

# Meccano triangles

<https://github.com/heptagons/meccano/nest>

## Abstract

We construct meccano triangles. Basic triangles has the three sides as integers and calculate the internal diagonal distances. Such diagonals then are used as the new side of more complicated triangles and then again we calculate new distances formed and so on. Eventually we expect to find certain angles joining the triangles which can be used to construct regular polygons or more figures.

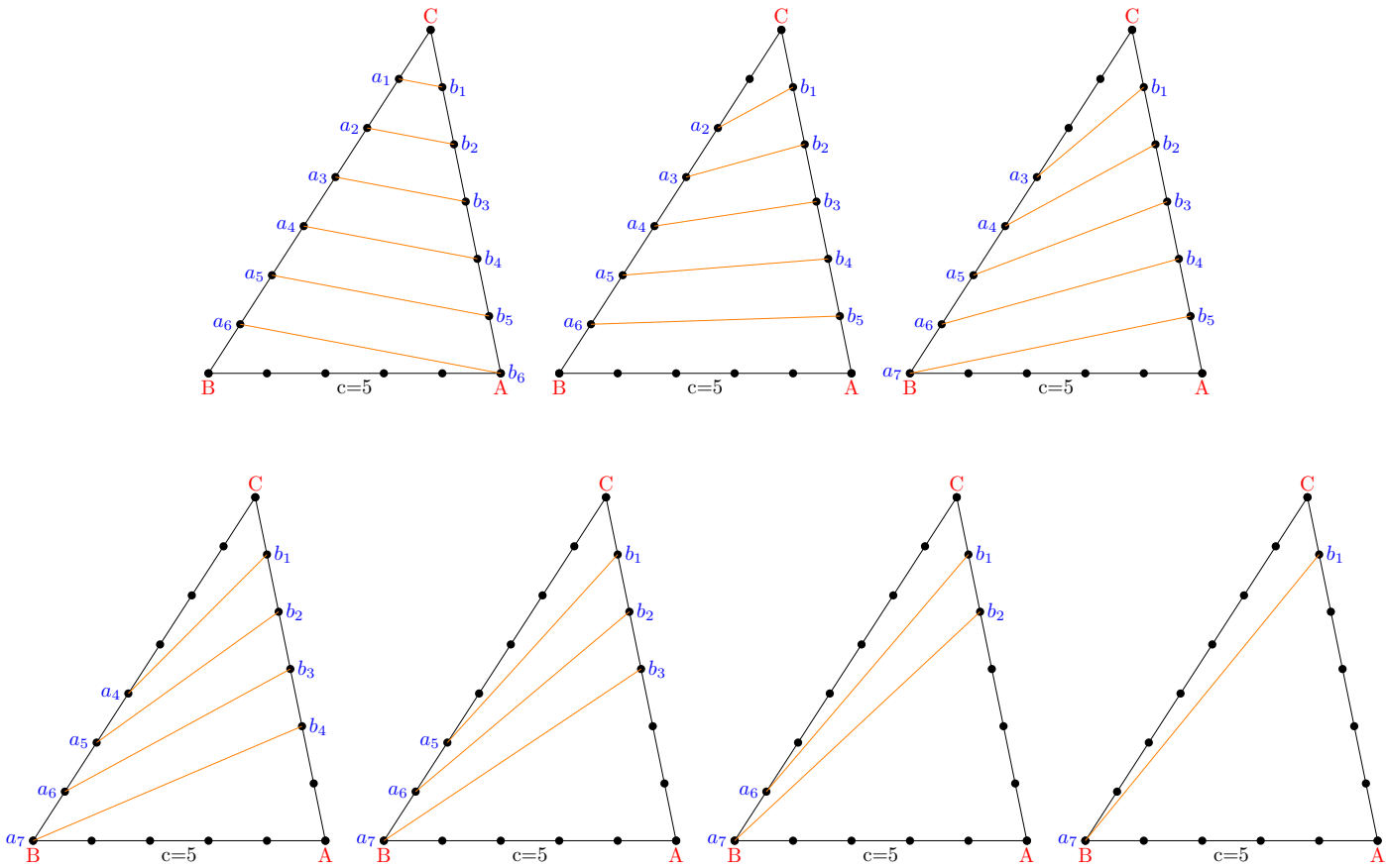


Figure 1: The meccano scalene basic triangle with sides 7, 6 and 5.

## 1 Basic triangle

A basic triangle has the tree sides  $a$ ,  $b$  and  $c$  where  $a, b, c \in \mathbb{N}$ . To avoid repetitions we consider only the cases  $a \geq b$ ,  $b \geq c$  and  $a \geq c$ . Valid triangles also need the condition  $a \geq b + c$ .

## 1.1 Basic triangle diagonals

Figure 1 show a basic triangle with sides  $a = 7$ ,  $b = 6$  and  $c = 5$ . For this particular triangle we have seven groups of diagonals shown as orange lines. The diagonals join points from side  $a$  nodes described as  $a_x$  to side  $b$  nodes described as  $b_y$  in all combinations. From top to bottom and left to right the groups are:

Group	$x - y$	diagonals
1	0	$a_1b_1$ to $a_6b_6$
2	1	$a_2b_1$ to $a_6b_5$
3	2	$a_3b_1$ to $a_7b_5$
4	3	$a_4b_1$ to $a_7b_4$
5	4	$a_5b_1$ to $a_7b_3$
6	5	$a_6b_1$ to $a_7b_2$
7	6	$a_7b_1$

To calculate the diagonals we start calculating  $\cos C$  where  $C$  is the opposite angle of side  $c = 5$ :

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \quad (1)$$

Then with the  $\cos C$  we can calculate every diagonal  $\overline{a_xb_y}$  with the law of cosines:

$$\overline{a_xb_y} = \sqrt{x^2 + y^2 - 2xy \cos C} \quad (2)$$

$$= \sqrt{x^2 + y^2 - 2xy \frac{a^2 + b^2 - c^2}{2ab}} \quad (3)$$

$$= \frac{\sqrt{a^2b^2(x^2 + y^2) - abxy(a^2 + b^2 - c^2)}}{ab} \quad (4)$$

where  $1 \leq x \leq a$ ,  $1 \leq y \leq b$  and  $x - y \geq 0$ .

By inspection we deduce that for basic meccano triangles:

$$a, b, c \in \mathbb{N} \quad (5)$$

$$\cos A, \cos B, \cos C \in \mathbb{Q} \quad (6)$$

$$\overline{a_xb_y}, \overline{b_y c_z}, \overline{a_x c_z} \in \mathbb{A} \quad (7)$$