# Horns unit

https://github.com/heptagons/meccano/units/horns

**Abstract**

Horns unit is a group of seven meccano [1] strips intended to build polygons.

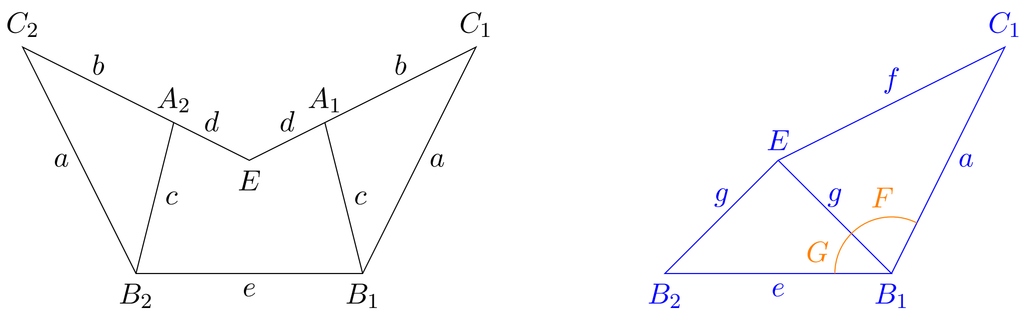


Figure 1: The **horn unit** has seven strips: Two of length $a$, two of length $b + d$, two of length $c$ and one of length $e$. We expect to build polygons with internal angle $C_1B_1B_2$ and perimeter including segments $a, e, a$.

## 1 Algebra

From figure 1 we start with triangle $\triangle A_1B_1C1$. At vertex $A_1$ we have angle $A$ and the supplement $A'$:

$$A \equiv \angle B_1A_1C_1 \tag{1}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \text{if and only if } a < b + c \tag{2}$$

$$A' \equiv \angle EA_1B_1 = \pi - A \tag{3}$$

$$\cos A' = \cos(\pi - A) = -\cos A = \frac{-b^2 - c^2 + a^2}{2bc} \tag{4}$$

[1] Meccano mathematics by ‘t Hooft

We define $f = b + d$ and $g \equiv \overline{EB_1}$ and with the law of cosines we have:

$$
\begin{aligned}
f &\equiv b + d && (5)\\
g^2 &= c^2 + d^2 - 2cd\cos A' && (6)\\
&= c^2 + d^2 - (2cd)\frac{-b^2 - c^2 + a^2}{2bc} && (7)\\
&= \frac{bc^2 + bd^2 + b^2d + c^2d - a^2d}{b} && (8)\\
&= \frac{(b+d)(bd + c^2) - a^2d}{b} && (9)\\
&= \frac{(bd + c^2)f - a^2d}{b} && (10)\\
&\equiv \frac{h}{b} \qquad \text{if and only if } -b < h < b && (11)\\
& && (12)
\end{aligned}
$$

We sum the angles $F$ and $G$ to get:

$$
\begin{aligned}
F &\equiv \angle C_1B1E && (13)\\
\cos F &= \frac{a^2 + g^2 - f^2}{2ag} && (14)\\
G &\equiv \angle B_2B_1E && (15)\\
\cos G &= \frac{e}{2g} && (16)\\
F + G &\equiv \angle B_2B_1C_1 && (17)\\
\cos(F + G) &= \cos F\cos G - \sin F\sin G && (18)\\
& && (19)
\end{aligned}
$$

We calculate cosines part and replacing $g^2$ with $h/b$ first in the denominator and finally in the numerator:

$$
\begin{aligned}
\cos F\cos G &= \frac{a^2 + g^2 - f^2}{2ag} \times \frac{e}{2g} && (20)\\
&= \frac{e(a^2 + g^2 - f^2)e}{4ag^2} && (21)\\
&= \frac{e(a^2b + bg^2 - bf^2)}{4ah} && (22)\\
&= \frac{e(a^2b + h - bf^2)}{4ah} && (23)
\end{aligned}
$$

We calculate the sines part squared. Replace $g^2$ with $h/b$ first in the denominator:

$$
\begin{aligned}
(\sin F\sin G)^2 &= (1 - cos^2F)(1 - cos^2G) && (24)\\
&= 1 - cos^2F - cos^2G + cos^2F\cos^2 G && (25)\\
&= 1 - \frac{(a^2 + g^2 - f^2)^2}{4a^2g^2} - \frac{e^2}{4g^2} + (\cos F\cos G)^2 && (26)\\
&= 1 - \frac{b(a^2 + g^2 - f^2)^2}{4a^2h} - \frac{be^2}{4h} + \frac{e^2(a^2b + h - bf^2)^2}{16a^2h^2} && (27)\\
&= \frac{16a^2h^2}{16a^2h^2} - \frac{4bh(a^2 + g^2 - f^2)^2}{16a^2h^2} - \frac{4a^2be^2h}{16a^2h^2} + \frac{e^2(a^2b + h - bf^2)^2}{16a^2h^2} && (28)\\
& && (29)
\end{aligned}
$$