Symmetry 9

https://github.com/heptagons/lenses

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Abstract

Symmetry 9

1 Rhombi

Rhombus	θ_1	θ_2	Area	Symmetry
$\boldsymbol{a}$	$\alpha/2$	$2\alpha/2$	$\sin(\alpha)$	$R_3(\frac{1}{2}, 1)$
$\boldsymbol{b}$	$\beta/2$	$4\beta/2$	$\sin(2\beta)$	$R_5(\frac{1}{2}, 2)$
$\boldsymbol{c}$	$2\beta/2$	$3\beta/2$	$\sin(\beta)$	$R_5(1, \frac{3}{2})$
$\boldsymbol{d}$	$\gamma/2$	$6\gamma/2$	$\sin(3\gamma)$	$R_7(\frac{1}{2}, 3)$
$\boldsymbol{e}$	$2\gamma/2$	$5\gamma/2$	$\sin(\gamma)$	$R_7(1, \frac{5}{2})$
$\boldsymbol{f}$	$3\gamma/2$	$4\gamma/2$	$\sin(2\gamma)$	$R_7(\frac{3}{2}, 2)$
$\boldsymbol{g}$	$\delta/2$	$8\delta/2$	$\sin(4\delta)$	$R_9(\frac{1}{2}, 4)$
$\boldsymbol{h}$	$2\delta/2$	$7\delta/2$	$\sin(\delta)$	$R_9(1, \frac{7}{2})$
$\boldsymbol{a}$	$3\delta/2$	$6\delta/2$	$\sin(3\delta) = \sin(\alpha)$	$R_9(\frac{3}{2}, 3)$
$\boldsymbol{i}$	$4\delta/2$	$5\delta/2$	$\sin(2\delta)$	$R_9(2, \frac{5}{2})$

Table 1: Rhombi for symmetries $\{3, 5, 7, 9\}$ internal angles $\theta_1 < \theta_2$ ($\theta_1 + \theta_2 = \pi$) and areas. $\alpha = 2\pi/3$, $\beta = 2\pi/5$, $\gamma = 2\pi/7$ and $\delta = 2\pi/9$ ($3\delta = \alpha$).

1.1 Stars from rhombi

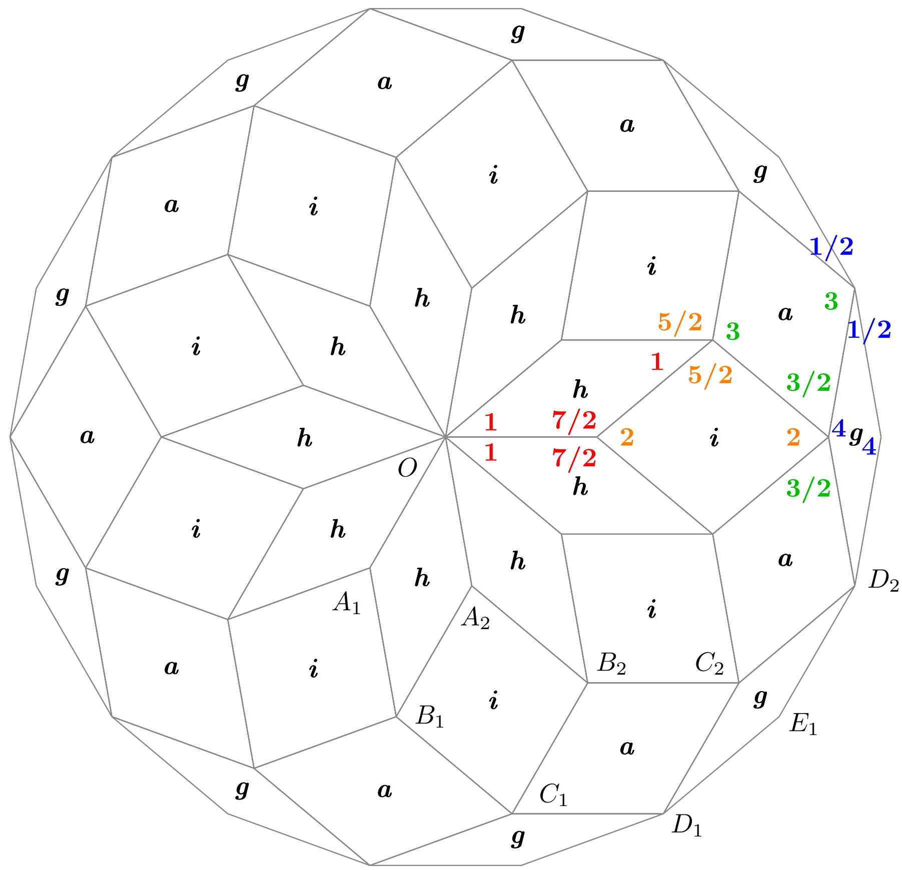

Figure 1: The symmetry 9 four rhombi $\{h, i, a, g\}$ produce the four stars $\{S_G, S_H, S_I, S_J\}$ respectivelly with areas $\{9(h+i+a+g), 9(h+i+a), 9(h+i), 9h)\}$.

2 Hexagons

2.1 Hexagons from stars

Figure 2 show the disposition of the symmetry 9 four stars $\{S_G, S_H, S_I, S_J\}$. We denote the 18 vertices of every star as $\{X_0, X_1, ..., X_{17}\}$ where $X = \{G, H, I, J\}$. Only some vertices are labeled in the figure. First we make coincident at vertice O all the vertices G_0, H_0, I_0, J_0. With the center at O we rotate star S_H to make coincident vertices G_{17} and H_{17}. Similarly we rotate stars S_I and S_J to make coincident vertices G_{17} and I_{17} and vertices G_{17} and J_{17}. The rotations also joined another different vertices.

First we add three new edges (in red) joining the stars S_G and S_H vertices: $\overline{G_3H_2}$, $\overline{G_5H_4}$ and $\overline{G_7H_6}$ dissecting the red region into four hexagons, two of them essentially different. The three consective angles of the two hexagons are shown: (**1,4,4**) and (**3,4,2**).

Then we add three new edges (in orange) joining the stars S_H and S_I vertices: $\overline{H_3I_2}$, $\overline{H_5I_4}$ and $\overline{H_7I_6}$ dissecting the orange region into four hexagons, two of them new. The three consective angles of the the two hexagons are show: (**1,5,3**) and (**3,3,3**).

Finally we add three more edges (in green) joining the stars S_I and S_J vertices: $\overline{I_3J_2}$, $\overline{I_5J_4}$ and $\overline{I_7J_6}$ dissecting the green region into four hexagons, two of them new. The three consective angles of the the two hexagons are show: (**1,6,2**) and (**2,5,2**).

The three consecutive angles of the hexagons are of the form (a, b, c) where $a + b + c = 9$. Table 2

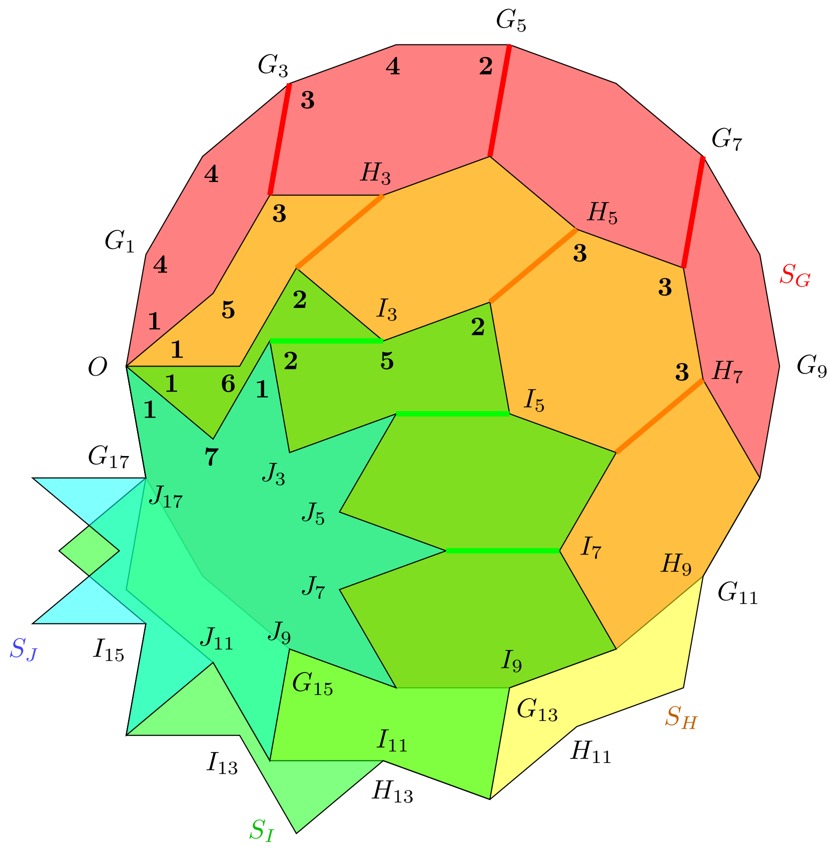

Figure 2: Symmetry 9 stars $\{S_G, S_H, S_I, S_J\}$ dissected with vectors to get symmetry-9 hexagonal hexagons.

Hexagon	**a**	**b**	**c**	Details
$H_9(1,1)$	1	1	7	self-intersecting
$H_9(1,2)$	1	2	6	Lense $\boldsymbol{H}^+$
$H_9(1,3)$	1	3	5	Lense $\boldsymbol{H}$
$H_9(1,4)$	1	4	4	Lense $\boldsymbol{G}$
$H_9(2,2)$	2	2	5	Lense $\boldsymbol{J}$
$H_9(2,3)$	2	3	4	Lense $\boldsymbol{I}$
$H_9(3,3)$	3	3	3	Lense $\boldsymbol{A}$ equal to $H_3(1,1)$

Table 2: Symmetry 9 hexagons with angles factors $a \leq b \leq c$.