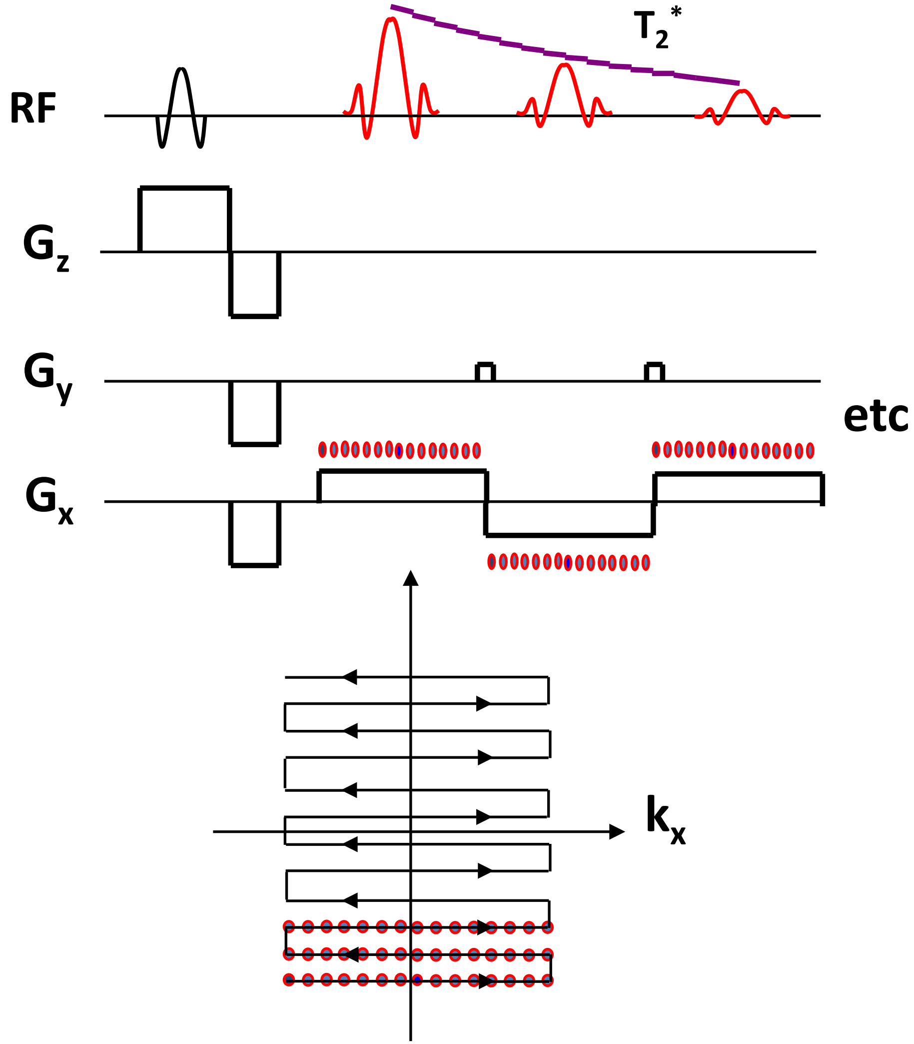
Steps of an MRI experiment

EPI: single shot imaging

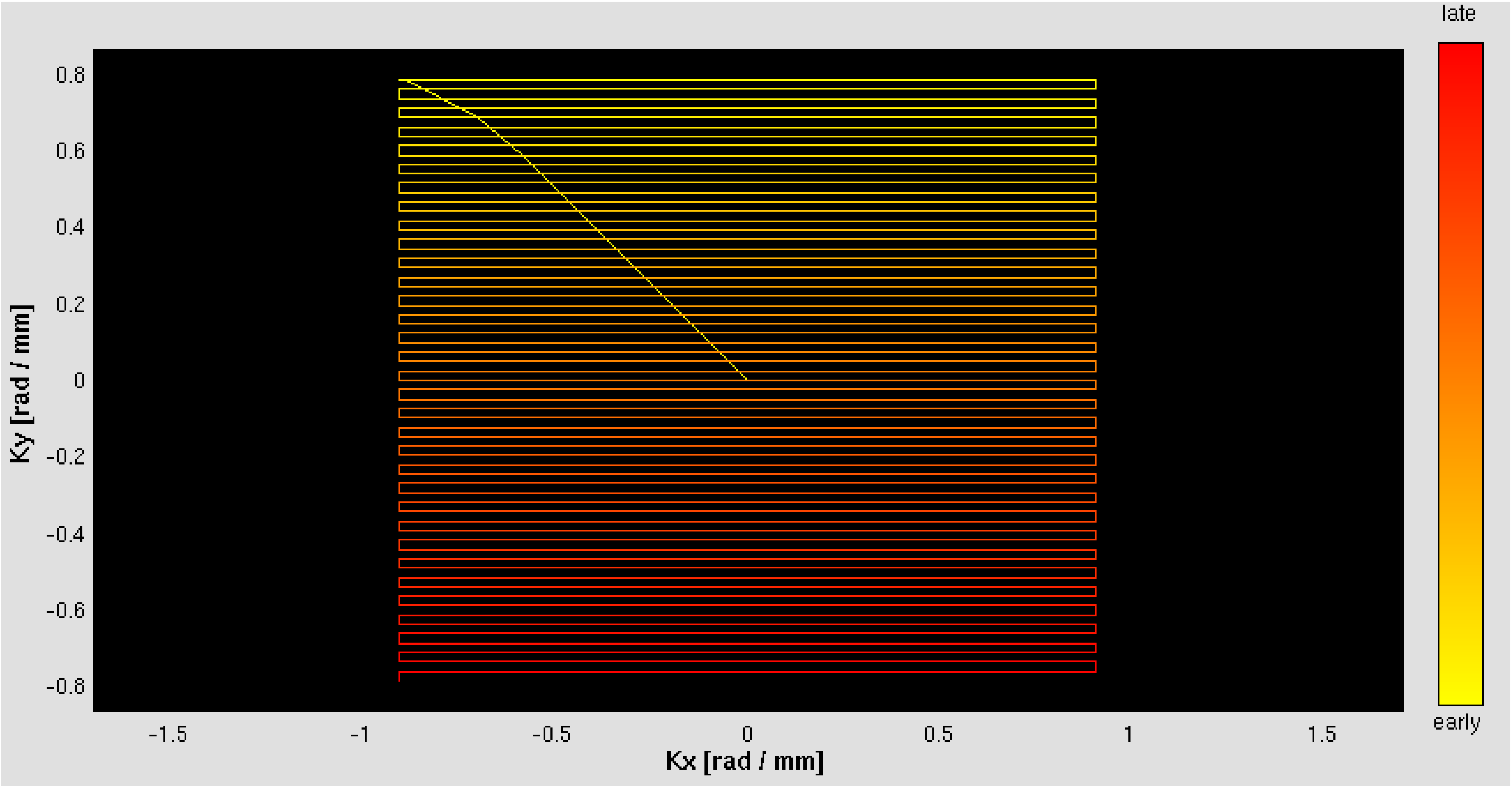
conventional MRI



Echo Planar Imaging

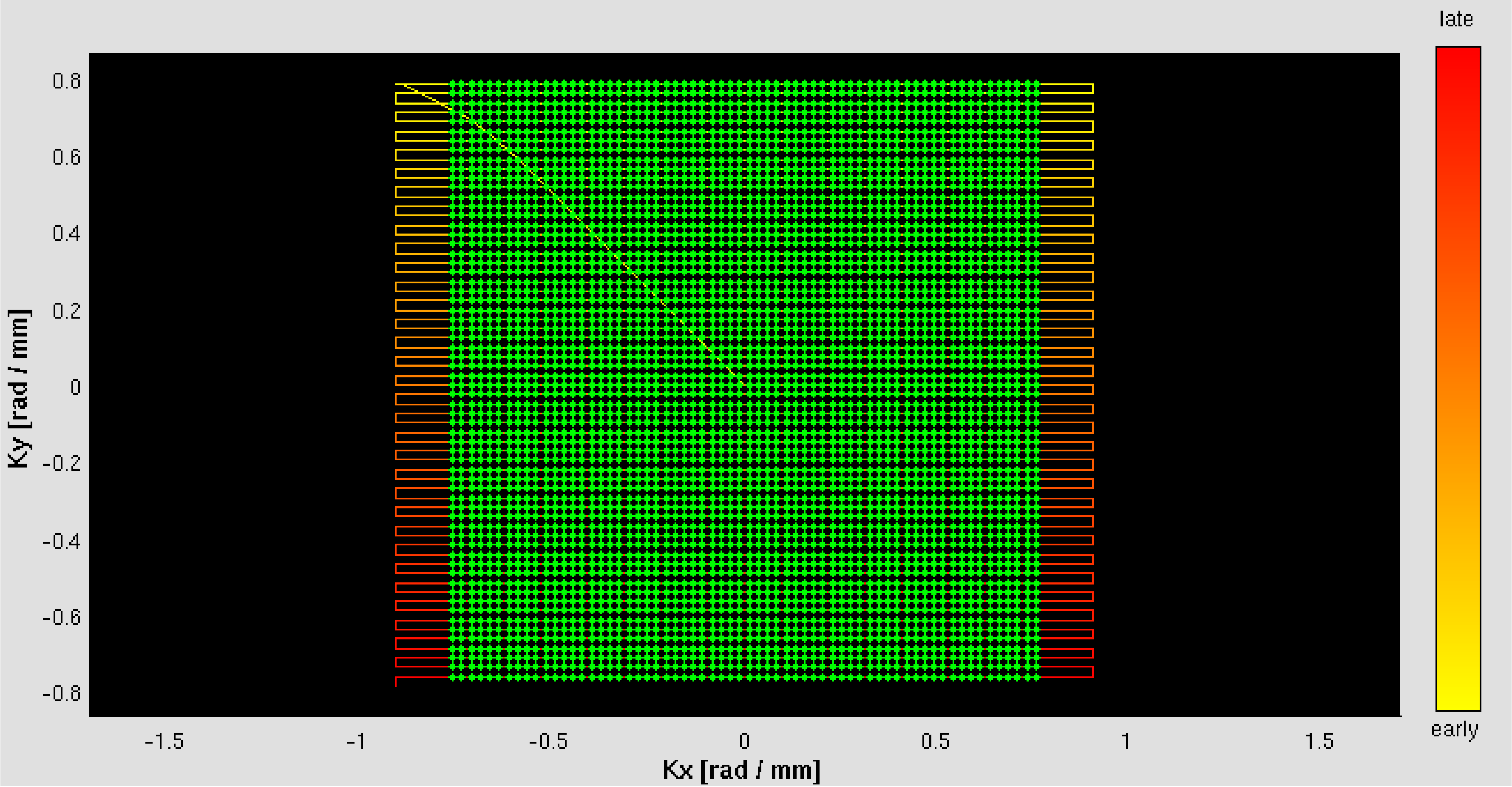


EPI (Echo-Planar-Imaging)



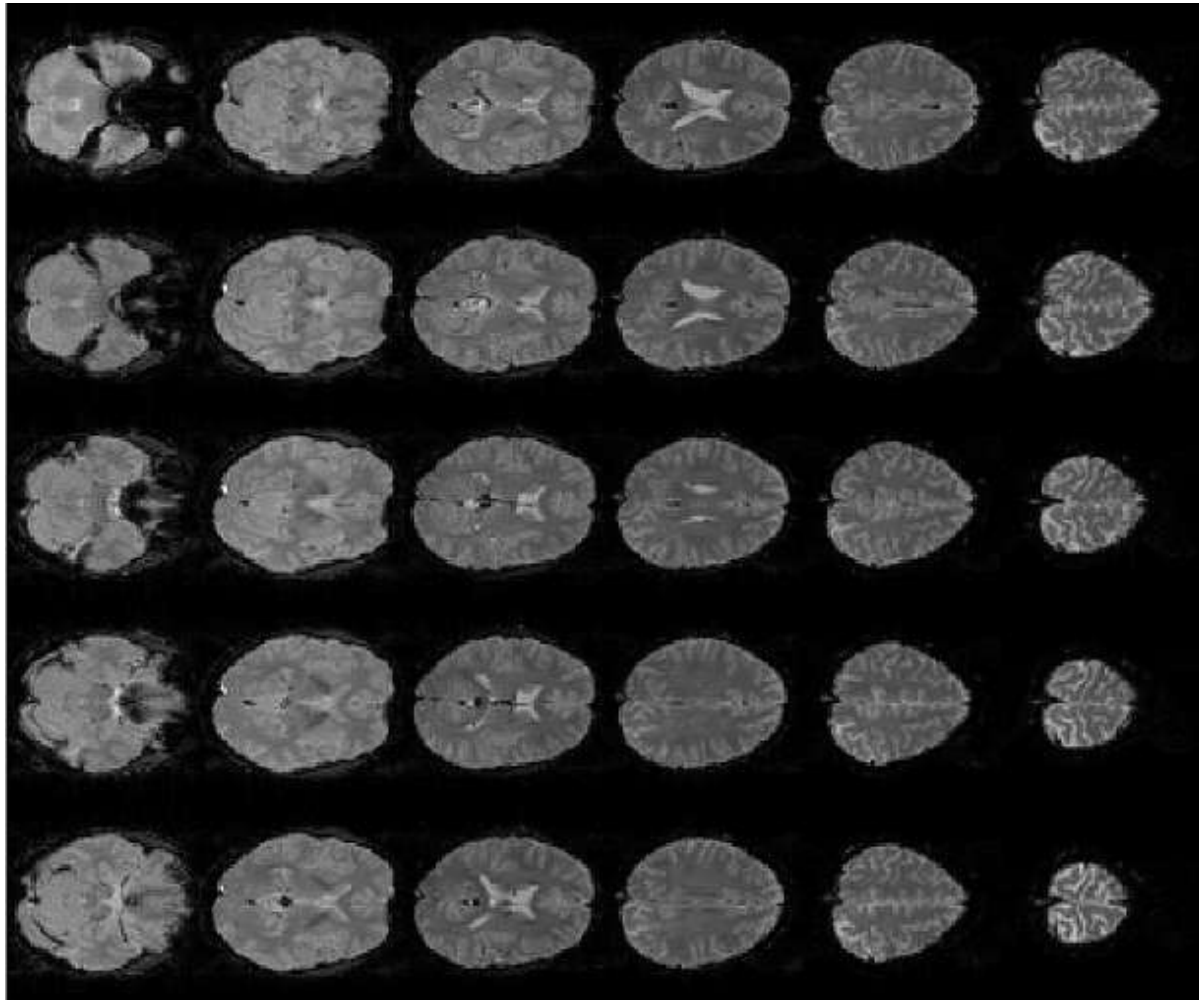
Single-shot EPI K-space trajectory

EPI (Echo-Planar-Imaging)



Single-shot EPI K-space trajectory + ADCs

EPI: workhorse for fMRI & dMRI



EPI with 30 slices,
each 3mm,
resolution of 64x64 pixels

= 122880 Voxel

T_2^* weighted

scan time ≈ 2 sec. !

Note on Oblique Slices
Logical and Physical Gradient System

The true physical orientation of the slice does not change the physics, and therefore we are dealing mainly with logical coordinates:

- Slice-Select Gradient: G_S
- Frequency-Encoding (Readout)Gradient: G_R
- Phase-Encoding Gradient: G_P

Transform from logical coordinate system to physical coordinate system with rotation matrices:

$$\begin{pmatrix} G_x \\ G_y \\ G_z \end{pmatrix} = \mathbf{R}_\phi(\alpha) \begin{pmatrix} G_R \\ G_P \\ G_S \end{pmatrix}$$

Often, G_x=G_R, G_y=G_P, G_z=G_S is used in textbooks.

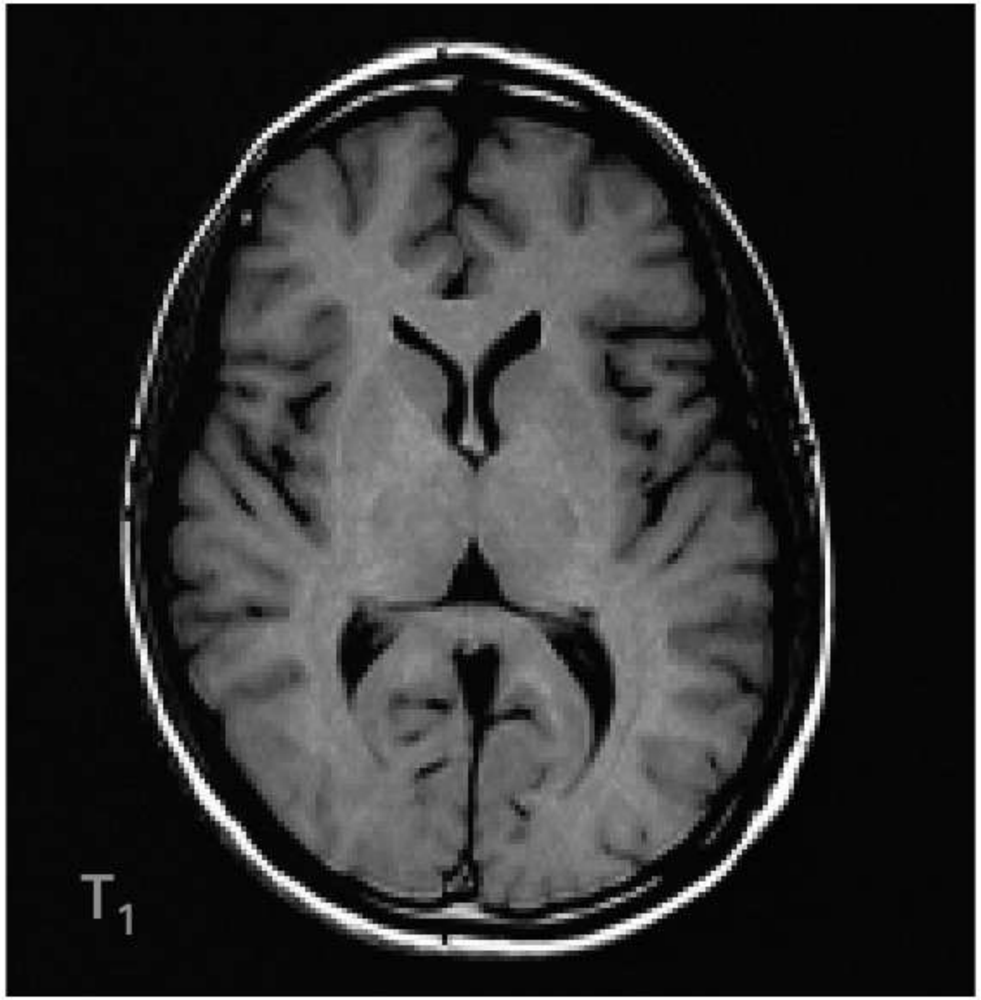
Summary

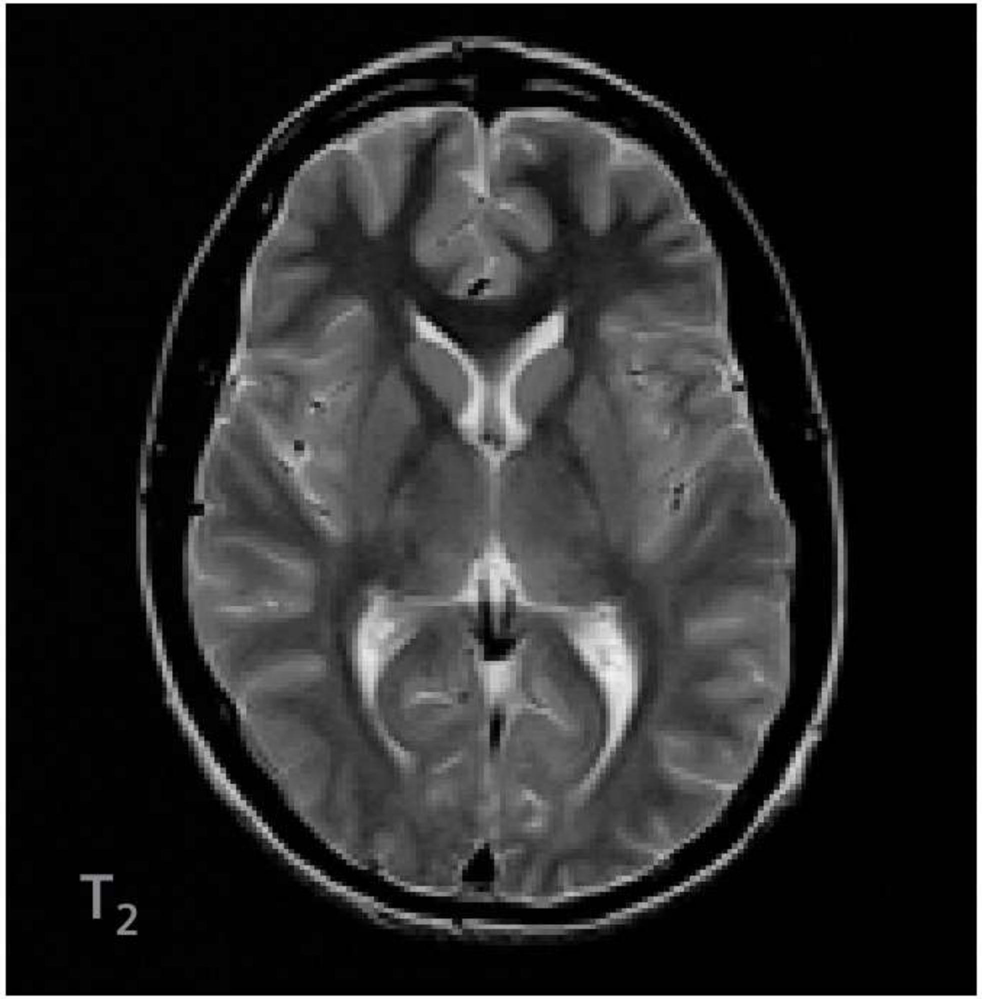
- MR Imaging uses **gradients** for a linear spatial variation of the main field
- **Slice Selection**: Gradient applied *during excitation*
- **Frequency Encoding**: Gradient pulse *during data acquisition*
- **Phase Encoding:** Repeated measurements; each time, a linearly varying phase-shift is encoded *between excitation and acquisition*
- **Echo Planar Imaging** acquires a full slice with a single excitation, at a reduced spatial resolution

3.4 Different Contrasts in MRI

Based on slides by Prof. Tony Stöcker

Reminder: Multi-Contrast (T1/T2-weighting)





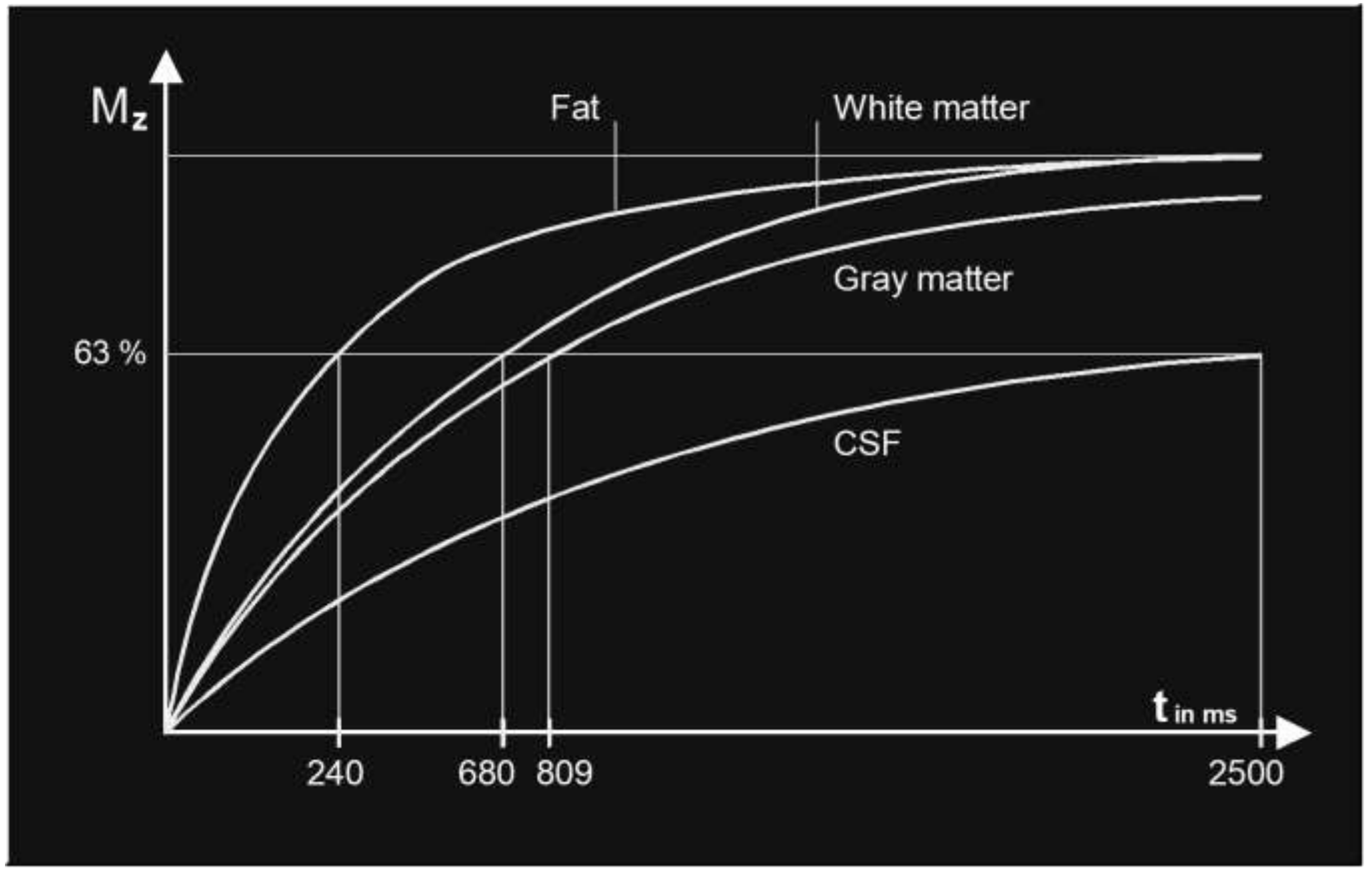
Signal Equation For Spin-Echo Sequence

$$S \propto \rho \exp\left(-\frac{TE}{T_2}\right)\left(1-\exp\left(-\frac{TR}{T_1}\right)\right)$$

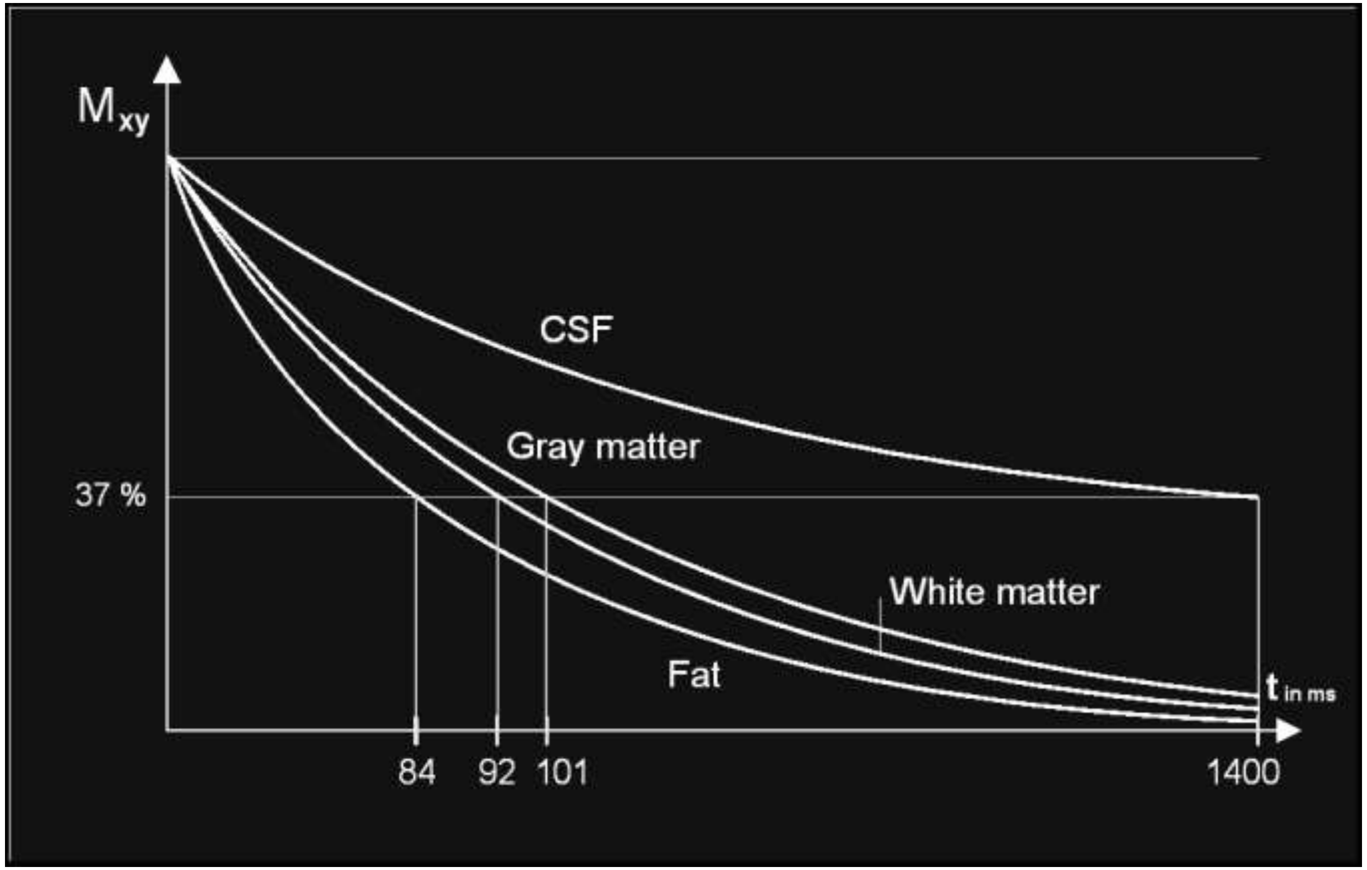
- $TE \rightarrow 0 \quad (TR \approx T_1)$ $\qquad S \propto \rho \cdot 1 \cdot \left(1-\exp\left(-\frac{TR}{T_1}\right)\right) \Rightarrow S = S(\rho, T_1)$
- $TR \rightarrow \infty \quad (TE \approx T_2)$ $\qquad S \propto \rho \cdot \exp\left(-\frac{TE}{T_2}\right) \cdot (1-0) \Rightarrow S = S(\rho, T_2)$
- $TR \rightarrow \infty$, $TE \rightarrow 0$ $\qquad S \propto \rho \cdot 1 \cdot (1-0) \Rightarrow S = S(\rho)$

Same for gradient echo with 90° flip angle, if you replace T2 with T2*!

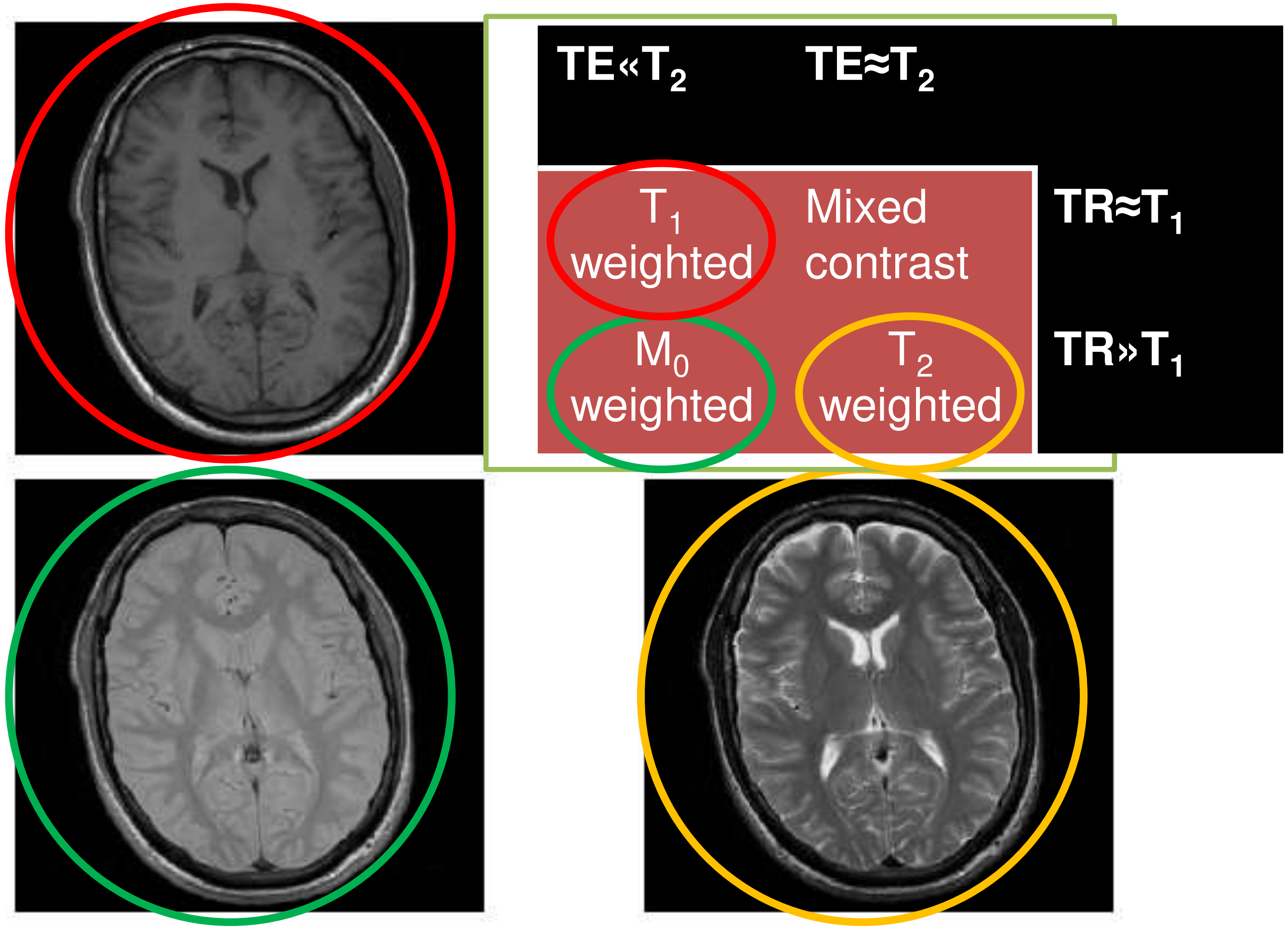
T_1 relaxation in the brain at 1T



T_2 relaxation in the brain at 1T



In vivo examples: different contrasts



3.5 Image Artifacts in MRI

Based on slides by Prof. Tony Stöcker

Real world imaging... artefacts

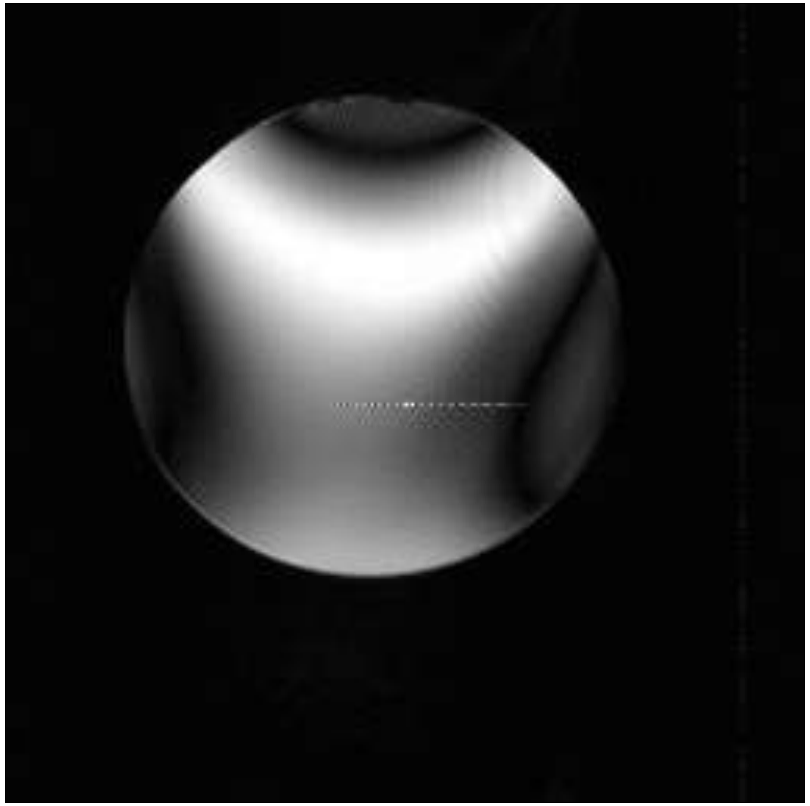
Artefact sources:

- Hardware (magnet, coils, gradients) imperfections
- Eddy currents caused by gradients
- Inhomogeneous B_0 caused by patient
- Human body is not rigid (motion, flow)
- Limits of the sequence itself (PSF, etc.)

→ influence on MR image

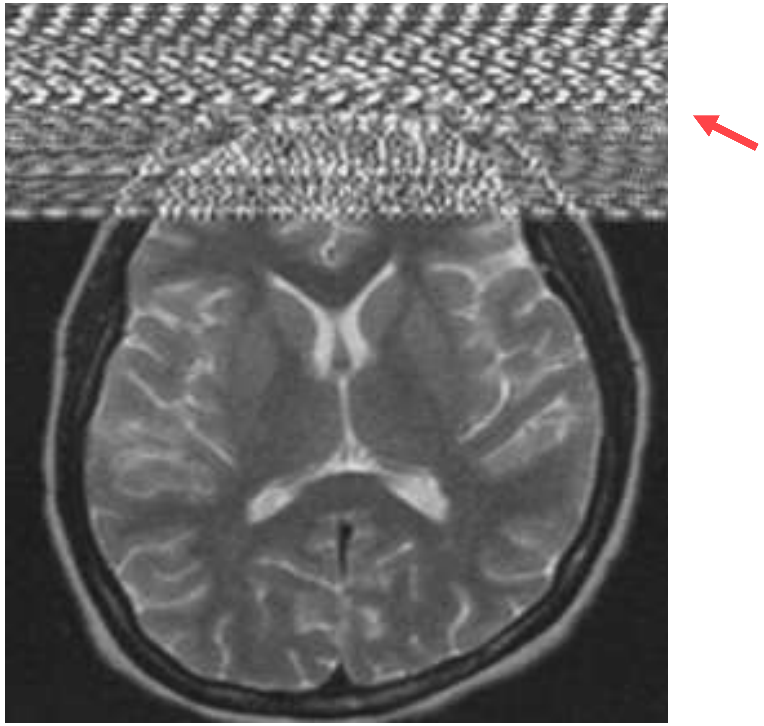
Inhomogenous Field

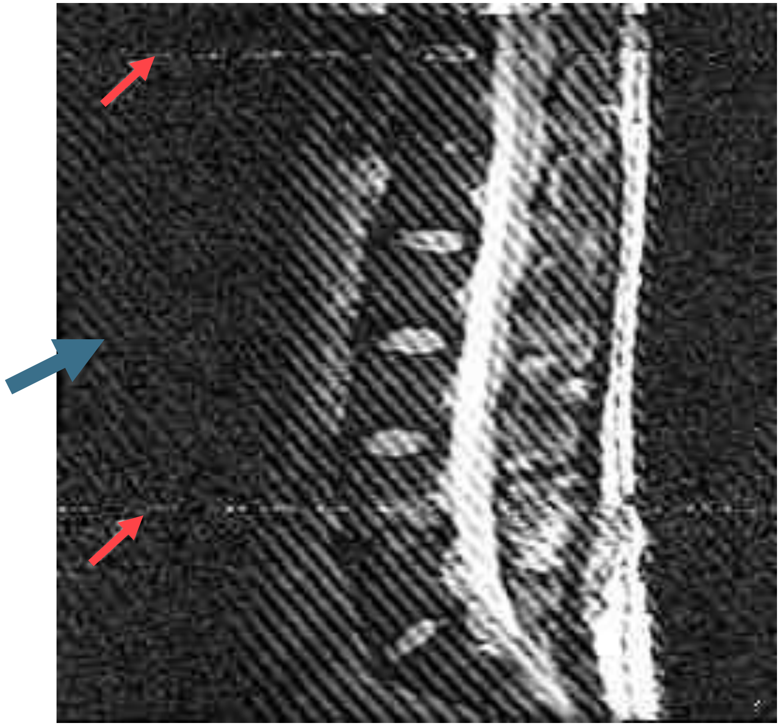
E.g. mis-set shim currents



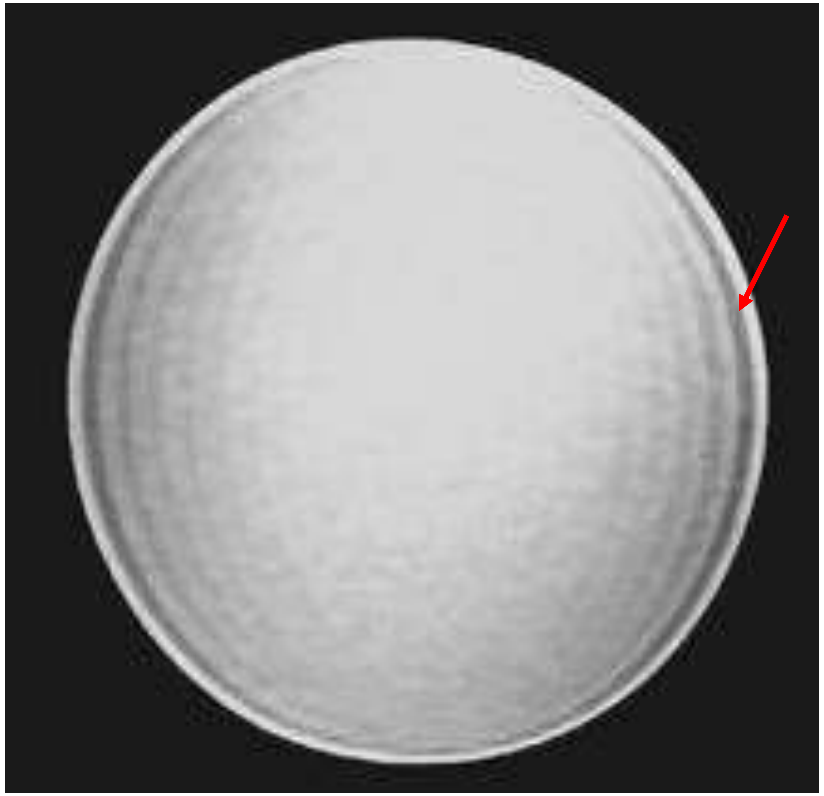
RF Noise

- Spurious RF signals inside the magnet room.
- E.g. from break-down of the shielding, or electric devices inside the room.
- Spikes (single peak at one time-point of the acquisition)
- Zipper (continuous RF interference)





Gibbs Ringing



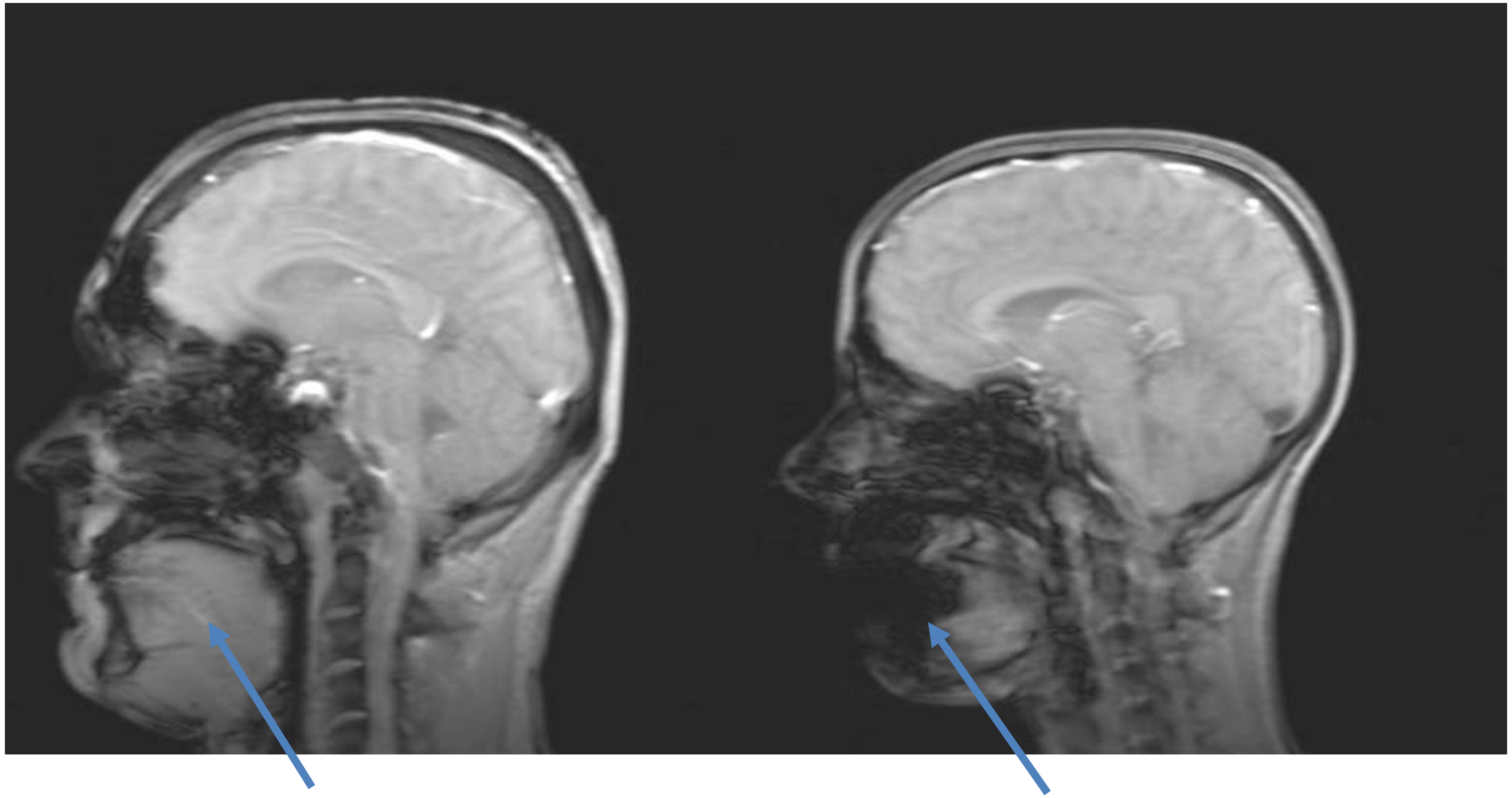
Truncation artifact due to finite sampling of k-space signal.
(Box-car transforms to sinc function!)

Can be suppressed by acquiring more lines and/or digital filtering.

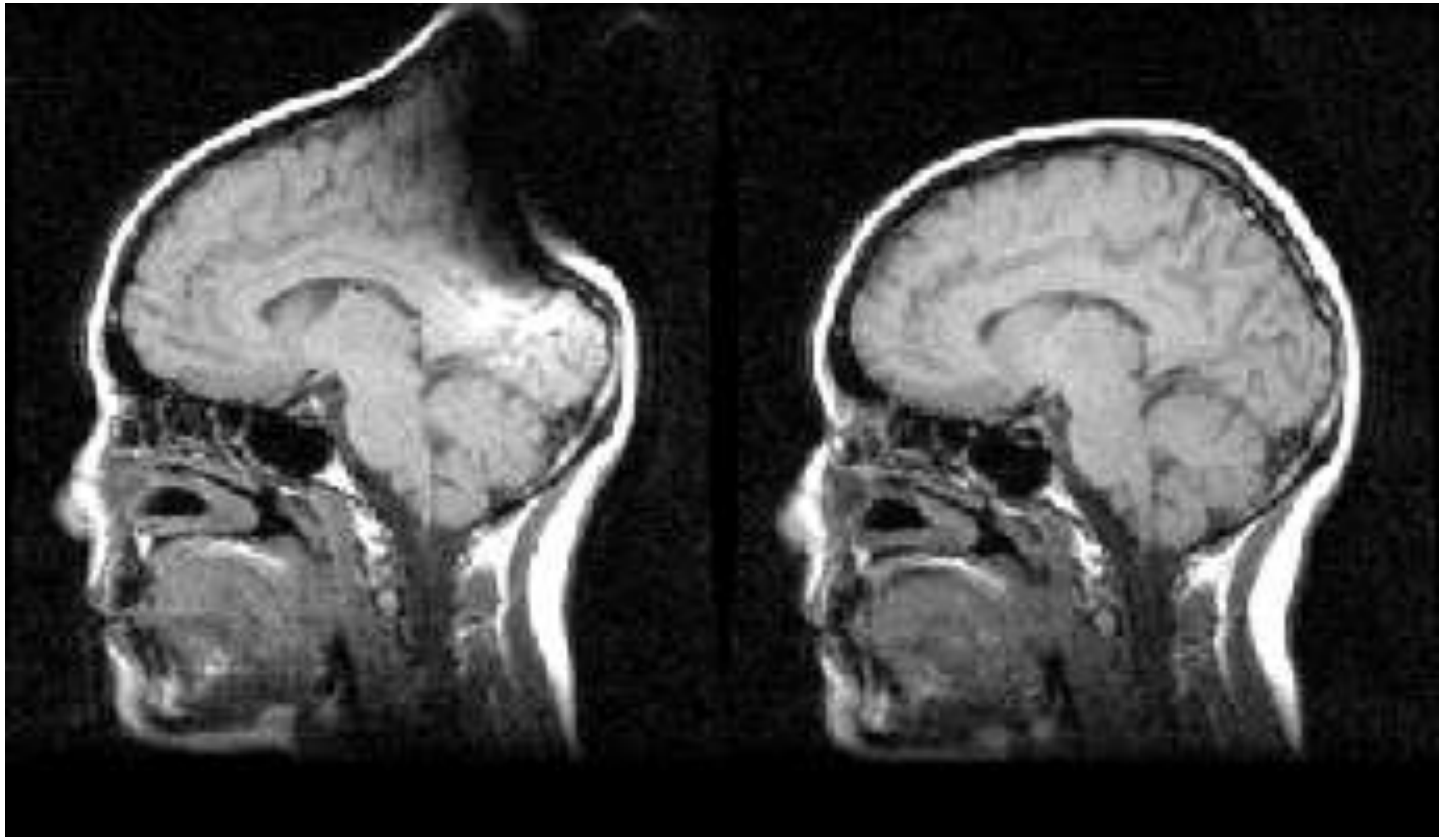
Metal artefacts

- Metals strongly distort the static magnetic field
- Therefore T_2* is extremely shortened
- Possible consequences for imaging:
  - Image distortions
  - Signal dropout
- Remedies:
  - Remove metal
  - Shimming (but: might not be sufficient)

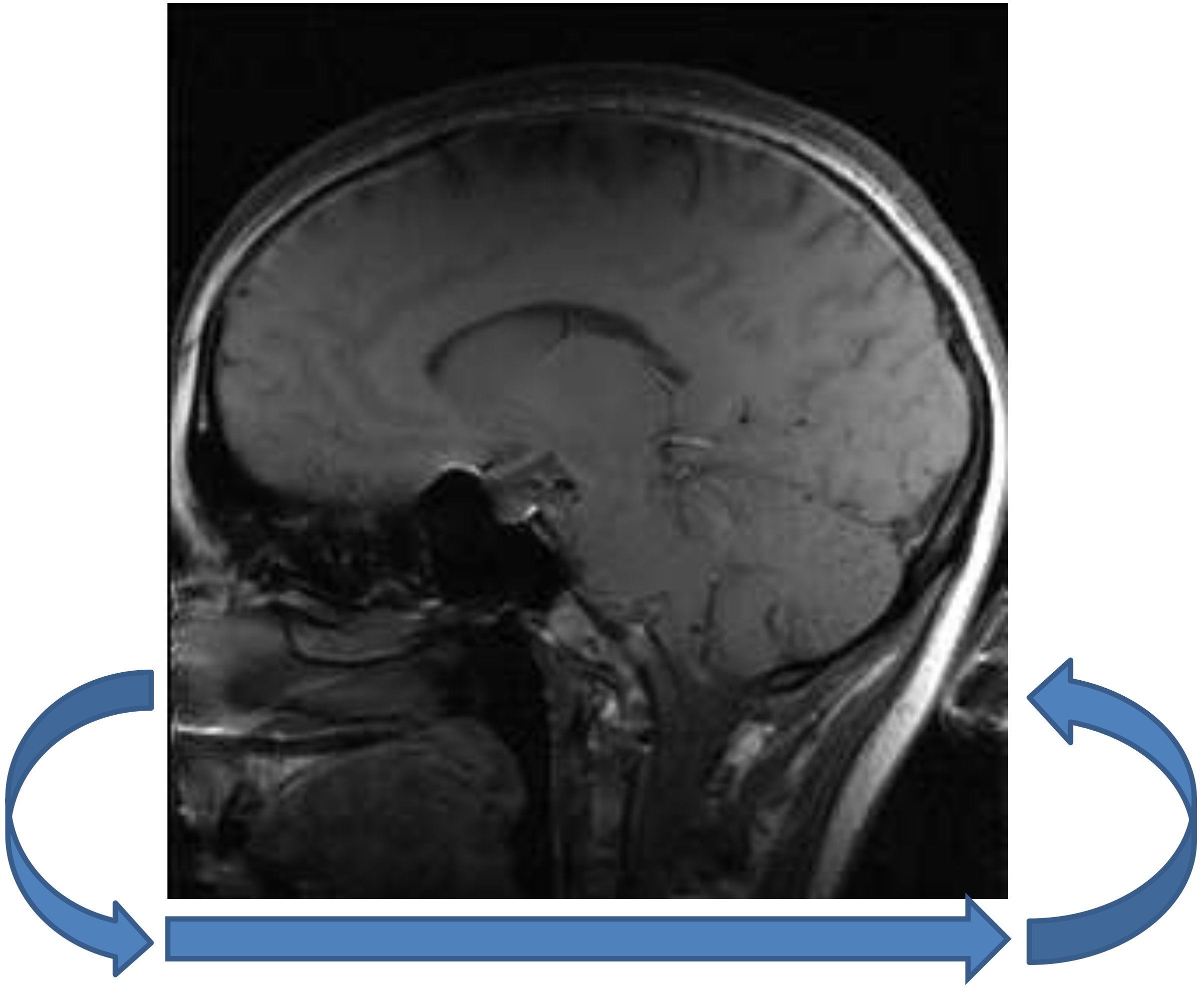
Metal artefacts: Retainer wire



Metal artefacts: Hair band



Fold-in artefacts



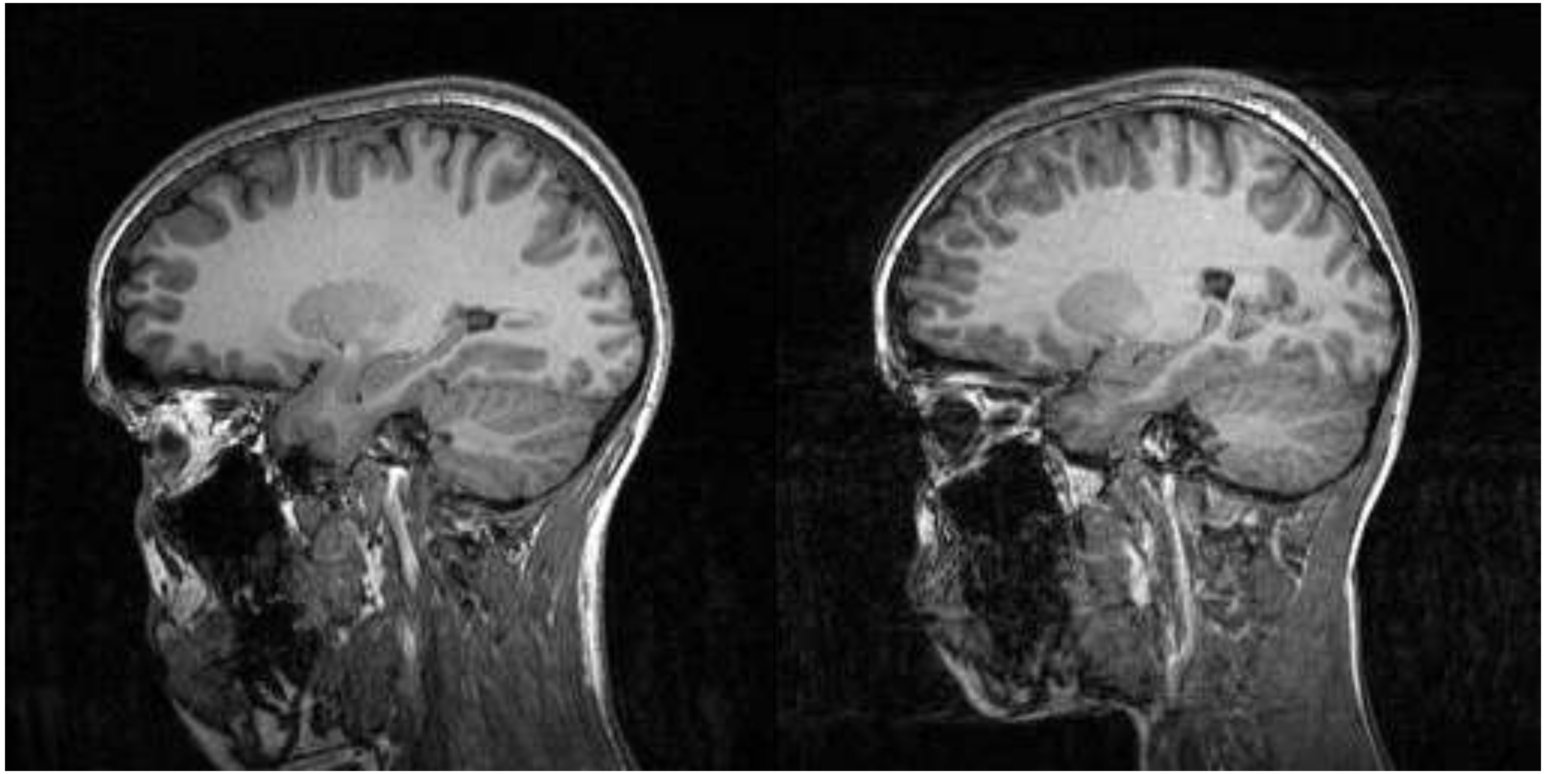
Motion artefacts

- General assumption: Patient does not move during measurement
- If motion occurs, the *k*-space to be measured changes
- Result: lines of different *k*-spaces are measured
- Possible consequences for imaging:
  - Blurring
  - Repetition of structures (ringing)
- Remedies:
  - Speed up image acquisition
  - Restrict patient movement

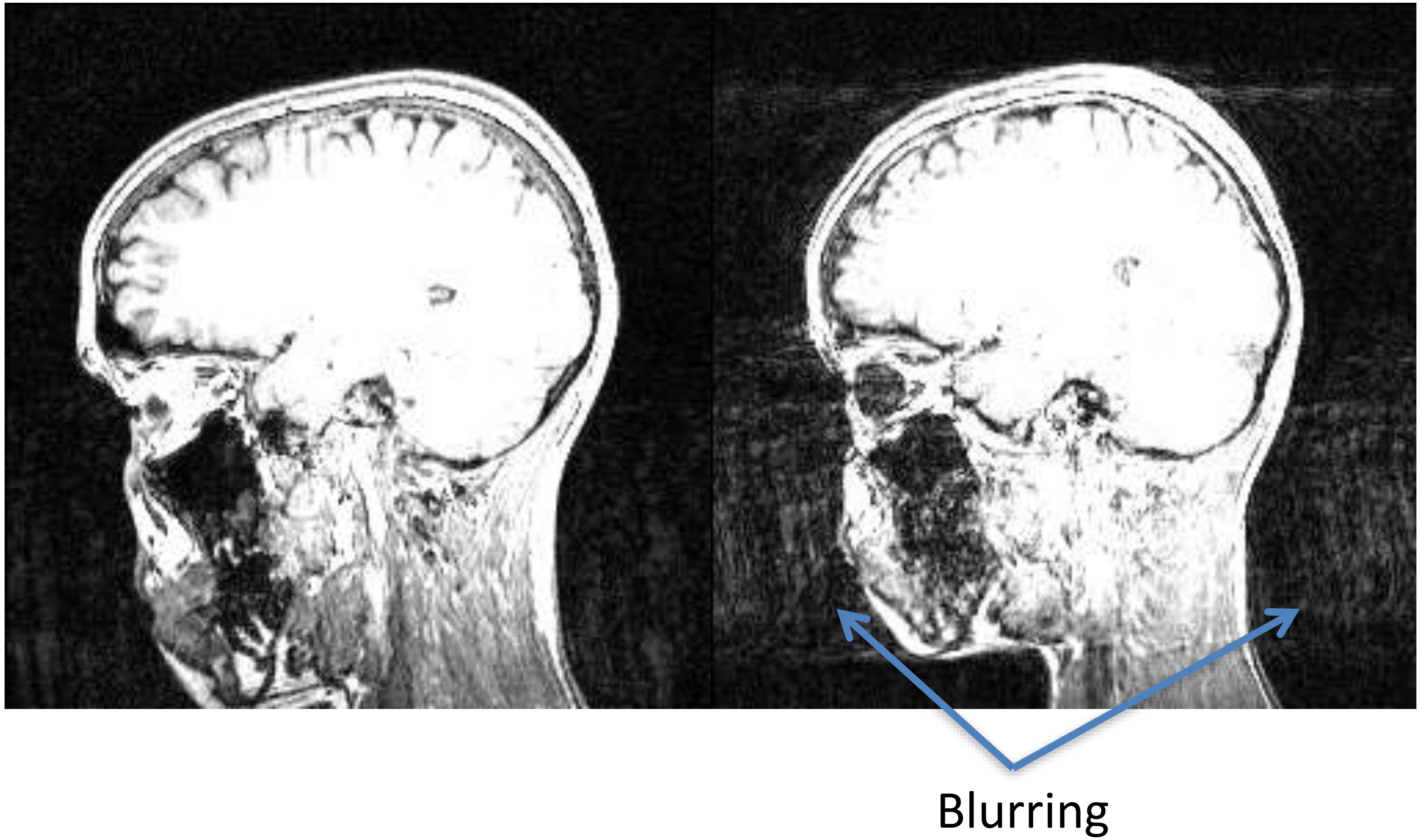
Motion artefacts

No motion

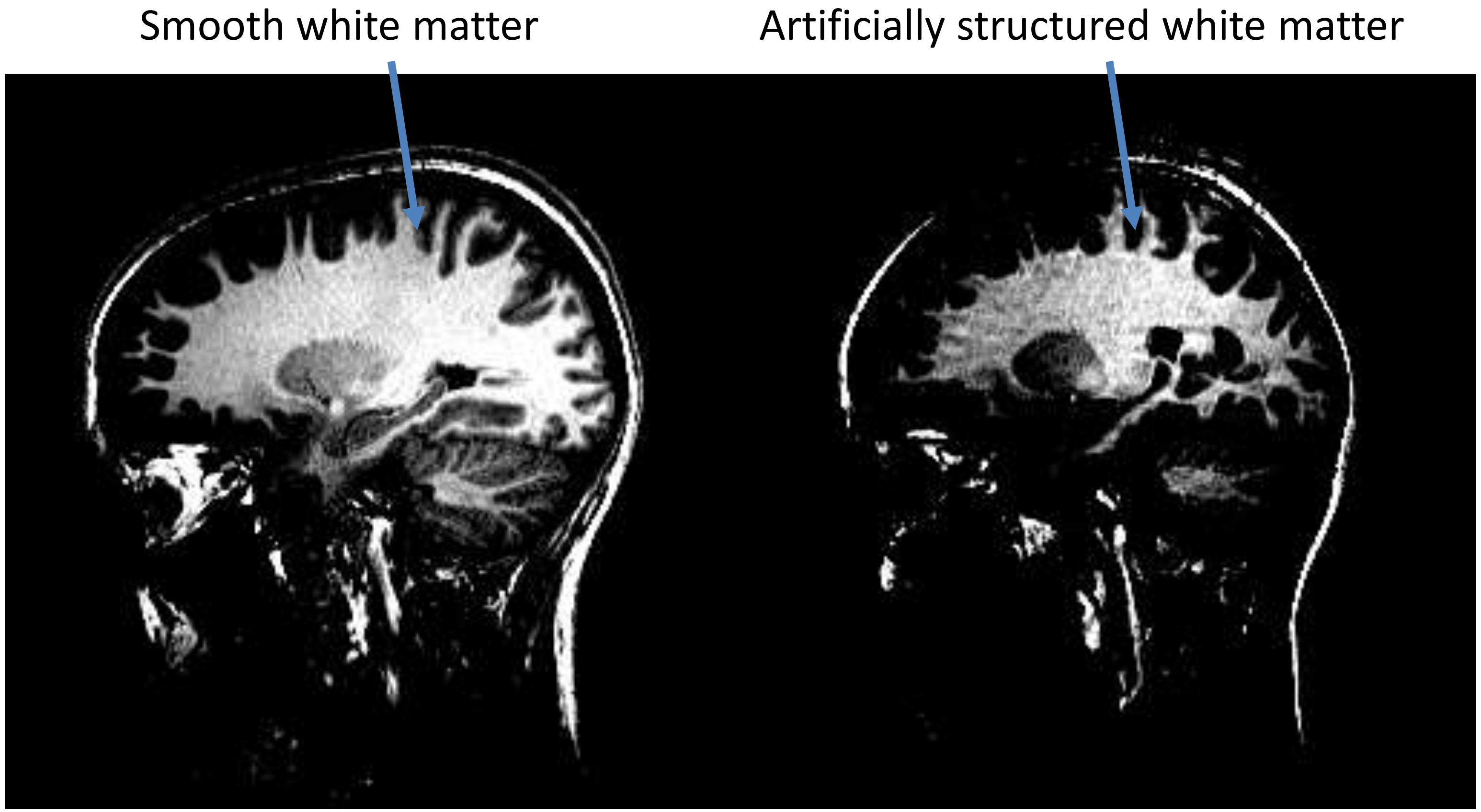
Motion artefacts



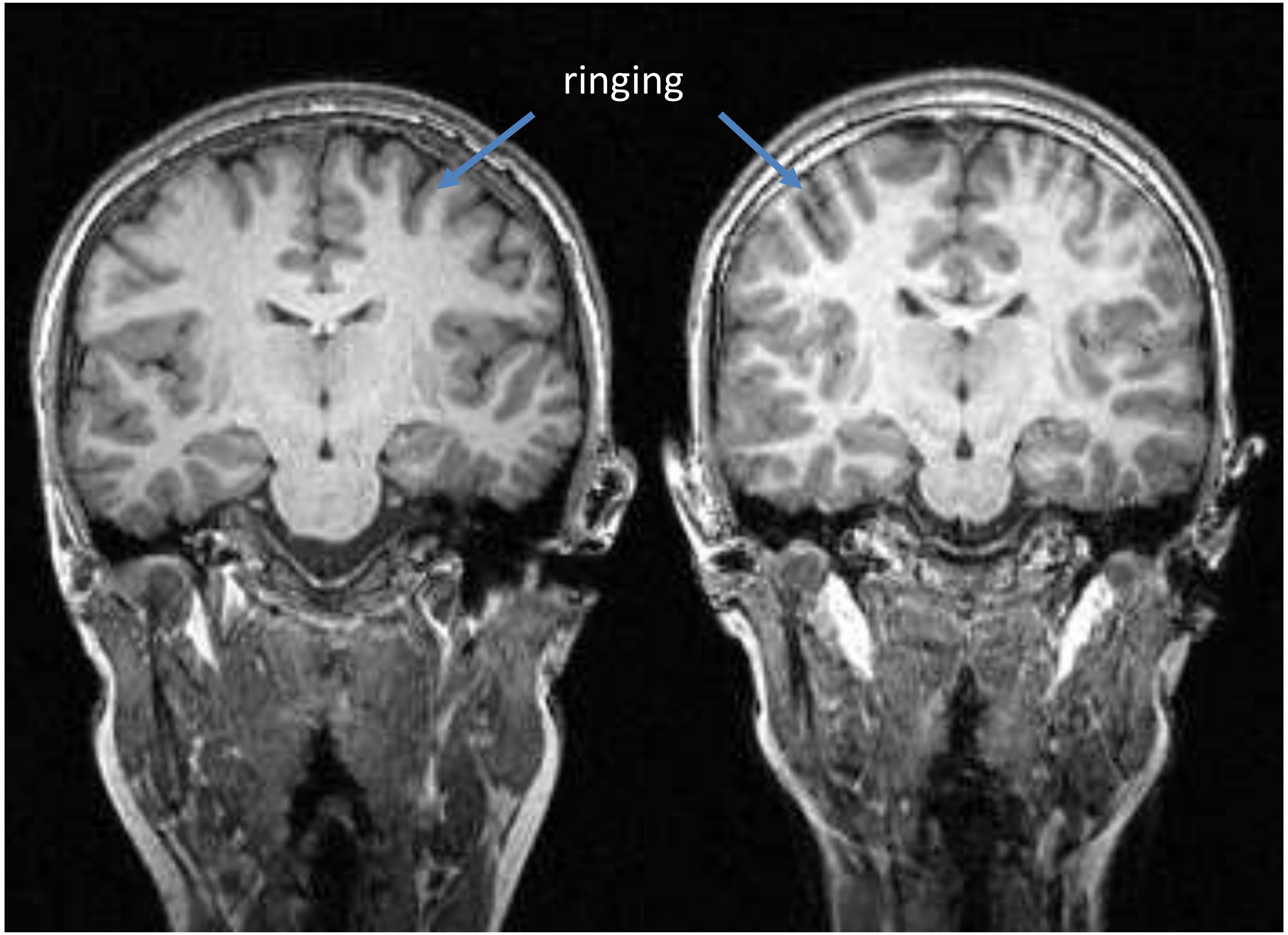
Motion artefacts 2



Motion artefacts 3

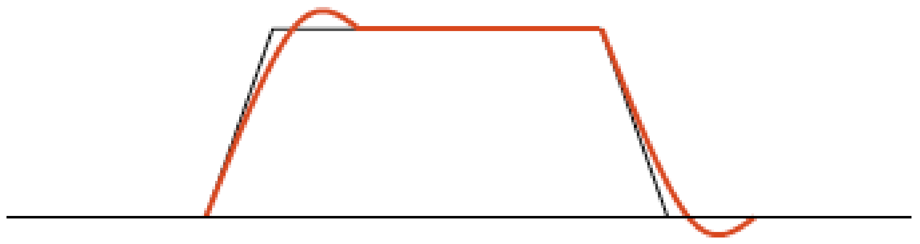


Motion 4

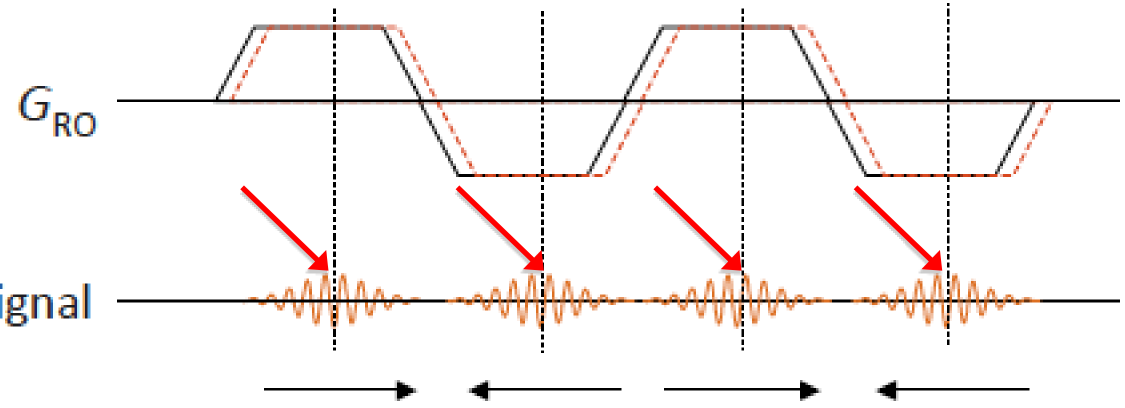


EPI image artefacts: “N/2 Ghost”

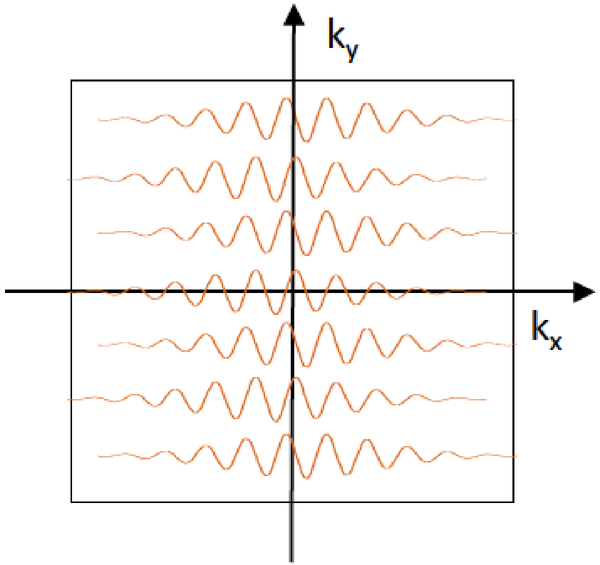
Because of Eddy currents, the gradients shape is modified:

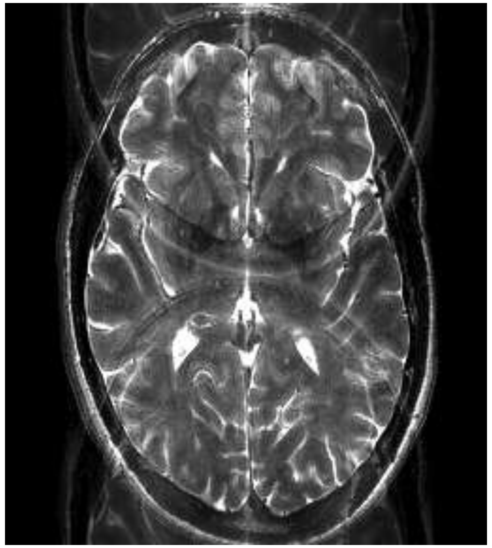


And so is the K-Space trajectory, and the signal echo doesn’t appear at the K-Space centre anymore:



EPI image artefacts: "N/2 Ghost"





The artefact is 2 lines-redundant in the K-Space:
(Period = 2 × Δk_y = $2/FoV_y$)

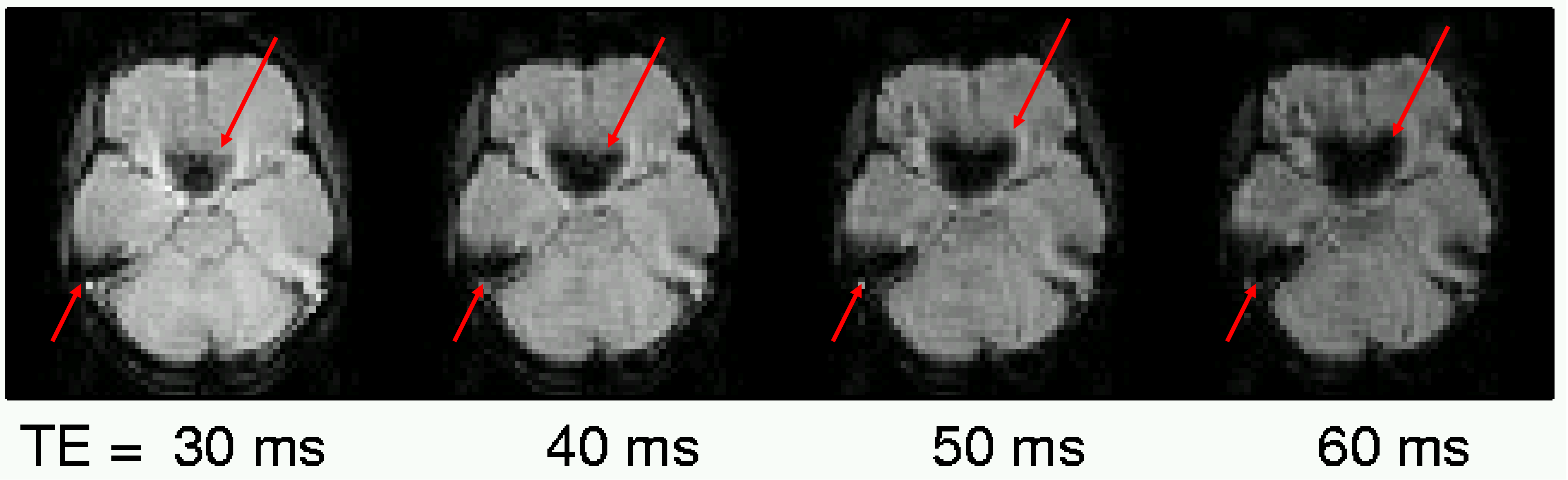
Image space periodicity
=
inverse of the frequency space (K-Space) periodicity

Artefact is
$1/(2/FoV_y) = FoV_y/2$
periodic on the image

EPI: susceptibility artefacts

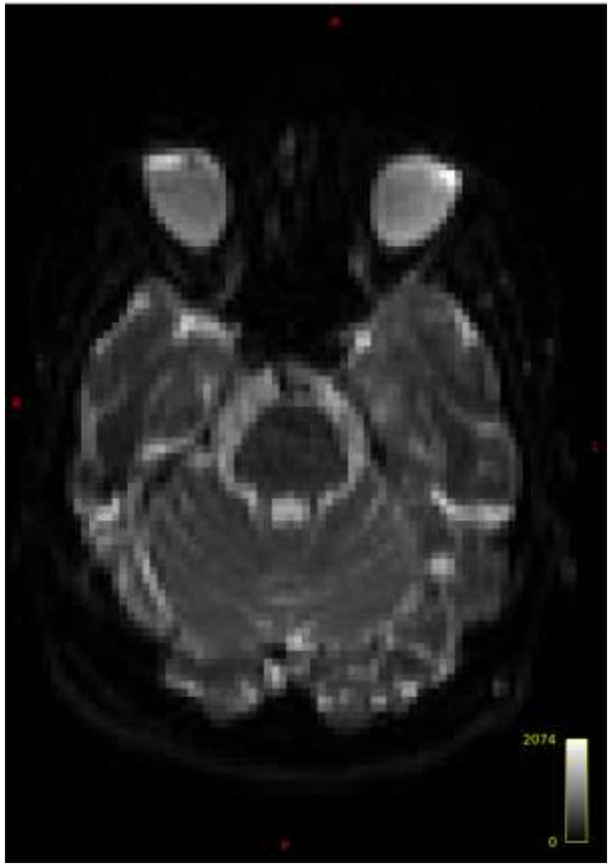
- Induced by local changes of the main magnetic field due to different magnetic properties of tissue.
- Strong effect at air-tissue boundaries (sinuses, ear canal)
- Especially pronounced in T_2^*-weighted sequences with long echo time (=> BOLD EPI)

EPI: susceptibility-induced signal dropout and image distortions

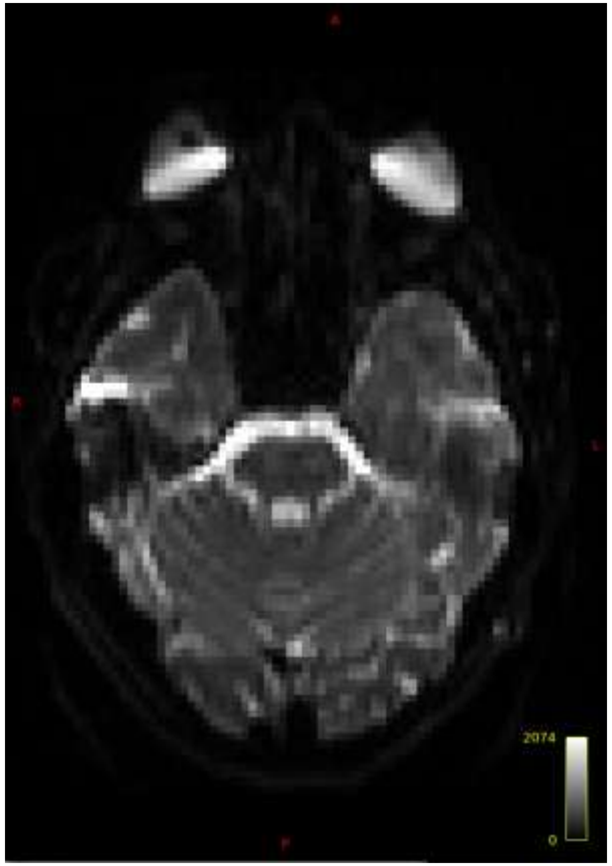


Example: Correcting Geometric EPI Distortions

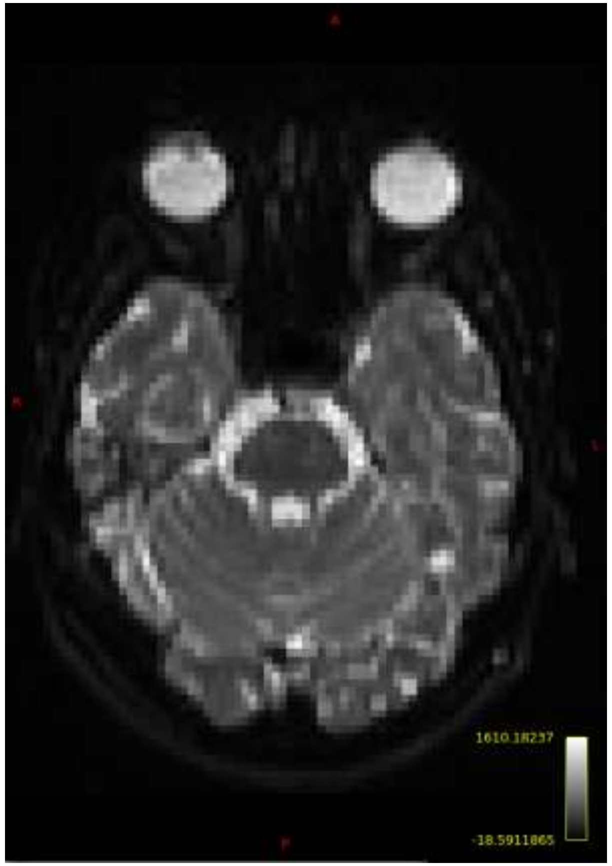
- EPI distortions depend on the polarity of the phase encoding
- We can reduce them by combining information from both polarities



A>>P



P>>A



Unwarped A>>P

Summary

- **Artifacts** that we will have to account for during MR image processing include:
  - **Measurement noise:** Additive complex Gaussian leads to Rician distribution on magnitude images
  - **Bias fields:** Low-frequency variations in image intensity
  - **Motion artifacts:**
    - Between acquisitions: Repositioning of head
    - During acquisitions: Distortions, signal loss, blurring, noise
  - **Susceptibility artifacts:** Signal loss, distortions

What to Take Away From This Lecture

- Basic principle of Nuclear Magnetic Resonance
  - Relevance of static vs. RF fields
  - Longitudinal vs. Transversal relaxation
- Basic echo sequences
  - Spin echo vs. Gradient echo
- Different types of contrast
  - Impact of relaxation times (T1, T2, T2*)
- MR image acquisition in k-space
  - Slice selection, frequency vs. phase encoding

References

- M. A. Flower (Ed): *Webb‘s Physics of Medical Imaging,* 2nd edition, CRC Press 2012
- *Magnets, Spins and Resonances,* Siemens Healthcare (available online)
- *Is Quantum Mechanics Necessary for Understanding Magnetic Resonance?*, L.G. Hanson, Conc. Magn. Res. A, Vol.32A(5), pp. 329-340, 2008
- B. Preim, C. Botha: *Visual Computing for Medicine: Theory, Algorithms, and Applications,* Morgan Kaufmann, 2014

Exam-like Questions

1. What do the $T1$, $T2$, and $T2*$ time constants in Magnetic Resonance Imaging have in common?
   Also explain how each of them differs from the others.
2. Name the three mechanisms that are used to encode the three spatial dimensions in classical Magnetic Resonance Imaging. For each of them, briefly state at which point of the acquisition sequence it is employed.
3. State an important difference between a classical MR image acquisition and Echo Planar Imaging (EPI). Mention two practical consequences of that difference.