Introduction to quantum computing and quantum circuit simulator

2020.1.15

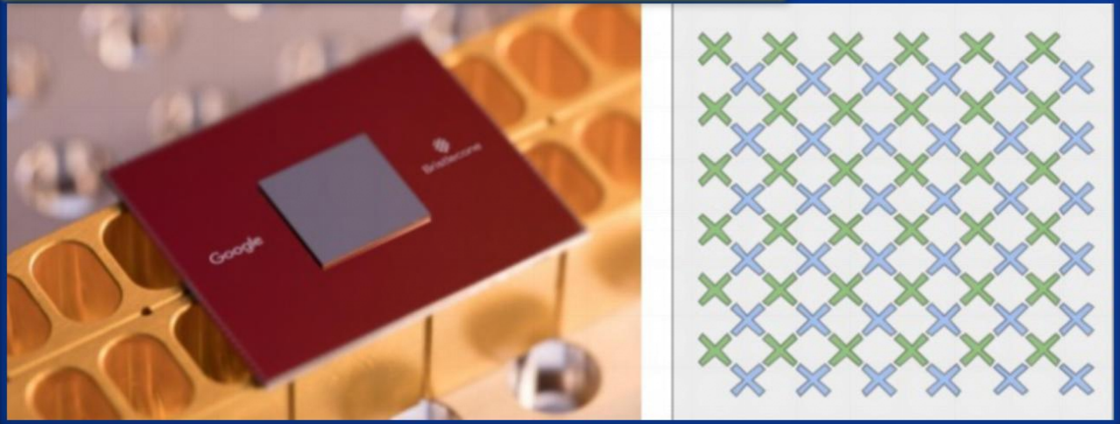
Outlook

- Background introduction
- Quantum computing model
- Quantum circuit simulator
- The end

Background introduction

Breaking news!

➢ Google's 53 qubits QPU [1]



NISQ

Noisy Intermediate-Scale Quantum technology

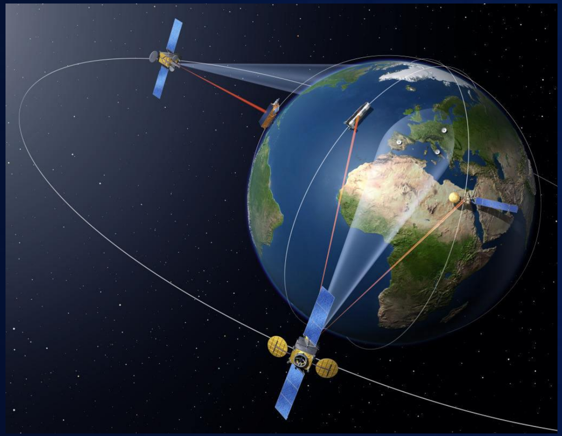
➢ IBM's quantum computer [2]

The 53-qubit quantum processor was able to perform a calculation in 200 seconds that would take the world's more powerfull supercomputer 10,000 years.

[1] Nature 24, 505 (2019)
[2] https://www.cnet.com/news/ibm-new-53-qubit-quantum-computer-is-its-biggest-yet/

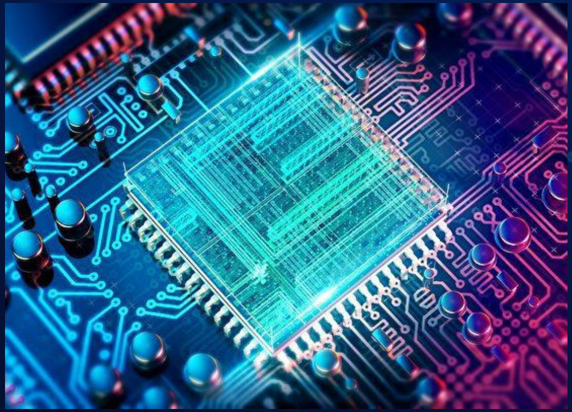
Background introduction

- Why quantum computing

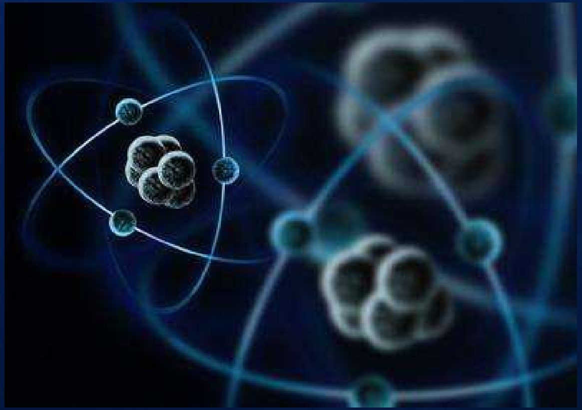


Unconditional
secure communication

Reveal the rules of
complex physical system



More powerful
computing performance

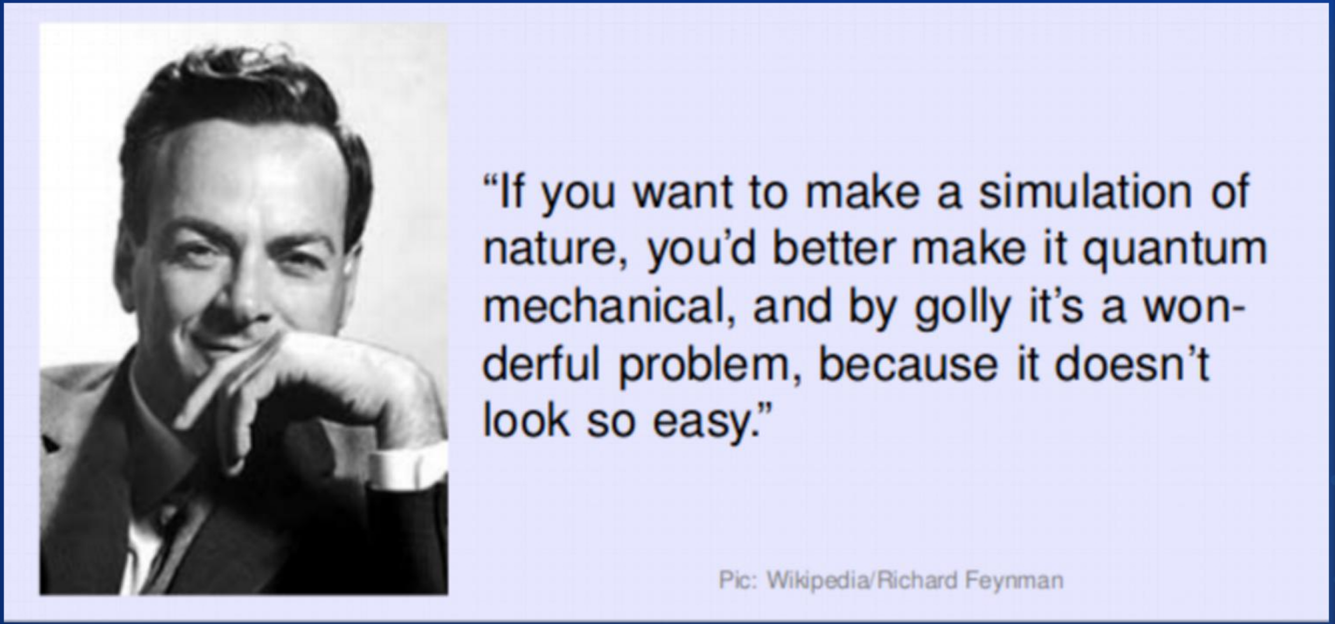


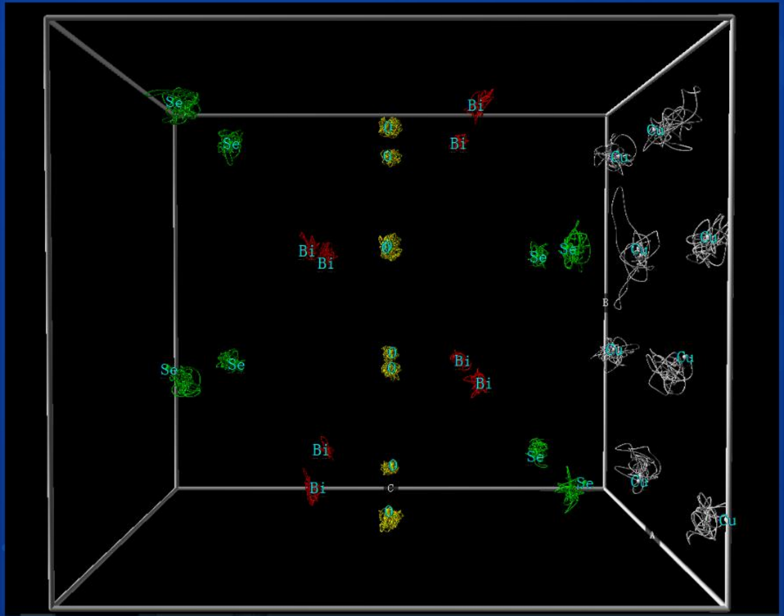
Precision measurement
beyond the classical limit

Background introduction

● Origin of quantum computing

In 1981, Faynman presented the following quandary: classical computers cannot simulate the evolution of quantum systems in an efficient way.





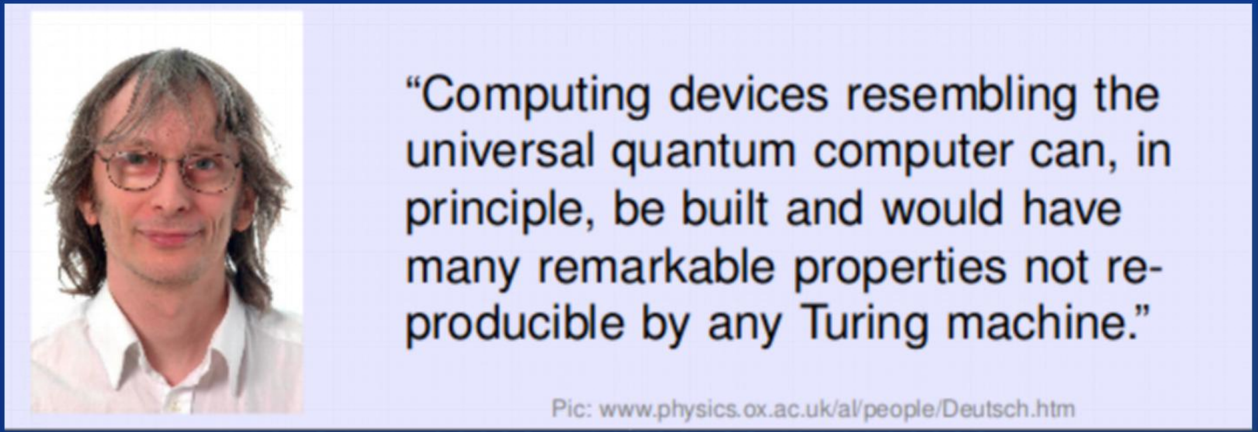
Based on the density functional theory(DFT), the maximum simulated system can only contain severval thousands of atoms.

- ➢ The nature is a big quantum machine!!!

Background introduction

● Origin of quantum algorithm

- In 1985, David Deutsch proposes the mathematical concept of the quantum Turing machine to model quantum computation.



Pic: www.physics.ox.ac.uk/al/people/Deutsch.htm

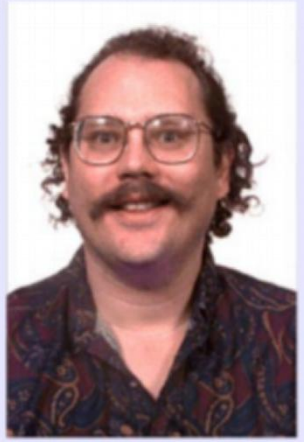
- In 1992, David Deutsch and Richard Jozsa give the first such example (The D-J algorithm).

- In 1994, Dan Simon shows that quantum computers can be exponentially faster than classical computers.

Background introduction

● Quantum algorithms with practical significance

- **Shor's Algorithm (1994):** the first quantum algorithm that people actually care about.

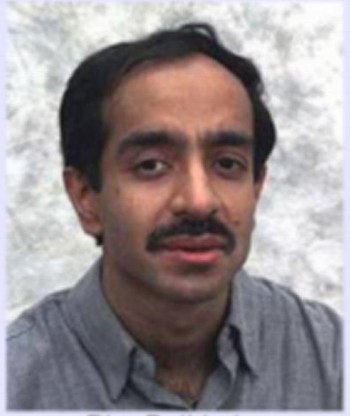


Pic: physik.uni-graz.at

Given an integer $N = p \times q$ for prime numbers p and q, Shor's algorithm outputs p and q.

No efficient classical algorithm for this task is known.

- **Grover's algorithm (1996)**: find with high probability the unique input to a black box function that produces a particular output value.



Pic: Bell Labs

Background introduction

- More and more quantum algorithms were proposed

Quantum machine learning:

- 2009, HHL, PRL [1]
- 2012, Quantum least square fitting, PRL
- 2014, Quantum PCA, Nature physics
- 2014, Quantum SVM, PRL
- 2016, Quantum Boltzmann machine, PRA
- 2016, Quantum reinforcement learning, PRL

Method	Speedup
Bayesian Inference [107, 108]	$O(\sqrt{N})$
Online Perceptron [109]	$O(\sqrt{N})$
Least squares fitting [9]	$O(\log N^{(*)})$
Classical BM [20]	$O(\sqrt{N})$
Quantum BM [22, 62]	$O(\log N^{(*)})$
Quantum PCA [11]	$O(\log N^{(*)})$
Quantum SVM [13]	$O(\log N^{(*)})$
Quantum reinforcement learning [30]	$O(\sqrt{N})$

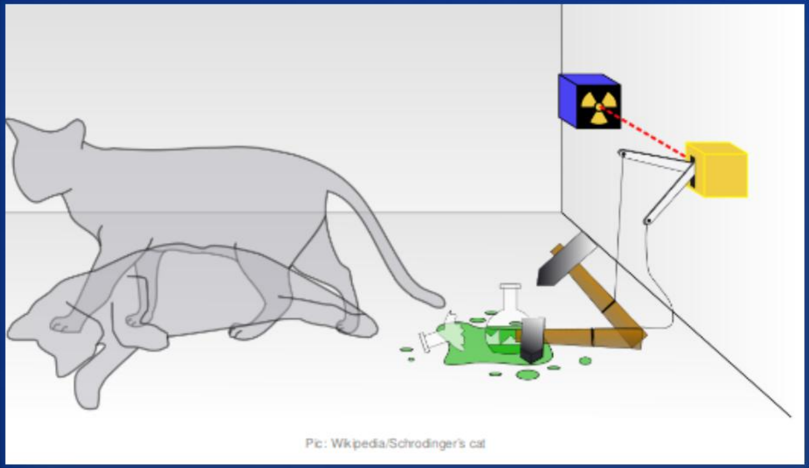
- Ref: Nature 549, 196

[1] PRL, Physical Review Letter

Background introduction

- Two unique properties used in quantum computing

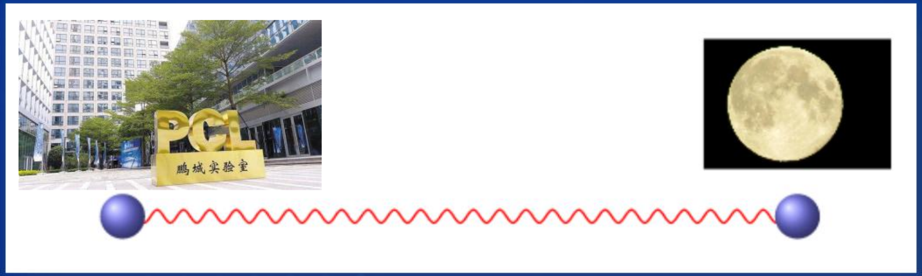
superposition



Schrodinger's cat

$$|\varphi\rangle = a_0|Dead\rangle + a_1|Live\rangle$$

Entanglement



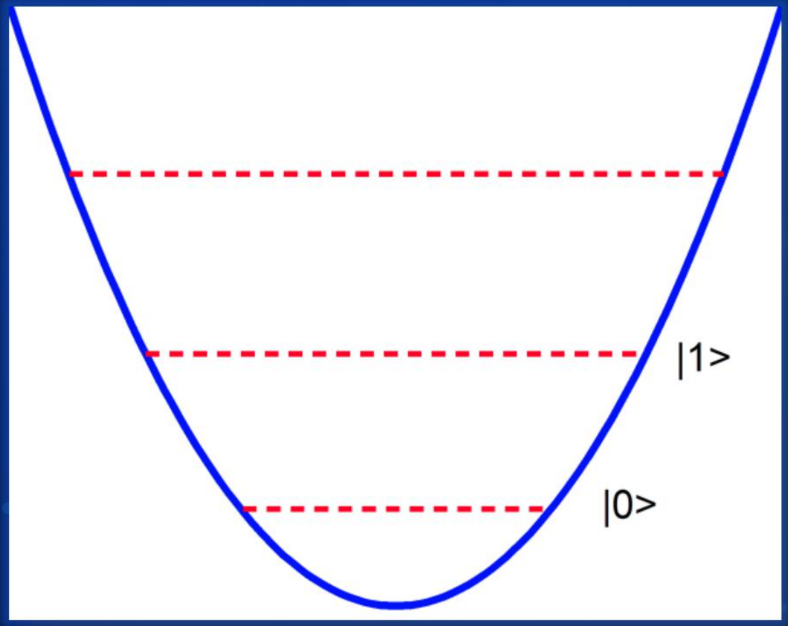
Spooky action at a distance!

---- Einstein

Background introduction

● Quantum systems that realize quantum computing

- ➢ Quantum dot
- ➢ Neclear magnetic resonance
- ➢ Nitrogen-Vacancy center
- ➢ Cold atom system
- ➢ **Superconducting system**
- ➢ Ion trap



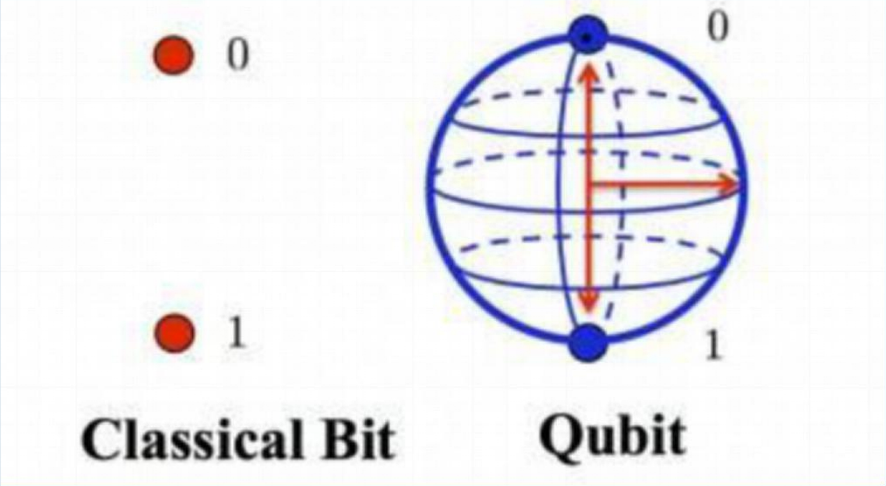
Outlook

- Background introduction
- Quantum computing model
- Quantum circuit simulator
- The end

Quantum computing model

- The basic computing unit

Quantum bit - qubit



$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Matrix representation:

$$|0\rangle \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |1\rangle \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

Quantum computing model

- The power of superposition

N=1 $$|\varphi\rangle = a_0|0\rangle + a_1|1\rangle$$

N=2 $$|\varphi\rangle = a_0|00\rangle + a_1|01\rangle + a_2|10\rangle + a_3|11\rangle$$

⋮ ⋮

N=50 1125899906842624 coefficients

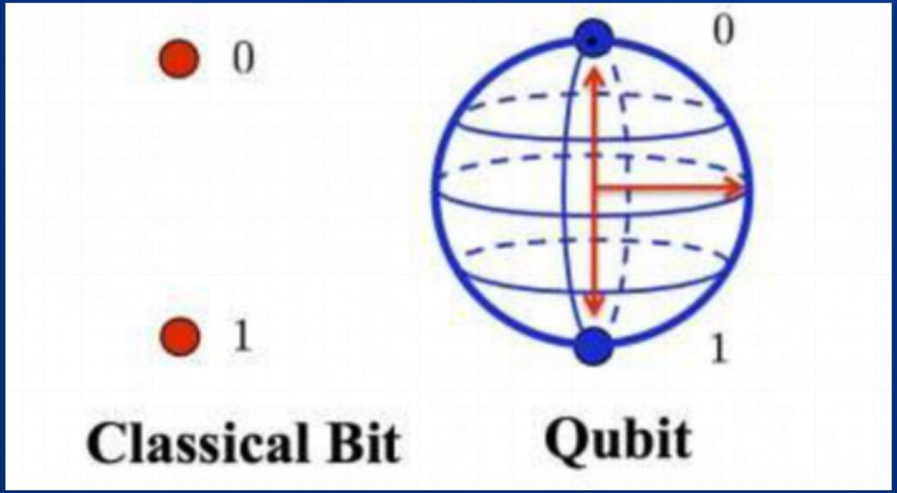
Classical computation: $$y = f(x)$$

Quantum computation: $$y' = f(|\varphi\rangle) = \underbrace{f(a_0)|00..0\rangle + f(a_1)|00..1\rangle + \ldots}_{2^n}$$

Quantum computing model

- Quantum measurement---the collapse of quantum state

Single qubit



$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\varphi\rangle \xrightarrow{\text{measure}} \begin{cases} 0 \text{ with probability } \|\alpha\|^2 \\ 1 \text{ with probability } \|\beta\|^2 \end{cases}$$

$$\|\alpha\|^2 + \|\beta\|^2 = 1$$

Quantum computing model

- **Quantum measurement---the collapse of quantum state**

Multiple qubits

$$|\varphi\rangle = a_0|00\rangle + a_1|01\rangle + a_2|10\rangle + a_3|11\rangle$$

$$\|a_0\|^2 + \|a_1\|^2 + \|a_2\|^2 + \|a_3\|^2 = 1$$

The probability of measuring the first qubit to be 0:

$$p(q_0 == 0) = \|a_0\|^2 + \|a_1\|^2$$

After the measurement, the quantum state reads:

$$|\varphi\rangle = \frac{1}{\sqrt{\|a_0\|^2 + \|a_1\|^2}}\left(a_0|00\rangle + a_1|01\rangle\right)$$

Quantum computing model

- The challenge of quantum algorithms

53 bit

2^{53} coefficients

|000000…00000000>

google's 53 qubit QPU

010101…01011000

Input

Quantum algorithm runs on quantum machine

Output

state preparation

result reading

Quantum computing model

● Building blocks for quantum computing

Quantum gates

X gate

$|0\rangle - \boxed{x} - |1\rangle$

$|1\rangle - \boxed{x} - |0\rangle$

Matrix representation

$X|0\rangle = |1\rangle$

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$X|\varphi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

$X|1\rangle = |0\rangle$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Quantum computing model

● Building blocks for quantum computing

Quantum gates

H gate

$$|0\rangle - \boxed{H} - \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|1\rangle - \boxed{H} - \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Matrix representation

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

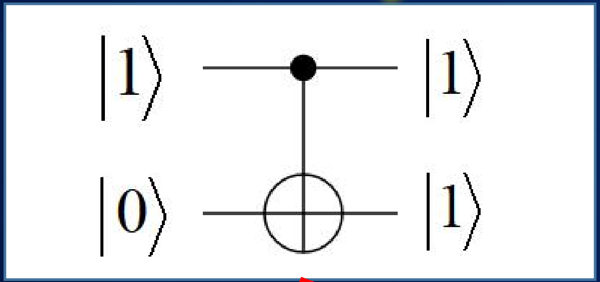
$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

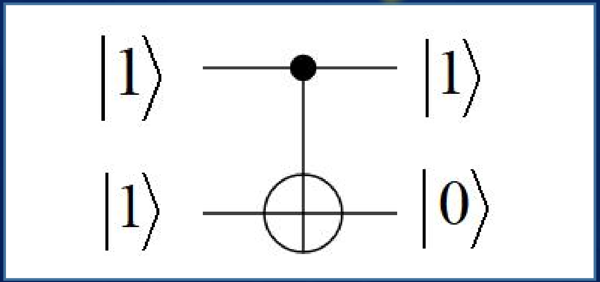
Quantum computing model

- Building blocks for quantum computing

Quantum gates

CNOT gate





Matrix representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathrm{CNOT}|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |11\rangle$$

Quantum computing model

Building blocks for quantum computing

Quantum gates

Single qubit gates:

Y gate: $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

Z gate: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

RX gate: $\begin{bmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ i\sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$

Two qubit gates:

CNOT gate: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Three qubit gate:

Toffli gate: $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Quantum computing model

- **Construct multiple qubit states**

Kronecker product

$$Mathmatic\ \ definition:\ A\otimes B=\begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix},$$

where A and B could be arbitrary vector or matrix.

$$|0\rangle\otimes|0\rangle=\begin{bmatrix}1\\0\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\quad |1\rangle\otimes|0\rangle=\begin{bmatrix}0\\1\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$$

$$\mathrm{CNOT}|10\rangle=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\begin{bmatrix}0\\0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\\1\end{bmatrix}=|11\rangle$$

Quantum computing model

- An example

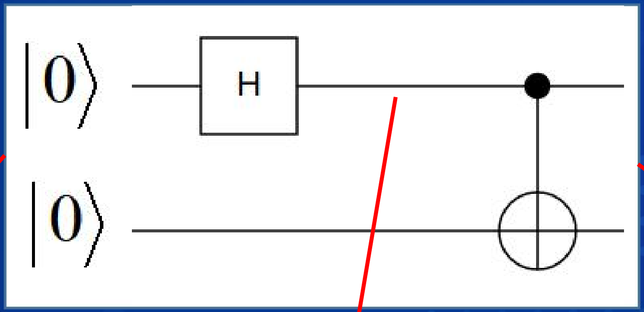
Bell state preparation

Spooky action at a distance!

---- Einstein

Bell pair:

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$



$$|\varphi\rangle = |0\rangle|0\rangle$$

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|0\rangle$$
$$= \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|0\rangle\right)$$

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right)$$
$$= \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

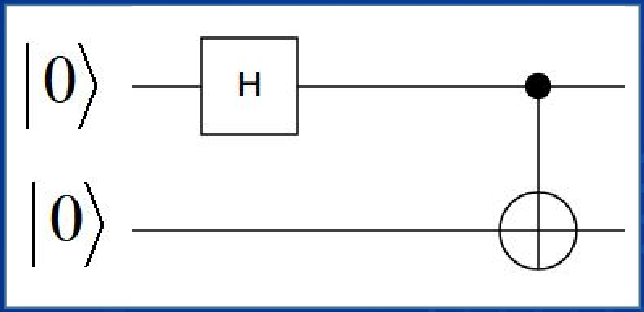
Quantum computing model

- An example

Bell state preparation

Bell pair:

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$



$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

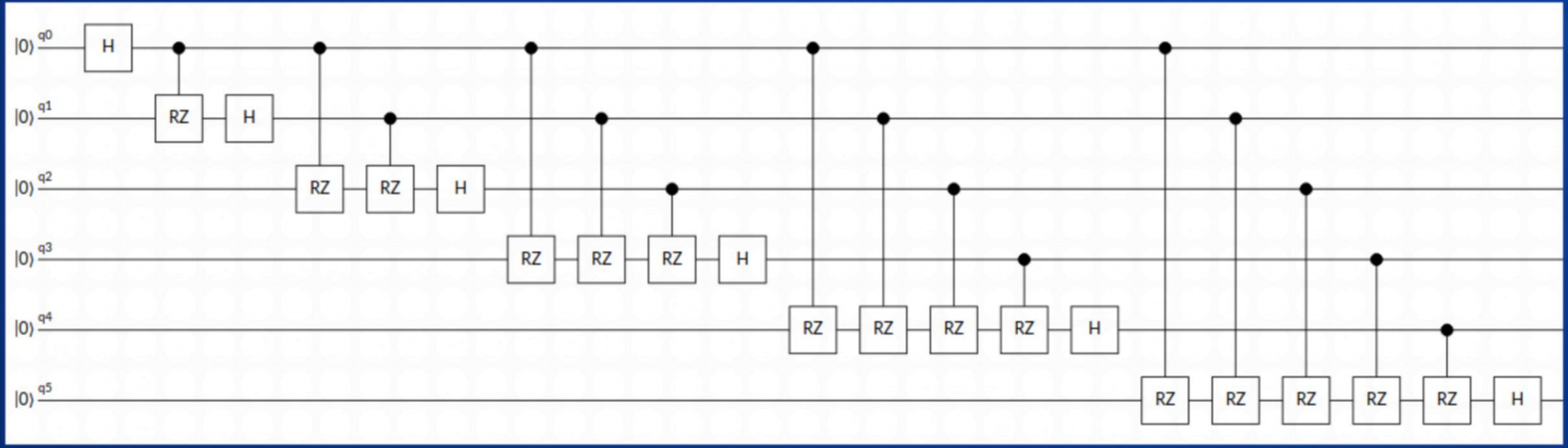
$$\mathrm{CNOT}\,\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 0\\ 1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 0\\ 0\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Outlook

- Background introduction
- Quantum computing model
- Quantum circuit simulator
- The end

Quantum circuit simulator

- QFT circuit



Given arbitrary number of qubits, apply arbitrary number of quantum gates, and get the final state!

Quantum circuit simulator

- **General principles**

State vector:

$$U|\varphi\rangle = |\varphi'\rangle$$

$$|\varphi\rangle = a_0|0\rangle + a_1|1\rangle + \ldots + a_{2^n-1}|2^n-1\rangle$$

$$\begin{bmatrix} U_{11} & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} a_0 \\ . \\ . \end{bmatrix} = \begin{bmatrix} a'_0 \\ . \\ . \end{bmatrix}$$

$$U(i) = I \otimes \ldots \otimes u_i \otimes \ldots \otimes I$$

$$U(i,j) = I \otimes \ldots u_i \otimes \ldots u_j \otimes \ldots \otimes I$$

$$2^n \times 2^n \ matrix$$

```
32×32 Array{Int64,2}:
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
```

CNOT gate for five qubit!

Quantum circuit simulator

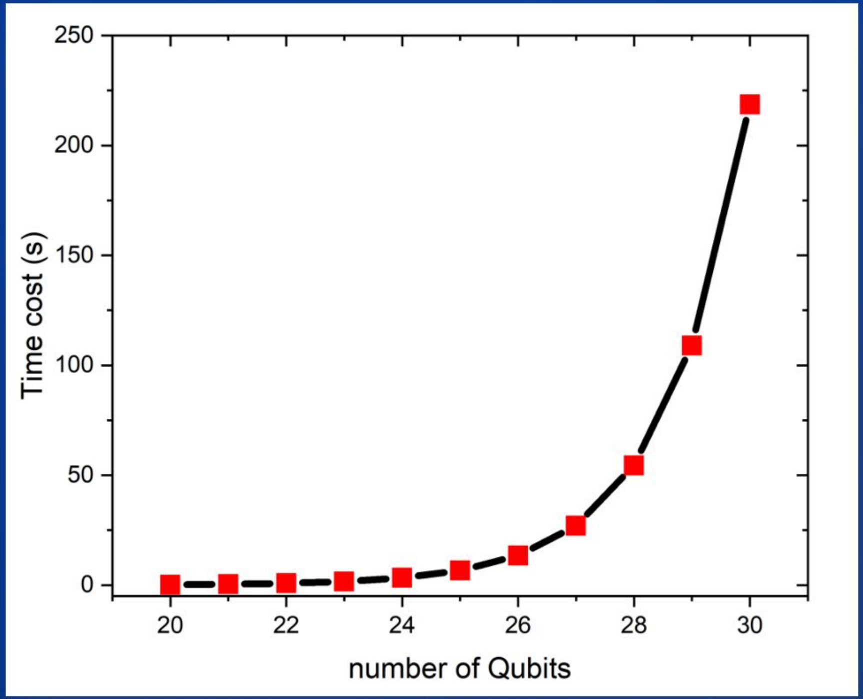
- Features of quantum circuit simulator

Memory cost: $O(2^N)$

Number of qubits	Memory cost
10	16 Kb
20	16 Mb
30	16 Gb
40	16 Tb
50	16 Pb
60	16 Eb
. . .	

quantum supremacy!

Time cost: $O(2^N)$



Quantum circuit simulator

- Some well-known quantum simulators

  - HiQ (Huawei): https://hiq.huaweicloud.com/
  - ProjectQ (ETH Zurich): http://projectq.ch/
  - Qiskit (IBM): https://www.qiskit.org/
  - QuEST (University of Oxford): https://quest.qtechtheory.org/
  - Yao: https://github.com/QuantumBFS/Yao.jl (julia)
  - Q# (Microsoft): https://www.microsoft.com/en-us/quantum/development-kit
  - Bracket (Amazon)

Outlook

- Background introduction
- Quantum computing model
- Quantum circuit simulator
- The end

ASC Intelligent Computing

Thanks for your attention!

I wish you all achieve the best in the competition!