Lenses

https://github.com/heptagons/lenses

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Abstract

Lenses are equilateral hexagons resembling concave and convex optical lenses. The hexagons consecutive six internal angles are $(\theta_1, \theta_2, \theta_3, \theta_1, \theta_2, \theta_3)$ where $\theta_1 = X\theta_0$, $\theta_2 = Y\theta_0$, and $\theta_3 = Z\theta_0$ where $\theta_0 = 2\pi/S$ is the base angle of symmetry S.

1 Lenses

2 Symmetry 5

Symmetry 5 uses as base the angle $\beta = \frac{2\pi}{5}$. Includes two rhombi $\boldsymbol{b}$ and $\boldsymbol{c}$ and two lenses $\boldsymbol{B}$ and $\boldsymbol{C}$.

2.1 Rhombi $\boldsymbol{b}$ and $\boldsymbol{c}$

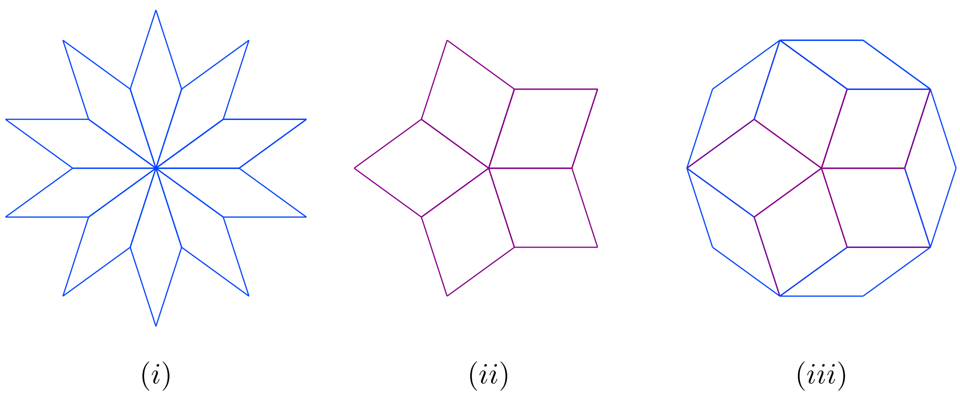

Figure 1: Rhombi of the types $\boldsymbol{b}$ and $\boldsymbol{c}$.

Figure 1 show rhombi $\boldsymbol{b}$ and $\boldsymbol{c}$. $\boldsymbol{b}$ is the rhombus with smallest internal angles equal to $\frac{\beta}{2} = \frac{\pi}{5}$. $\boldsymbol{c}$ is the rhombus with smallest internal angles equal to $\beta = \frac{2\pi}{5}$. Figure (i) show a dissected star whose area equals to $10\boldsymbol{b}$. Figure (ii) show a dissected star whose area equals to $5\boldsymbol{c}$. Figure (iii) show a dissected regular decagon whose area equals to $5\boldsymbol{b} + 5\boldsymbol{c}$.

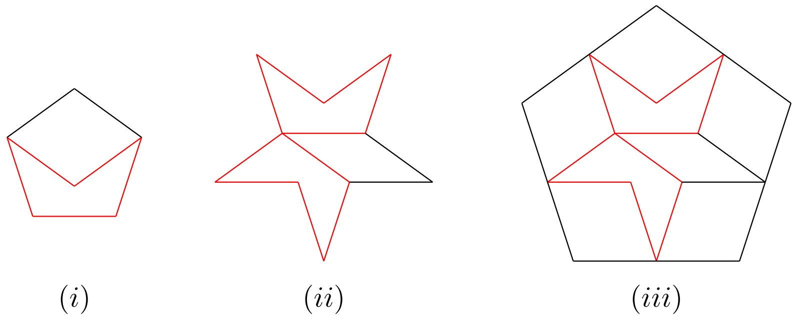

Figure 2: Regular pentagon and pentagram. Concave pentagon (in red).

Figure 2 show regular pentagon and pentagram dissected with rhombi $\boldsymbol{b}$ and $\boldsymbol{c}$ and a concave pentagon (in red). We calculate the areas of the regular pentagon, pentagram in function of rhombi $\boldsymbol{b}, \boldsymbol{c}$. Let x be the concave pentagon area. From the figures we note pentagon P_2 at (iii) is double the side and then four times the area of pentagon at (i) P_1. From the figures we note the area of P_1 equals to $x + \boldsymbol{c}$ and the area of P_2 equals to $2x + \boldsymbol{b} + 5\boldsymbol{c}$, then we compare the pentagons and isolate x to get:

$$\begin{aligned} 4P_1 &= P_2 \\ 4(x + \boldsymbol{c}) &= 2x + \boldsymbol{b} + 5\boldsymbol{c} \\ x &= \frac{\boldsymbol{b} + \boldsymbol{c}}{2} \end{aligned} \tag{1}$$

Then we can calculate the area of the pentagon P_1 and the pentagram P_G shown in figure (ii):

$$P_1 = x + \boldsymbol{c} = \frac{\boldsymbol{b} + \boldsymbol{c}}{2} + \boldsymbol{c} = \frac{\boldsymbol{b} + 3\boldsymbol{c}}{2} \tag{2}$$

$$P_G = 2x + \boldsymbol{b} = \frac{2(\boldsymbol{b} + \boldsymbol{c})}{2} + \boldsymbol{b} = 2\boldsymbol{b} + \boldsymbol{c} \tag{3}$$

2.2 Lenses $\boldsymbol{B}$ and $\boldsymbol{C}$

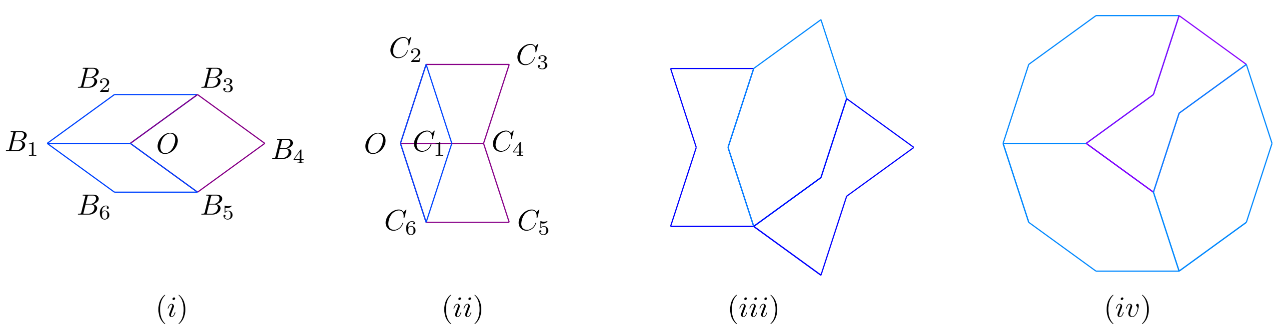

Figure 3: Lenses of types $\boldsymbol{B}$ and $\boldsymbol{C}$.

Figure 3 show lenses $\boldsymbol{B}$ and $\boldsymbol{C}$. Figure (i) show the lense $\boldsymbol{B}$ with perimeter $\overline{B_1 ... B_6}$ which is formed adding two rhombi $\boldsymbol{b}$ and adding one rhombus $\boldsymbol{c}$ so its area equals to $2\boldsymbol{b} + \boldsymbol{c}$. Figure (ii) show the lense $\boldsymbol{C}$ with perimeter $\overline{C_1 ... C_6}$ which is formed adding two rhombi $\boldsymbol{c}$ and substracting one rhombus $\boldsymbol{b}$ so its area equals to $2\boldsymbol{c} - \boldsymbol{b}$. Figure (iii) show a dissected star whose area equals to $2\boldsymbol{C} + \boldsymbol{B} = 5\boldsymbol{c}$. Figure (iv) show a dissected regular decagon whose area equals to $3\boldsymbol{B} + \boldsymbol{C} = 5\boldsymbol{b} + 5\boldsymbol{c}$.