Basis Function Type	Form / Definition	Comments / Parameters
Polynomial	$\phi_j(x) = x^j$	Simple, global, often used in regression. Susceptible to overfitting at high degrees.
Gaussian (RBF)	$\phi_j(x) = \exp\left(-\frac{(x-\mu_j)^2}{2s^2}\right)$	Radial basis function. μ_j is center, s is scale. Localized.
Sigmoid (Logistic)	$\phi_j(x) = \sigma\left(\frac{x-\mu_j}{s}\right), \quad \sigma(a) = \frac{1}{1+e^{-a}}$	Smooth, bounded. Related to logistic regression.
Tanh	$\phi_j(x) = \tanh(a_j x + b_j)$	Centered sigmoid-like, often used in neural networks.
Spline Functions	Piecewise low-degree polynomials	Smooth joins at "knots", e.g., B-splines, cubic splines. Good for localized fit.
Fourier Basis	$\phi_j(x) = \sin(2\pi jx),\ \cos(2\pi jx)$	Global, periodic functions. Useful in signal analysis.
Wavelet Basis	$\phi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$	Localized in time and frequency. j, k control scale/translation.
Piecewise Polynomial	$\phi_j(x) = \begin{cases} (x-\kappa_j)^p & x > \kappa_j \\ 0 & \text{else} \end{cases}$	Hinge functions; used in regression splines and MARS.
Step Function	$\phi_j(x) = \mathbb{I}(x > \kappa_j)$	Binary, non-continuous. Can partition space into regions.
Indicator / Binary Basis	$\phi_j(x) = \mathbb{I}(x \in A_j)$	Useful for categorical features or binning.
ReLU	$\phi_j(x) = \max(0, x - \kappa_j)$	Popular in neural networks; piecewise linear.
ELU / Softplus	$\phi_j(x) = \log(1 + e^x)$ or ELU variant	Smoothed version of ReLU. Used in deep learning.
ArcTan Basis	$\phi_j(x) = \arctan(a_j x + b_j)$	Smooth, bounded. Similar to sigmoid in shape.
Exponential Decay	$\phi_j(x) = \exp(-\lambda_j x)$	Monotonic decaying functions. Often in survival models.
Chebyshev Polynomials	$T_j(x)$, recursively defined	Orthogonal polynomials on $[-1, 1]$. Better numerical properties than standard polynomials.
Legendre Polynomials	$P_j(x)$, orthogonal on $[-1, 1]$	Alternative polynomial basis. Often used in spectral methods.
Hermite Polynomials	$H_j(x)$, used in quantum mechanics	Orthogonal w.r.t. Gaussian weight. Related to Fourier–Hermite series.
Bump Functions	$\phi_j(x) = \begin{cases} \exp\left(-\frac{1}{1-(x-\mu_j)^2/s^2}\right) & \lvert x-\mu_j \rvert < s \\ 0 & \text{otherwise} \end{cases}$	Infinitely differentiable, compactly supported. Used in smooth approximation.

Table 1: Overview of common basis functions for linear-in-parameters regression models.