

Meccano pentagons

<https://github.com/heptagons/meccano/penta>

Abstract

We show how to construct two types of meccano¹ regular pentagons. The process as in another meccano constructions of this site is to build the polygon perimeter and attach **internal diagonals** to make the polygon rigid.

We prepare general layouts to look for the use of only valid **meccano rods** testing by increasing the sides lengths and by rotating the diagonals.

The last diagonal most of the time is not an integer, so the purpose of this study is to find when it is as the rest of diagonals and tabulate a so called solution. Formulas are prepared exactly for each type instead of using floating numbers to prevent skip solutions by rounding errors. Finally a program is run using the algebraic conditions and formulas to iterate over a given range of sides to store and print solutions without repetitions by scaling.

From the two types of pentagons two conjectures emerges. **First conjecture** is that the first type of pentagon seems to have a **unique** solution after testing pentagons sides somehow large.

Second conjecture appears in second type of pentagon. For this type we got a lot of solutions but by inspecting the numeric value of the last diagonal called e seems to be always in the form $10x + 1$ for $x = 1, 2, 3..$ something unrelated at the moment by the formulas used.

1 Regular pentagon type 1

1.1 Type 1 equations

Figure 1 show the layout of the meccano regular pentagon of type 1. Let define the side of the pentagon as a and define other three variables b , c and d :

$$a = \overline{BC}$$

$$b = \overline{BF}$$

$$c = \overline{FI}$$

$$d = \overline{CI}$$

Angles $\angle LBC$ and $\angle JFI$ are equal to $\frac{2\pi}{5}$ so:

$$\alpha = \frac{2\pi}{5}$$

$$\overline{BL} = a \cos \alpha$$

$$\overline{CL} = a \sin \alpha$$

$$\overline{FJ} = c \cos \alpha$$

$$\overline{IJ} = c \sin \alpha$$

¹ Meccano mathematics by 't Hooft

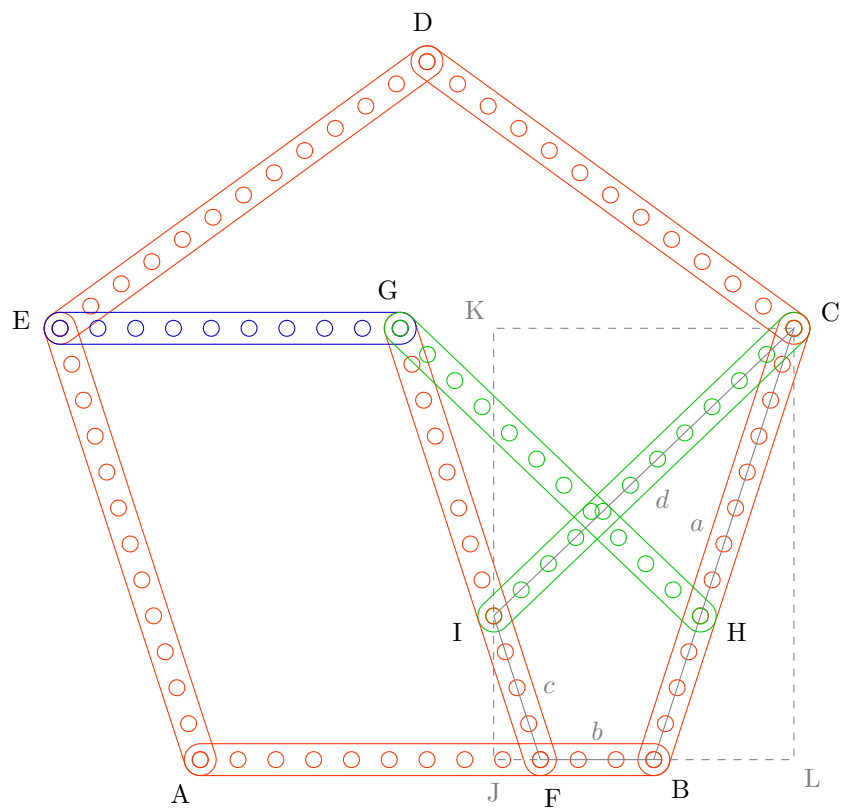


Figure 1: Pentagon of type 1.

Let calculate d in function of a , b and c :

$$\begin{aligned}
d^2 &= (\overline{CI})^2 \\
&= (\overline{CK})^2 + (\overline{IK})^2 \\
&= (\overline{BL} + \overline{BF} + \overline{FJ})^2 + (\overline{CL} - \overline{IJ})^2 \\
&= (a \cos \alpha + b + c \cos \alpha)^2 + (a \sin \alpha - c \sin \alpha)^2 \\
&= ((a + c) \cos \alpha + b)^2 + ((a - c) \sin \alpha)^2 \\
&= (a + c)^2 \cos^2 \alpha + 2(a + c)b \cos \alpha + b^2 + (a - c)^2 \sin^2 \alpha \\
&= (a^2 + c^2)(\cos^2 \alpha + \sin^2 \alpha) + 2ac(\cos^2 \alpha - \sin^2 \alpha) + 2(a + c)b \cos \alpha + b^2 \\
&= (a^2 + c^2) + 2ac(\cos^2 \alpha - \sin^2 \alpha) + 2(a + c)b \cos \alpha + b^2
\end{aligned}$$

For $\alpha = \frac{2\pi}{5}$ we have these regular pentagon identities:

$$\begin{aligned}
\cos \alpha &= \frac{-1 + \sqrt{5}}{4} \\
\cos^2 \alpha &= \frac{3 - \sqrt{5}}{8} \\
\sin^2 \alpha &= \frac{5 + \sqrt{5}}{8} \\
\cos^2 \alpha - \sin^2 \alpha &= -\frac{1 + \sqrt{5}}{4}
\end{aligned}$$

Applying the identities to the last equation of d we get:

$$\begin{aligned}
d^2 &= a^2 + c^2 - \left(\frac{1 + \sqrt{5}}{2}\right)ac + \left(\frac{-1 + \sqrt{5}}{2}\right)(a + c)b + b^2 \\
&= a^2 + c^2 - \frac{ac}{2} - \frac{(a + c)b}{2} + b^2 + \left[-\frac{ac}{2} + \frac{(a + c)b}{2}\right]\sqrt{5} \\
&= a^2 + b^2 + c^2 - \frac{ac + (a + c)b}{2} + \left[\frac{-ac + (a + c)b}{2}\right]\sqrt{5}
\end{aligned}$$

Let define two variables p and q such that $d^2 = p + q\sqrt{5}$ so we have:

$$\begin{aligned}
d^2 &= p + q\sqrt{5} \\
q &= \frac{-ac + (a + c)b}{2} \\
p &= a^2 + b^2 + c^2 - \frac{ac + (a + c)b}{2} \\
&= a^2 + b^2 + c^2 - \frac{-ac + (a + c)b}{2} - ac \\
&= a^2 + b^2 + c^2 - q - ac
\end{aligned}$$

For a meccano pentagon we need d to be an integer. If we let the integer $q > 0$ then $d = \sqrt{p + q\sqrt{5}}$ will never be an integer for p and q integers. If we force q to be zero then $d = \sqrt{p}$ has possibilities to be an integer. So before calculating d we force the condition that $q = 0$ or that is the same $-ac + (a + c)b = 0$:

$$\begin{aligned}
a &> b \\
a &> c \\
ac &= (a + c)b \\
d &= \sqrt{a^2 + b^2 + c^2 + ac}
\end{aligned}$$

1.2 Type 1 program

First we write a **go** struct called **Sols** to collect and print solutions found for the following searching function of any pentagon type. The struct **Add** function prevent solutions duplicated by scaling:

```
1 type Sols struct {
2     sols [][]int
3 }
4
5 func (s *Sols) Add(rods ...int) {
6     if len(rods) < 0 {
7         return
8     }
9     const RODS = "abcdefghijkl"
10    for _, s := range s.sols {
11        a := rods[0]
12        if a % s[0] != 0 {
13            continue
14        }
15        // new a is a factor of previous a
16        f := a / s[0]
17        cont := false
18        for r := 1; r < len(rods); r++ {
19            if s[r] == 0 {
20                continue
21            }
22            b := rods[r]
23            if t := b % s[r] == 0 && b / s[r] == f; !t {
24                cont = true
25                break
26            }
27        }
28        if cont {
29            continue // scaled solution already found (reject)
30        }
31        return
32    }
33    // solution!
34    if s.sols == nil {
35        s.sols = make([][]int, 0)
36    }
37    s.sols = append(s.sols, rods)
38    fmt.Printf("%3d ", len(s.sols))
39    for i, r := range rods {
40        fmt.Printf(" %c=%3d", RODS[i], r)
41    }
42    fmt.Println()
43 }
```

Next **go** function called **pentagons_type_1** iterate over three variables $a \leq max$, $1 \leq b \leq a$, $0 \leq c \leq a$ (lines 15, 16 and 17). The $q = 0$ condition is tested (line 18) and only when is valid we check d to be an integer (see internal function at line 5). Only when d is an integer we call function **sorts Add** (in line 11) to print and store a prospect of solution.

```
1 func pentagons_type_1(max int) {
2
3     sols := &Sols{}
4 }
```

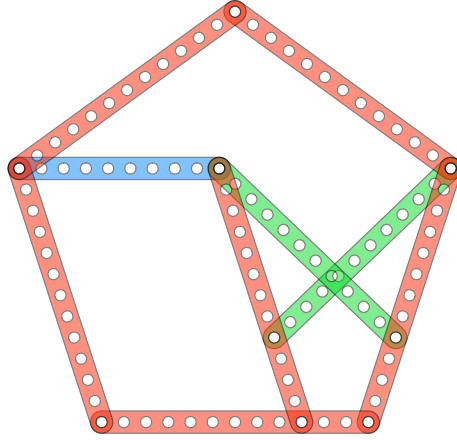


Figure 2: The smallest and maybe unique (?) of pentagons of type 1. Is composed of 6 rods of length $a = 12$ in color red, two rods of length $d = 11$ in green and one rod of size $a - b = 9$ in blue.

```

5  check := func(a, b, c int) {
6      f := float64(a*a + b*b + c*c - a*c)
7      if f < 0 {
8          return
9      }
10     if d := int(math.Sqrt(f)); math.Pow(float64(d), 2) == f {
11         sols.Add(a, b, c, d)
12     }
13 }
14
15 for a := 1; a < max; a++ {
16     for b := 1; b <= a; b++ {
17         for c := 0; c <= a; c++ {
18             if a*c == (a + c)*b {
19                 check(a, b, c)
20             }
21         }
22     }
23 }
24 }
25 }

```

1.3 Type 1 results

After searching for values of $a \leq 5000$ we found a single result:

```

1  1  a= 12 b=  3 c=  4 d= 11

```

Figure 2 shows the first (unique?) pentagon of type 1 with values $a = 12$, $b = 3$, $c = 4$ and $d = 11$.

1.4 Type 1 conjecture

There is only a single case for the type 1 with values $a = 12$, $b = 3$, $c = 4$ and $d = 11$.

2 Regular pentagon type 2

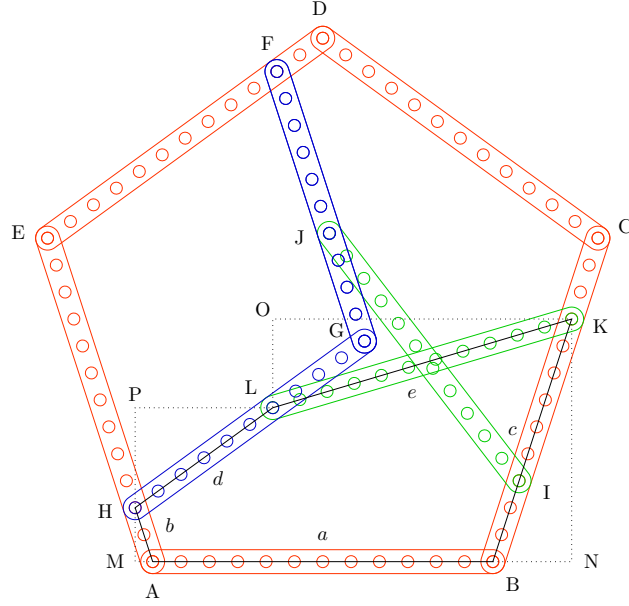


Figure 3: Pentagon of type 2.

2.1 Type 2 equations

Figure 3 show the layout of the meccano regular pentagon of type 2. Let define the side of the pentagon as a and define other four variables b , c , d and e :

$$a = \overline{AB}$$

$$b = \overline{AH}$$

$$c = \overline{BK}$$

$$d = \overline{HL}$$

$$e = \overline{KL}$$

Angles $\angle NBC$ and $\angle MAH$ are equal to $\frac{2\pi}{5}$ so:

$$\alpha = \frac{2\pi}{5}$$

$$\overline{BN} = b \cos \alpha$$

$$\overline{KN} = b \sin \alpha$$

$$\overline{AM} = c \cos \alpha$$

$$\overline{HM} = c \sin \alpha$$

Angle $\angle PLH$ is equal to $\frac{\pi}{5}$ so:

$$\begin{aligned}\beta &= \frac{\pi}{5} \\ \overline{LP} &= d \cos \beta \\ \overline{HP} &= d \sin \beta\end{aligned}$$

Our goal is to find e as integer as function of variables a , b , c and d . e^2 equals $(\overline{KO})^2 + (\overline{LO})^2$ so we first calculate \overline{KO} and \overline{LO} . From figure 3:

$$\begin{aligned}\overline{KO} &= \overline{AM} + \overline{AB} + \overline{BN} - \overline{LP} \\ &= b \cos \alpha + a + c \cos \alpha - d \cos \beta \\ &= (b + c) \cos \alpha + a - d \cos \beta \\ \overline{LO} &= \overline{KN} - \overline{HM} - \overline{HP} \\ &= c \sin \alpha - b \sin \alpha - d \sin \beta \\ &= (c - b) \sin \alpha - d \sin \beta\end{aligned}$$

So by adding the squares we get:

$$\begin{aligned}e^2 &= (\overline{KO})^2 + (\overline{LO})^2 \\ &= ((b + c) \cos \alpha)^2 + 2(b + c) \cos \alpha (a - d \cos \beta) + (a - d \cos \beta)^2 \\ &\quad + ((c - b) \sin \alpha)^2 - 2(c - b) \sin \alpha d \sin \beta + (d \sin \beta)^2 \\ &= (b^2 + c^2)(\cos^2 \alpha + \sin^2 \alpha) + 2bc(\cos^2 \alpha - \sin^2 \alpha) \\ &\quad + 2a(b + c) \cos \alpha - 2(b + c)d \cos \alpha \cos \beta - 2(c - b)d \sin \alpha \sin \beta \\ &\quad + a^2 - 2ad \cos \beta + d^2(\cos^2 \beta + \sin^2 \beta)\end{aligned}$$

Calculate the α and β identities that appear in the last equation:

$$\begin{aligned}\cos^2 \alpha - \sin^2 \alpha &= -\frac{1 + \sqrt{5}}{4} \\ \cos \alpha &= \frac{-1 + \sqrt{5}}{4} \\ \cos \alpha \cos \beta &= \frac{1}{4} \\ \sin \alpha \sin \beta &= \frac{\sqrt{5}}{4} \\ \cos \beta &= \frac{1 + \sqrt{5}}{4}\end{aligned}$$

Replace the identities:

$$\begin{aligned}
e^2 &= (b^2 + c^2)(1) + 2bc(-\frac{1 + \sqrt{5}}{4}) \\
&\quad + 2a(b + c)(\frac{-1 + \sqrt{5}}{4}) - 2(b + c)d(\frac{1}{4}) - 2(c - b)d(\frac{\sqrt{5}}{4}) \\
&\quad + a^2 - 2ad(\frac{1 + \sqrt{5}}{4}) + d^2(1) \\
&= b^2 + c^2 - bc(\frac{1 + \sqrt{5}}{2}) \\
&\quad + a(b + c)(\frac{-1 + \sqrt{5}}{2}) - (b + c)d(\frac{1}{2}) - (c - b)d(\frac{\sqrt{5}}{2}) \\
&\quad + a^2 - ad(\frac{1 + \sqrt{5}}{2}) + d^2 \\
&= a^2 + b^2 + c^2 + d^2 - (b + c)d(\frac{1}{2}) \\
&\quad - (ad + bc)(\frac{1 + \sqrt{5}}{2}) + a(b + c)(\frac{-1 + \sqrt{5}}{2}) - (c - b)d(\frac{\sqrt{5}}{2}) \\
&= a^2 + b^2 + c^2 + d^2 - \frac{(b + c)d}{2} \\
&\quad - \frac{(ad + bc)(1 + \sqrt{5})}{2} + \frac{a(b + c)(-1 + \sqrt{5})}{2} - \frac{(c - b)d\sqrt{5}}{2}
\end{aligned}$$

Let define two variables p and q such that $e^2 = p + q\sqrt{5}$:

$$\begin{aligned}
p &= a^2 + b^2 + c^2 + d^2 - \frac{(b + c)d}{2} - \frac{ad + bc}{2} + \frac{-a(b + c)}{2} \\
&= a^2 + b^2 + c^2 + d^2 - \frac{bd + cd + ad + bc + ab + ac}{2} \\
&= a^2 + b^2 + c^2 + d^2 - \frac{(a + b)(c + d) + ab + cd}{2} \\
q &= -\frac{ad + bc}{2} + \frac{a(b + c)}{2} - \frac{(c - b)d}{2} \\
&= \frac{-ad - bc + ab + ac - cd + bd}{2} \\
&= \frac{(a - b)(c - d) + ab - cd}{2}
\end{aligned}$$

For a meccano pentagon we need e to be an integer. If we let the integer $q > 0$ then $e = \sqrt{p + q\sqrt{5}}$ will never be an integer for p and q integers. If we force q to be zero then $e = \sqrt{p}$ has possibilities to be an integer. So before calculating e we force the condition that $q = 0$ or that is the same $cd = (a - b)(c - d) + ab$:

$$\begin{aligned}
a &> b \\
a &> c \\
cd &= (a - b)(c - d) + ab
\end{aligned}$$

From the condition $q = 0$ we know that $cd = (a - b)(c - d) + ab$, and use this cd value in in the equation

for p to get:

$$\begin{aligned}
p &= a^2 + b^2 + c^2 + d^2 - \frac{(a+b)(c+d) + ab + cd}{2} \\
&= a^2 + b^2 + c^2 + d^2 - \frac{(a+b)(c+d) + ab + (a-b)(c-d) + ab}{2} \\
&= a^2 + b^2 + c^2 + d^2 - ac - bd - ab
\end{aligned}$$

So finally, when $q = 0$ we calculate $e = \sqrt{p}$ expecting to be an integer:

$$e = \sqrt{a^2 + b^2 + c^2 + d^2 - ac - bd - ab}$$

Another solution is:

$$e = \sqrt{a^2 + b^2 + c^2 + d^2 - ad - bc - cd}$$

2.2 Type 2 first program

With the last equations, next go function search for solutions of type 2. Is called `pentagons_type_2`, iterates over the integer values of rods a (line 15), b (line 16), c (line 17) and d (line 18) to discover a rod e with integer length too. First we check condition $q == 0$ is true (line 19) and square root is integer (line 10):

```

1 func pentagons_type_2(max int) {
2
3     sols := &Sols{}
4
5     check := func(a, b, c, d int) {
6         f := float64(a*a + b*b + c*c + d*d - a*c - b*d - a*b)
7         if f < 0 {
8             return
9         }
10        if e := int(math.Sqrt(f)); math.Pow(float64(e), 2) == f {
11            sols.Add(a, b, c, d, e)
12        }
13    }
14
15    for a := 1 ; a < max; a++ {
16        for b := 1; b < a; b++ {
17            for c := 0; c < a; c++ {
18                for d := 1; d < a; d++ {
19                    if ((a - b)*(c - d) + a*b == c*d) {
20                        check(a, b, c, d)
21                    }
22                }
23            }
24        }
25    }
26 }
```

2.3 Type 2 first results

The program found 19 pentagons of type 2 for $a \leq 100$. While we found a single solution for type 1, type 2 has several.

```

1  1  a= 12 b=  2 c=  9 d=  6 e= 11
2  2  a= 12 b=  3 c=  0 d=  4 e= 11
3  3  a= 12 b=  6 c=  3 d= 10 e= 11
4  4  a= 31 b=  4 c= 28 d= 16 e= 31
5  5  a= 31 b= 15 c=  3 d= 27 e= 31
6  6  a= 38 b= 12 c= 18 d= 21 e= 31
7  7  a= 38 b= 17 c= 20 d= 26 e= 31
8  8  a= 48 b=  8 c= 24 d= 21 e= 41
9  9  a= 48 b= 12 c=  9 d= 20 e= 41
10 10 a= 48 b= 27 c= 24 d= 40 e= 41
11 11 a= 48 b= 28 c= 39 d= 36 e= 41
12 12 a= 72 b= 21 c= 48 d= 40 e= 61
13 13 a= 72 b= 24 c= 16 d= 39 e= 61
14 14 a= 72 b= 32 c= 24 d= 51 e= 61
15 15 a= 72 b= 33 c= 56 d= 48 e= 61
16 16 a= 78 b= 27 c=  4 d= 42 e= 71
17 17 a= 78 b= 36 c= 74 d= 51 e= 71
18 18 a= 87 b= 28 c= 36 d= 48 e= 71
19 19 a= 87 b= 39 c= 51 d= 59 e= 71

```

2.4 Type 2 simpler program

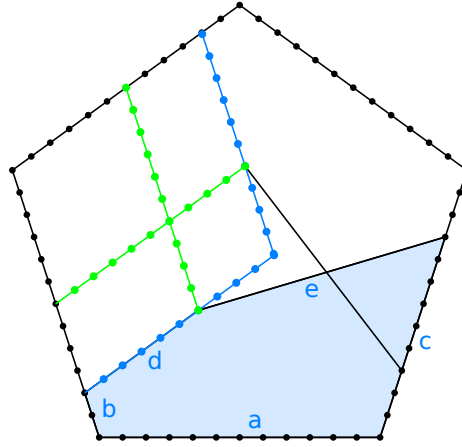


Figure 4: Pentagon of type 2 has a symmetry where pair bars green and blue can be switched leaving the bars e lengths and positions unmodified. This symmetry appears in the first program when two solutions have same a and same e .

Figure 4 show what happens when the first program reports two solutions with the same a and the same e . The type 2 symmetry can be taken into account to simplify the first program to reduce the search space and report only the half of symmetries. Next go function called `pentagons_type_2_half` first iterates over $1 \leq a \leq \max$ (line 4), then over $1 \leq b < a$ (line 6), then over $1 \leq d < (a - b)$ (line 8) and finally over $0 \leq c < a$ (line 10).

```

1  func pentagons_type_2_half(max int) {
2      sols := &Sols{}
3      aa, a_b, ab, bb, dd, ad, bc, c_d, cd, cc := 0,0,0,0,0,0,0,0,0,0
4      for a := 1; a <= max; a++ {

```

```

5  aa = a*a
6  for b := 1; b < a; b++ {
7      a_b, ab, bb = a - b, a*b, b*b
8      for d := 1; d < (a-b); d++ {
9          dd, ad = d*d, a*d
10         for c := 0; c < a; c++ {
11             bc, c_d, cd, cc = b*c, c - d, c*d, c*c
12             if a_b * c_d + ab == cd {
13                 if f := float64(aa + bb + cc + dd - ad - bc - cd); f > 0 {
14                     if e := int(math.Sqrt(f)); math.Pow(float64(e), 2) == f {
15                         sols.Add(a, b, c, d, e)
16                     }
17                 }
18             }
19         }
20     }
21 }
22 }
23 }

```

2.5 Type 2 simpler results

The second type 2 program found 139 solutions iterating over $1 \leq a \leq 1000$:

```

1  1 a= 12 b= 2 c= 9 d= 6 e= 11
2  2 a= 12 b= 3 c= 0 d= 4 e= 11
3  3 a= 31 b= 4 c= 28 d= 16 e= 31
4  4 a= 38 b= 12 c= 18 d= 21 e= 31
5  5 a= 48 b= 8 c= 24 d= 21 e= 41
6  6 a= 48 b= 12 c= 9 d= 20 e= 41
7  7 a= 72 b= 21 c= 48 d= 40 e= 61
8  8 a= 72 b= 24 c= 16 d= 39 e= 61
9  9 a= 78 b= 27 c= 4 d= 42 e= 71
10 10 a= 87 b= 28 c= 36 d= 48 e= 71
11 11 a=111 b= 39 c= 99 d= 67 e=101
12 12 a=121 b= 33 c= 33 d= 57 e=101
13 13 a=128 b= 8 c= 89 d= 56 e=121
14 14 a=138 b= 12 c= 54 d= 47 e=121
15 15 a=145 b= 45 c= 39 d= 75 e=121
16 16 a=147 b= 43 c= 51 d= 75 e=121
17 17 a=151 b= 19 c= 73 d= 61 e=131
18 18 a=156 b= 43 c= 96 d= 84 e=131
19 19 a=165 b= 36 c=132 d= 88 e=151
20 20 a=179 b= 15 c=177 d= 93 e=191
21 21 a=183 b= 66 c= 62 d=108 e=151
22 22 a=201 b= 9 c= 13 d= 21 e=191
23 23 a=204 b= 21 c=112 d= 84 e=181
24 24 a=216 b= 48 c=111 d=104 e=181
25 25 a=236 b= 80 c= 20 d=125 e=211
26 26 a=249 b= 45 c= 75 d= 95 e=211
27 27 a=264 b= 76 c= 3 d=108 e=241
28 28 a=285 b= 73 c= 27 d=111 e=251
29 29 a=296 b=104 c=128 d=173 e=241
30 30 a=303 b= 51 c= 29 d= 81 e=271
31 31 a=304 b= 76 c=133 d=148 e=251
32 32 a=312 b= 36 c= 93 d=100 e=271
33 33 a=315 b= 24 c=160 d=120 e=281

```

34	34	a=324	b= 64	c=204	d=159	e=281
35	35	a=343	b= 7	c=115	d= 91	e=311
36	36	a=352	b= 3	c=240	d=144	e=341
37	37	a=354	b= 53	c= 60	d=102	e=311
38	38	a=368	b= 36	c=219	d=156	e=331
39	39	a=369	b= 37	c= 27	d= 63	e=341
40	40	a=370	b= 1	c=172	d=118	e=341
41	41	a=375	b= 15	c=191	d=135	e=341
42	42	a=378	b= 21	c= 84	d= 86	e=341
43	43	a=384	b=120	c=312	d=223	e=341
44	44	a=390	b= 84	c= 50	d=135	e=341
45	45	a=390	b= 87	c=228	d=194	e=331
46	46	a=392	b=119	c=296	d=224	e=341
47	47	a=392	b=128	c= 56	d=203	e=341
48	48	a=393	b= 98	c= 54	d=156	e=341
49	49	a=396	b=138	c= 73	d=222	e=341
50	50	a=399	b= 70	c=210	d=180	e=341
51	51	a=403	b= 78	c=114	d=156	e=341
52	52	a=404	b= 89	c=104	d=164	e=341
53	53	a=408	b= 16	c=312	d=183	e=401
54	54	a=408	b= 84	c=167	d=180	e=341
55	55	a=411	b=123	c=243	d=227	e=341
56	56	a=435	b= 96	c=400	d=240	e=421
57	57	a=450	b= 92	c=438	d=249	e=451
58	58	a=468	b=173	c= 24	d=276	e=431
59	59	a=480	b= 80	c= 75	d=144	e=421
60	60	a=486	b=180	c= 18	d=287	e=451
61	61	a=488	b= 72	c= 15	d= 96	e=451
62	62	a=488	b=132	c=423	d=276	e=451
63	63	a=488	b=152	c=269	d=272	e=401
64	64	a=495	b=135	c=415	d=279	e=451
65	65	a=502	b= 93	c= 36	d=138	e=451
66	66	a=507	b= 18	c=366	d=220	e=491
67	67	a=507	b= 60	c= 84	d=128	e=451
68	68	a=509	b=150	c= 42	d=228	e=451
69	69	a=516	b=114	c=169	d=222	e=431
70	70	a=520	b= 36	c=225	d=180	e=461
71	71	a=525	b=185	c=399	d=315	e=451
72	72	a=525	b=189	c=105	d=305	e=451
73	73	a=528	b= 80	c=171	d=192	e=451
74	74	a=540	b=150	c=321	d=290	e=451
75	75	a=543	b=123	c=221	d=249	e=451
76	76	a=546	b=135	c=228	d=262	e=451
77	77	a=552	b=179	c=288	d=312	e=451
78	78	a=553	b=180	c=276	d=312	e=451
79	79	a=560	b=200	c=344	d=335	e=461
80	80	a=565	b= 69	c=153	d=177	e=491
81	81	a=588	b=104	c= 12	d=135	e=541
82	82	a=600	b= 65	c=240	d=216	e=521
83	83	a=600	b=120	c= 96	d=205	e=521
84	84	a=617	b= 89	c=533	d=317	e=601
85	85	a=632	b=113	c=152	d=224	e=541
86	86	a=652	b= 58	c=235	d=214	e=571
87	87	a=661	b=109	c= 37	d=157	e=601
88	88	a=684	b=237	c=192	d=388	e=571
89	89	a=699	b= 84	c=564	d=344	e=671
90	90	a=701	b=254	c=698	d=428	e=671

91	91	a=713	b=234	c=582	d=420	e=631
92	92	a=715	b=211	c=655	d=415	e=671
93	93	a=720	b=216	c=712	d=423	e=701
94	94	a=724	b=147	c= 72	d=228	e=641
95	95	a=728	b= 21	c=192	d=168	e=661
96	96	a=729	b= 36	c=428	d=288	e=671
97	97	a=732	b= 18	c=681	d=358	e=781
98	98	a=732	b= 42	c=111	d=134	e=671
99	99	a=744	b=228	c=155	d=372	e=631
100	100	a=746	b=164	c= 38	d=233	e=671
101	101	a=755	b=123	c=267	d=291	e=641
102	102	a=756	b= 69	c=168	d=196	e=671
103	103	a=762	b= 73	c=372	d=294	e=671
104	104	a=765	b= 30	c=354	d=260	e=691
105	105	a=777	b=234	c=118	d=372	e=671
106	106	a=781	b=108	c=348	d=312	e=671
107	107	a=784	b=192	c=189	d=336	e=661
108	108	a=800	b=164	c=263	d=332	e=671
109	109	a=804	b=177	c=272	d=348	e=671
110	110	a=805	b=202	c=238	d=364	e=671
111	111	a=810	b=276	c=510	d=475	e=671
112	112	a=819	b=136	c=216	d=288	e=701
113	113	a=824	b=276	c=363	d=468	e=671
114	114	a=826	b=315	c=420	d=510	e=671
115	115	a=840	b=196	c=777	d=468	e=811
116	116	a=845	b=285	c=465	d=489	e=691
117	117	a=859	b=130	c=502	d=388	e=751
118	118	a=861	b=126	c= 66	d=196	e=781
119	119	a=863	b=303	c=711	d=519	e=761
120	120	a=864	b= 24	c=349	d=264	e=781
121	121	a=873	b=137	c=453	d=381	e=751
122	122	a=879	b=231	c= 63	d=343	e=781
123	123	a=885	b=206	c=642	d=468	e=781
124	124	a=885	b=309	c= 13	d=477	e=821
125	125	a=892	b=112	c=196	d=259	e=781
126	126	a=896	b=144	c=528	d=411	e=781
127	127	a=896	b=332	c=725	d=548	e=781
128	128	a=904	b=328	c=640	d=547	e=761
129	129	a=905	b=161	c=185	d=305	e=781
130	130	a=912	b=168	c=507	d=424	e=781
131	131	a=915	b=135	c=345	d=349	e=781
132	132	a=928	b=319	c=232	d=520	e=781
133	133	a=938	b=252	c=270	d=441	e=781
134	134	a=947	b=306	c=558	d=540	e=781
135	135	a=948	b=342	c=589	d=570	e=781
136	136	a=949	b=273	c=495	d=507	e=781
137	137	a=960	b=195	c=760	d=504	e=881
138	138	a=961	b=249	c=633	d=513	e=821
139	139	a=987	b=350	c=594	d=588	e=811

2.6 Type 2 conjecture

The last report of 139 pentagons shows all e values have the form $10x + 1$ for x integer. So the conjecture is that e always is of the form $10x + 1$ for x integer.

Next go function called `pentagons_type_2_half_with_conjecture` is an adaptation of the previous one and instead checking for a square root to be an integer, only checks for $e^2 = (10x + 1)^2$ for small $x > 1$. The

results of this program is exactly the same result of the program checking the square root, up to $a \leq 1000$.

```

1 func pentagons_type_2_half_with_conjecture(max int) {
2   sols := &Sols{}
3   aa, a_b, ab, bb, dd, ad, bc, c_d, cd, cc := 0,0,0,0,0,0,0,0,0,0
4   for a := 1; a <= max; a++ {
5     aa = a*a
6     for b := 1; b < a; b++ {
7       a_b, ab, bb = a - b, a*b, b*b
8       for d := 1; d < (a-b); d++ {
9         dd, ad = d*d, a*d
10        for c := 1; c < a; c++ {
11          bc, c_d, cd, cc = b*c, c - d, c*d, c*c
12          if a_b * c_d + ab == cd {
13
14            e2 := aa + bb + cc + dd - ad - bc - cd
15
16            x := 1
17            for {
18              if e := 10*x + 1; e*e == e2 {
19                sols.Add(a, b, c, d, e)
20                break
21              } else if e*e > e2 {
22                break
23              }
24              x++
25            }
26          }
27        }
28      }
29    }
30  }
31 }

```

2.7 Type 2 examples

Figures 5, 6 and 7 show some of the pentagons of type 2 found.

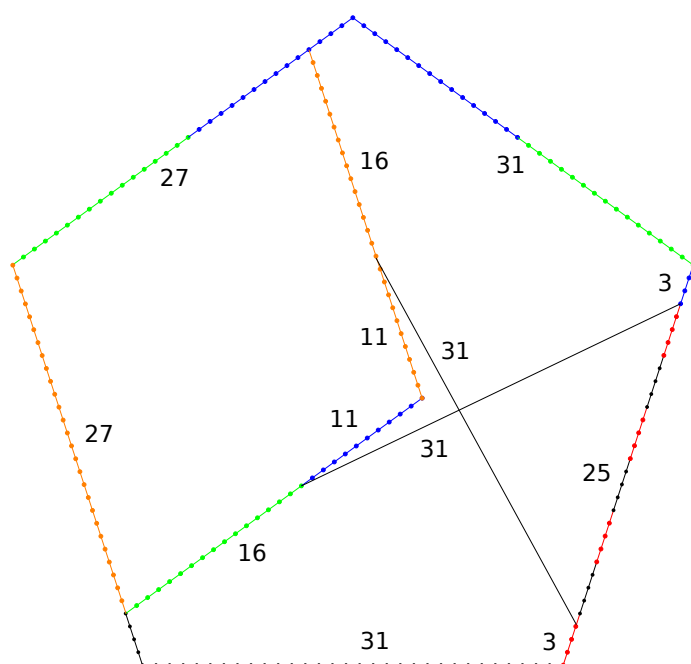


Figure 5: Pentagon of type 2 with $a = 31$. This construction requires 7 rods of length 31 and 2 rods of length 27.

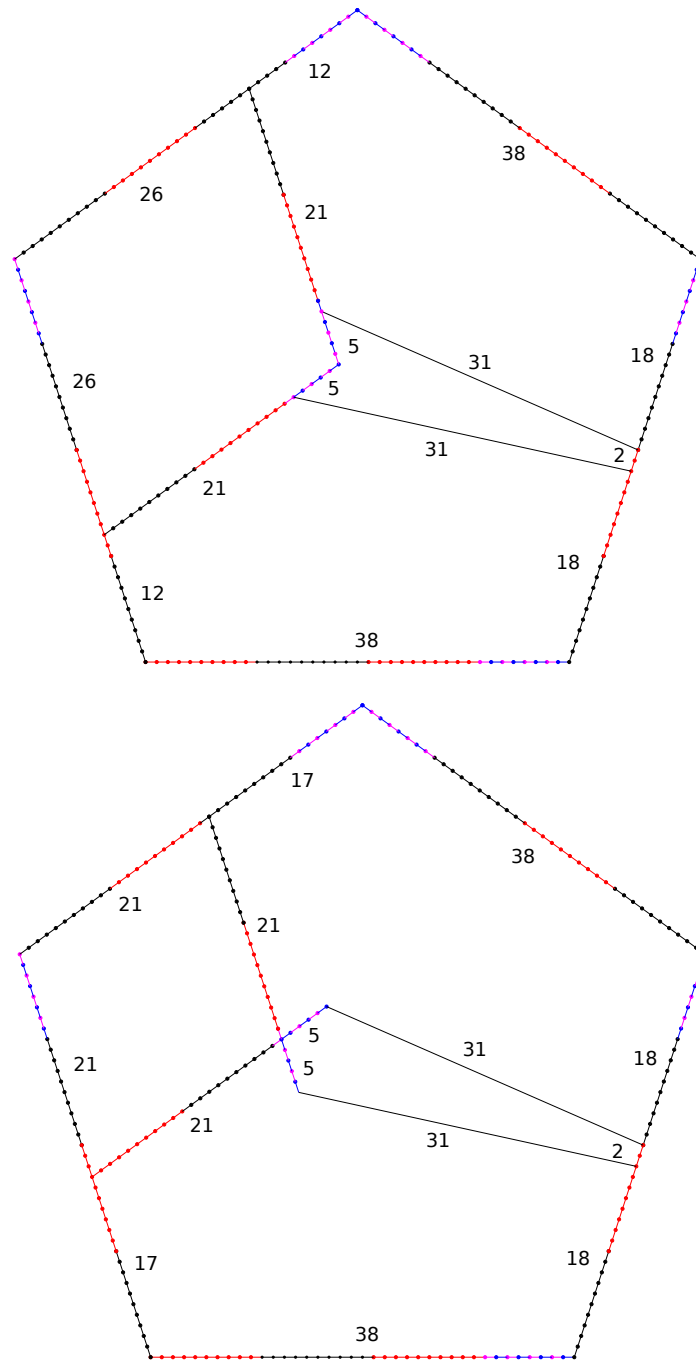


Figure 6: Pentagons of type 2 with $a = 38$. Each construction requires 5 rods of length 38, 2 rods of length 31 and 2 rods of length 26

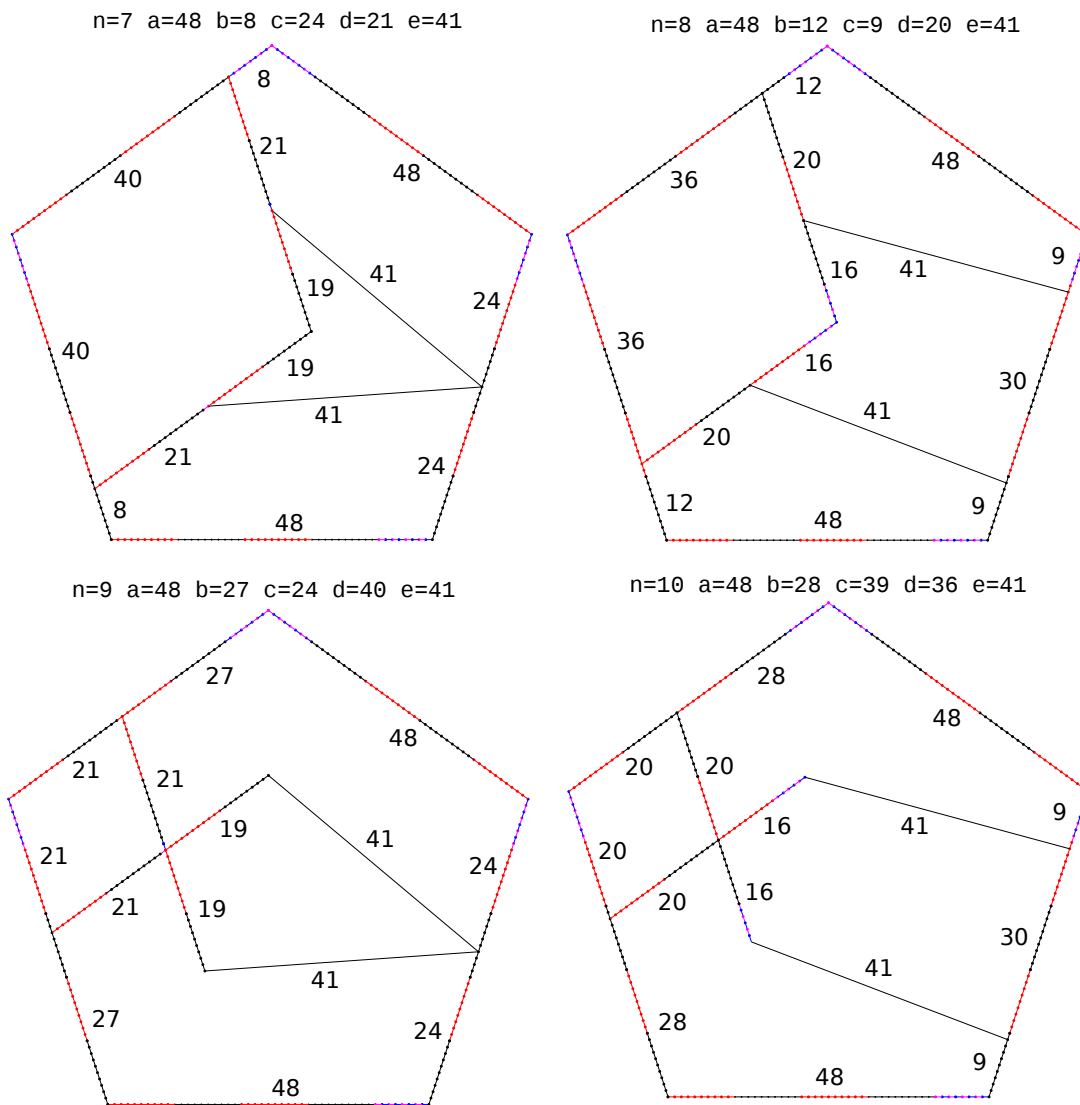


Figure 7: Pentagons of type 2 with $a = 48$