Lenses

https://github.com/heptagons/lenses

2023/12/30

Abstract

Lenses are equilateral hexagons resembling concave and convex optical lenses. The hexagons consecutive six internal angles are $(\theta_1, \theta_2, \theta_3, \theta_1, \theta_2, \theta_3)$ where $\theta_1 = X\theta_0$, $\theta_2 = Y\theta_0$, and $\theta_3 = Z\theta_0$ where $\theta_0 = 2\pi/S$ is the base angle of symmetry $S = X + Y + Z$.

1 Lenses

2 Symmetry 5

Symmetry 5 uses as base the angle $\beta = \dfrac{2\pi}{5}$. Includes the rhombi $(\boldsymbol{b}, \boldsymbol{c})$ and the lenses $(\boldsymbol{B}, \boldsymbol{C})$.

2.1 Rhombi $(\boldsymbol{b}, \boldsymbol{c})$

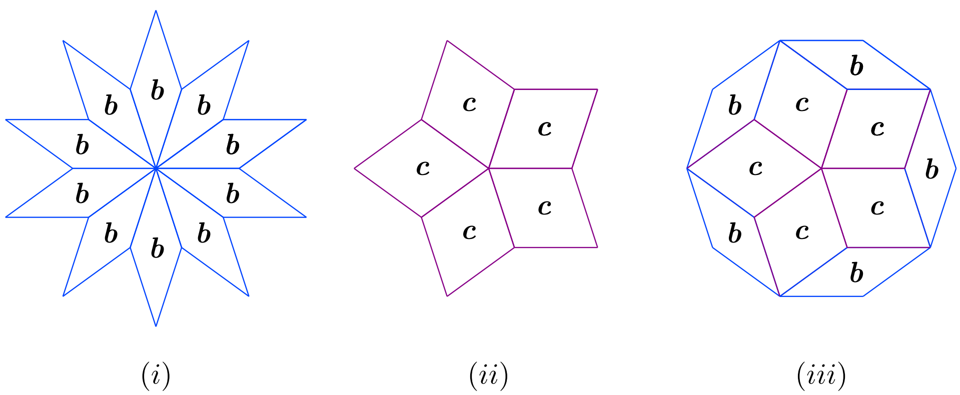

Figure 1: Rhombi $(\boldsymbol{b}, \boldsymbol{c})$.

Rhombus	θ_1	θ_2
$\boldsymbol{b}$	$\beta/2$	$4\beta/2$
$\boldsymbol{c}$	$2\beta/2$	$3\beta/2$

Table 1: Rhombi $(\boldsymbol{b}, \boldsymbol{c})$ internal angles $\theta_1 + \theta_2 = \pi$ where $\beta = 2\pi/5$.

Figure 1 show rhombi $(\boldsymbol{b}, \boldsymbol{c})$. Inspecting the figures we get these areas: Star at (i) is $10\boldsymbol{b}$, star at (ii) is $5\boldsymbol{c}$ and regular pentagon at (iii) is $5\boldsymbol{b} + 5\boldsymbol{c}$. Table 1 show the angles.

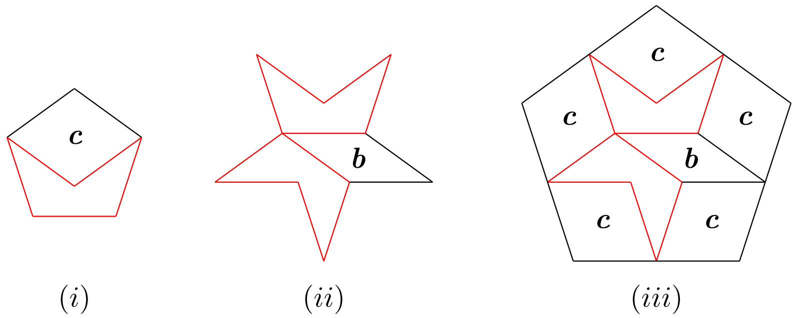

Figure 2: Regular pentagon and pentagram. Concave pentagon (in red).

Figure 2 show regular pentagon and pentagram dissected with rhombi $(\boldsymbol{b}, \boldsymbol{c})$ and a concave pentagon (in red). We calculate the areas of the regular pentagon P_1 at (i) and pentagram P_G at (ii) in function of rhombi $(\boldsymbol{b}, \boldsymbol{c})$. Let x be the concave pentagon area. From the figures we note pentagon P_2 at (iii) is double the side and then four times the area of pentagon P_1. From the figures we note the area of P_1 is $x + \boldsymbol{c}$ and the area of P_2 is $2x + \boldsymbol{b} + 5\boldsymbol{c}$, so we compare the pentagons and solve for x to get:

$$\begin{aligned} 4P_1 &= P_2 \\ 4(x + \boldsymbol{c}) &= 2x + \boldsymbol{b} + 5\boldsymbol{c} \\ x &= \frac{\boldsymbol{b} + \boldsymbol{c}}{2} \end{aligned} \tag{1}$$

Then the areas of the pentagon and the pentagram are:

$$P_1 = x + \boldsymbol{c} = \frac{\boldsymbol{b} + \boldsymbol{c}}{2} + \boldsymbol{c} = \frac{\boldsymbol{b} + 3\boldsymbol{c}}{2} \tag{2}$$

$$P_G = 2x + \boldsymbol{b} = \frac{2(\boldsymbol{b} + \boldsymbol{c})}{2} + \boldsymbol{b} = 2\boldsymbol{b} + \boldsymbol{c} \tag{3}$$

2.2 Lenses $(\boldsymbol{B}, \boldsymbol{C})$

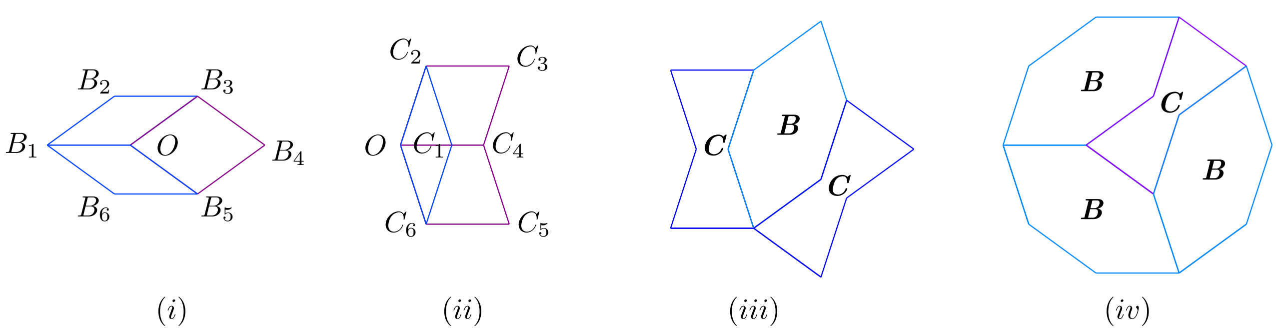

Figure 3: Lenses $(\boldsymbol{B}, \boldsymbol{C})$.

Rhombus	θ_1	θ_2	θ_3
$\boldsymbol{B}$	β	2β	2β
$\boldsymbol{C}$	β	β	3β

Table 2: Lenses $(\boldsymbol{B}, \boldsymbol{C})$ internal angles $\theta_1 + \theta_2 + \theta_3 = 2\pi$ where $\beta = 2\pi/5$.

Figure 3 show lenses ($\boldsymbol{B}$, $\boldsymbol{C}$). Figure (i) show the lense $\boldsymbol{B}$ with perimeter $\overline{B_1...B_6}$ is formed adding two rhombi $\boldsymbol{b}$ and adding one rhombus $\boldsymbol{c}$ so its area is $2\boldsymbol{b}+\boldsymbol{c}$. Figure ($ii$) show the lense $\boldsymbol{C}$ with perimeter $\overline{C_1...C_6}$ is formed adding two rhombi $\boldsymbol{c}$ and substracting one rhombus $\boldsymbol{b}$ so its area is $2\boldsymbol{c}-\boldsymbol{b}$. From the figures we see the area of star (iii) is $\boldsymbol{B}+2\boldsymbol{C}=5\boldsymbol{c}$ and the area of regular decagon (iv) is $3\boldsymbol{B}+\boldsymbol{C}=5\boldsymbol{b}+5\boldsymbol{c}$.

3 Symmetry 7

Symmetry 7 uses as base the angle $\gamma = \dfrac{2\pi}{7}$. Includes the rhombi ($\boldsymbol{d}, \boldsymbol{f}, \boldsymbol{e}$) and the lenses ($\boldsymbol{D}, \boldsymbol{E}, \boldsymbol{F}$).

3.1 Rhombi ($\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}$)

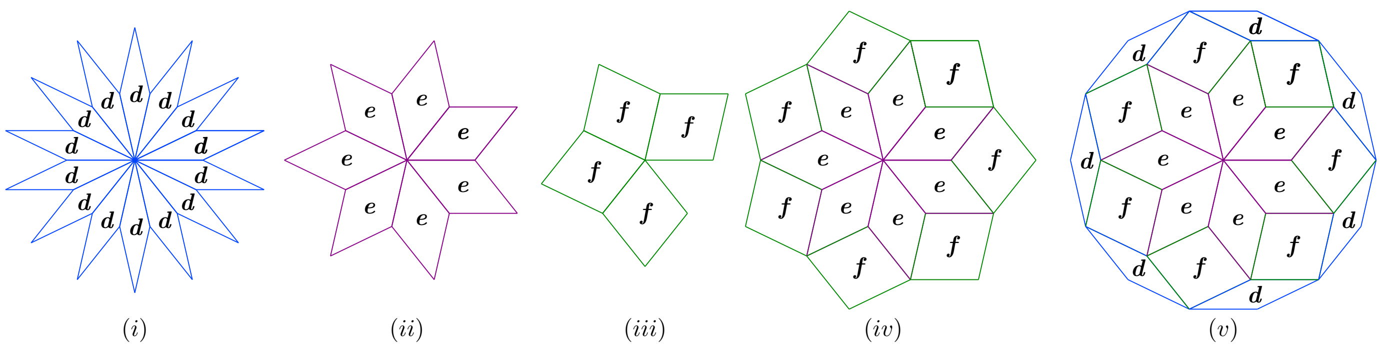

Figure 4: Rhombi ($\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}$).

Rhombus	θ_1	θ_2
$\boldsymbol{d}$	$\gamma/2$	$6\gamma/2$
$\boldsymbol{e}$	$2\gamma/2$	$5\gamma/2$
$\boldsymbol{f}$	$3\gamma/2$	$4\gamma/2$

Table 3: Rhombi ($\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}$) internal angles $\theta_1+\theta_2=\pi$ where $\gamma = 2\pi/7$.

Figure 4 show rhombi ($\boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}$). Inspecting the figures we get these areas: Star at (i) is $14\boldsymbol{d}$, star at (ii) is $7\boldsymbol{e}$ and irregular shape (iii) is $4\boldsymbol{f}$. The star at (iv) is $7(\boldsymbol{e}+\boldsymbol{f})$ and the regular 14-gon area is $7(\boldsymbol{d}+\boldsymbol{e}+\boldsymbol{f})$. Table 3 show the angles.