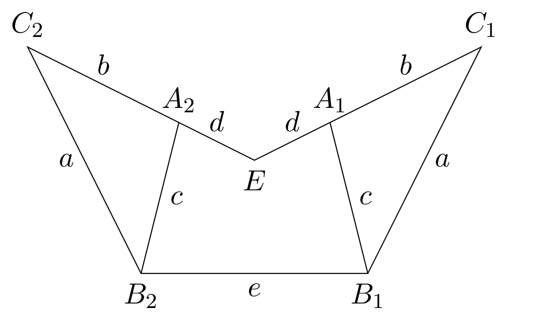
Horns unit

https://github.com/heptagons/meccano/units/horns

Abstract

Horns unit is a group of seven meccano [1] strips intended to build polygons.



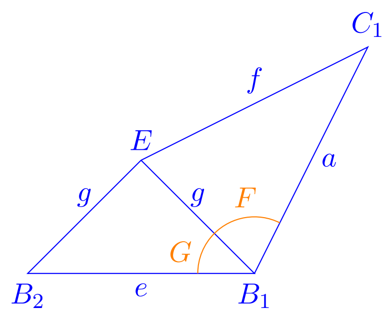


Figure 1: The **horn unit** has seven strips: Two of length a, two of length $b+d$, two of length c and one of length e. We expect to build polygons with internal angle $C_1B_1B_2$ and perimeter including segments a, e, a.

1 Algebra

From figure 1 we start with triangle $\triangle A_1B_1C1$. At vertex A_1 we have angle A and the supplement A':

$$A \equiv \angle B_1A_1C_1 \tag{1}$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc} \qquad \text{if and only if } a < b+c \tag{2}$$

$$A' \equiv \angle EA_1B_1 = \pi - A \tag{3}$$

$$\cos A' = \cos(\pi - A) = -\cos A = \frac{-b^2-c^2+a^2}{2bc} \tag{4}$$

We define $f = b + d$ and $g \equiv \overline{EB_1}$ and with the law of cosines we have:

$$f \equiv b+d \tag{5}$$

$$g^2 = c^2+d^2-2cd\cos A' \tag{6}$$

$$= c^2+d^2-(2cd)\frac{-b^2-c^2+a^2}{2bc} \tag{7}$$

$$= \frac{bc^2+bd^2+b^2d+c^2d-a^2d}{b} \tag{8}$$

$$= \frac{(b+d)(bd+c^2)-a^2d}{b} \tag{9}$$

$$= \frac{(bd+c^2)f-a^2d}{b} \equiv \frac{h}{b} \qquad \text{if and only if } 0 < h < b \tag{10}$$

[1] Meccano mathematics by ‘t Hooft

We sum the angles F and G to get:

$$F \equiv \angle C_1 B1E \tag{11}$$

$$\cos F = \frac{a^2 + g^2 - f^2}{2ag} = \boxed{\frac{a^2 b - bf^2 + h}{2abg}} \tag{12}$$

$$G \equiv \angle B_2 B_1 E \tag{13}$$

$$\cos G = \boxed{\frac{e}{2g}} \tag{14}$$

$$F + G \equiv \angle B_2 B_1 C_1 \tag{15}$$

$$\cos(F + G) = \cos F \cos G - \sin F \sin G \tag{16}$$

We calculate cosines squared and multiplied:

$$\cos^2 F = \frac{(a^2 b - bf^2 + h)^2}{4a^2 b^2 g^2} = \boxed{\frac{(a^2 b - bf^2 + h)^2}{4a^2 bh}} \tag{17}$$

$$\cos^2 G = \frac{e^2}{4g^2} = \boxed{\frac{be^2}{4h}} \tag{18}$$

$$\cos^2 F \cos^2 G = \frac{be^2 (a^2 b - bf^2 + h)^2}{16a^2 bh^2} = \boxed{\frac{e^2 (a^2 b - bf^2 + h)^2}{16a^2 h^2}} \tag{19}$$

$$\cos F \cos G = \boxed{\frac{e(a^2 b - bf^2 + h)}{4ah}} \tag{20}$$

We calculate the sines part squared. Replace g^2 with h/b first in the denominator:

$$(\sin F \sin G)^2 = (1 - cos^2 F)(1 - cos^2 G) \tag{21}$$

$$= cos^2 F \cos^2 G - cos^2 F - cos^2 G + 1 \tag{22}$$

$$= \frac{e^2 (a^2 b - bf^2 + h)^2}{16a^2 h^2} - \frac{(a^2 b - bf^2 + h)^2}{4a^2 bh} - \frac{be^2}{4h} + 1 \tag{23}$$