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Discrete and Computational Geometry

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Assignment 6

Problem 1: (3 Points)

For a point set with $n \geq d+1$ points, suppose there are no $d+1$ points in the same hyperplane ($d-1$-flat) in R^d.
Prove that this implies there are no $k+1$ points in any $k-1$ flat, $1 \leq k < d$.

Problem 2: (3+3 Points)

Let $e_1, \ldots, e_5$ be vectors of the standard orthonormal basis in $\mathbb{R}^5$, and let e_0 stand for the zero vector. For $i = 0, 1, 2$ and $j = 1, 2, \ldots, n$, let $p_{i,j} = e_{2i} + \frac{j}{n} e_{2i+1}$.

a) Prove that for every choice of $j_0, j_1, j_2 \in \{1, 2, \ldots, n\}$, there is a point q in $\mathbb{R}^5$ with

$$q = \left(\frac{j_0}{n}, x_1, \frac{j_1}{n}, x_2, \frac{j_2}{n} \right)$$

for $x_i = \frac{1+(j_0/n)^2-(j_i/n)^2}{2}$, such that the points which are nearest to q among the $p_{i,j}$ are exactly $p_{0,j_0}, p_{1,j_1}, p_{2,j_2}$.

b) Conclude that the total number of faces of dimension 3 in the Voronoi diagram of the set

$$P = \{p_{i,j} \mid 0 \leq i \leq 2 \text{ and } 1 \leq j \leq n\}$$

is in $\Omega(n^3)$.

Problem 3: (6 Points)

Let $P \subset R^4 = conv\{\pm e_i \pm e_j \mid i, j = 1, 2, 3, 4, i \neq j\}$, where $e_1, \ldots, e_4$ is the standard basis (this P is called the 24-cell).
Describe the face lattice of P, and, for each $0 \leq k \leq 4$, show the bijections between the k-faces of P and the $d-k-1$-faces of its dual.