Meccano heptagons

https://github.com/heptagons/meccano/hepta

1 Meccano heptagons

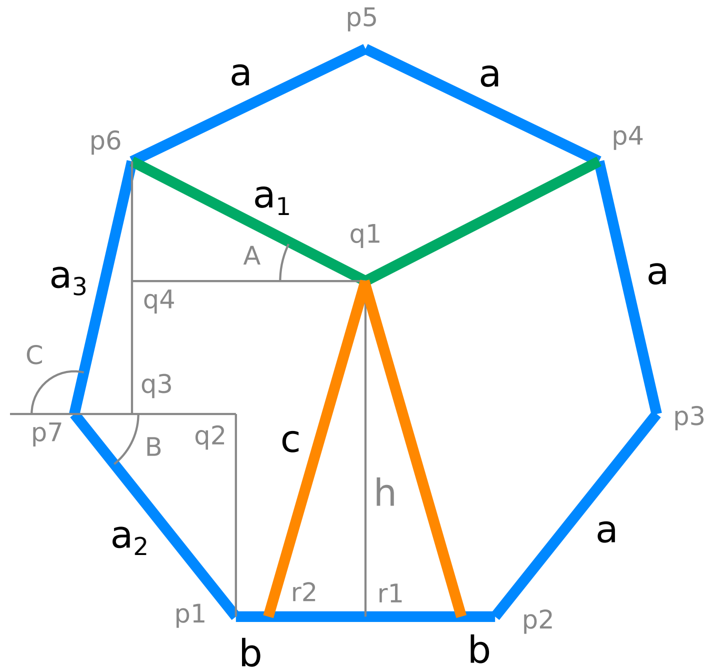

Figure 1: The meccano heptagon is defined with two integers $a > b$. A third value c is checked to be an integer too to form a valid heptagon.

Consider the heptagon in figure 1. First start with the three angles A, B and C:

$$
\begin{aligned}
A &= \pi/7 \\
B &= 2\pi/7 \\
C &= 3\pi/7
\end{aligned}
$$

Then find the sines, nothing that the regular heptagon side is $a = a_1 = a_2 = a_3$:

$$
\begin{aligned}
sin(A) &= \frac{\overline{q_4p_6}}{a_1} \\
sin(B) &= \frac{\overline{p_1q_2}}{a_2} \\
sin(C) &= \frac{\overline{p_6q_3}}{a_3}
\end{aligned}
$$

From the figure the height h corresponds to:

$$
\begin{aligned}
h &= \overline{p_1q_2} + \overline{p_6q_3} + \overline{p_6q_4} \\
&= a(sin(A) + sin(B) + sin(C))
\end{aligned}
$$

According to `https://en.wikipedia.org/wiki/Heptagonal_triangle`

$$
sin(A) + sin(B) + sin(C) = -\frac{\sqrt{7}}{2}
$$

So

$$
h = \frac{\sqrt{7}a}{2}
$$

Finally we get the c length as a function of lengths a and b:

$$
\begin{aligned}
c^2 &= \overline{r_1r_2}^2 + h^2 \\
&= \frac{(a-b)^2}{4} + \frac{7a^2}{4} \\
&= \frac{8a^2 - 2ab + b^2}{4}
\end{aligned}
$$

A valid meccano heptagon will have the three lenghts a, b and c as integers. With a software routine we look for c to be integer by incrementing the values of $a > b$. Figures 2 and 3 show the first two cases satisfying such condition.

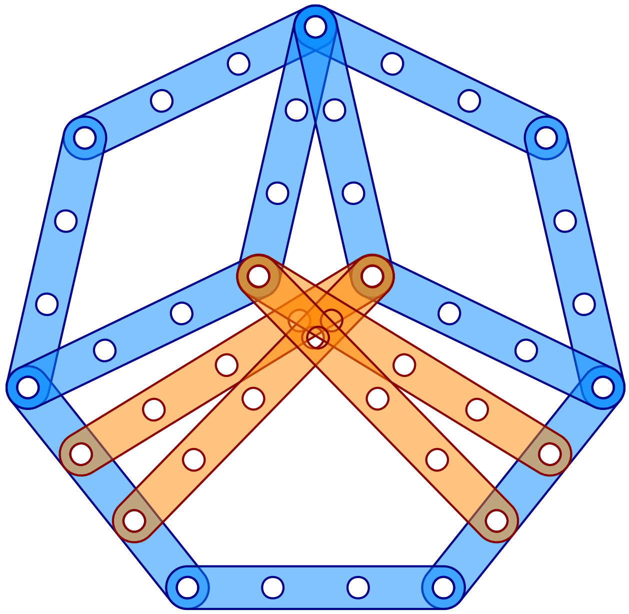

Figure 2: The first meccano heptagon with values $a = 3$, $b = 1$ and $c = 4$.

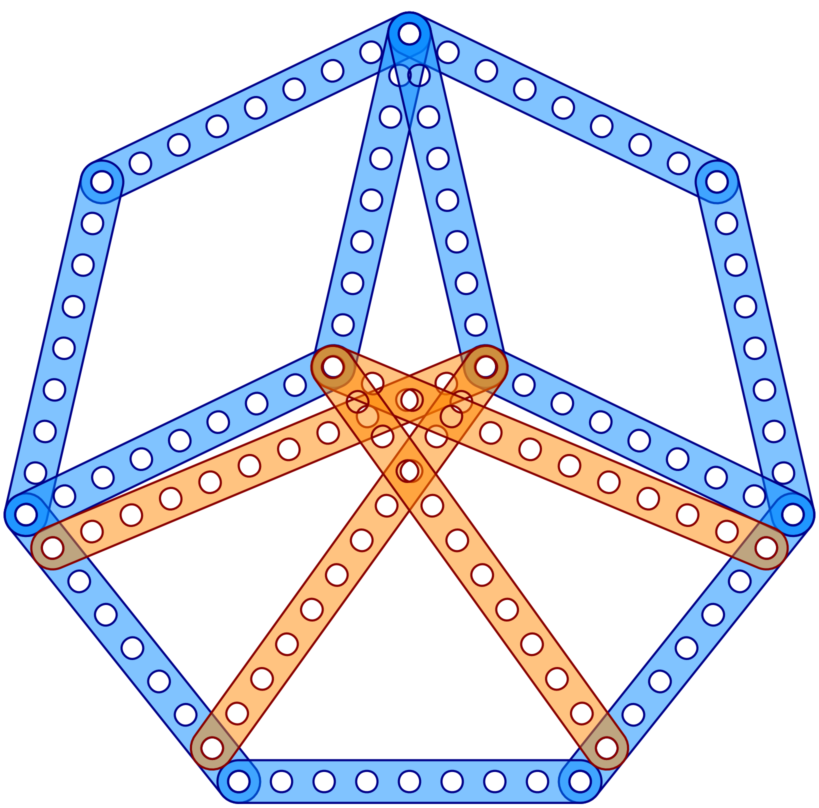

Figure 3: The second meccano heptagon with values $a = 8$, $b = 1$ and $c = 11$.