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Algorithmic Game Theory

Summer Term 2025

Exercise Set 6

If you would like to present one of the solutions in class, please also send an email to rlehming@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated on a first-come-first-served basis, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1:
Recall the *Greedy-by-Value* and *Greedy-by-Sqrt-Value-Density* algorithms for single-minded combinatorial auctions of lecture 12. Let us analyse another greedy algorithm that looks as follows.

Greedy-by-Value-Density

- Re-order the bids such that $\frac{b_1^*}{|S_1^*|} \geq \frac{b_2^*}{|S_2^*|} \geq \cdots \geq \frac{b_n^*}{|S_n^*|}$.
- Initialize the set of winning bidders to $W = \emptyset$.
- For $i = 1$ to n do: If $S_i^* \cap \bigcup_{j \in W} S_j^* = \emptyset$, then $W = W \cup \{i\}$.

Let $d = \max_{i \in \mathcal{N}} |S_i^*|$. Show that the given algorithm yields a d-approximation.

Exercise 2:
As seen in lecture 13, let $f \colon V \to X$ be a function that maximizes declared welfare, i.e., $f(b) \in \arg\max_{x \in X} \sum_i b_i(x)$ for all $b \in V$. For each i, let h_i be an arbitrary function $b_{-i} \mapsto h_i(b_{-i})$ which does not depend on b_i. We define a mechanism $\mathcal{M} = (f, p)$ by setting

$$p_i(b) = h_i(b_{-i}) - \sum_{j \neq i} b_j(f(b)) \ .$$

Prove that $\mathcal{M}$ is a truthful mechanism.

The following exercises rely on lectures 13 and 14.

Exercise 3:
Consider the following *Procurement Auction*. It's being attempted to buy a certain item. There are n vendors who are able to manufacture the wanted item. Vendor i incurs a cost of c_i for crafting the item. Now, the vendors are asked to state their costs for crafting the item and a vendor with lowest cost shall be chosen. The latter potentially gets a payment for it. The stated problem can be formalized by the general model of the lecture: Each vendor i is interpreted as a bidder who has negative valuation v_i, if he/she is chosen to craft the item, that is, $v_i(x) = -c_i$, if i is chosen in x.

(a) The results of the lecture concerning VCG are applicable in this situation. Make use of them in order to state a truthful mechanism.

(b) Use your results from the previous exercise to make the mechanism individually rational.

Exercise 4:
We consider a single-item auction via a mechanism which follows the spirit of Lecture 14, Section 2: All bidders submit their bids b_i. Fix a price of p (may depend on b) for the item. Approach bidders in order $1, \ldots, n$. As we consider bidder i: if the item is not allocated yet, assign the item for a price of p if $b_i - p \geq 0$.

(a) If $b = v$, show that the social welfare obtained by this auction is at least

$$\max_i v_i \mathbb{1}_{\text{item not allocated}} + p\left(\mathbb{1}_{\text{item allocated}} - \mathbb{1}_{\text{item not allocated}}\right) .$$

(b) Use your result from (a) to set a price obtaining a social welfare of at least $\frac{1}{2} \max_i v_i$ if $b = v$.