Hierarchy of Probability Distributions

A simplified categorization of common probability distributions can be summarized as follows (note that these categories are not always mutually exclusive, but give a sense of how distributions relate to each other):

- **Discrete distributions**
  - *Bernoulli family*: Bernoulli, Binomial, Negative binomial, Geometric
  - *Poisson*
  - *Discrete uniform*
  - *Hypergeometric family*: Hypergeometric, Multivariate hypergeometric
  - *Categorical family*: Bernoulli (as $K = 2$), Categorical, Multinomial
  - Others: Beta-binomial, etc.
- **Continuous distributions**
  - *Uniform* (continuous)
  - *Exponential family:* Exponential, Gamma, Chi-squared, Beta, Dirichlet, Wishart
  - *Normal family*: Normal (Gaussian), Log-normal, Chi-squared (sum of normals^2), t, F, etc.
  - *Power-law family*: Pareto
  - *Extreme value distributions*: Gumbel, Fréchet, Weibull
  - *Others*: Rayleigh, Rice (Rician), Beta prime, Logistic, etc.
- **Multivariate or matrix-variate distributions**
  - Multinomial, Multivariate hypergeometric
  - Multivariate normal, Wishart, Dirichlet
  - Others, e.g. Inverse-Wishart, Matrix-variate t-distributions, etc.
- **Conjugate priors** in Bayesian inference:
  - Beta (conjugate to Bernoulli/Binomial)
  - Gamma (conjugate to Poisson/exponential rate)
  - Dirichlet (conjugate to Categorical/Multinomial)
  - Wishart (conjugate to inverse covariance of multivariate normal)

Reference Table of Distributions

Below is a reference table covering many of these distributions, with their probability mass function (PMF) or probability density function (PDF), support, mean, variance, and a few extra notes. Some new distributions (e.g. Weibull, Gumbel, Beta prime, Logistic) have been added as illustrative examples.

Table 1: Distributions, their PMF/PDF, support, mean, variance, and further notes.

Distribution	**PMF / PDF**	**Support**	**Mean**	**Variance**
Normal (Gaussian)	$f(x)=\frac{1}{\sqrt{2\pi}\,\sigma}\,\exp\Big(-\frac{(x-\mu)^2}{2\sigma^2}\Big)$	$x\in(-\infty,\infty)$	μ	σ^2
Exponential	$f(x)=\lambda\,e^{-\lambda x},\ x\ge 0$	$x\ge 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Log-normal	$f(x)=\frac{1}{x\,\sigma\sqrt{2\pi}}\,\exp\Big(-\frac{(\ln x-\mu)^2}{2\sigma^2}\Big),\ x>0$	$x>0$	$e^{\mu+\frac{1}{2}\,\sigma^2}$	$e^{2\mu+\sigma^2}\big(e^{\sigma^2}-1\big)$
Pareto	$f(x)=\alpha\,x_m^\alpha\,x^{-(\alpha+1)},\ x\ge x_m$	$x\ge x_m>0$	$\frac{\alpha\,x_m}{\alpha-1},\ \alpha>1$	$\frac{\alpha\,x_m^2}{(\alpha-1)^2(\alpha-2)},\ \alpha>2$
Weibull	$f(x)=\frac{k}{\lambda}\Big(\frac{x}{\lambda}\Big)^{k-1}e^{-(\frac{x}{\lambda})^k},\ x\ge 0$	$x\ge 0$	$\lambda\,\Gamma\big(1+\frac{1}{k}\big)$	$\lambda^2\Big[\Gamma\big(1+\frac{2}{k}\big)-\Gamma\big(1+\frac{1}{k}\big)^2\Big]$
Gumbel	$f(x)=\frac{1}{\beta}\,\exp\Big[-\Big(\frac{x-\mu}{\beta}\Big)-\exp\big(-\frac{x-\mu}{\beta}\big)\Big]$	$x\in\mathbb{R}$	$\mu+\gamma\,\beta\quad(\gamma\approx 0.5772)$	$\frac{\pi^2}{6}\,\beta^2$
Beta prime (Inverted Beta)	$f(x)=\frac{x^{\alpha-1}\,(1+x)^{-\alpha-\beta}}{B(\alpha,\beta)},\ x>0$	$x>0$	$\frac{\alpha}{\beta-1}$, for $\beta>1$	$\frac{\alpha(\alpha+\beta-1)}{(\beta-1)^2(\beta-2)}$, for $\beta>2$
Logistic	$f(x)=\frac{\exp\big(-\frac{x-\mu}{s}\big)}{s\big(1+\exp\big(-\frac{x-\mu}{s}\big)\big)^2},\ x\in\mathbb{R}$	$x\in\mathbb{R}$	μ	$\frac{\pi^2}{3}\,s^2$
Discrete uniform	$P(X=k)=\frac{1}{n},\quad k=1,\ldots,n$	$\{1,\ldots,n\}$	$\frac{n+1}{2}$	$\frac{n^2-1}{12}$
Continuous uniform	$f(x)=\frac{1}{b-a},\ a\le x\le b$	$[a,b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Bernoulli	$P(X=1)=p,\ P(X=0)=1-p$	$\{0,1\}$	p	$p(1-p)$
Binomial	$P(X=k)=\binom{n}{k}\,p^k\,(1-p)^{n-k}$	$\{0,\ldots,n\}$	$n\,p$	$n\,p(1-p)$
Negative binomial	$P(X=k)=\binom{k+r-1}{k}\,p^r(1-p)^k,\ k=0,1,\ldots$	$\{0,1,2,\ldots\}$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$
Geometric	$P(X=k)=(1-p)^k\,p,\ k=0,1,2,\ldots$	$\{0,1,2,\ldots\}$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
Hypergeometric	$P(X=k)=\frac{\binom{K}{k}\,\binom{N-K}{n-k}}{\binom{N}{n}}$	$\{0,\ldots,n\}$	$n\,\frac{K}{N}$	$n\,\frac{K}{N}\big(1-\frac{K}{N}\big)\,\frac{N-n}{N-1}$
Beta-binomial	$P(X=k)=\binom{n}{k}\,\frac{B(\alpha+k,\beta+n-k)}{B(\alpha,\beta)\,B(\alpha+\beta,n)}$	$\{0,\ldots,n\}$	$n\,\frac{\alpha}{\alpha+\beta}$	$n\,p\,(1-p)\,\frac{n+\alpha+\beta}{\alpha+\beta+1},\quad p=\frac{\alpha}{\alpha+\beta}$
Categorical	$P(X=i)=p_i,\ \sum p_i=1$	$\{1,\ldots,K\}$	—	—
Multinomial	$P(\mathbf{X}=\mathbf{k})=\frac{n!}{k_1!\cdots k_K!}\,\prod p_i^{k_i},\ \sum k_i=n$	$\{(k_1,\ldots,k_K)\}$	$\mathbb{E}[X_i]=n\,p_i$	$\mathrm{Var}(X_i)=n\,p_i(1-p_i)$
Multivariate hypergeometric	$P(\mathbf{X}=\mathbf{k})=\frac{\prod_{i=1}^K\binom{K_i}{k_i}}{\binom{N}{n}},\ \sum k_i=n$	$\{(k_1,\ldots,k_K)\}$	$n\,\frac{K_i}{N}$	$\mathrm{Cov}(X_i,X_j)=-n\,\frac{K_i}{N}\,\frac{K_j}{N}\,\frac{N-n}{N-1}$
Poisson	$P(X=k)=\frac{\lambda^k e^{-\lambda}}{k!}$	$\{0,1,2,\ldots\}$	λ	λ
Gamma	$f(x)=\frac{\beta^\alpha}{\Gamma(\alpha)}\,x^{\alpha-1}\,e^{-\beta x},\ x>0$	$x>0$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Rayleigh	$f(r)=\frac{r}{\sigma^2}\,\exp\Big(-\frac{r^2}{2\sigma^2}\Big),\ r\ge 0$	$r\ge 0$	$\sigma\,\sqrt{\frac{\pi}{2}}$	$\frac{4-\pi}{2}\,\sigma^2$
Rice (Rician)	$f(r)=\frac{r}{\sigma^2}\,\exp\Big(-\frac{r^2+\nu^2}{2\sigma^2}\Big)\,I_0\Big(\frac{r\,\nu}{\sigma^2}\Big)$	$r\ge 0$	No simple closed form	No simple closed form
Chi-squared	$f(x)=\frac{1}{2^{k/2}\Gamma(\frac{k}{2})}\,x^{\frac{k}{2}-1}e^{-x/2},\ x>0$	$x>0$	k	$2k$
Student's t	$f(x)=\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}\Big(1+\frac{x^2}{\nu}\Big)^{-\frac{\nu+1}{2}}$	$\mathbb{R}$	0, for $\nu>1$	$\frac{\nu}{\nu-2}$, for $\nu>2$

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Distribution	PMF / PDF		Support	Mean	Variance
F-distribution	$f(x) = \dfrac{\sqrt{\frac{(d_1 x)^{d_1}\, d_2^{d_2}}{(d_1 x + d_2)^{d_1+d_2}}}}{x\, B(\frac{d_1}{2}, \frac{d_2}{2})}$		$x > 0$	$\frac{d_2}{d_2-2}$, for $d_2 > 2$	$\dfrac{2\, d_2^2\, (d_1 + d_2 - 2)}{d_1\, (d_2-2)^2\, (d_2-4)}$, for $d_2 > 4$
Beta	$f(x) = \dfrac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)},\ 0 \le x \le 1$		$0 \le x \le 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\, \beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Dirichlet	$f(x_1, \ldots, x_K)$ $\dfrac{1}{B(\boldsymbol{\alpha})} \prod x_i^{\alpha_i - 1},\ \sum x_i = 1,\ x_i \ge 0$	$=$	Probability simplex	$\mathbb{E}[X_i] = \frac{\alpha_i}{\sum_j \alpha_j}$	Covariance more involved
Wishart	$f(\mathbf{W})$ $\det(\mathbf{W})^{\frac{\nu-p-1}{2}}\, \exp\!\big(-\frac{1}{2}\mathrm{tr}(\Sigma^{-1}\mathbf{W})\big)$	$\propto$	Sym. pos. definite matrices	$\mathbb{E}[\mathbf{W}] = \nu\, \Sigma$	Matrix-variate

Notes / Reminders

- **Support**: Domain where the random variable (or vector) takes values.
- Some distributions require certain parameter constraints (e.g. $\alpha > 1$ for Pareto's mean to exist) to ensure mean or variance is finite.
- For Categorical-like distributions (Categorical, Multinomial, etc.), the "mean" is better thought of as a probability vector in one-hot space (multidimensional).
- **Processes vs. Distributions**: Some items (e.g. Poisson process, linear growth, etc.) are *processes* rather than single distributions. For example, a Poisson process has *increments* that are Poisson-distributed, and *interarrival times* that are exponential or gamma-distributed.
- **Absolute values of normal vectors**: yield Rayleigh or Rice (Rician) distributions.
- **Sum/ratios of squared normals**: yield chi-squared, t, and F distributions, etc.
- **Bayesian conjugate priors**: Beta, Gamma, Dirichlet, and Wishart appear frequently in Bayesian updating for binomial/Poisson/multinomial/multivariate-normal models.
- **Extreme value distributions**: Gumbel, Fréchet, and Weibull are common for modeling maxima or minima.

Contents

1 Normal (Gaussian) Distribution

The Normal (Gaussian) distribution with mean μ and standard deviation σ has

$$f(x) \;=\; \frac{1}{\sqrt{2\pi}\,\sigma}\,\exp\!\Big(-\frac{(x-\mu)^2}{2\sigma^2}\Big), \quad x \in (-\infty, \infty).$$

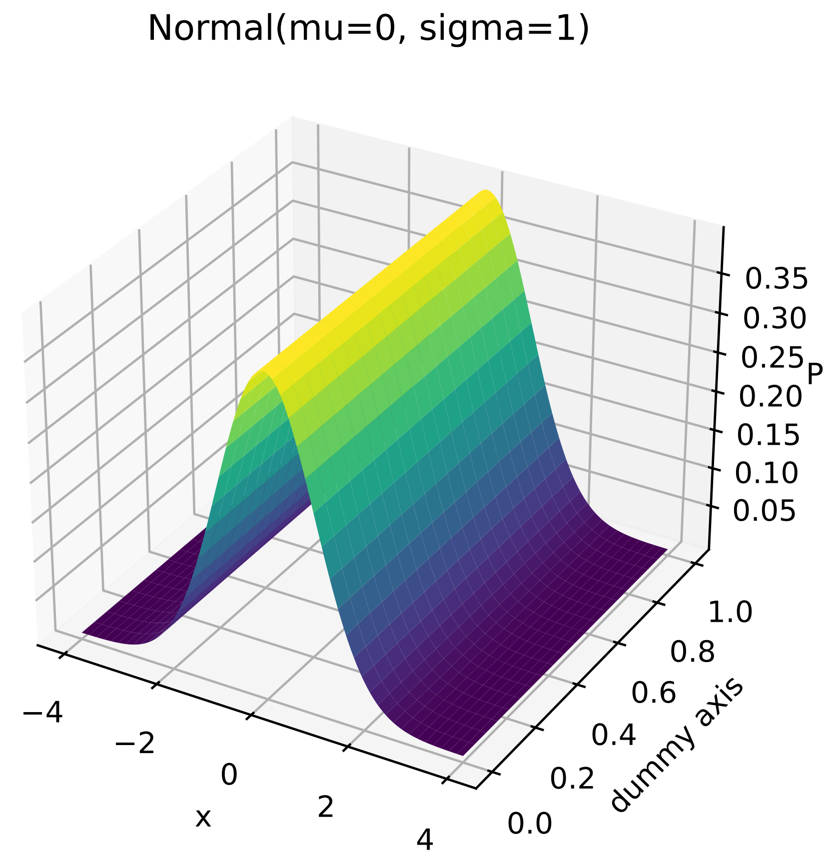

Figure 1: 3D surface plot of the Normal(μ, σ^2) PDF.

2 Exponential Distribution

The Exponential distribution with rate λ has

$$f(x) \;=\; \lambda\, e^{-\lambda x}, \quad x \geq 0.$$

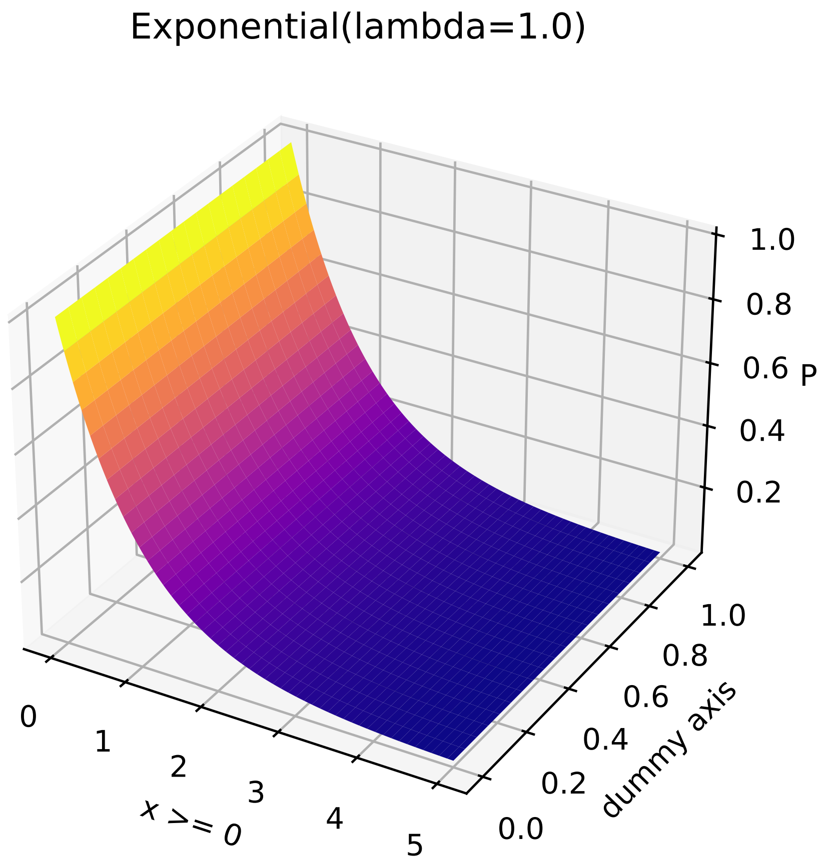

Figure 2: 3D surface plot of Exponential(λ) PDF.

3 Log-normal Distribution

If $Y \sim \text{Normal}(\mu, \sigma^2)$, then $X = e^Y$ is said to be Log-normal. Its PDF is

$$f(x) \;=\; \frac{1}{x\,\sigma\,\sqrt{2\pi}}\,\exp\!\Big(-\frac{(\ln x-\mu)^2}{2\sigma^2}\Big), \quad x>0.$$

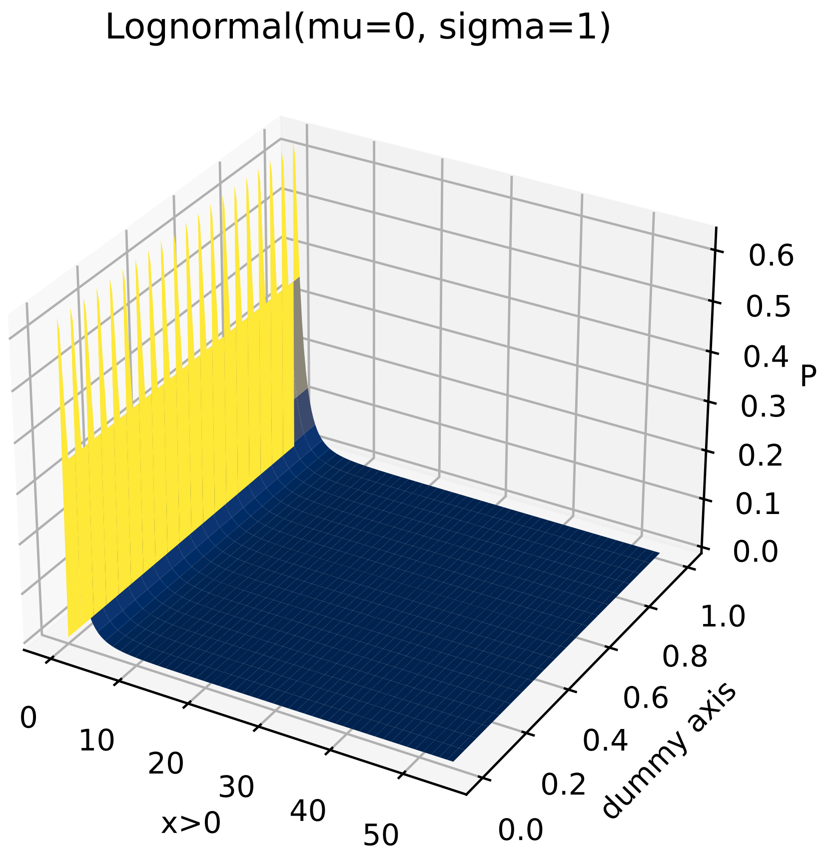

Figure 3: 3D surface plot of the Log-normal(μ, σ^2) PDF.

4 Pareto Distribution

The Pareto distribution with shape α and scale x_m has

$$f(x) \;=\; \alpha\, x_m^{\alpha}\, x^{-(\alpha+1)}, \quad x \ge x_m.$$

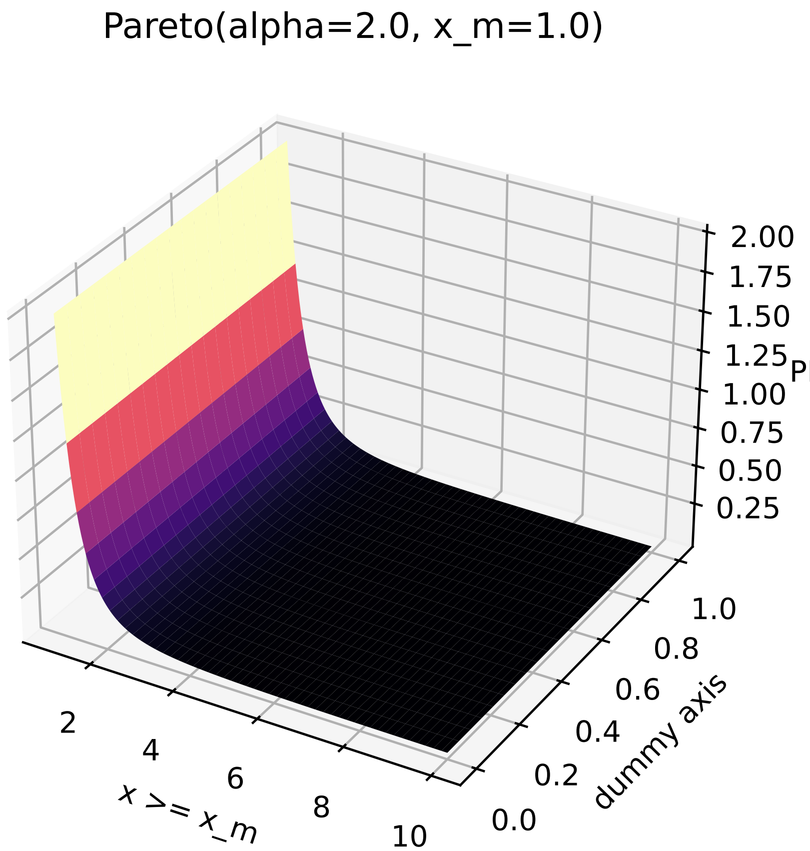

Figure 4: 3D surface plot of Pareto(α, x_m) PDF.

5 Weibull Distribution

The Weibull distribution with parameters k and λ has

$$f(x) \;=\; \frac{k}{\lambda}\Big(\frac{x}{\lambda}\Big)^{k-1}\exp\!\Big(-\Big(\frac{x}{\lambda}\Big)^{k}\Big), \quad x \geq 0.$$

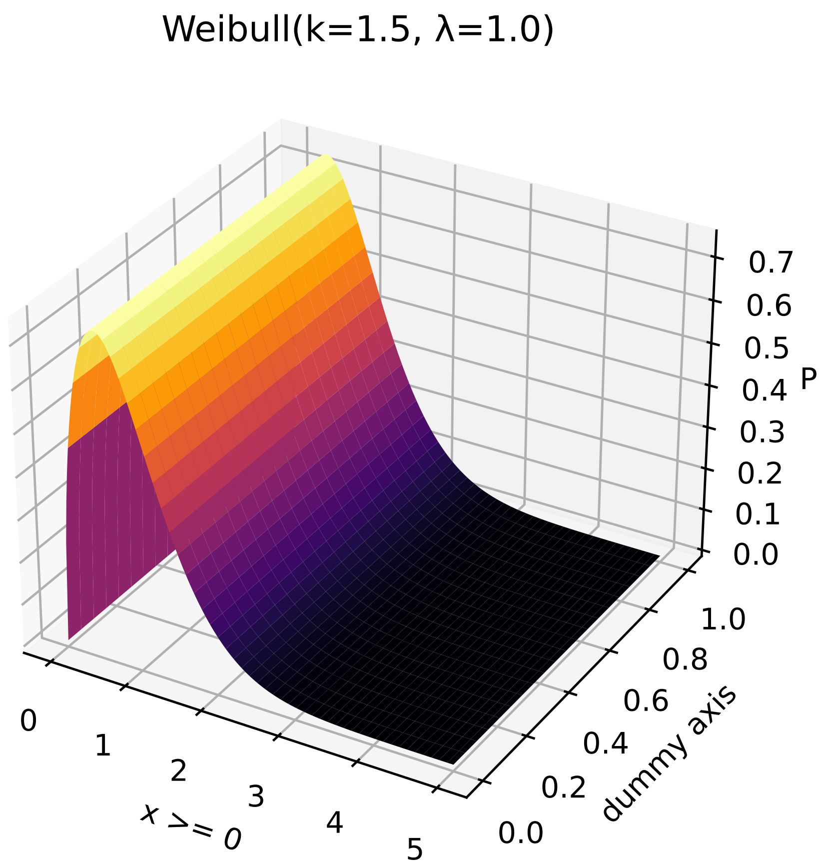

Figure 5: 3D surface plot of Weibull(k, λ) PDF.

6 Gumbel Distribution

The Gumbel (Type-I extreme value) distribution with location μ and scale β has

$$f(x) \;=\; \frac{1}{\beta}\,\exp\Bigl[-\left(\frac{x-\mu}{\beta}\right)\;-\;\exp\bigl(-\tfrac{x-\mu}{\beta}\bigr)\Bigr].$$

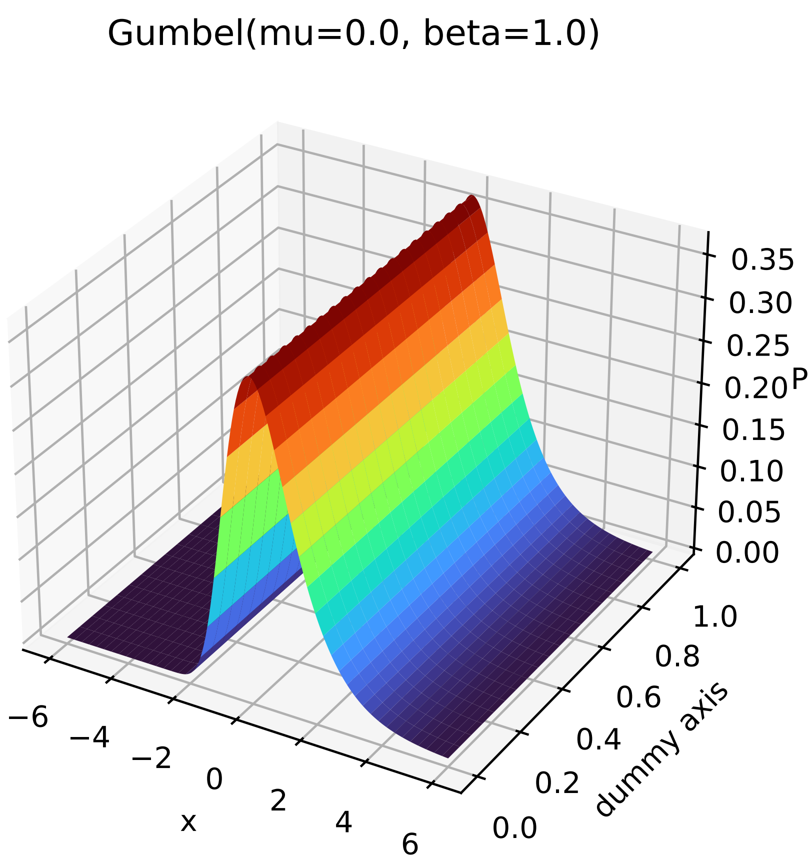

Figure 6: 3D surface plot of Gumbel(μ, β) PDF.

7 Beta Prime (Inverted Beta) Distribution

Sometimes called the inverted Beta distribution, with parameters α, β:

$$f(x) = \frac{x^{\alpha-1}(1+x)^{-\alpha-\beta}}{B(\alpha,\beta)}, \quad x > 0.$$

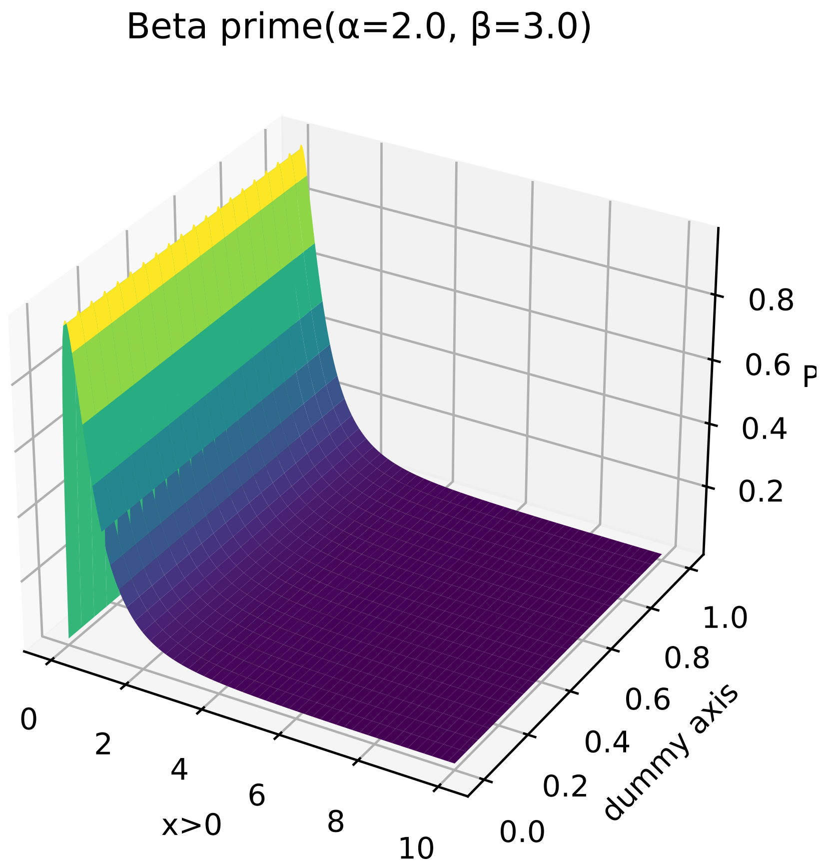

Figure 7: 3D surface plot of Beta prime(α, β) PDF.

8 Logistic Distribution

The Logistic distribution with parameters μ (location) and s (scale):

$$f(x) \;=\; \frac{\exp\!\left(-\frac{x-\mu}{s}\right)}{s\Big(1+\exp\!\left(-\frac{x-\mu}{s}\right)\Big)^2}, \quad x \in \mathbb{R}.$$

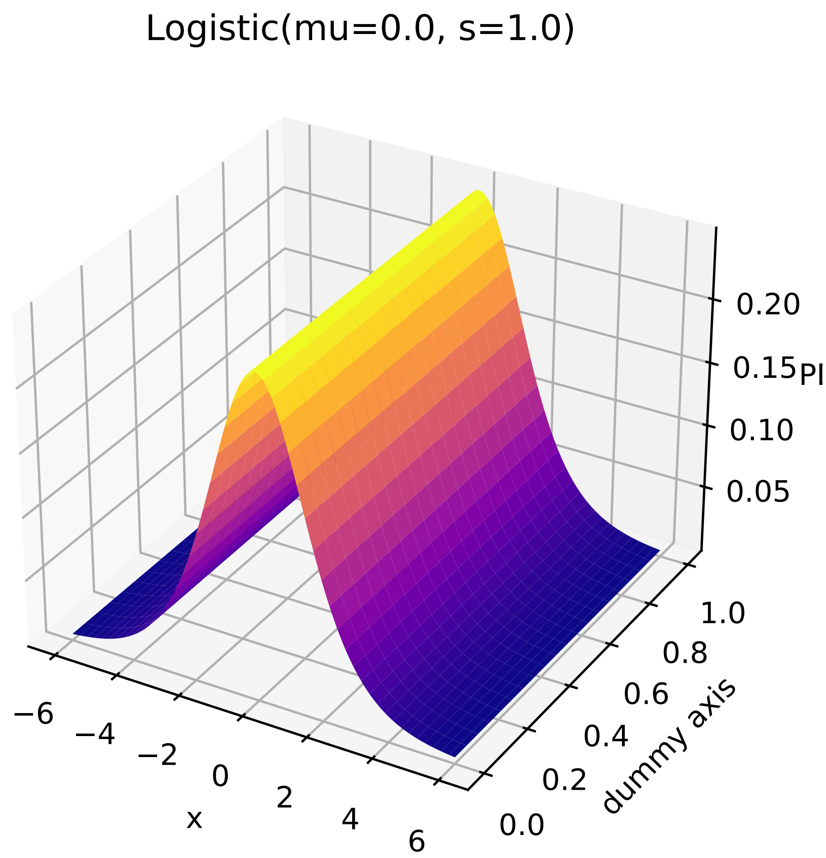

Figure 8: 3D surface plot of Logistic(μ, s) PDF.

9 Discrete Uniform Distribution

A discrete uniform distribution over $\{1, \ldots, n\}$ has

$$P(X = k) \;=\; \frac{1}{n}, \quad k = 1, \ldots, n.$$

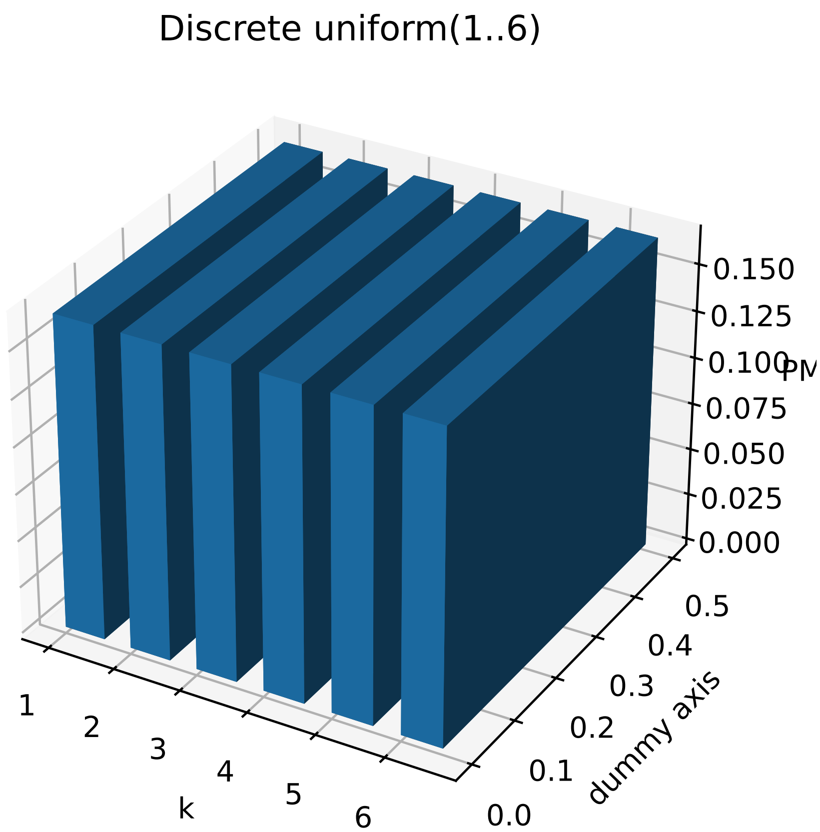

Figure 9: 3D bar chart of Discrete Uniform$\{1, \ldots, n\}$.

10 Continuous Uniform Distribution

A continuous uniform distribution on $[a, b]$:

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b.$$

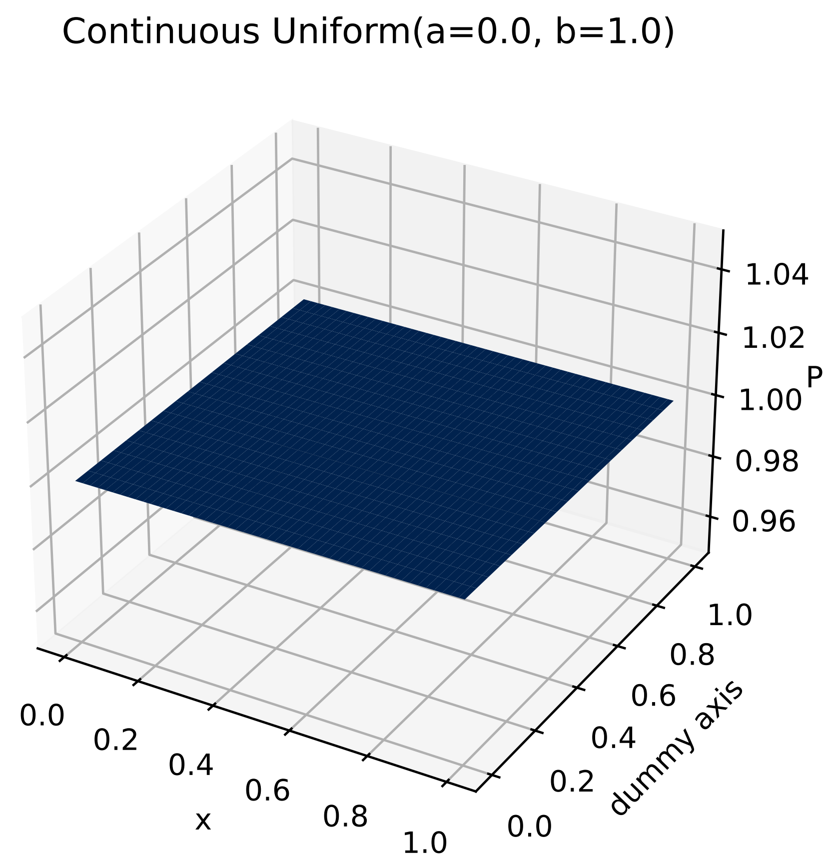

Figure 10: 3D surface plot of Uniform(a, b) PDF.

11 Bernoulli Distribution

A Bernoulli distribution with parameter p has

$$P(X = 1) \;=\; p, \quad P(X = 0) \;=\; 1 - p.$$

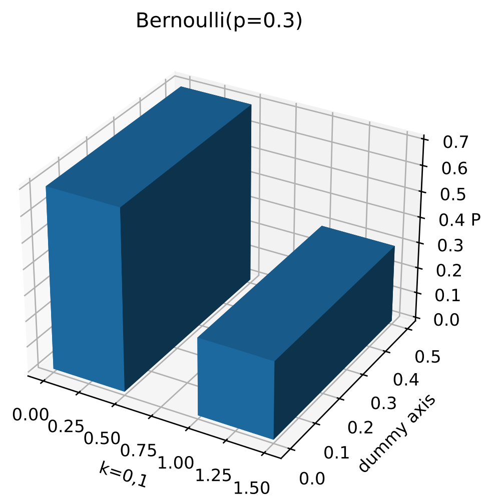

Figure 11: 3D bar chart of Bernoulli(p) PMF.

12 Binomial Distribution

A Binomial(n, p) distribution has

$$P(X = k) \;=\; \binom{n}{k}\, p^k\,(1-p)^{n-k}, \quad k = 0, \ldots, n.$$

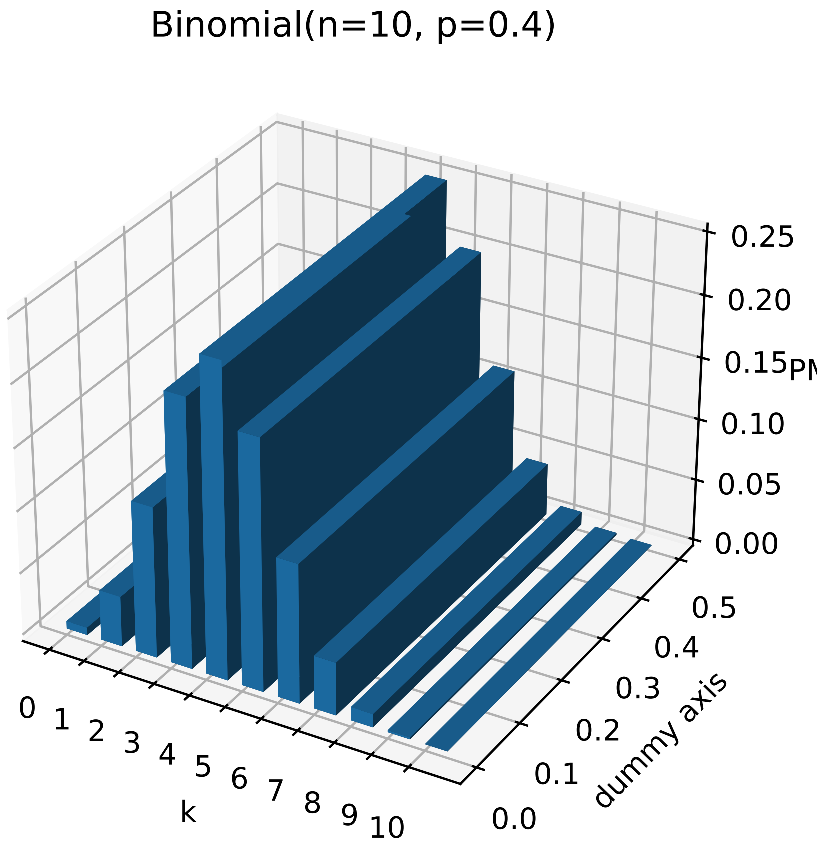

Figure 12: 3D bar chart of Binomial(n, p) PMF.

13 Negative Binomial Distribution

A Negative binomial with parameters (r, p):

$$P(X = k) \;=\; \binom{k + r - 1}{k}\, p^r\, (1 - p)^k, \quad k = 0, 1, \ldots$$

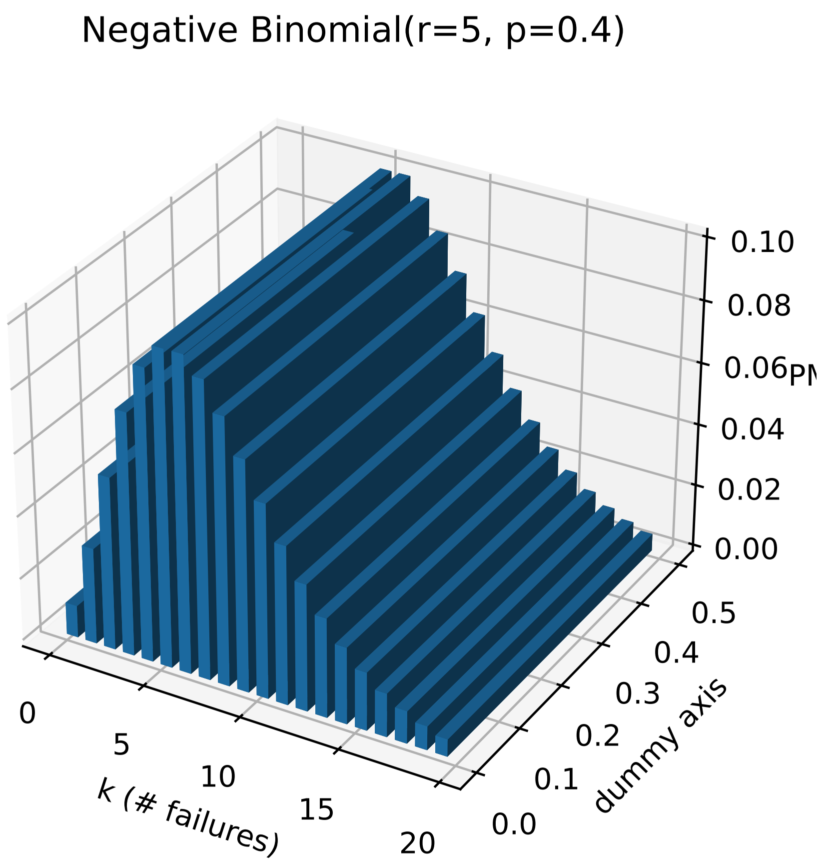

Figure 13: 3D bar chart of Negative Binomial(r, p) PMF.

14 Geometric Distribution

A special case of Negative binomial with $r = 1$:

$$P(X = k) \;=\; (1-p)^k\, p, \quad k = 0, 1, 2, \ldots$$

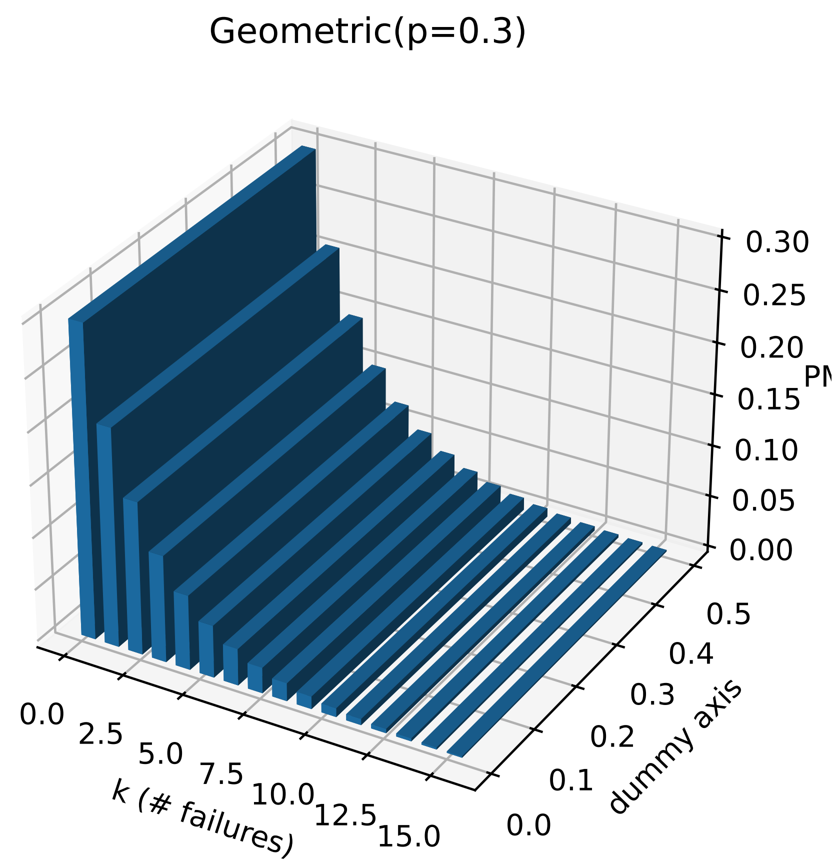

Figure 14: 3D bar chart of Geometric(p) PMF.

15 Hypergeometric Distribution

A Hypergeometric(N, K, n) distribution:

$$P(X = k) \;=\; \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}.$$

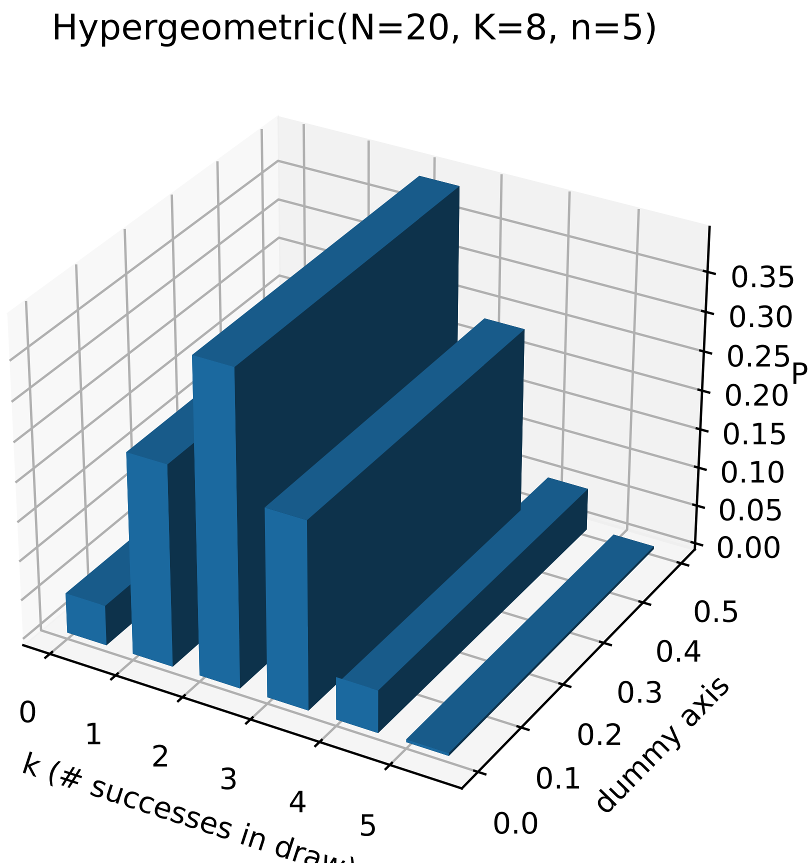

Figure 15: 3D bar chart of Hypergeometric(N, K, n) PMF.

16 Beta-binomial Distribution

A conjugate extension of Binomial with parameters (n, α, β):

$$P(X = k) = \binom{n}{k} \frac{B(\alpha + k,\ \beta + n - k)}{B(\alpha, \beta)\, B(\alpha + \beta, n)}.$$

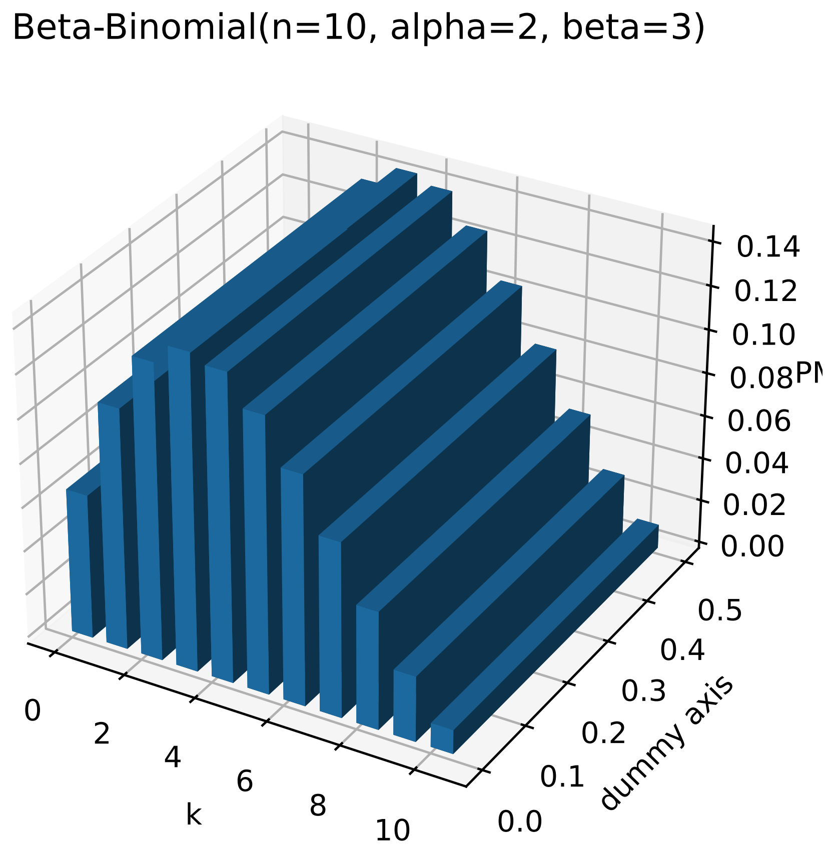

Figure 16: 3D bar chart of Beta-binomial(n, α, β) PMF.

17 Categorical Distribution

For K categories with probabilities p_i,

$$P(X = i) \;=\; p_i, \quad \sum_{i=1}^{K} p_i = 1.$$

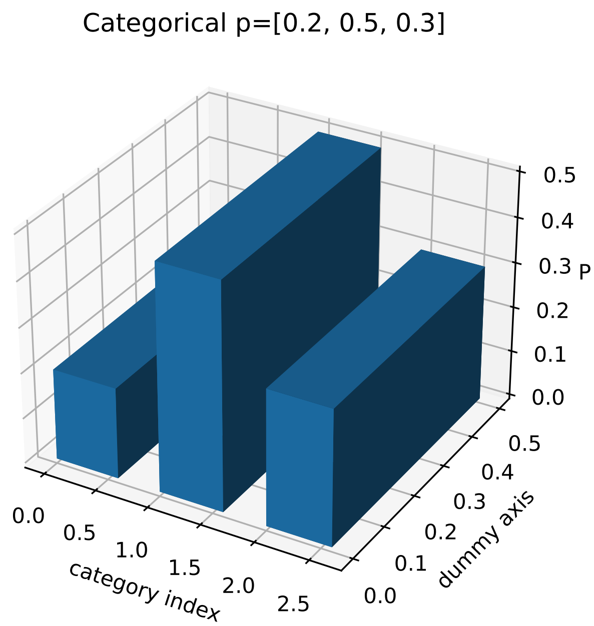

Figure 17: 3D bar chart of a Categorical distribution.

18 Multinomial Distribution

Generalizing Binomial to K categories:

$$P(X_1 = k_1, \ldots, X_K = k_K) \;=\; \frac{n!}{k_1! \cdots k_K!} \prod_{i=1}^{K} p_i^{k_i}, \quad \sum k_i = n.$$

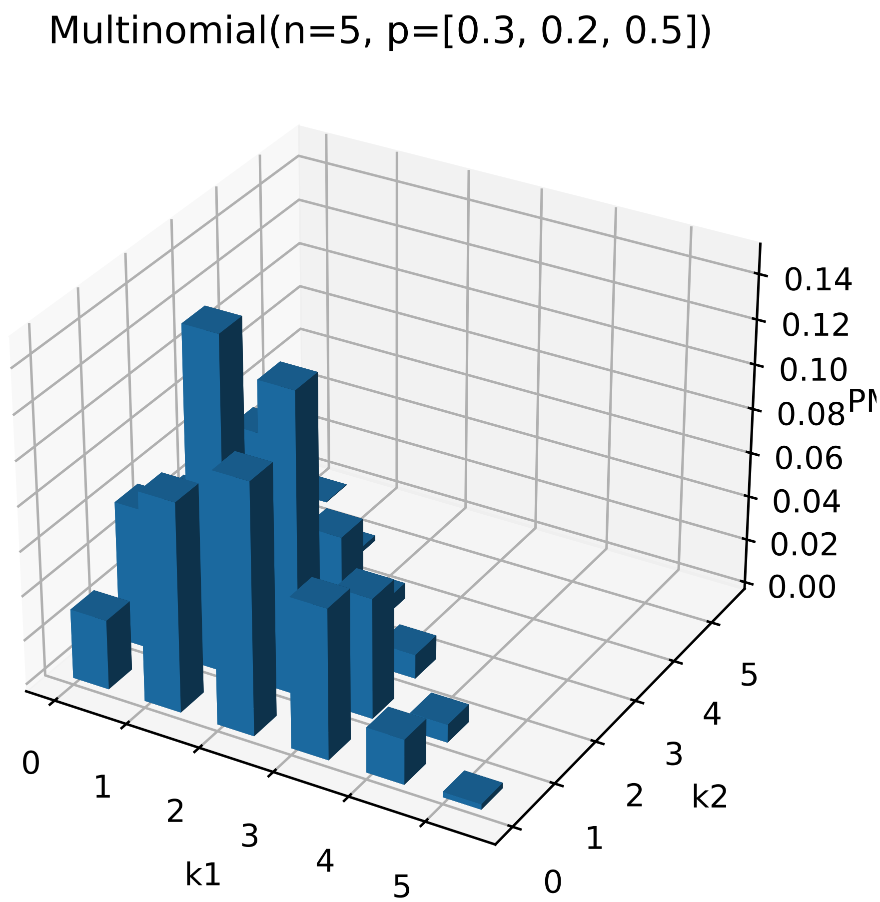

Figure 18: 3D bar chart of Multinomial$(n, p_1, \ldots, p_K)$ PMF for $K = 3$.

19 Multivariate Hypergeometric Distribution

A generalization of Hypergeometric to multiple categories. With K_i items in category i,

$$P(\mathbf{X}=\mathbf{k}) \;=\; \frac{\prod_{i=1}^{K}\binom{K_i}{k_i}}{\binom{N}{n}}, \quad \sum_{i=1}^{K} k_i = n.$$

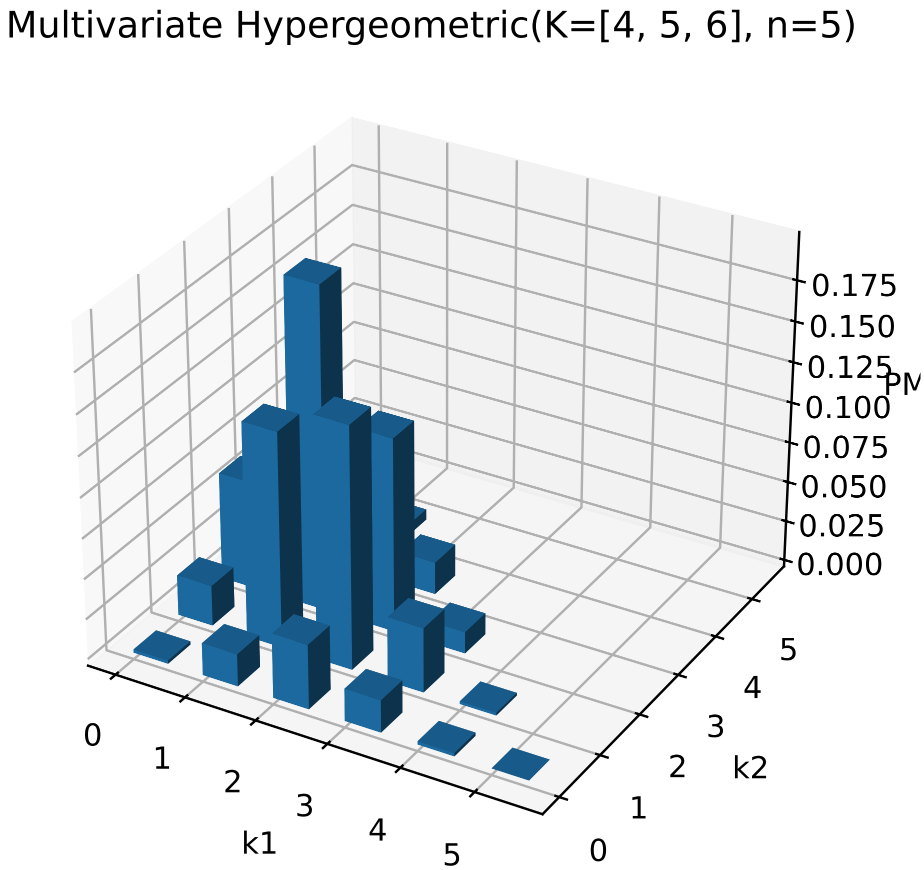

Figure 19: 3D bar chart for the Multivariate Hypergeometric distribution with $K = 3$.

20 Poisson Distribution

A Poisson(λ) distribution:

$$P(X = k) \;=\; \frac{\lambda^k\, e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \ldots$$

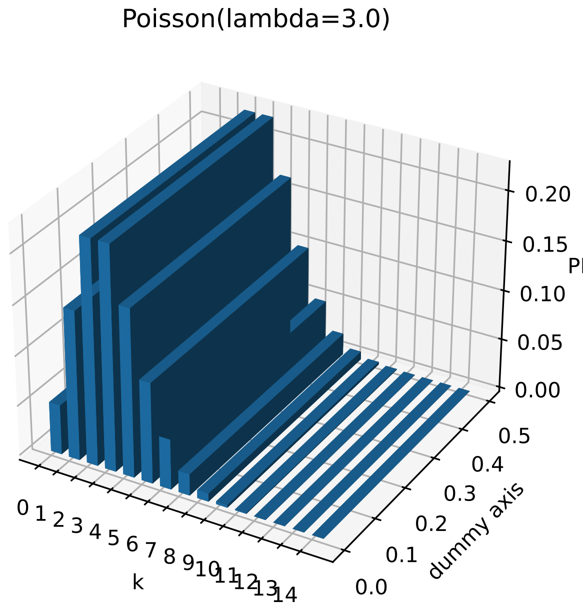

Figure 20: 3D bar chart of Poisson(λ) PMF.

21 Gamma Distribution

With shape α and rate β:

$$f(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x > 0.$$

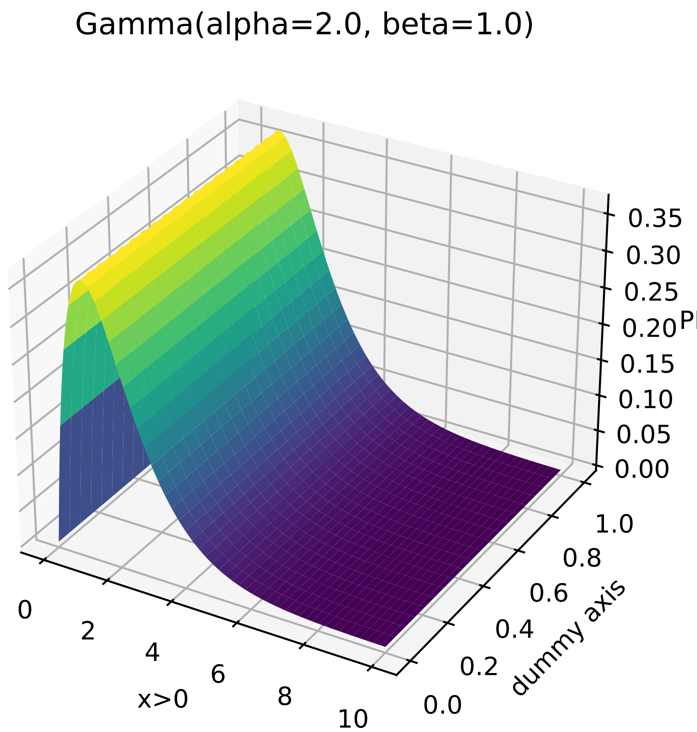

Figure 21: 3D surface plot of Gamma(α, β) PDF.

22 Rayleigh Distribution

Rayleigh(σ):

$$f(r) = \frac{r}{\sigma^2} \exp\!\Big(-\frac{r^2}{2\sigma^2}\Big), \quad r \geq 0.$$

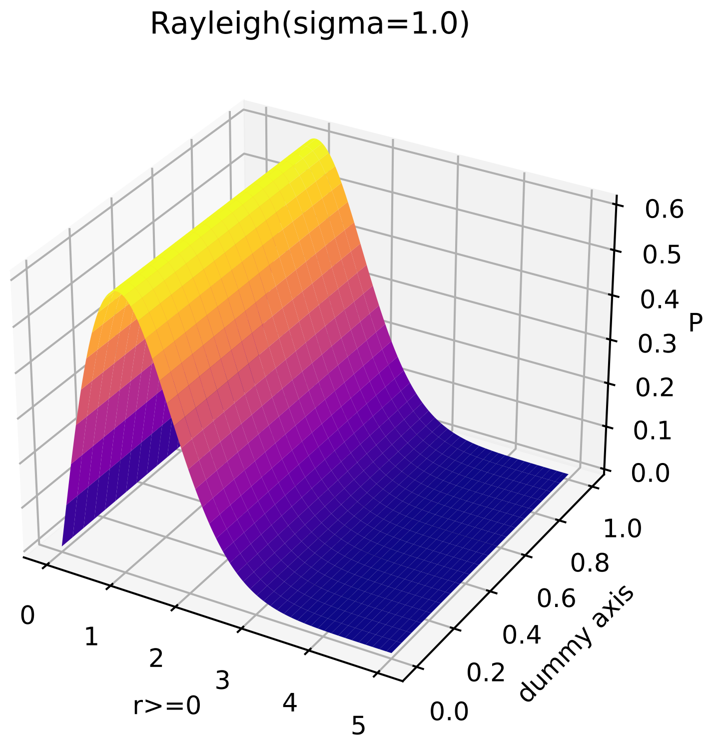

Figure 22: 3D surface plot of Rayleigh(σ) PDF.

23 Rice (Rician) Distribution

Rice(ν, σ):

$$f(r) \;=\; \frac{r}{\sigma^2}\exp\!\Big(-\frac{r^2+\nu^2}{2\sigma^2}\Big)\, I_0\!\Big(\frac{r\,\nu}{\sigma^2}\Big), \quad r \geq 0.$$

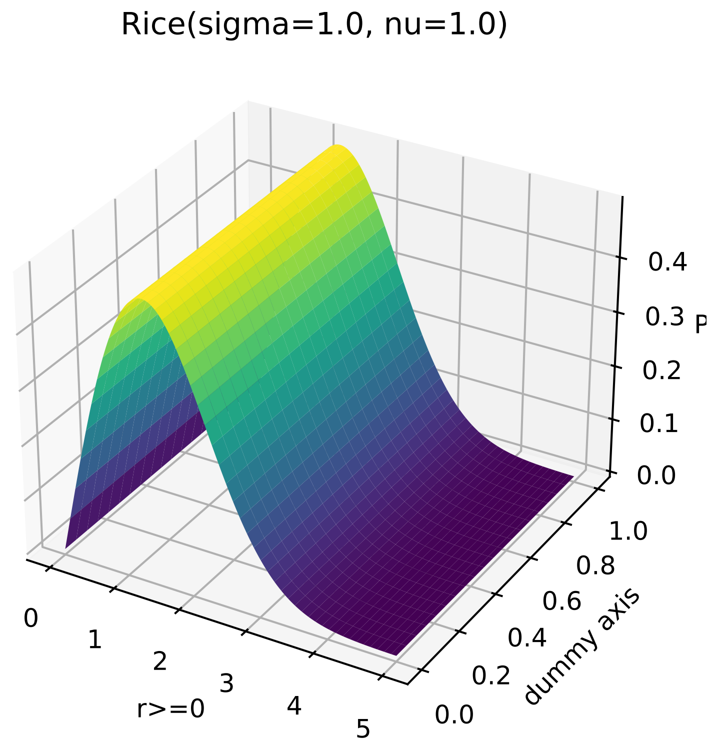

Figure 23: 3D surface plot of Rice(ν, σ) PDF.

24 Chi-squared Distribution

Chi-squared with k degrees of freedom:

$$f(x) \;=\; \frac{1}{2^{k/2}\Gamma(\frac{k}{2})}\, x^{\frac{k}{2}-1}\, e^{-x/2}, \quad x > 0.$$

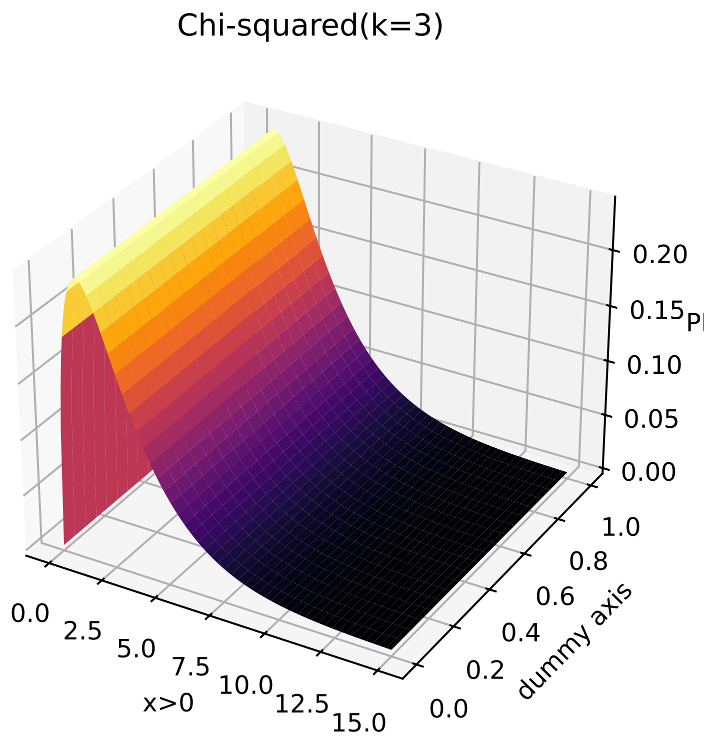

Figure 24: 3D surface plot of Chi-squared(k) PDF.

25 Student's t Distribution

With ν degrees of freedom:

$$f(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \quad x \in \mathbb{R}.$$

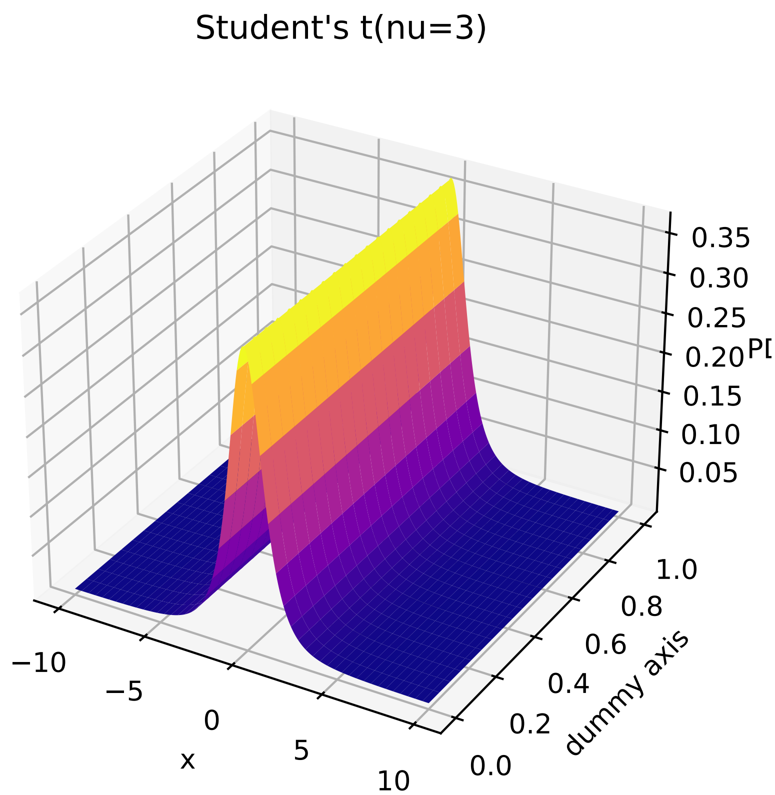

Figure 25: 3D surface plot of Student's $t(\nu)$ PDF.

26 F-distribution

With degrees of freedom d_1, d_2:

$$f(x) = \frac{\sqrt{\frac{(d_1\,x)^{d_1}\,d_2^{d_2}}{(d_1\,x+d_2)^{d_1+d_2}}}}{x\,B\!\left(\frac{d_1}{2},\frac{d_2}{2}\right)}, \quad x > 0.$$

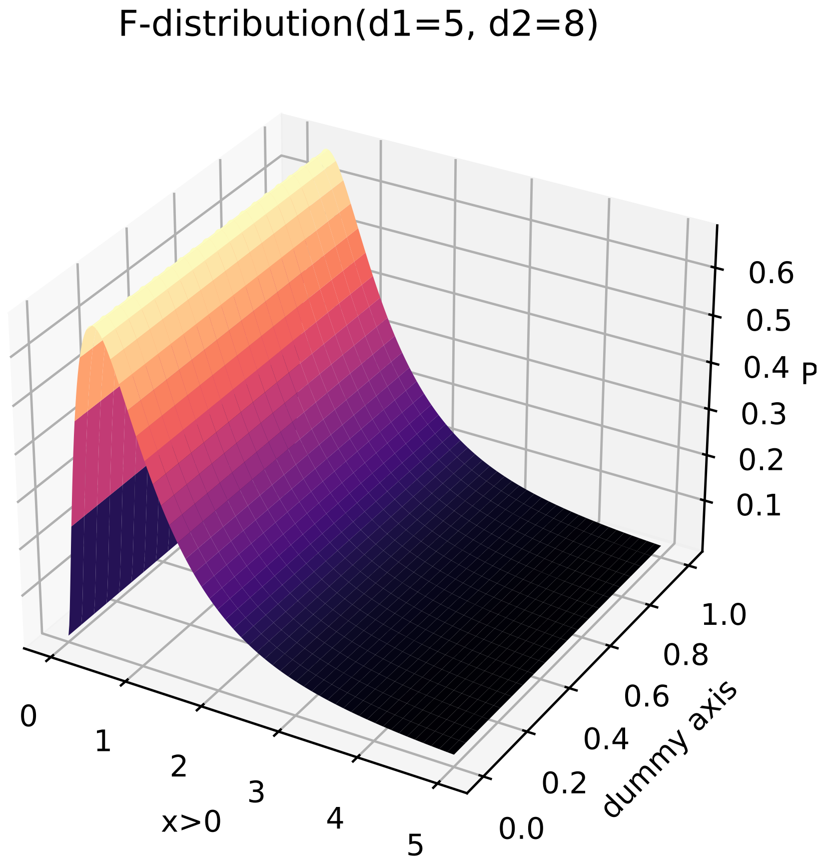

Figure 26: 3D surface plot of $F(d_1, d_2)$ PDF.

27 Beta Distribution

With parameters α, β on $[0,1]$:

$$f(x) \;=\; \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, \quad 0 \le x \le 1.$$

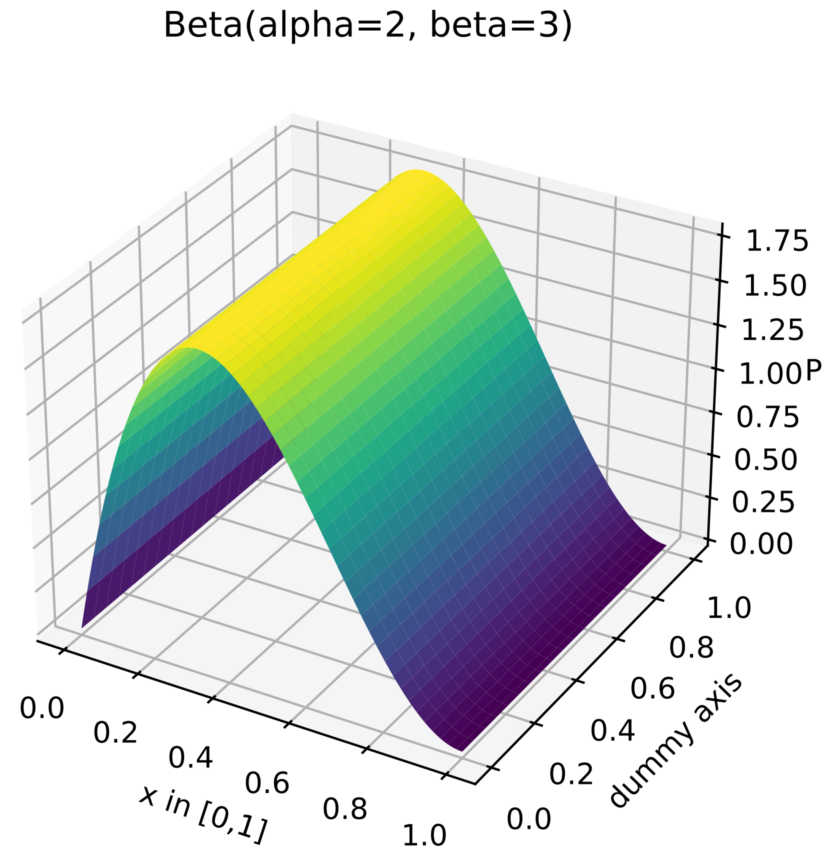

Figure 27: 3D surface plot of Beta(α, β) PDF.

28 Dirichlet Distribution

A distribution over the probability simplex $x_1 + x_2 + \cdots + x_K = 1, x_i \geq 0$:

$$f(x_1, \ldots, x_K) = \frac{1}{B(\boldsymbol{\alpha})} \prod_{i=1}^{K} x_i^{\alpha_i - 1}, \quad \sum_{i=1}^{K} x_i = 1.$$

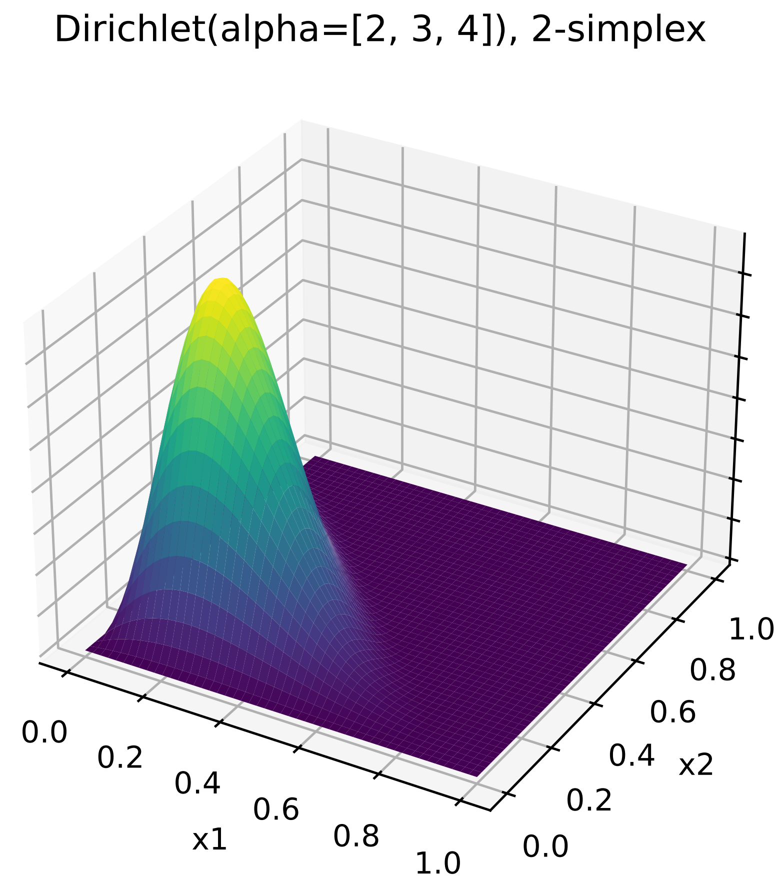

Figure 28: 3D surface plot of Dirichlet$(\alpha_1, \alpha_2, \alpha_3)$ along 2-simplex.

29 Wishart Distribution (Placeholder Slice)

The Wishart distribution is matrix-valued. For a $p \times p$ positive-definite matrix $\mathbf{W}$:

$$f(\mathbf{W}) \propto \det(\mathbf{W})^{\frac{\nu-p-1}{2}} \exp\bigl(-\tfrac{1}{2}\mathrm{tr}(\Sigma^{-1}\mathbf{W})\bigr).$$

Here, we show a *slice* for a 2×2 Wishart (diagonal only), as a demonstration:

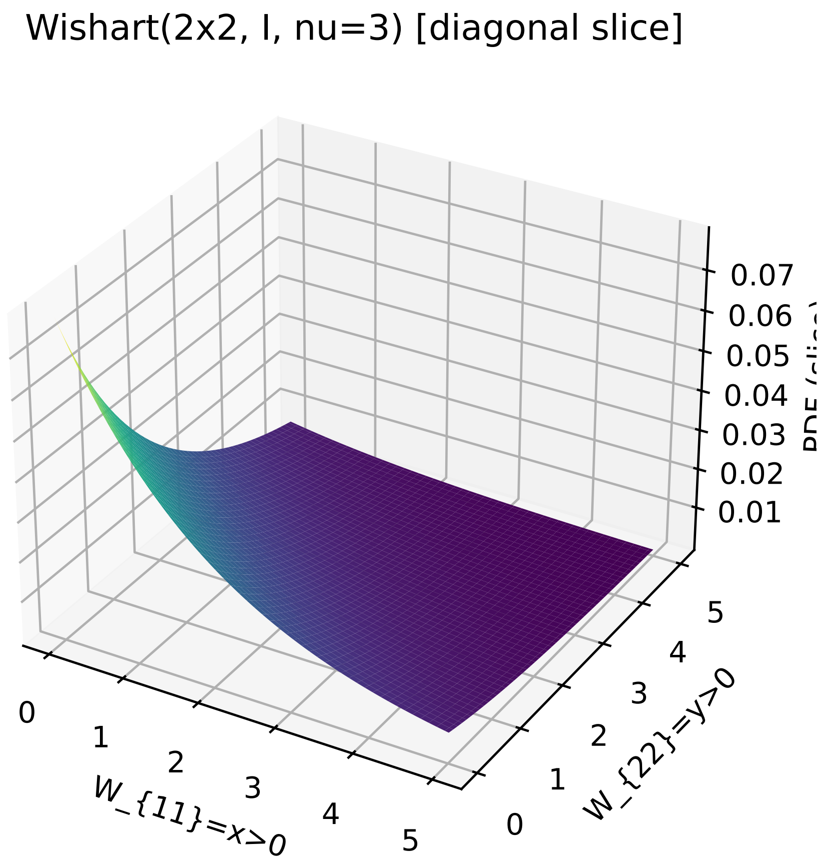

Figure 29: A 3D "slice" of Wishart$(\nu, \Sigma = I)$ in 2D, restricting $\mathbf{W}$ to diagonal.

30 Normal (Gaussian) Distribution

The Normal (Gaussian) distribution with mean μ and standard deviation σ has

$$f(x) \;=\; \frac{1}{\sqrt{2\pi}\,\sigma}\,\exp\!\Big(-\frac{(x-\mu)^2}{2\sigma^2}\Big), \quad x \in (-\infty, \infty).$$

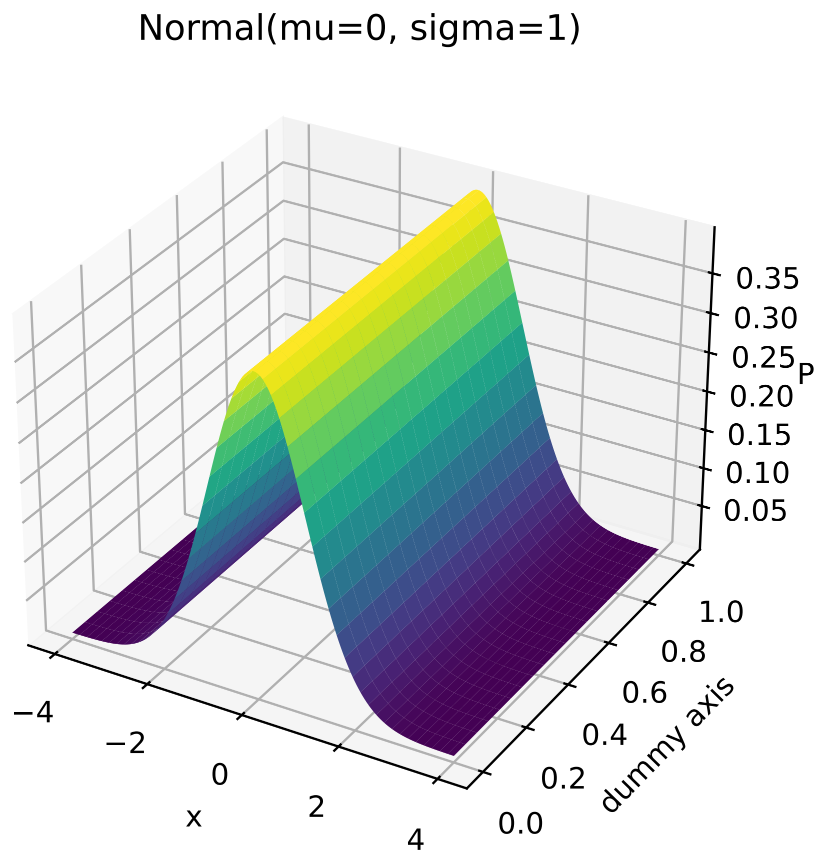

Figure 30: 3D surface plot of the Normal(μ, σ^2) PDF.

31 Exponential Distribution

The Exponential distribution with rate λ has

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0.$$

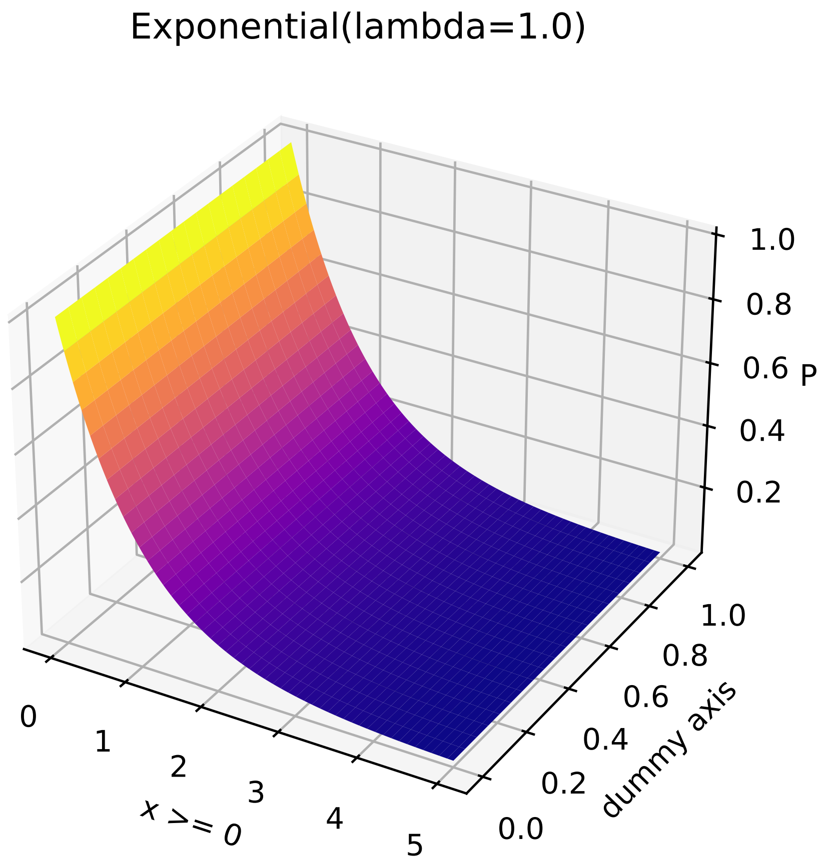

Figure 31: 3D surface plot of Exponential(λ) PDF.

32 Log-normal Distribution

If $Y \sim \mathrm{Normal}(\mu, \sigma^2)$, then $X = e^Y$ is said to be Log-normal. Its PDF is

$$f(x) \;=\; \frac{1}{x\,\sigma\,\sqrt{2\pi}}\,\exp\!\Big(-\frac{(\ln x - \mu)^2}{2\sigma^2}\Big), \quad x > 0.$$

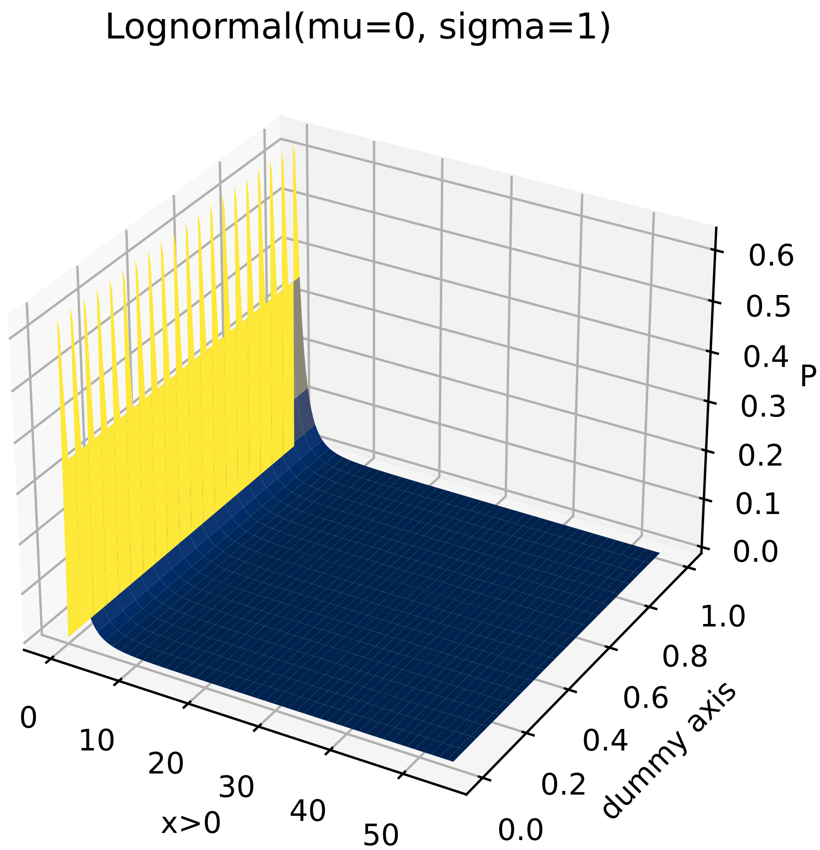

Figure 32: 3D surface plot of the Log-normal(μ, σ^2) PDF.

33 Pareto Distribution

The Pareto distribution with shape α and scale x_m has

$$f(x) \;=\; \alpha\, x_m^{\alpha}\, x^{-(\alpha+1)}, \quad x \geq x_m.$$

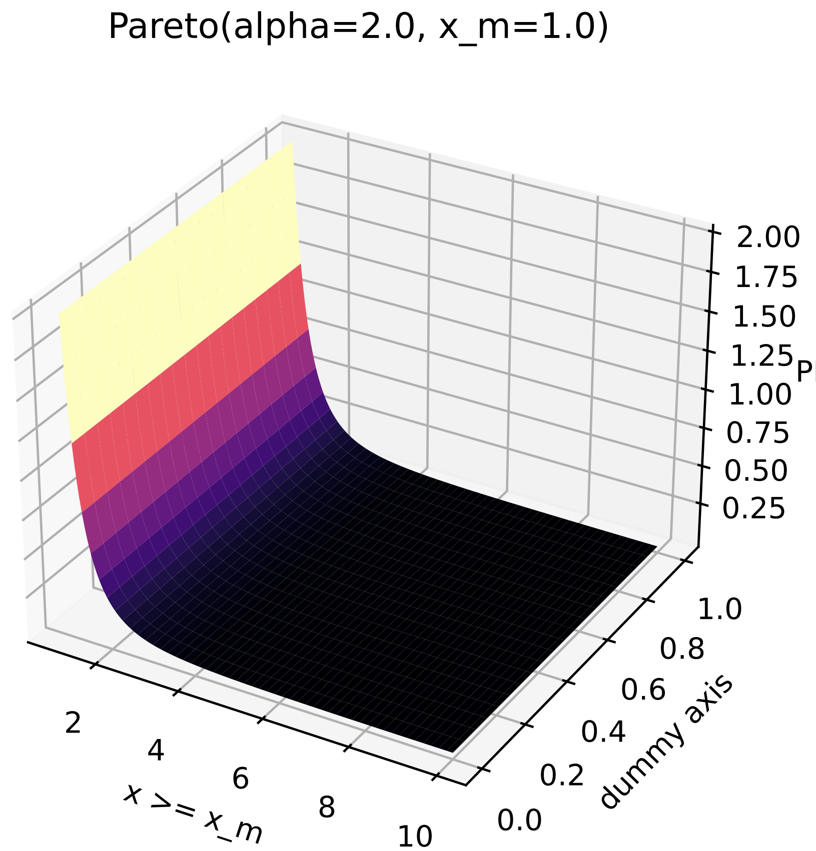

Figure 33: 3D surface plot of Pareto(α, x_m) PDF.

34 Weibull Distribution

The Weibull distribution with parameters k and λ has

$$f(x) = \frac{k}{\lambda}\Big(\frac{x}{\lambda}\Big)^{k-1}\exp\Big(-\Big(\frac{x}{\lambda}\Big)^{k}\Big), \quad x \geq 0.$$

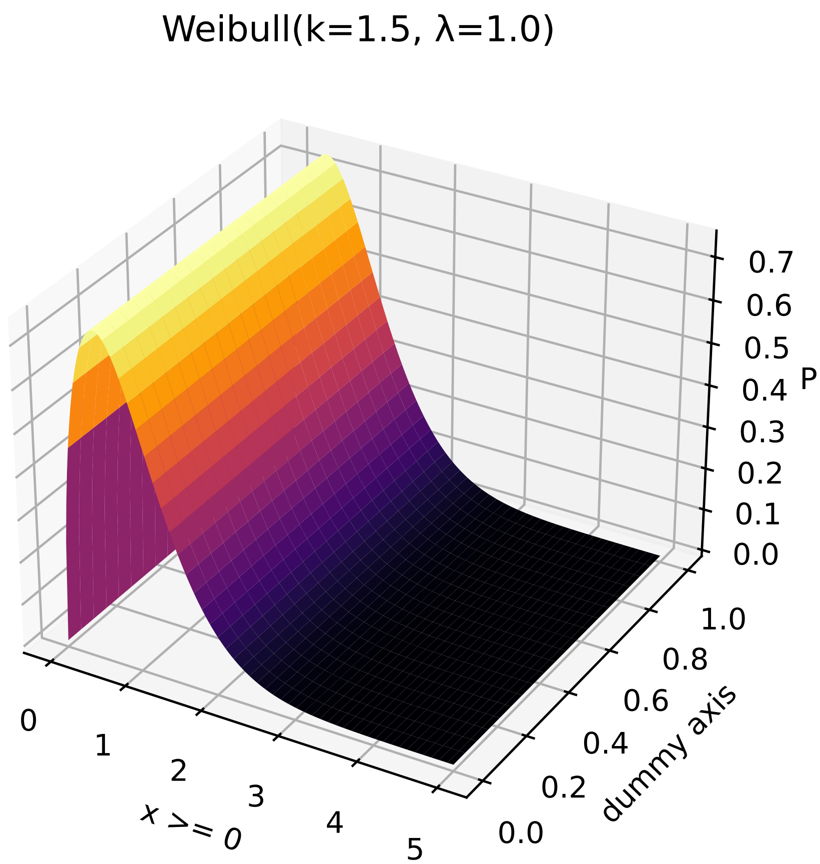

Figure 34: 3D surface plot of Weibull(k, λ) PDF.

35 Gumbel Distribution

The Gumbel (Type-I extreme value) distribution with location μ and scale β has

$$f(x) = \frac{1}{\beta} \exp\Big[-\Big(\frac{x-\mu}{\beta}\Big) - \exp\big(-\tfrac{x-\mu}{\beta}\big)\Big].$$

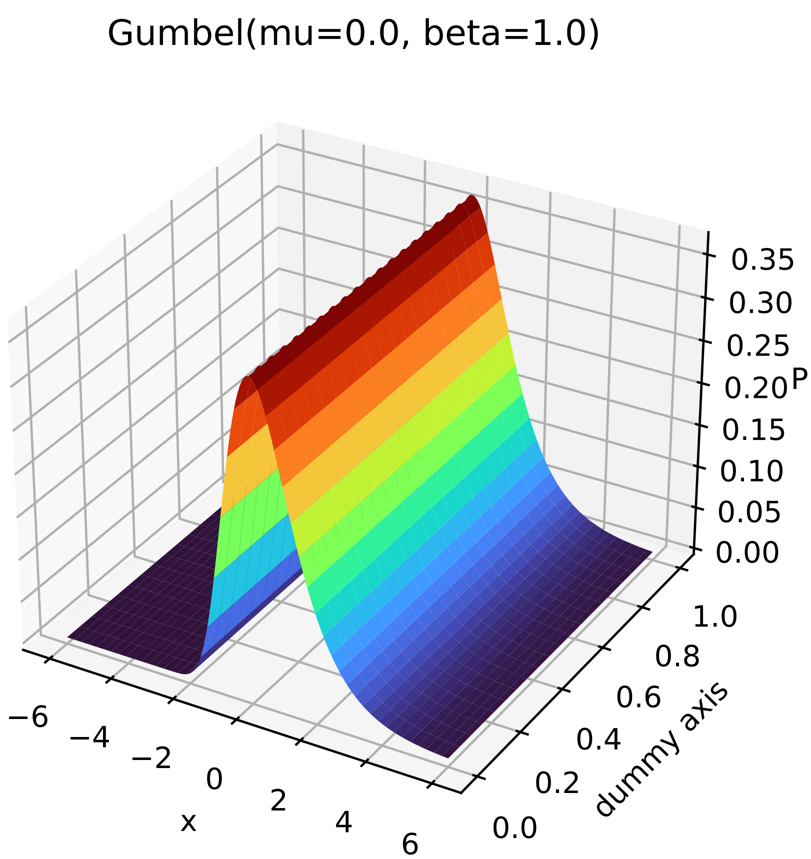

Figure 35: 3D surface plot of Gumbel(μ, β) PDF.

36 Beta Prime (Inverted Beta) Distribution

Sometimes called the inverted Beta distribution, with parameters α, β:

$$f(x) = \frac{x^{\alpha-1}(1+x)^{-\alpha-\beta}}{B(\alpha,\beta)}, \quad x > 0.$$

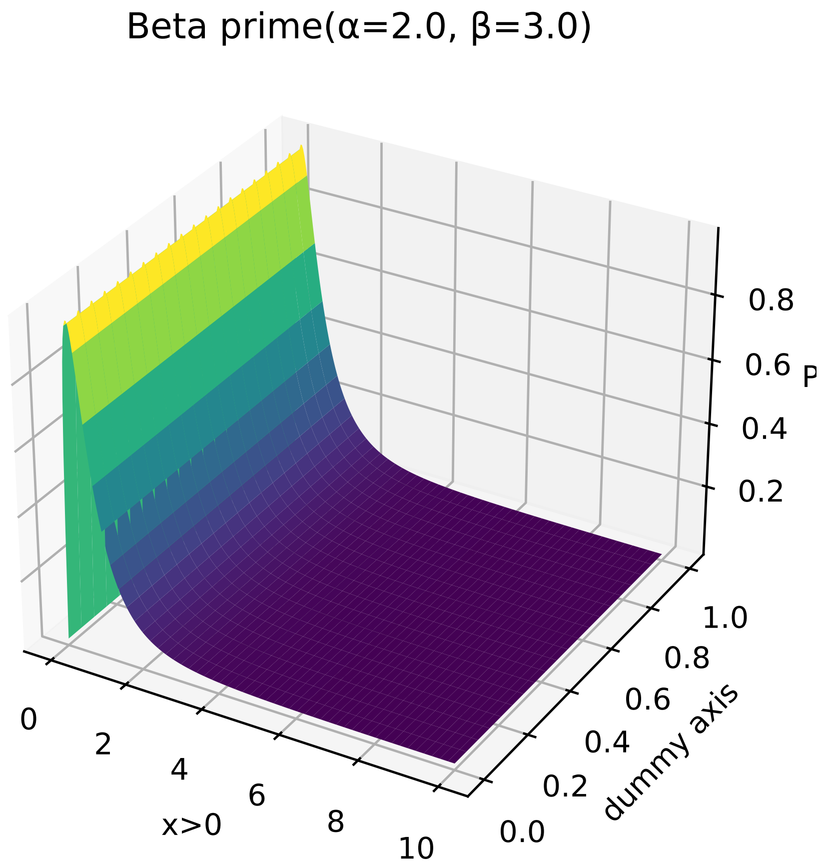

Figure 36: 3D surface plot of Beta prime(α, β) PDF.

37 Logistic Distribution

The Logistic distribution with parameters μ (location) and s (scale):

$$f(x) = \frac{\exp\left(-\frac{x-\mu}{s}\right)}{s\left(1+\exp\left(-\frac{x-\mu}{s}\right)\right)^2}, \quad x \in \mathbb{R}.$$

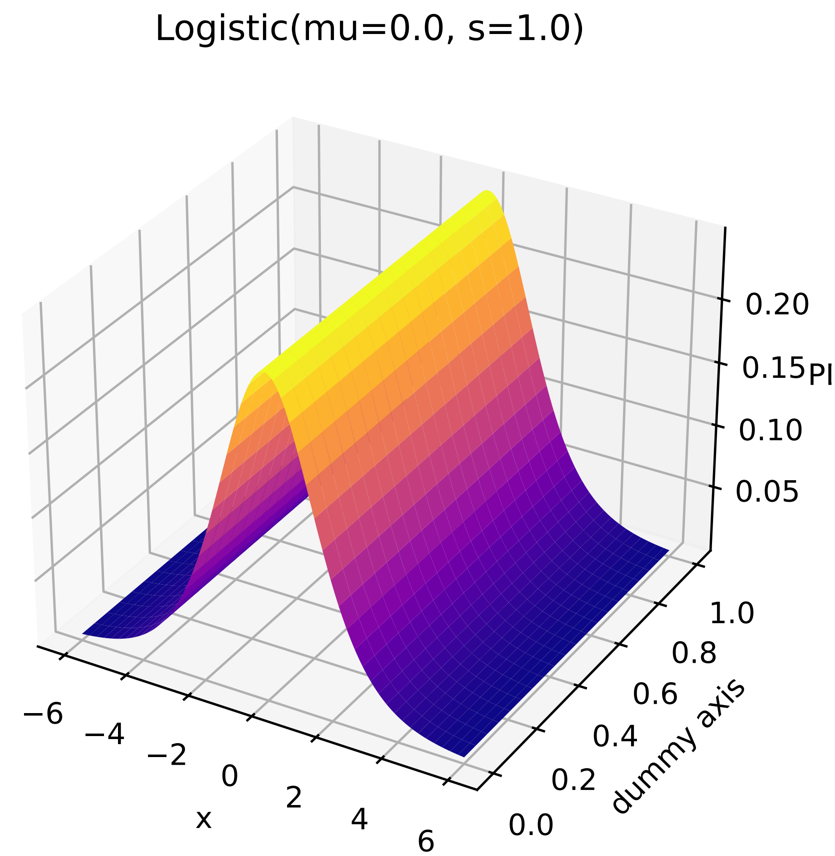

Figure 37: 3D surface plot of Logistic(μ, s) PDF.

38 Discrete Uniform Distribution

A discrete uniform distribution over $\{1, \ldots, n\}$ has

$$P(X = k) \;=\; \frac{1}{n}, \quad k = 1, \ldots, n.$$

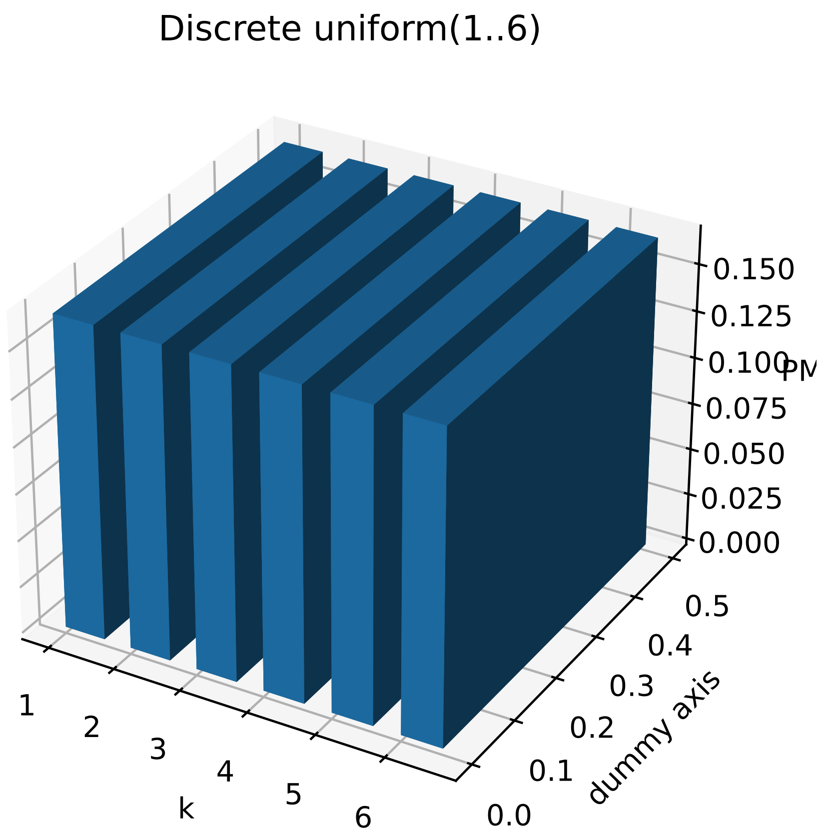

Figure 38: 3D bar chart of Discrete Uniform$\{1, \ldots, n\}$.

39 Continuous Uniform Distribution

A continuous uniform distribution on $[a, b]$:

$$f(x) = \frac{1}{b-a}, \quad a \le x \le b.$$

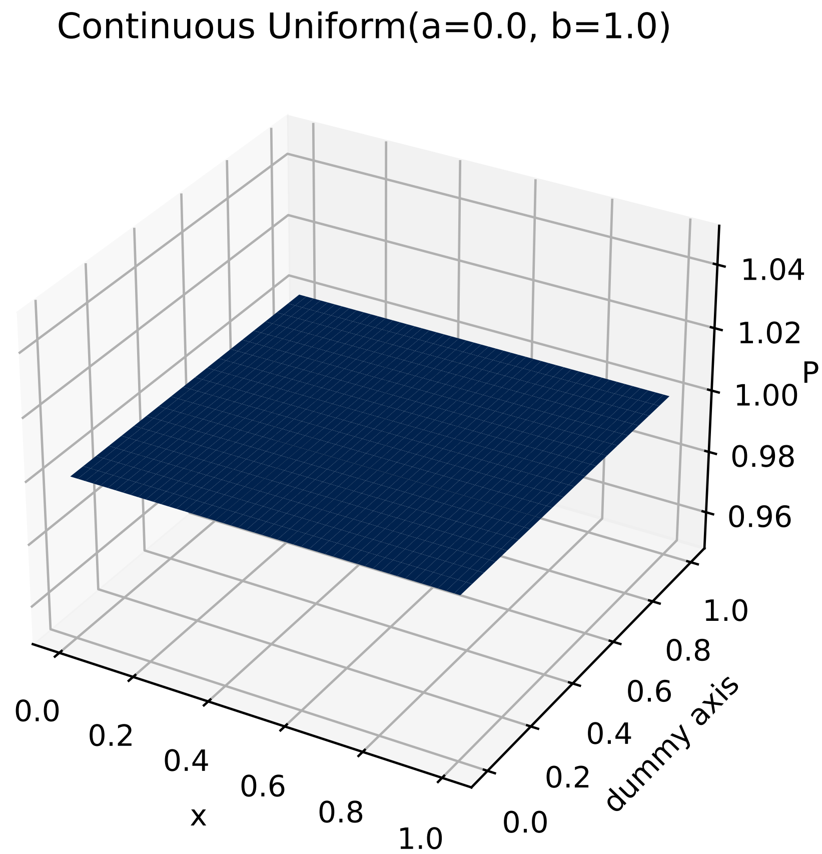

Figure 39: 3D surface plot of Uniform(a, b) PDF.

40 Bernoulli Distribution

A Bernoulli distribution with parameter p has

$$P(X = 1) = p, \quad P(X = 0) = 1 - p.$$

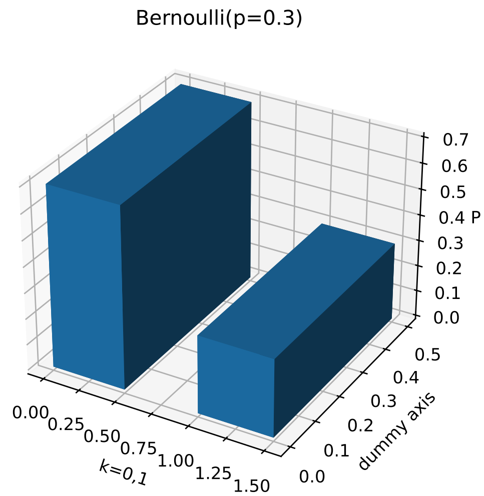

Figure 40: 3D bar chart of Bernoulli(p) PMF.

41 Binomial Distribution

A Binomial(n, p) distribution has

$$P(X = k) \ = \ \binom{n}{k} \, p^k \, (1-p)^{n-k}, \quad k = 0, \ldots, n.$$

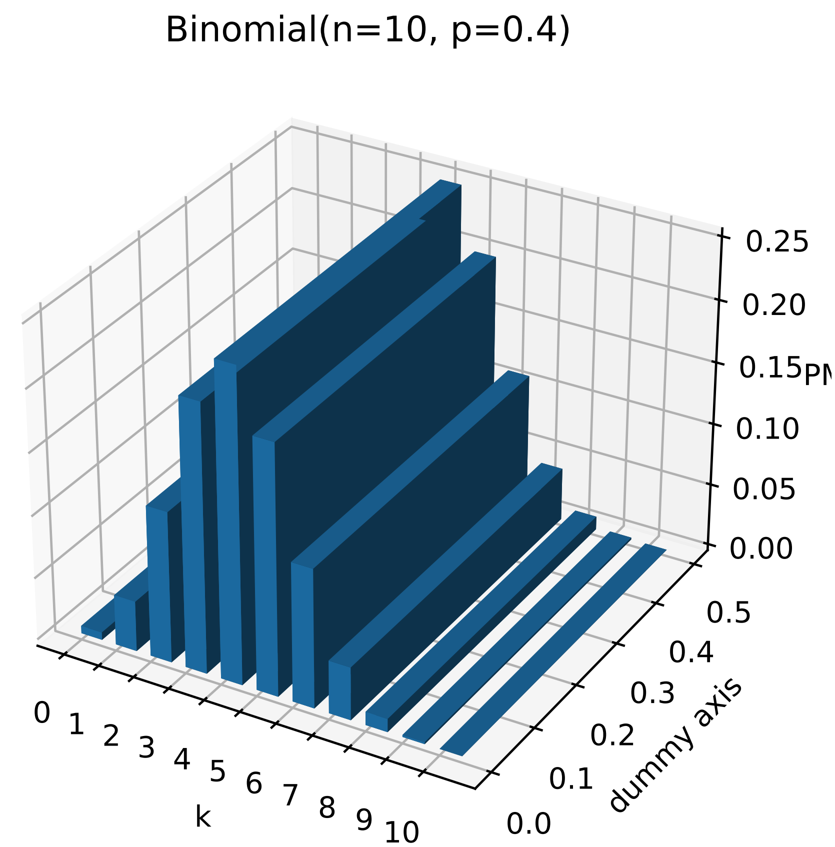

Figure 41: 3D bar chart of Binomial(n, p) PMF.

42 Negative Binomial Distribution

A Negative binomial with parameters (r, p):

$$P(X = k) \; = \; \binom{k + r - 1}{k} \, p^r \, (1 - p)^k, \quad k = 0, 1, \ldots$$

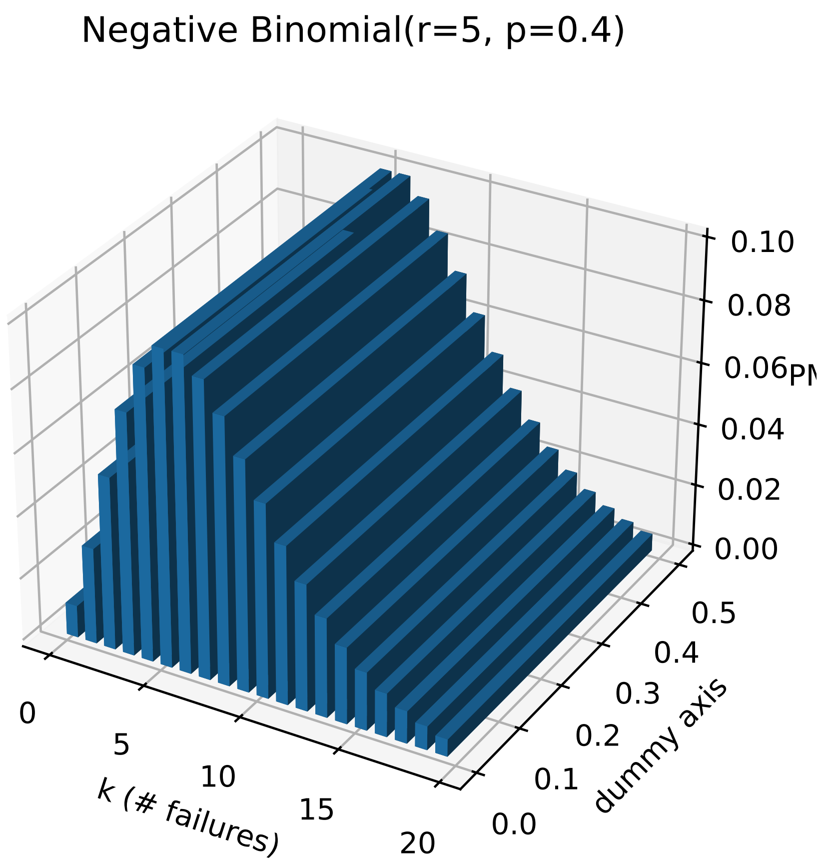

Figure 42: 3D bar chart of Negative Binomial(r, p) PMF.

43 Geometric Distribution

A special case of Negative binomial with $r = 1$:

$$P(X = k) \; = \; (1 - p)^k \, p, \quad k = 0, 1, 2, \ldots$$

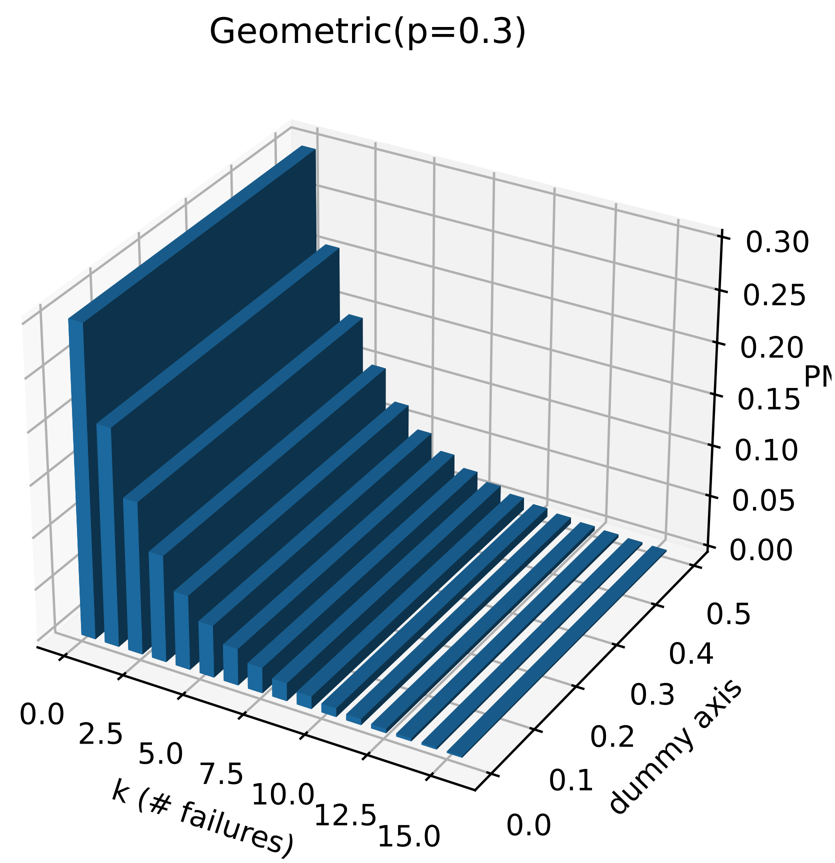

Figure 43: 3D bar chart of Geometric(p) PMF.

44 Hypergeometric Distribution

A Hypergeometric(N, K, n) distribution:

$$P(X = k) \; = \; \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}.$$

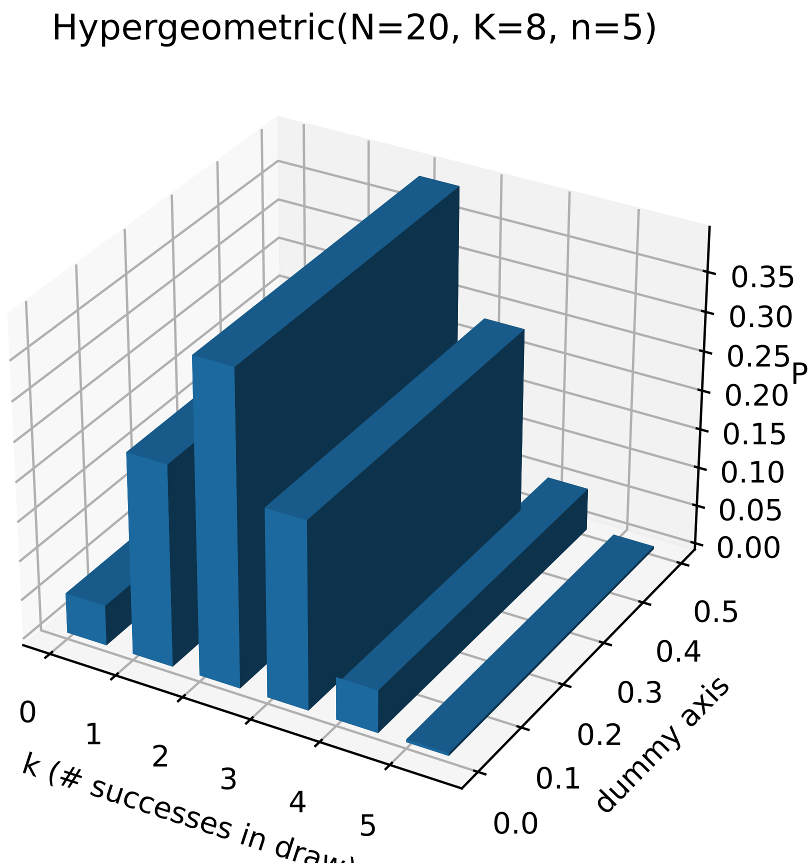

Figure 44: 3D bar chart of Hypergeometric(N, K, n) PMF.

45 Beta-binomial Distribution

A conjugate extension of Binomial with parameters (n, α, β):

$$P(X = k) \;=\; \binom{n}{k} \frac{B(\alpha + k,\ \beta + n - k)}{B(\alpha, \beta)\, B(\alpha + \beta, n)}.$$

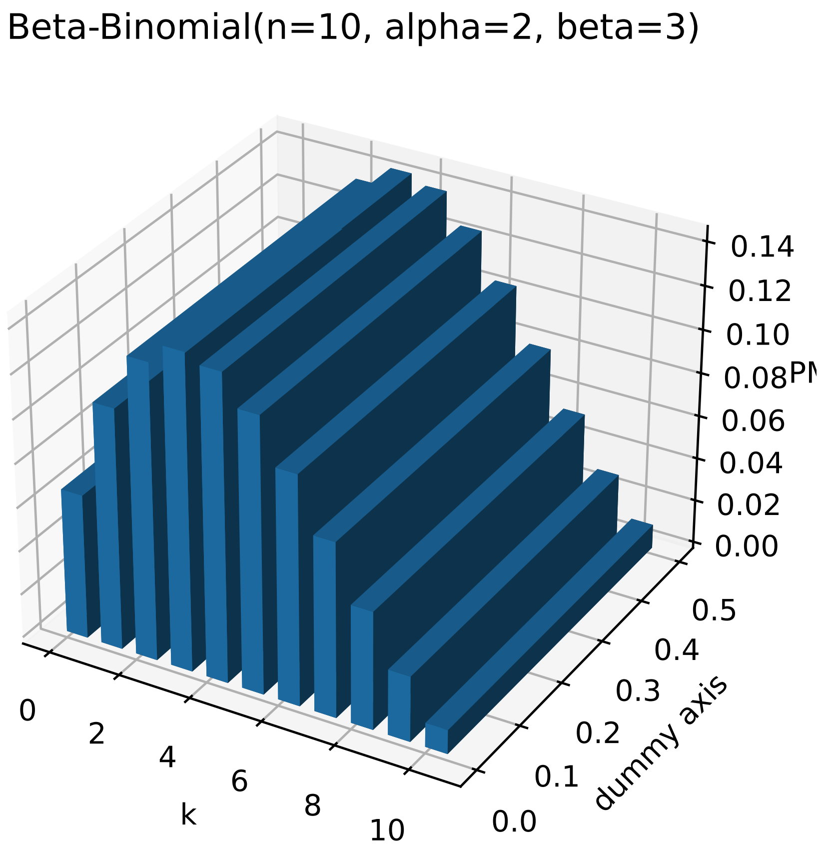

Figure 45: 3D bar chart of Beta-binomial(n, α, β) PMF.

46 Categorical Distribution

For K categories with probabilities p_i,

$$P(X = i) \; = \; p_i, \quad \sum_{i=1}^{K} p_i = 1.$$

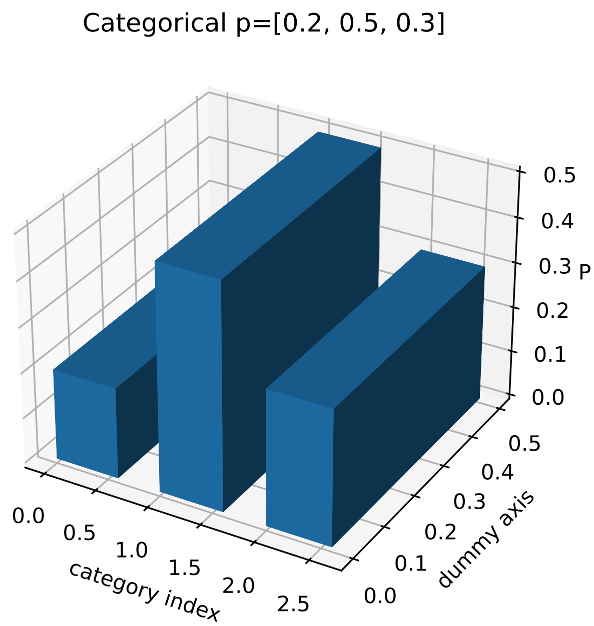

Figure 46: 3D bar chart of a Categorical distribution.

47 Multinomial Distribution

Generalizing Binomial to K categories:

$$P(X_1 = k_1, \dots, X_K = k_K) = \frac{n!}{k_1! \cdots k_K!} \prod_{i=1}^{K} p_i^{k_i}, \quad \sum k_i = n.$$

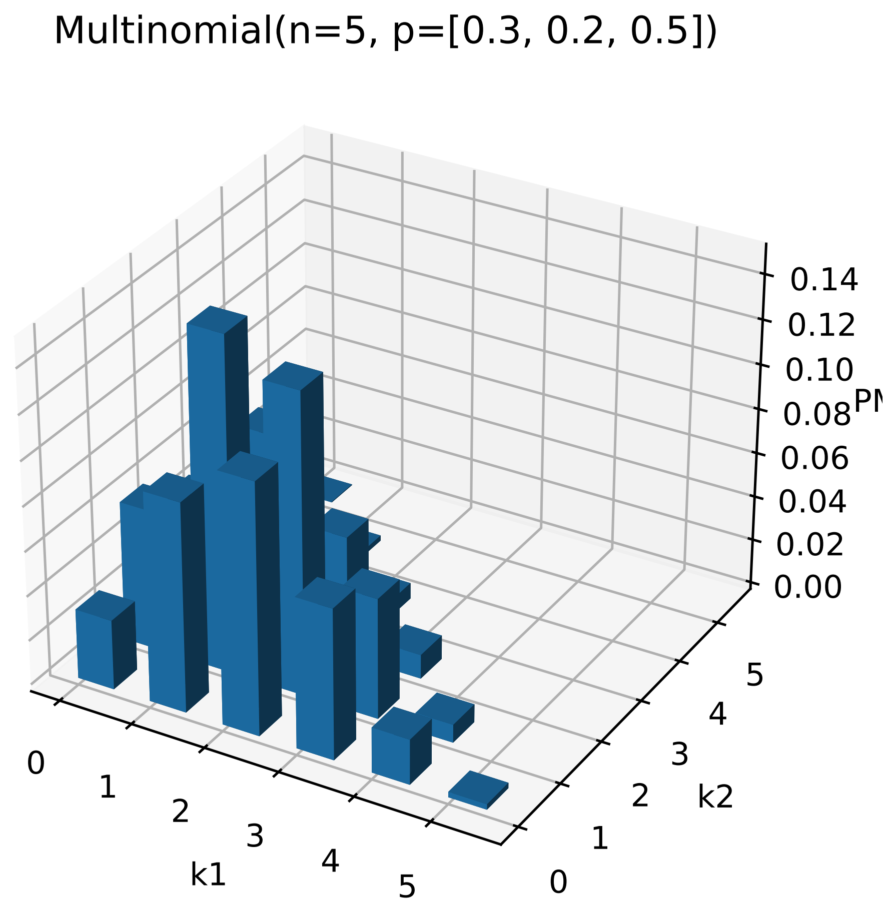

Figure 47: 3D bar chart of Multinomial$(n, p_1, \dots, p_K)$ PMF for $K = 3$.

48 Multivariate Hypergeometric Distribution

A generalization of Hypergeometric to multiple categories. With K_i items in category i,

$$P(\mathbf{X} = \mathbf{k}) \;=\; \frac{\prod_{i=1}^{K} \binom{K_i}{k_i}}{\binom{N}{n}}, \quad \sum_{i=1}^{K} k_i = n.$$

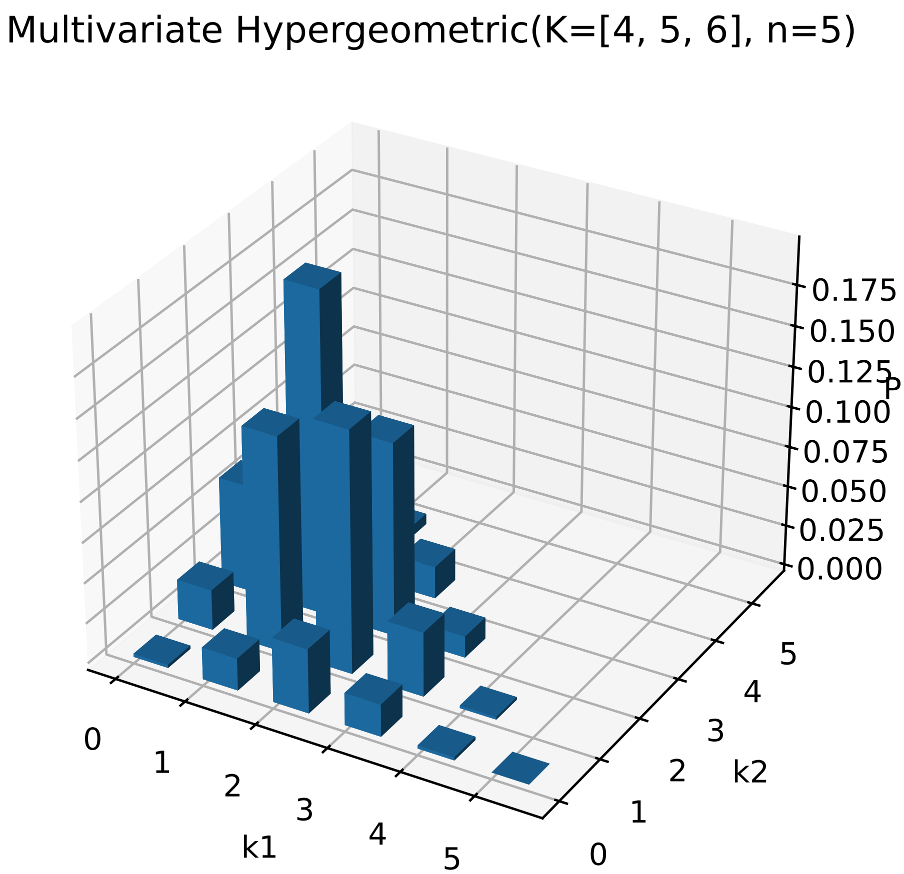

Figure 48: 3D bar chart for the Multivariate Hypergeometric distribution with $K = 3$.

49 Poisson Distribution

A Poisson(λ) distribution:

$$P(X = k) \;=\; \frac{\lambda^k \, e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \ldots$$

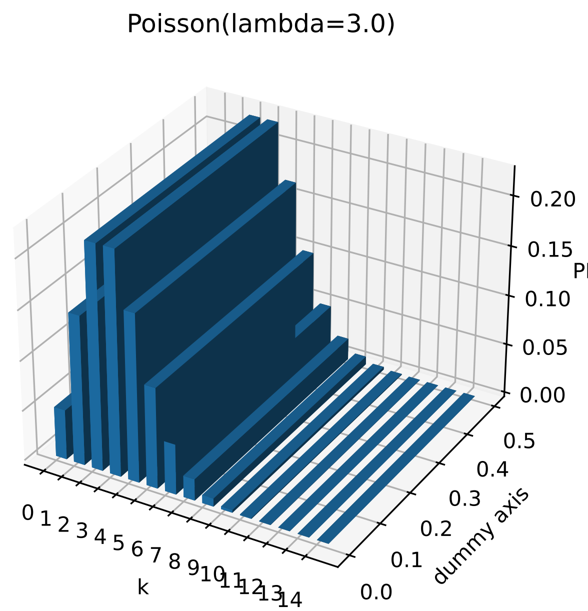

Figure 49: 3D bar chart of Poisson(λ) PMF.

50 Gamma Distribution

With shape α and rate β:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x > 0.$$

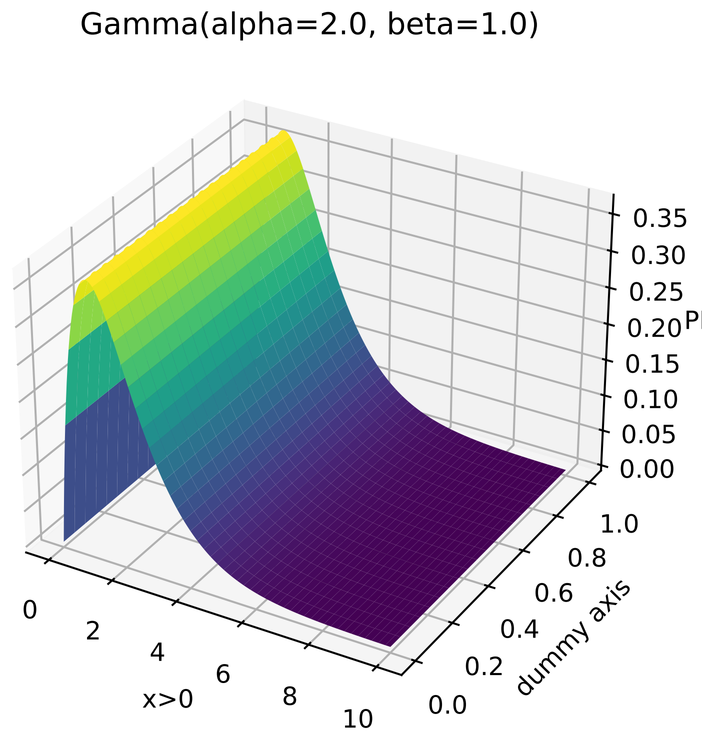

Figure 50: 3D surface plot of Gamma(α, β) PDF.

51 Rayleigh Distribution

Rayleigh(σ):

$$f(r) = \frac{r}{\sigma^2} \exp\Bigl(-\frac{r^2}{2\sigma^2}\Bigr), \quad r \geq 0.$$

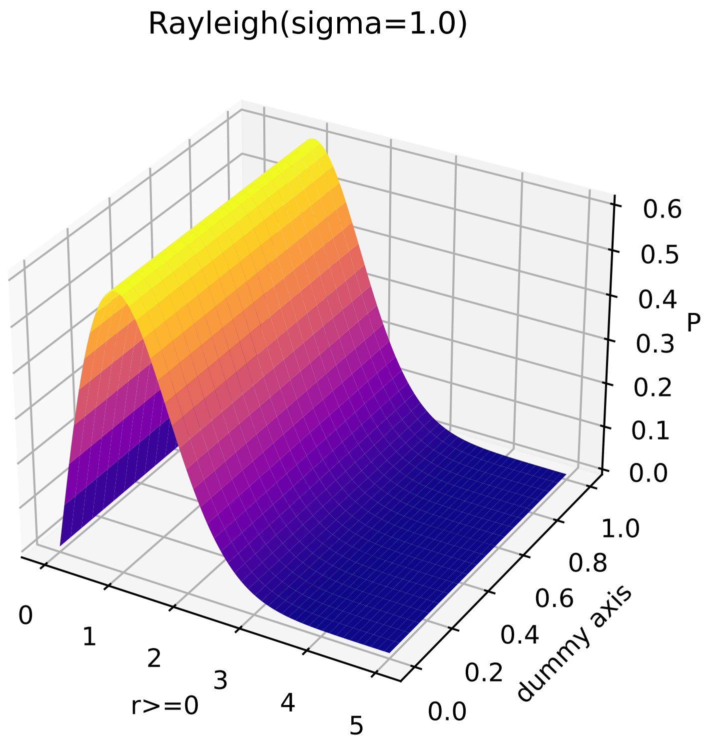

Figure 51: 3D surface plot of Rayleigh(σ) PDF.

52 Rice (Rician) Distribution

Rice(ν, σ):

$$f(r) \;=\; \frac{r}{\sigma^2}\exp\!\Big(-\frac{r^2+\nu^2}{2\sigma^2}\Big)\, I_0\!\Big(\frac{r\,\nu}{\sigma^2}\Big), \quad r \ge 0.$$

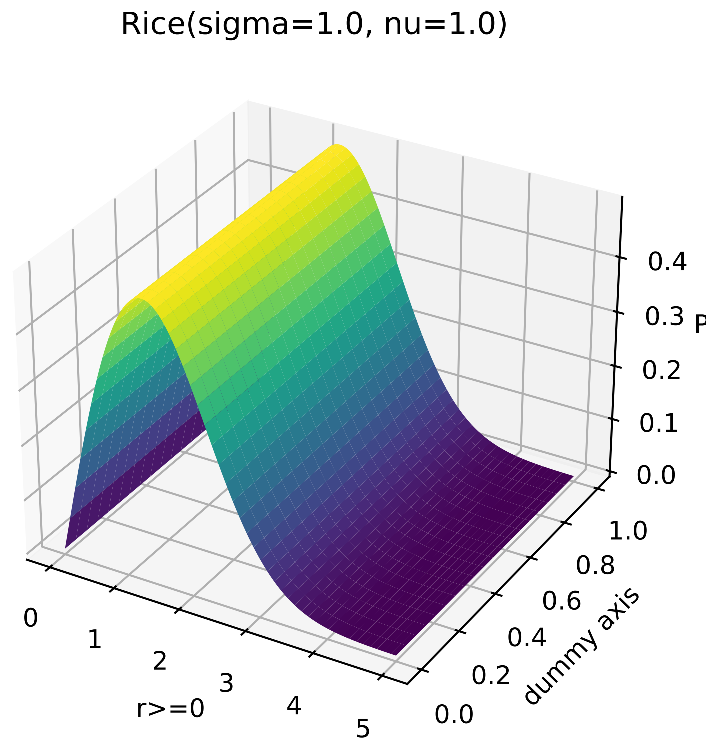

Figure 52: 3D surface plot of Rice(ν, σ) PDF.

53 Chi-squared Distribution

Chi-squared with k degrees of freedom:

$$f(x) \;=\; \frac{1}{2^{k/2}\Gamma(\frac{k}{2})}\, x^{\frac{k}{2}-1}\, e^{-x/2}, \quad x > 0.$$

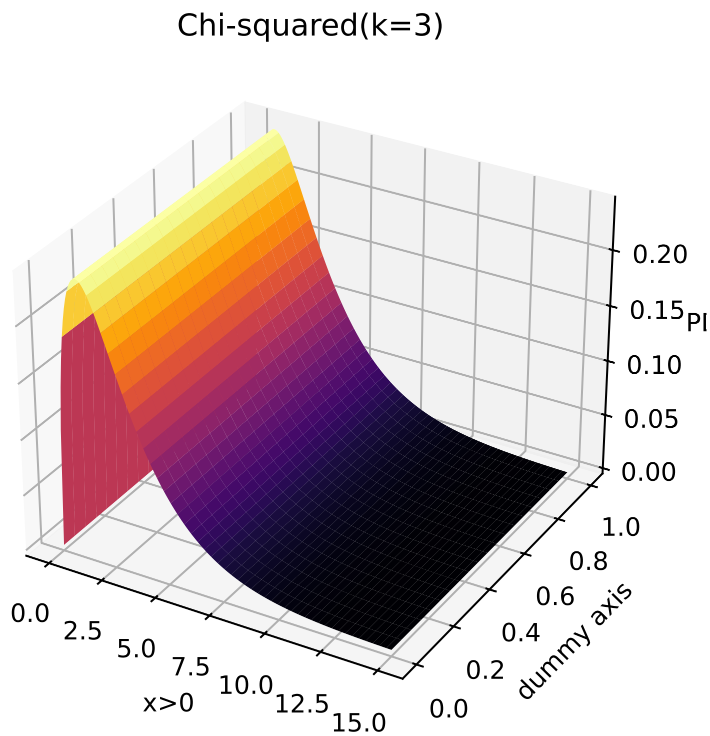

Figure 53: 3D surface plot of Chi-squared(k) PDF.

54 Student's t Distribution

With ν degrees of freedom:

$$f(x) \;=\; \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}\Big(1+\frac{x^2}{\nu}\Big)^{-\frac{\nu+1}{2}}, \quad x \in \mathbb{R}.$$

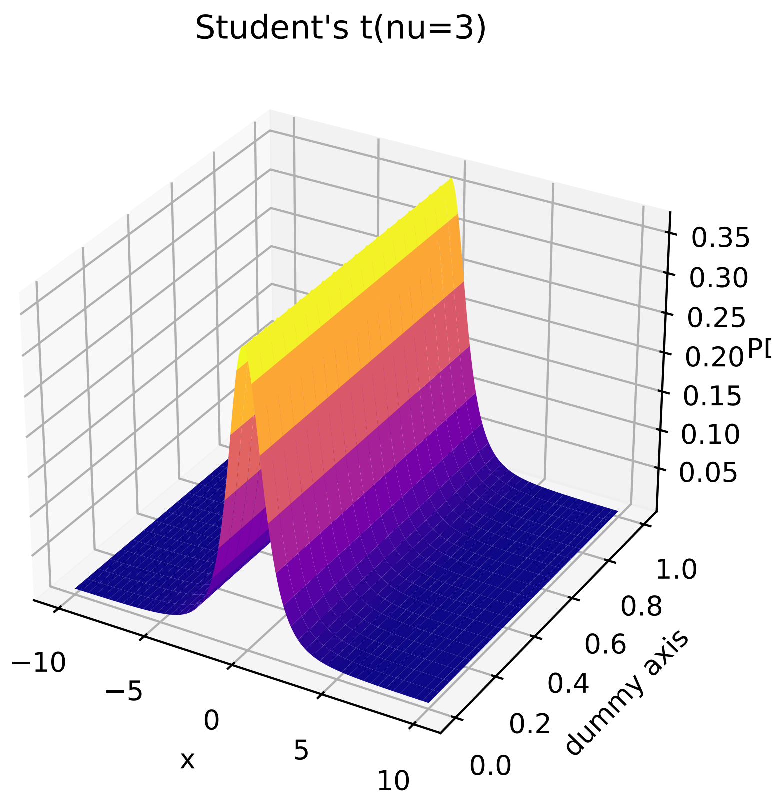

Figure 54: 3D surface plot of Student's $t(\nu)$ PDF.

55 F-distribution

With degrees of freedom d_1, d_2:

$$f(x) = \frac{\sqrt{\frac{(d_1\,x)^{d_1}\,d_2^{d_2}}{(d_1\,x+d_2)^{d_1+d_2}}}}{x\,B\left(\frac{d_1}{2},\frac{d_2}{2}\right)}, \quad x > 0.$$

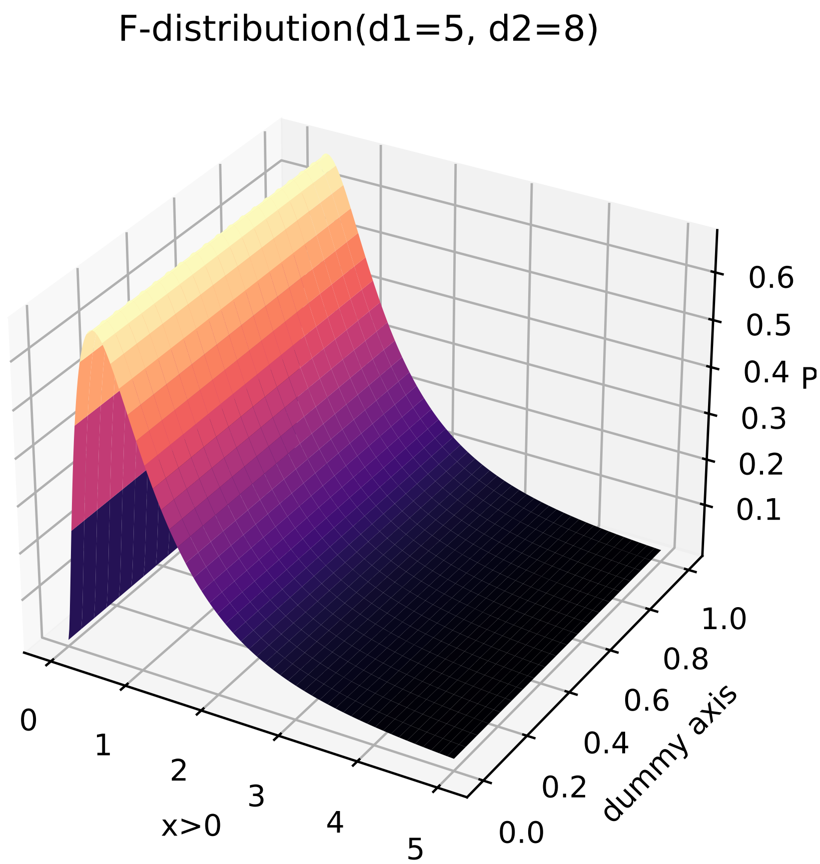

Figure 55: 3D surface plot of $F(d_1, d_2)$ PDF.

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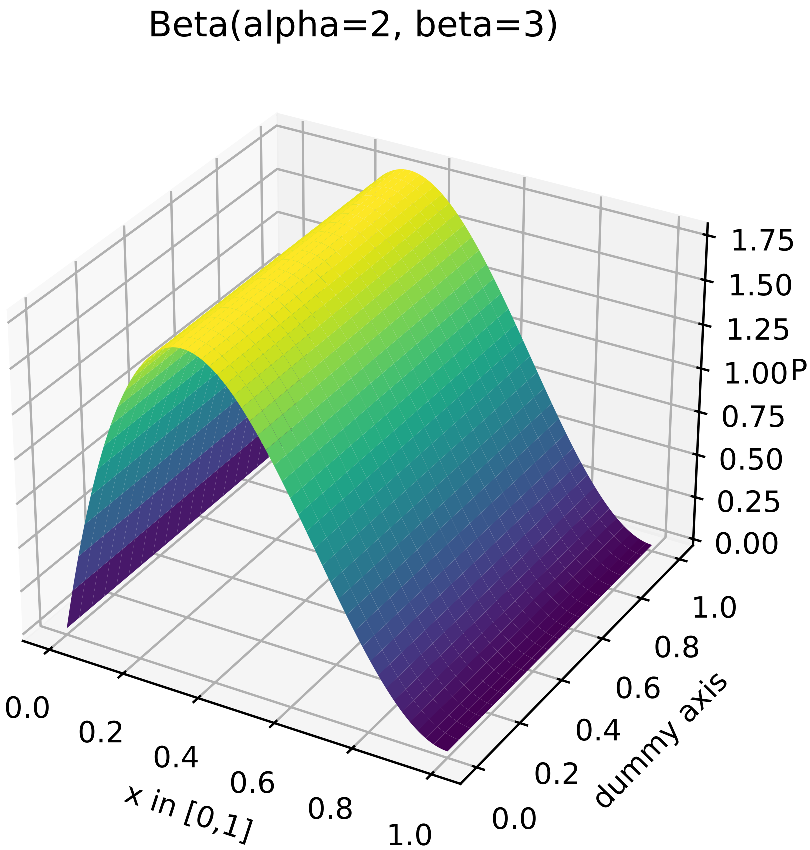

Figure 56: 3D surface plot of Beta(α, β) PDF.

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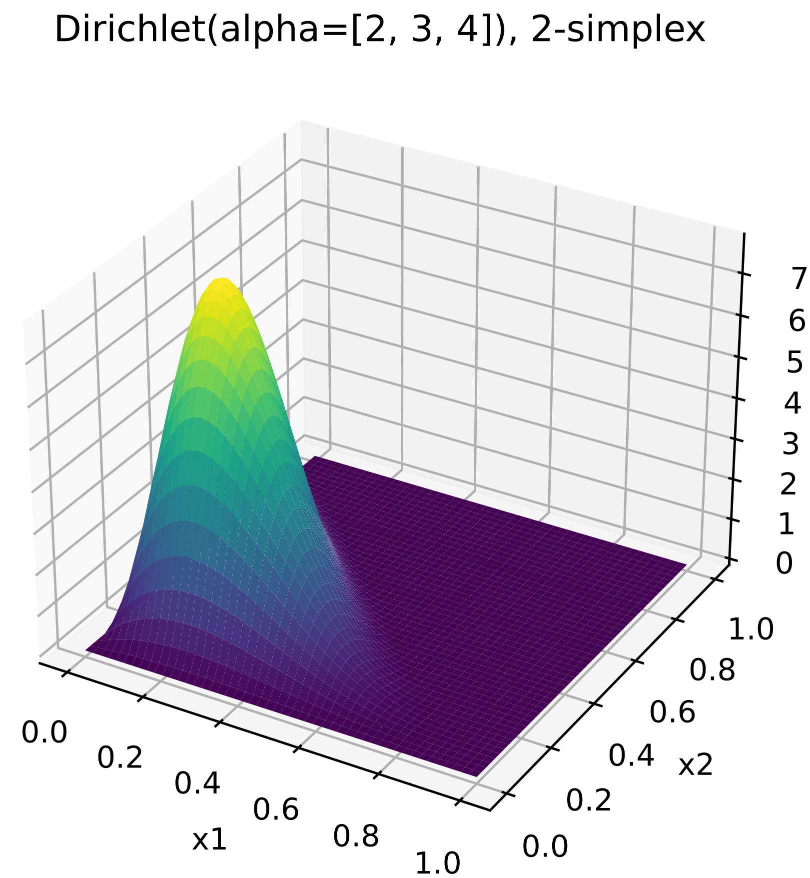

Figure 57: 3D surface plot of Dirichlet$(\alpha_1, \alpha_2, \alpha_3)$ along 2-simplex.

58 Wishart Distribution (Placeholder Slice)

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Here, we show a *slice* for a 2×2 Wishart (diagonal only), as a demonstration:

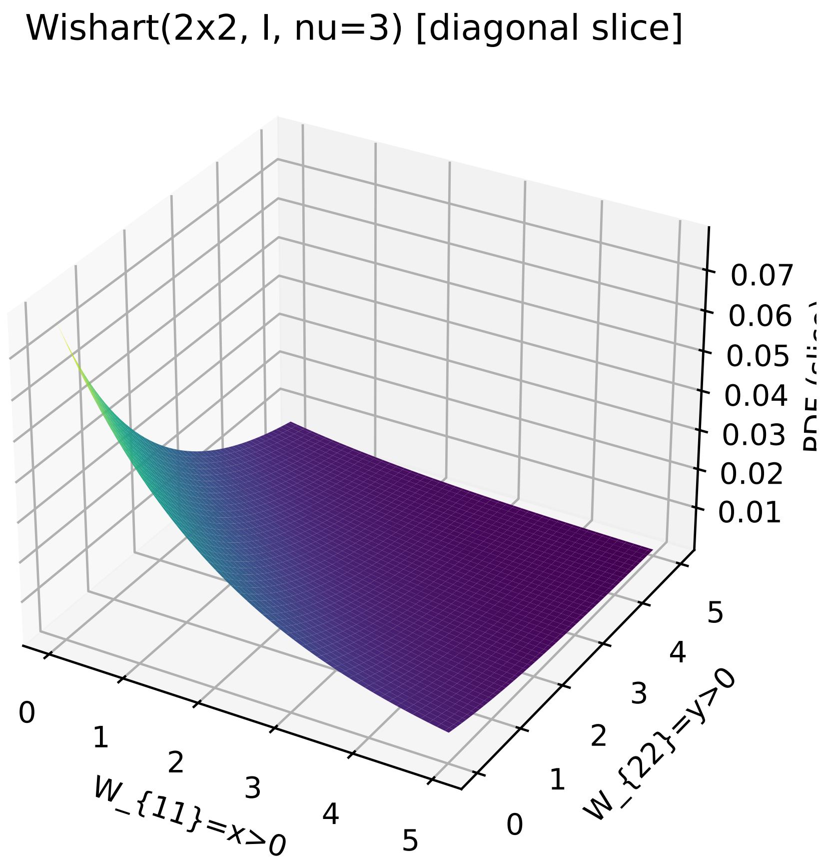

Figure 58: A 3D "slice" of Wishart$(\nu, \Sigma = I)$ in 2D, restricting $\mathbf{W}$ to diagonal.