

Meccano pentagons gallery

<https://github.com/heptagons/meccano/penta>

Abstract

We show constructions of small meccano pentagons from side 12 to 3. We restrict all internal strips remain internal and do not overlap.

1 Pentagons of size 12

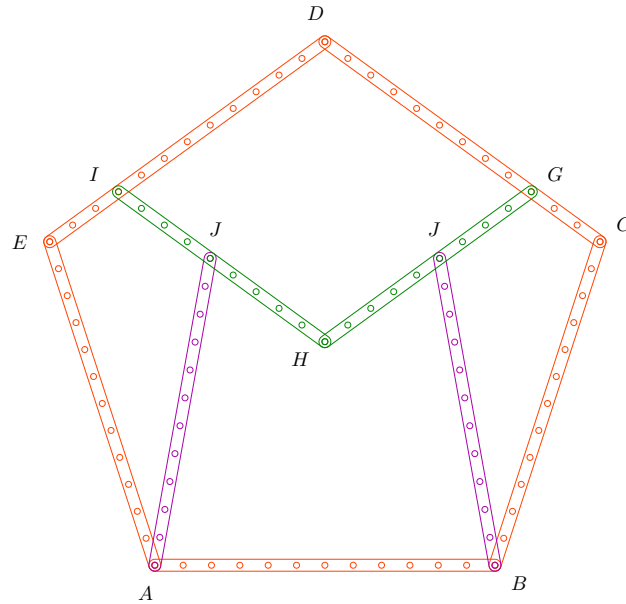


Figure 1: Pentagon size 12.

From figure 1 suppose we have a regular pentagon of side 12 A, B, C, D, E and a rhombus D, I, H, G of side 9. Assume vertex A is at coordinates $(0,0)$ and calculate relative coordinates of vertex J . First we calculate the abscissas going through vertices A, E, I, J subtracting when we move to the left and adding when we move to the right:

$$AJ_x = AE_x + EI_x + IJ_x \quad (1)$$

$$\begin{aligned}
&= -\overline{AE} \cos\left(\frac{2\pi}{5}\right) + \overline{EI} \cos\left(\frac{\pi}{5}\right) + \overline{IJ} \cos\left(\frac{\pi}{5}\right) \\
&= -12 \left(\frac{\sqrt{5}-1}{4}\right) + 3 \left(\frac{1+\sqrt{5}}{4}\right) + 4 \left(\frac{1+\sqrt{5}}{4}\right) = \frac{19-5\sqrt{5}}{4}
\end{aligned} \tag{2}$$

Then we calculate the ordinates going to the same order of vertices adding when we go up and subtracting when we go down:

$$AJ_y = AE_y + EI_y + IJ_y \quad (3)$$

$$\begin{aligned} &= \overline{AE} \sin\left(\frac{2\pi}{5}\right) + \overline{EI} \sin\left(\frac{\pi}{5}\right) - \overline{IJ} \sin\left(\frac{\pi}{5}\right) \\ &= 12 \left(\frac{\sqrt{10+2\sqrt{5}}}{4} \right) + 3 \left(\frac{\sqrt{10-2\sqrt{5}}}{4} \right) - 4 \left(\frac{\sqrt{10-2\sqrt{5}}}{4} \right) \\ &= \frac{12\sqrt{10+2\sqrt{5}} - \sqrt{10-2\sqrt{5}}}{4} = \frac{\sqrt{1450+190\sqrt{5}}}{4} \end{aligned} \quad (4)$$

Finally we calculate the distance \overline{AJ} wich coincides with strip size 11:

$$\overline{AJ} = \sqrt{(AJ_x)^2 + (AJ_y)^2} \quad (5)$$

$$\begin{aligned} &= \sqrt{\left(\frac{19-5\sqrt{5}}{4} \right)^2 + \frac{1450+190\sqrt{5}}{16}} \\ &= \sqrt{\frac{486-190\sqrt{5}}{16} + \frac{1450+190\sqrt{5}}{16}} = \sqrt{121} = 11 \end{aligned} \quad (6)$$