Meccano triangles

https://github.com/heptagons/meccano/nest

Abstract

We construct meccano triangles with integers sides and calculate the diagonal distances. Such diagonals then are used as the new side of more complicated triangles and then again we calculate new distances formed and so on. Eventually we expect to find certain angles which can be used to construct regular polygons or something else.

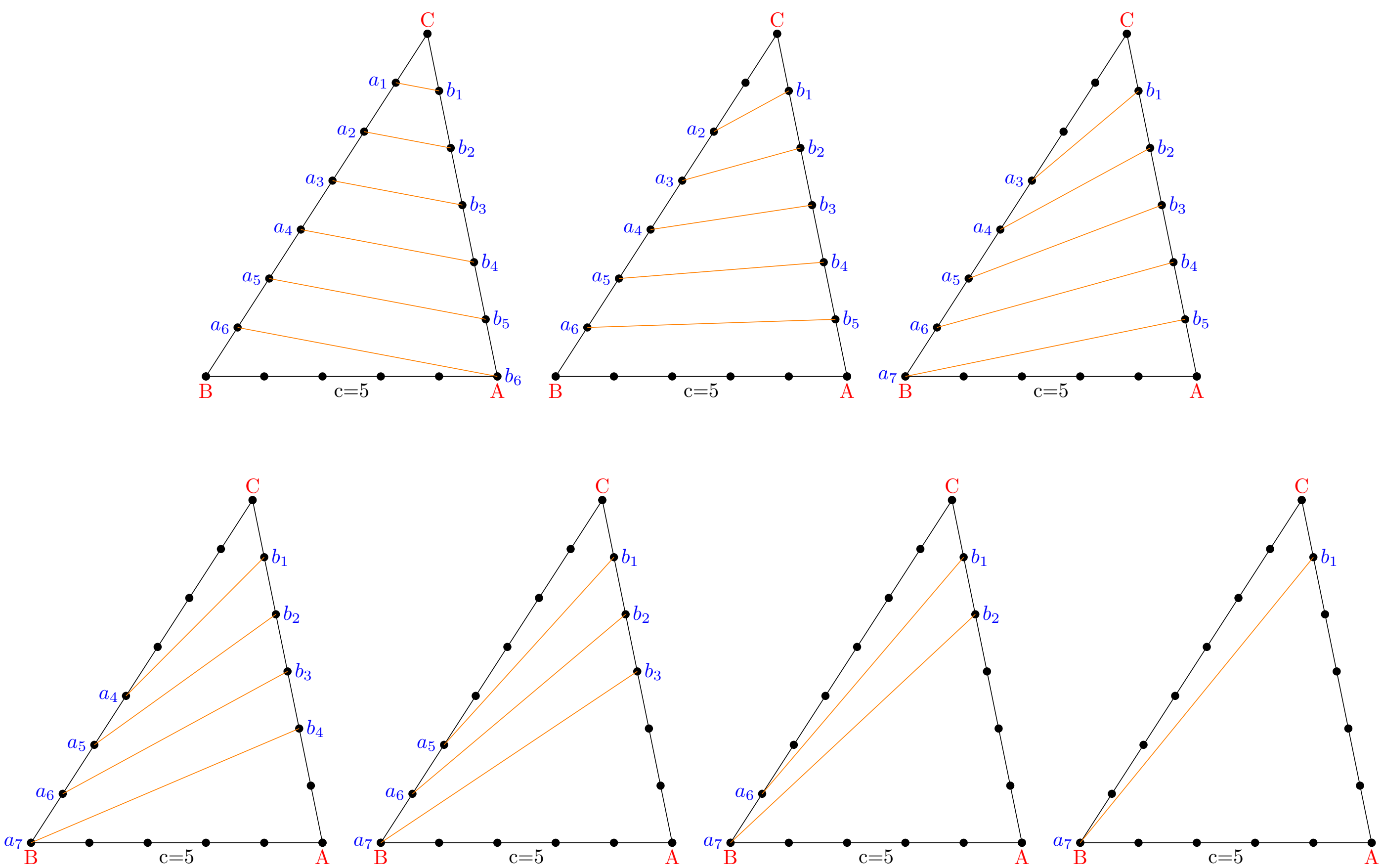

Figure 1: The meccano scalene triangle with sides 7, 6 and 5.

1 Meccano triangle

A meccano triangle has the tree sides a, b and c where $a, b, c \in \mathbb{N}$. To avoid repetitions we consider only the cases $a \geq b$, $b \geq c$ and $a \geq c$. Valid triangles also need the condition $a \geq b + c$.

1.1 Triangle diagonals

Figure 1 show a meccano triangle with sides $a = 7$, $b = 6$ and $c = 5$. We have seven groups of diagonals shown as orange lines. The diagonals join points from side a to side b in all combinations. From top to bottom and left to right the groups are:

Group	$a - b$	diagonals
1	0	a_1b_1 to a_6b_6
2	1	a_2b_1 to a_6b_5
3	2	a_3b_1 to a_7b_5
4	3	a_4b_1 to a_7b_5
5	4	a_5b_1 to a_7b_3
6	5	a_6b_1 to a_7b_2
7	6	a_7b1

To calculate the diagonals we start calculating $\cos C$ where C is the opposite angle of side $c = 5$:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \tag{1}$$
$$= \frac{7^2 + 6^2 - 5^2}{2 \times 7 \times 6} = \frac{4}{21} \tag{2}$$

Then with the $\cos C$ we can calculate every diagonal $\overline{a_x b_y}$ with:

$$\overline{a_x b_y} = \sqrt{x^2 + y^2 - 2xy \cos C} \tag{3}$$
$$= \sqrt{x^2 + y^2 - 2xy \frac{a^2 + b^2 - c^2}{2ab}} \tag{4}$$

where $1 \le x \le a$, $1 \le y \le b$ and $x - y \ge 0$.
By inspection we deduce that for first meccano triangles:

$$a, b, c \in \mathbb{N} \tag{5}$$
$$\cos A, \cos B, \cos C \in \mathbb{Q} \tag{6}$$
$$\overline{a_x b_y}, \overline{b_y c_z}, \overline{a_x c_z} \in \mathbb{A} \tag{7}$$