Meccano octagons

https://github.com/heptagons/meccano/octa

Abstract

We construct meccano [1] regular octagons using eight equal strips to build the a perimeter and attaching **internal diagonals** to make the polygon regular and rigid. The attached diagonals form angles of 135° to fix consecutive sides. We prepare a three variables formula $z = f(x, y)$ and catalog solutions when the variables are integers.

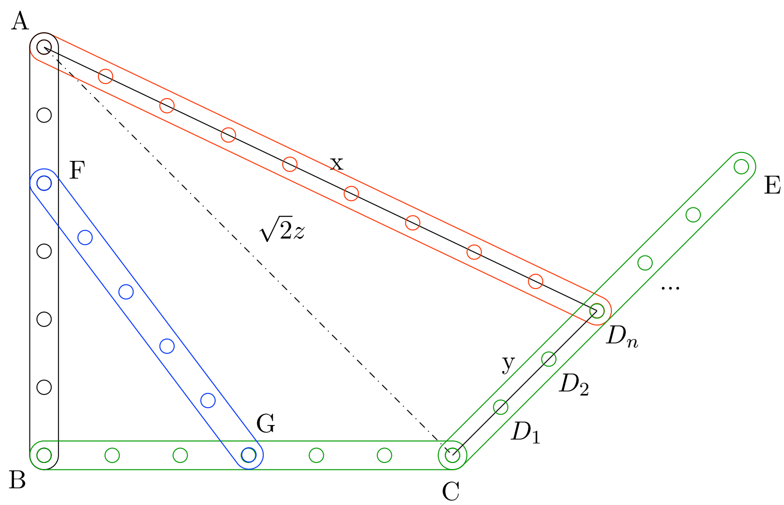

Figure 1: Construction of a 135° angle with meccano strips.

1 Meccano regular octagons

We build meccano regular octagons forming rigid strips with internal angles of 135°. Figure 1 show the internal diagonals construction. First, we build two triangles with adjacent angles adding 135°. Consider triangle ABC with $\angle BCA = 45°$ and triangle ACD_n with $\angle ACD_n = 90°$, so $\angle BCE = 135°$. Lets define:

$$x = \overline{AD_n}$$
$$y = \overline{CD_n}$$
$$z = \overline{AB} = \overline{BC} = \overline{CE}$$

The internal diagonal we are looking for is the integer x (shown in red) and the two adjacent octagon sides are $z = \overline{BC} = \overline{CE}$ (shown in green). The two angles common hyphotenuse is shown as a dashed line with a length of $\overline{AC} = \sqrt{2}z$ so:

$$(\sqrt{2}z)^2 = x^2 - y^2$$
$$2z^2 = x^2 - y^2$$
$$z = \sqrt{\frac{x^2 - y^2}{2}}$$

[1] Meccano mathematics by ‘t Hooft

We run a program using the above formula to iterate over integers pair $x > y$ and expecting to find z as an integer too.

1.1 Program

Next golang listing program finds valid octagons diagonals a and sides $\max(b, c)$. First we iterate diagonals from 1 to a given maximum (line 2). Then we iterate over integer b from 1 to a (line 3). We calculate $2c^2$ and check if its even (lines 4, 5) and the we check if c is an integer (line 8). To prevent repetitions by scaling we check the greatest common divisor to be 1 (line 9) and print a valid result where the octagon size is the maximum value of b or c. (line 11).

```
1  func Angles135(max int) {
2    for x := 1; x < max; x++ {
3      for y := 1; y < x; y++ {
4        if zz := x*x - y*y; zz % 2 == 0 {
5          f := math.Sqrt(float64(zz / 2))
6          if z := int(f); f == float64(z) {
7            if meccano.Gcd(z, meccano.Gcd(x, y)) == 1 {
8              a := int(math.Max(float64(y), f))
9              fmt.Printf("a=%3d x=%3d y=%3d z=%3d\n", a, x, y, z)
10           }
11         }
12       }
13     }
14   }
15 }
```

1.2 Results

First results for $x < 200$ are shown in the next listing. The octagon size is $a = max(y, z)$.

```
a=  2 x=  3 y=  1 z=  2
a=  7 x=  9 y=  7 z=  4
a=  7 x= 11 y=  7 z=  6
a= 12 x= 17 y=  1 z= 12
a= 17 x= 19 y= 17 z=  6
a= 23 x= 27 y= 23 z= 10
a= 20 x= 33 y= 17 z= 20
a= 31 x= 33 y= 31 z=  8
a= 24 x= 41 y= 23 z= 24
a= 30 x= 43 y=  7 z= 30
a= 47 x= 51 y= 47 z= 14
a= 49 x= 51 y= 49 z= 10
a= 40 x= 57 y=  7 z= 40
a= 41 x= 57 y= 41 z= 28
a= 41 x= 59 y= 41 z= 30
a= 42 x= 67 y= 31 z= 42
a= 71 x= 73 y= 71 z= 12
a= 56 x= 81 y= 17 z= 56
a= 79 x= 83 y= 79 z= 18
a= 73 x= 89 y= 73 z= 36
a= 60 x= 97 y= 47 z= 60
a= 70 x= 99 y=  1 z= 70
a= 97 x= 99 y= 97 z= 14
a= 89 x=107 y= 89 z= 42
a= 72 x=113 y= 49 z= 72
a= 84 x=121 y= 23 z= 84
a= 73 x=123 y= 73 z= 70
a=119 x=123 y=119 z= 22
a=113 x=129 y=113 z= 44
a=127 x=129 y=127 z= 16
a= 90 x=131 y= 31 z= 90
a=119 x=137 y=119 z= 48
a=103 x=139 y=103 z= 66
a= 89 x=153 y= 89 z= 88
a=103 x=153 y=103 z= 80
a=161 x=163 y=161 z= 18
a=110 x=171 y= 71 z=110
a=167 x=171 y=167 z= 26
a=112 x=177 y= 79 z=112
a=161 x=177 y=161 z= 52
a=126 x=179 y= 17 z=126
a=137 x=187 y=137 z= 90
a=151 x=187 y=151 z= 78
a=132 x=193 y= 49 z=132
```

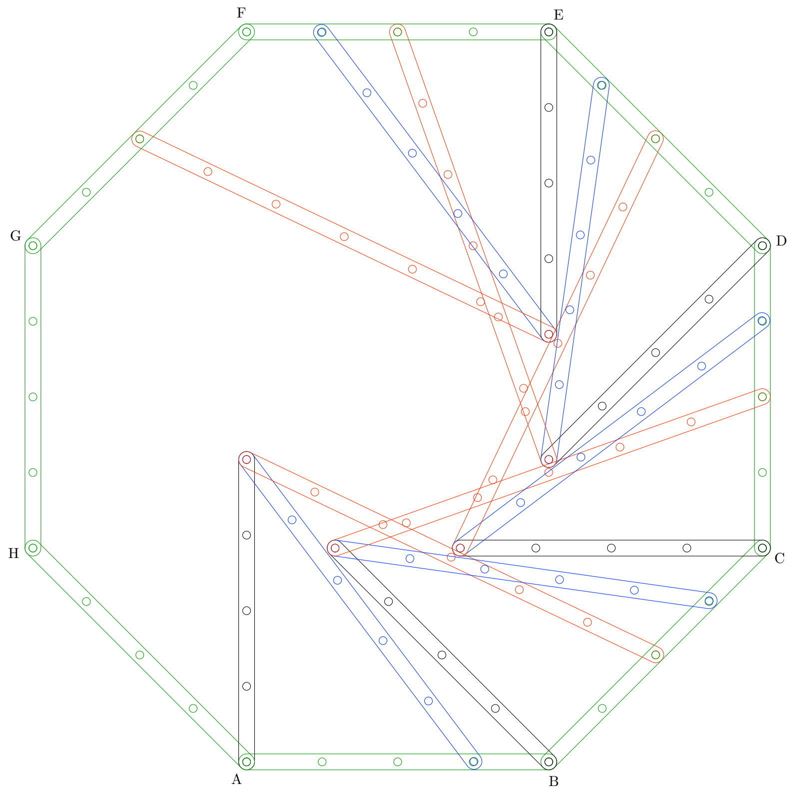

Figure 2: The smallest octagon with diagonals $x = 6$ and sides $z = 4$. In order to prevent bolts collisions we need to use strips with holes well separated. Four 135° units are needed to fix six consecutive rigid sides. This complex figure is simplified in next figure.

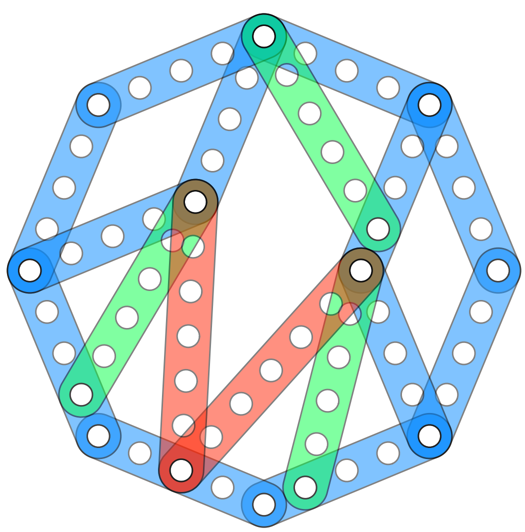

Figure 3: The smallest octagon again with diagonals $x = 6$ and sides $z = 4$ but with fewer extra strips. This construction is of the size of fig 2 but we put only two adjacent x red bars face to face and complete the rest with two $z = 4$ in blue and one more pythagorean green strip of size 5.

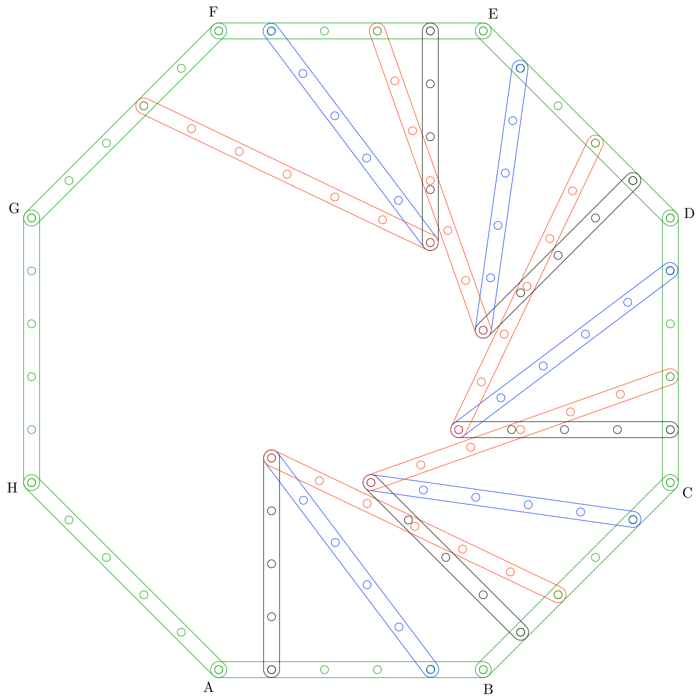

Figure 4: Octagon with diagonals $x = 6$ and sides $z + 1 = 5$.

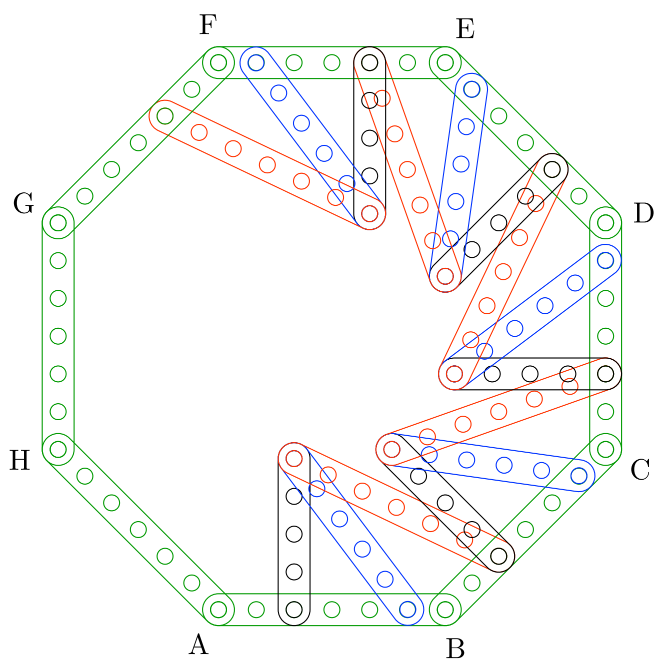

Figure 5: Octagon with diagonals $x = 6$ and sides $z + 2 = 6$.

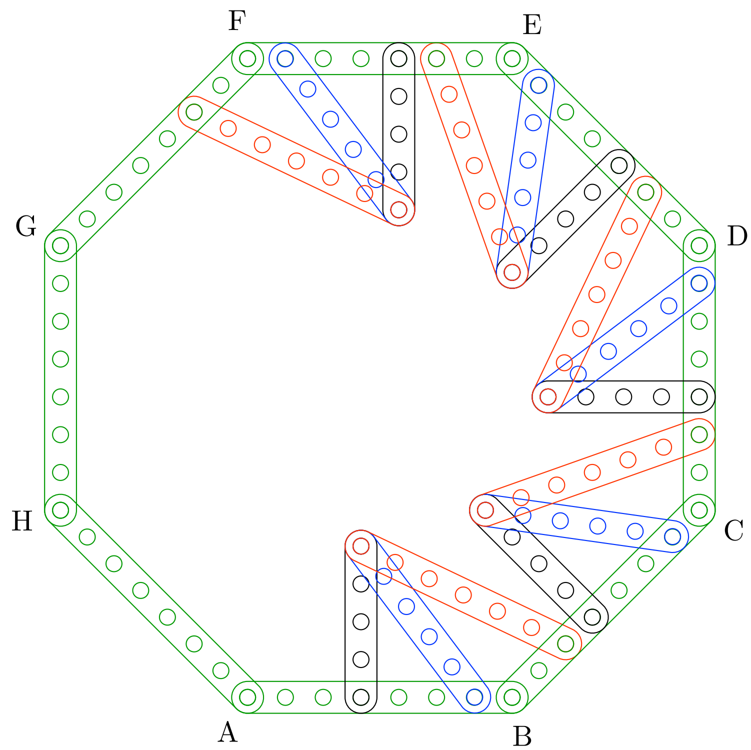

Figure 6: Octagon with diagonals $x = 6$ and sides $z + 3 = 7$.

1.3 Examples with diagonal $x = 6$

We can't use the first result:

$$a = 2, x = 3, y = 1, z = 2$$

because the octagon size is to small, is only $z = 2$. At least the octagon size should be 4 because the smallest right angle can be made with the pythagorean triplet 3, 4, 5. To make rigid the angle of 90° of triangle ABC in figure 1 we need a rod of size 5 at least, as shown the points F and G. Multiplying the first result by 2 we get:

$$a = 4, x = 6, y = 2, z = 4$$

and this will be the smallest octagon since $z = 4$ can hold a $3 - 4 - 5$ triplet. Figure 2 is the smallest octagon; we need to scale the bars in order the bolts don't collapse with others, the complexity of bars can be simplified symmetrically as is show in figure 3. In figure 4 we increase the side from 4 to 5 but keeping the same diagonal of 6. In figure 5 we increase the side to 6 and in figure 6 the side is increased to 7.

1.4 Examples with diagonal $x = 9$

With the second result:

$$a = 7, x = 9, y = 7, z = 4$$

we can form a second group of octagons. Figure 7 shows the smallest octagon with diagonals 9 and sides 7. Figures 8, 9, 10 and 11 show octagons with diagonals 9 with sides of 8, 9, 10 and 11.

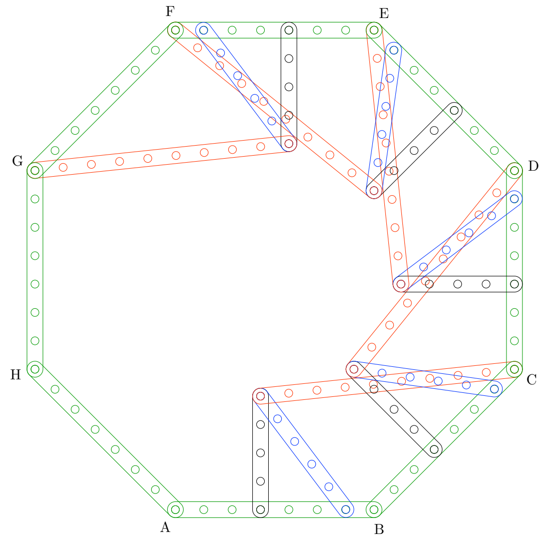

Figure 7: Octagon with diagonal $x = 9$ and side $y = 7$.

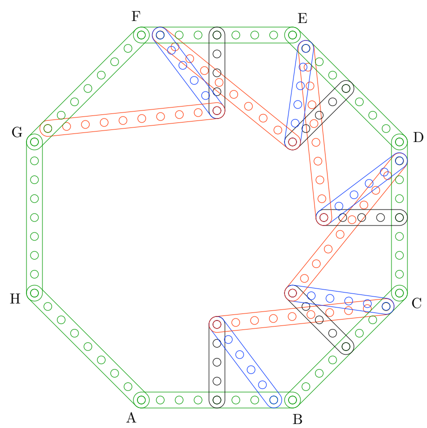

Figure 8: Octagon with diagonal $x = 9$ and side $y + 1 = 8$.

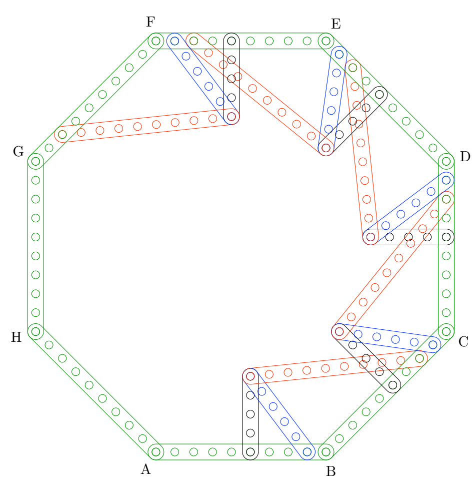

Figure 9: Octagon with diagonal $x = 9$ and side $y + 2 = 9$.

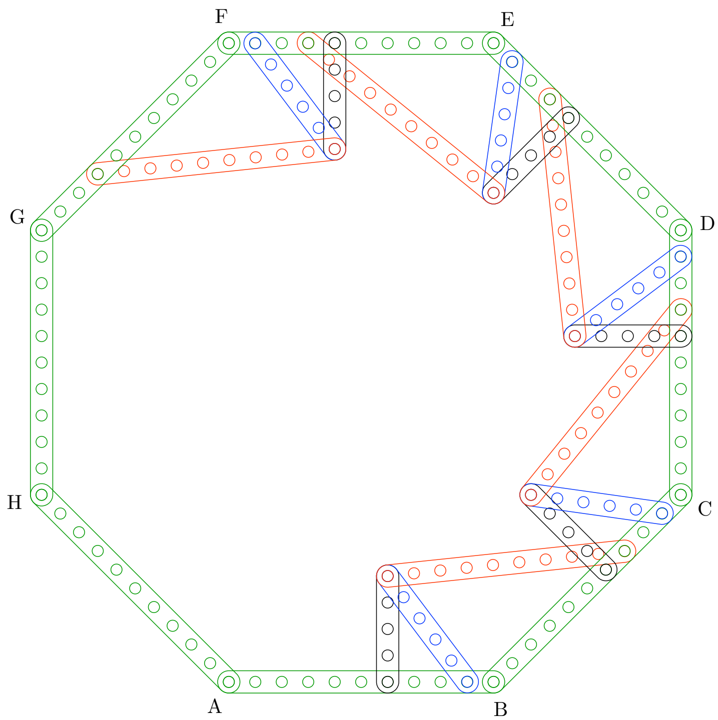

Figure 10: Octagon with diagonal $x = 9$ and side $y + 3 = 10$.

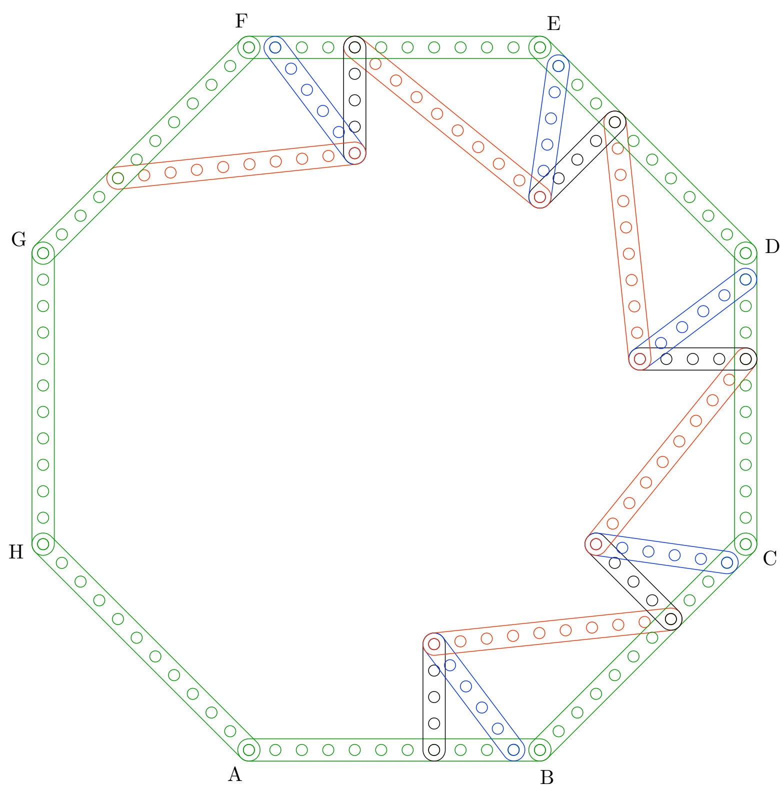

Figure 11: Octagon with diagonal $x = 9$ and side $y + 4 = 11$.

1.5 Example with double diagonals $x = 6$, $x = 9$

By comparing figures 6 and 10 both with sides=$y = 7$ we can make use of two diagonals at the same time and omit the rod FG of figure 1 used until now to make the 90° angle. Figure 12 show a octagon angle with two diagonals. In other words, for two results or their scaling, when both have the same y, we can use the two diagonals an omit the 5 rod.

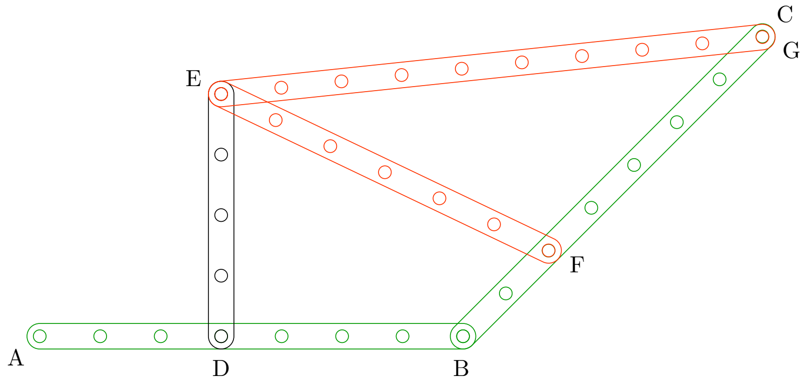

Figure 12: Octagon angle ABC fixed with two diagonals. The union of rods $\overline{BG}$, $\overline{EF}$ and $\overline{EG}$ is rigid. Adding two rods $\overline{AB}$ and $\overline{DE}$ remains rigid.