Meccano pentagons diagonals

https://github.com/heptagons/meccano/penta

Abstract

We construct meccano [1] regular pentagons internal diagonals.

1 Regular pentagon diagonals

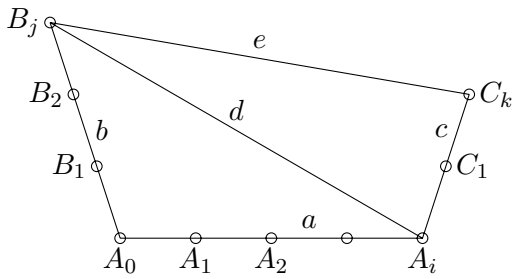

Figure 1: Regular pentagon basic diagonals d and e from sides segments $a \geq\ b \geq c$.

From figure 1 we know the regular internal pentagons angles equal $3\pi/5$:

$$\alpha = \angle A_0 A_i B_j \tag{1}$$
$$\beta = \angle B_j A_i C_k \tag{2}$$
$$\theta = \angle B_j A_0 A_i \tag{3}$$
$$= \alpha + \beta \tag{4}$$
$$\cos\theta = \frac{1-\sqrt{5}}{4} \tag{5}$$

From the law of cosines we calculate distance d from integers a, b which equal respectively to iterators i, j:

$$\begin{aligned} d &= \sqrt{a^2 + b^2 - 2ab\cos\theta} \\ &= \sqrt{a^2 + b^2 - 2ab\left(\frac{1-\sqrt{5}}{4}\right)} \\ &= \frac{\sqrt{4a^2 + 4b^2 - 2ab + 2ab\sqrt{5}}}{2} \end{aligned} \tag{6}$$

We define two integers m, n to simplify last equation and obtain:

$$m = 4a^2 + 4b^2 - 2ab \tag{7}$$
$$n = 2ab \tag{8}$$
$$d = \frac{\sqrt{m + n\sqrt{5}}}{2} \tag{9}$$

[1] Meccano mathematics by 't Hooft