Symmetry 9

https://github.com/heptagons/lenses

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Abstract

Symmetry 9

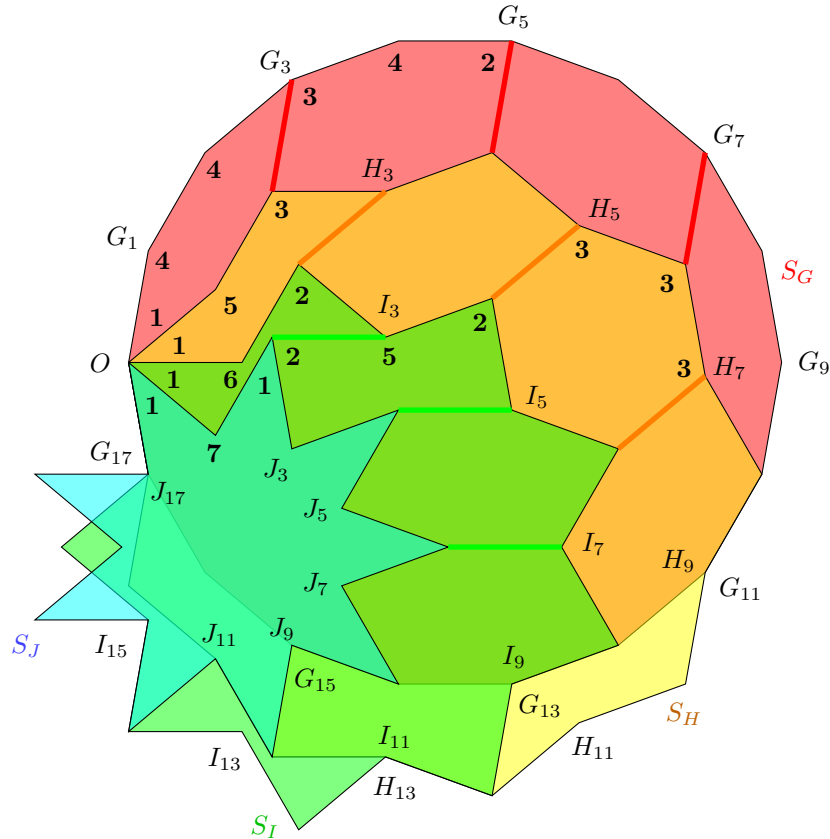# 1 Rhombus

2 Lenses

2.1 Lenses from stars

Figure 1: Symmetry 9 stars $\{S_G, S_H, S_I, S_J\}$ dissected with vectors to get symmetry-9 hexagonal lenses.

Figure 1 show the disposition of the symmetry 9 four stars $\{S_G, S_H, S_I, S_J\}$. We denote the 18 vertices of every star as $\{X_0, X_1, ..., X_{17}\}$ where $X = \{G, H, I, J\}$. Only some vertices are labeled in the figure. First we make coincident at vertice O all the vertices G_0, H_0, I_0, J_0. With the center at O we rotate star S_H

to make coincident vertices G_{17} and H_{17}. Similarly we rotate stars S_I and S_J to make coincident vertices G_{17} and I_{17} and vertices G_{17} and J_{17}. The rotations also joined another different vertices.

First we add three new edges (in red) joining the stars S_G and S_H vertices: $\overline{G_3H_2}$, $\overline{G_5H_4}$ and $\overline{G_7H_6}$ dissecting the red region into four hexagons, two of them essentially different. The three consective angles of the two hexagons are shown: (**1,4,4**) and (**3,4,2**).

Then we add three new edges (in orange) joining the stars S_H and S_I vertices: $\overline{H_3I_2}$, $\overline{H_5I_4}$ and $\overline{H_7I_6}$ dissecting the orange region into four hexagons, two of them new. The three consective angles of the the two hexagons are show: (**1,5,3**) and (**3,3,3**).

Finally we add three more edges (in green) joining the stars S_I and S_J vertices: $\overline{I_3J_2}$, $\overline{I_5J_4}$ and $\overline{I_7J_6}$ dissecting the green region into four hexagons, two of them new. The three consective angles of the the two hexagons are show: (**1,6,2**) and (**2,5,2**).

The three consecutive angles of the hexagons are of the form (a, b, c) where $a + b + c = 9$. Table 1

Hexagon	a	b	c	Details
$H_9(1,1)$	1	1	7	self-intersecting
$H_9(1,2)$	1	2	6	Lense $\boldsymbol{H^+}$
$H_9(1,3)$	1	3	5	Lense $\boldsymbol{H}$
$H_9(1,4)$	1	4	4	Lense $\boldsymbol{G}$
$H_9(2,2)$	2	2	5	Lense $\boldsymbol{J}$
$H_9(2,3)$	2	3	4	Lense $\boldsymbol{I}$
$H_9(3,3)$	3	3	3	Lense $\boldsymbol{A}$ equal to $H_3(1,1)$

Table 1: Symmetry 9 hexagons with angles factors $a \leq b \leq c$.