Lenses

https://github.com/heptagons/lenses

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Abstract

Lenses are equilateral hexagons resembling concave and convex optical lenses. Lenses consecutive six internal angles are $(\theta_1, \theta_2, \theta_3, \theta_1, \theta_2, \theta_3)$ where $\theta_1 = X\theta_0$, $\theta_2 = Y\theta_0$, and $\theta_3 = Z\theta_0$ where $\theta_0 = 2\pi/S$ is the base angle of symmetry $S = X + Y + Z$.

1 Lenses

2 Symmetry 5

Symmetry 5 uses as base the angle $\beta = \dfrac{2\pi}{5}$ and produces the two rhombi $(\boldsymbol{b}, \boldsymbol{c})$ and the two lenses $(\boldsymbol{B}, \boldsymbol{C})$.

2.1 Rhombi $(\boldsymbol{b},\boldsymbol{c})$

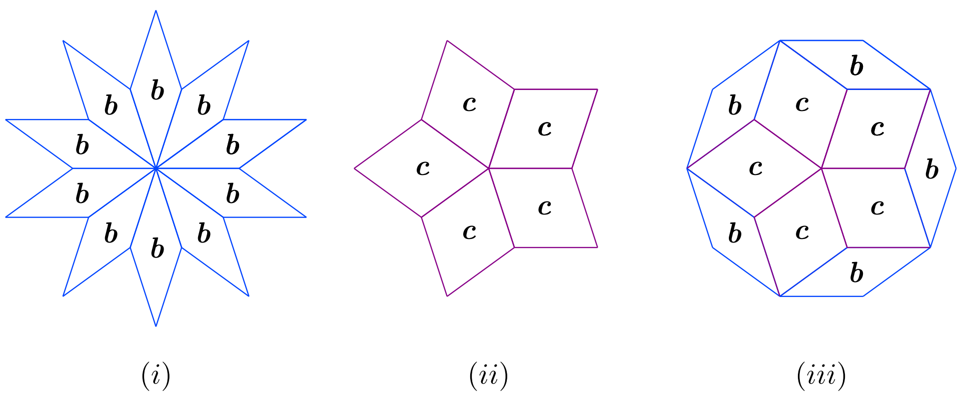


Figure 1: Rhombi $(\boldsymbol{b}, \boldsymbol{c})$.

Rhombus	θ_1	θ_2
$\boldsymbol{b}$	$\beta/2$	$4\beta/2$
$\boldsymbol{c}$	$2\beta/2$	$3\beta/2$

Table 1: Rhombi $(\boldsymbol{b}, \boldsymbol{c})$ internal angles $\theta_1 + \theta_2 = \pi$ where $\beta = 2\pi/5$.

Figure 1 show rhombi $(\boldsymbol{b}, \boldsymbol{c})$. From the figures we calculate the areas adding the rhombi: Star (i) area is $10\boldsymbol{b}$, star (ii) area is $5\boldsymbol{c}$ and regular decagon (iii) area is $5\boldsymbol{b} + 5\boldsymbol{c}$. Table 1 show the rhombi angles.

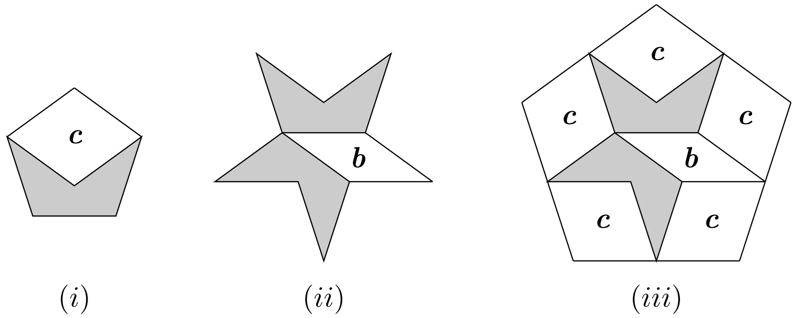


Figure 2: Pentagon {5/1} and pentagram {5/2}.

Figure 2 show regular pentagon and pentagram dissected with rhombi ($\boldsymbol{b}, \boldsymbol{c}$) and a concave pentagon (in gray). We calculate the areas of the regular pentagon P_1 at (i) and pentagram P_2 at (ii) in function of rhombi ($\boldsymbol{b}, \boldsymbol{c}$). Let x be the concave pentagon area. From the figures we note pentagon P_4 at (iii) is double the side and then four times the area of pentagon P_1. From the figures we note the area of P_1 is $\boldsymbol{c} + x$ and the area of P_4 is $\boldsymbol{b} + 5\boldsymbol{c} + 2x$, so we compare the pentagons and solve for x to get:

$$
\begin{aligned}
4P_1 &= P_4 \\
4(\boldsymbol{c} + x) &= \boldsymbol{b} + 5\boldsymbol{c} + 2x \\
x &= \frac{\boldsymbol{b} + \boldsymbol{c}}{2}
\end{aligned} \tag{1}
$$

We use x to get the areas of pentagon and pentagram:

$$P_1 = \boldsymbol{c} + x = \boldsymbol{c} + \frac{\boldsymbol{b} + \boldsymbol{c}}{2} = \frac{\boldsymbol{b} + 3\boldsymbol{c}}{2} \tag{2}$$

$$P_2 = \boldsymbol{b} + 2x = \boldsymbol{b} + \frac{2(\boldsymbol{b} + \boldsymbol{c})}{2} = 2\boldsymbol{b} + \boldsymbol{c} \tag{3}$$

2.2 Lenses ($\boldsymbol{B}, \boldsymbol{C}$)

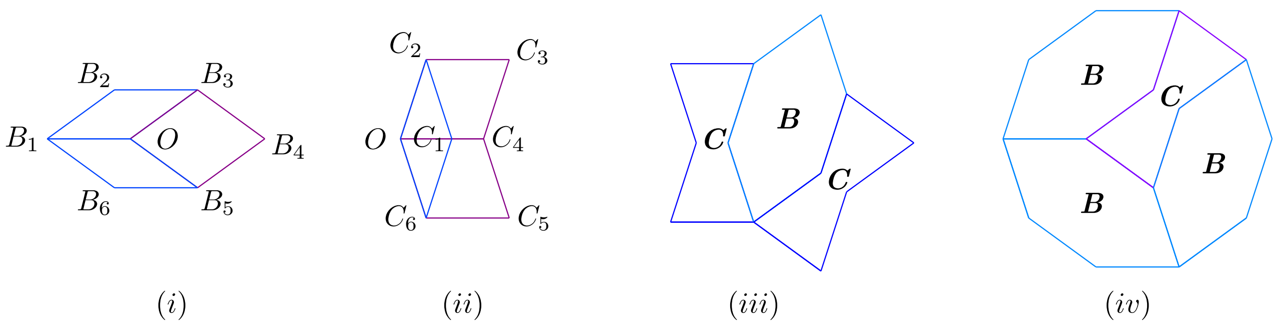


Figure 3: Lenses ($\boldsymbol{B}$, $\boldsymbol{C}$).

Rhombus	θ_1	θ_2	θ_3
$\boldsymbol{B}$	β	2β	2β
$\boldsymbol{C}$	β	β	3β

Table 2: Lenses ($\boldsymbol{B}, \boldsymbol{C}$) internal angles $\theta_1 + \theta_2 + \theta_3 = 2\pi$ where $\beta = 2\pi/5$.

Figure 3 show lenses $(\boldsymbol{B}, \boldsymbol{C})$. Figure (i) show the lense $\boldsymbol{B}$ with perimeter $\overline{B_1 ... B_6}$ is formed adding two rhombi $\boldsymbol{b}$ and adding one rhombus $\boldsymbol{c}$ so its area is $2\boldsymbol{b} + \boldsymbol{c}$. Figure (ii) show the lense $\boldsymbol{C}$ with perimeter $\overline{C_1 ... C_6}$ is formed adding two rhombi $\boldsymbol{c}$ and substracting one rhombus $\boldsymbol{b}$ so its area is $2\boldsymbol{c} - \boldsymbol{b}$. From the figures we see the area of star (iii) is $\boldsymbol{B}+2\boldsymbol{C} = 5\boldsymbol{c}$ and the area of regular decagon (iv) is $3\boldsymbol{B}+\boldsymbol{C} = 5\boldsymbol{b}+5\boldsymbol{c}$.

3 Symmetry 7

Symmetry 7 uses as base the angle $\gamma = \dfrac{2\pi}{7}$ and produces the three rhombi $(\boldsymbol{d}, \boldsymbol{f}, \boldsymbol{e})$ and the three lenses $(\boldsymbol{D}, \boldsymbol{E}, \boldsymbol{F})$.

3.1 Rhombi $(\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f})$

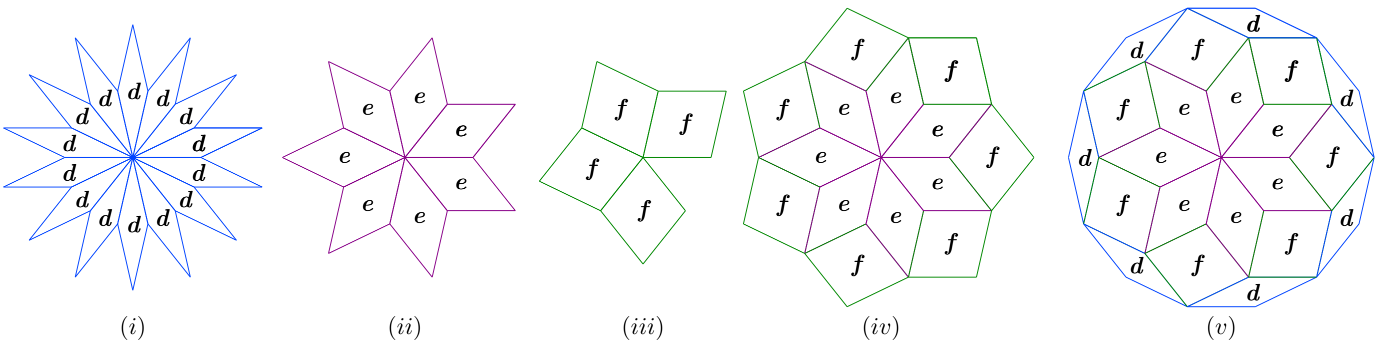
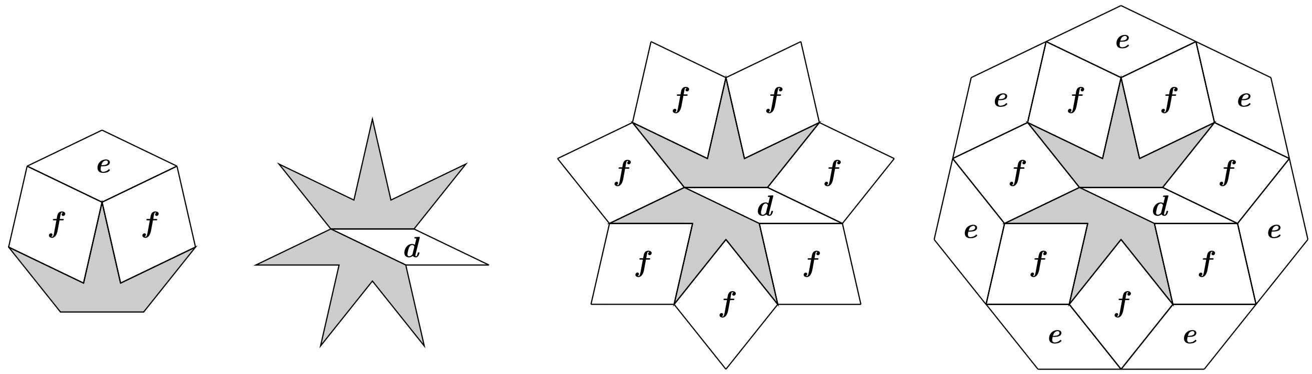


Figure 4: Rhombi $(\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f})$.

Rhombus	θ_1	θ_2
$\boldsymbol{d}$	$\gamma/2$	$6\gamma/2$
$\boldsymbol{e}$	$2\gamma/2$	$5\gamma/2$
$\boldsymbol{f}$	$3\gamma/2$	$4\gamma/2$

Table 3: Rhombi $(\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f})$ internal angles $\theta_1 + \theta_2 = \pi$ where $\gamma = 2\pi/7$.

Figure 4 show rhombi $(\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f})$. Inspecting the figures we get the areas simply adding the rhombi: Star (i) area is $14\boldsymbol{d}$, star (ii) area is $7\boldsymbol{e}$. The star (iv) area is $7(\boldsymbol{e}+\boldsymbol{f})$ and the regular 14-gon area is $7(\boldsymbol{d}+\boldsymbol{e}+\boldsymbol{f})$. Table 3 show the symmetry 7 rhombi angles.



(i) (ii) (iii) (iv)

Figure 5: Heptagon $\{7/1\}$ at (i) and heptagrams $\{7/2\}$ at (ii) and $\{7/3\}$ at (iii).

Figure 5 show regular heptagon and heptagrams dissected with rhombi ($\boldsymbol{c}$, $\boldsymbol{d}$, $\boldsymbol{e}$) and with one equilateral concave heptagon (in gray). Let $\boldsymbol{x}$ the area of such gray piece. By inspection the area of regular heptagon at (i) is $H_1 = \boldsymbol{e} + 2\boldsymbol{f} + \boldsymbol{x}$ while the area of regular heptagon at (iv) is $H_2 = \boldsymbol{d} + 7(\boldsymbol{e} + \boldsymbol{f}) + 2\boldsymbol{x}$. Since the side of H_2 is the double of H_1 its area is four times so we can get the value of $\boldsymbol{x}$:

$$\begin{aligned} 4H_1 &= H_2 \\ 4(\boldsymbol{e} + 2\boldsymbol{f} + \boldsymbol{x}) &= \boldsymbol{d} + 7(\boldsymbol{e} + \boldsymbol{f}) + 2\boldsymbol{x} \\ \boldsymbol{x} &= \frac{\boldsymbol{d} + 3\boldsymbol{e} - \boldsymbol{f}}{2} \end{aligned} \tag{4}$$

We use the value of $\boldsymbol{x}$ to calculate the areas of heptagon (i) and heptagrams (ii) and (iii) in function of $(\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f})$:

$$\begin{aligned} A\{7/1\} &= \boldsymbol{e} + 2\boldsymbol{f} + \boldsymbol{x} \\ &= \frac{\boldsymbol{d} + 5\boldsymbol{e} + 3\boldsymbol{f}}{2} && (5) \\ A\{7/2\} &= \boldsymbol{d} + 2\boldsymbol{x} \\ &= 2\boldsymbol{d} + 3\boldsymbol{e} - \boldsymbol{f} && (6) \\ A\{7/3\} &= A\{7/2\} + 7\boldsymbol{f} \\ &= 2\boldsymbol{d} + 3\boldsymbol{e} + 6\boldsymbol{f} && (7) \end{aligned}$$