Meccano heptagons

https://github.com/heptagons/meccano/hepta

Abstract

We construct meccano[1] regular heptagons. We use seven equal strips to build the polygon perimeter and then we attach **internal diagonals** to make the polygon regular and rigid. Using some heptagonal identities we deduce three variables a, b and c corresponding to the heptagon side, an internal point of the side and an internal diagonal. Then we find a closed formula $c = f(a, b)$ and use a program to iterate over a and b to get c integer which produces prime solutions.

1 Meccano heptagons

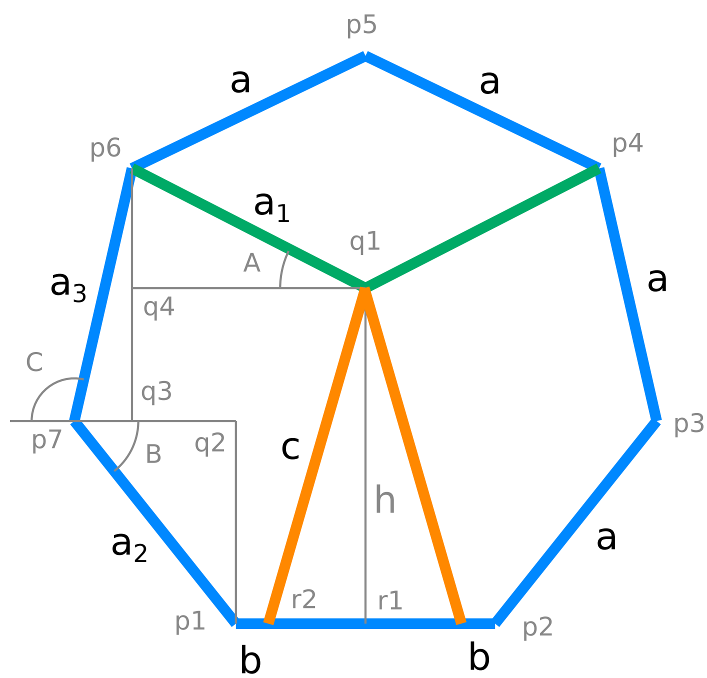

Figure 1: A meccano regular heptagon layout. First we define two integers a and b where $a > 2b$. We look for a third integer c to make the heptagon.

Consider the regular heptagon in figure 1. By inspection we identify three angles A, B and C:

$$A = \frac{\pi}{7}, B = \frac{2\pi}{7}, C = \frac{4\pi}{7}$$

Then we find the sines of the angles, noticing that the regular heptagon side is $a = a_1 = a_2 = a_3$:

[1] Meccano mathematics by 't Hooft

$$sinA = \frac{\overline{p_6q_4}}{a_1}, sinB = \frac{\overline{p_1q_2}}{a_2}, sinC = \frac{\overline{p_6q_3}}{a_3}$$

From the figure the height h corresponds to:

$$\begin{aligned} h &= \overline{p_1q_2} + \overline{p_6q_3} - \overline{p_6q_4} \\ &= a_2 sinB + a_3 sinC - a_1 sinA \\ &= a(-sinA + sinB + sinC) \end{aligned}$$

According to *heptagonal triangles*[2]

$$\begin{aligned} sinA - sinB - sinC &= -\frac{\sqrt{7}}{2} \\ \frac{h}{a} &= \frac{\sqrt{7}}{2} \\ h &= \frac{\sqrt{7}a}{2} \end{aligned}$$

Finally we get the c length as a function of lengths a and b:

$$\begin{aligned} c^2 &= h^2 + \overline{r_1r_2}^2 \\ &= (\frac{\sqrt{7}a}{2})^2 + (\frac{a-2b}{2})^2 \\ &= \frac{7a^2}{4} + \frac{a^2-4ab+4b^2}{4} + \\ &= \frac{8a^2-4ab+4b^2}{4} \\ &= 2a^2 - ab + b^2 \end{aligned}$$

1.1 Heptagons search

A valid meccano heptagon needs to have the three lenghts a, b and c as integers. With a software routine we look for c to be integer by incrementing the values of a and b, where $b < \lceil a/2 \rceil$.

1.2 Code

Following code function `Diagonals` (line 9) find several integer diagonals c in function of variables a and b. We iterate over $2 \leq a \leq max$ (line 14), then we iterate over $1 \leq b < \lceil a/2 \rceil$ (line 17). Then we check c is and integer using the previous formula (lines 18 and 19). We store a solution not counting repetitions by scaling (line 20).

```
1 package hepta
2
3 import (
4   "math"
5
```

[2] https://en.wikipedia.org/wiki/Heptagonal_triangle

```
  "github.com/heptagons/meccano"
)

func Diagonals(max int) *meccano.Sols {

  sols := &meccano.Sols{}

  bMax, aa := 0, 0
  for a := 2; a <= max; a++ {
    bMax = int(math.Ceil(float64(a) / 2))
    aa = 2*a*a
    for b := 1; b < bMax; b++ {
      f := float64(aa - a*b + b*b)
      if c := int(math.Sqrt(f)); math.Pow(float64(c), 2) == f {
        sols.Add(a, b, c)
      }
    }
  }
  return sols
}
```

1.3 Results

Heptagons prime solutions for $a \leq 450$:

```
  1  a=  3 b=  1 c=  4
  2  a=  8 b=  1 c= 11
  3  a= 33 b=  2 c= 46
  4  a= 40 b= 17 c= 53
  5  a= 55 b= 14 c= 74
  6  a= 65 b= 31 c= 86
  7  a= 85 b= 14 c=116
  8  a= 91 b=  2 c=128
  9  a= 95 b=  1 c=134
 10  a= 96 b= 47 c=127
 11  a=105 b= 23 c=142
 12  a=119 b= 46 c=158
 13  a=120 b= 23 c=163
 14  a=133 b= 62 c=176
 15  a=144 b= 41 c=193
 16  a=153 b=  7 c=214
 17  a=161 b= 34 c=218
 18  a=171 b= 34 c=232
 19  a=175 b= 17 c=242
 20  a=176 b= 79 c=233
 21  a=207 b= 94 c=274
 22  a=208 b= 47 c=281
 23  a=225 b= 98 c=298
 24  a=240 b=  7 c=337
 25  a=240 b= 89 c=319
 26  a=253 b= 47 c=344
 27  a=261 b= 62 c=352
 28  a=264 b=  1 c=373
 29  a=275 b= 82 c=368
 30  a=279 b= 41 c=382
 31  a=297 b=119 c=394
 32  a=312 b= 73 c=421
 33  a=319 b=158 c=422
 34  a=320 b= 79 c=431
 35  a=325 b= 23 c=452
 36  a=341 b=142 c=452
 37  a=351 b= 62 c=478
 38  a=360 b= 31 c=499
 39  a=377 b=103 c=506
 40  a=403 b=146 c=536
 41  a=407 b= 73 c=554
 42  a=408 b=167 c=541
 43  a=429 b= 46 c=592
 44  a=429 b=191 c=568
 45  a=435 b= 34 c=604
 46  a=448 b=137 c=599
```

1.4 Examples

Figures 2 and 3 show the first two heptagons. Figure 4 show the first six heptagons for comparison of growing.

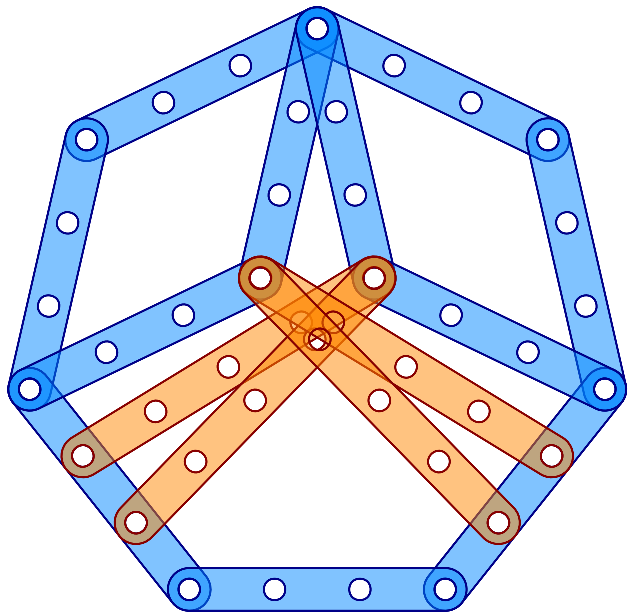

Figure 2: The first meccano heptagon with values $a = 3$, $b = 1$ and $c = 4$.

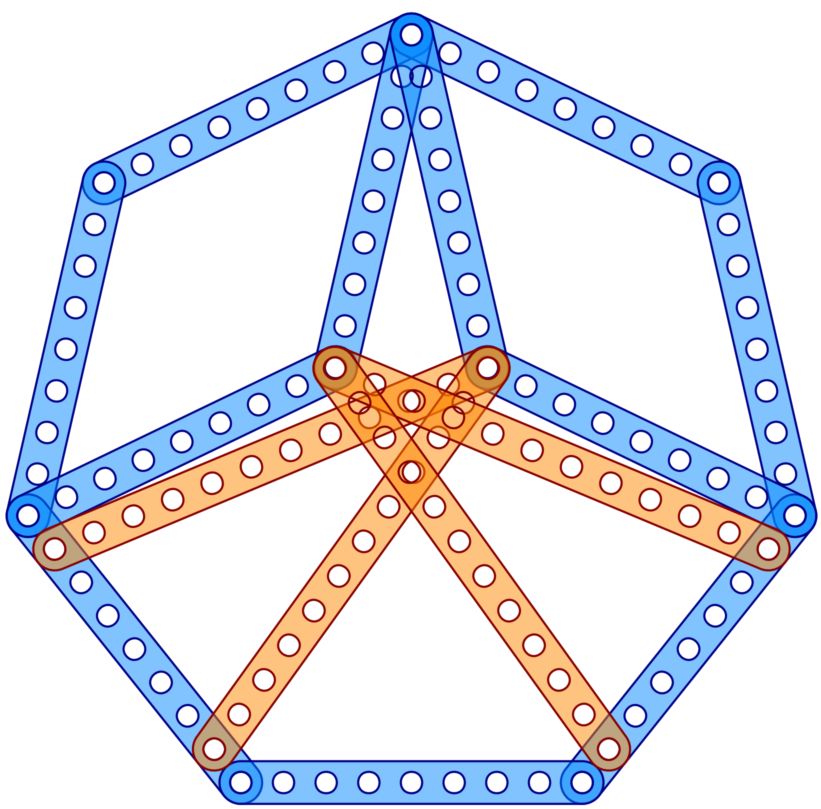

Figure 3: The second meccano heptagon with values $a = 8$, $b = 1$ and $c = 11$.

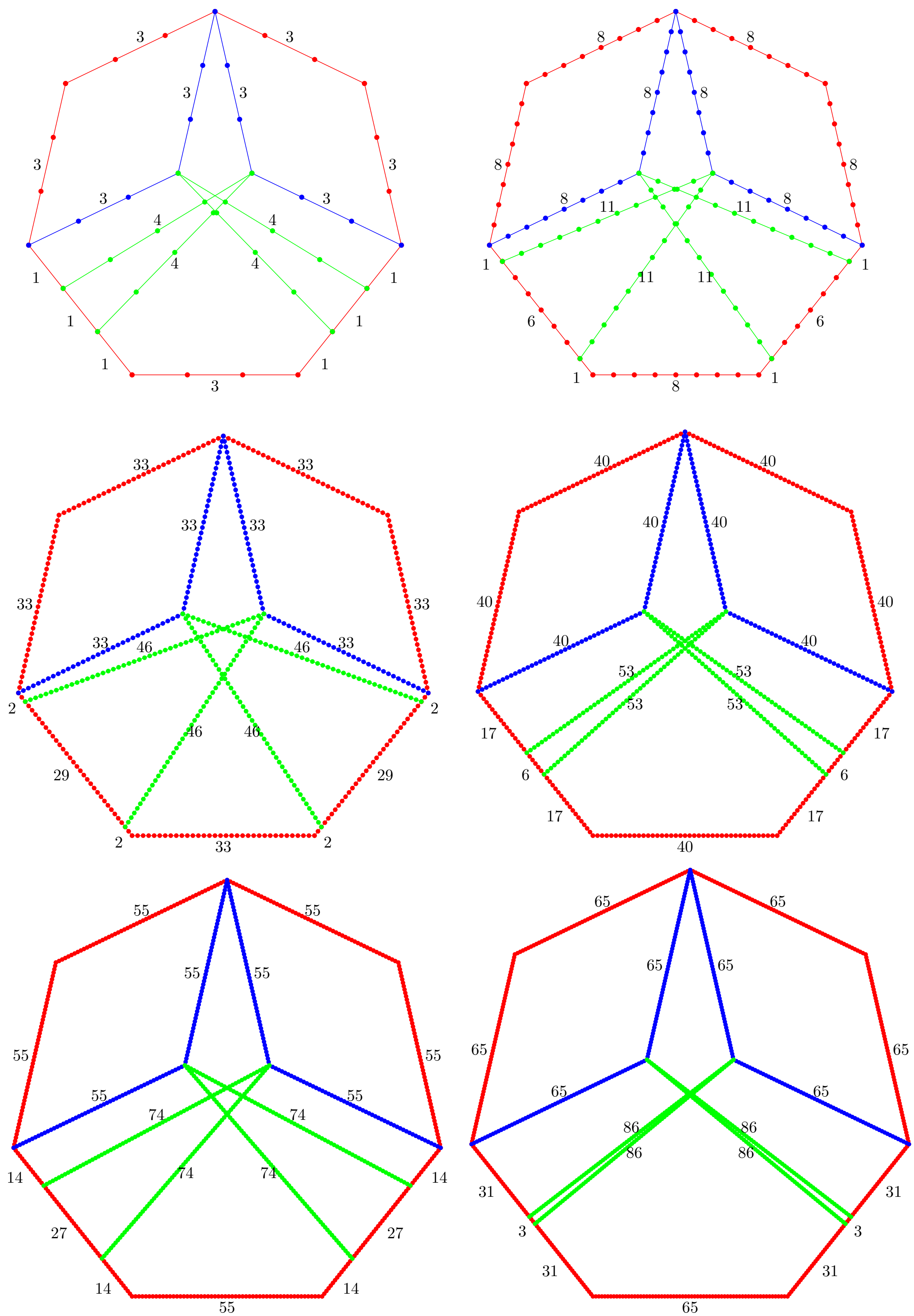

Figure 4: The first six prime hexagons $a = 3$, 8, 33, 40, 55 and 65.