# Exponential Powers - GMAT Math Study Guide

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## Definitions

- **Base - the number that is multiplied by itself a certain quantity of times.**

  For example, in the expression $2^3$, the number 2 is the base.
- **Exponent - the number of times a quantity is multiplied by itself.**

  For example, in the expression $2^3$, the number 3 is the exponent.
- **Power - a synonym for exponent.**

  For example, if one were to say, "raise 3 to the 4th power," the base would be 3 and the exponent (or power to which 3 is raised) would be 4.

An exponential expression is simply multiplication repeated (e.g., $2^3=2*2*2$).

### Table Clarifying Definitions

The following chart breaks down the parts in an exponential expression, clarifying exactly which number is the exponential power.

| Expression | Long Hand Expression | Base | Exponent | Value |
|---|---|---|---|---|
| $2^3$ | 2 * 2 * 2 | 2 | 3 | 8 |
| $4^6$ | 4 * 4 * 4 * 4 * 4 * 4 | 4 | 6 | 4096 |
| $3^2$ | 3 * 3 | 3 | 2 | 9 |

| | | | | |
|---|---|---|---|---|
| $6^4$ | 6 * 6 * 6 * 6 | 6 | 4 | 1296 |

In reading math problems, expressions with exponential powers such as $3^2$ are often pronounced "three to the second power." Alternatively, exponential expressions such as $3^2$ are often read as "the second power of three."

## Rules of Exponents

$$\frac{x^n}{x^m} = x^{n-m}$$

$$x^n x^m = x^{n+m}$$

$$x^n y^n = (xy)^n$$

$$\left[\frac{x}{y}\right]^n = \frac{x^n}{y^n}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(x^y)^z = x^{(y*z)}$$

$1^n = 1$

$x^0 = 1$

$0^n = 0$

Note: During the past decade, mathematicians argued extensively about the value of $0^0$. Some answer that $0^0 = 1$ while others answer that $0^0$ is undefined. In the unlikely event that this question appears in some format or is a required intermediary calculation, the correct answer is more likely that $0^0 = 1$

**Examples of the Rules of Exponents**

$$\frac{2^4}{2^2} = \frac{2(2)(2)(2)}{(2)(2)} = 2^{4-2} = 2^2$$

$2^3 2^4 = 2^{3+4} = 2^7$

$2^4 3^4 = (2*3)^4 = 6^4$

$$\left[\frac{3}{2}\right]^4 = \frac{3^4}{2^4}$$

$$3^{-2} = \frac{1}{3^2}$$

$(3^2)^4 = 3^{(2*4)} = 3^8$

## Exponential Powers Grow Expressions Rapidly

Exponential powers increase the value of an expression at an incredibly large rate. In order to see this, consider the following example:

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

In each instance, the value is doubling--which makes sense since it is being multiplied by 2 another time for each increase in the value of the exponent.

## Types of Exponents

### Positive Exponents

The most basic and common type of exponent is a positive exponent. The expression $x^y$ has a positive exponent if $y > 0$. The following are all examples of positive exponents.

$10^{20}$

$2^1$

$3^2$

### Negative Exponents

Although it is most common to see an exponential expression with a base raised to a positive power, a base can just as easily be raised to a negative power. The expression $x^y$ has a positive exponent if $y < 0$. In working with negative exponential powers, it is extremely important to remember the following formula.

$$x^{-n} = \frac{1}{x^n}$$

As a result of this formula, an exponential equation can often be simplified. Consider the following examples:

Note: The first example could be solved using the formula: $x^n x^m = x^{n+m}$

$$2^6 2^{-4} = \frac{2^6}{2^4} = 2^2$$

$$2(3^{-4}) = \frac{2}{3^4}$$

**Fractional Exponents**

While many exponential expressions are raised to an integer power, nothing prevents a base from being raised to a fractional power. Although all fractional exponents follow the aforementioned rules and therefore are alike, it is often helpful to break down fractional exponents in a separate lesson since the use of radicals and roots is involved.

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**Exponent of Zero**

Any number raised to the 0 power is one.

$x^0 = 1$

**Exponent of 1**

Any number raised to the first power is simply that number.

$x^1 = x$

**Recursive Exponents**

A recursive exponential expression is one in which multiple exponents are nested within each other. For example:

$$2^{2^3}$$

As per the order of operations (order_of_operations), you evaluate an expression such as this by first computing the value inside parenthesis (there are none here) and then performing exponential expressions by working from left to right. Consequently, the expression above is evaluated in the following manner.

1. $2^2 = 4$
2. $4^3 = 64$

In the above example, it would be wrong to first compute $2^3 = 8$ and then compute $2^8 = 256$