Lenses

https://github.com/heptagons/lenses

2023/12/31

Abstract

Lenses are equilateral hexagons resembling concave and convex optical lenses. Lenses consecutive six internal angles are $(\theta_1, \theta_2, \theta_3, \theta_1, \theta_2, \theta_3)$ where $\theta_1 = X\theta_0$, $\theta_2 = Y\theta_0$, and $\theta_3 = Z\theta_0$ where $\theta_0 = 2\pi/S$ is the base angle of symmetry $S = X + Y + Z$.

1 Lenses

2 Symmetry 5

Symmetry 5 is based in angle $\beta = \frac{2\pi}{5}$ and produces the two rhombi $(\boldsymbol{b}, \boldsymbol{c})$ and the two lenses $(\boldsymbol{B}, \boldsymbol{C})$.

2.1 Rhombi $(\boldsymbol{b}, \boldsymbol{c})$

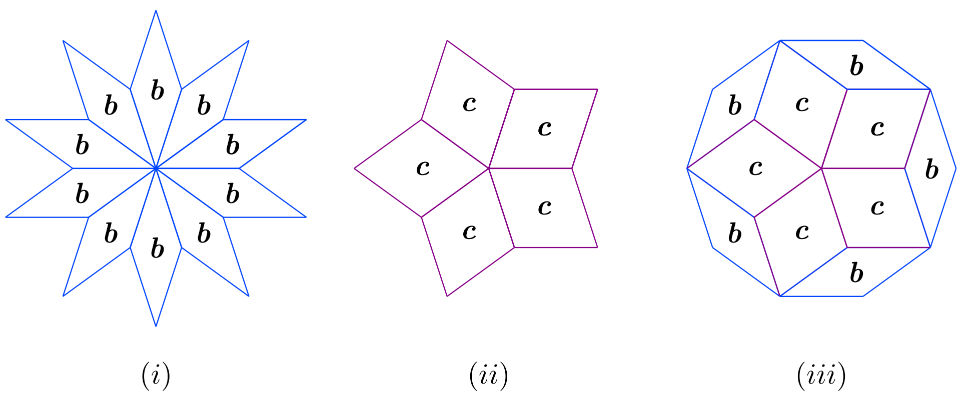

Figure 1: Rhombi $(\boldsymbol{b}, \boldsymbol{c})$.

Rhombus	θ_1	θ_2
$\boldsymbol{b}$	$\beta/2$	$4\beta/2$
$\boldsymbol{c}$	$2\beta/2$	$3\beta/2$

Table 1: Rhombi $(\boldsymbol{b}, \boldsymbol{c})$ internal angles $\theta_1 + \theta_2 = \pi$ where $\beta = 2\pi/5$.

Figure 1 show rhombi $(\boldsymbol{b}, \boldsymbol{c})$. From the figures we calculate the areas adding the rhombi: Star (i) area is $10\boldsymbol{b}$, star (ii) area is $5\boldsymbol{c}$ and regular decagon (iii) area is $5\boldsymbol{b} + 5\boldsymbol{c}$. Table 1 show the rhombi angles.

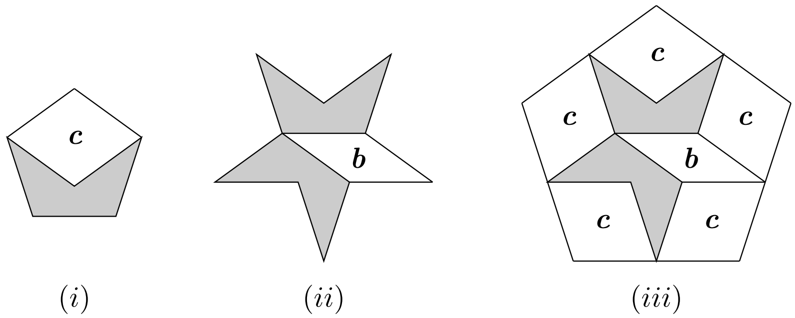

Figure 2: Pentagon $\{5/1\}$ at (i) and pentagram $\{5/2\}$ at (ii).

Figure 2 show regular pentagon and pentagram dissected with rhombi $(\boldsymbol{b}, \boldsymbol{c})$ and a concave pentagon (in gray). Let $\boldsymbol{x}$ be the area of such gray piece. By inspection the area of regular pentagon at (i) is $A_1 = \boldsymbol{c} + \boldsymbol{x}$ and the area of regular pentagon at (iii) is $P_2 = \boldsymbol{b} + 5\boldsymbol{c} + 2\boldsymbol{x}$. Since the side of A_2 is the double of A_1 its area is four times so we can get the value of $\boldsymbol{x}$ in terms of $(\boldsymbol{b}, \boldsymbol{c})$

$$
\begin{aligned}
4P_1 &= P_4 \\
4(\boldsymbol{c} + x) &= \boldsymbol{b} + 5\boldsymbol{c} + 2x \\
x &= \frac{\boldsymbol{b} + \boldsymbol{c}}{2} \qquad (1)
\end{aligned}
$$

We use the value of $\boldsymbol{x}$ to get the areas of pentagon (i) and pentagram (ii):

$$
\begin{aligned}
A\{5/1\} &= \boldsymbol{c} + \boldsymbol{x} \\
&= \frac{\boldsymbol{b} + 3\boldsymbol{c}}{2} \qquad (2) \\
A\{5/2\} &= \boldsymbol{b} + 2\boldsymbol{x} \\
&= 2\boldsymbol{b} + \boldsymbol{c} \qquad (3)
\end{aligned}
$$

2.2 Lenses $(\boldsymbol{B}, \boldsymbol{C})$

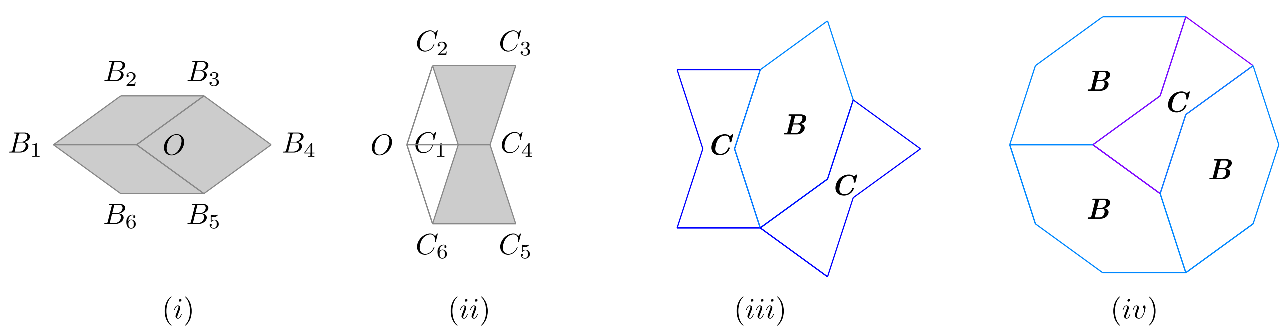

Figure 3: Lenses $(\boldsymbol{B},\ \boldsymbol{C})$.

Rhombus	θ_1	θ_2	θ_3
$\boldsymbol{B}$	β	2β	2β
$\boldsymbol{C}$	β	β	3β

Table 2: Lenses $(\boldsymbol{B}, \boldsymbol{C})$ internal angles $\theta_1 + \theta_2 + \theta_3 = 2\pi$ where $\beta = 2\pi/5$.

Figure 6 show lenses $(\boldsymbol{B}, \boldsymbol{C})$. Figure (i) show the lense $\boldsymbol{B}$ with perimeter $\overline{B_1...B_6}$ is formed adding two rhombi $\boldsymbol{b}$ and adding one rhombus $\boldsymbol{c}$ so its area is $2\boldsymbol{b}+\boldsymbol{c}$. Figure (ii) show the lense $\boldsymbol{C}$ with perimeter $\overline{C_1...C_6}$ is formed adding two rhombi $\boldsymbol{c}$ and substracting one rhombus $\boldsymbol{b}$ so its area is $2\boldsymbol{c}-\boldsymbol{b}$. From the figures we see the area of star (iii) is $\boldsymbol{B}+2\boldsymbol{C}=5\boldsymbol{c}$ and the area of regular decagon (iv) is $3\boldsymbol{B}+\boldsymbol{C}=5(\boldsymbol{b}+\boldsymbol{c})$.

3 Symmetry 7

Symmetry 7 is based in angle $\gamma = \dfrac{2\pi}{7}$ and produces the three rhombi $(\boldsymbol{d}, \boldsymbol{f}, \boldsymbol{e})$ and the three lenses $(\boldsymbol{D}, \boldsymbol{E}, \boldsymbol{F})$.

3.1 Rhombi $(\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f})$

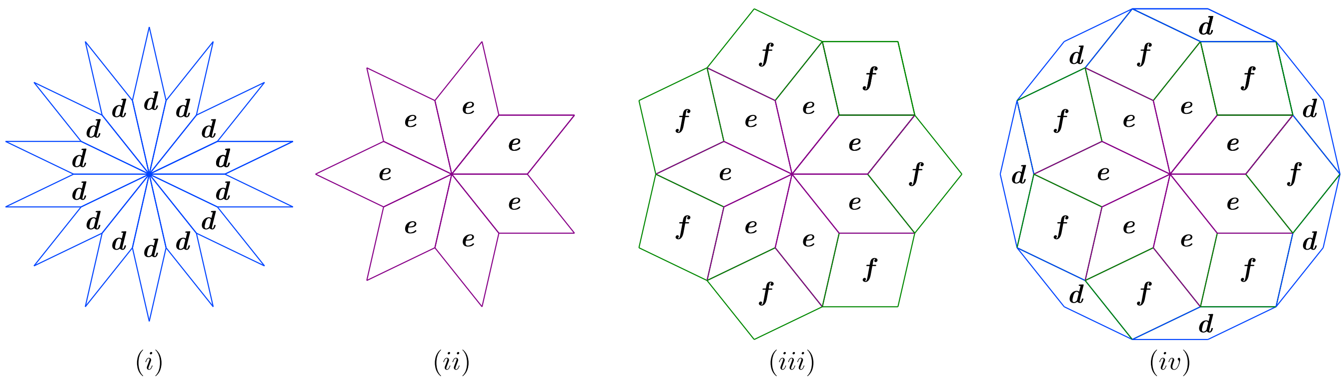

Figure 4: Rhombi $(\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f})$.

Rhombus	θ_1	θ_2
$\boldsymbol{d}$	$\gamma/2$	$6\gamma/2$
$\boldsymbol{e}$	$2\gamma/2$	$5\gamma/2$
$\boldsymbol{f}$	$3\gamma/2$	$4\gamma/2$

Table 3: Rhombi $(\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f})$ internal angles $\theta_1 + \theta_2 = \pi$ where $\gamma = 2\pi/7$.

Figure 4 show rhombi $(\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f})$. Inspecting the figures we get the areas simply adding the rhombi: Star (i) area is $14\boldsymbol{d}$, star (ii) area is $7\boldsymbol{e}$. The star at (iii) area is $7(\boldsymbol{e}+\boldsymbol{f})$ and the regular 14-gon area at (iv) is $7(\boldsymbol{d}+\boldsymbol{e}+\boldsymbol{f})$. Table 3 show the symmetry 7 rhombi angles.

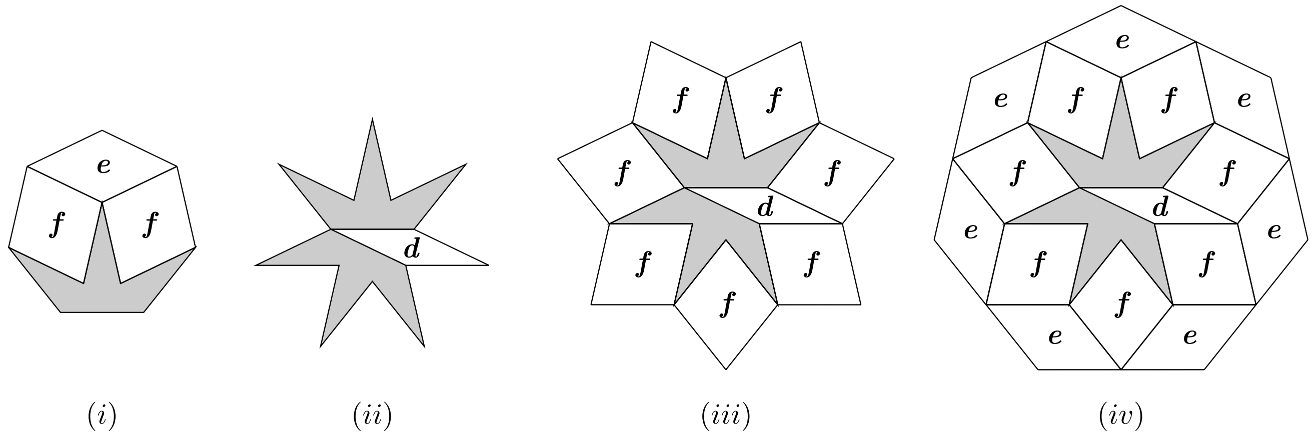

Figure 5: Heptagon {7/1} at (*i*) and heptagrams {7/3} at (*ii*) and {7/2} at (*iii*).

Figure 5 show regular heptagon and heptagrams dissected with rhombi ($\boldsymbol{c}$, $\boldsymbol{d}$, $\boldsymbol{e}$) and with one equilateral concave heptagon (in gray). Let $\boldsymbol{x}$ be the area of such gray piece. By inspection the area of regular heptagon at (*i*) is $A_1 = \boldsymbol{e} + 2\boldsymbol{f} + \boldsymbol{x}$ while the area of regular heptagon at (*iv*) is $A_2 = \boldsymbol{d} + 7(\boldsymbol{e} + \boldsymbol{f}) + 2\boldsymbol{x}$. Since the side of A_2 is the double of A_1 its area is four times so we can get the value of $\boldsymbol{x}$ in terms of ($\boldsymbol{d}$, $\boldsymbol{e}$, $\boldsymbol{f}$):

$$\begin{aligned} 4A_1 &= A_2 \\ 4(\boldsymbol{e} + 2\boldsymbol{f} + \boldsymbol{x}) &= \boldsymbol{d} + 7(\boldsymbol{e} + \boldsymbol{f}) + 2\boldsymbol{x} \\ \boldsymbol{x} &= \frac{\boldsymbol{d} + 3\boldsymbol{e} - \boldsymbol{f}}{2} \qquad (4) \end{aligned}$$

We use the value of $\boldsymbol{x}$ to calculate the areas of heptagon (*i*) and heptagrams (*ii*) and (*iii*) in terms of ($\boldsymbol{d}$, $\boldsymbol{e}$, $\boldsymbol{f}$):

$$\begin{aligned} A\{7/1\} &= \boldsymbol{e} + 2\boldsymbol{f} + \boldsymbol{x} \\ &= \frac{\boldsymbol{d} + 5\boldsymbol{e} + 3\boldsymbol{f}}{2} \qquad (5) \\ A\{7/3\} &= \boldsymbol{d} + 2\boldsymbol{x} \\ &= 2\boldsymbol{d} + 3\boldsymbol{e} - \boldsymbol{f} \qquad (6) \\ A\{7/2\} &= A\{7/3\} + 7\boldsymbol{f} \\ &= 2\boldsymbol{d} + 3\boldsymbol{e} + 6\boldsymbol{f} \qquad (7) \end{aligned}$$

3.2 Lenses ($\boldsymbol{D}$,$\boldsymbol{E}$,$\boldsymbol{F}$)

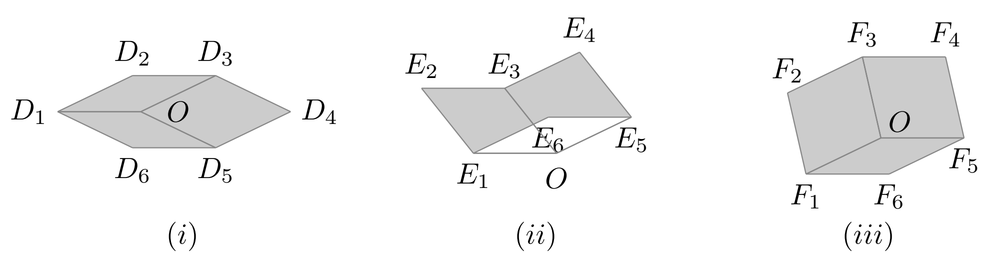

Figure 6: Lenses ($\boldsymbol{D}$,$\boldsymbol{E}$,$\boldsymbol{F}$).