Let define algebraic A as this irreducible:

$$A \equiv \frac{\pm a\sqrt{c \pm e\sqrt{f}} \pm d}{b}$$

when we are limited to 32-bit integers:

$$0 \le a, c, d, e, f \le 4,294,967,295$$
$$1 \le b \le 4,294,967,295$$

When $a > 0$ and $c = d = e = f = 0$ we have a rational Rat. When $a, c > 0$ and $c = d = e = 0$ we have an algebraic Alg^c. When $a, c, d > 0$ and $e = 0$ we have an algebraic Alg^c. When $a, c, d, e, f > 0$ we have an algebraic Alg^f.

$$\begin{aligned}
Rat &\equiv \frac{\pm a}{b} && c = d = e = 0 \\
Alg^c &\equiv \frac{\pm a\sqrt{c}}{b} && d = e = 0 \\
Alg^d &\equiv \frac{\pm a\sqrt{c} \pm d}{b} && e = 0 \\
Alg^f &\equiv \frac{\pm a\sqrt{c \pm e\sqrt{f}} \pm d}{b} && e, f > 0
\end{aligned}$$

1 Operations

1.1 Rationals Rat

1.1.1 $Rat_1 + Rat_2 = Rat_3$

$$\begin{aligned}
Rat_1 + Rat_2 &= \frac{\pm a_1}{b_1} + \frac{\pm a_2}{b_2} \\
&= \frac{\pm a_1 b_2 \pm a_2 b_1}{b_1 b_2} \\
&= \frac{\pm a_3}{b_3}
\end{aligned}$$

1.1.2 $Rat_1 \times Rat_2 = Rat_3$

$$\begin{aligned}
Rat_1 \times Rat_2 &= \frac{\pm a_1}{b_1} \times \frac{\pm a_2}{b_2} \\
&= \frac{\pm a_1 a_2}{b_1 b_2} \\
&= \frac{\pm a_3}{b_3}
\end{aligned}$$

1.1.3 $1/Rat_1 = Rat_2, a > 0$

$$\begin{aligned}\frac{1}{Rat_1} &= \frac{1}{\frac{\pm a_1}{b_1}} \\ &= \frac{\pm b_1}{a_1} \\ &= \frac{\pm a_2}{b_2}\end{aligned}$$

1.2 Algebraics A^c

1.2.1 $\sqrt{Rat_1} = A_2^c$

$$\begin{aligned}\sqrt{Rat_1} &= \sqrt{\frac{a_1}{b_1}} \\ &= \frac{\sqrt{a_1 b_1}}{b_1} \\ &= \frac{m\sqrt{c_2}}{b_1} && a_1 b_1 = m^2 c_2 \\ &= \frac{a_2\sqrt{c_2}}{b_2}\end{aligned}$$

1.2.2 $A_1^c \times A_2^c = A_3^c$

$$\begin{aligned}A_1^c \times A_2^c &= \frac{\pm a_1\sqrt{c_1}}{b_1} \times \frac{\pm a_2\sqrt{c_2}}{b_2} \\ &= \frac{\pm a_1 a_2\sqrt{c_1 c_2}}{b_1 b_2} \\ &= \frac{\pm a_1 a_2 m\sqrt{c_3}}{b_1 b_2} && c_1 c_2 = m^2 c_3 \\ &= \frac{\pm a_3\sqrt{c_3}}{b_3}\end{aligned}$$

1.3 Algebraics A^d

1.3.1 $A_1^c + A_2^c = A_3^d$

$$\begin{aligned}
A_1^c + A_2^c &= \frac{\pm a_1\sqrt{c_1}}{b_1} + \frac{\pm a_2\sqrt{c_2}}{b_2} \\
&= \frac{\pm a_1 b_2\sqrt{c_1} \pm a_2 b_1\sqrt{c_2}}{b_1 b_2} \\
&= \frac{\pm m\sqrt{c_1} \pm n\sqrt{c_2}}{o} \\
&= \frac{\pm\sqrt{m^2 c_1 + n^2 c_2 \pm 2mn\sqrt{c_1 c_2}}}{o} \\
&= \frac{\pm\sqrt{q \pm 2mnp\sqrt{f_3}}}{o} && c_1 c_2 = p^2 f_3 \\
&= \frac{\pm r\sqrt{c_3 \pm e_3\sqrt{f_3}}}{o} && q = r^2 c_3, 2mnp = r^2 e_3 \\
&= \frac{\pm a_3\sqrt{c_3 \pm e_3\sqrt{f_3}}}{b_3}
\end{aligned}$$

1.3.2 $\sqrt{A_1^c} = A_2^d$