

Meccano fox-surd frame

<https://github.com/heptagons/meccano/frames/fox-surd>

Abstract

Meccano ¹ fox-surd frame is a generalization of fox-frame² where at least one of the frame's strips size is no longer an integer but a surd.

1 Pentagons fox-surd

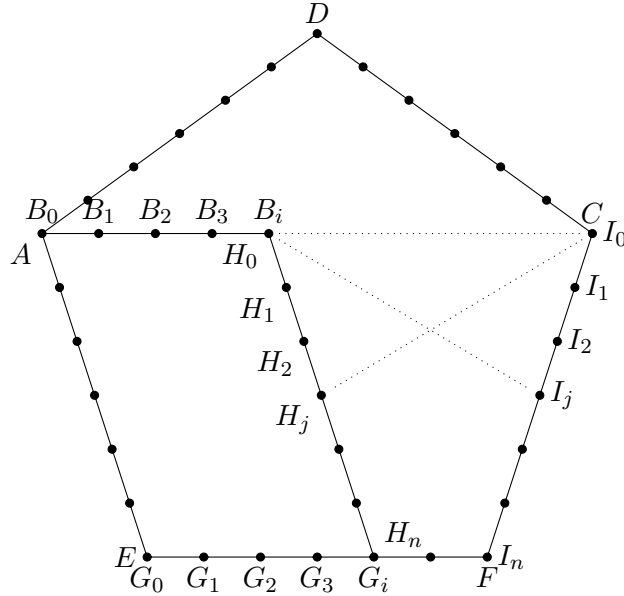


Figure 1: Pentagon of size n where each segment separated by circles represents a unit. We have a surd frame formed by the six points: B_i , I_0 , H_j , I_j , H_n and I_n . By iterating the values $i, j = 0, \dots, n$ we'll get diverse frames.

From figure 1 the fox-surd frame has three real strips of integer size:

- $\overline{B_i G_i}$ of size n .
- $\overline{G_i I_n}$ of size $n - i$, where $i = 0, \dots, n$.
- $\overline{I_0 I_n}$ of size n .

The other two strips are generic in the sense the sizes can be surds:

- $\overline{B_i I_j}$ of size to be determined $f(n, i, j)$, where $i, j = 0, \dots, n$.
- $\overline{H_j I_0}$ of equal size of $\overline{B_i I_j}$.

¹ Meccano mathematics by 't Hooft

² Meccano fox frame

From the regular pentagon we know the main diagonal \overline{AC} equals $\frac{1+\sqrt{5}}{2}n$ where n is the pentagon side size. We can calculate different segments of the main diagonal iterating $i = 0, \dots, n$:

$$\begin{aligned} B_0 &\equiv A \\ \overline{B_0C} &= \frac{1+\sqrt{5}}{2}n \end{aligned} \tag{1}$$

$$\begin{aligned} \overline{B_iC} &= \frac{1+\sqrt{5}}{2}n - i \\ &= \frac{n-2i}{2} + \frac{n\sqrt{5}}{2}, \quad i = 0, \dots, n \\ &= \frac{x_i}{2} + \frac{n\sqrt{5}}{2}, \quad x_i = n - 2i \end{aligned} \tag{2}$$

From the regular pentagon we know the angle B_iCI_j equals $2\pi/5$ so we have:

$$\theta \equiv \angle B_iCI_j \tag{3}$$

$$\cos \theta = \cos \frac{2\pi}{5} = \frac{\sqrt{5}-1}{4} \tag{4}$$

1.1 Pentagon surds sizes

Using the law of cosines we can calculate one of the frame surds $s_{ij} \equiv \overline{B_iI_j}$. We notice the value of $\overline{CI_j}$ equals j , and we'll use the values of $\overline{B_iC}$ from equation 2, and the cosine value from equation 4 to get:

$$s_{ij}^2 \equiv \overline{B_iI_j}^2 \tag{5}$$

$$\begin{aligned} &= \overline{CI_j}^2 + \overline{B_iC}^2 - 2\overline{CI_j} \times \overline{B_iC} \cos \theta \\ &= j^2 + \left(\frac{x_i}{2} + \frac{n\sqrt{5}}{2}\right)^2 - 2j \left(\frac{x_i}{2} + \frac{n\sqrt{5}}{2}\right) \left(\frac{\sqrt{5}-1}{4}\right) \end{aligned} \tag{6}$$

$$= j^2 + \frac{1}{4} \left(x_i + n\sqrt{5}\right)^2 - \frac{j}{4} \left(x_i + n\sqrt{5}\right) (\sqrt{5}-1) \tag{7}$$

We multiply both sides by 4 and simplify:

$$(2s_{ij})^2 = 4j^2 + (x_i + n\sqrt{5})^2 - j(x_i + n\sqrt{5})(\sqrt{5}-1) \tag{8}$$

$$= 4j^2 + x_i^2 + 2x_in\sqrt{5} + 5n^2 - j(x_i\sqrt{5} - x_i + 5n - n\sqrt{5}) \tag{9}$$

$$= 4j^2 + x_i^2 + 5n^2 + x_ij - 5nj + (2nx_i - x_ij + nj)\sqrt{5} \tag{10}$$

In order to have a simpler $(4s_i)^2 = u + v\sqrt{5}$ we define two variables u and v . We replace again $x_i = n - 4i$ defined in equation 2:

$$\begin{aligned} u &\equiv 4j^2 + x_i^2 + 5n^2 + x_ij - 5nj \\ v &\equiv 2nx_i - x_ij + nj \end{aligned} \tag{11}$$

Finally we have s_{ij} in function of n the side:

$$\begin{aligned} s_{ij} &= \frac{\sqrt{u + v\sqrt{5}}}{4} \\ &= \frac{\sqrt{(n-5i)^2 + 16(j^2 - i^2) + 5 - i^2 + 2((2i-1)^2 - (n-1)(i-1))\sqrt{5}}}{4} \end{aligned} \tag{12}$$

1.2 Pentagons surds simplification

If value v from equation ?? is zero s_{ij} simplifies to $\frac{\sqrt{u}}{4}$:

$$\begin{aligned}v &= 0 \\2((2i - 1)^2 - (n - 1)(i - 1)) &= 0 \\(2i - 1)^2 &= (n - 1)(i - 1)\end{aligned}\tag{13}$$