Meccano hexagons gallery

https://github.com/heptagons/meccano/hexa/gallery

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Abstract

We build rigid meccano [1] regular pentagons from sides 4 to 24. We restrict all internal strips to remain inside the hexagons's perimeter and don't permit they overlap with others. The internal strips must not be parallel to any side of the hexagon.

1 Internal strips

We run a software program to look strips that can make rigid two consecutive internal sides of any hexagon. Figure 1 show the smaller four cases found. Consider the figure at top left, the internal hexagon angle is $\theta \equiv \angle GBC = 2\pi/3$ and the hexagon side is $p \equiv \overline{BC}$. Consider the triangle $\triangle GBC$ and define the other two sides as $b \equiv \overline{GB}$ and $c = \overline{GC}$. By the law of cosines we know that:

$$\begin{aligned} c &= \sqrt{b^2 + p^2 - 2bp\cos\theta} \\ &= \sqrt{b^2 + p^2 - 2bp\left(-\frac{1}{2}\right)} \\ &= \sqrt{b^2 + p^2 + bp} \end{aligned} \tag{1}$$

By defining $a \equiv p + b$ we get:

$$c = \sqrt{a^2 + b^2 - ab} \quad \texttt{where } a > b \tag{2}$$

a	b	c	p
8	3	7	5
15	7	13	8
21	5	19	16
35	11	31	24
40	7	37	33
48	13	43	35

Table 1: Triplets of sides of triangle with angle $2\pi/3$ where $c > p > b$.

We run a software to iterate first over $0 < a < max$ and then by $1 < b < a$ and filtering c to be integer we get the first rows of triangles with sides $c > p > b$ in table 1.

Figure 1 shows hexagons of sizes $p = 5, 8, 16, 24$ with perimeter strips in orange made rigid adding three internal green strips of lenght $c = 7, 13, 19, 31$. In the figure we have also an equilateral triangle $\triangle GCY$

[1] Meccano mathematics by 't Hooft

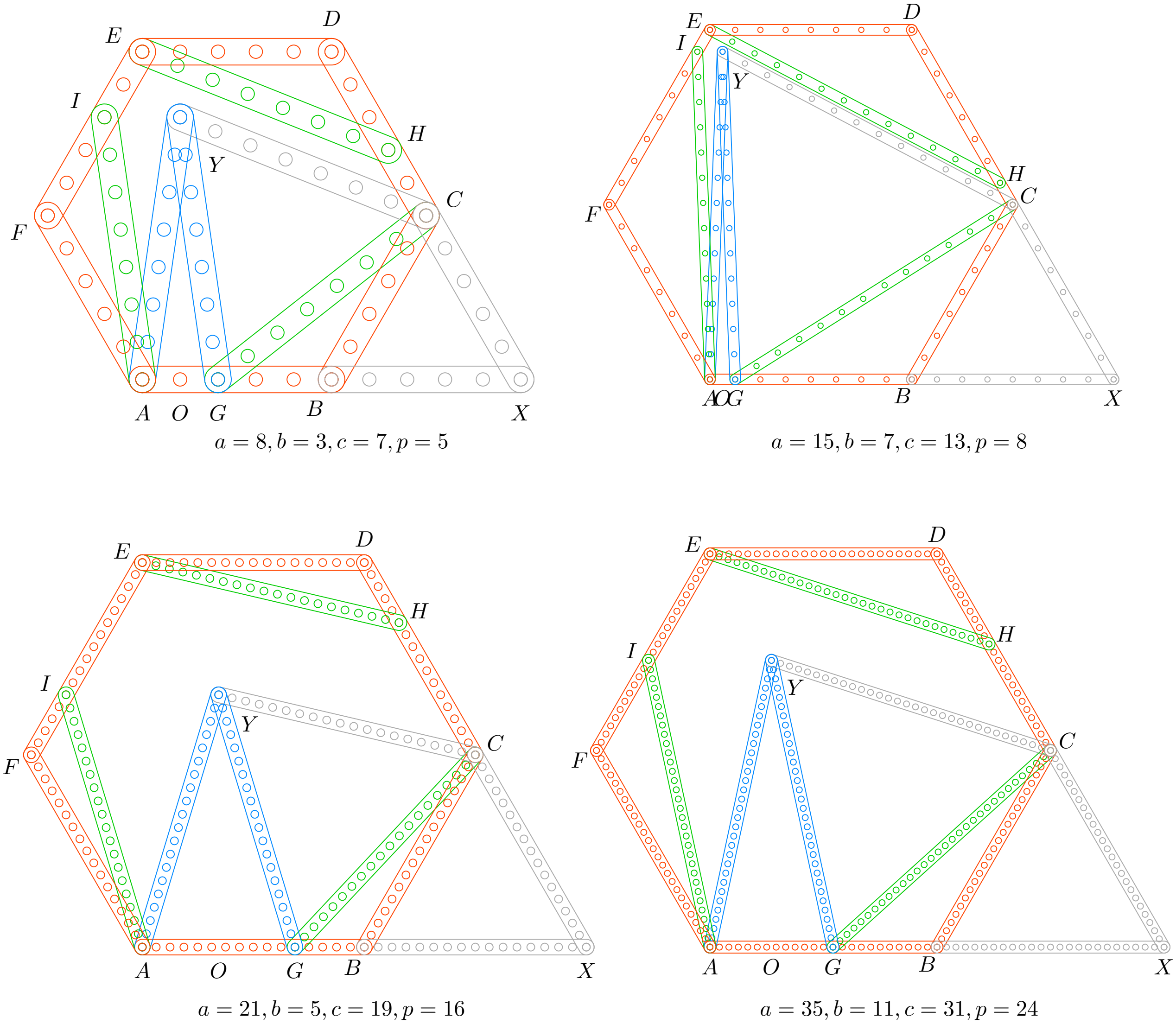

Figure 1: First four cases where internal strip $c = \overline{GC}$ is an integer and makes rigid two consecutive regular hexagon sides $p = \overline{AB} = \overline{BC}$. We use two numbers to identify every solution a and b where $b = \overline{GB}$ and $a = p - b$ and $c = \sqrt{a^2 + b^2 - ab}$.

and an isoscelles triangle $\triangle AGY$. The base of the isoscelles triangle is $x \equiv \overline{AG} = \overline{AB} - \overline{GB} = p - b$ and the equals sides $\overline{AY} = \overline{GY} = c$. So we can calculate the height $y \equiv \overline{OY}$:

$$
\begin{aligned}
y &= \sqrt{(\overline{GY})^2 - (\overline{AO})^2} \\
&= \sqrt{c^2 - \left(\frac{p-b}{2}\right)^2} \\
&= \sqrt{b^2 + p^2 + bp - \left(\frac{p-b}{2}\right)^2} = \frac{(p+s)\sqrt{3}}{2} = \frac{a\sqrt{3}}{2} \qquad (3)
\end{aligned}
$$

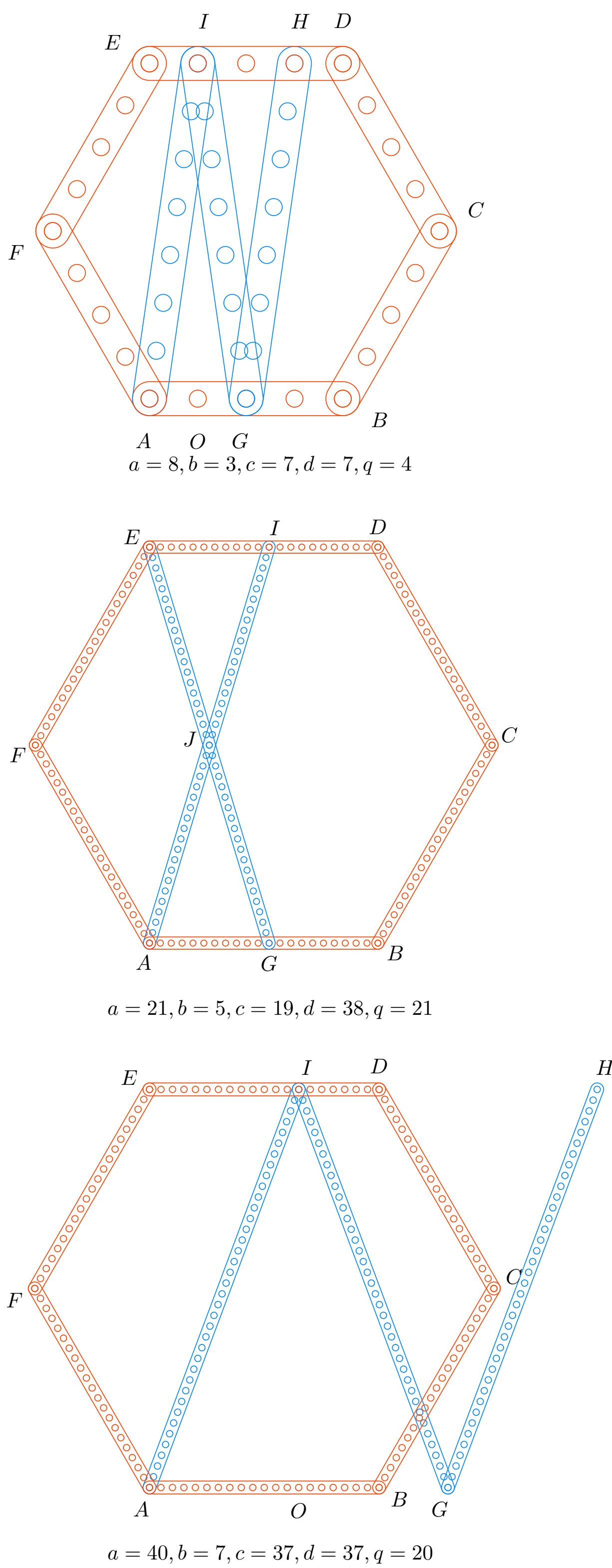

Figure 2: First six cases of integral distances c. When the distance $p - b = \overline{AG}$ is even, we use the strips $d = c$ to join opposites sides of hexagons of side $q = a/2$. When is odd, we use the strips $d = 2c$ to join opposites sides of hexagons of side $q = a$.

We know $\dfrac{a\sqrt{3}}{2}$ is the height of the regular hexagon of side $\dfrac{a}{2}$ so we can use the blue strips to connect opposite sides. Figure 2 show the smaller hexagons that have integer strips connecting opposites sides.

Through the gallery we will use the green and blue strips and scaled copies of such strips to make rigid regular hexagons from size 4 to 24. We prioritize minimum number of bolts, minimum number of strips and the largest strips sizes as possible.

2 Hexagons of size 13

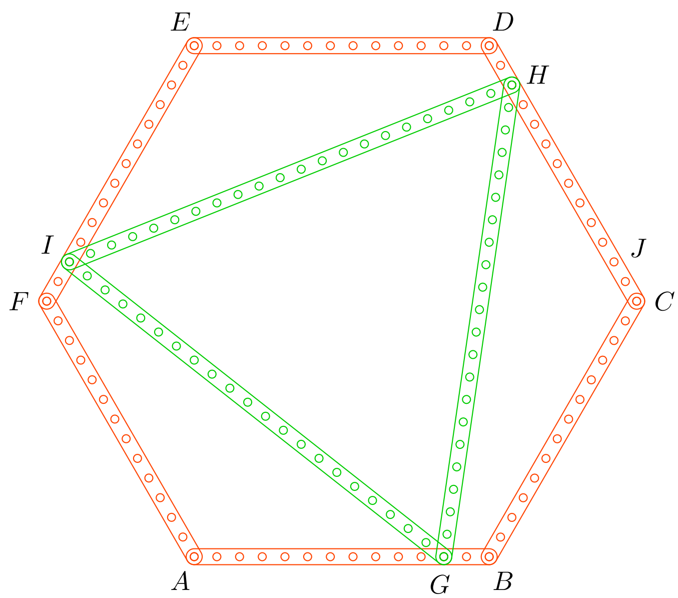

Figure 3: Hexagon of size $s = 13$. Diagonals $c = \overline{GH} = \overline{HI} = \overline{IG} = 21$.

Figure 3 show hexagon of size $s = 13$. First we detect an offset $o \equiv \overline{DH}$ which we use to calculate the sides p', b', c' of triangle $\triangle GJH$:

$$\begin{aligned} o &\equiv \overline{DH} = \overline{CJ} = 2 \\ p' &\equiv \overline{GJ} = \overline{BC} + o = 13 + 2 = 15 \\ b' &\equiv \overline{JH} = \overline{CD} - 2o = 13 - 2(2) = 9 \\ c' &\equiv \overline{GH} = 21 \end{aligned} \tag{4}$$

We confirm triplet c', p', b' is three times $(n = 3)$ valid hexagonal triplet $c = 7, p = 5, b = 3$. This case has an equilaterl triangle $\triangle GHI$ inside the hexagon because is a special of the general case when:

$$\begin{aligned} nb &= s - 2o \\ np &= s + o \\ nc &= \sqrt{(nb)^2 + (np)^2 - (nb)(np)} \\ &= \sqrt{(s-2o)^2 + (s+o)^2 + (s-2o)(s+o)} \\ &= \sqrt{3s^2 - 3so + 3o^2} \end{aligned} \tag{5}$$

First terms of this case is shown in the next table:

s	o	c	p	b
13	2	21	15	9
23	1	39	24	21
37	11	57	48	15
59	13	93	72	33
73	26	111	99	21
83	22	129	105	39
94	23	147	117	48

Table 2: Equilateral triangles side=c inside regular hexagons side=s.