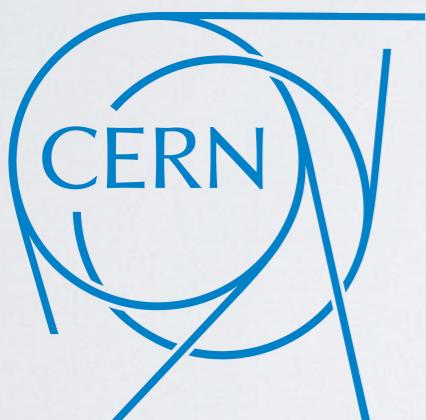


# GRAPH GANS FOR HIGH ENERGY PHYSICS DATA GENERATION

Raghav Kansal, Javier Duarte, Breno Orzari, Thiago Tomei, Maurizio Pierini, Mary Touranakou, Jean-Roch Vlimant, Dimitrios Gunopulos

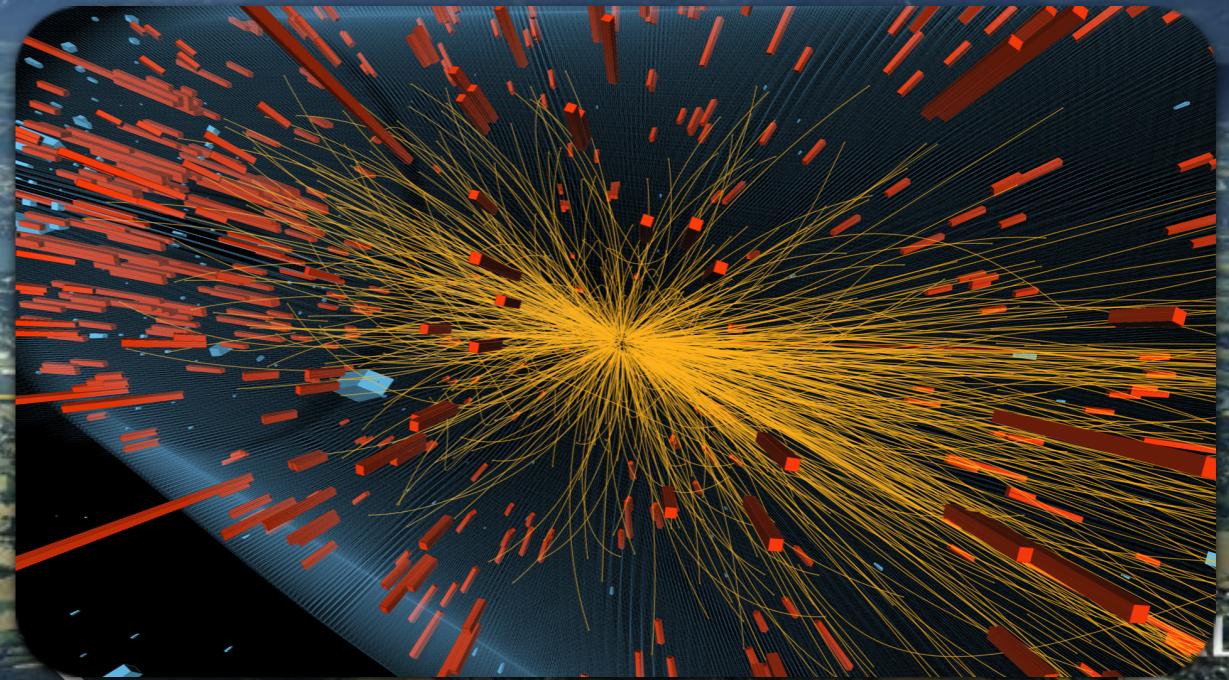
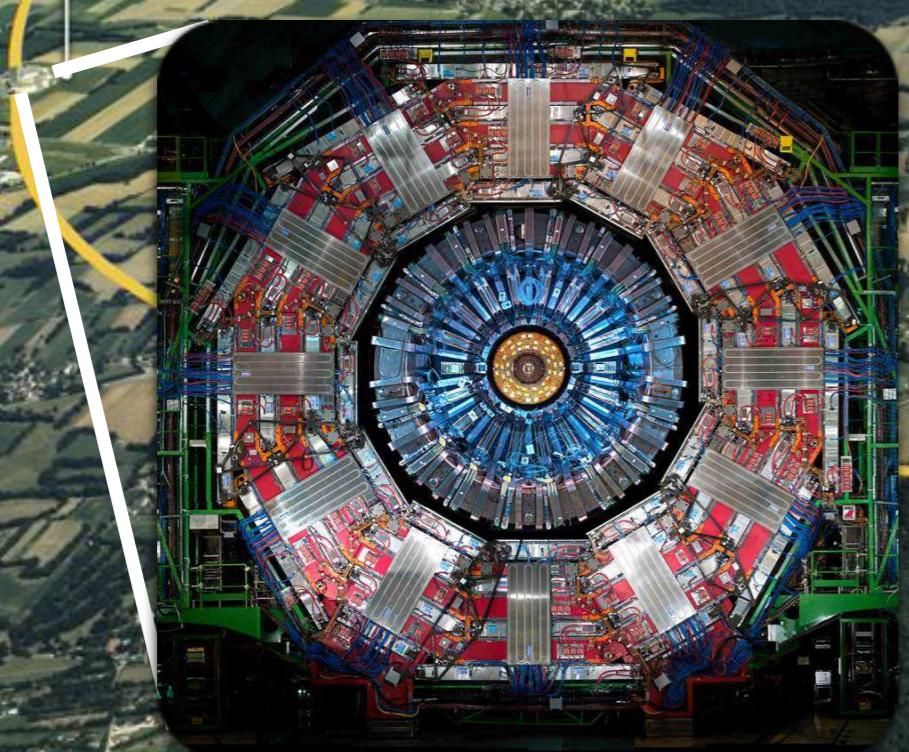


Berkeley Deep Generative Models for Fundamental Physics Meeting  
17/3/21

# LARGE HADRON COLLIDER

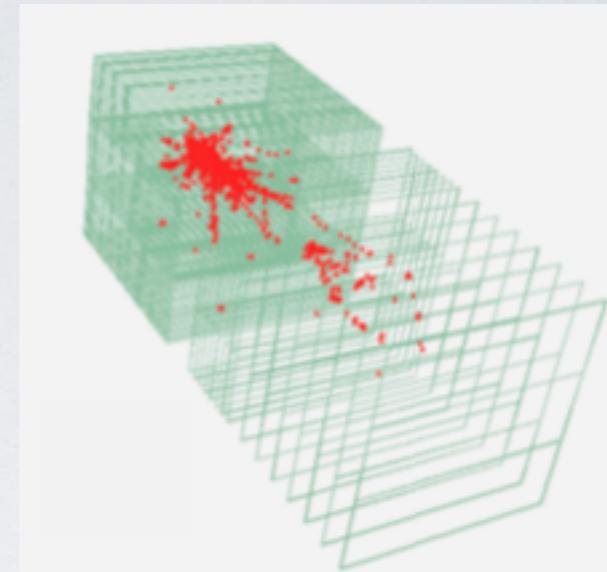
high energy particle collisions

Using deep learning,  
simulation can go from  
 $\mathcal{O}(\text{min})$  to  $\mathcal{O}(\text{ms})$  per event



# WHY GRAPHS?

- Properties of HEP data:
  - High granularity
  - Sparsity
  - Irregular geometry



Raw/Reconstructed Detector Data

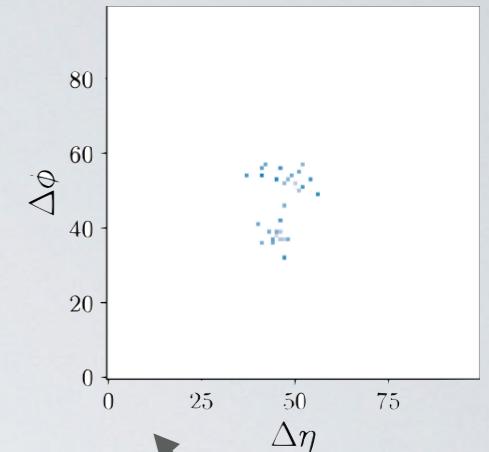
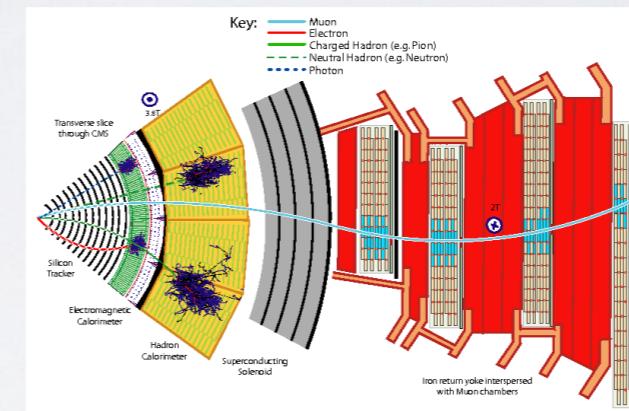
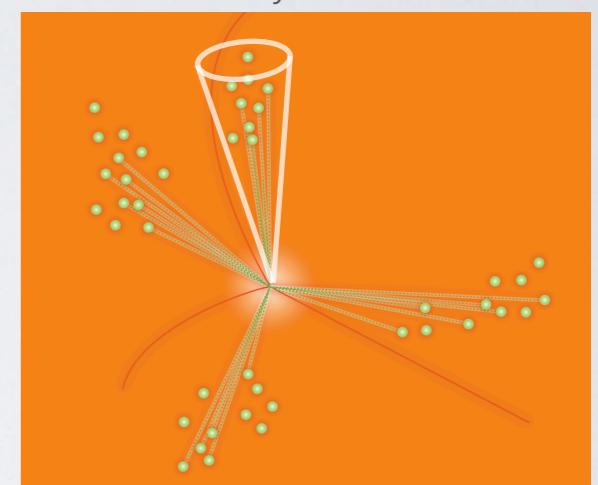


Image repr. for CNNs

Jets



# RELATED WORK

LAGAN (Oliveira et al. 2017)

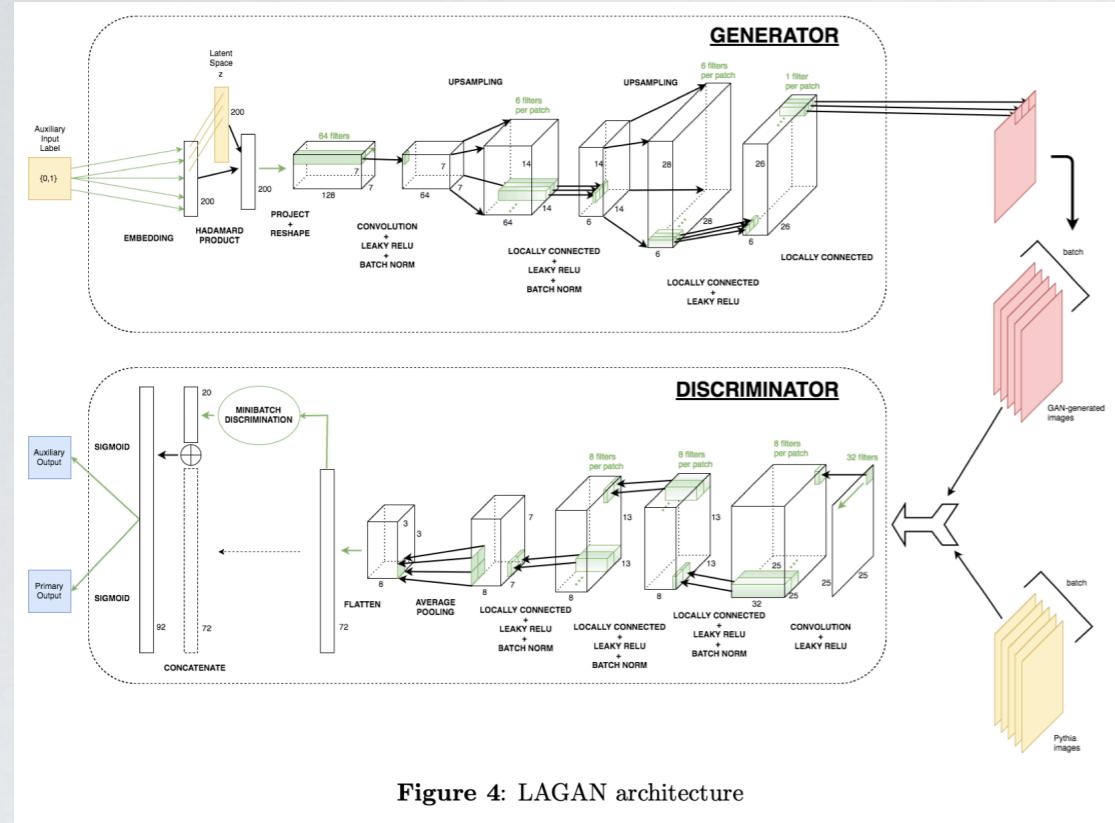
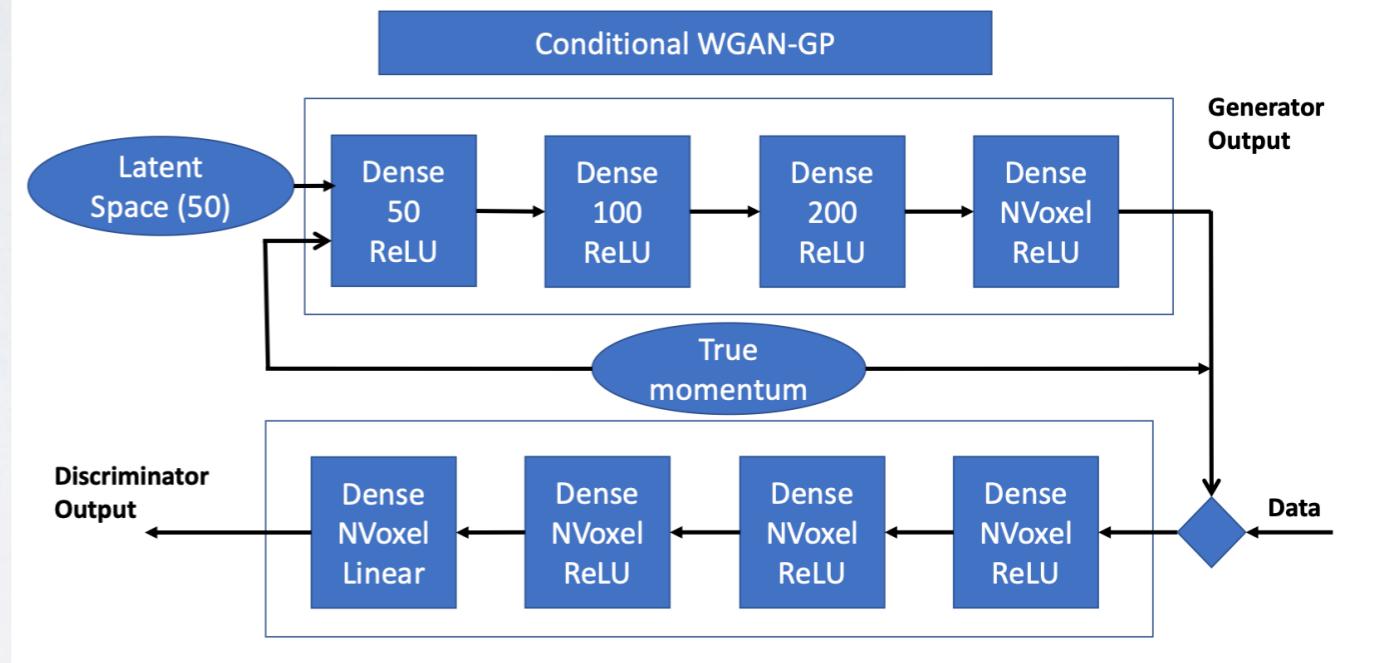
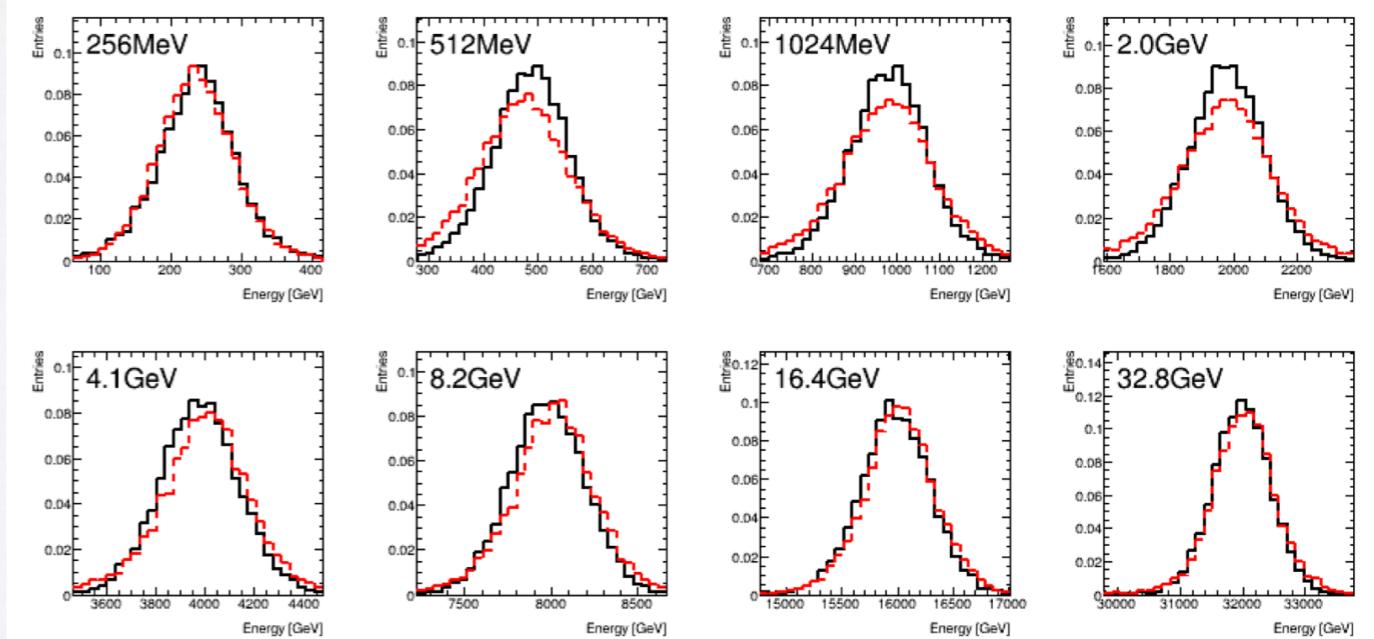
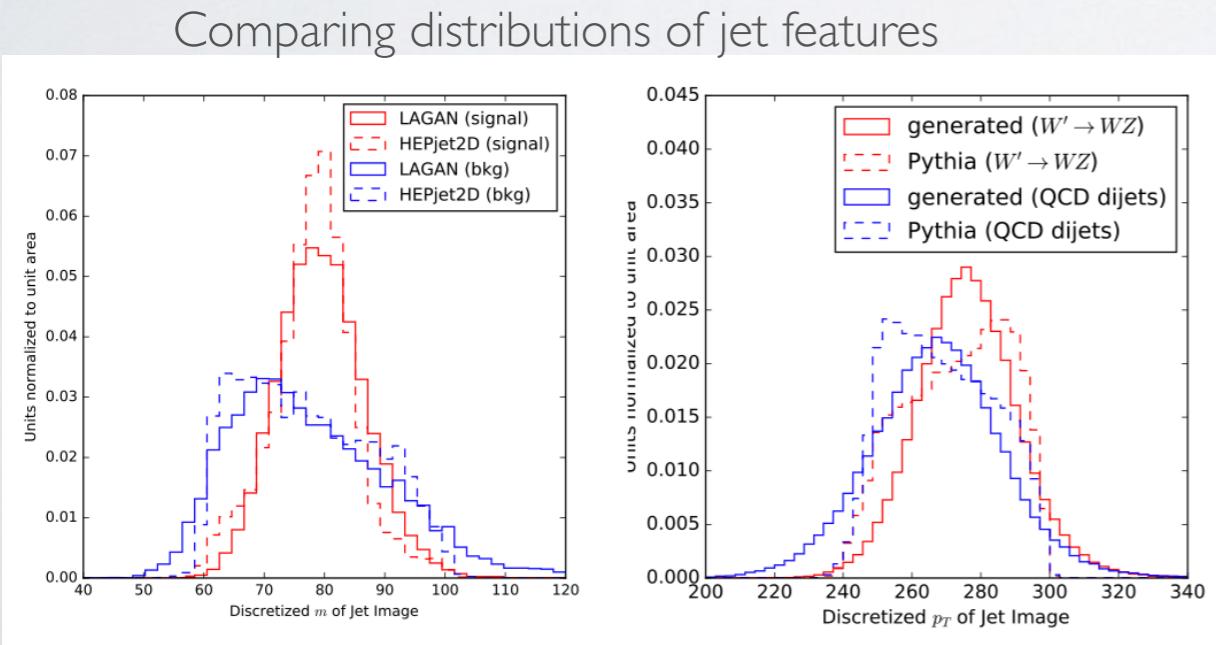


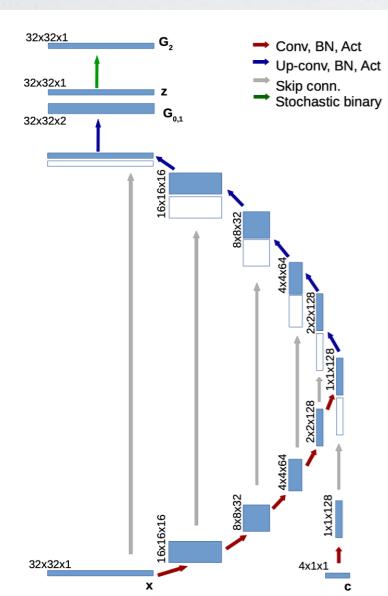
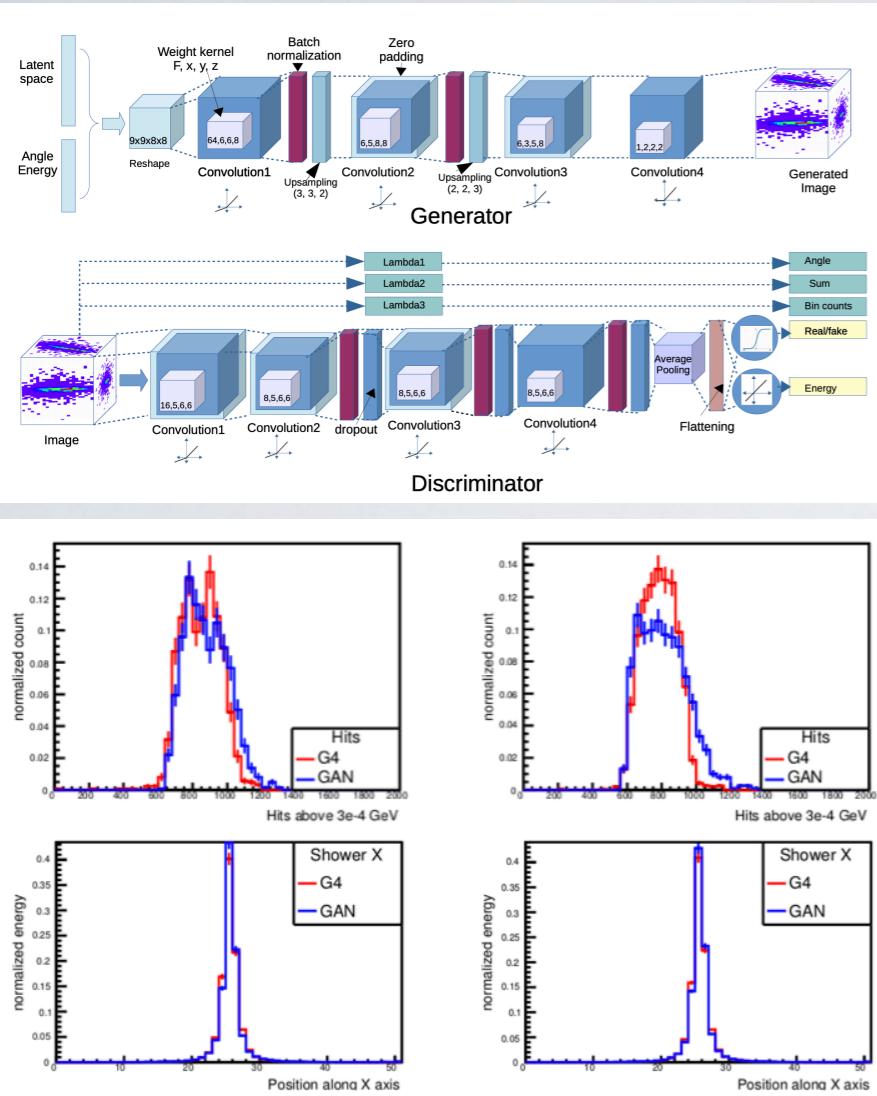
Figure 4: LAGAN architecture

FastCaloGAN (ATLAS Collab. 2020)



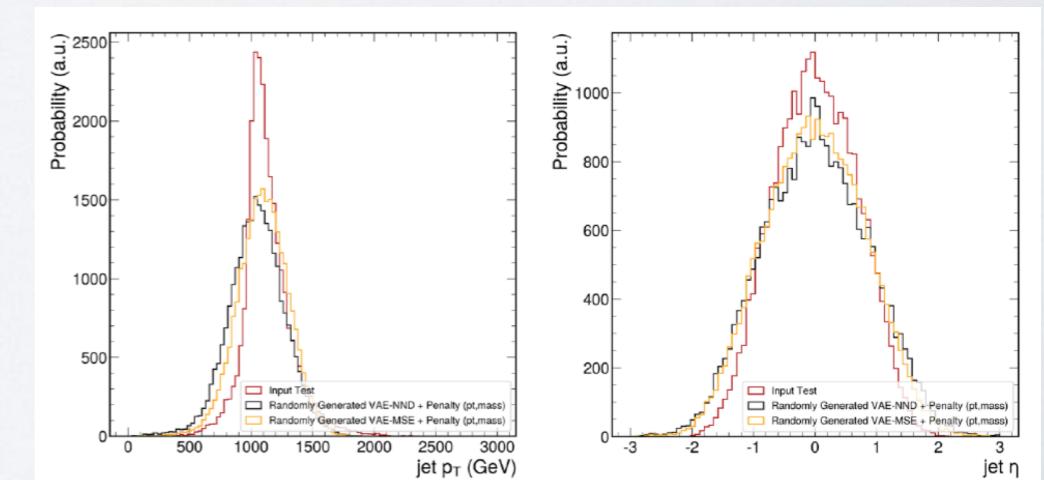
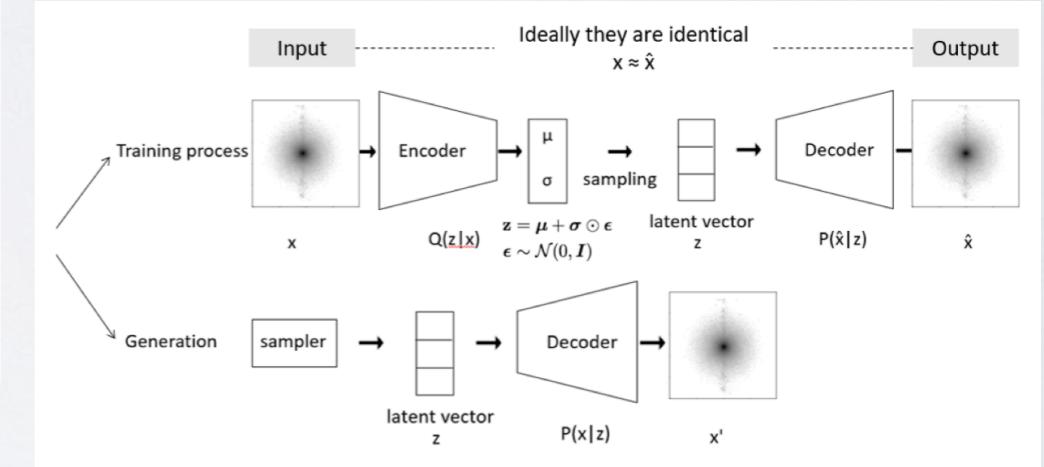
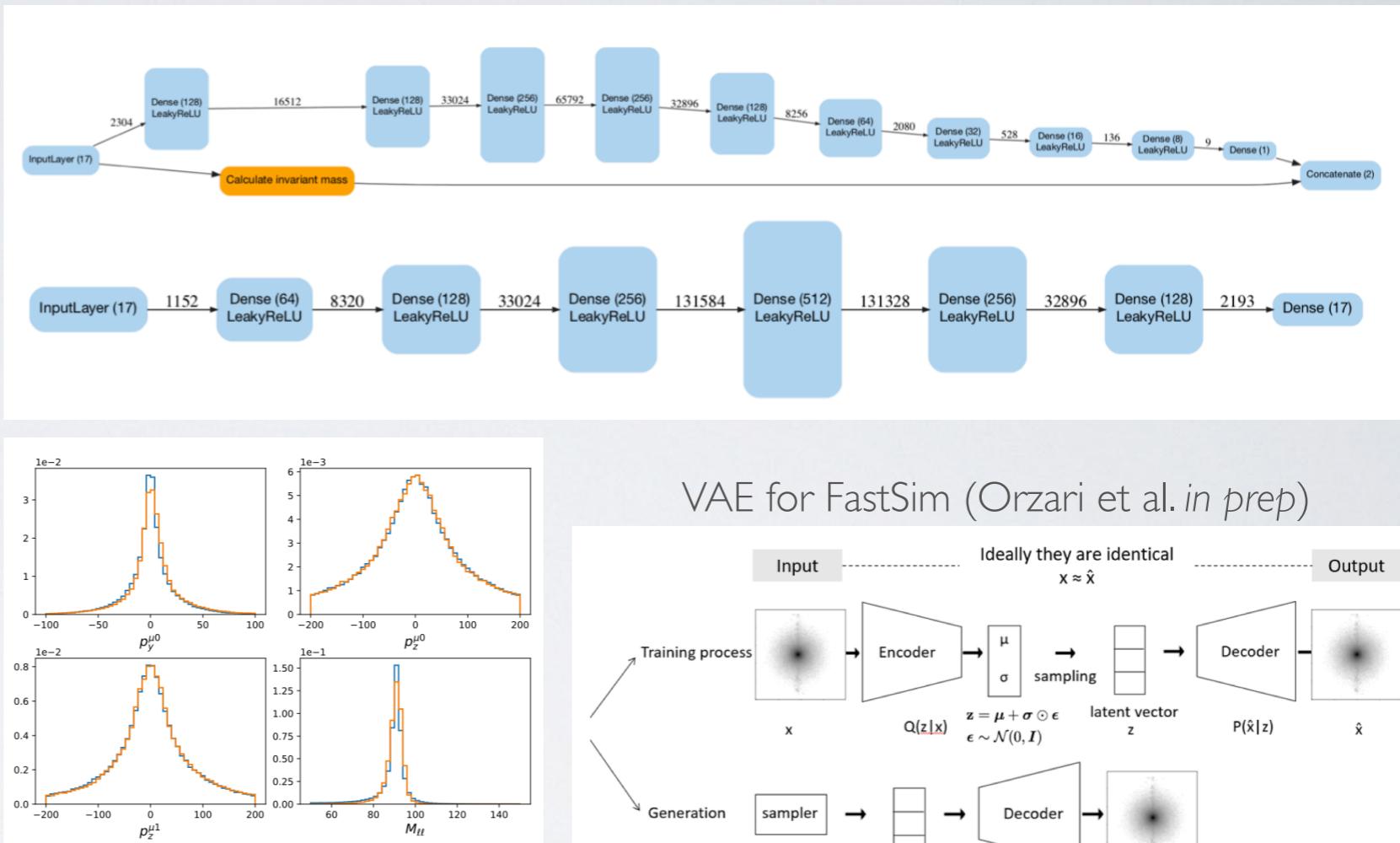
Comparing distributions of jet features





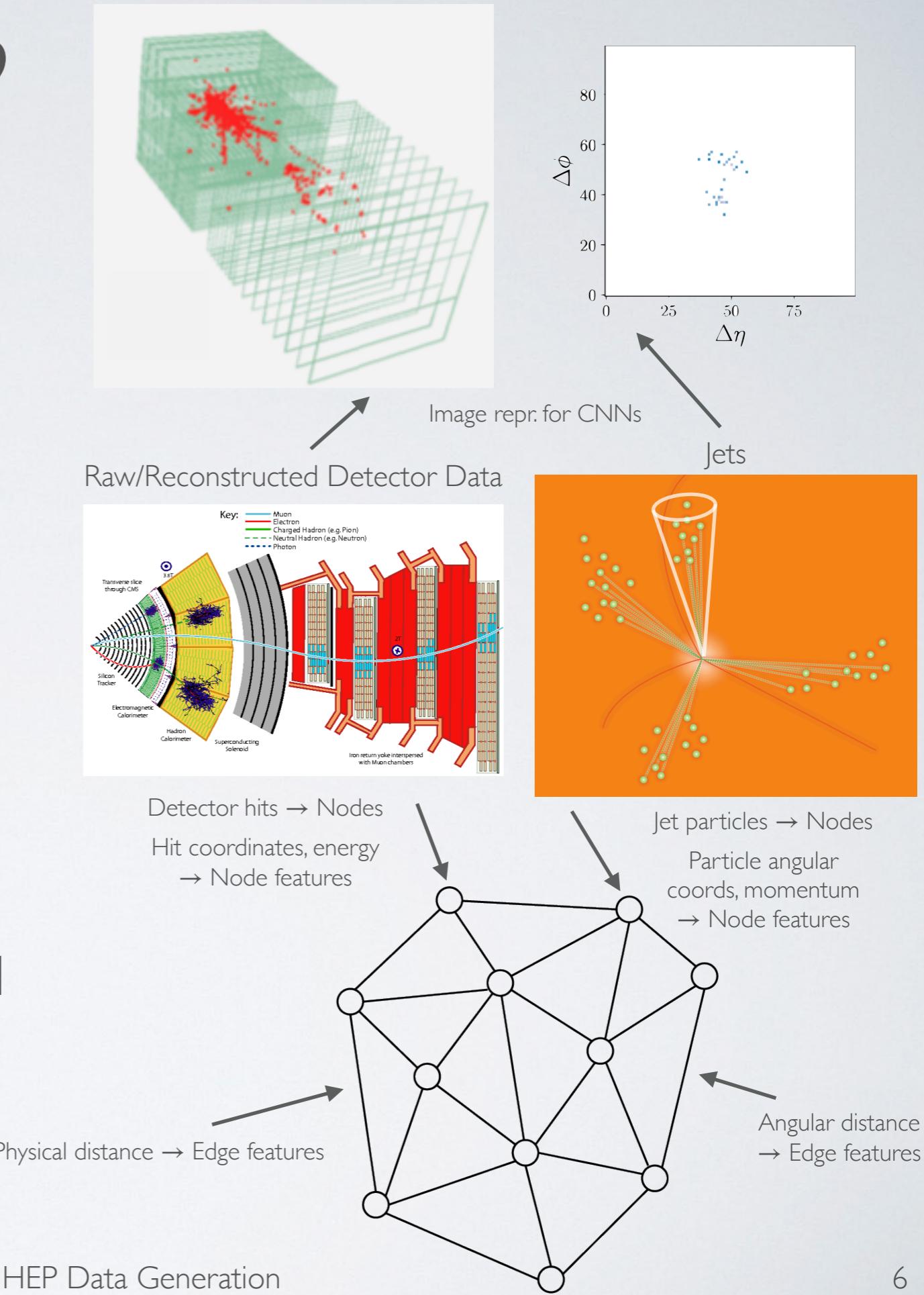
# RELATED WORK

Hashemi et al. 2019



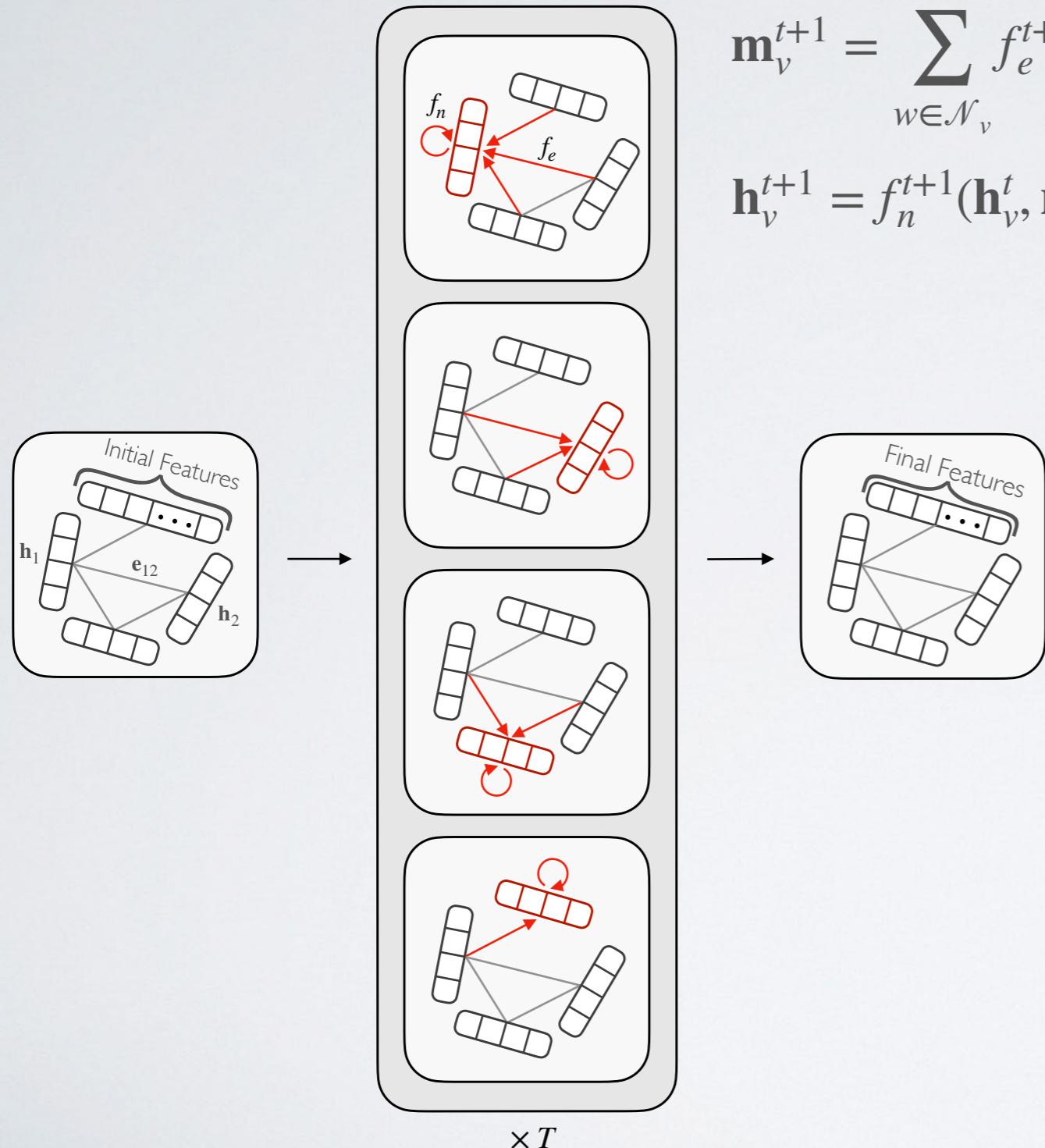
# WHY GRAPHS?

- Properties of HEP data:
  - High granularity
  - Sparsity
  - Irregular geometry
- → Graphs furnish a more natural representation of such data than images
- Our goal is to design a graph-based GAN, which may be more efficient, flexible, and better suited to data



# GAN ARCHITECTURE

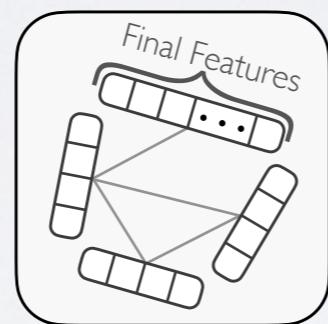
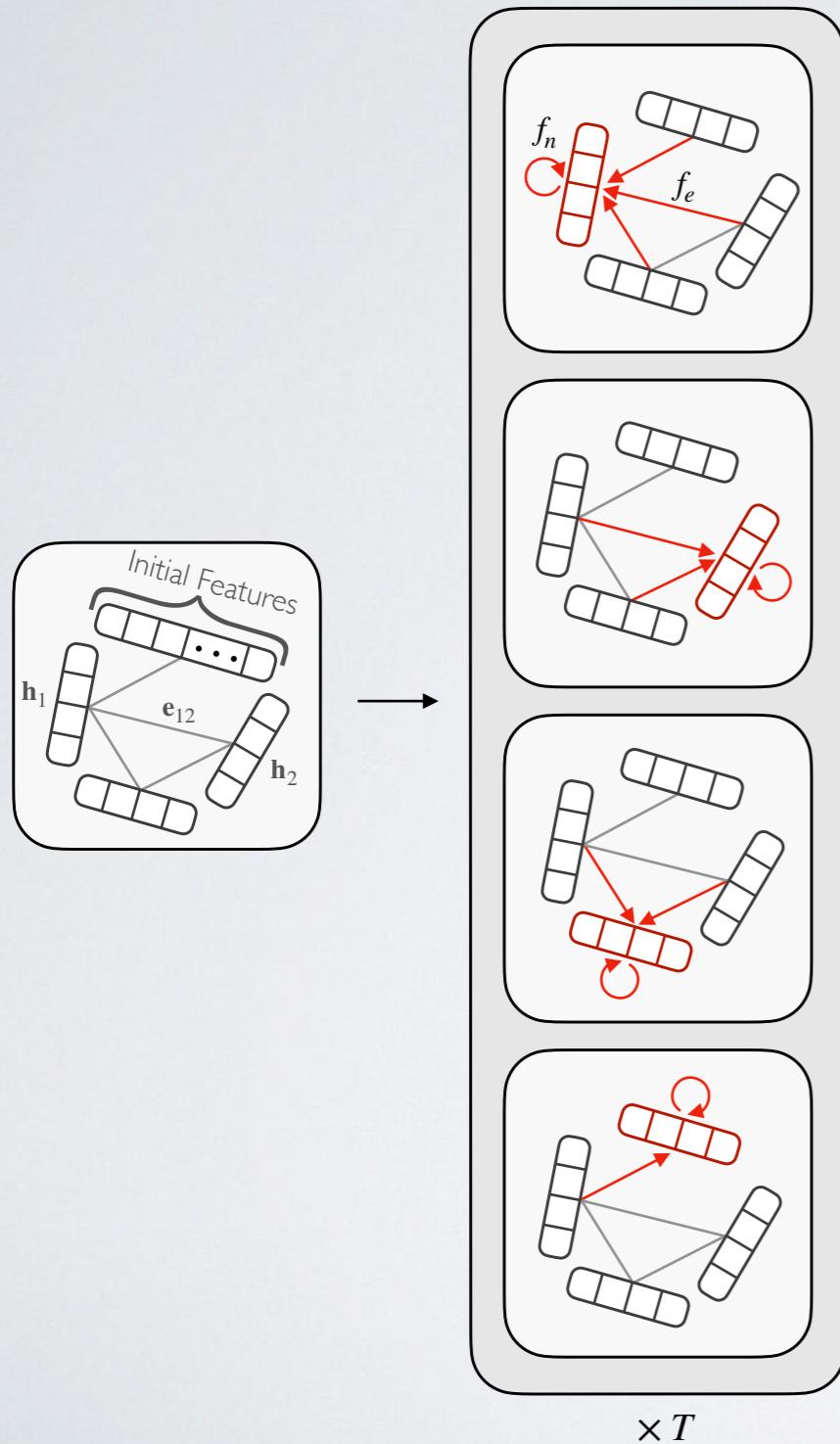
Nodes in the graph learn via ‘message passing’ between their neighbours:



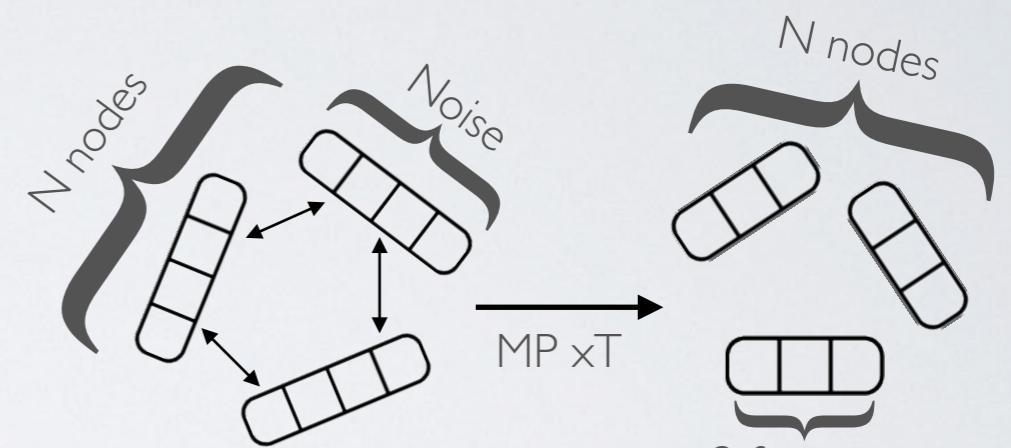
$$\mathbf{m}_v^{t+1} = \sum_{w \in \mathcal{N}_v} f_e^{t+1}(\mathbf{h}_v^t, \mathbf{h}_w^t, \mathbf{e}_{vw}^t)$$
$$\mathbf{h}_v^{t+1} = f_n^{t+1}(\mathbf{h}_v^t, \mathbf{m}_v^{t+1})$$

# GAN ARCHITECTURE

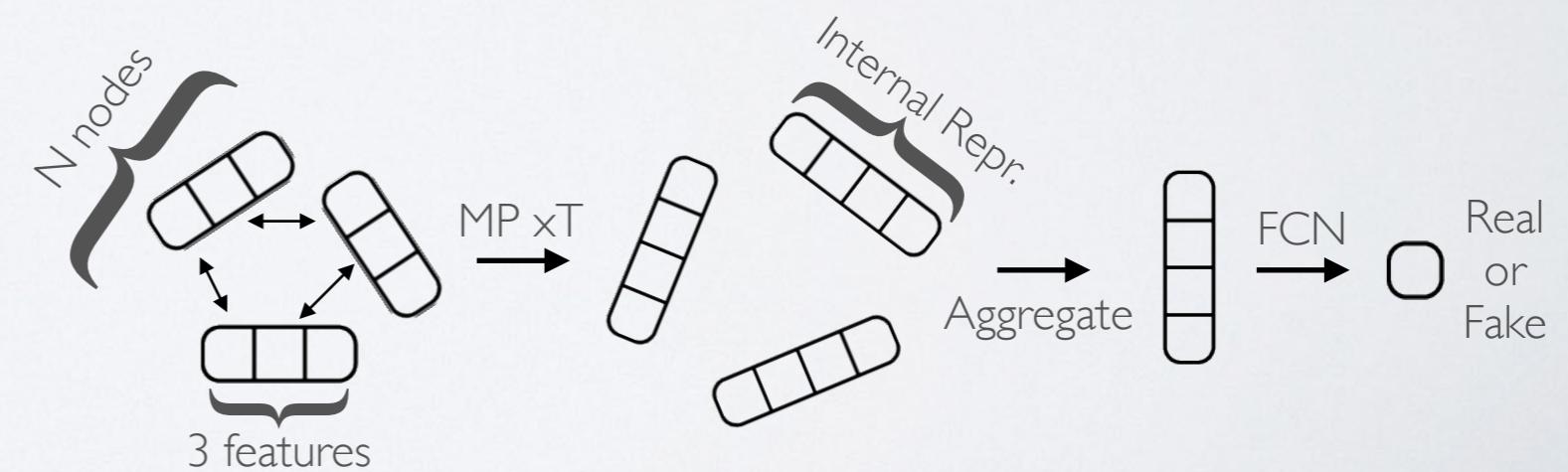
Such a message passing model is used in both the generator and discriminator:



Generator



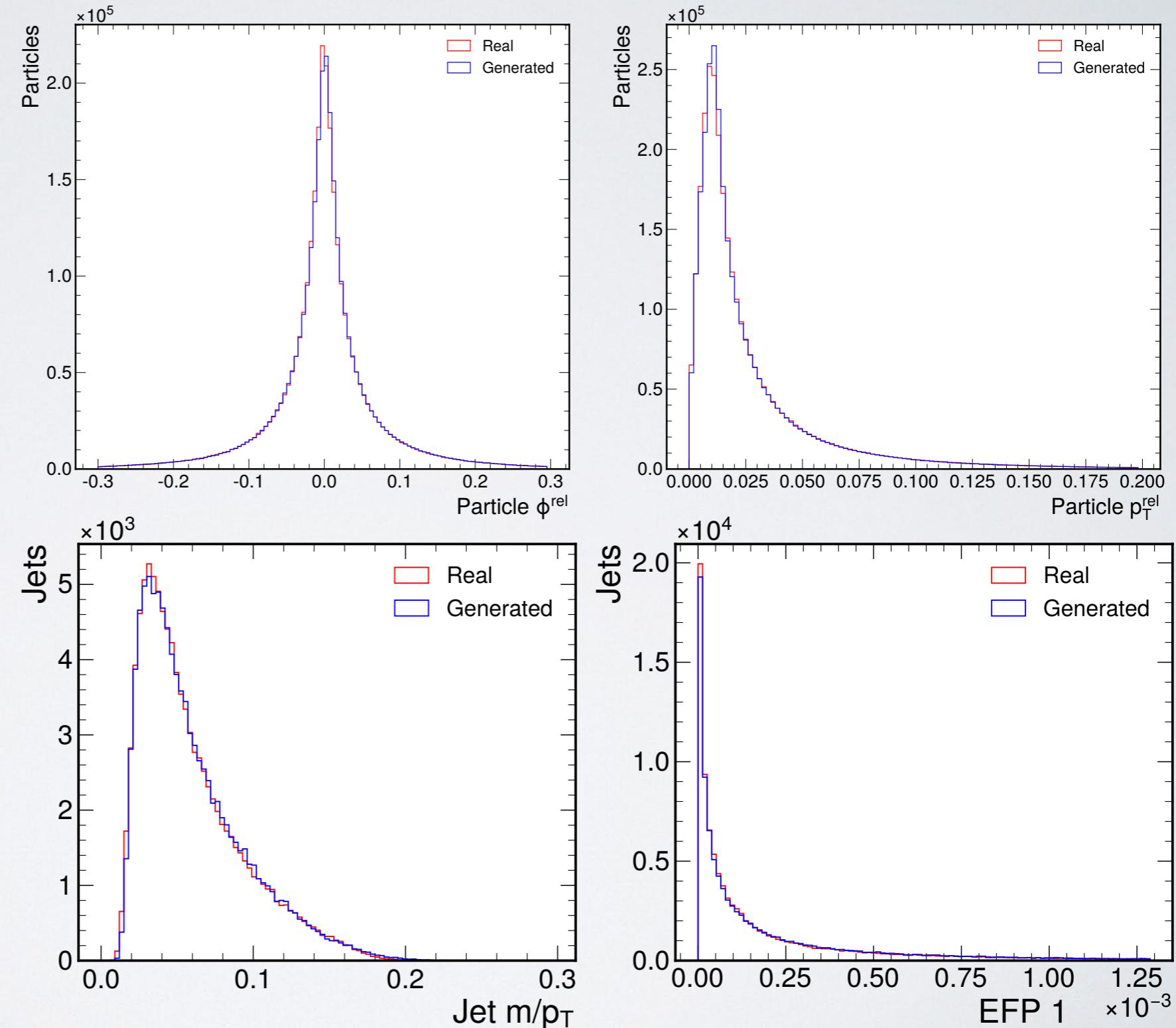
Discriminator



# RESULTS

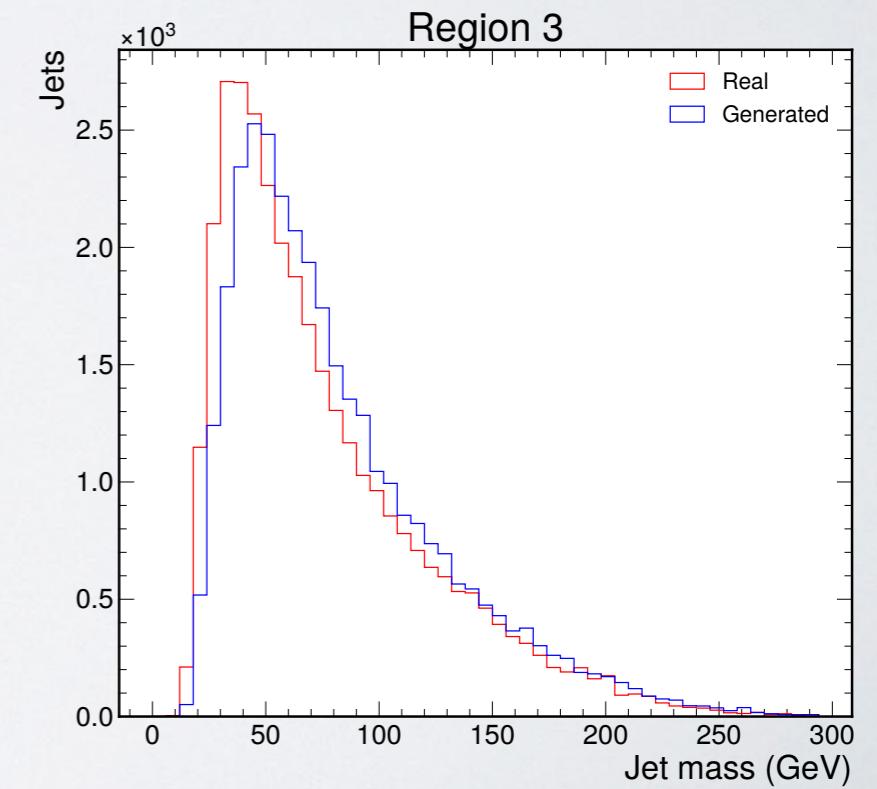
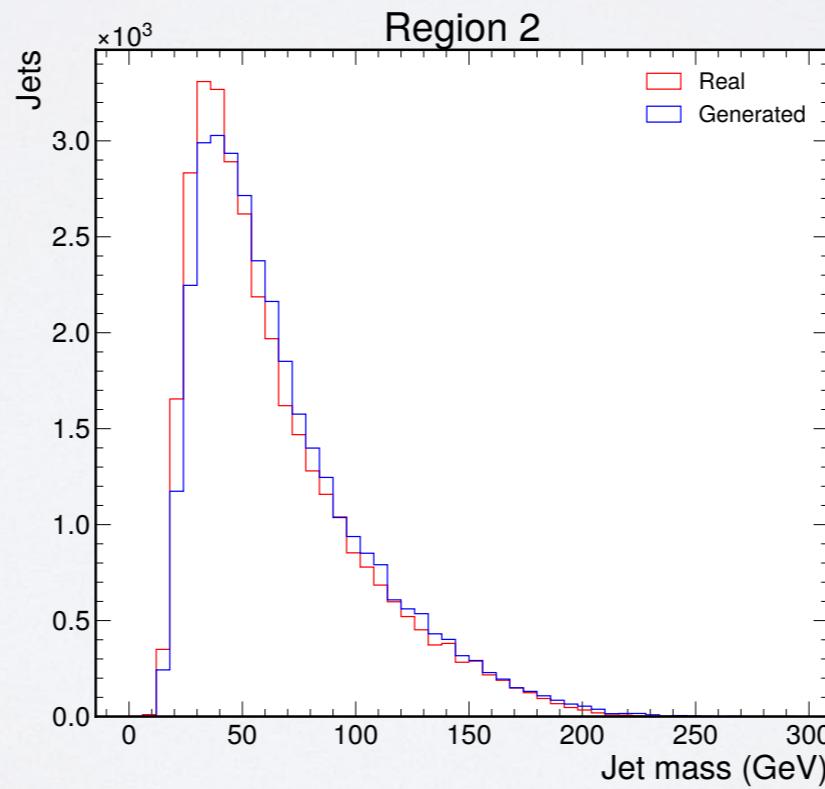
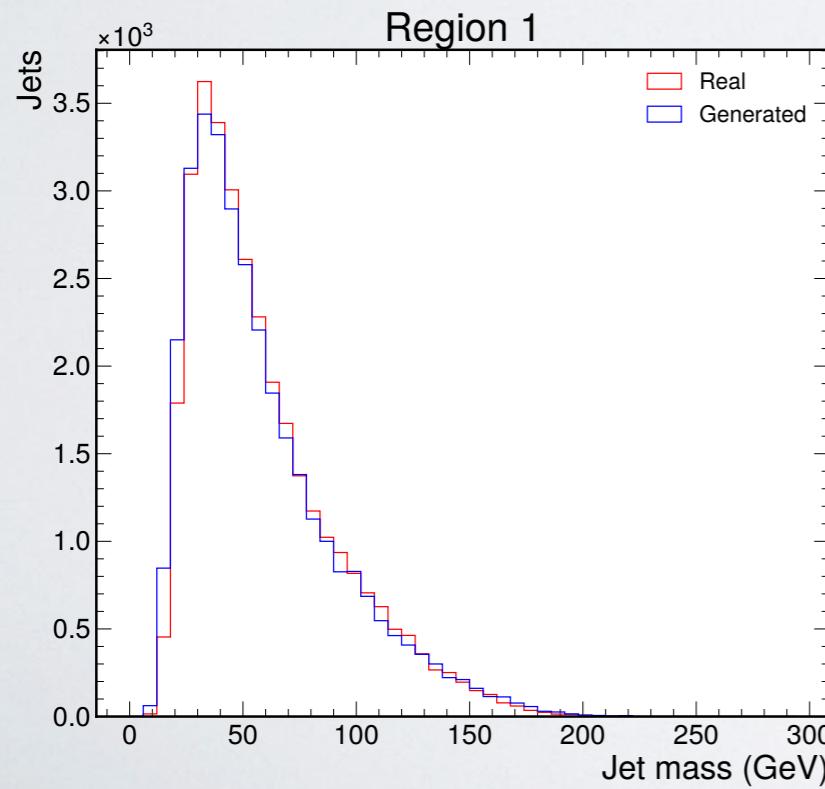
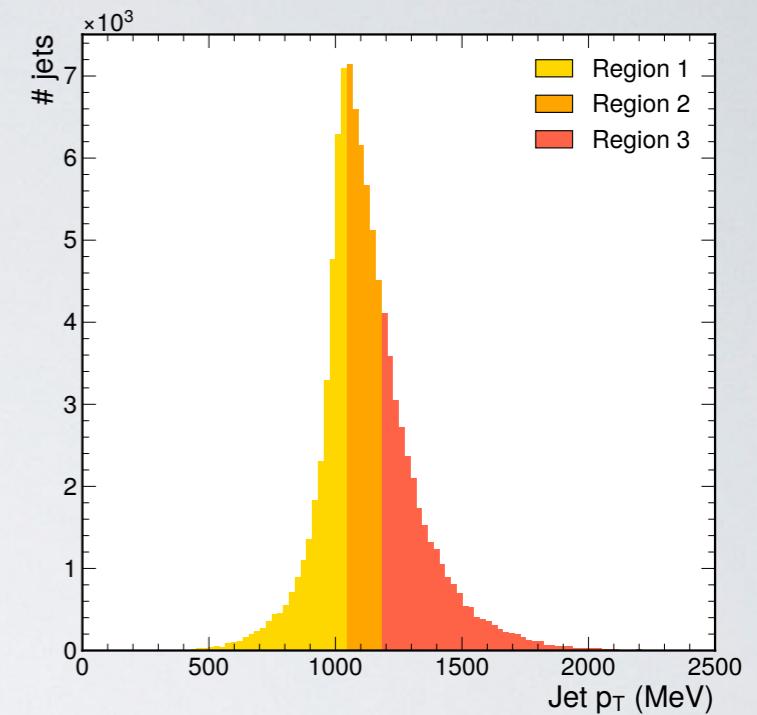
Up to 30 particles (zero-padding when needed) per **gluon** jets of  $p_T \sim 1 \text{ TeV}$

- Real distributions are reproduced with high fidelity:
- We look at jet mass and energy flow polynomials (EFPs) (Komiske et al. 2017) - high-order particle correlations which form a basis for all useful jet observables
- Remarkable that complex jet features are learnt without any specific guidance



# CONDITIONAL GAN

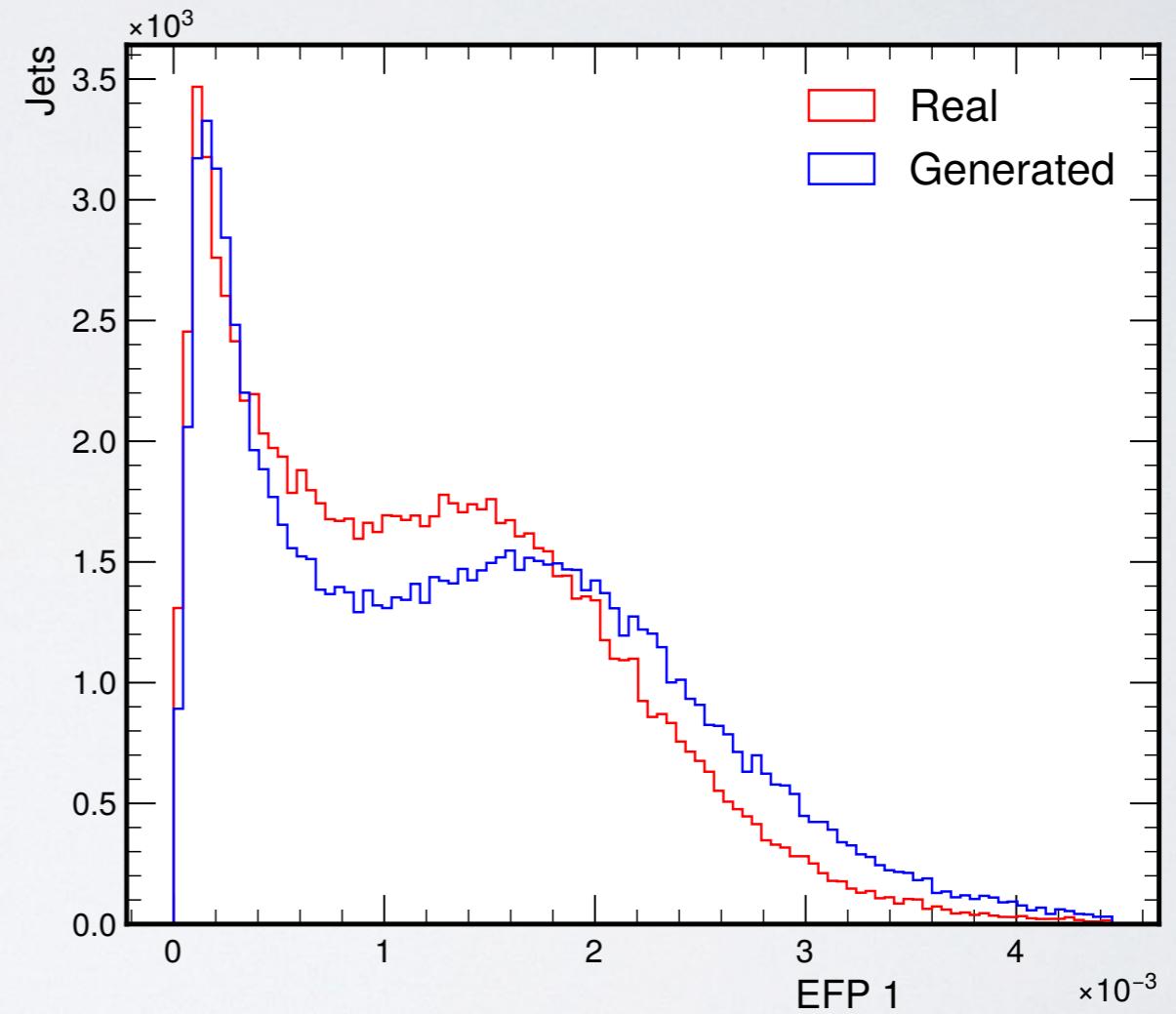
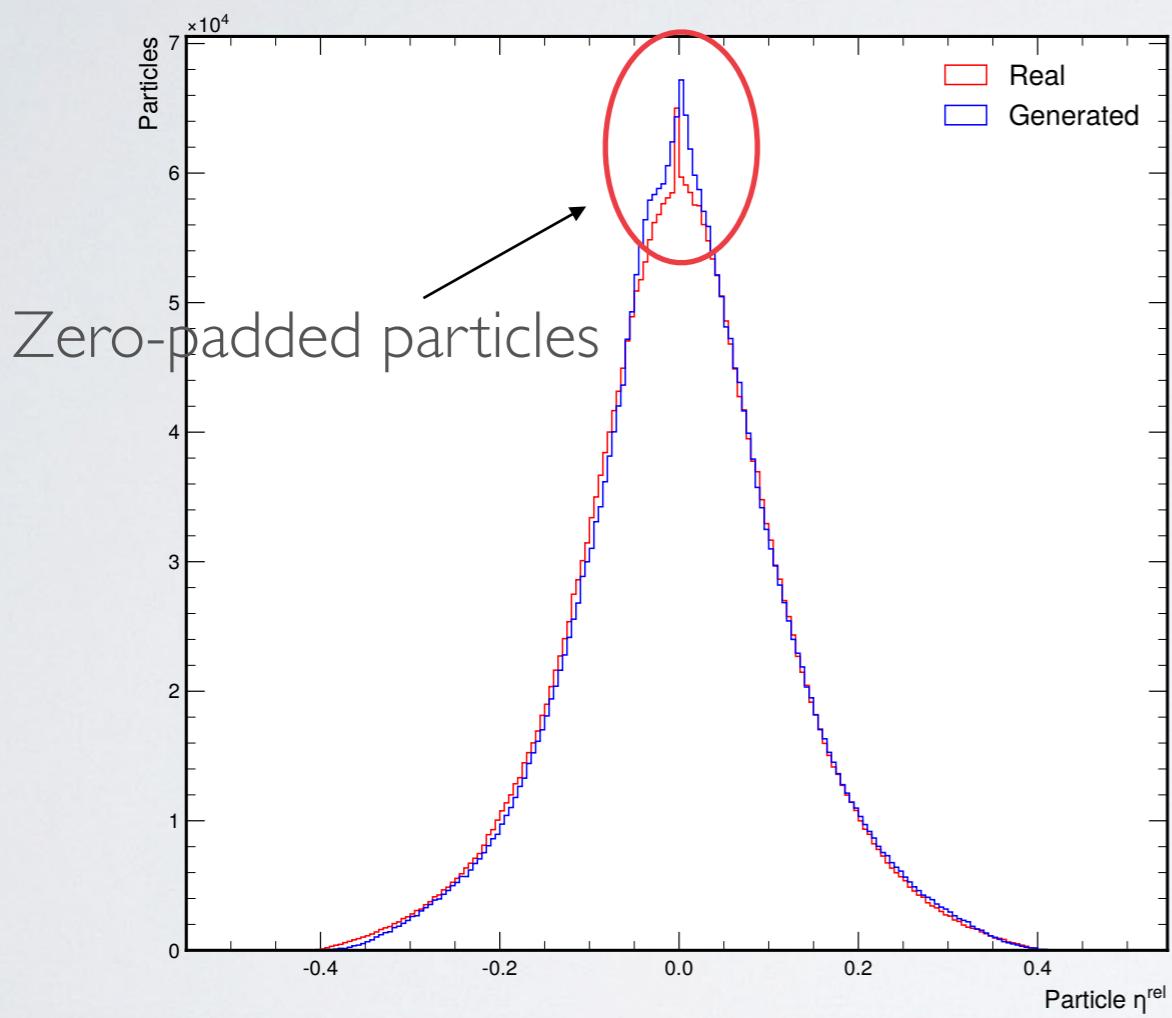
- We want to be able to condition the generator on jet  $p_T, \eta$
- We modified the message passing architecture to incorporate **graph-level features**, and trained a GAN conditioned on  $p_T$
- For evaluation the jets have been split into three regions with  $\sim 33k$  jets each
- Distributions well-matched in all three regions, but some degradation for higher  $p_T$ :



# VARIABLE-SIZED GRAPHS

Up to 30 particles (zero-padding when needed) per **top** jets of  $p_T \sim 1 \text{ TeV}$

- Some issues with generation because of zero-padded particles:

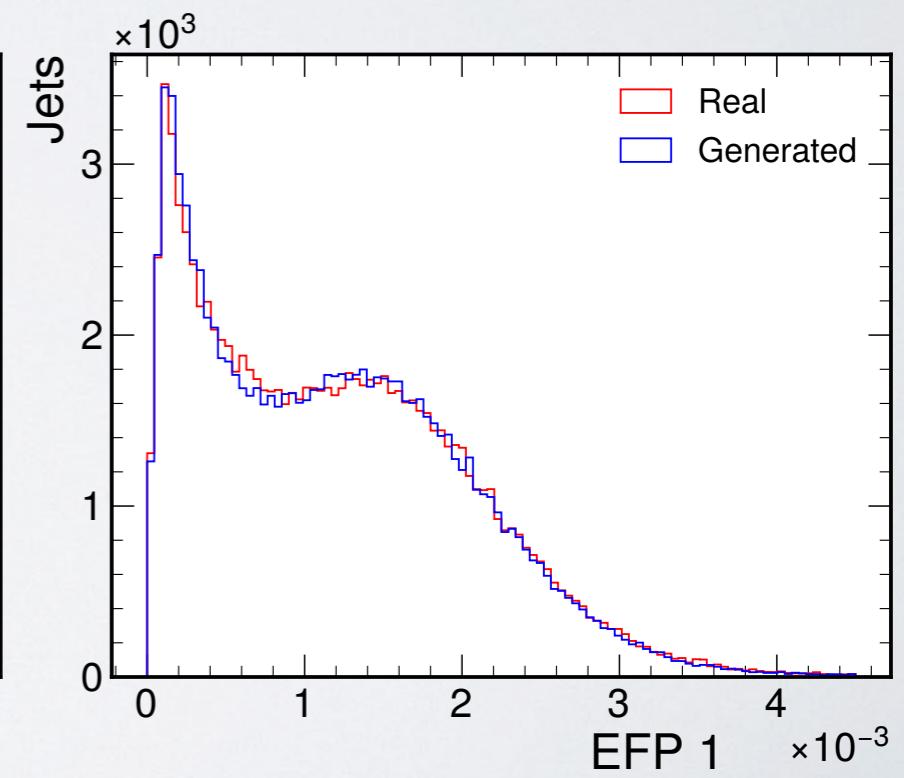
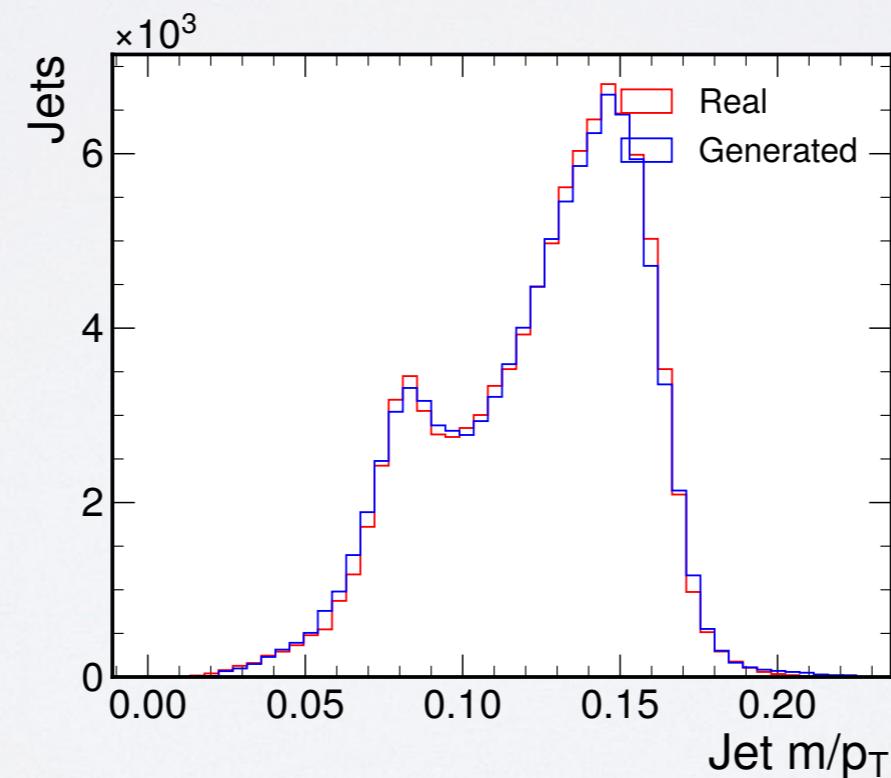
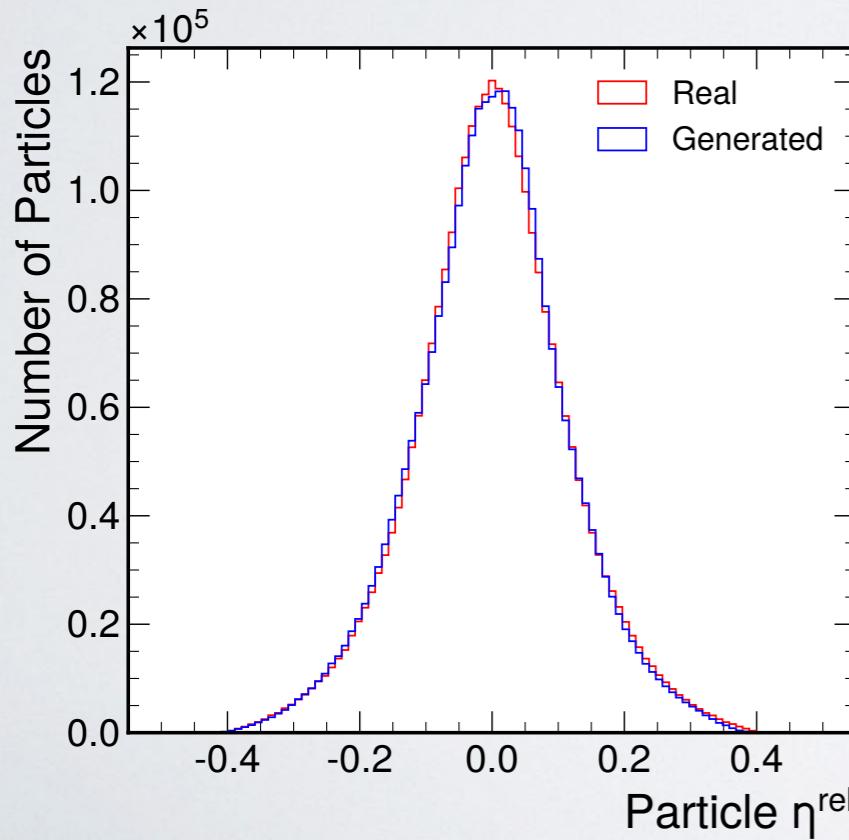


- (Also top jets are more complicated:  $t \rightarrow Wb \rightarrow qqb$  so 2 or 3 subjets)

# VARIABLE-SIZED GRAPHS

Up to 30 particles per **top** jets of  $p_T \sim 1 \text{ TeV}$

- We ‘mask’ the particles with an extra binary feature, telling the model if they’re real or zero-padded - effectively allowing for variable sized graphs
- With our masking strategy, distributions are well-matched even for complex top jets:



# SUMMARY

Paper: <https://arxiv.org/abs/2012.00173>  
Code: <https://github.com/rkansal47/graph-gan>  
Email: [rkansal@ucsd.edu](mailto:rkansal@ucsd.edu)



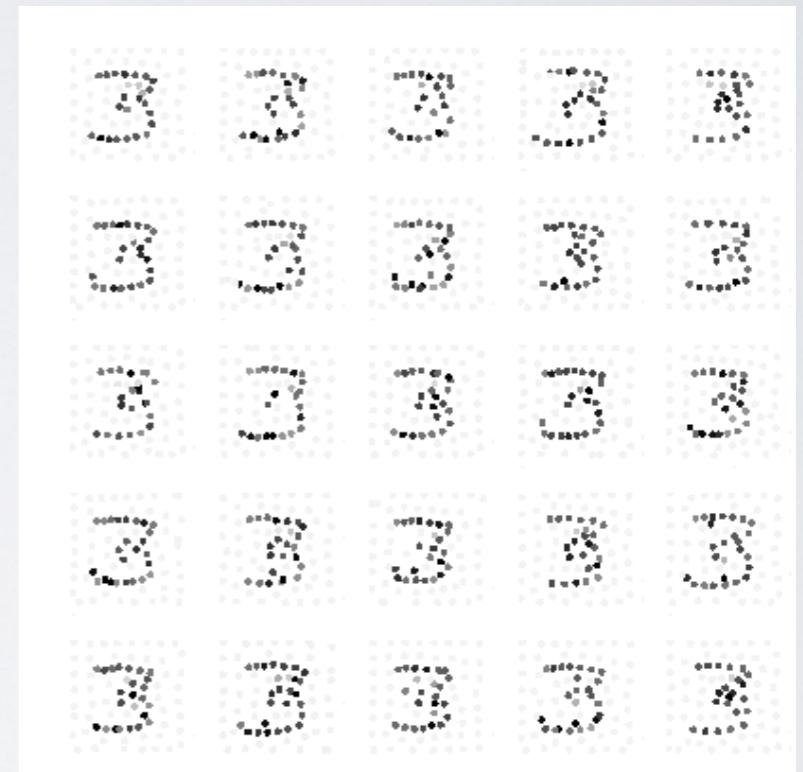
- We developed a new graph-based GAN, which may be naturally suited for HEP data
- It has been successful in producing jets (as well as graphical MNIST data)
- Currently exploring conditioning and variable-sized graphs
- Future work
  - Further applications to HEP datasets (calorimeter data) and beyond
  - More sophisticated architectures e.g. sequential generation, Lorentz group equivariant networks

# BACKUP

# GAN TRAINING

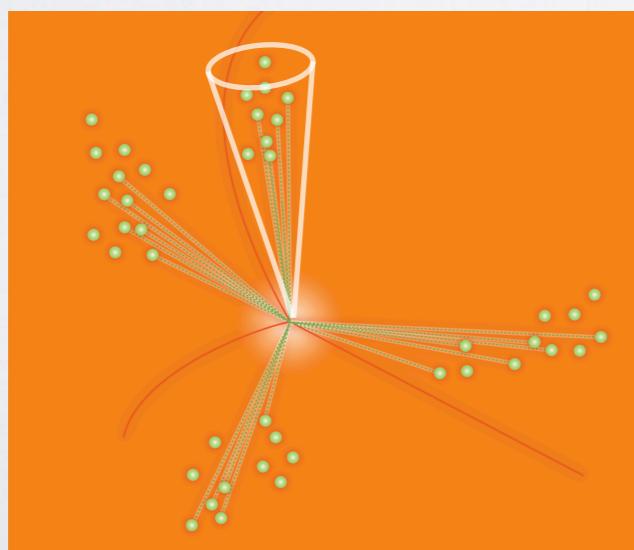
- Some GAN training techniques attempted, mostly to improve mode collapse on the MNIST Superpixels dataset (final choices in bold):
  - **Least squares**, binary cross entropy, Wasserstein, hinge and AGCD losses
  - **RMSProp**, Adam, Adadelta, SGD
  - Two-time update rules (LR of  $10^{-3}$  and  $3 \times 10^{-3}$  for G and D respectively)
  - Dropout (0.5 in D only)
  - Batch Normalisation
  - Gradient Penalty
  - Spectral Normalisation
  - Noisy labels, label smoothing
  - Data augmentation
  - Calculating quantitative metrics such as **I-Wasserstein score** and **Graph Fréchet Distance** for model evaluation and optimisation

Early sample outputs of models trained on Superpixels 3s, showing clear ‘mode collapse’



# JETS

- Quarks, gluons, W, Z bosons are produced often at LHC but are never **directly** detected
- This is because they decay or hadronise far too quickly to reach the detector (lifetime of  $\sim 10^{-23}$ s)
- Instead after decaying/hadronising (multiple times sometimes) they produce a set of stable particles with distinct features and geometries, called a jet
- From this we can infer the originating particle



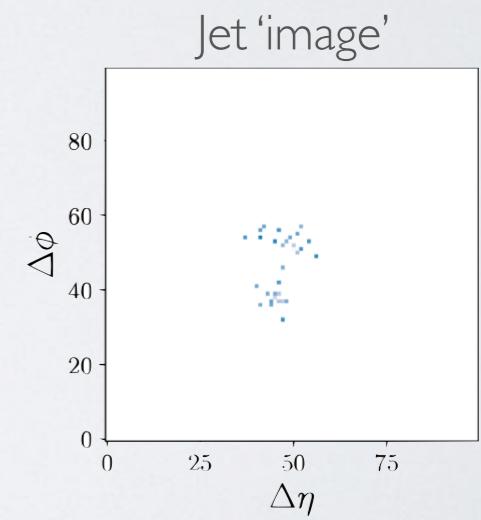
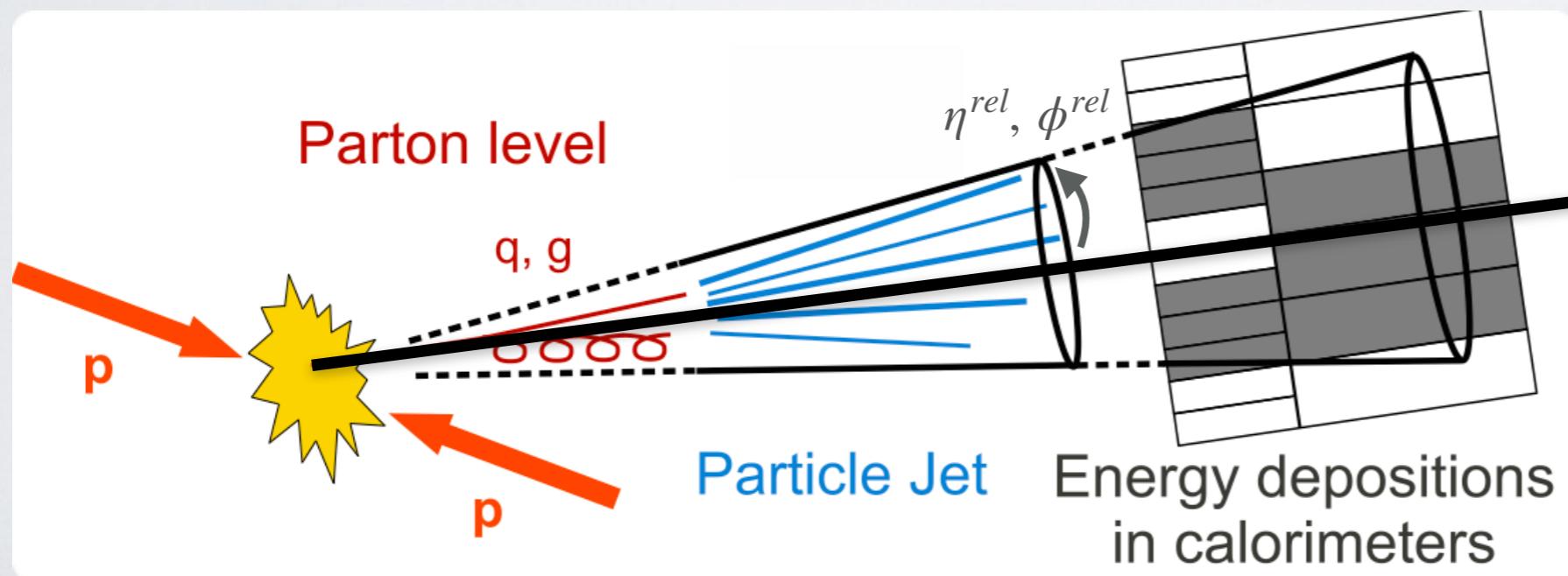
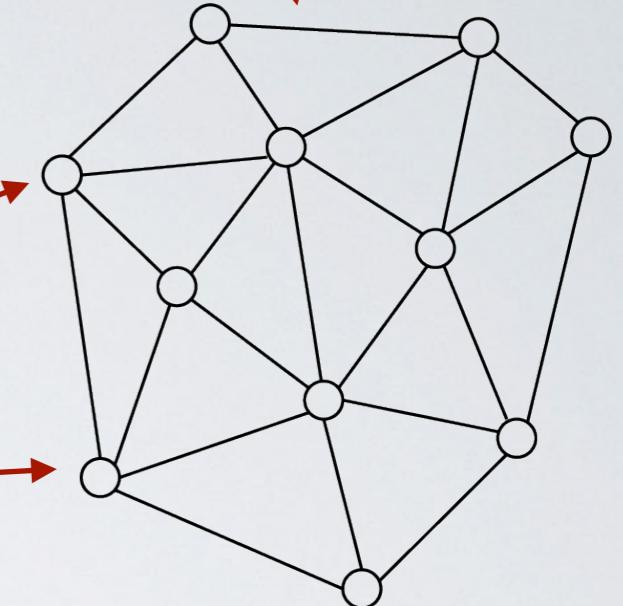
## STANDARD MODEL OF ELEMENTARY PARTICLES

<b>QUARKS</b>	<b>UP</b> mass $2,3 \text{ MeV}/c^2$ charge $\frac{2}{3}$ spin $\frac{1}{2}$ <b>u</b>	<b>CHARM</b> $1,275 \text{ GeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$ <b>c</b>	<b>TOP</b> $173,07 \text{ GeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$ <b>t</b>	<b>GLUON</b> 0 0 1 <b>g</b>	<b>HIGGS BOSON</b> $126 \text{ GeV}/c^2$ 0 0 <b>H</b>
	<b>DOWN</b> $4,8 \text{ MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ <b>d</b>	<b>STRANGE</b> $95 \text{ MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ <b>s</b>	<b>BOTTOM</b> $4,18 \text{ GeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ <b>b</b>	<b>PHOTON</b> 0 0 1 <b>γ</b>	
<b>LEPTONS</b>	<b>ELECTRON</b> $0,511 \text{ MeV}/c^2$ $-1$ $\frac{1}{2}$ <b>e</b>	<b>MUON</b> $105,7 \text{ MeV}/c^2$ $-1$ $\frac{1}{2}$ <b>μ</b>	<b>TAU</b> $1,777 \text{ GeV}/c^2$ $-1$ $\frac{1}{2}$ <b>τ</b>	<b>Z BOSON</b> $91,2 \text{ GeV}/c^2$ 0 1 <b>Z</b>	
	<b>ELECTRON NEUTRINO</b> $<2,2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ <b>ν<sub>e</sub></b>	<b>MUON NEUTRINO</b> $<0,17 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ <b>ν<sub>μ</sub></b>	<b>TAU NEUTRINO</b> $<15,5 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ <b>ν<sub>τ</sub></b>	<b>W BOSON</b> $80,4 \text{ GeV}/c^2$ $\pm 1$ 1 <b>W</b>	
<b>GAUGE BOSONS</b>					

# JETS

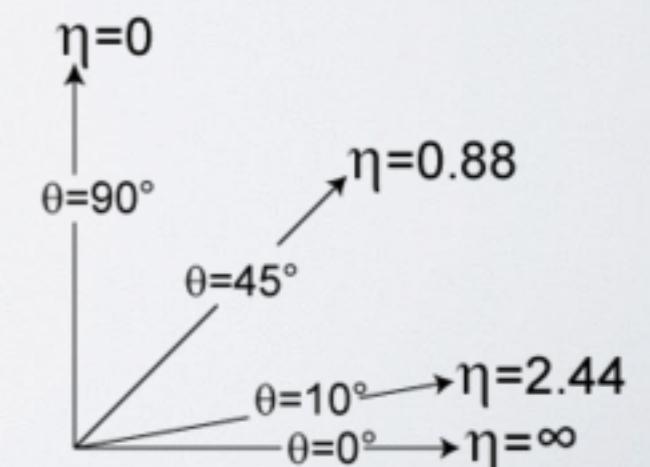
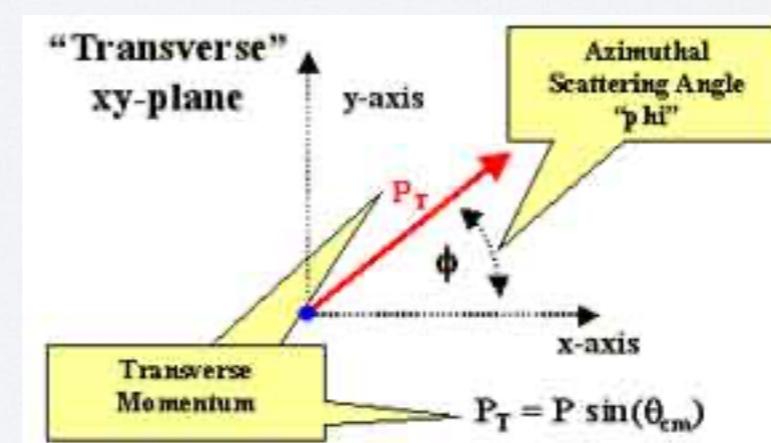
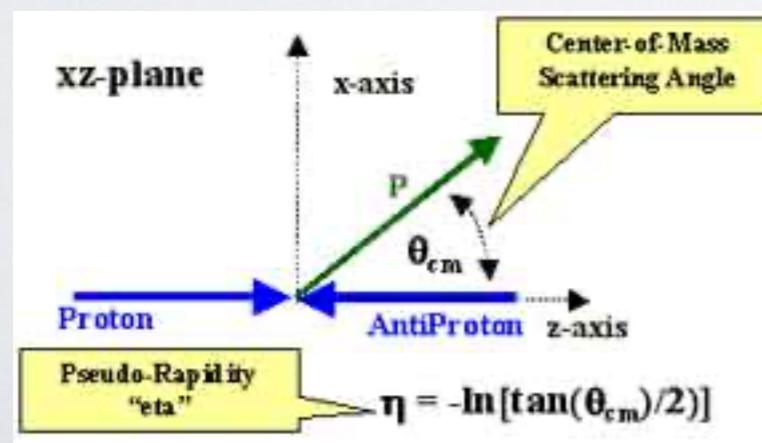
- Simulated high transverse momentum ( $p_T$ ) jets
- In each jet we consider the N highest  $p_T$  particle constituents (zero padding when needed), and 3 particle features ( $\eta^{rel}$ ,  $\phi^{rel}$ ,  $p_T^{rel}$ )

Edge features are angular distances  
 $R = \sqrt{\eta^2 + \phi^2}$  between nodes



# LHC GEOMETRY

- Variables are chosen carefully to be invariant to ‘boosts’ along the ‘beam-line’ (z-axis), so we can always boost, or switch, to the centre of mass or some other convenient frame of the collision
- Instead of momentum  $p$ , transverse momentum  $p_T$
- Instead of  $\theta$ ,  $\eta = -\ln[\tan(\theta/2)] \Rightarrow$  that  $\Delta\eta$  is invariant to boosts so we can freely translate in  $\eta$  space (also defined so that the angle perpendicular to beam axis  $\theta = \pi/2 \rightarrow \eta = 0$ )



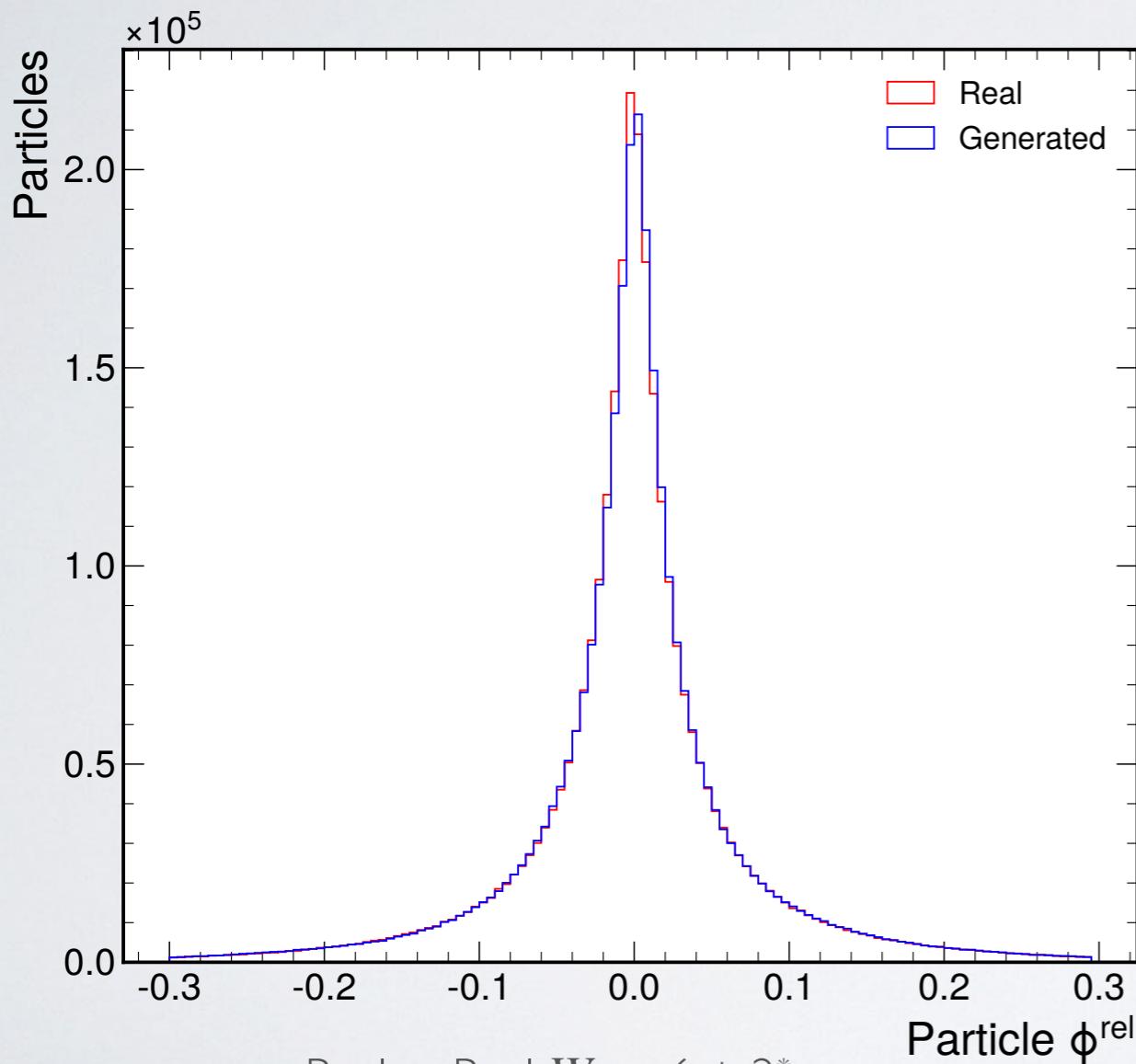
# EVALUATION: JETS

- We look at particle  $\eta^{rel}$ ,  $\phi^{rel}$ ,  $p_T^{rel}$ , and jet mass and Energy Flow Polynomials ([Komiske et al.](#)) distributions
  - EFPs calculate n-particle correlations per jet
  - Span the set of useful\* jet observables
- We use  $l$ -Wasserstein ( $W_1$ ) metric (minimum work needed to transform one dist. to another)
  - Baseline: average  $W_1$  between different sets of samples of real jets
  - Score: average  $W_1$  between sets of real and fake samples

# RESULTS: JETS (PARTICLE LEVEL)

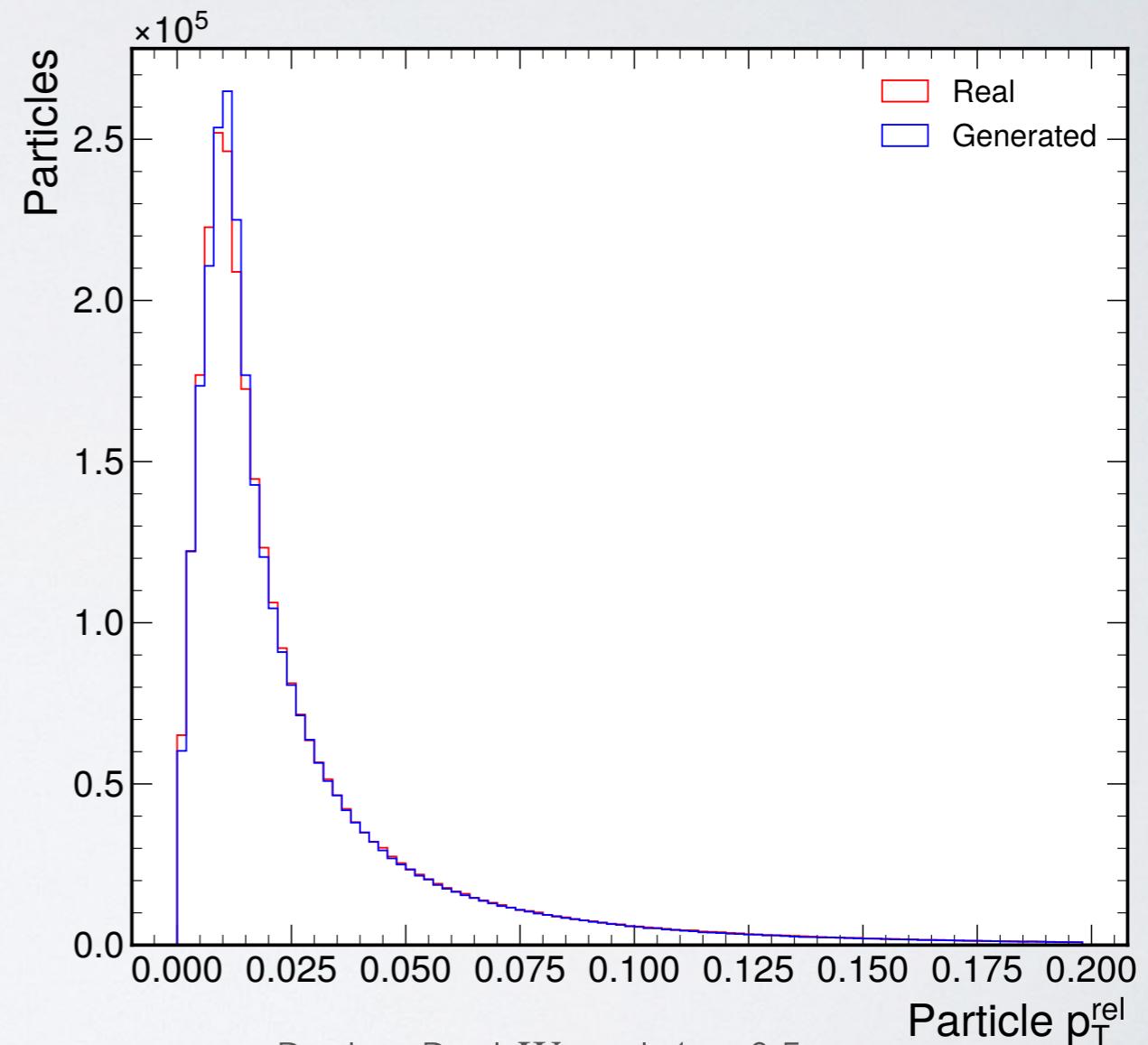
Jets Dataset (Up to 30 particles per **gluon** jets of  $p_T \sim 1 \text{ TeV}$ )

- Real distributions are reproduced with high fidelity:



Real vs Real  $W_1 = 6 \pm 2^*$

Real vs Fake  $W_1 = 11 \pm 4$

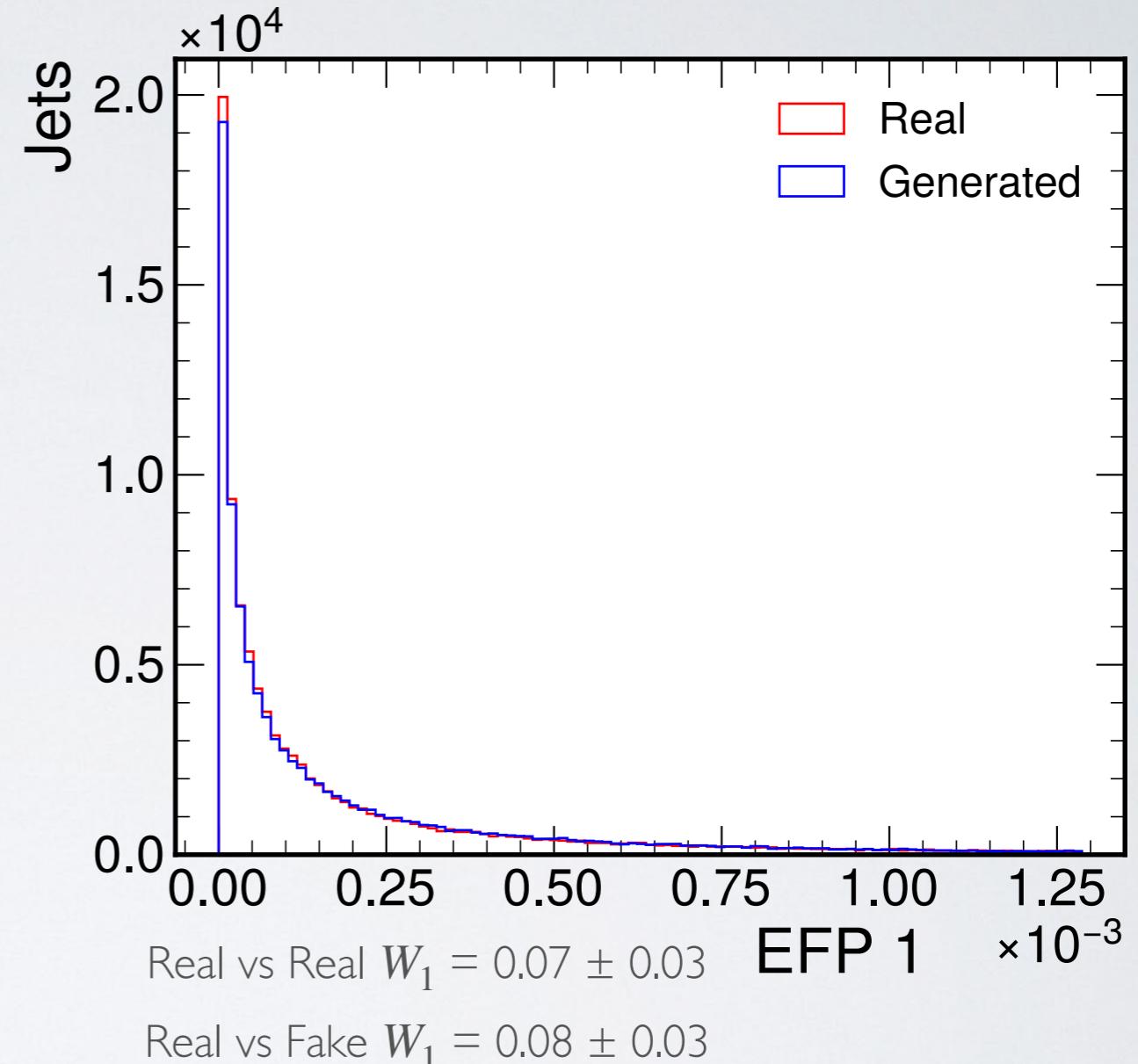
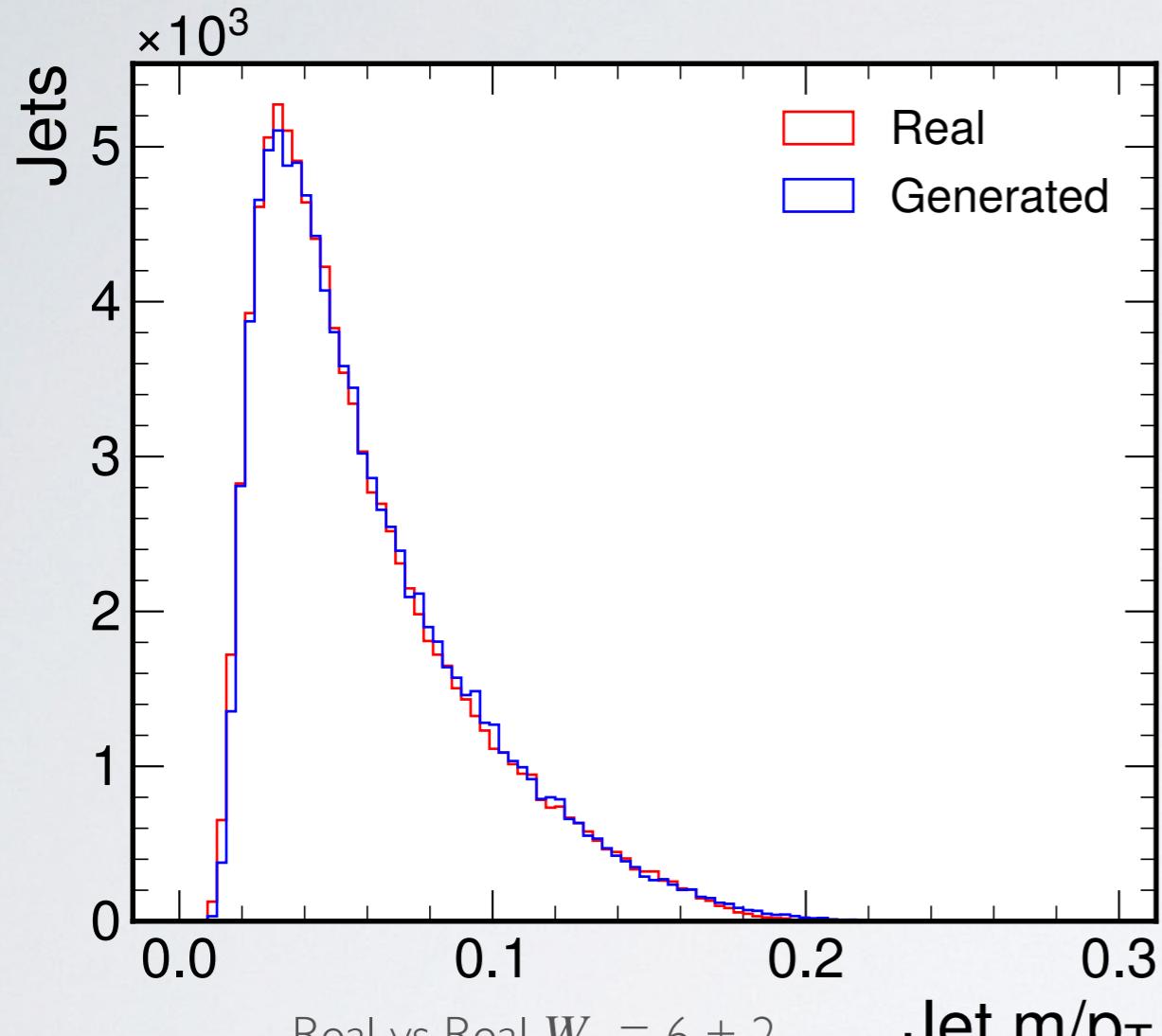


Real vs Real  $W_1 = 1.4 \pm 0.5$

Real vs Fake  $W_1 = 2 \pm 1$

# RESULTS: JETS (JET LEVEL)

- Real distributions are reproduced with high fidelity (4-particle correlation EFP):

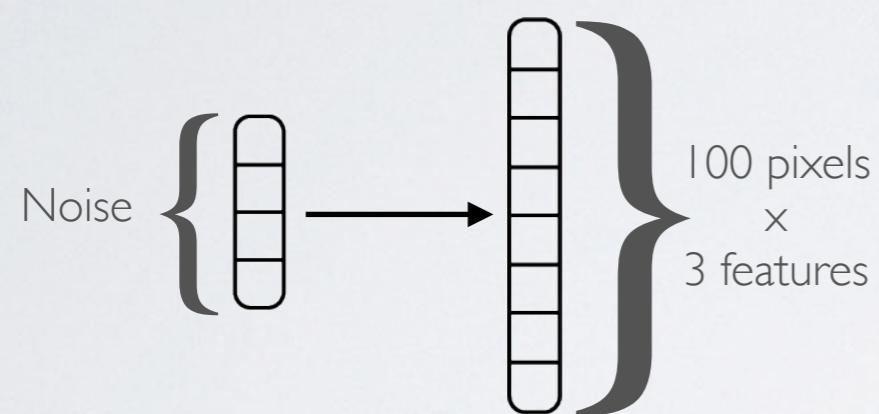


- Remarkable that without specific guidance complex physics is learnt
- Learning tails well  $\Rightarrow$  little evidence of mode collapse

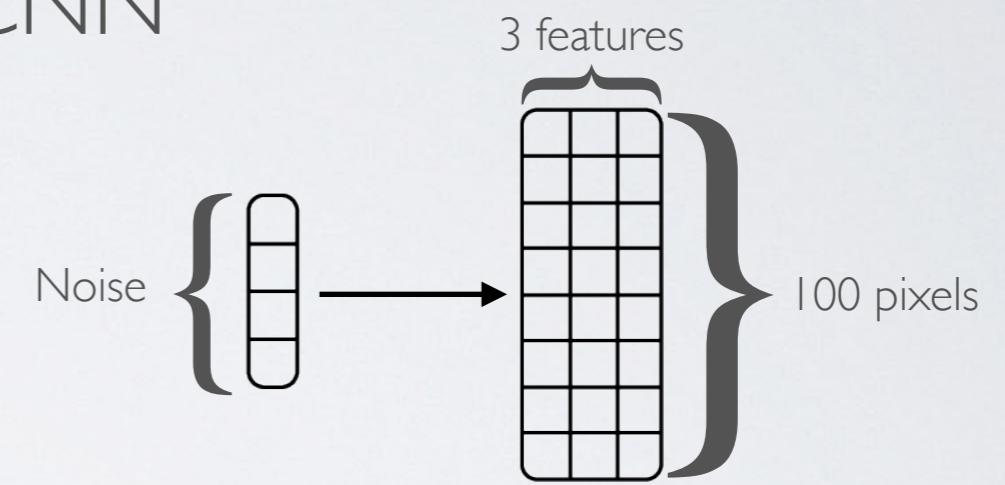
# GAN ARCHITECTURES (NON-GRAFH)

- First, baseline attempt at using our sparse data structure with standard NN and CNN based GANs:

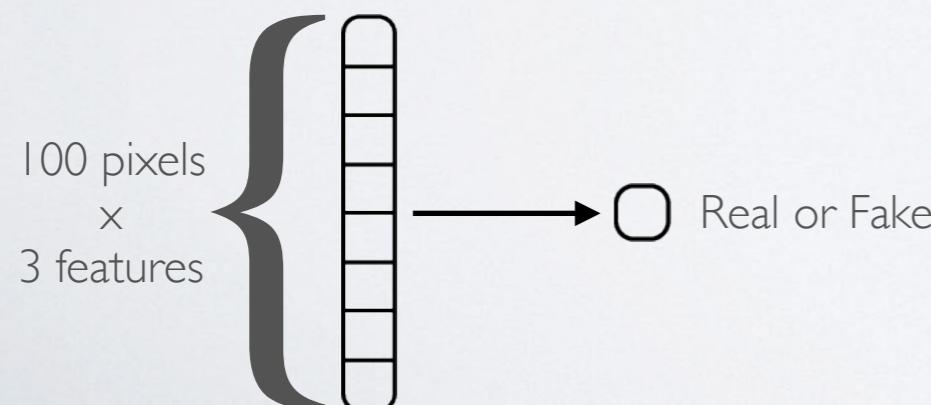
Generator  
Neural Network



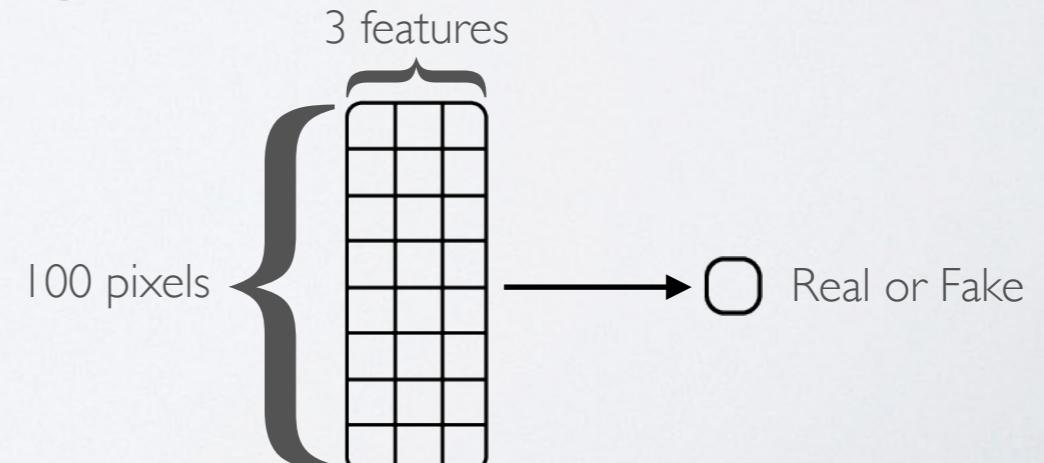
CNN



Discriminator  
Neural Network



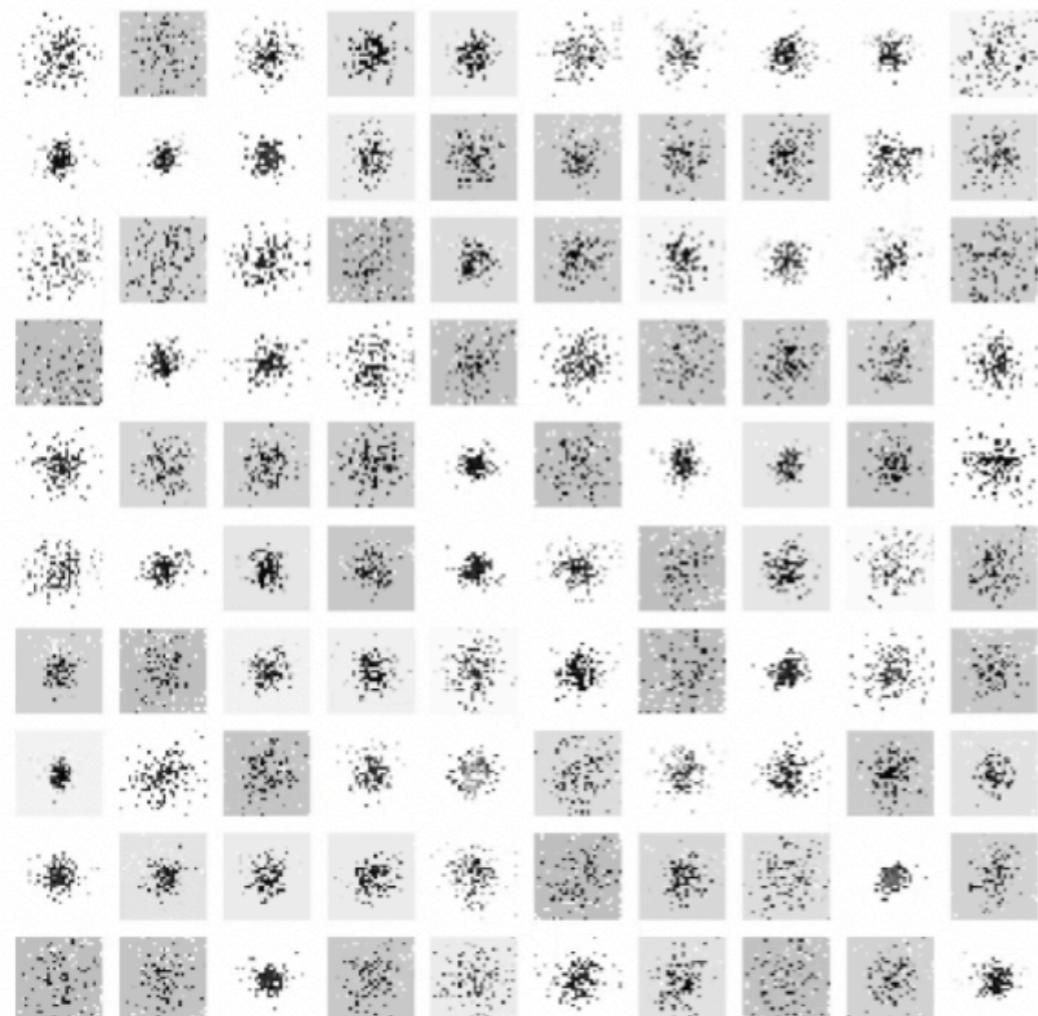
CNN



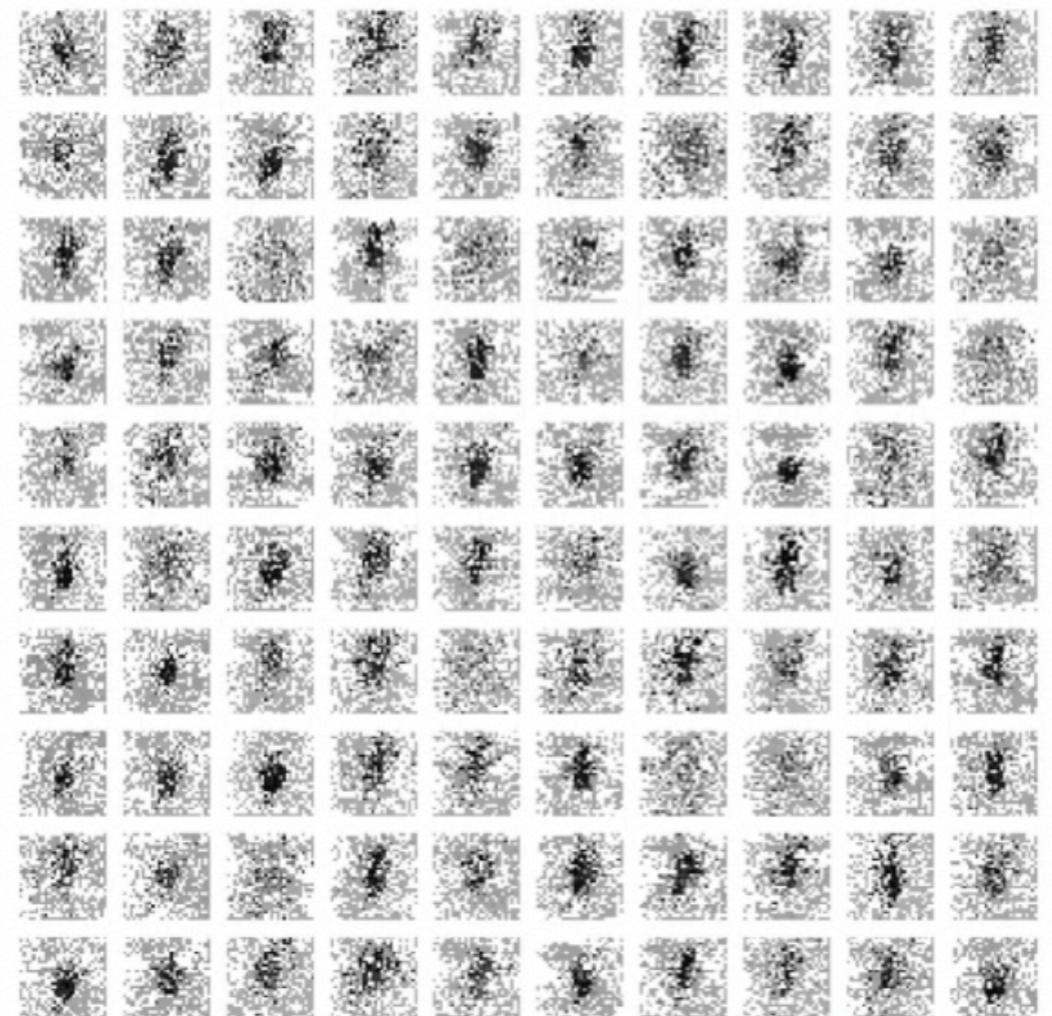
# GAN ARCHITECTURES (NON-GRAFH)

Outputs (on Sparse MNIST) were just noise:

NN



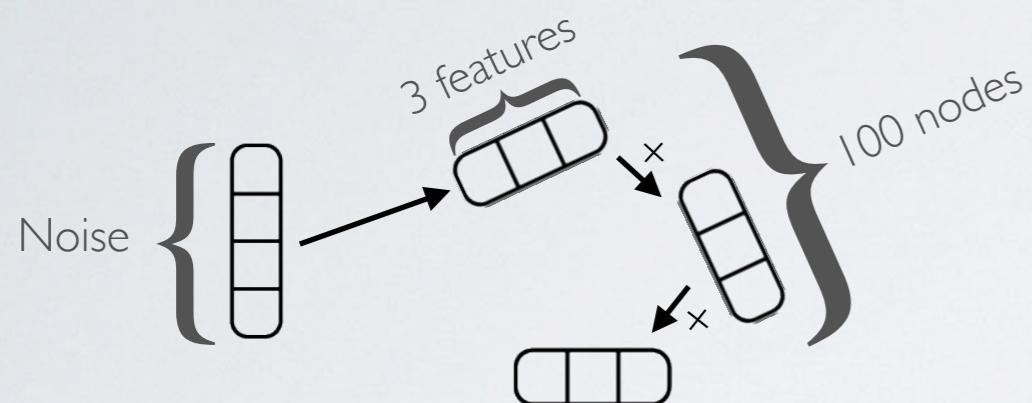
CNN



# GAN ARCHITECTURES

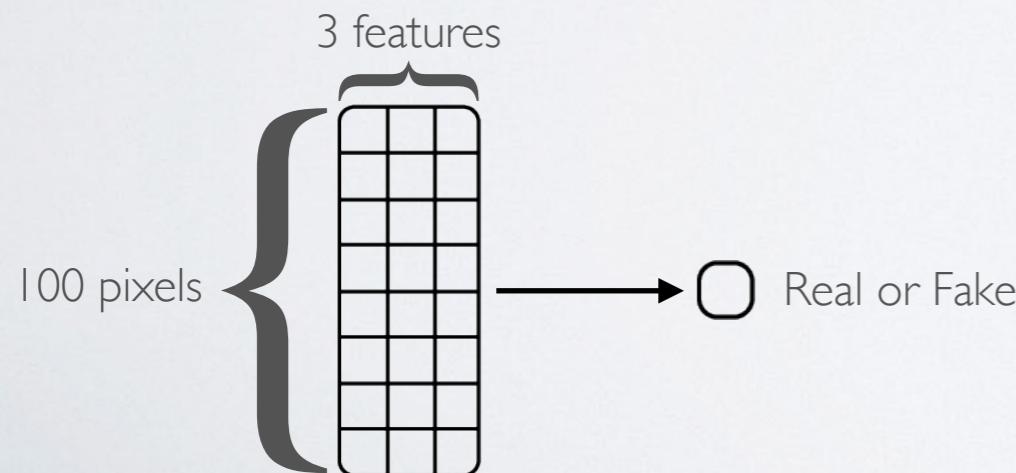
## Generator

Node generation via RNN



## Discriminator

CNN

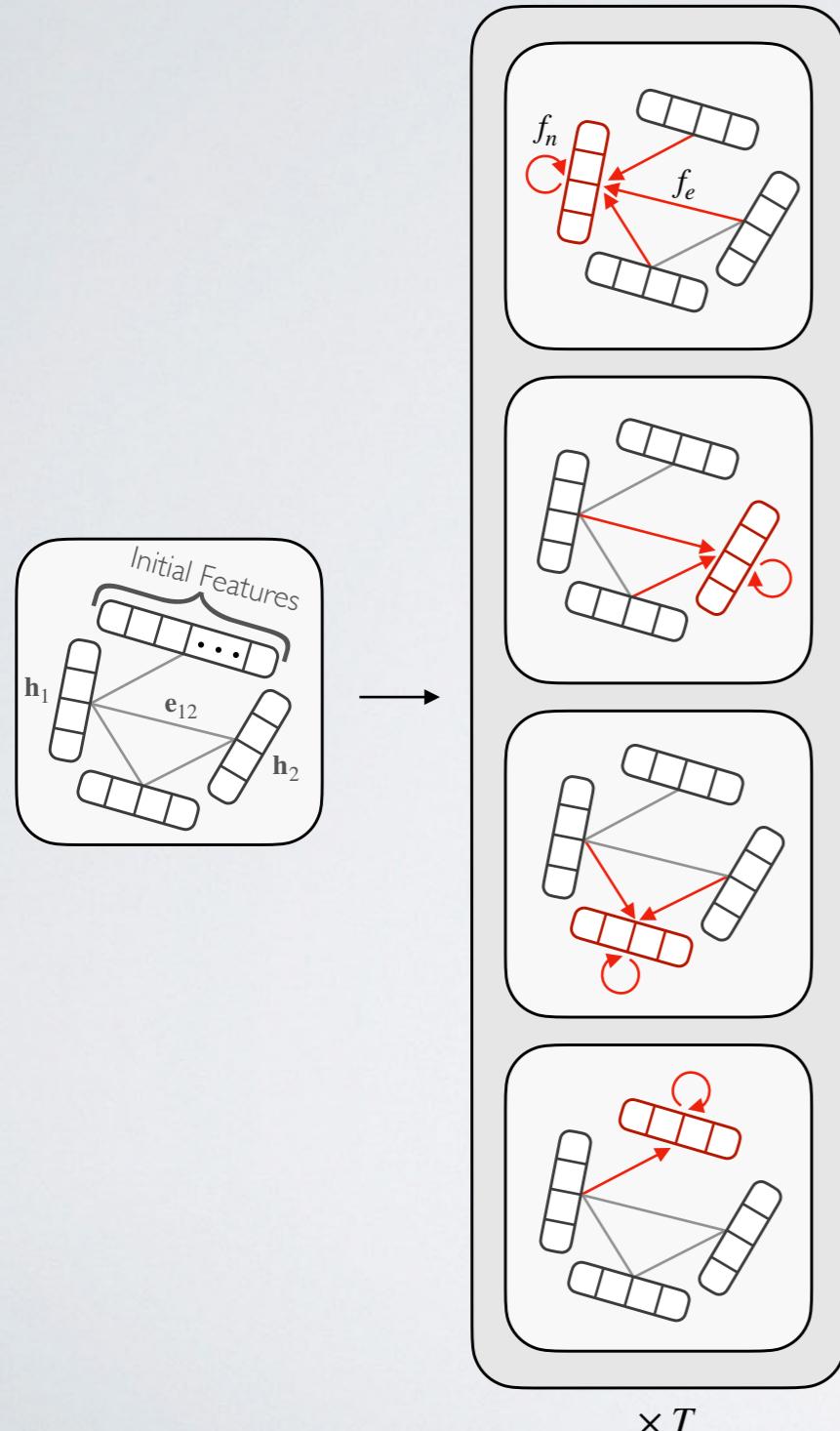


- Next, recurrent neural network (RNN) based generator which can produce graphs with arbitrary numbers of nodes
- But while outputs (on Sparse MNIST, below) showed learning of some structure, this architecture was unable to reproduce real samples



# GAN ARCHITECTURE

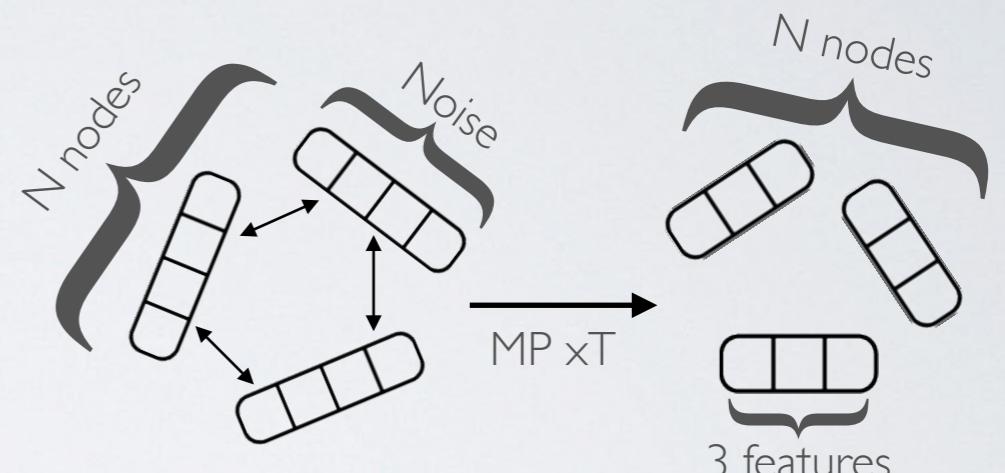
End goal is a generator which takes graph/jet-level variables into account, so we try a modified 'conditional' GAN/generalised MPNN:



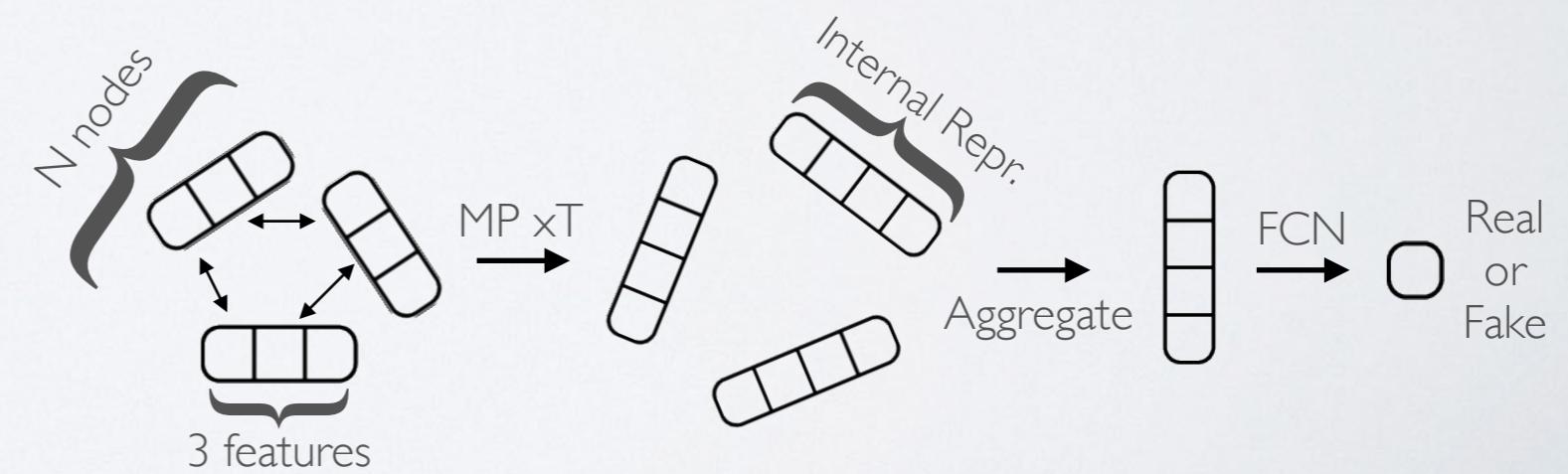
$$\mathbf{m}_v^{t+1} = \sum_{w \in \mathcal{N}_v} f_e^{t+1}(\mathbf{h}_v^t, \mathbf{h}_w^t, \mathbf{e}_{vw}^t)$$

$$\mathbf{h}_v^{t+1} = f_n^{t+1}(\mathbf{h}_v^t, \mathbf{m}_v^{t+1})$$

Generator

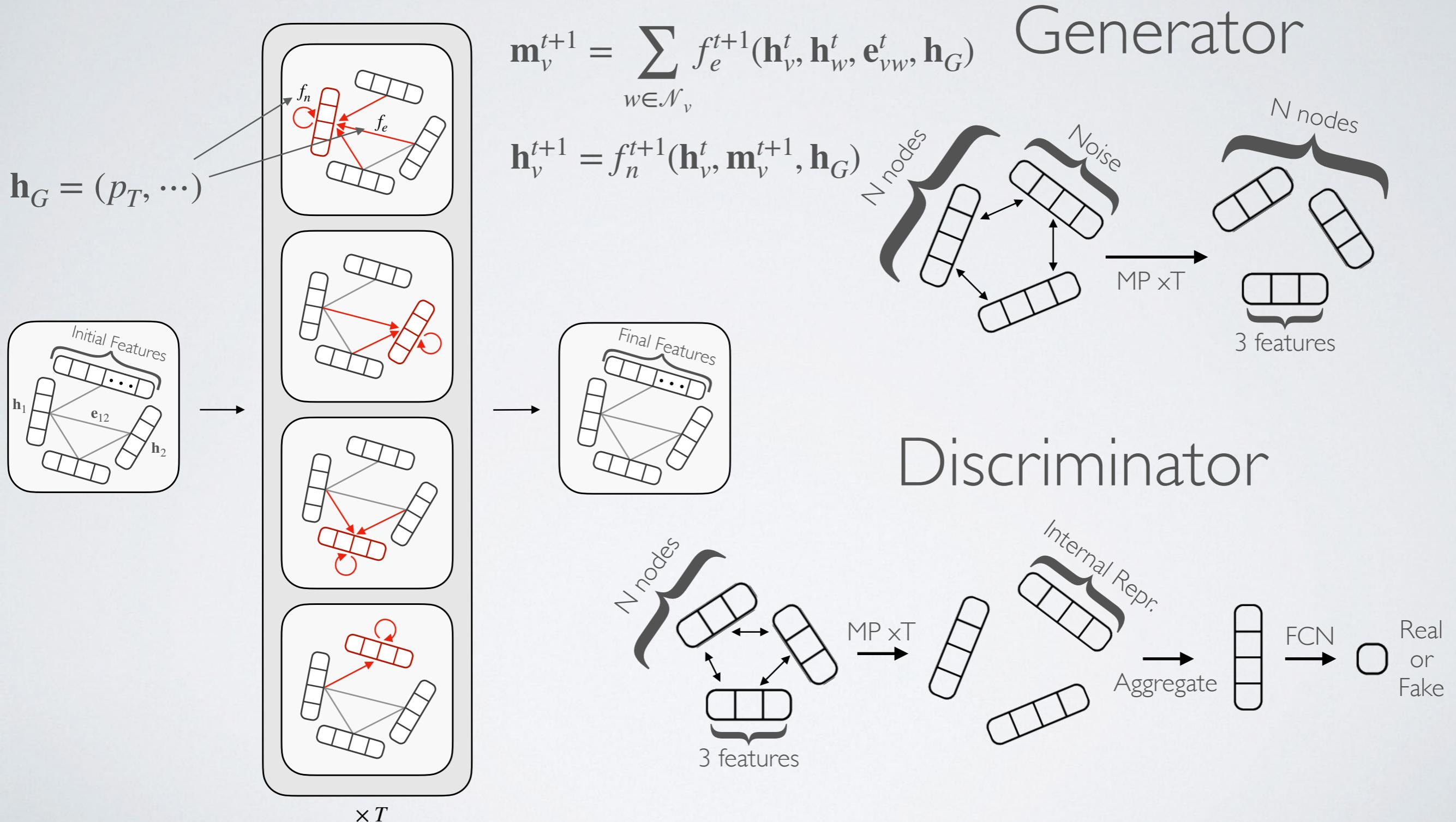


Discriminator



# GAN ARCHITECTURE

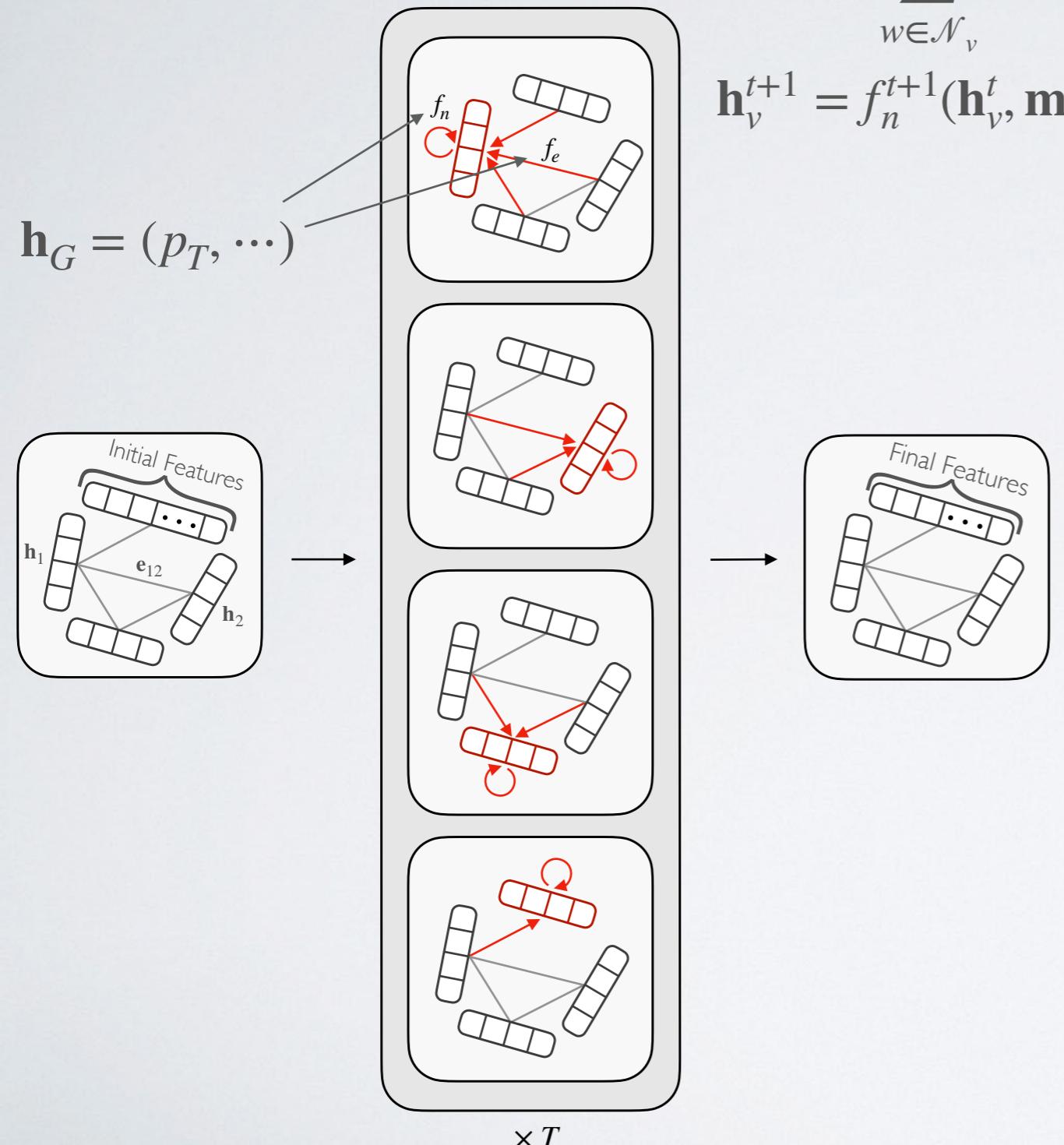
End goal is a generator which takes graph/jet-level variables into account, so we try a modified ‘conditional’ GAN/generalised MPNN:



# GAN ARCHITECTURE

To allow for variable sized graphs we ‘mask’ extra nodes with an additional binary feature:

$$\mathbf{m}_v^{t+1} = \sum_{w \in \mathcal{N}_v} f_e^{t+1}(\mathbf{h}_v^t, \mathbf{h}_w^t, \mathbf{e}_{vw}^t, \mathbf{h}_G)$$
$$\mathbf{h}_v^{t+1} = f_n^{t+1}(\mathbf{h}_v^t, \mathbf{m}_v^{t+1}, \mathbf{h}_G)$$



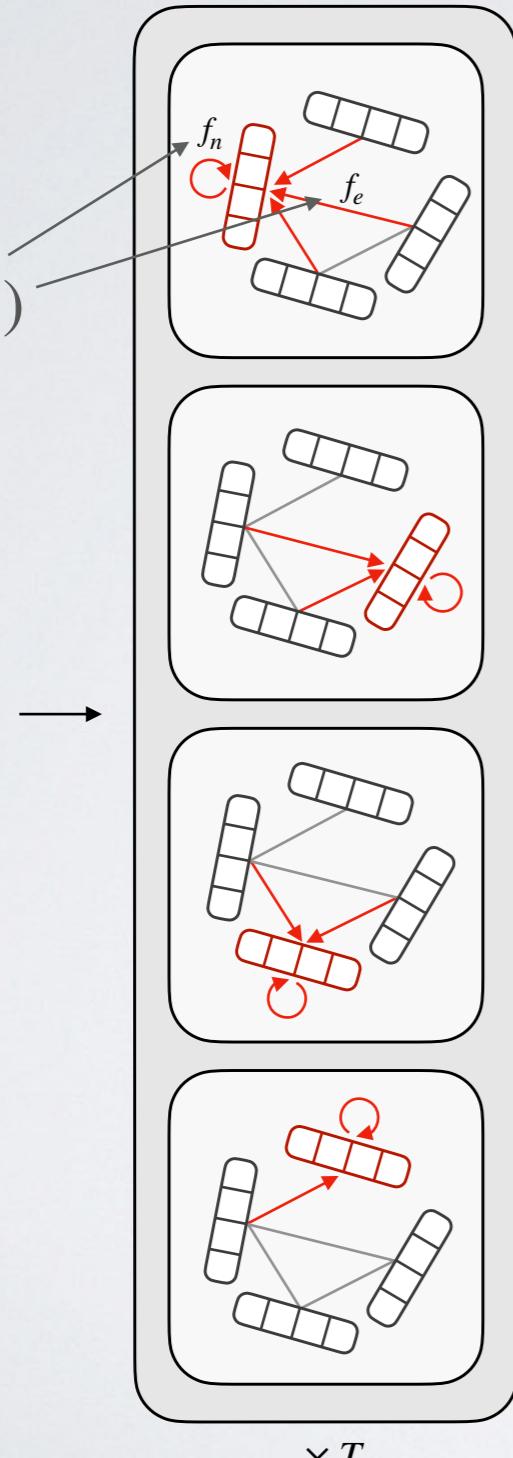
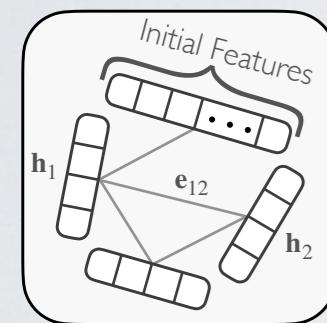
# GAN ARCHITECTURE

To allow for variable sized graphs we ‘mask’ extra nodes with an additional binary feature:

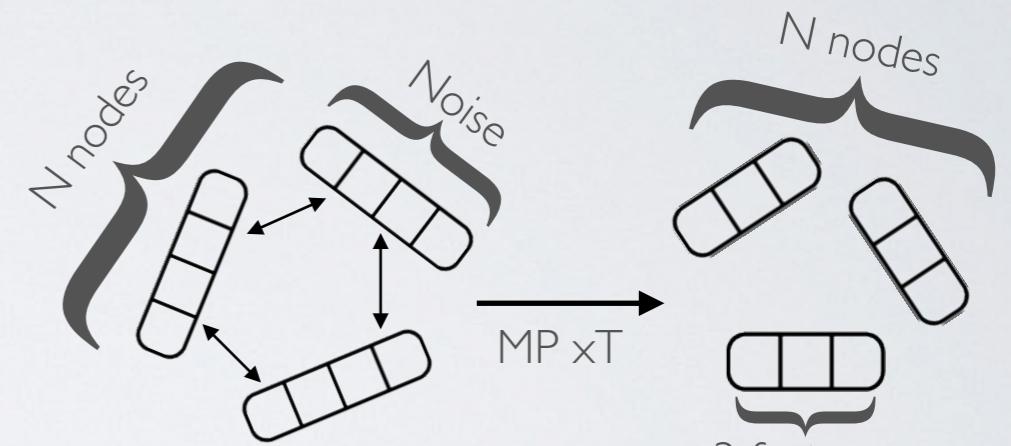
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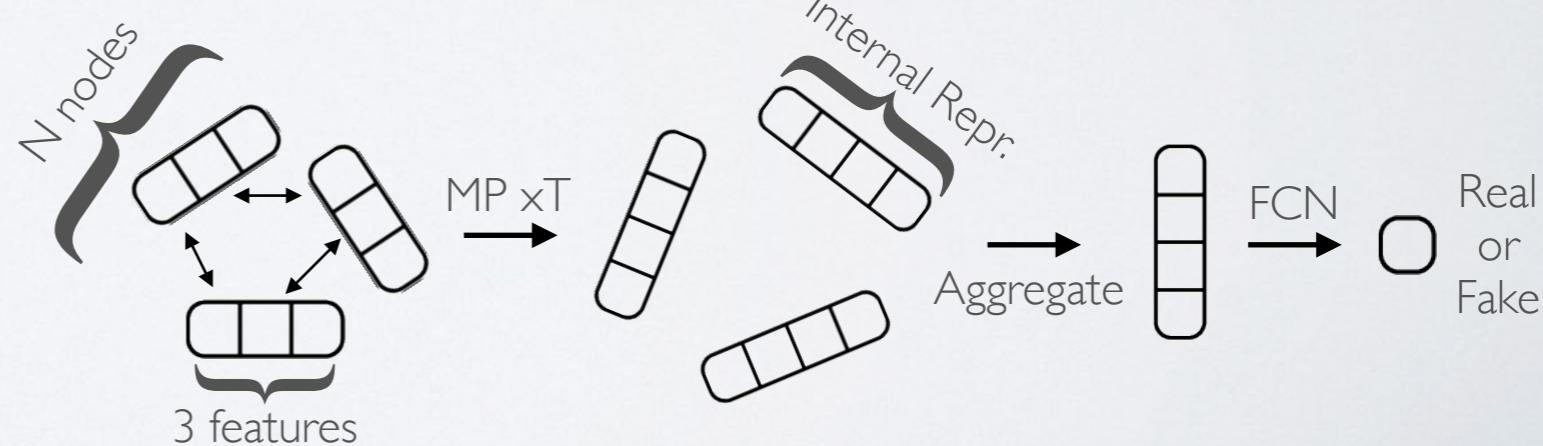
$$\mathbf{h}_G = (p_T, \dots)$$



Generator



Discriminator

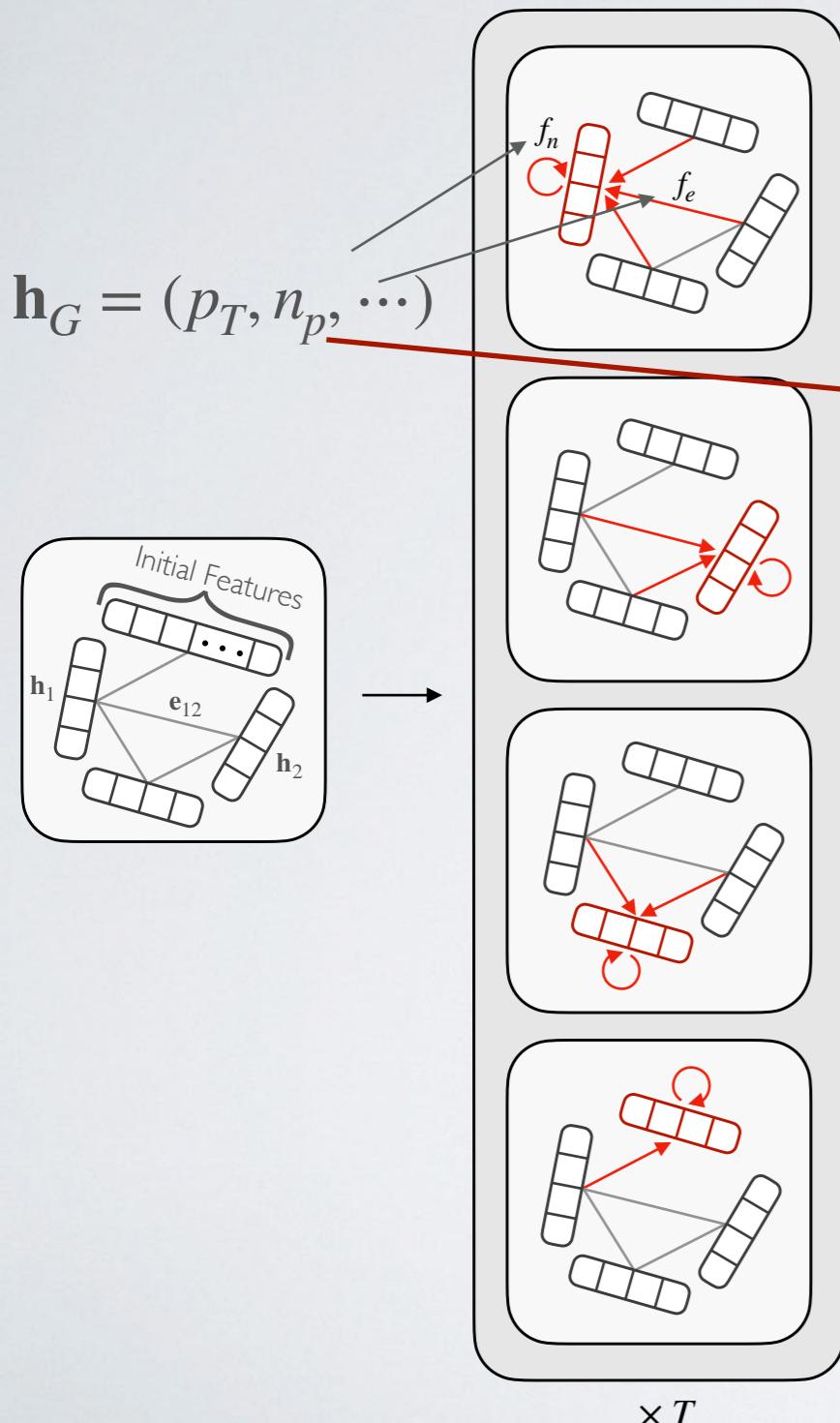


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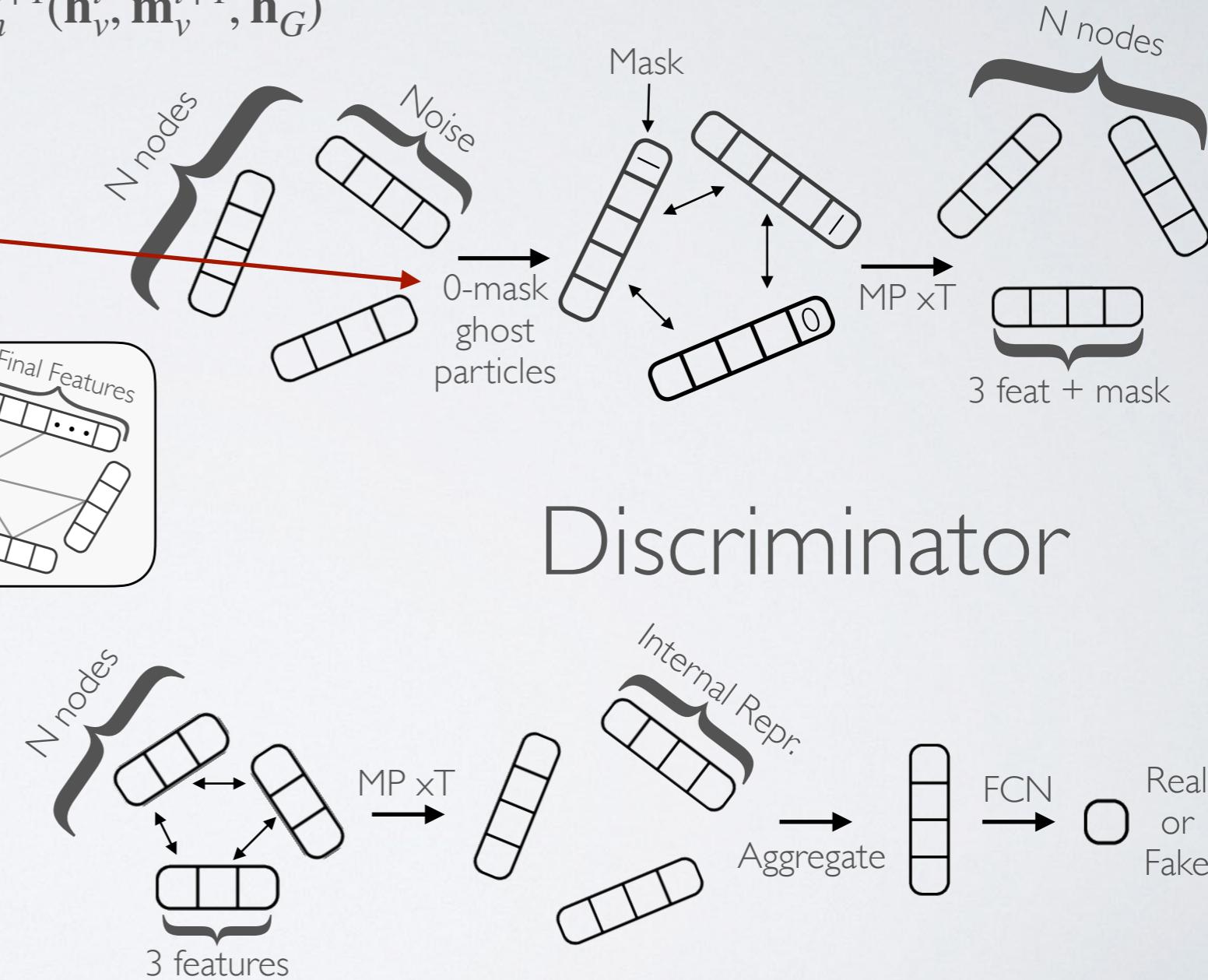
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Generator

Discriminator

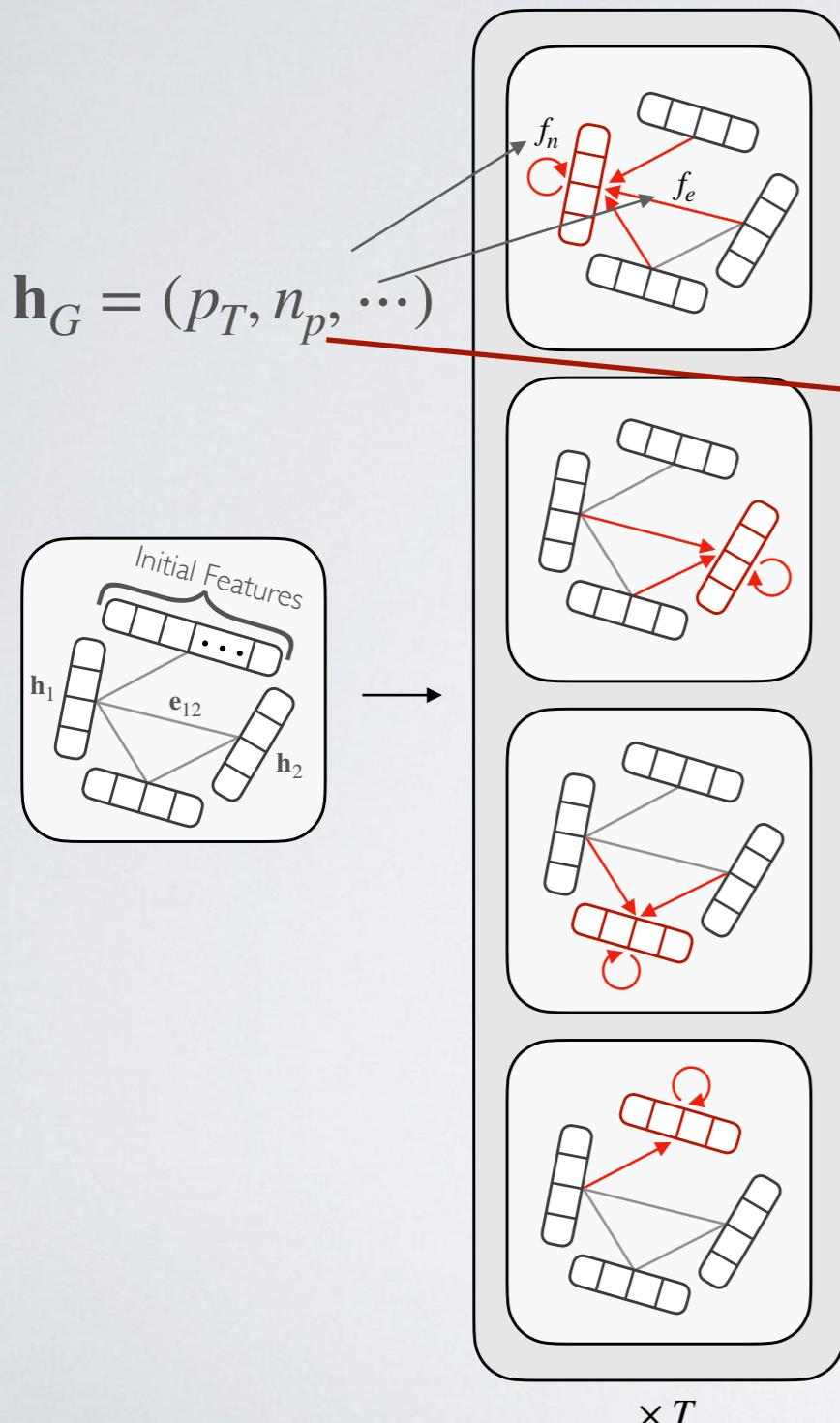


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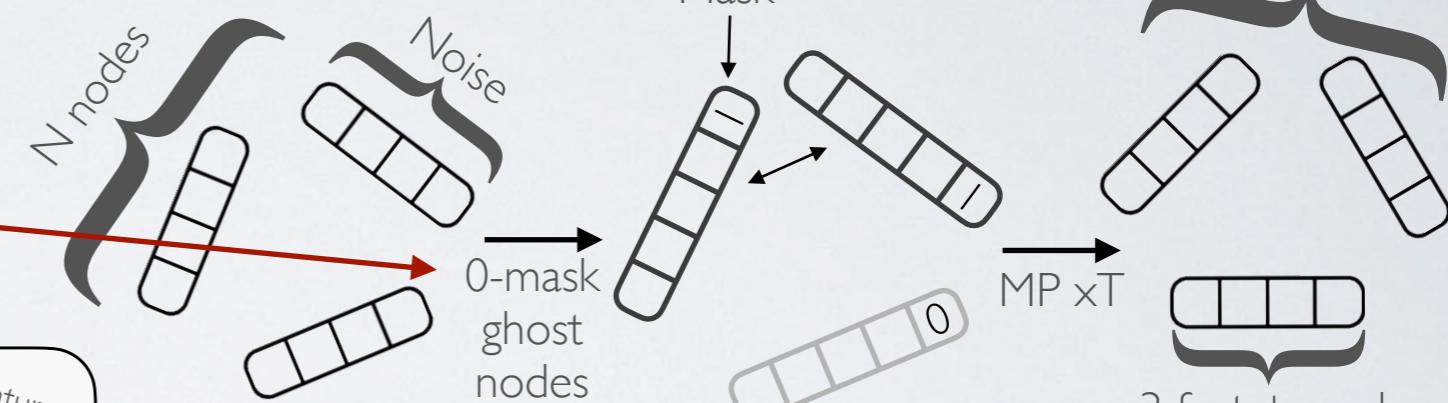
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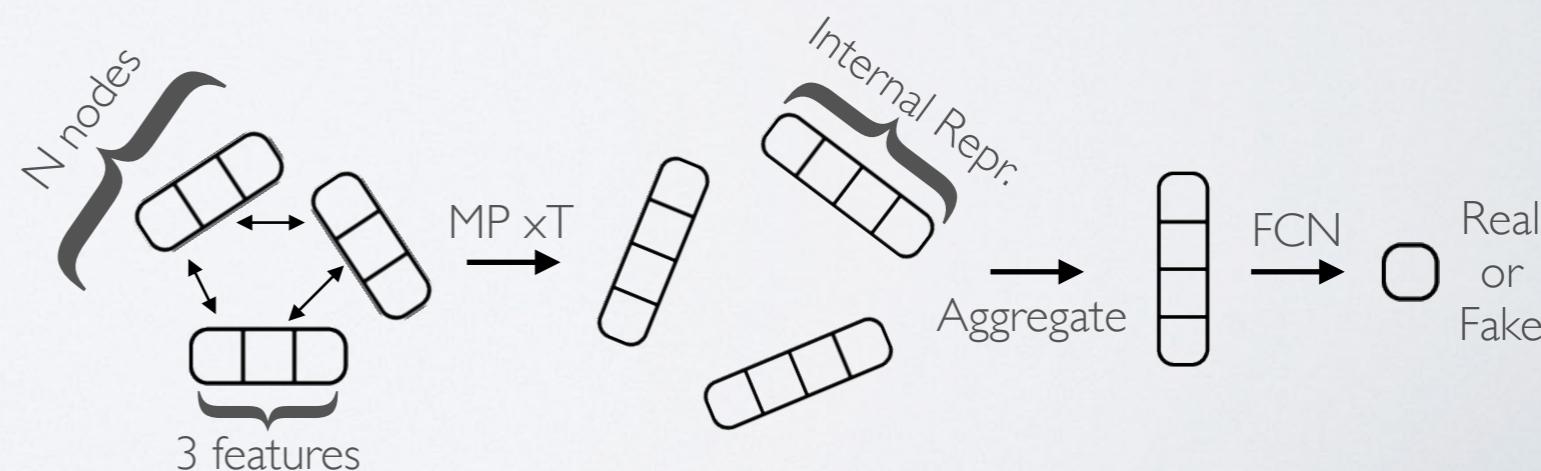
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Generator



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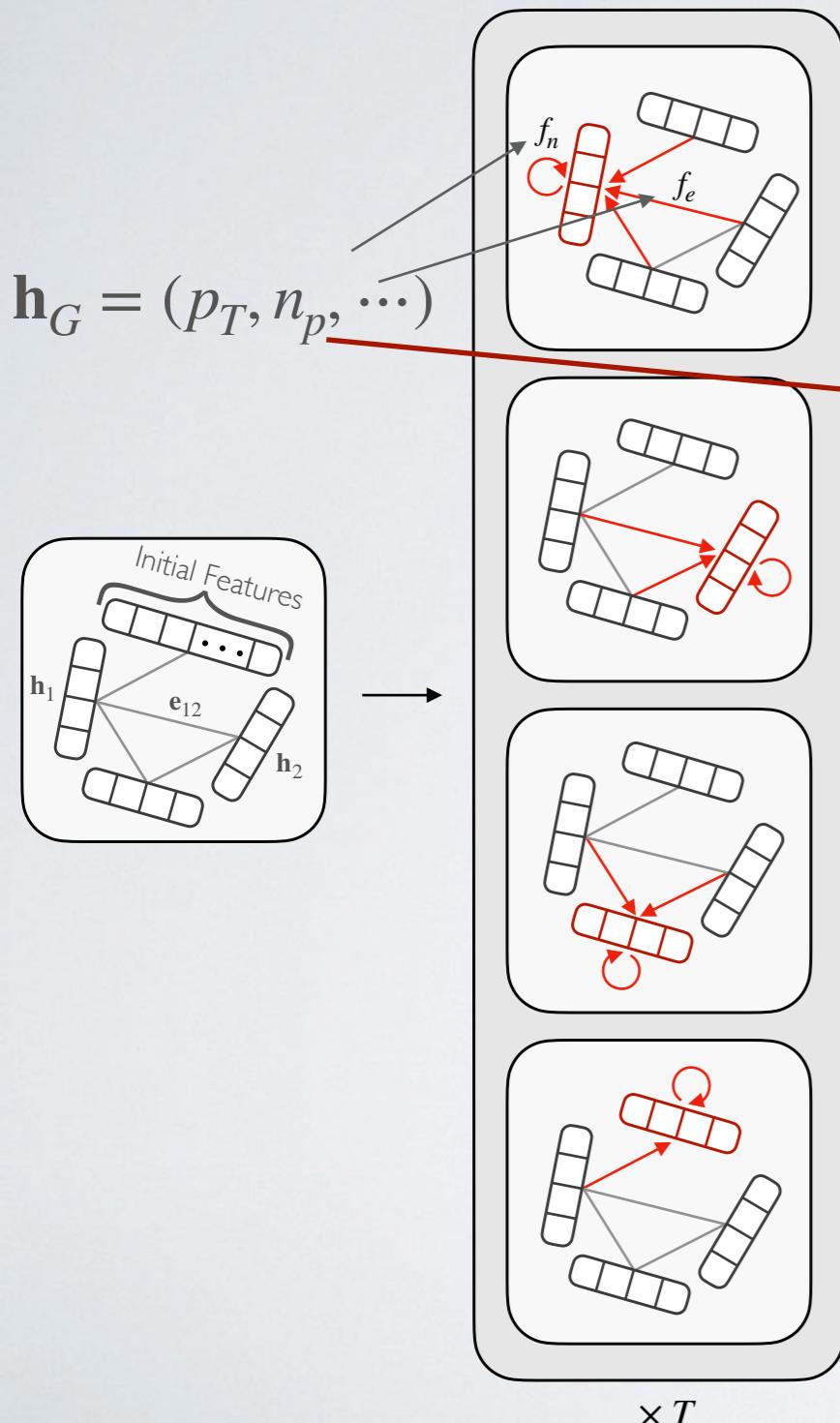


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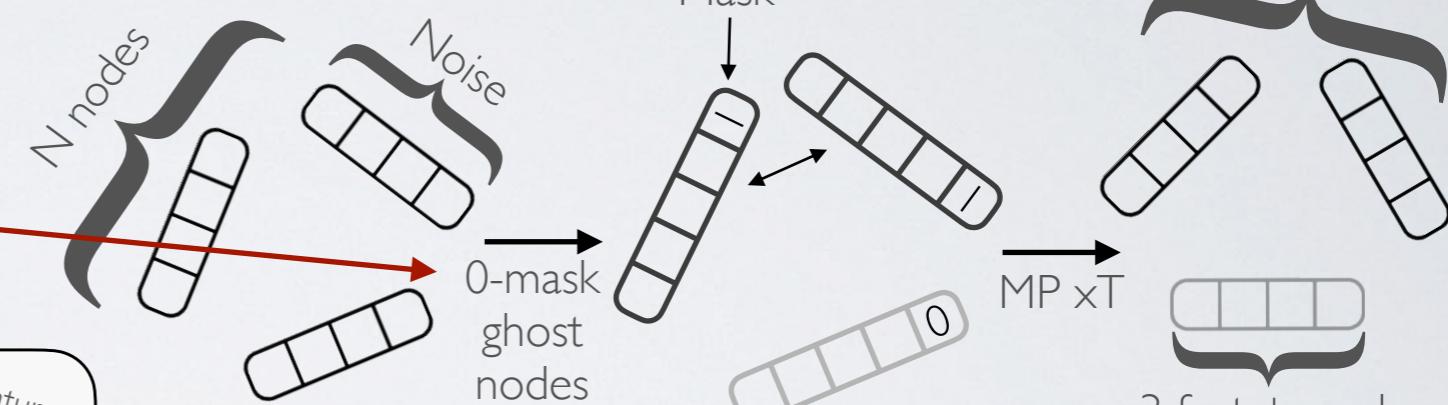
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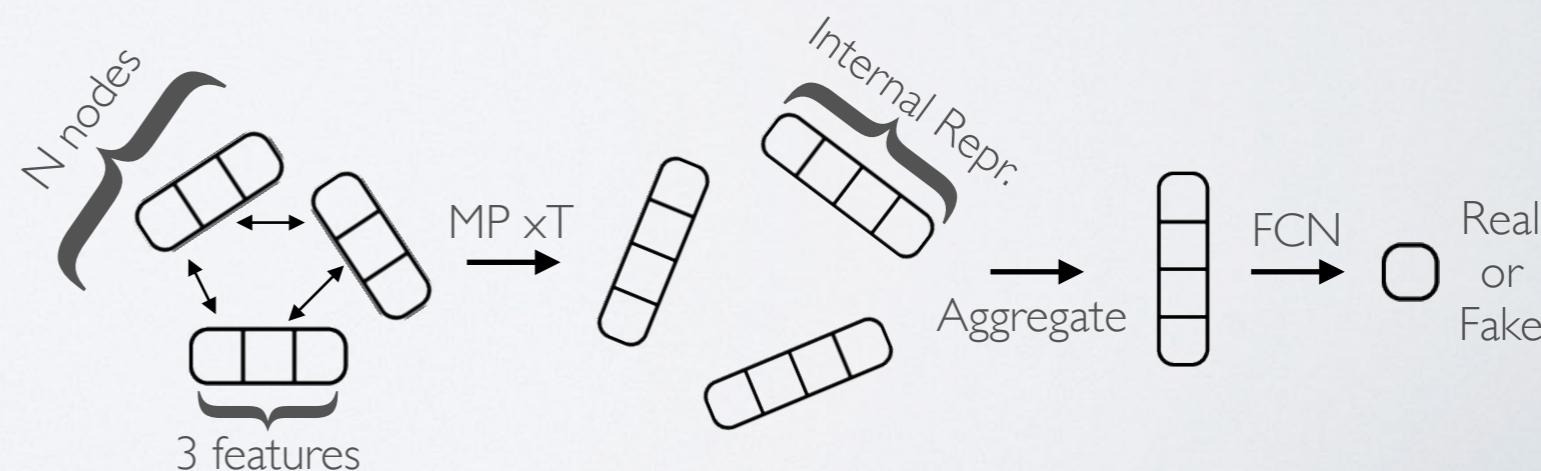
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Generator



Discriminator

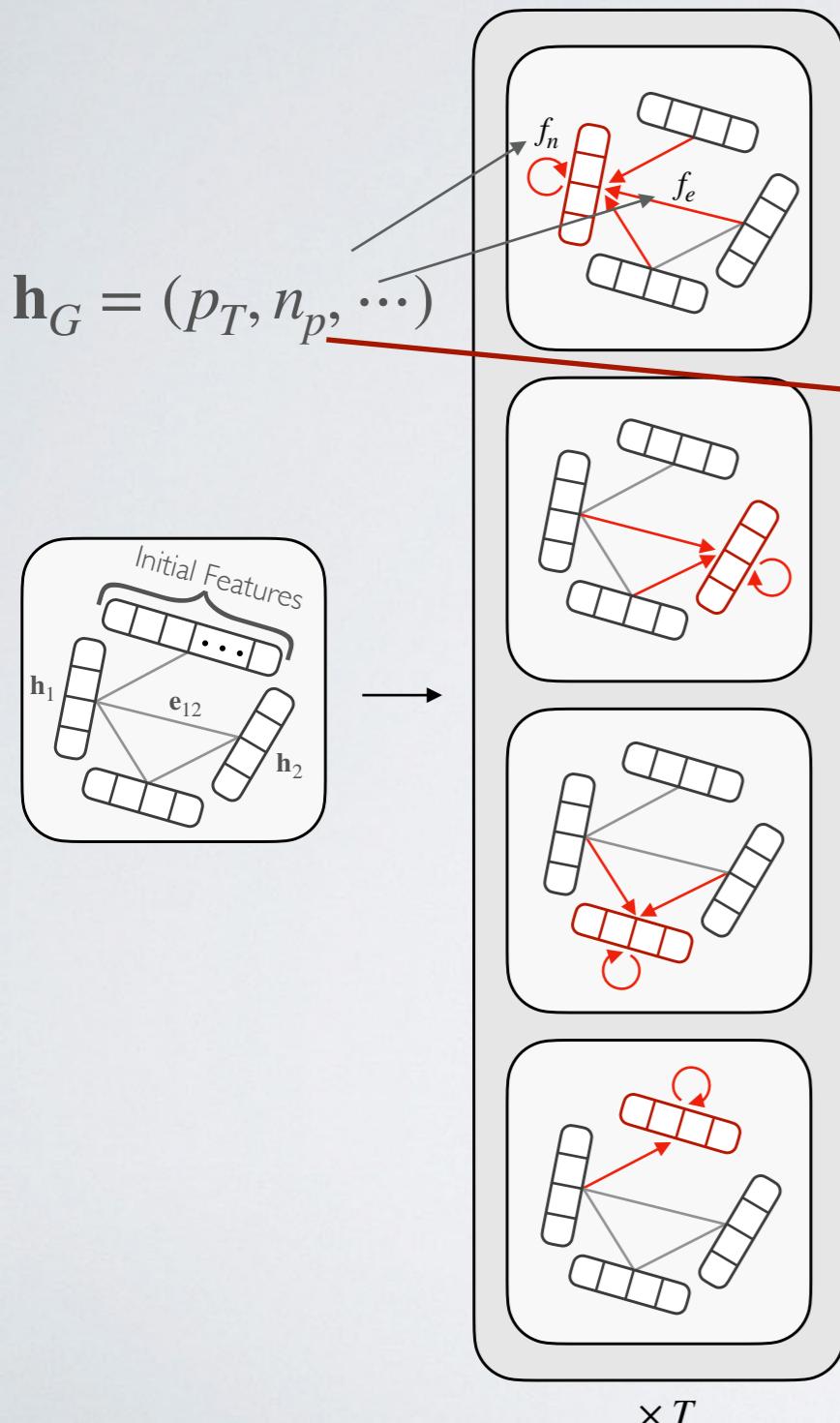


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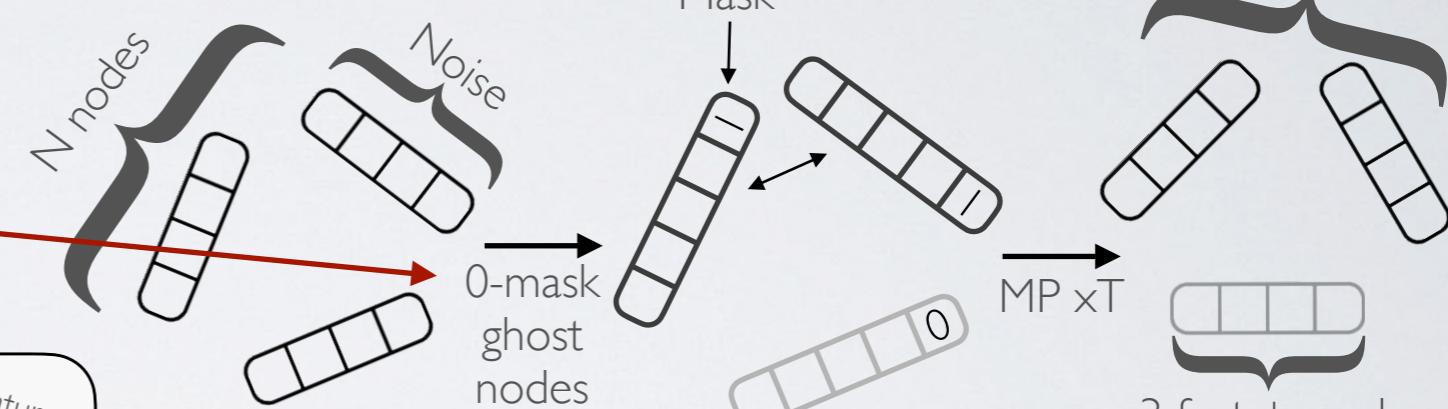
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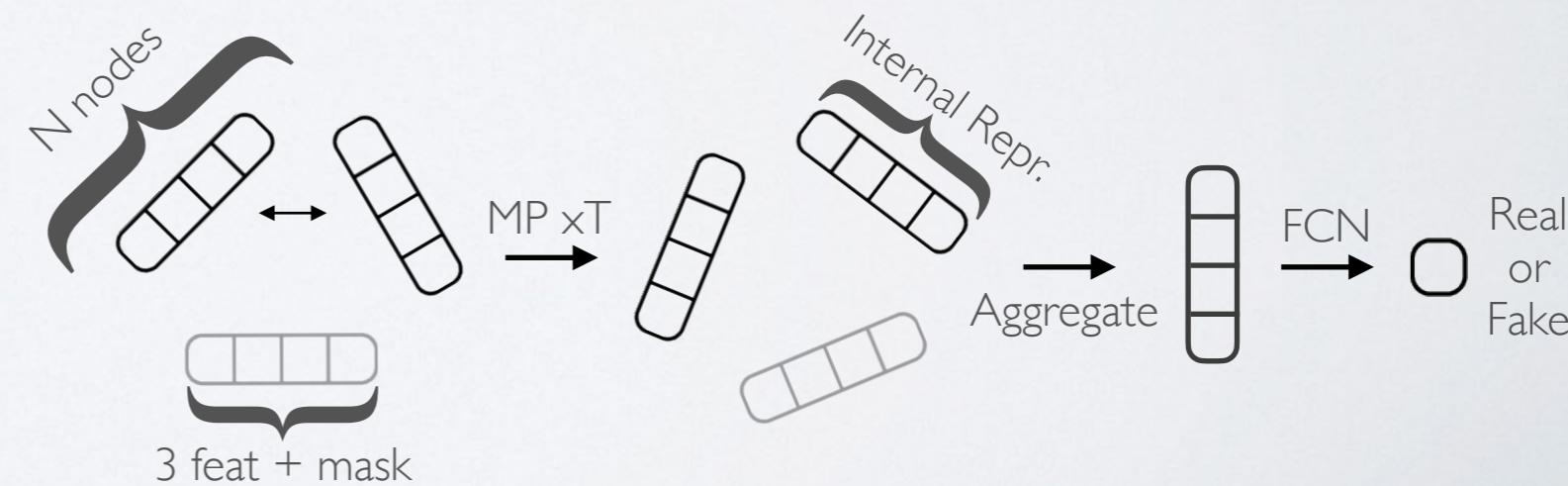
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## Generator



## Discriminator

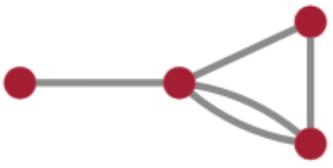


# ENERGY FLOW POLYNOMIALS

$$z_i = \frac{p_{T,i}}{p_{T,J}}, \quad p_{T,J} \equiv \sum_{i=1}^M p_{T,i},$$

$$\theta_{ij} = (\Delta y_{ij}^2 + \Delta \phi_{ij}^2)^{\beta/2},$$

$$\bullet_j \iff \sum_{i_j=1}^M z_{i_j}, \quad k \xrightarrow{\hspace{1cm}} \ell \iff \theta_{i_k i_\ell}.$$



$$= \sum_{i_1=1}^M \sum_{i_2=1}^M \sum_{i_3=1}^M \sum_{i_4=1}^M z_{i_1} z_{i_2} z_{i_3} z_{i_4} \theta_{i_1 i_2} \theta_{i_2 i_3} \theta_{i_2 i_4}^2 \theta_{i_3 i_4}.$$

EFPs form a complete basis for IRC observables

e.g. (relative) jet mass:

$$\frac{m_J^2}{p_{TJ}^2} = \sum_{i_1=1}^M \sum_{i_2=1}^M z_{i_1} z_{i_2} (\cosh(\Delta y_{i_1 i_2}) - \cos(\Delta \phi_{i_1 i_2})) = \frac{1}{2} \times \text{Diagram} + \dots \quad (2.7)$$

Degree	Connected Multigraphs
$d = 0$	
$d = 1$	
$d = 2$	
$d = 3$	
$d = 4$	
$d = 5$	