

Symmetry 9

<https://github.com/heptagons/lenses>

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Abstract

Symmetry 9

1 Rhombi

Rhombus	Name	θ_1	θ_2	Area
$R_3(\frac{1}{2}, 1)$	a	$\alpha/2$	$2\alpha/2$	$\sin(\alpha) \approx 0.866$
$R_5(\frac{1}{2}, 2)$	b	$\beta/2$	$4\beta/2$	$\sin(2\beta) \approx 0.587$
$R_5(1, \frac{3}{2})$	c	$2\beta/2$	$3\beta/2$	$\sin(\beta) \approx 0.951$
$R_7(\frac{1}{2}, 3)$	d	$\gamma/2$	$6\gamma/2$	$\sin(3\gamma) \approx 0.433$
$R_7(1, \frac{5}{2})$	e	$2\gamma/2$	$5\gamma/2$	$\sin(\gamma) \approx 0.781$
$R_7(\frac{3}{2}, 2)$	f	$3\gamma/2$	$4\gamma/2$	$\sin(2\gamma) \approx 0.974$
$R_9(\frac{1}{2}, 4)$	g	$\delta/2$	$8\delta/2$	$\sin(4\delta) \approx 0.342$
$R_9(1, \frac{7}{2})$	h	$2\delta/2$	$7\delta/2$	$\sin(\delta) \approx 0.642$
$R_9(\frac{3}{2}, 3)$	a	$3\delta/2$	$6\delta/2$	$\sin(3\delta) \approx 0.866$
$R_9(2, \frac{5}{2})$	i	$4\delta/2$	$5\delta/2$	$\sin(2\delta) \approx 0.984$

Table 1: Rhombi for symmetries $\{3, 5, 7, 9\}$ internal angles $\theta_1 < \theta_2$ ($\theta_1 + \theta_2 = \pi$) and areas. $\alpha = 2\pi/3$, $\beta = 2\pi/5$, $\gamma = 2\pi/7$ and $\delta = 2\pi/9$ ($3\delta = \alpha$).

Star	Name	Area	Polygon
$S_3(\frac{1}{2}, 1)$	-	$6a$	$ 6/2 $ hexagram
$S_3(1, 1)$	\mathcal{A}	$3a$	Regular hexagon
$S_5(\frac{1}{2}, 4)$	-	$5b$	$ 10/4 $ decagram
$S_5(1, 3)$	\mathcal{B}	$5c$	$ (5/2)_\alpha $ decagram
$S_5(2, 2)$	\mathcal{C}	$5(c + b)$	Regular decagon
$S_7(\frac{1}{2}, 6)$	-	$7d$	$ 14/6 $ 14-gram
$S_7(1, 5)$	\mathcal{D}	$7e$	$ (7/4)_\alpha $ 14-gram
$S_7(2, 4)$	\mathcal{E}	$7(e + f)$	$ (7/2)_\alpha $ 14-gram
$S_7(3, 3)$	\mathcal{F}	$7(e + f + d)$	Regular 14-gon
$S_9(\frac{1}{2}, 7)$	-	$9g$	$ 18/8 $ 18-gram
$S_9(1, 6)$	\mathcal{G}	$9h$	$ (9/6)_\alpha $ 18-gram
$S_9(2, 5)$	\mathcal{H}	$9(h + i)$	$ (9/4)_\alpha $ 18-gram
$S_9(3, 4)$	\mathcal{I}	$9(h + i + a)$	$ (9/2)_\alpha $ 18-gram
$S_9(4, 4)$	\mathcal{J}	$9(h + i + a + g)$	Regular 18-gon

Table 2: Stars $\{\mathcal{A}, \mathcal{B}, \dots, \mathcal{J}\}$ for symmetries $\{3, 5, 7, 9\}$.

1.1 Stars from rhombi

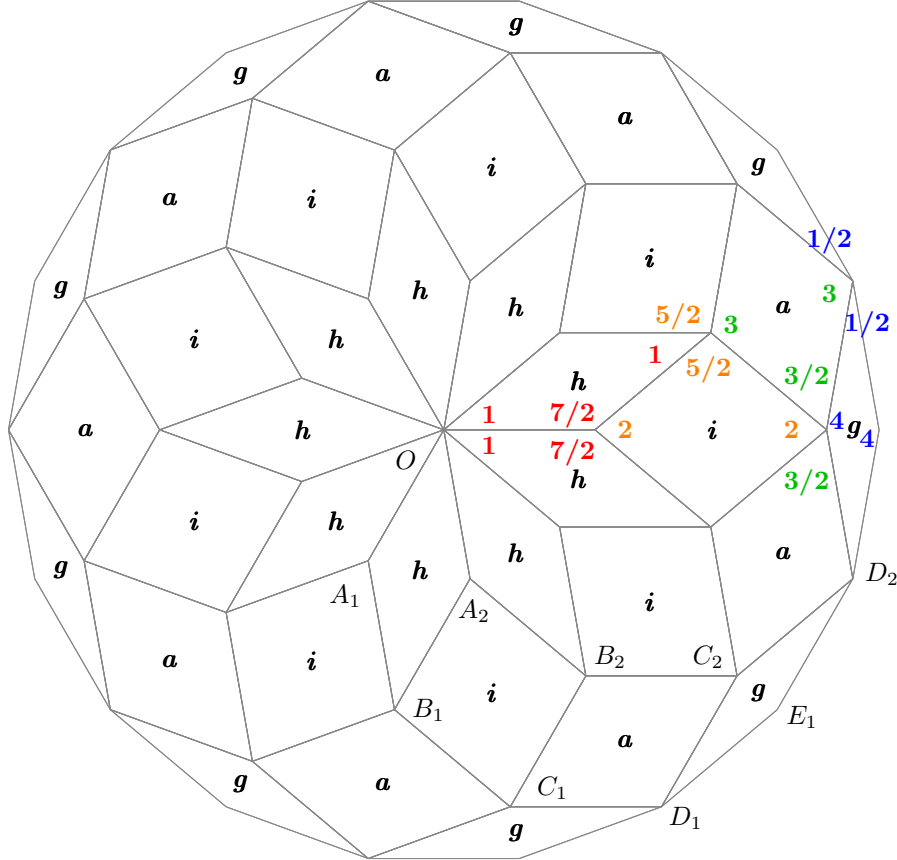


Figure 1: The symmetry 9 four rhombi $\{h, i, a, g\}$ produce the four stars $\{\mathcal{G}, \mathcal{H}, \mathcal{I}, \mathcal{J}\}$ with areas $9h$, $9(h + i)$, $9(h + i + a)$ and $9(h + i + a + g)$ respectively.

Figure 1 show nine copies of symmetry-9 rhombi $\{h, i, a, g\}$ to form four stars.

2 Hexagons

2.1 Hexagons from stars

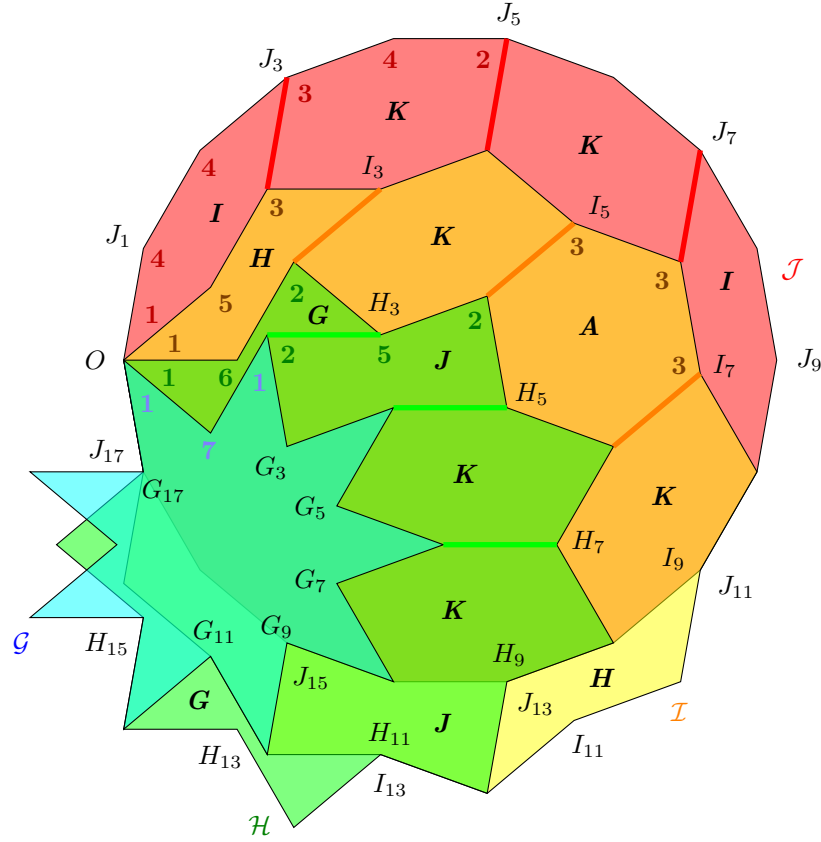


Figure 2: Symmetry 9 stars $\{\mathcal{G}, \mathcal{H}, \mathcal{I}, \mathcal{J}\}$ dissected to get the six hexagons $\{G, H, I, J, K, A\}$.

Figure 2 show the disposition of the symmetry 9 four stars. We denote the 18 vertices of stars $\{\mathcal{G}, \mathcal{H}, \mathcal{I}, \mathcal{J}\}$ as $\{G_0, G_1, \dots, G_{17}\}$, $\{H_0, H_1, \dots, H_{17}\}$, $\{I_0, I_1, \dots, I_{17}\}$ and $\{J_0, J_1, \dots, J_{17}\}$ respectively. For simplification only some vertices are labeled in the figure. First we make coincident at vertex O all the vertices G_0, H_0, I_0, J_0 . With the center at O we rotate all stars to make coincidents G_{17}, H_{17}, I_{17} and J_{17} . The rotations also joined another different vertices.

First we add three new edges (in red) joining the stars \mathcal{J} and \mathcal{I} vertices: $\overline{J_3 I_2}$, $\overline{J_5 I_4}$ and $\overline{J_7 I_6}$ dissecting the red region into four hexagons, two of them essentially different. The three consecutive angles of the two hexagons are shown: **I** (1,4,4) and **K** (3,4,2).

Then we add three new edges (in orange) joining the stars \mathcal{I} and \mathcal{H} vertices: $\overline{I_3 H_2}$, $\overline{I_5 H_4}$ and $\overline{I_7 H_6}$ dissecting the orange region into four hexagons, two of them new. The three consecutive angles of the the two hexagons are show: **H** (1,5,3) and **A** (3,3,3).

Finally we add three more edges (in green) joining the stars \mathcal{H} and \mathcal{G} vertices: $\overline{H_3 G_2}$, $\overline{H_5 G_4}$ and $\overline{H_7 G_6}$ dissecting the green region into four hexagons, two of them new. The three consecutive angles of the the two hexagons are show: **G** (1,6,2) and **J** (2,5,2).

2.2 Hexagons from rhombi

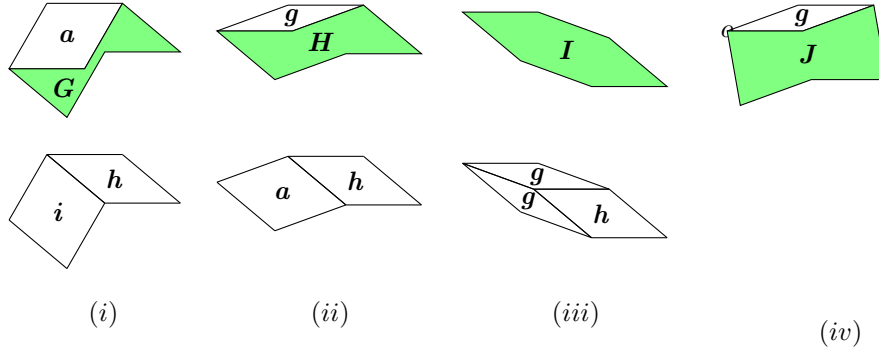


Figure 3: .

2.3 Hexagons angles

The three consecutive angles of the hexagons are of the form (a, b, c) where $a + b + c = 9$. Table 3

Hexagon	Name	(a, b, c)	Polygon
$H_3(1, 1)$	A	(1, 1, 1)	Regular hexagon
$H_5(1, 1)$	B	(1, 1, 3)	Sormeh Dan Girih tile
$H_5(1, 2)$	C	(1, 2, 2)	Shesh Band Girih tite
$H_7(1, 1)$	-	(1, 1, 5)	self-intersecting
$H_7(1, 2)$	D	(1, 2, 4)	
$H_7(1, 3)$	E	(1, 3, 3)	
$H_7(2, 2)$	F	(2, 2, 3)	
$H_9(1, 1)$	-	(1, 1, 7)	self-intersecting
$H_9(1, 2)$	G	(1, 2, 6)	
$H_9(1, 3)$	H	(1, 3, 5)	
$H_9(1, 4)$	I	(1, 4, 4)	
$H_9(2, 2)$	J	(2, 2, 5)	
$H_9(2, 3)$	K	(2, 3, 4)	
$H_9(3, 3)$	A	(3, 3, 3)	symmetry 3 hexagon

Table 3: Hexagons of symmetries $\{3, 5, 7, 9\}$ with angles factors $\mathbf{a} \leq \mathbf{b} \leq \mathbf{c}$.