GoSam 1.0: 
$$\bar{d}d \rightarrow He^+e^-g$$

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#### Abstract

This process consists of 2 tree-level diagrams and 27 NLO diagrams. Golem has identified 6 groups of NLO diagrams by analyzing their one-loop integrals.

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# 1 Helicities

Index	1	2	3	4	5	6
0	+	_	0	+	_	_
1	_	+	0	+	_	_
2	+	_	0	_	+	_
3	_	+	0	_	+	_
$4 \to 0$	+	_	0	+	_	+
$5 \to 1$	_	+	0	+	_	+
$6 \rightarrow 2$	+	_	0	_	+	+
$7 \rightarrow 3$	_	+	0	_	+	+

# 2 Wave Functions

In this section, we use  $l_i = k_i$  for massless particles; in spinors  $|i\rangle$  (resp. |i|) denote  $|l_i\rangle$  (resp.  $|l_i|$ ). For the massive particles we have:

$$l_3 = k_3 - \frac{mH^2}{2k_3 \cdot k_2} k_2 \tag{1}$$

All helicity amplitudes are defined in terms of the following wave functions:

•  $\bar{d}(k_1)$ 

$$\bar{v}_{+}(k_1) = \langle 1| \tag{2}$$

$$\bar{v}_{-}(k_1) = [1] \tag{3}$$

•  $d(k_2)$ 

$$u_+(k_2) = |2\rangle \tag{4}$$

$$u_{-}(k_2) = |2| \tag{5}$$

•  $H(k_3)$ 

$$\epsilon(k_3) = 1 \tag{6}$$

•  $e^+(k_4)$ 

$$v_{+}(k_4) = |4] (7)$$

$$v_{-}(k_4) = |4\rangle \tag{8}$$

•  $e^-(k_5)$ 

$$\bar{u}_{+}(k_{5}) = [5] \tag{9}$$

$$\bar{u}_{-}(k_5) = \langle 5| \tag{10}$$

•  $g(k_6)$ 

$$\varepsilon_{+}^{\mu}(k_{6})^{*} = \frac{[2|\gamma^{\mu}|6\rangle}{\sqrt{2}[2|6]} \tag{11}$$

$$\varepsilon_{-}^{\mu}(k_6)^* = \frac{\langle 2|\gamma^{\mu}|6]}{\sqrt{2}\langle 6|2\rangle} \tag{12}$$

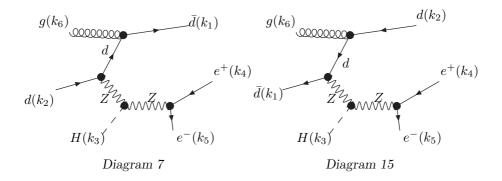
# 3 Colour Basis

$$|c_1\rangle = \bar{q}_{i_1}^{(1)} q_{i_2}^{(2)} g_{(6)}^{A_6} (T^{A_6})_{i_1 i_2}$$
 (13)

# 4 Tree Diagrams

### QGraf Setup

```
qgraf - 3.1.2
output = 'diagrams - 0.hh';
style = 'form.sty';
model = 'model';
in = Dbar[k1], D[k2];
out = H[k3], ep[k4], em[k5], g[k6];
loops=0;
loop_momentum=p;
options=onshell, notadpole, nosnail;
true=iprop[ep,em,ne,nebar,phim,phip,A,H,0,0];
true=vsum[QCD, 1, 1];
  24P ---- 7+ 17- ---- 5N+ 2C+ 17C-
          -3^142 4^27
             -- 0 diagrams
              - 0 diagrams
                 20 diagrams
  total = 20 diagrams
```



# 5 One-Loop Diagrams

# **General Information**

### QGraf Setup

```
qgraf - 3.1.2
output = 'diagrams - 1.hh';
style = 'form.sty';
model = 'model';
in = Dbar[k1], D[k2];
out = H[k3], ep[k4], em[k5], g[k6];
loops=1;
loop_momentum=p;
options=onshell, notadpole, nosnail;
true=iprop[ep,em,ne,nebar,phim,phip,A,H,0,0];
true=vsum[QCD, 3, 3];
            7 + 17 -
                           5N+
                                2C+ 17C-
  169V -
          -3^142 4^27
                 0 diagrams
                 0 diagrams
                 0 diagrams
                 620 diagrams
  total = 620 diagrams
```

Loop diagrams are grouped into sets of diagrams which share loop-propagators. A loop integral can be written as

$$\int \frac{\mathrm{d}^n k}{i\pi^{\frac{n}{2}}} \frac{\mathcal{N}(q)}{\prod_{j=1} N \left[ (k+r_j)^2 - (m_j^2 - im_j \Gamma_j) + i\delta \right]}.$$
 (14)

For each group we list  $r_j$ ,  $m_j$  and  $\Gamma_j$ . For  $m_j$  and  $\Gamma_j$  only non-vanishing symbols are listed. Furthermore, we give the matrix S which is defined as

$$S_{\alpha\beta} = (r_{\alpha} - r_{\beta})^2 - (m_{\alpha}^2 - im_{\alpha}\Gamma_{\alpha}) - (m_{\beta}^2 - im_{\beta}\Gamma_{\beta}). \tag{15}$$

For each diagram we denote how the matrix S' for the specific diagram is obtained from the original S. The notation

$$S' = S_{Q \to q'}^{\{l_1, l_2, \dots\}} \tag{16}$$

means, that the rows and columns labeled by  $l_1, l_2, \ldots$  should be removed from S (likewise  $r_{l_1}, r_{l_2}, \ldots$  are removed from the list of propagators) and  $\mathcal{N}(q)$  has to be replaced by  $\mathcal{N}(q')$ . The maximum effective rank of a group is the rank that has to be passed to SAMURAI if the whole group is reduced at once; this number is calculated as

$$\max_{\text{diagrams}} \{ (\text{rank of diagram}) + (\text{number of pinches}) \}.$$
 (17)

Diagrams with massless closed quark lines are multiplied by a factor Nfrat = Nf/Nfgen. This multiplication is indicated by the symbol  $N_f$  following the rank. By default Nfrat evaluates to one but can be changed by modifying Nf or Nfgen in the model file.

# 5.1 Group 0 (4-Point)

#### General Information

The maximum effective rank in this group is 3.

$$r_1 = -k_2 \tag{18a}$$

$$r_2 = 0 ag{18b}$$

$$r_3 = -k_6 \tag{18c}$$

$$r_4 = k_3 - k_2 + k_5 + k_4 \tag{18d}$$

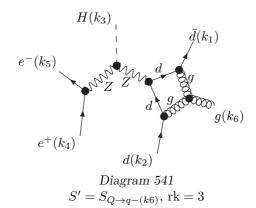
$$S = \begin{pmatrix} 0 & 0 & S_{1,3} & S_{1,4} \\ 0 & 0 & 0 & S_{2,4} \\ S_{3,1} & 0 & 0 & 0 \\ S_{4,1} & S_{4,2} & 0 & 0 \end{pmatrix}$$
 (19)

$$S_{1,3} = s_{345} - s_{61} - s_{12} (20a)$$

$$S_{1,4} = s_{345} (20b)$$

$$S_{2.4} = s_{61} \tag{20c}$$

#### 5.1.1 Diagrams (1)



#### Group 1 (4-Point) 5.2

# General Information

The maximum effective rank in this group is 4.

$$r_1 = -k_6 \tag{21a}$$

$$r_2 = 0 (21b)$$

$$r_3 = -k_2$$
 (21c)  
 $r_4 = -k_3 - k_6 - k_5 - k_4$  (21d)

$$r_4 = -k_3 - k_6 - k_5 - k_4 \tag{21d}$$

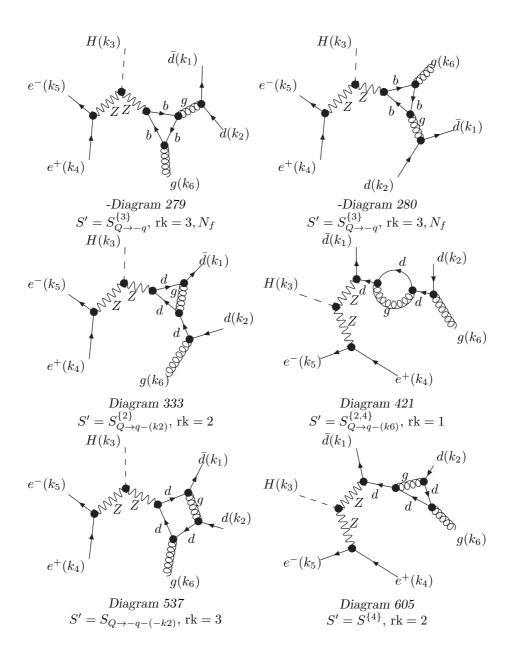
$$S = \begin{pmatrix} 0 & 0 & S_{1,3} & S_{1,4} \\ 0 & 0 & 0 & S_{2,4} \\ S_{3,1} & 0 & 0 & 0 \\ S_{4,1} & S_{4,2} & 0 & 0 \end{pmatrix}$$
 (22)

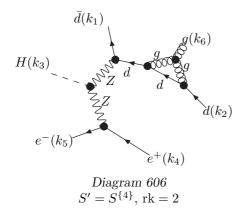
$$S_{1,3} = s_{345} - s_{61} - s_{12} (23a)$$

$$S_{1,4} = s_{345} \tag{23b}$$

$$S_{2,4} = s_{12} (23c)$$

# 5.2.1 Diagrams (7)





# 5.3 Group 2 (4-Point)

# General Information

The maximum effective rank in this group is 3.

$$r_1 = -k_3 - k_5 - k_4 \tag{24a}$$

$$r_2 = -k_3 - k_6 - k_5 - k_4 \tag{24b}$$

$$r_3 = -k_2 \tag{24c}$$

$$r_4 = 0 (24d)$$

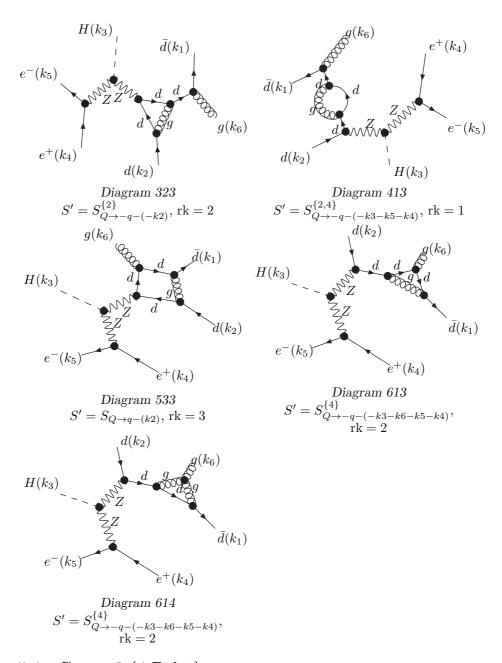
$$S = \begin{pmatrix} 0 & 0 & S_{1,3} & S_{1,4} \\ 0 & 0 & 0 & S_{2,4} \\ S_{3,1} & 0 & 0 & 0 \\ S_{4,1} & S_{4,2} & 0 & 0 \end{pmatrix}$$
 (25)

$$S_{1,3} = s_{61}$$
 (26a)

$$S_{1,4} = s_{345} (26b)$$

$$S_{2,4} = s_{12} (26c)$$

# 5.3.1 Diagrams (5)



#### Group 3 (4-Point) **5.4**

### General Information

The maximum effective rank in this group is 4.

$$r_1 = -k_3 - k_5 - k_4, \quad m_1 = m_t$$
 (27a)  
 $r_2 = -k_3, \quad m_2 = m_t$  (27b)  
 $r_3 = 0, \quad m_3 = m_t$  (27c)

$$r_2 = -k_3, \quad m_2 = m_t$$
 (27b)

$$r_3 = 0, \quad m_3 = m_t$$
 (27c)

$$r_4 = k_6, \quad m_4 = m_t$$
 (27d)

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix}$$

$$(28)$$

$$S_{1,1} = -2m_t^2 (29a)$$

$$S_{1,2} = -2m_t^2 + s_{45} (29b)$$

$$S_{1,3} = s_{345} - 2m_t^2 (29c)$$

$$S_{1,4} = -2m_t^2 + s_{12} (29d)$$

$$S_{2,2} = -2m_t^2 (29e)$$

$$S_{2,3} = -2m_t^2 + m_H^2 (29f)$$

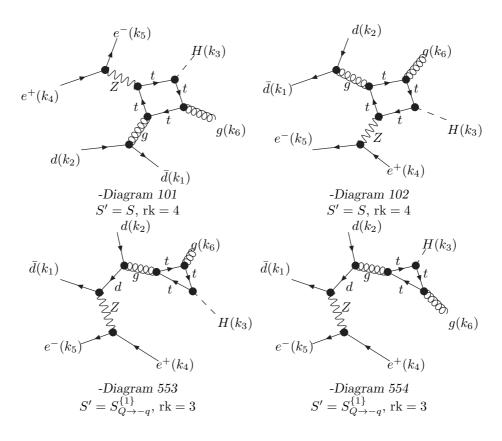
$$S_{2,4} = -2m_t^2 - s_{345} + m_H^2 + s_{45} + s_{12} - s_{123}$$
 (29g)

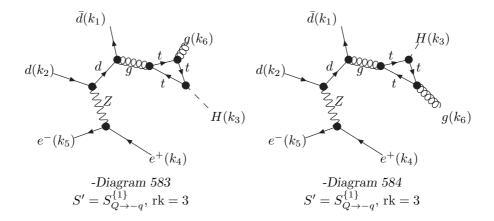
$$S_{3,3} = -2m_t^2 (29h)$$

$$S_{3,4} = -2m_t^2 (29i)$$

$$S_{4,4} = -2m_t^2 (29j)$$

# 5.4.1 Diagrams (6)





# 5.5 Group 4 (4-Point)

#### **General Information**

The maximum effective rank in this group is 4.

$$r_1 = k_6 + k_5 + k_4, \quad m_1 = m_t$$
 (30a)

$$r_2 = k_6, \quad m_2 = m_t$$
 (30b)

$$r_3 = 0, \quad m_3 = m_t$$
 (30c)

$$r_4 = -k_3, \quad m_4 = m_t$$
 (30d)

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix}$$
(31)

$$S_{1,1} = -2m_t^2 (32a)$$

$$S_{1,2} = -2m_t^2 + s_{45} (32b)$$

$$S_{1,3} = s_{123} - 2m_t^2 (32c)$$

$$S_{1,4} = -2m_t^2 + s_{12} (32d)$$

$$S_{2,2} = -2m_t^2 (32e)$$

$$S_{2,3} = -2m_t^2 (32f)$$

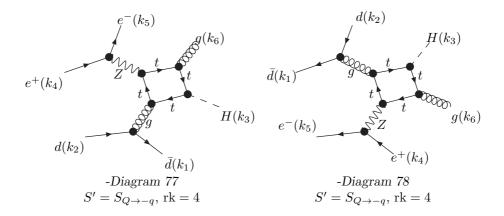
$$S_{2,4} = -2m_t^2 - s_{345} + m_H^2 + s_{45} + s_{12} - s_{123}$$
 (32g)

$$S_{3,3} = -2m_t^2 (32h)$$

$$S_{3,4} = -2m_t^2 + m_H^2 (32i)$$

$$S_{4,4} = -2m_t^2 (32j)$$

### 5.5.1 Diagrams (2)



# 5.6 Group 5 (4-Point)

# General Information

The maximum effective rank in this group is 4.

$$r_1 = k_6 + k_5 + k_4, \quad m_1 = m_t$$
 (33a)

$$r_2 = k_5 + k_4, \quad m_2 = m_t$$
 (33b)

$$r_3 = 0, \quad m_3 = m_t$$
 (33c)

$$r_4 = -k_3, \quad m_4 = m_t$$
 (33d)

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix}$$
(34)

$$S_{1,1} = -2m_t^2 (35a)$$

$$S_{1,2} = -2m_t^2 (35b)$$

$$S_{1,3} = s_{123} - 2m_t^2 (35c)$$

$$S_{1,4} = -2m_t^2 + s_{12} (35d)$$

$$S_{2,2} = -2m_t^2 (35e)$$

$$S_{2,3} = -2m_t^2 + s_{45} (35f)$$

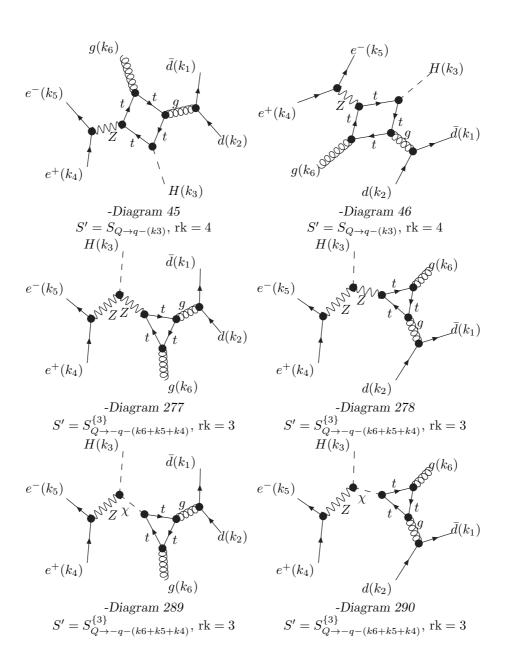
$$S_{2,4} = s_{345} - 2m_t^2 (35g)$$

$$S_{3,3} = -2m_t^2 (35h)$$

$$S_{3,4} = -2m_t^2 + m_H^2 (35i)$$

$$S_{4,4} = -2m_t^2 (35j)$$

### 5.6.1 Diagrams (6)



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### 6 Related Work

If you publish results obtained by using this matrix element code please cite the appropriate papers in the bibliography of this document.

Scientific publications prepared using the present version of GoSAM or any modified version of it or any code linking to GoSAM or parts of it should make a clear reference to the publication [15].

For graph generation we use QGraf [1]. The Feynman diagrams are further processed with the symbolic manipulation program FORM [2] using the FORM library SPINNEY [3]. The Fortran 90 code is generated using HAGGIES [4]. For the reduction of the tensor integrals the code uses the implementation of the OPP method [5, 6] and extensions thereof from the package SAMURAI [7]. For the reduction of the tensor integrals, the code uses the package GOLEM95 [8, 9]. The tensor coefficients are obtained using tensorial reconstruction at the integrand level [14].

Please, make sure, you also give credit to the authors of the scalar loop libraries, if you configured the amplitude code such that it calls other libraries than the ones mentioned so far. Depending on your configuration you might use one or more of the following programs for the evaluation of the scalar integrals:

- OneLOop [10],
- QCDLoop [11], which uses FF [12],
- LoopTools [13], which uses FF [12].
- GOLEM95 [8] which uses OneLOop [10] and may be configured such that it uses LoopTools [13, 12].

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