

# Lenses

<https://github.com/heptagons/lenses>

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## Abstract

Lenses are equilateral hexagons resembling concave and convex optical lenses. Lenses consecutive six internal angles are  $(\theta_1, \theta_2, \theta_3, \theta_1, \theta_2, \theta_3)$  where  $\theta_1 = X\theta_0$ ,  $\theta_2 = Y\theta_0$ , and  $\theta_3 = Z\theta_0$  where  $\theta_0 = 2\pi/S$  is the base angle of symmetry  $S = X + Y + Z$ .

## 1 Lenses

## 2 Stars

## 3 Symmetry 5

Symmetry 5 is based in angle  $\beta = \frac{2\pi}{5}$  and produces the two rhombi  $(\mathbf{b}, \mathbf{c})$  and the two lenses  $(\mathbf{B}, \mathbf{C})$ .

### 3.1 Rhombi $(\mathbf{b}, \mathbf{c})$

Rhombus	$\theta_1$	$\theta_2$
$\mathbf{b}$	$\beta/2$	$4\beta/2$
$\mathbf{c}$	$2\beta/2$	$3\beta/2$

Table 1: Rhombi  $(\mathbf{b}, \mathbf{c})$  internal angles.  $\theta_1 + \theta_2 = \pi$  and  $\beta = 2\pi/5$ .

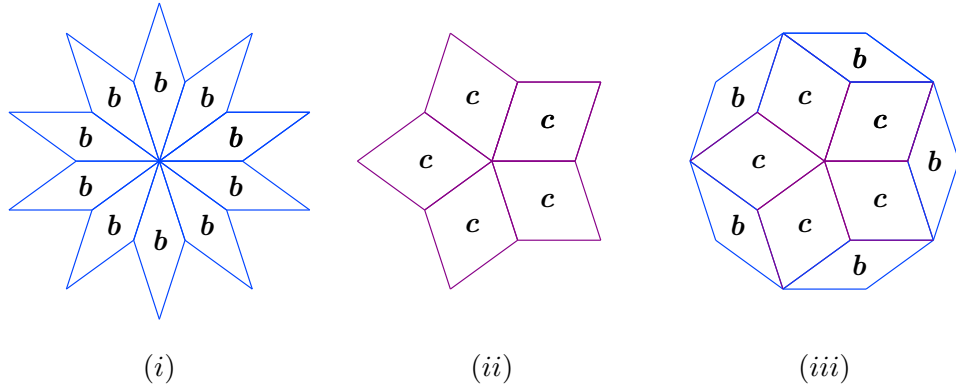


Figure 1: Rhombi  $(\mathbf{b}, \mathbf{c})$  from dissecting stars  $S_{10}$ .

Table 1 show the rhombi  $(\mathbf{b}, \mathbf{c})$  internal angles in terms of angle  $\beta = 2\pi/5$ . Figure 1 show the rhombi  $(\mathbf{b}, \mathbf{c})$ . Inspecting the stars we get the areas simply adding their rhombi. At (i) the star  $S_{10}(1,8)$  with area

$A = 10\mathbf{b}$ . At (ii) the star  $S_{10}(2, 6) = S_5(1, 3)$  with area  $A = 5\mathbf{c}$ . At (iii) the regular decagon equivalent to stars  $S_{10}(4, 4) = S_5(2, 2)$  with area  $A = 5\mathbf{b} + 5\mathbf{c}$ .

### 3.2 Regular pentagon and star $|5/2|$

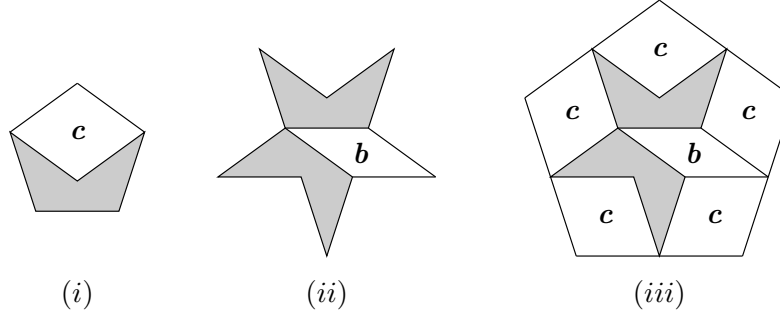


Figure 2: Regular pentagon  $|5/1|$  at (i). Star  $|5/2|$  at (ii). Double pentagon at (iii).

Figure 2 show regular pentagon and isotoxal star  $|5/2|$  dissected with rhombi  $(\mathbf{b}, \mathbf{c})$  plus concave pentagons (in gray). Let  $\mathbf{x}$  be the area of such gray piece. By inspection the area of regular pentagon at (i) is  $A_1 = \mathbf{c} + \mathbf{x}$  and the area of regular pentagon at (iii) is  $P_2 = \mathbf{b} + 5\mathbf{c} + 2\mathbf{x}$ . Since the side of  $A_2$  is the double of  $A_1$  its area is four times so we can get the value of  $\mathbf{x}$  in terms of  $(\mathbf{b}, \mathbf{c})$

$$\begin{aligned} 4P_1 &= P_4 \\ 4(\mathbf{c} + \mathbf{x}) &= \mathbf{b} + 5\mathbf{c} + 2\mathbf{x} \\ \mathbf{x} &= \frac{\mathbf{b} + \mathbf{c}}{2} \end{aligned} \tag{1}$$

We use the value of  $\mathbf{x}$  to get the areas of pentagon (i) and star (ii):

$$\begin{aligned} A|5/1| &= \mathbf{c} + \mathbf{x} \\ &= \frac{\mathbf{b} + 3\mathbf{c}}{2} \end{aligned} \tag{2}$$

$$\begin{aligned} A|5/2| &= \mathbf{b} + 2\mathbf{x} \\ &= 2\mathbf{b} + \mathbf{c} \end{aligned} \tag{3}$$

### 3.3 Lenses $(\mathbf{B}, \mathbf{C})$

Lense	$\theta_1$	$\theta_2$	$\theta_3$
$\mathbf{B}$	$\beta$	$2\beta$	$2\beta$
$\mathbf{C}$	$\beta$	$\beta$	$3\beta$

Table 2: Lenses  $(\mathbf{B}, \mathbf{C})$  internal angles  $\theta_1 + \theta_2 + \theta_3 = 2\pi$  where  $\beta = 2\pi/5$ .

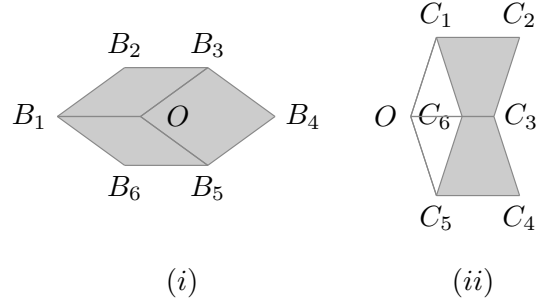


Figure 3: Lenses ( $\mathbf{B}$ ,  $\mathbf{C}$ ) build with rhombi ( $\mathbf{b}$ ,  $\mathbf{c}$ ).

Table 2 show the lenses ( $\mathbf{B}$ ,  $\mathbf{C}$ ) internal angles. Figure 3 show lenses ( $\mathbf{B}$ ,  $\mathbf{C}$ ) construction and two stars formed with them. At (i) we form the lense  $\mathbf{B}$  with perimeter  $\overline{B_1...B_6}$  adding two rhombi  $\mathbf{b}$  ( $\overline{B_1B_2B_3O}$  and  $\overline{B_1OB_5B_6}$ ) and adding one rhombus  $\mathbf{c}$  ( $\overline{OB_3B_4B_5}$ ) so its area is  $2\mathbf{b} + \mathbf{c}$ . Lense  $\mathbf{B}$  is equivalent to the hexagon  $H_5(1, 2, 2)$ . At (ii) we form the lense  $\mathbf{C}$  with perimeter  $\overline{C_1...C_6}$  adding two rhombi  $\mathbf{c}$  ( $\overline{OC_1C_2C_3}$  and  $\overline{OC_3C_4C_5}$ ) and substracting one rhombus  $\mathbf{b}$  ( $\overline{OC_1C_6C_5}$ ) so its area is  $2\mathbf{c} - \mathbf{b}$ . Lense  $\mathbf{C}$  is equivalent to the hexagon  $H_5(1, 1, 3)$ .

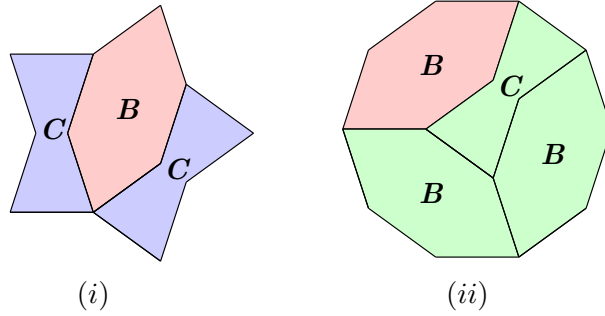


Figure 4: Two stars dissected with lenses ( $\mathbf{B}$ ,  $\mathbf{C}$ ).

Figure 4 show two stars dissected with lenses ( $\mathbf{B}$ ,  $\mathbf{C}$ ). At (i) the star  $S_5(1, 3)$  dissection implies its area is  $A = \mathbf{B} + 2\mathbf{C} = 5\mathbf{c}$ . At (ii) the regular decagon or star  $S_5(2, 2)$  dissection implies its area is  $A = 3\mathbf{B} + \mathbf{C} = 5(\mathbf{b} + \mathbf{c})$ .

## 4 Symmetry 7

Symmetry 7 is based in angle  $\gamma = \frac{2\pi}{7}$  and produces the three rhombi ( $\mathbf{d}$ ,  $\mathbf{f}$ ,  $\mathbf{e}$ ) and the three lenses ( $\mathbf{D}$ ,  $\mathbf{E}$ ,  $\mathbf{F}$ ).

### 4.1 Rhombi ( $\mathbf{d}$ , $\mathbf{e}$ , $\mathbf{f}$ )

Rhombus	$\theta_1$	$\theta_2$
$\mathbf{d}$	$\gamma/2$	$6\gamma/2$
$\mathbf{e}$	$2\gamma/2$	$5\gamma/2$
$\mathbf{f}$	$3\gamma/2$	$4\gamma/2$

Table 3: Rhombi ( $\mathbf{d}$ ,  $\mathbf{e}$ ,  $\mathbf{f}$ ) internal angles.  $\theta_1 + \theta_2 = \pi$  and  $\gamma = 2\pi/7$ .

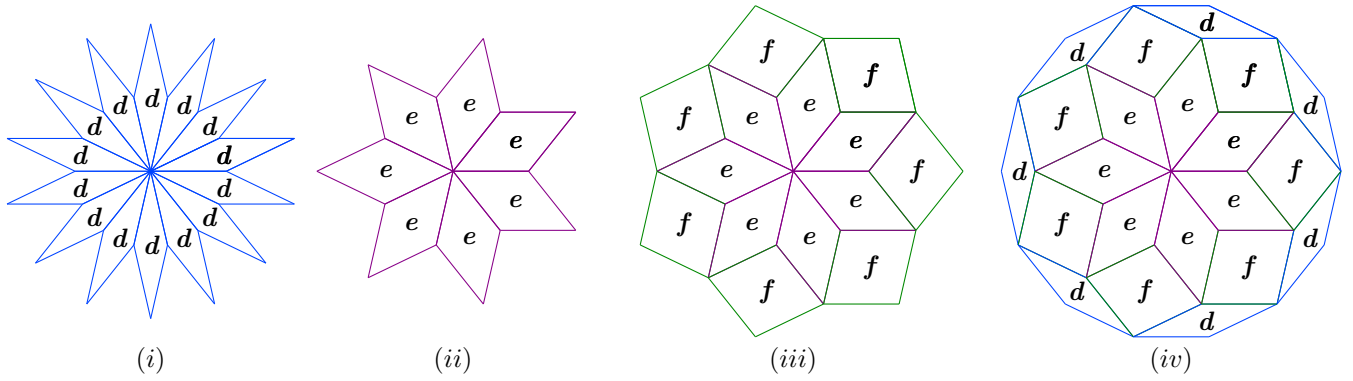


Figure 5: Rhombi  $(d, e, f)$  from dissected stars  $S_{14}$ .

Table 3 show the symmetry 7 lenses internal angles based in angle  $\gamma = 2\pi/7$ . Figure 5 show rhombi  $(d, e, f)$ . Inspecting the stars we get the areas simply adding their rhombi. At (i) the star  $S_{14}(1, 12)$  with area  $A = 14d$ . At (ii) the star  $S_{14}(2, 10) = S_7(1, 5)$  with area  $A = 7e$ . At (iii) the star  $S_{14}(4, 8) = S_7(2, 4)$  with area  $A = 7(e + f)$ . At (iv) the regular 14-gon equivalent to stars  $S_{14}(6, 6) = S_7(3, 3)$  with area  $A = 7(d + e + f)$ .

#### 4.2 Regular heptagon and stars $|7/3|$ and $|7/2|$

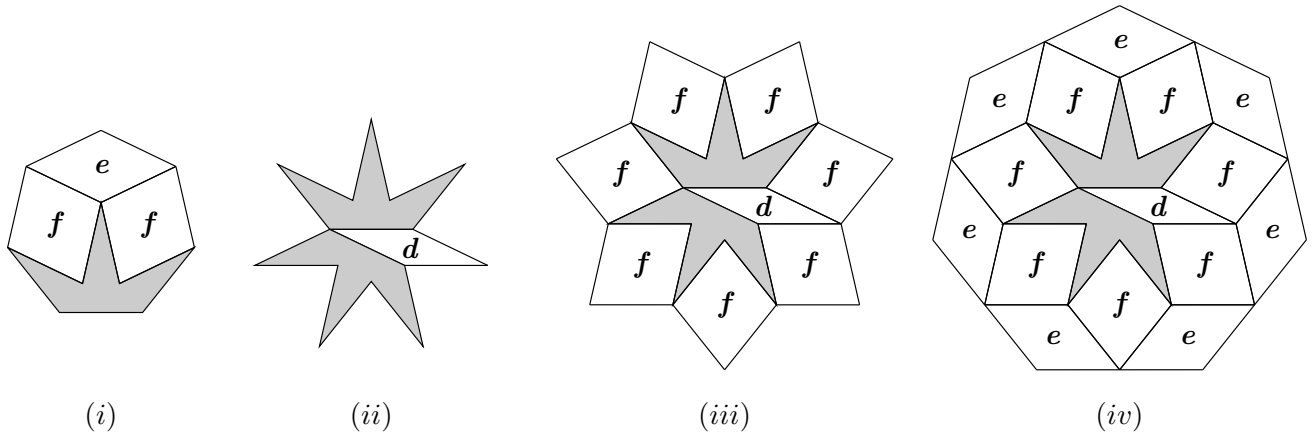


Figure 6: Heptagon  $|7/1|$  at (i). Star  $|7/3|$  at (ii). Star  $|7/2|$  at (iii). Double heptagon at (iv).

Figure 6 show regular heptagon and heptagrams dissected with rhombi  $(c, d, e)$  plus equilateral concave heptagons (in gray). Let  $y$  be the area of such gray piece. By inspection the area of regular heptagon at (i) is  $A_1 = e + 2f + y$  while the area of regular heptagon at (iv) is  $A_2 = d + 7(e + f) + 2y$ . Since the side of  $A_2$  is the double of  $A_1$  its area is four times so we can get the value of  $y$  in terms of  $(d, e, f)$ :

$$\begin{aligned}
 4A_1 &= A_2 \\
 4(e + 2f + y) &= d + 7(e + f) + 2y \\
 y &= \frac{d + 3e - f}{2}
 \end{aligned} \tag{4}$$

We use the value of  $y$  to calculate the areas of heptagon (i) and stars (ii) and (iii) in terms of  $(d, e, f)$ :

$$\begin{aligned} A|7/1| &= e + 2f + y \\ &= \frac{d + 5e + 3f}{2} \end{aligned} \quad (5)$$

$$\begin{aligned} A|7/3| &= d + 2y \\ &= 2d + 3e - f \end{aligned} \quad (6)$$

$$\begin{aligned} A|7/2| &= A\{7/3\} + 7f \\ &= 2d + 3e + 6f \end{aligned} \quad (7)$$

### 4.3 Lenses $(D, E, F)$

Lense	$\theta_1$	$\theta_2$	$\theta_3$
<b>D</b>	$\gamma$	$3\gamma$	$3\gamma$
<b>E</b>	$\gamma$	$2\gamma$	$4\gamma$
<b>F</b>	$2\gamma$	$2\gamma$	$3\gamma$

Table 4: Lenses  $(D, E, F)$  internal angles.  $\theta_1 + \theta_2 + \theta_3 = 2\pi$  and  $\gamma = 2\pi/7$ .

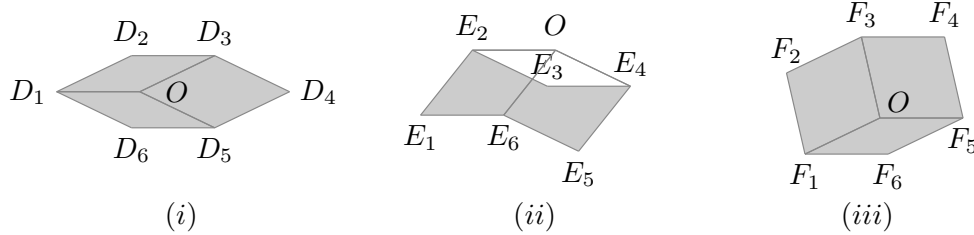


Figure 7: Lenses  $(D, E, F)$  build from rhombi  $(d, e, f)$ .

Table 4 show the lenses  $(D, E, F)$  internal angles. Figure 7 show lenses  $(B, C)$  construction. At (i) we form the lense **D** with perimeter  $\overline{D_1...D_6}$  adding two rhombi  $d$  ( $\overline{D_1D_2D_3O}$  and  $\overline{D_1OD_5D_6}$ ) and adding one rhombus  $e$  ( $\overline{OD_3D_4D_5}$ ) so its area is  $2d + e$ . Lense **D** is equivalent to the hexagon  $H_7(1, 3, 3)$ .

At (ii) we form the lense **E** with perimeter  $\overline{E_1...E_6}$  adding one rhombus  $e$  ( $\overline{E_1E_2OE_6}$ ) adding one rhombus  $f$  ( $\overline{OE_4E_5E_6}$ ) and subtracting one rhombus  $d$  ( $\overline{E_2OE_4E_3}$ ) so its area is  $-d + e + f$ . Lense **E** is equivalent to the hexagon  $H_7(1, 2, 4)$ .

At (iii) we form the lense **F** with perimeter  $\overline{F_1...F_6}$  adding two rhombi  $f$  ( $\overline{F_1F_2F_3O}$  and  $\overline{F_3F_4F_5O}$ ) and adding one rhombus  $d$  ( $\overline{F_1OF_5F_6}$ ) so its area is  $d + 2f$ . Lense **F** is equivalent to the hexagon  $H_7(2, 2, 3)$ .