Meccano triangles

https://github.com/heptagons/meccano/nest

Abstract

We construct meccano triangles. Basic triangles has the three sides as integers and calculate the internal diagonal distances. Such diagonals then are used as the new side of more complicated triangles and then again we calculate new distances formed and so on. Eventually we expect to find certain angles joining the triangles which can be used to construct regular polygons or more figures.

1 Basic triangle

A basic triangle has the tree sides a, b and c where $a, b, c \in \mathbb{N}$. To avoid repetitions we consider only the cases $a \geq b$, $b \geq c$ and $a \geq c$. Valid triangles also need the condition $a \geq b + c$.

1.1 Basic triangle diagonals

To calculate the diagonals from side a to side b we start calculating $\cos C$ where C is the opposite angle of side c:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \tag{1}$$

Then with the $\cos C$ we can calculate every diagonal $\overline{a_x b_y}$ with the law of cosines:

$$\begin{aligned}
\overline{a_x b_y} &= \sqrt{x^2 + y^2 - 2xy \cos C} && (2)\\
&= \sqrt{x^2 + y^2 - 2xy \frac{a^2 + b^2 - c^2}{2ab}} && (3)\\
&= \frac{\sqrt{a^2 b^2 (x^2 + y^2) - abxy(a^2 + b^2 - c^2)}}{ab} && (4)
\end{aligned}$$

where $1 \leq x \leq a$, $1 \leq y \leq b$ and $x - y \geq 0$.
By inspection we deduce that for basic meccano triangles:

$$\begin{aligned}
a, b, c &\in \mathbb{N} && (5)\\
\cos A, \cos B, \cos C &\in \mathbb{Q} && (6)\\
\overline{a_x b_y}, \overline{b_y c_z}, \overline{a_x c_z} &\in \mathbb{A} && (7)
\end{aligned}$$

Figure 1 show a basic triangle with sides $a = 7$, $b = 6$ and $c = 5$. For this particular triangle we have seven groups of diagonals shown as orange lines. The diagonals join points from side a nodes described as a_x to side b nodes described as b_y in all combinations. From top to bottom and left to right the groups are:

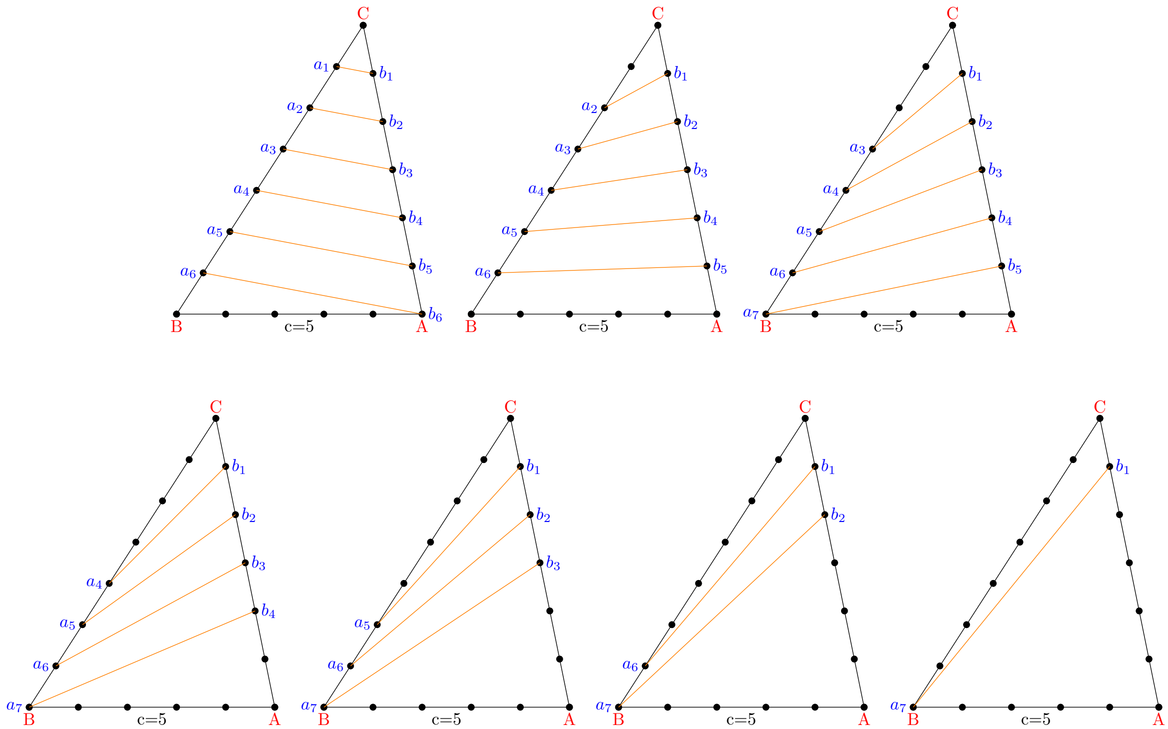

Figure 1: Basic triangle $[7, 6, 5]$, $a_x b_x$ diagonals.

$$a_x b_y = \begin{bmatrix} \frac{2\sqrt{7}}{7} & \frac{\sqrt{105}}{7} & \frac{2\sqrt{70}}{7} & \frac{\sqrt{553}}{7} & \frac{2\sqrt{231}}{7} & \frac{\sqrt{1393}}{7} & 2\sqrt{10} \\ & \frac{4\sqrt{7}}{7} & \frac{\sqrt{217}}{7} & \frac{2\sqrt{105}}{7} & \frac{\sqrt{721}}{7} & \frac{4\sqrt{70}}{7} & \sqrt{33} \\ & & \frac{6\sqrt{7}}{7} & \frac{\sqrt{385}}{7} & \frac{2\sqrt{154}}{7} & \frac{3\sqrt{105}}{7} & 2\sqrt{7} \\ & & & \frac{8\sqrt{7}}{7} & \frac{\sqrt{609}}{7} & \frac{2\sqrt{217}}{7} & 5 \\ & & & & \frac{10\sqrt{7}}{7} & \frac{\sqrt{889}}{7} & 2\sqrt{6} \\ & & & & & \frac{12\sqrt{7}}{7} & \end{bmatrix}$$