Meccano four frame

https://github.com/heptagons/meccano/frames/four

Abstract

Four frame is a group of four rigid meccano [1] strips.

1 Four frame

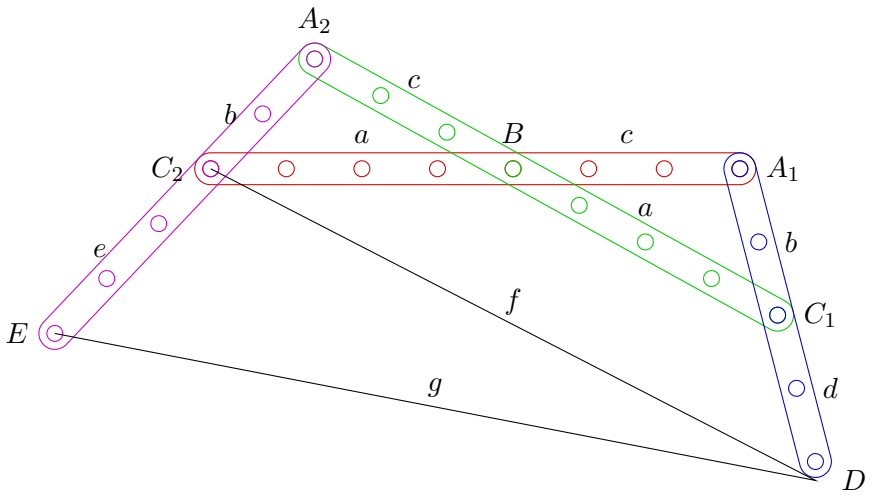

Figure 1: Antisymmetric four frame.

Figure 1 show the antisymmetric four-strips frame. From the figure we define $\alpha \equiv \angle BA_1C_1$ and define integers $m = b^2 + c^2 - a^2$ and $n = 2bc$ using the law of cosines, then we calculate $\cos\alpha$ and $\sin\alpha$:

$$(\alpha, m, n) \equiv (\angle BA_1C_1, b^2 + c^2 - a^2, 2bc) \tag{1}$$

$$\cos\alpha = \frac{m}{n} \tag{2}$$

$$\sin\alpha = \frac{\sqrt{n^2 - m^2}}{n} \tag{3}$$

We calculate the distance $f = \overline{C_2D}$ with the law of cosines using angle α and defining integers $x = a + c$ and $y = b + d$:

$$x \equiv a + c \tag{4}$$

$$y \equiv b + d \tag{5}$$

$$f^2 = (a + c)^2 + (b + d)^2 - 2(a + c)(b + d)\cos\alpha \tag{6}$$

$$= x^2 + y^2 - \frac{2mxy}{n} \tag{7}$$

$$f = \frac{\sqrt{n^2x^2 + n^2y^2 - 2mnxy}}{n} \tag{8}$$

[1] Meccano mathematics by 't Hooft