

GoSam 1.0:  $\bar{u}u \rightarrow He^+e^-g$

gionata

2013-05-14 (19:26:34)

**Abstract**

This process consists of 2 tree-level diagrams and 27 NLO diagrams. Golem has identified 6 groups of NLO diagrams by analyzing their one-loop integrals.

# Contents

<b>1</b>	<b>Helicities</b>	<b>3</b>
<b>2</b>	<b>Wave Functions</b>	<b>3</b>
<b>3</b>	<b>Colour Basis</b>	<b>4</b>
<b>4</b>	<b>Tree Diagrams</b>	<b>4</b>
<b>5</b>	<b>One-Loop Diagrams</b>	<b>5</b>
5.1	Group 0 (4-Point) . . . . .	6
5.1.1	Diagrams (1) . . . . .	6
5.2	Group 1 (4-Point) . . . . .	7
5.2.1	Diagrams (7) . . . . .	7
5.3	Group 2 (4-Point) . . . . .	9
5.3.1	Diagrams (5) . . . . .	9
5.4	Group 3 (4-Point) . . . . .	10
5.4.1	Diagrams (6) . . . . .	11
5.5	Group 4 (4-Point) . . . . .	12
5.5.1	Diagrams (2) . . . . .	12
5.6	Group 5 (4-Point) . . . . .	13
5.6.1	Diagrams (6) . . . . .	13
<b>6</b>	<b>Related Work</b>	<b>14</b>

## 1 Helicities

Index	1	2	3	4	5	6
0	+	−	0	+	−	−
1	−	+	0	+	−	−
2	+	−	0	−	+	−
3	−	+	0	−	+	−
4 → 0	+	−	0	+	−	+
5 → 1	−	+	0	+	−	+
6 → 2	+	−	0	−	+	+
7 → 3	−	+	0	−	+	+

## 2 Wave Functions

In this section, we use  $l_i = k_i$  for massless particles; in spinors  $|i\rangle$  (resp.  $|i]$ ) denote  $|l_i\rangle$  (resp.  $|l_i]$ ). For the massive particles we have:

$$l_3 = k_3 - \frac{mH^2}{2k_3 \cdot k_2} k_2 \quad (1)$$

All helicity amplitudes are defined in terms of the following wave functions:

- $\bar{u}(k_1)$

$$\bar{v}_+(k_1) = \langle 1| \quad (2)$$

$$\bar{v}_-(k_1) = [1| \quad (3)$$

- $u(k_2)$

$$u_+(k_2) = |2\rangle \quad (4)$$

$$u_-(k_2) = |2] \quad (5)$$

- $H(k_3)$

$$\epsilon(k_3) = 1 \quad (6)$$

- $e^+(k_4)$

$$v_+(k_4) = |4] \quad (7)$$

$$v_-(k_4) = |4\rangle \quad (8)$$

- $e^-(k_5)$

$$\bar{u}_+(k_5) = [5| \quad (9)$$

$$\bar{u}_-(k_5) = \langle 5| \quad (10)$$

- $g(k_6)$

$$\varepsilon_+^\mu(k_6)^* = \frac{[2|\gamma^\mu|6\rangle}{\sqrt{2}[2|6]} \quad (11)$$

$$\varepsilon_-^\mu(k_6)^* = \frac{\langle 2|\gamma^\mu|6]}{\sqrt{2}\langle 6|2]} \quad (12)$$

### 3 Colour Basis

$$|c_1\rangle = \bar{q}_{i_1}^{(1)} q_{i_2}^{(2)} g_{(6)}^{A_6} (T^{A_6})_{i_1 i_2} \quad (13)$$

### 4 Tree Diagrams

#### QGraf Setup

---

qgraf - 3.1.2

---

```
output = 'diagrams-0.hh';
style = 'form.sty';
model = 'model';
in = Ubar[k1], U[k2];
out = H[k3], ep[k4], em[k5], g[k6];
loops=0;
loop_momentum=p;
options=onshell, notadpole, nosnail;
true=iprop[ep,em,ne,nebar,phim,phip,A,H,0,0];
true=vsum[QCD,1,1];
```

---

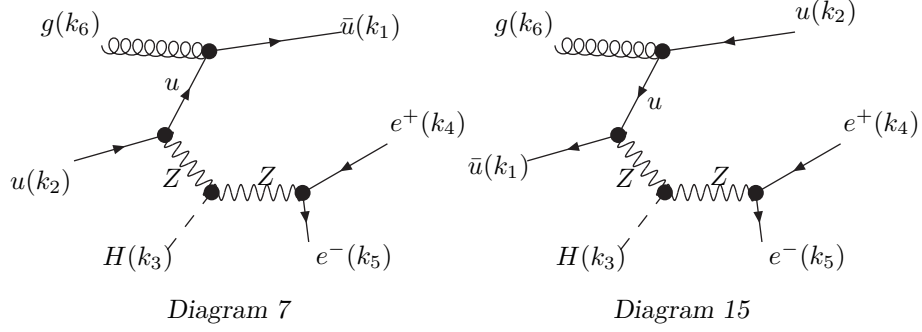
24P — 7+ 17- — 5N+ 2C+ 17C-

169V — 3^142 4^27

---

-	4^2	—	0 diagrams
3^2	4^1	—	0 diagrams
3^4	-	—	20 diagrams

total = 20 diagrams



## 5 One-Loop Diagrams

### General Information

#### QGraf Setup

qgraf-3.1.2

```
output = 'diagrams-1.hh';
style = 'form.sty';
model = 'model';
in = Ubar[k1], U[k2];
out = H[k3], ep[k4], em[k5], g[k6];
loops=1;
loop_momentum=p;
options=onshell, notadpole, nosnail;
true=iprop[ep,em,ne,nebar,phim,phip,A,H,0,0];
true=vsum[QCD,3,3];
```

24P — 7+ 17- — 5N+ 2C+ 17C-

169V — 3^142 4^27

— 4^3 — 0 diagrams  
3^2 4^2 — 0 diagrams  
3^4 4^1 — 0 diagrams  
3^6 — — 620 diagrams

total = 620 diagrams

Loop diagrams are grouped into sets of diagrams which share loop-propagators. A loop integral can be written as

$$\int \frac{d^n k}{i\pi^{\frac{n}{2}}} \frac{\mathcal{N}(q)}{\prod_{j=1}^N [(k+r_j)^2 - (m_j^2 - im_j\Gamma_j) + i\delta]}. \quad (14)$$

For each group we list  $r_j$ ,  $m_j$  and  $\Gamma_j$ . For  $m_j$  and  $\Gamma_j$  only non-vanishing symbols are listed. Furthermore, we give the matrix  $S$  which is defined as

$$S_{\alpha\beta} = (r_\alpha - r_\beta)^2 - (m_\alpha^2 - im_\alpha\Gamma_\alpha) - (m_\beta^2 - im_\beta\Gamma_\beta). \quad (15)$$

For each diagram we denote how the matrix  $S'$  for the specific diagram is obtained from the original  $S$ . The notation

$$S' = S_{Q \rightarrow q'}^{\{l_1, l_2, \dots\}} \quad (16)$$

means, that the rows and columns labeled by  $l_1, l_2, \dots$  should be removed from  $S$  (likewise  $r_{l_1}, r_{l_2}, \dots$  are removed from the list of propagators) and  $\mathcal{N}(q)$  has to be replaced by  $\mathcal{N}(q')$ . The maximum effective rank of a group is the rank that has to be passed to SAMURAI if the whole group is reduced at once; this number is calculated as

$$\max_{\text{diagrams}} \{(\text{rank of diagram}) + (\text{number of pinches})\}. \quad (17)$$

Diagrams with massless closed quark lines are multiplied by a factor **Nfrat** = **Nf**/**Nfgen**. This multiplication is indicated by the symbol  $N_f$  following the rank. By default **Nfrat** evaluates to one but can be changed by modifying **Nf** or **Nfgen** in the model file.

## 5.1 Group 0 (4-Point)

### General Information

The maximum effective rank in this group is 3.

$$r_1 = -k_2 \quad (18a)$$

$$r_2 = 0 \quad (18b)$$

$$r_3 = -k_6 \quad (18c)$$

$$r_4 = k_3 - k_2 + k_5 + k_4 \quad (18d)$$

$$S = \begin{pmatrix} 0 & 0 & S_{1,3} & S_{1,4} \\ 0 & 0 & 0 & S_{2,4} \\ S_{3,1} & 0 & 0 & 0 \\ S_{4,1} & S_{4,2} & 0 & 0 \end{pmatrix} \quad (19)$$

$$S_{1,3} = s_{345} - s_{61} - s_{12} \quad (20a)$$

$$S_{1,4} = s_{345} \quad (20b)$$

$$S_{2,4} = s_{61} \quad (20c)$$

#### 5.1.1 Diagrams (1)

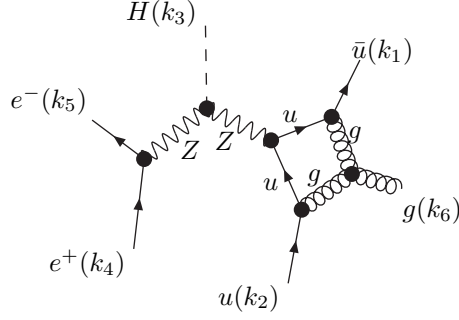


Diagram 541  
 $S' = S_{Q \rightarrow q-(k_6)}, \text{rk} = 3$

## 5.2 Group 1 (4-Point)

### General Information

The maximum effective rank in this group is 4.

$$r_1 = -k_6 \quad (21a)$$

$$r_2 = 0 \quad (21b)$$

$$r_3 = -k_2 \quad (21c)$$

$$r_4 = -k_3 - k_6 - k_5 - k_4 \quad (21d)$$

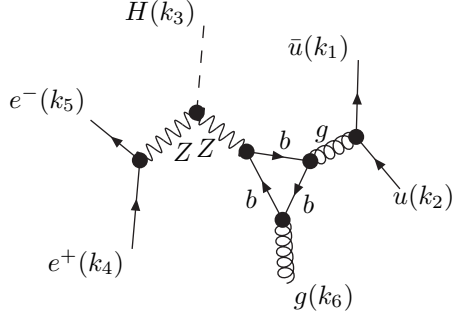
$$S = \begin{pmatrix} 0 & 0 & S_{1,3} & S_{1,4} \\ 0 & 0 & 0 & S_{2,4} \\ S_{3,1} & 0 & 0 & 0 \\ S_{4,1} & S_{4,2} & 0 & 0 \end{pmatrix} \quad (22)$$

$$S_{1,3} = s_{345} - s_{61} - s_{12} \quad (23a)$$

$$S_{1,4} = s_{345} \quad (23b)$$

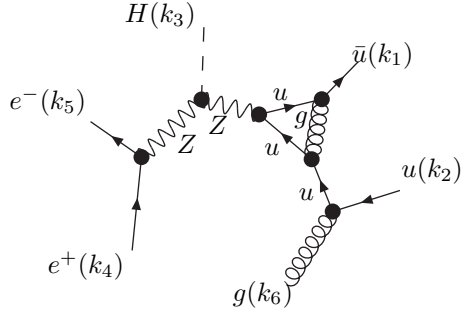
$$S_{2,4} = s_{12} \quad (23c)$$

### 5.2.1 Diagrams (7)



-Diagram 279

$$S' = S_{Q \rightarrow -q}^{\{3\}}, \text{rk} = 3, N_f$$



-Diagram 280

$$S' = S_{Q \rightarrow -q}^{\{3\}}, \text{rk} = 3, N_f$$

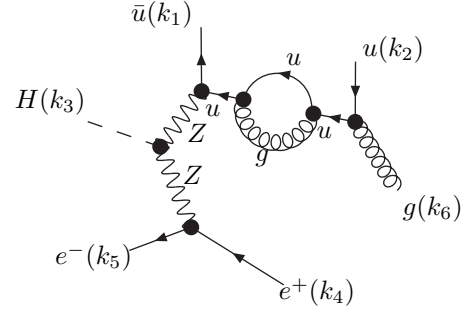


Diagram 333

$$S' = S_{Q \rightarrow q-(k2)}^{\{2\}}, \text{rk} = 2$$

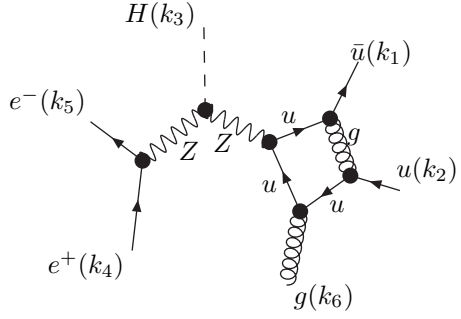


Diagram 421

$$S' = S_{Q \rightarrow q-(k6)}^{\{2,4\}}, \text{rk} = 1$$

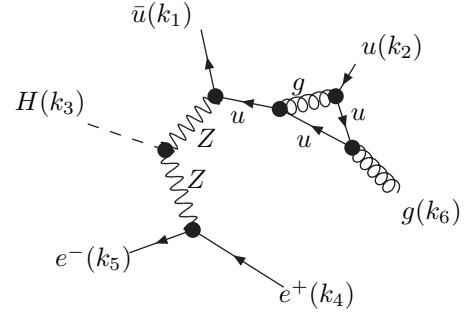


Diagram 537

$$S' = S_{Q \rightarrow -q-(-k2)}, \text{rk} = 3$$

Diagram 605

$$S' = S^{\{4\}}, \text{rk} = 2$$



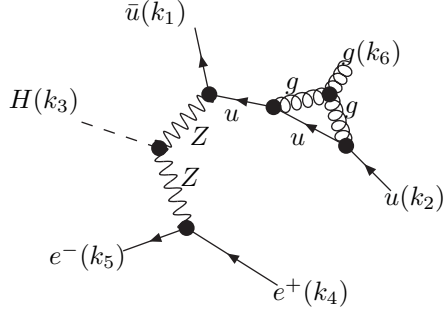


Diagram 606  
 $S' = S^{\{4\}}$ ,  $\text{rk} = 2$

### 5.3 Group 2 (4-Point)

#### General Information

The maximum effective rank in this group is 3.

$$r_1 = -k_3 - k_5 - k_4 \quad (24a)$$

$$r_2 = -k_3 - k_6 - k_5 - k_4 \quad (24b)$$

$$r_3 = -k_2 \quad (24c)$$

$$r_4 = 0 \quad (24d)$$

$$S = \begin{pmatrix} 0 & 0 & S_{1,3} & S_{1,4} \\ 0 & 0 & 0 & S_{2,4} \\ S_{3,1} & 0 & 0 & 0 \\ S_{4,1} & S_{4,2} & 0 & 0 \end{pmatrix} \quad (25)$$

$$S_{1,3} = s_{61} \quad (26a)$$

$$S_{1,4} = s_{345} \quad (26b)$$

$$S_{2,4} = s_{12} \quad (26c)$$

#### 5.3.1 Diagrams (5)

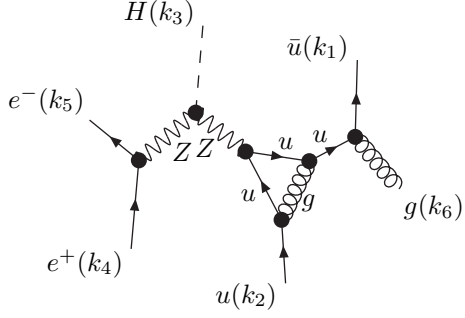


Diagram 323

$$S' = S_{Q \rightarrow q-(-k_2)}^{\{2\}}, \text{rk} = 2$$

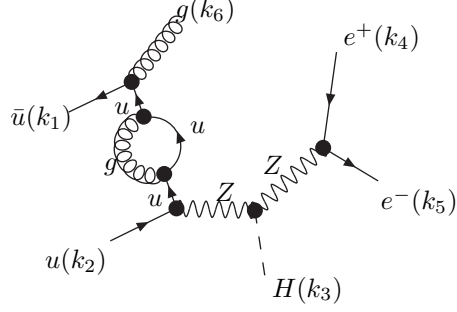


Diagram 413

$$S' = S_{Q \rightarrow q-(-k_3-k_5-k_4)}^{\{2,4\}}, \text{rk} = 1$$

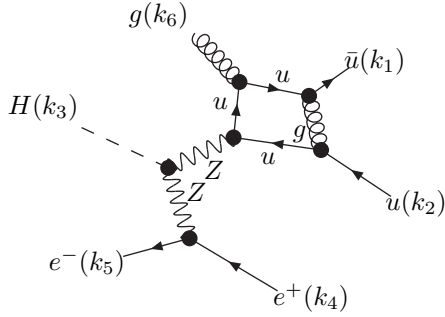


Diagram 533

$$S' = S_{Q \rightarrow q-(k_2)}, \text{rk} = 3$$

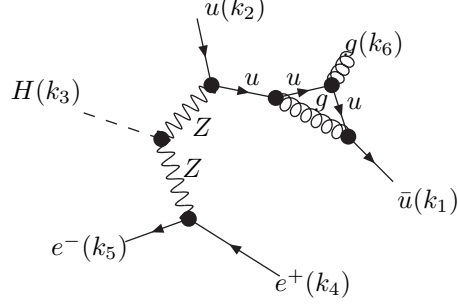


Diagram 613

$$S' = S_{Q \rightarrow q-(-k_3-k_6-k_5-k_4)}^{\{4\}}, \text{rk} = 2$$

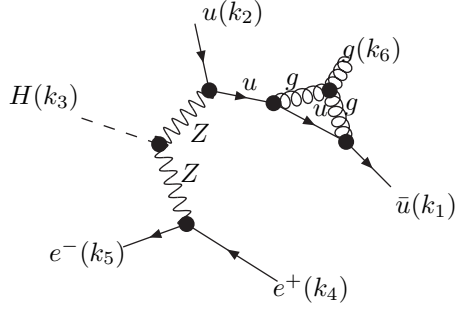


Diagram 614

$$S' = S_{Q \rightarrow q-(-k_3-k_6-k_5-k_4)}^{\{4\}}, \text{rk} = 2$$

## 5.4 Group 3 (4-Point)

### General Information

The maximum effective rank in this group is 4.

$$r_1 = -k_3 - k_5 - k_4, \quad m_1 = m_t \quad (27a)$$

$$r_2 = -k_3, \quad m_2 = m_t \quad (27b)$$

$$r_3 = 0, \quad m_3 = m_t \quad (27c)$$

$$r_4 = k_6, \quad m_4 = m_t \quad (27d)$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix} \quad (28)$$

$$S_{1,1} = -2m_t^2 \quad (29a)$$

$$S_{1,2} = -2m_t^2 + s_{45} \quad (29b)$$

$$S_{1,3} = s_{345} - 2m_t^2 \quad (29c)$$

$$S_{1,4} = -2m_t^2 + s_{12} \quad (29d)$$

$$S_{2,2} = -2m_t^2 \quad (29e)$$

$$S_{2,3} = -2m_t^2 + m_H^2 \quad (29f)$$

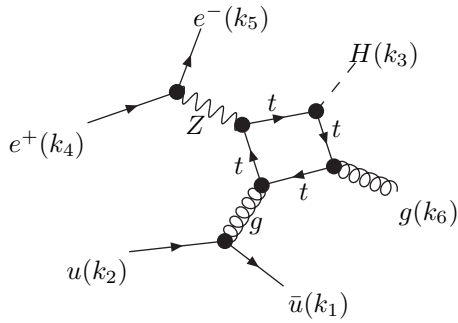
$$S_{2,4} = -2m_t^2 - s_{345} + m_H^2 + s_{45} + s_{12} - s_{123} \quad (29g)$$

$$S_{3,3} = -2m_t^2 \quad (29h)$$

$$S_{3,4} = -2m_t^2 \quad (29i)$$

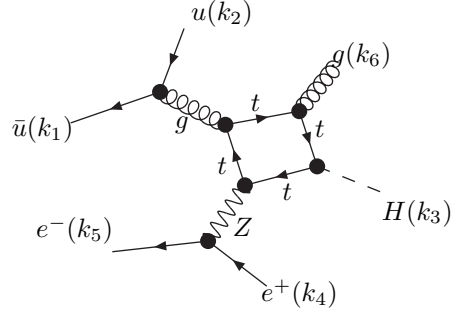
$$S_{4,4} = -2m_t^2 \quad (29j)$$

#### 5.4.1 Diagrams (6)



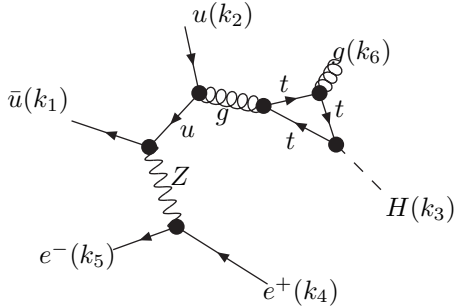
-Diagram 101

$S' = S, \text{rk} = 4$



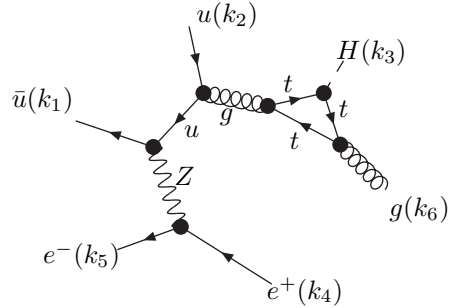
-Diagram 102

$S' = S, \text{rk} = 4$



-Diagram 553

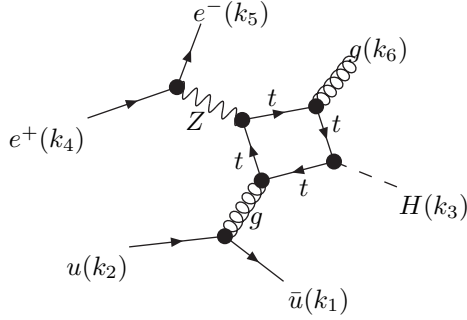
$S' = S_{Q \rightarrow -q}^{\{1\}}, \text{rk} = 3$



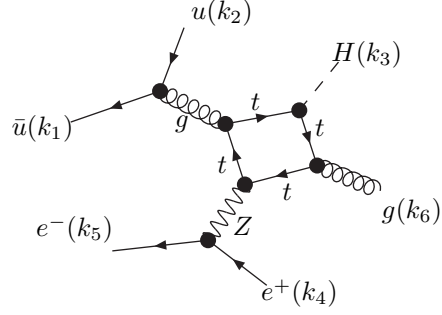
-Diagram 554

$S' = S_{Q \rightarrow -q}^{\{1\}}, \text{rk} = 3$





-Diagram 77  
 $S' = S_{Q \rightarrow -q}, \text{rk} = 4$



-Diagram 78  
 $S' = S_{Q \rightarrow -q}, \text{rk} = 4$

## 5.6 Group 5 (4-Point)

### General Information

The maximum effective rank in this group is 4.

$$r_1 = k_6 + k_5 + k_4, \quad m_1 = m_t \quad (33a)$$

$$r_2 = k_5 + k_4, \quad m_2 = m_t \quad (33b)$$

$$r_3 = 0, \quad m_3 = m_t \quad (33c)$$

$$r_4 = -k_3, \quad m_4 = m_t \quad (33d)$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix} \quad (34)$$

$$S_{1,1} = -2m_t^2 \quad (35a)$$

$$S_{1,2} = -2m_t^2 \quad (35b)$$

$$S_{1,3} = s_{123} - 2m_t^2 \quad (35c)$$

$$S_{1,4} = -2m_t^2 + s_{12} \quad (35d)$$

$$S_{2,2} = -2m_t^2 \quad (35e)$$

$$S_{2,3} = -2m_t^2 + s_{45} \quad (35f)$$

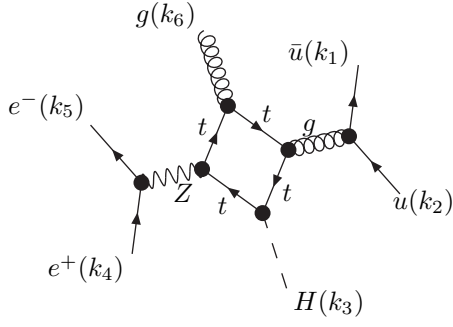
$$S_{2,4} = s_{345} - 2m_t^2 \quad (35g)$$

$$S_{3,3} = -2m_t^2 \quad (35h)$$

$$S_{3,4} = -2m_t^2 + m_H^2 \quad (35i)$$

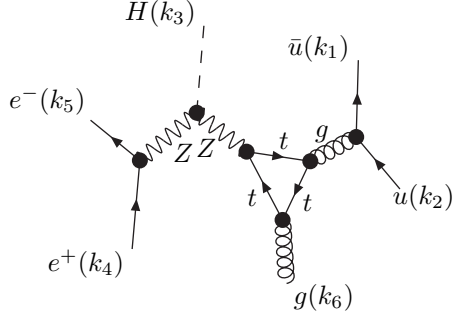
$$S_{4,4} = -2m_t^2 \quad (35j)$$

### 5.6.1 Diagrams (6)



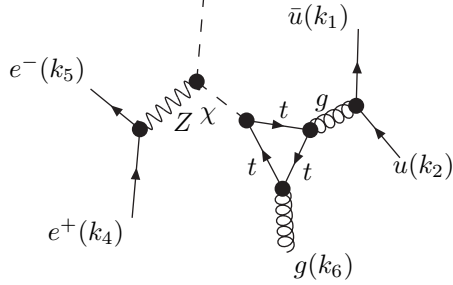
-Diagram 45

$$S' = S_{Q \rightarrow q-(k_3)}, \text{rk} = 4$$



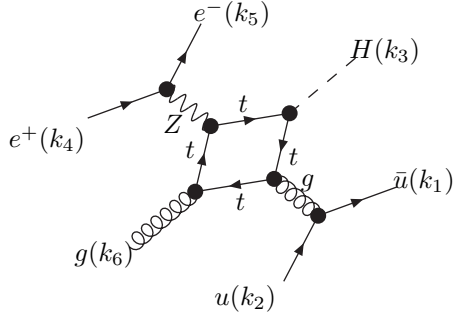
-Diagram 277

$$S' = S_{Q \rightarrow -q-(k_6+k_5+k_4)}^{\{3\}}, \text{rk} = 3$$



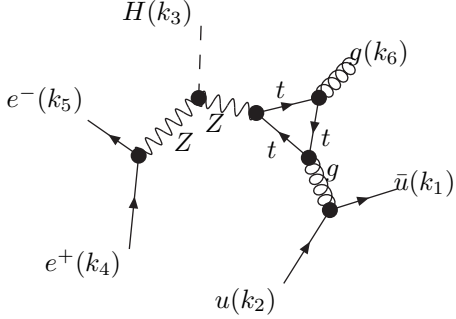
-Diagram 289

$$S' = S_{Q \rightarrow -q-(k_6+k_5+k_4)}^{\{3\}}, \text{rk} = 3$$



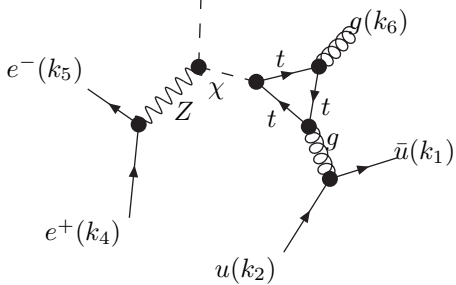
-Diagram 46

$$S' = S_{Q \rightarrow q-(k_3)}, \text{rk} = 4$$



-Diagram 278

$$S' = S_{Q \rightarrow -q-(k_6+k_5+k_4)}^{\{3\}}, \text{rk} = 3$$



-Diagram 290

$$S' = S_{Q \rightarrow -q-(k_6+k_5+k_4)}^{\{3\}}, \text{rk} = 3$$

## Index of all Loop Diagrams

Diagram 45 (Group 5).....	14
Diagram 46 (Group 5).....	14
Diagram 77 (Group 4).....	13
Diagram 78 (Group 4).....	13
Diagram 101 (Group 3).....	11
Diagram 102 (Group 3).....	11
Diagram 277 (Group 5).....	14
Diagram 278 (Group 5).....	14
Diagram 279 (Group 1).....	8
Diagram 280 (Group 1).....	8
Diagram 289 (Group 5).....	14
Diagram 290 (Group 5).....	14
Diagram 323 (Group 2).....	10
Diagram 333 (Group 1).....	8
Diagram 413 (Group 2).....	10
Diagram 421 (Group 1).....	8
Diagram 533 (Group 2).....	10
Diagram 537 (Group 1).....	8
Diagram 541 (Group 0).....	7
Diagram 553 (Group 3).....	11
Diagram 554 (Group 3).....	11
Diagram 583 (Group 3).....	12
Diagram 584 (Group 3).....	12
Diagram 605 (Group 1).....	8
Diagram 606 (Group 1).....	9
Diagram 613 (Group 2).....	10
Diagram 614 (Group 2).....	10

## 6 Related Work

If you publish results obtained by using this matrix element code please cite the appropriate papers in the bibliography of this document.

Scientific publications prepared using the present version of GOSAM or any modified version of it or any code linking to GOSAM or parts of it should make a clear reference to the publication [15].

For graph generation we use QGraf [1]. The Feynman diagrams are further processed with the symbolic manipulation program FORM [2] using the FORM library SPINNEY [3]. The Fortran 90 code is generated using HAGGIES [4]. For the reduction of the tensor integrals the code uses the implementation of the OPP method [5, 6] and extensions thereof from the package SAMURAI [7]. For the reduction of the tensor integrals, the code uses the package GOLEM95 [8, 9]. The tensor coefficients are obtained using tensorial reconstruction at the integrand level [14].

Please, make sure, you also give credit to the authors of the scalar loop libraries, if you configured the amplitude code such that it calls other libraries than the ones mentioned so far. Depending on your configuration you might use one or more of the following programs for the evaluation of the scalar integrals:

- OneLOop [10],
- QCDDLoop [11], which uses FF [12],
- LoopTools [13], which uses FF [12].
- GOLEM95 [8] which uses OneLOop [10] and may be configured such that it uses LoopTools [13, 12].

## References

- [1] P. Nogueira, “Automatic Feynman graph generation,” *J. Comput. Phys.* **105** (1993) 279.
- [2] J. A. M. Vermaseren, “New features of FORM,” *arXiv:math-ph/0010025*.
- [3] G. Cullen, M. Koch-Janusz and T. Reiter, “spinney: A Form Library for Helicity Spinors,” *arXiv:1008.0803 [hep-ph]*.
- [4] T. Reiter, “Optimising Code Generation with haggies,” *Comput. Phys. Commun.* **181** (2010) 1301 [*arXiv:0907.3714 [hep-ph]*].
- [5] G. Ossola, C. G. Papadopoulos, R. Pittau, “Reducing full one-loop amplitudes to scalar integrals at the integrand level,” *Nucl. Phys.* **B763** (2007) 147-169. [*hep-ph/0609007*].
- [6] G. Ossola, C. G. Papadopoulos, R. Pittau, “Numerical evaluation of six-photon amplitudes,” *JHEP* **0707** (2007) 085. [*arXiv:0704.1271 [hep-ph]*].
- [7] P. Mastrolia, G. Ossola, T. Reiter and F. Tramontano, “Scattering Amplitudes from Unitarity-based Reduction Algorithm at the Integrand-level,” *arXiv:1006.0710 [Unknown]*.



- [8] T. Binoth, J. P. Guillet, G. Heinrich, E. Pilon and T. Reiter, *Comput. Phys. Commun.* **180** (2009) 2317 [arXiv:0810.0992 [hep-ph]].
- [9] G. Cullen, J. P. Guillet, G. Heinrich, T. Kleinschmidt, E. Pilon, T. Reiter, M. Rodgers, *Comput. Phys. Commun.* **182** (2011) 2276-2284. [arXiv:1101.5595 [hep-ph]].
- [10] A. van Hameren, “OneLOop: For the evaluation of one-loop scalar functions,” [arXiv:1007.4716 [hep-ph]].
- [11] R. K. Ellis, G. Zanderighi, “Scalar one-loop integrals for QCD,” *JHEP* **0802** (2008) 002. [arXiv:0712.1851 [hep-ph]].
- [12] G. J. van Oldenborgh, “FF: A Package to evaluate one loop Feynman diagrams,” *Comput. Phys. Commun.* **66** (1991) 1-15.
- [13] T. Hahn, M. Perez-Victoria, “Automatized one loop calculations in four-dimensions and D-dimensions,” *Comput. Phys. Commun.* **118** (1999) 153-165. [hep-ph/9807565].
- [14] G. Heinrich, G. Ossola, T. Reiter, F. Tramontano, “Tensorial Reconstruction at the Integrand Level,” *JHEP* **1010** (2010) 105. [arXiv:1008.2441 [hep-ph]].
- [15] G. Cullen *et al.*, “Automated One-Loop Calculations with GoSam,” arXiv:1111.2034 [hep-ph].