QCDLoop

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The f77 library QL, in combination with the library FF, provides a seamless way of evaluating divergent or finite one-loop scalar integrals. We work in the Bjorken-Drell metric so that $l^2 = l_0^2 - l_1^2 - l_2^2 - l_3^2$. As illustrated in Fig. 1 the definition of the integrals is as follows

$$
\begin{aligned}
&I_1^D(m_1^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \, \frac{1}{(l^2 - m_1^2 + i\varepsilon)} \,, \\
&I_2^D(p_1^2; m_1^2, m_2^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int d^D l \, \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l+q_1)^2 - m_2^2 + i\varepsilon)} \,, \\
&I_3^D(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \\
&\times \int d^D l \, \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l+q_1)^2 - m_2^2 + i\varepsilon)((l+q_2)^2 - m_3^2 + i\varepsilon)} \,, \\
&I_4^D(p_1^2, p_2^2, p_3^2, p_4^2; s_{12}, s_{23}; m_1^2, m_2^2, m_3^2, m_4^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \\
&\times \int d^D l \, \frac{1}{(l^2 - m_1^2 + i\varepsilon)((l+q_1)^2 - m_2^2 + i\varepsilon)((l+q_2)^2 - m_3^2 + i\varepsilon)((l+q_3)^2 - m_4^2 + i\varepsilon)} \,,
\end{aligned}
$$

The divergences of the integrals are regulated by dimensional regularization, $D = 4 - 2\epsilon$ and we have removed the overall constant which occurs in D-dimensional integrals

$$
r_\Gamma \equiv \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)} = \frac{1}{\Gamma(1-\epsilon)} + \mathcal{O}(\epsilon^3) = 1 - \epsilon\gamma + \epsilon^2 \left[\frac{\gamma^2}{2} - \frac{\pi^2}{12}\right] + \mathcal{O}(\epsilon^3) \,.
$$

The numerical code returns the three complex coefficients in the Laurent series, for any N-point integral

$$
I_N^D = \frac{a_{-2}}{\epsilon^2} + \frac{a_{-1}}{\epsilon^1} + a_0 \,, \qquad N \leq 4 \,.
$$

As a first thing one should call the ql-initialization subroutine

```
subroutine qlinit
```

which calls the ff-initialization subroutine as well.

The five scalar integrals, (tadpole, bubble, derivative of bubble, triangle, box) can then be evaluated by calling the following four fortran functions. The arguments of those four functions are defined as follows,

```
      double complex function qlI1(m1,mu2,ep)
      double precision msq,musq
      integer ep
C     mi=m(i)^2 is the square of the mass of the propagator i
C     mu2 is the square of the scale mu
C     ep=-2,-1,0 chooses the term in the Laurent series.
      .....

      double complex function qlI2(p1,m1,m2,mu2,ep)
      double precision p1,m1,m2,mu2
      integer ep
!     p1=psq(1) is the squared four-momentum of the external particle i
!     mi=m(i)^2, i=1,2 are the squares of the mass of the propagator i
!     mu2 is the square of the scale mu
!     ep=-2,-1,0 chooses the appropriate term in the Laurent series.
      .....

      double complex function qlI2p(p1,m1,m2,mu2,ep)
      double precision p1,m1,m2,mu2
!     (or double precision p1,mu2 and double complex m1,m2)
      integer ep
!     function returned is the derivative d/p1 qlI2(p1,m1,m2,mu2,ep)
!     p1=psq(1) is the squared four-momentum of the external particle i
!     mi=m(i)^2, i=1,2 are the squares of the mass of the propagator i
!     mu2 is the square of the scale mu
!     ep=-2,-1,0 chooses the appropriate term in the Laurent series.
      .....

      double complex function qlI3(p1,p2,p3,m1,m2,m3,mu2,ep)
      double precision p1,p2,p3,m1,m2,m3,mu
      integer ep
!     pi=p(i)^2, i=1,2,3 are the four-momentum squared of the external lines
!     mi=m(i)^2, i=1,2,3,are the squares of the masses of the internal propagators
!     mu2 is the square of the scale mu
!     ep=-2,-1,0 chooses the term in the Laurent series.
      .....

      double complex function qlI4(p1,p2,p3,p4,s12,s23,m1,m2,m3,m4,mu2,ep)
      double precision p1,p2,p3,p4,s12,s23,m1,m2,m3,m4,mu2
      integer ep
!     pi=p(i)^2, i=1,2,3,4 are the four-momentum squared of the external lines
!     mi=m(i)^2  i=1,2,3,4 are the squares of the masses of the internal propagators
!     sij=(pi+pj)^2 are external invariants
!     mu2 is the square of the scale mu
!     ep=-2,-1,0 chooses the term in the Laurent series.
      .....
```

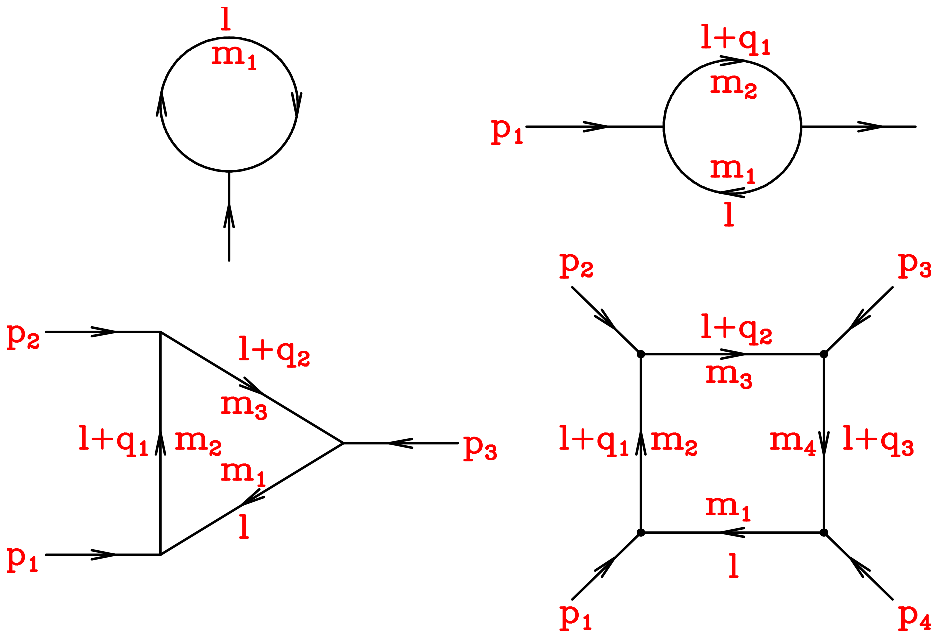

Figure 1: Tadpole, bubble, triangle and box scalar integrals

Thus the call to the function

```
qlI1(10d0,20d0,-1)
```

returns the coefficient of the single pole for the tadpole with internal mass squared $m^2 = 10$ and $\mu^2 = 20$.

A full description of the method and analytic results are given in ref. [1]. The code relies heavily on the library FF, ref. [2]. In addition, some utility routines, (the evaluation of complex logarithms and dilogarithms) have been taken from the looptools distribution, ref. [3]. The header on the file containing those routines indicates that these are adapted from routines of Denner.

References

[1] R. K. Ellis and G. Zanderighi, One-loop Scalar integrals for QCD, Fermilab preprint-PUB-633-T, OUTP-07/16P.

[2] G. J. van Oldenborgh, Comput. Phys. Commun. **66**, 1 (1991).

[3] T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. **118**, 153 (1999) [arXiv:hep-ph/9807565].