

Part (a)

```
for(int i=n-1; i>=0; i--){
  for(int k=0; k<=i; k++){
    // do something that takes O(1) time
  }
}
```

$$c + (n) * (1) + (n-1)c + (n-2)c + \dots + mc + 0$$

$$= \frac{n(n+1)}{2}c = \frac{n^2}{2}c$$

the "fine" parts cancel, which leaves:

$$T(n) = n^2 c \cdot \left(\frac{1}{2}\right) = n^2 c$$

$f(n)$ must be n^2
for $\Theta(f(n))$ in order for
 $\lim_{n \rightarrow \infty} \frac{T(n)}{f(n)} = C$
 $= \Theta(n^2)$

Part (b)

Assume A is an array of size $n+1$.

```
for(int i=1; i<=n; i++){
  for(int k=1; k<=n; k++){
    if(A[k] == 1){
      for(int m=1; m<=n; m=m/2){
        // do something that takes O(1) time
        // Assume the contents of the A[] array are not changed
      }
    }
  }
}
```

$\log(n)$ // every step in a way "halves" the distance towards n .
// in other words $\left[\frac{n}{2} \mid \frac{n}{4} \mid \dots \right]$
// then $\left[\frac{n}{2} \mid \frac{n}{4} \mid \dots \mid 1 \right]$
this takes $\log(n)$ steps

$$C_1 \log(n) + C_2$$

if runs c .
when it's true, it runs the $\log(n)$ for

$$T(n) = C_1 n \log(n) + n C_2$$

$$= n \log(n) + n C_2$$

$$\Theta(f(n)) = \Theta(n \log n)$$

$f(n)$ must be $n \log(n)$

$$\Theta(n \log n)$$

$$\lim_{n \rightarrow \infty} \frac{T(n)}{f(n)} = C$$

```
}
}
}
}
```

Part (c)

```
void f3(int* A, int n)
{
  if(n <= 1) return;
  else {
    f3(A, n-2);
    // do something that takes O(1) time
    f3(A, n-2);
  }
}
```

$$f_3(A, n) = f_3(A, n-2) + f_3(A, n-2) + C$$

$$= 2f_3(A, n-2) + C_1$$

$$= 2(f_3(A, n-4) + f_3(A, n-4) + C_2) + C_1$$

$$= 4f_3(A, n-4) + C_3$$

$\frac{n}{2}$ steps

Part (d)

Notice that this code is somewhat similar to what happens if you keep inserting into a Vector.

$$K f_3(A, 1) + C_4$$

$$T(n) = C_5 * \left(\frac{n}{2}\right) = C n$$

$$f(n) = n$$

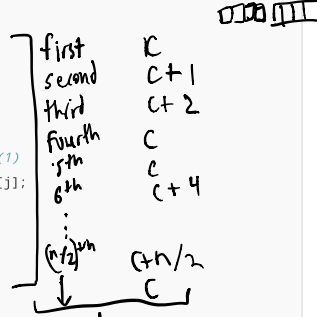
$$\text{for } \lim_{n \rightarrow \infty} \frac{T(n)}{f(n)} = C$$

$$\Theta(n)$$

Part (d)

Notice that this code is somewhat similar to what happens if you keep inserting into a vector.

```
int *a = new int [10]; // new is O(1)
int size = 10;
for (int i = 0; i < n; i++)
{
    if (i == size)
    {
        int newsize = 3*size/2;
        int *b = new int [newsize]; // new is O(1)
        for (int j = 0; j < size; j++) b[j] = a[j];
        delete [] a; // delete is O(1)
        a = b;
        size = newsize;
    }
    a[i] = i*i;
}
```



$$Cn + \frac{n}{2} + \frac{n}{4} + \frac{n}{8} \cdots 2 + 1 = Cn = T(n)$$

Problem 5 (Linked Lists, Recursion, 10%)

Consider the following C++ code. What linked list is returned if funcA is called with the input

$$f(n) \text{ must be } \lim_{n \rightarrow \infty} \frac{T(n)}{f(n)} = \frac{cn}{n} = c$$

$$\underline{\underline{O(n)}}$$