

Triple unit

<https://github.com/heptagons/meccano/units/triple>

Abstract

Triple unit is a group of five meccano ¹ strips a, b, c, d, e intended to build regular polygons three consecutive perimeter sides. This unit has three angles equal to the polygon internal angle θ . Triple unit has been using to build the pentagon type 2 mentioned in pentagons paper².

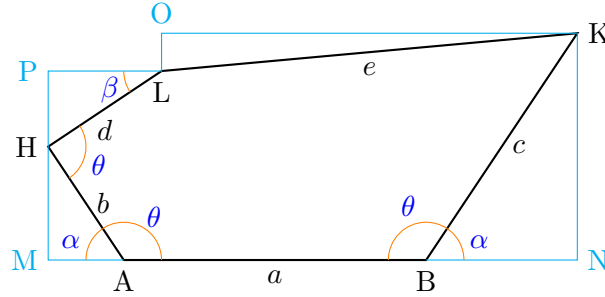


Figure 1: Triple unit has five strips a, b, c, d, e

From the fig 1 we have:

$$\theta = \pi - \alpha \quad \text{in nodes } A, B \quad (1)$$

$$\theta = \alpha + \beta \quad \text{in node } H \quad (2)$$

$$\alpha = \pi - \theta \quad (3)$$

$$\beta = \theta - \alpha = \theta - (\pi - \theta) = 2\theta - \pi \quad (4)$$

We calculate horizontal segment \overline{OK} :

$$\begin{aligned} \overline{OK} &= \overline{MA} + a + \overline{BN} - \overline{PL} \\ &= b \cos \alpha + a + c \cos \alpha - d \cos \beta \\ &= a + (b + c) \cos \alpha - d \cos \beta \\ &= a + (b + c) \cos (\pi - \theta) - d \cos (2\theta - \pi) \\ &= a - (b + c) \cos \theta + d \cos (2\theta) \end{aligned} \quad (5)$$

And vertical segment \overline{OL} :

$$\begin{aligned} \overline{OL} &= \overline{KN} - \overline{PH} - \overline{HM} \\ &= c \sin \alpha - d \sin \beta - b \sin \alpha \\ &= (c - b) \sin \alpha - d \sin \beta \\ &= (c - b) \sin (\pi - \theta) - d \sin (2\theta - \pi) \\ &= (c - b) \sin \theta - d \sin (2\theta) \end{aligned} \quad (6)$$

¹ Meccano mathematics by 't Hooft

² Meccano pentagons

So we can express e in function of a, b, c, d and angles α, β :

$$\begin{aligned}
e^2 &= (\overline{OK})^2 + (\overline{OL})^2 \\
&= (a - (b + c) \cos \theta + d \cos (2\theta))^2 + ((c - b) \sin \theta - d \sin (2\theta))^2 \\
&= a^2 + (b^2 + 2bc + c^2) \cos^2 \theta + d^2 \cos^2 (2\theta) - 2a(b + c) \cos \theta + 2ad \cos (2\theta) - 2(b + c)d \cos \theta \cos (2\theta) \\
&\quad (c^2 - 2cb + b^2) \sin^2 \theta - 2(c - b)d \sin \theta \sin (2\theta) + d^2 \sin^2 (2\theta) \\
&= a^2 + (b^2 + c^2)(\cos^2 \theta + \sin^2 \theta) + d^2(\cos^2 (2\theta) + \sin^2 (2\theta))
\end{aligned} \tag{7}$$