Meccano pentagons

https://github.com/heptagons/meccano/penta

Abstract

We show how to construct two types of meccano regular pentagons. The process as in another meccano constructions of this site is to build the polygon perimeter and attach **internal diagonals** to make the polygon rigid.

We prepare general layouts to look for the use of only valid **meccano rods** testing by increasing the sides lengths and by rotating the diagonals.

The last diagonal most of the time is not an integer, so the purpose of this study is to find when it is as the rest of diagonals and tabulate a so called solution. Formulas are prepared exactly for each type instead of using floating numbers to prevent skip solutions by rounding errors. Finally a program is run using the algebraic conditions and formulas to iterate over a given range of sides to store and print solutions without repetitions by scaling.

From the two types of pentagons two conjectures emerges. **First conjecture** is that the first type of pentagon seems to have a **unique** solution after testing pentagons sides somehow large.

Second conjecture appears in second type of pentagon. For this type we got a lot of solutions but by inspecting the numeric value of the last diagonal called e seems to be always in the form $10x + 1$ for $x = 1, 2, 3..$ something unrelated at the moment by the formulas used.

1 Regular pentagon type 1

1.1 Type 1 equations

Figure 1 show the layout of the meccano regular pentagon of type 1. Let define the side of the pentagon as a and define other three variables b, c and d:

$$
\begin{aligned}
a &= \overline{BC} \\
b &= \overline{BF} \\
c &= \overline{FI} \\
d &= \overline{CI}
\end{aligned}
$$

Angles $\angle LBC$ and $\angle JFI$ are equal to $\frac{2\pi}{5}$ so:

$$
\begin{aligned}
\alpha &= \frac{2\pi}{5} \\
\overline{BL} &= a\cos\alpha \\
\overline{CL} &= a\sin\alpha \\
\overline{FJ} &= c\cos\alpha \\
\overline{IJ} &= c\sin\alpha
\end{aligned}
$$

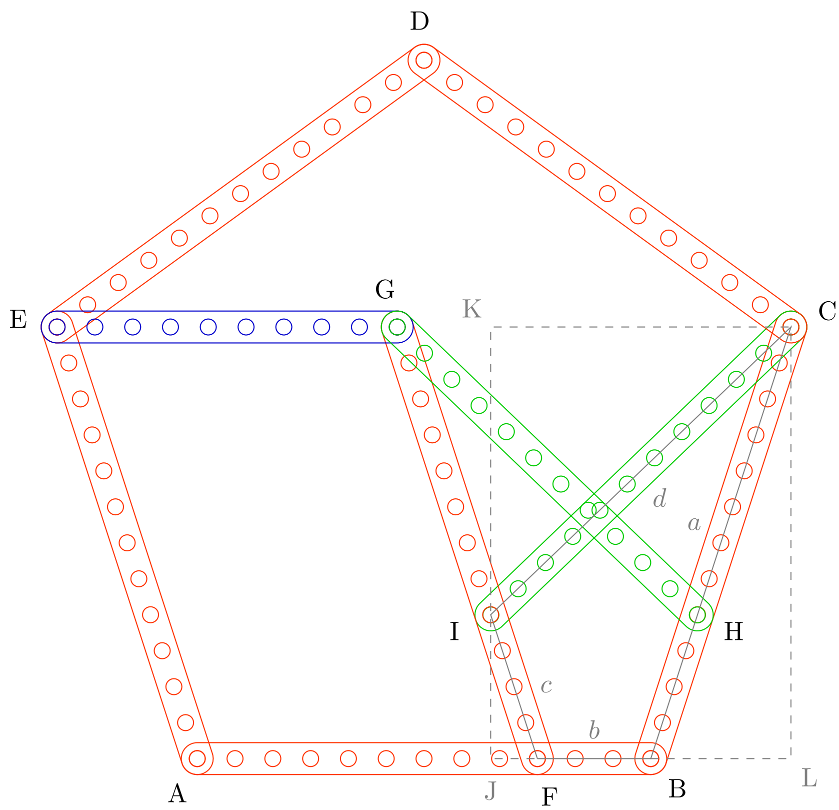

Figure 1: Pentagon of type 1.

Let calculate d in function of a, b and c:

$$
\begin{aligned}
d^2 &= (\overline{CI})^2 \\
&= (\overline{CK})^2 + (\overline{IK})^2 \\
&= (\overline{BL} + \overline{BF} + \overline{FJ})^2 + (\overline{CL} - \overline{IJ})^2 \\
&= (a\cos\alpha + b + c\cos\alpha)^2 + (a\sin\alpha - c\sin\alpha)^2 \\
&= ((a+c)\cos\alpha + b)^2 + ((a-c)\sin\alpha)^2 \\
&= (a+c)^2\cos^2\alpha + 2(a+c)b\cos\alpha + b^2 + (a-c)^2\sin^2\alpha \\
&= (a^2+c^2)(cos^2\alpha + sin^2\alpha) + 2ac(cos^2\alpha - \sin^2\alpha) + 2(a+c)b\cos\alpha + b^2 \\
&= (a^2+c^2) + 2ac(cos^2\alpha - \sin^2\alpha) + 2(a+c)b\cos\alpha + b^2
\end{aligned}
$$

For $\alpha = \frac{2\pi}{5}$ we have these regular pentagon identities:

$$
\begin{aligned}
cos\alpha &= \frac{-1+\sqrt{5}}{4} \\
cos^2\alpha &= \frac{3-\sqrt{5}}{8} \\
sin^2\alpha &= \frac{5+\sqrt{5}}{8} \\
cos^2\alpha - sin^2\alpha &= -\frac{1+\sqrt{5}}{4}
\end{aligned}
$$

Applying the identities to the last equation of d we get:

$$
\begin{aligned}
d^2 &= a^2 + c^2 - (\frac{1+\sqrt{5}}{2})ac + (\frac{-1+\sqrt{5}}{2})(a+c)b + b^2 \\
&= a^2 + c^2 - \frac{ac}{2} - \frac{(a+c)b}{2} + b^2 + [-\frac{ac}{2} + \frac{(a+c)b}{2}]\sqrt{5} \\
&= a^2 + b^2 + c^2 - \frac{ac + (a+c)b}{2} + [\frac{-ac + (a+c)b}{2}]\sqrt{5}
\end{aligned}
$$

Let define two variables p and q such that $d^2 = p + q\sqrt{5}$ so we have:

$$
\begin{aligned}
d^2 &= p + q\sqrt{5} \\
q &= \frac{-ac + (a+c)b}{2} \\
p &= a^2 + b^2 + c^2 - \frac{ac + (a+c)b}{2} \\
&= a^2 + b^2 + c^2 - \frac{-ac + (a+c)b}{2} - ac \\
&= a^2 + b^2 + c^2 - q - ac
\end{aligned}
$$

For a meccano pentagon we need d to be an integer. If we let the integer $q > 0$ then $d = \sqrt{p + q\sqrt{5}}$ will never be an integer for p and q integers. If we force q to be zero then $d = \sqrt{p}$ has possibilities to be an integer. So before calculating d we force the condition that $q = 0$ or that is the same $-ac + (a+c)b = 0$:

$$
\begin{aligned}
a &> b \\
a &> c \\
ac &= (a+c)b \\
d &= \sqrt{a^2 + b^2 + c^2 + ac}
\end{aligned}
$$

1.2 Type 1 program

Next **go** program iterate over three variables $a \leq max$, $b \leq a$, $c \leq a$ (lines 30, 31, 32). The $q = 0$ condition is tested (line 33) and only when is valid we check d to be an integer (call in line 34, function in line 20). Only when d is an integer we call function add (call in line 26, function in line 5) to print and store a solution to be used later to prevent repetitions by scaling.

```
1  func pentagons_type_1(max int) {
2  
3    sols := make([][]int, 0)
4  
5    add := func(a, b, c, d int) {
6      for _, s := range sols {
7        if a % s[0] != 0 { continue }
8        // new a is a factor of previous a
9        f := a / s[0]
10       if t := b % s[1] == 0 && b / s[1] == f; !t { continue }
11       if t := c % s[2] == 0 && c / s[2] == f; !t { continue }
12       if t := d % s[3] == 0 && d / s[3] == f; !t { continue }
13       return // scaled solution already found (reject)
14     }
15     // solution!
16     sols = append(sols, []int{ a, b, c, d })
17     fmt.Printf("%3d a=%2d b=%2d c=%2d d=%2d\n", len(sols), a, b, c, d)
18   }
19 
20   check := func(a, b, c int) {
21     f := float64(a*a + b*b + c*c - a*c)
22     if f < 0 {
23       return
24     }
25     if d := int(math.Sqrt(f)); math.Pow(float64(d), 2) == f {
26       add(a, b, c, d)
27     }
28   }
29 
30   for a := 1; a < max; a++ {
31     for b := 1; b <= a; b++ {
32       for c := 0; c <= a; c++ {
33         if a*c == (a + c)*b {
34           check(a, b, c)
35         }
36       }
37     }
38   }
39 }
```

1.3 Type 1 results

After serching for values of $a <= 5000$ we found a single result:

```
a=12 b=3 c=4 d=11
```

Figure 2 shows the first (unique?) pentagon of type 1 with values $a = 12$, $b = 3$, $c = 4$ and $d = 11$.

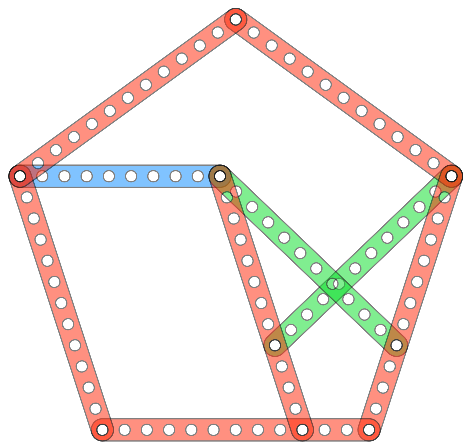

Figure 2: The smallest and maybe unique (?) of pentagons of type 1. Is composed of 6 rods of length $a = 12$ in color red, two rods of length $d = 11$ in green and one rod of size $a - b = 9$ in blue.

1.4 Type 1 conjecture

There is only a single case for the type 1 with values $a = 12$, $b = 3$, $c = 4$ and $d = 11$.

2 Regular pentagon type 2

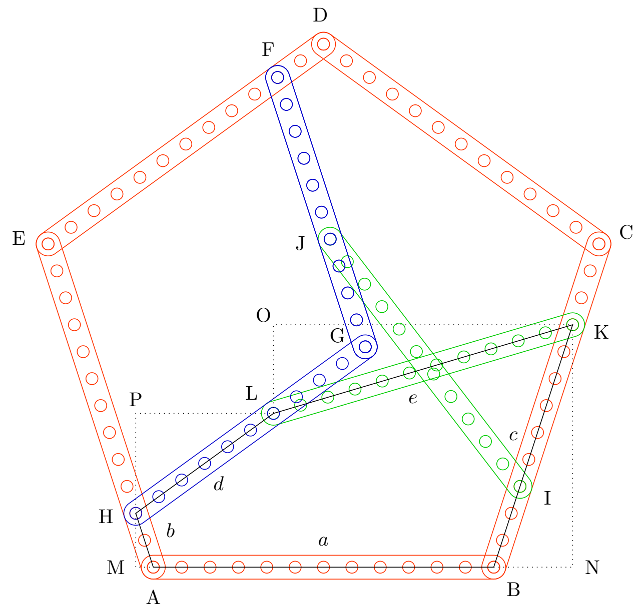

Figure 3: Pentagon of type 2.

2.1 Type 2 equations

Figure 3 show the layout of the meccano regular pentagon of type 2. Let define the side of the pentagon as a and define other four variables b, c, d and e:

$$a = \overline{AB}$$
$$b = \overline{AH}$$
$$c = \overline{BK}$$
$$d = \overline{HL}$$
$$e = \overline{KL}$$

Angles $\angle NBC$ and $\angle MAH$ are equal to $\frac{2\pi}{5}$ so:

$$\alpha = \frac{2\pi}{5}$$
$$\overline{BN} = b \cos \alpha$$
$$\overline{KN} = b \sin \alpha$$
$$\overline{AM} = c \cos \alpha$$
$$\overline{HM} = c \sin \alpha$$

Angle $\angle PLH$ is equal to $\frac{\pi}{5}$ so:

$$
\begin{aligned}
\beta &= \frac{\pi}{5} \\
\overline{LP} &= d\cos\beta \\
\overline{HP} &= d\sin\beta
\end{aligned}
$$

Our goal is to find e as integer as funcion of variables a, b, c and d. e^2 equals $(\overline{KO})^2 + (\overline{LO})^2$ so we first calculate $\overline{KO}$ and $\overline{LO}$. From figure 3:

$$
\begin{aligned}
\overline{KO} &= \overline{AM} + \overline{AB} + \overline{BN} - \overline{LP} \\
&= b\cos\alpha + a + c\cos\alpha - d\cos\beta \\
&= (b+c)\cos\alpha + a - d\cos\beta \\
\overline{LO} &= \overline{KN} - \overline{HM} - \overline{HP} \\
&= c\sin\alpha - b\sin\alpha - d\sin\beta \\
&= (c-b)\sin\alpha - d\sin\beta
\end{aligned}
$$

So by adding the squares we get:

$$
\begin{aligned}
e^2 &= (\overline{KO})^2 + (\overline{LO})^2 \\
&= ((b+c)\cos\alpha)^2 + 2(b+c)\cos\alpha(a - d\cos\beta) + (a - d\cos\beta)^2 \\
&\quad + ((c-b)\sin\alpha)^2 - 2(c-b)\sin\alpha d\sin\beta + (d\sin\beta)^2 \\
&= (b^2+c^2)(cos^2\alpha + sin^2\alpha) + 2bc(cos^2\alpha - sin^2\alpha) \\
&\quad + 2a(b+c)cos\alpha - 2(b+c)d\cos\alpha\cos\beta - 2(c-b)d\sin\alpha\sin\beta \\
&\quad + a^2 - 2ad\cos\beta + d^2(\cos^2\beta + \sin^2\beta)
\end{aligned}
$$

Calculate the α and β identities that appear in the last equation:

$$
\begin{aligned}
\cos^2\alpha - \sin^2\alpha &= -\frac{1+\sqrt{5}}{4} \\
\cos\alpha &= \frac{-1+\sqrt{5}}{4} \\
\cos\alpha\cos\beta &= \frac{1}{4} \\
\sin\alpha\sin\beta &= \frac{\sqrt{5}}{4} \\
\cos\beta &= \frac{1+\sqrt{5}}{4}
\end{aligned}
$$

Replace the identities:

$$\begin{aligned}
e^2 &= (b^2+c^2)(1) + 2bc(-\frac{1+\sqrt{5}}{4}) \\
&\quad + 2a(b+c)(\frac{-1+\sqrt{5}}{4}) - 2(b+c)d(\frac{1}{4}) - 2(c-b)d(\frac{\sqrt{5}}{4}) \\
&\quad + a^2 - 2ad(\frac{1+\sqrt{5}}{4}) + d^2(1) \\
&= b^2 + c^2 - bc(\frac{1+\sqrt{5}}{2}) \\
&\quad + a(b+c)(\frac{-1+\sqrt{5}}{2}) - (b+c)d(\frac{1}{2}) - (c-b)d(\frac{\sqrt{5}}{2}) \\
&\quad + a^2 - ad(\frac{1+\sqrt{5}}{2}) + d^2 \\
&= a^2 + b^2 + c^2 + d^2 - (b+c)d(\frac{1}{2}) \\
&\quad - (ad+bc)(\frac{1+\sqrt{5}}{2}) + a(b+c)(\frac{-1+\sqrt{5}}{2}) - (c-b)d(\frac{\sqrt{5}}{2}) \\
&= a^2 + b^2 + c^2 + d^2 - \frac{(b+c)d}{2} \\
&\quad - \frac{(ad+bc)(1+\sqrt{5})}{2} + \frac{a(b+c)(-1+\sqrt{5})}{2} - \frac{(c-b)d\sqrt{5}}{2}
\end{aligned}$$

Let define two variables p and q such that $e^2 = p + q\sqrt{5}$:

$$\begin{aligned}
p &= a^2 + b^2 + c^2 + d^2 - \frac{(b+c)d}{2} - \frac{ad+bc}{2} + \frac{-a(b+c)}{2} \\
&= a^2 + b^2 + c^2 + d^2 - \frac{bd+cd+ad+bc+ab+ac}{2} \\
&= a^2 + b^2 + c^2 + d^2 - \frac{(a+b)(c+d)+ab+cd}{2} \\
q &= -\frac{ad+bc}{2} + \frac{a(b+c)}{2} - \frac{(c-b)d}{2} \\
&= \frac{-ad-bc+ab+ac-cd+bd}{2} \\
&= \frac{(a-b)(c-d)+ab-cd}{2}
\end{aligned}$$

For a meccano pentagon we need e to be an integer. If we let the integer $q > 0$ then $e = \sqrt{p+q\sqrt{5}}$ will never be an integer for p and q integers. If we force q to be zero then $e = \sqrt{p}$ has possibilities to be an integer. So before calculating e we force the condition that $q = 0$ or that is the same $cd = (a-b)(c-d)+ab$:

$$\begin{aligned}
a &> b \\
a &> c \\
cd &= (a-b)(c-d) + ab
\end{aligned}$$

From the condition $q = 0$ we know that $cd = (a-b)(c-d)+ab$, and use this cd value in in the equation for p to get:

$$\begin{aligned} p &= a^2 + b^2 + c^2 + d^2 - \frac{(a+b)(c+d) + ab + cd}{2} \\ &= a^2 + b^2 + c^2 + d^2 - \frac{(a+b)(c+d) + ab + (a-b)(c-d) + ab}{2} \\ &= a^2 + b^2 + c^2 + d^2 - ac - bd - ab \end{aligned}$$

So finally, when $q = 0$ we calculate $e = \sqrt{p}$ expecting to be an integer:

$$e = \sqrt{a^2 + b^2 + c^2 + d^2 - ac - bd - ab}$$

Another solution is:

$$e = \sqrt{a^2 + b^2 + c^2 + d^2 - ad - bc - cd}$$

2.2 Type 2 first program

With the last equations, a new program for the pentagon type 2, iterates over the integer values of rods a, b, c and d to discover a rod e with integer length too. Next **go** program finds type 2 pentagons.

```
func pentagons_type_2(max int) {

  sols := make([][]int, 0)

  add := func(a, b, c, d, e int) {
    for _, s := range sols {
      if a % s[0] != 0 { continue }
      // new a is a factor of previous a
      f := a / s[0]
      if t := b % s[1] == 0 && b / s[1] == f; !t { continue }
      if t := c % s[2] == 0 && c / s[2] == f; !t { continue }
      if t := d % s[3] == 0 && d / s[3] == f; !t { continue }
      if t := e % s[4] == 0 && e / s[4] == f; !t { continue }
      return // scaled solution already found (reject)
    }
    // solution!
    sols = append(sols, []int{ a, b, c, d, e })
    fmt.Printf("%3d a=%3d b=%3d c=%3d d=%3d e=%3d\n", len(sols), a, b, c, d, e)
  }

  check := func(a, b, c, d int) {
    f := float64(a*a + b*b + c*c + d*d - a*d - b*c - c*d)
    if f < 0 {
      return
    }
    if e := int(math.Sqrt(f)); math.Pow(float64(e), 2) == f {
      add(a, b, c, d, e)
    }
  }

    for a := 1 ; a < max; a++ {
      for b := 1; b < a; b++ {
          for c := 1; c < a; c++ {
              for d := 1; d < a; d++ {
                if ((a - b)*(c - d) + a*b == c*d) {
                    check(a, b, c, d)
```

```
                  }
                }
              }
          }
      }
}
```

2.3 Type 2 first results

The program found 124 pentagons of type 2 for $a \leq 488$.

```
  1 a= 12 b=  2 c=  9 d=  6 e= 11
  2 a= 12 b=  6 c=  3 d= 10 e= 11
  3 a= 31 b=  4 c= 28 d= 16 e= 31
  4 a= 31 b= 15 c=  3 d= 27 e= 31
  5 a= 38 b= 12 c= 18 d= 21 e= 31
  6 a= 38 b= 17 c= 20 d= 26 e= 31
  7 a= 48 b=  8 c= 24 d= 21 e= 41
  8 a= 48 b= 12 c=  9 d= 20 e= 41
  9 a= 48 b= 27 c= 24 d= 40 e= 41
 10 a= 48 b= 28 c= 39 d= 36 e= 41
 11 a= 72 b= 21 c= 48 d= 40 e= 61
 12 a= 72 b= 24 c= 16 d= 39 e= 61
 13 a= 72 b= 32 c= 24 d= 51 e= 61
 14 a= 72 b= 33 c= 56 d= 48 e= 61
 15 a= 78 b= 27 c=  4 d= 42 e= 71
 16 a= 78 b= 36 c= 74 d= 51 e= 71
 . . .
 . . .
119 a=488 b= 72 c= 15 d= 96 e=451
120 a=488 b=132 c=423 d=276 e=451
121 a=488 b=152 c=269 d=272 e=401
122 a=488 b=212 c= 65 d=356 e=451
123 a=488 b=216 c=219 d=336 e=401
124 a=488 b=392 c=473 d=416 e=451
```

2.4 Type 2 simpler program

Figure 4 show what happens when the first program reports two solutions with the same a and the same e. The type 2 symmetry can be taken into account to simplify the first program to reduce the search space and report only the half. This simpler program first iterates over $1 \leq a \leq max$ (line 4), then over $1 \leq b < a$ (line 6), then over $1 \leq d < (a - b)$ (line 8) and finally over $1 \leq c < a$ (line 10).

```
1  func pentagons_type_2_half(max int) {
2    sols := &Sols{}
3    aa, a_b, ab, bb, dd, ad, bc, c_d, cd, cc := 0,0,0,0,0,0,0,0,0,0
4    for a := 1; a <= max; a++ {
5      aa = a*a
6      for b := 1; b < a; b++ {
7        a_b, ab, bb = a - b, a*b, b*b
8        for d := 1; d < (a-b); d++ {
9          dd, ad = d*d, a*d
10         for c := 1; c < a; c++ {
11           bc, c_d, cd, cc = b*c, c - d, c*d, c*c
12           if a_b * c_d + ab == cd {
13             if f := float64(aa + bb + cc + dd - ad - bc - cd); f > 0 {
```

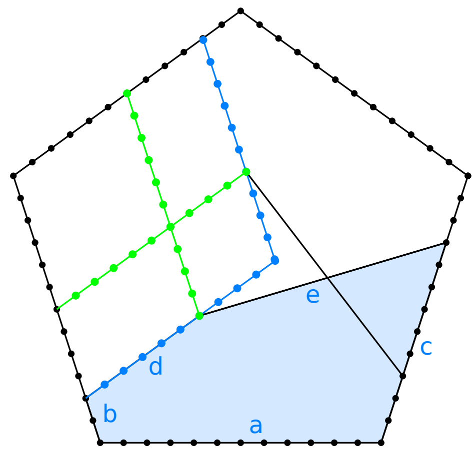

Figure 4: Pentagon of type 2 has a symmetry where pair bars green and blue can be switched leaving the bars e lengths and positions unmodified. This symmetry appears in the first program when two solutions have same a and same e.

```
              if e := int(math.Sqrt(f)); math.Pow(float64(e), 2) == f {
                sols.Add(a, b, c, d, e)
              }
            }
          }
        }
      }
    }
  }
}
```

2.5 Type 2 simpler results

The secont type 2 program found 138 solutions iterating over $1 \leq a \leq 1000$:

```
  1 a= 12 b=  2 c=  9 d=  6 e= 11
  2 a= 31 b=  4 c= 28 d= 16 e= 31
  3 a= 38 b= 12 c= 18 d= 21 e= 31
  4 a= 48 b=  8 c= 24 d= 21 e= 41
  5 a= 48 b= 12 c=  9 d= 20 e= 41
  6 a= 72 b= 21 c= 48 d= 40 e= 61
  7 a= 72 b= 24 c= 16 d= 39 e= 61
  8 a= 78 b= 27 c=  4 d= 42 e= 71
  9 a= 87 b= 28 c= 36 d= 48 e= 71
 10 a=111 b= 39 c= 99 d= 67 e=101
 11 a=121 b= 33 c= 33 d= 57 e=101
 12 a=128 b=  8 c= 89 d= 56 e=121
 13 a=138 b= 12 c= 54 d= 47 e=121
 14 a=145 b= 45 c= 39 d= 75 e=121
 15 a=147 b= 43 c= 51 d= 75 e=121
 16 a=151 b= 19 c= 73 d= 61 e=131
 17 a=156 b= 43 c= 96 d= 84 e=131
 18 a=165 b= 36 c=132 d= 88 e=151
```

```
19 a=179 b= 15 c=177 d= 93 e=191
20 a=183 b= 66 c= 62 d=108 e=151
21 a=201 b=  9 c= 13 d= 21 e=191
22 a=204 b= 21 c=112 d= 84 e=181
23 a=216 b= 48 c=111 d=104 e=181
24 a=236 b= 80 c= 20 d=125 e=211
25 a=249 b= 45 c= 75 d= 95 e=211
26 a=264 b= 76 c=  3 d=108 e=241
27 a=285 b= 73 c= 27 d=111 e=251
28 a=296 b=104 c=128 d=173 e=241
29 a=303 b= 51 c= 29 d= 81 e=271
30 a=304 b= 76 c=133 d=148 e=251
31 a=312 b= 36 c= 93 d=100 e=271
32 a=315 b= 24 c=160 d=120 e=281
33 a=324 b= 64 c=204 d=159 e=281
34 a=343 b=  7 c=115 d= 91 e=311
35 a=352 b=  3 c=240 d=144 e=341
36 a=354 b= 53 c= 60 d=102 e=311
37 a=368 b= 36 c=219 d=156 e=331
38 a=369 b= 37 c= 27 d= 63 e=341
39 a=370 b=  1 c=172 d=118 e=341
40 a=375 b= 15 c=191 d=135 e=341
41 a=378 b= 21 c= 84 d= 86 e=341
42 a=384 b=120 c=312 d=223 e=341
43 a=390 b= 84 c= 50 d=135 e=341
44 a=390 b= 87 c=228 d=194 e=331
45 a=392 b=119 c=296 d=224 e=341
46 a=392 b=128 c= 56 d=203 e=341
47 a=393 b= 98 c= 54 d=156 e=341
48 a=396 b=138 c= 73 d=222 e=341
49 a=399 b= 70 c=210 d=180 e=341
50 a=403 b= 78 c=114 d=156 e=341
51 a=404 b= 89 c=104 d=164 e=341
52 a=408 b= 16 c=312 d=183 e=401
53 a=408 b= 84 c=167 d=180 e=341
54 a=411 b=123 c=243 d=227 e=341
55 a=435 b= 96 c=400 d=240 e=421
56 a=450 b= 92 c=438 d=249 e=451
57 a=468 b=173 c= 24 d=276 e=431
58 a=480 b= 80 c= 75 d=144 e=421
59 a=486 b=180 c= 18 d=287 e=451
60 a=488 b= 72 c= 15 d= 96 e=451
61 a=488 b=132 c=423 d=276 e=451
62 a=488 b=152 c=269 d=272 e=401
63 a=495 b=135 c=415 d=279 e=451
64 a=502 b= 93 c= 36 d=138 e=451
65 a=507 b= 18 c=366 d=220 e=491
66 a=507 b= 60 c= 84 d=128 e=451
67 a=509 b=150 c= 42 d=228 e=451
68 a=516 b=114 c=169 d=222 e=431
69 a=520 b= 36 c=225 d=180 e=461
70 a=525 b=185 c=399 d=315 e=451
71 a=525 b=189 c=105 d=305 e=451
72 a=528 b= 80 c=171 d=192 e=451
73 a=540 b=150 c=321 d=290 e=451
74 a=543 b=123 c=221 d=249 e=451
75 a=546 b=135 c=228 d=262 e=451
```

```
 76 a=552 b=179 c=288 d=312 e=451
 77 a=553 b=180 c=276 d=312 e=451
 78 a=560 b=200 c=344 d=335 e=461
 79 a=565 b= 69 c=153 d=177 e=491
 80 a=588 b=104 c= 12 d=135 e=541
 81 a=600 b= 65 c=240 d=216 e=521
 82 a=600 b=120 c= 96 d=205 e=521
 83 a=617 b= 89 c=533 d=317 e=601
 84 a=632 b=113 c=152 d=224 e=541
 85 a=652 b= 58 c=235 d=214 e=571
 86 a=661 b=109 c= 37 d=157 e=601
 87 a=684 b=237 c=192 d=388 e=571
 88 a=699 b= 84 c=564 d=344 e=671
 89 a=701 b=254 c=698 d=428 e=671
 90 a=713 b=234 c=582 d=420 e=631
 91 a=715 b=211 c=655 d=415 e=671
 92 a=720 b=216 c=712 d=423 e=701
 93 a=724 b=147 c= 72 d=228 e=641
 94 a=728 b= 21 c=192 d=168 e=661
 95 a=729 b= 36 c=428 d=288 e=671
 96 a=732 b= 18 c=681 d=358 e=781
 97 a=732 b= 42 c=111 d=134 e=671
 98 a=744 b=228 c=155 d=372 e=631
 99 a=746 b=164 c= 38 d=233 e=671
100 a=755 b=123 c=267 d=291 e=641
101 a=756 b= 69 c=168 d=196 e=671
102 a=762 b= 73 c=372 d=294 e=671
103 a=765 b= 30 c=354 d=260 e=691
104 a=777 b=234 c=118 d=372 e=671
105 a=781 b=108 c=348 d=312 e=671
106 a=784 b=192 c=189 d=336 e=661
107 a=800 b=164 c=263 d=332 e=671
108 a=804 b=177 c=272 d=348 e=671
109 a=805 b=202 c=238 d=364 e=671
110 a=810 b=276 c=510 d=475 e=671
111 a=819 b=136 c=216 d=288 e=701
112 a=824 b=276 c=363 d=468 e=671
113 a=826 b=315 c=420 d=510 e=671
114 a=840 b=196 c=777 d=468 e=811
115 a=845 b=285 c=465 d=489 e=691
116 a=859 b=130 c=502 d=388 e=751
117 a=861 b=126 c= 66 d=196 e=781
118 a=863 b=303 c=711 d=519 e=761
119 a=864 b= 24 c=349 d=264 e=781
120 a=873 b=137 c=453 d=381 e=751
121 a=879 b=231 c= 63 d=343 e=781
122 a=885 b=206 c=642 d=468 e=781
123 a=885 b=309 c= 13 d=477 e=821
124 a=892 b=112 c=196 d=259 e=781
125 a=896 b=144 c=528 d=411 e=781
126 a=896 b=332 c=725 d=548 e=781
127 a=904 b=328 c=640 d=547 e=761
128 a=905 b=161 c=185 d=305 e=781
129 a=912 b=168 c=507 d=424 e=781
130 a=915 b=135 c=345 d=349 e=781
131 a=928 b=319 c=232 d=520 e=781
132 a=938 b=252 c=270 d=441 e=781
```

```
133 a=947 b=306 c=558 d=540 e=781
134 a=948 b=342 c=589 d=570 e=781
135 a=949 b=273 c=495 d=507 e=781
136 a=960 b=195 c=760 d=504 e=881
137 a=961 b=249 c=633 d=513 e=821
138 a=987 b=350 c=594 d=588 e=811
```

2.6 Type 2 conjecture

Last listing of 138 pentagons report all e values having the form $10x + 1$ for x integer. So the conjecture is that e always is of the form $10x + 1$ for x integer. Next program is an adaptation of the previous one and instead checking for a square root to be an integer, only checks for $e^2 = (10x + 1)^2$ for small xs. The results of this program is exactly the same result of the program checking the square root, up to $a \leq 1000$.

```
func pentagons_type_2_half_with_conjecture(max int) {
  sols := &Sols{}
  aa, a_b, ab, bb, dd, ad, bc, c_d, cd, cc := 0,0,0,0,0,0,0,0,0,0
  for a := 1; a <= max; a++ {
    aa = a*a
    for b := 1; b < a; b++ {
      a_b, ab, bb = a - b, a*b, b*b
      for d := 1; d < (a-b); d++ {
        dd, ad = d*d, a*d
        for c := 1; c < a; c++ {
          bc, c_d, cd, cc = b*c, c - d, c*d, c*c
          if a_b * c_d + ab == cd {

            e2 := aa + bb + cc + dd - ad - bc - cd

            x := 1
            for {
              if e := 10*x + 1; e*e == e2 {
                sols.Add(a, b, c, d, e)
                break
              } else if e*e > e2 {
                break
              }
              x++
            }
          }
        }
      }
    }
  }
}
```

2.7 Type 2 examples

Figures 5, 6 and 7 show some of the pentagons of type 2 found.

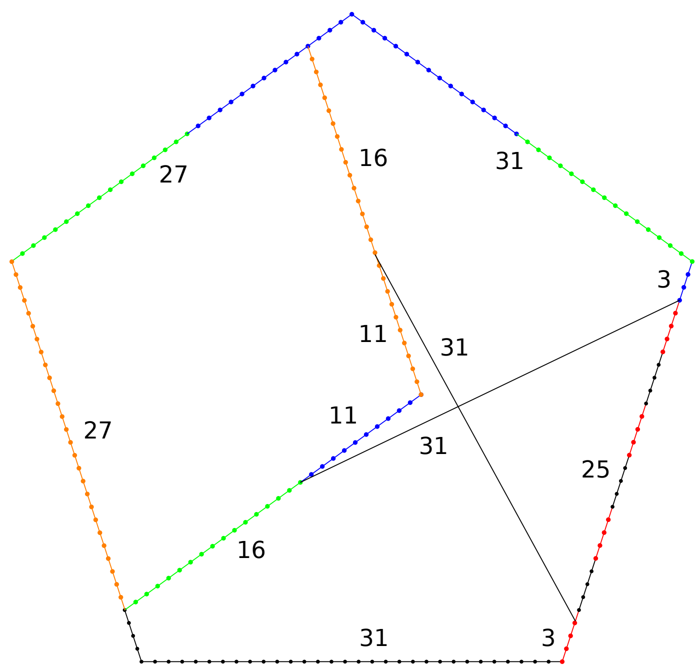

Figure 5: Pentagon of type 2 with $a = 31$. This construction requires 7 rods of length 31 and 2 rods of length 27.

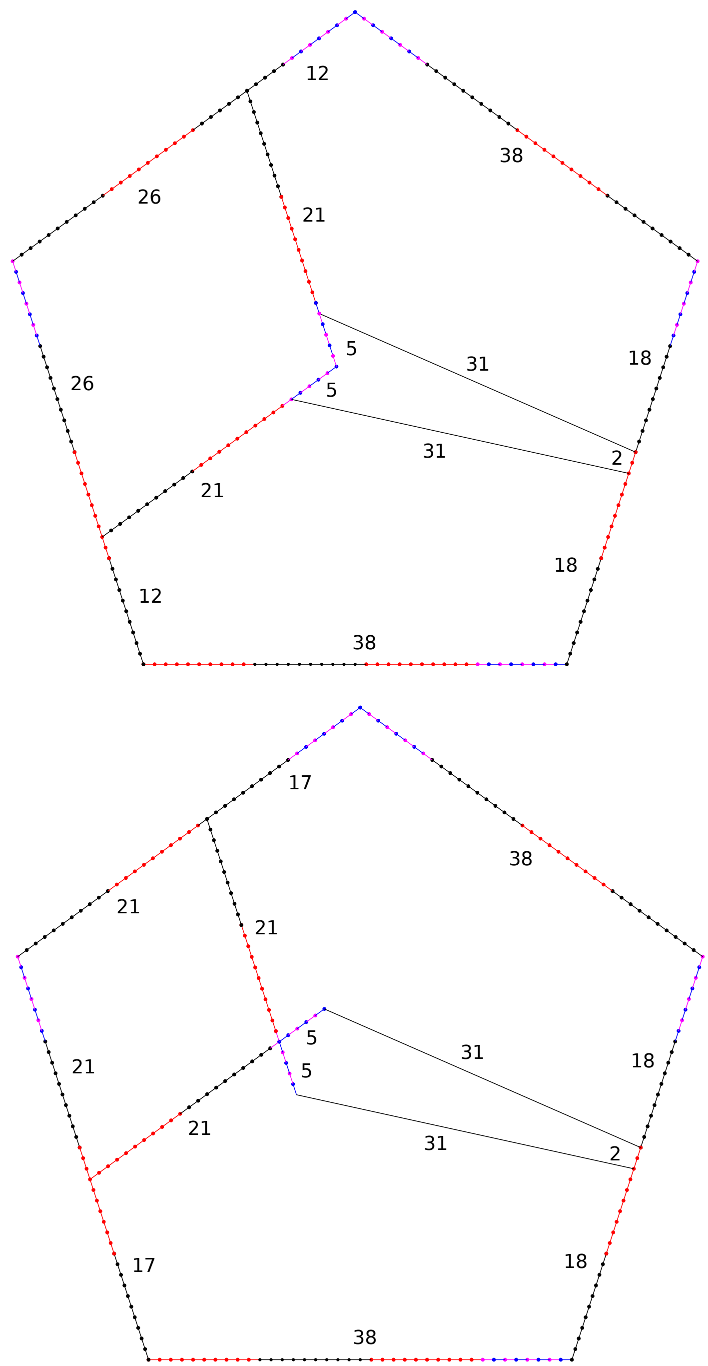

Figure 6: Pentagons of type 2 with $a = 38$. Each construction requires 5 rods of length 38, 2 rods of length 31 and 2 rods of length 26

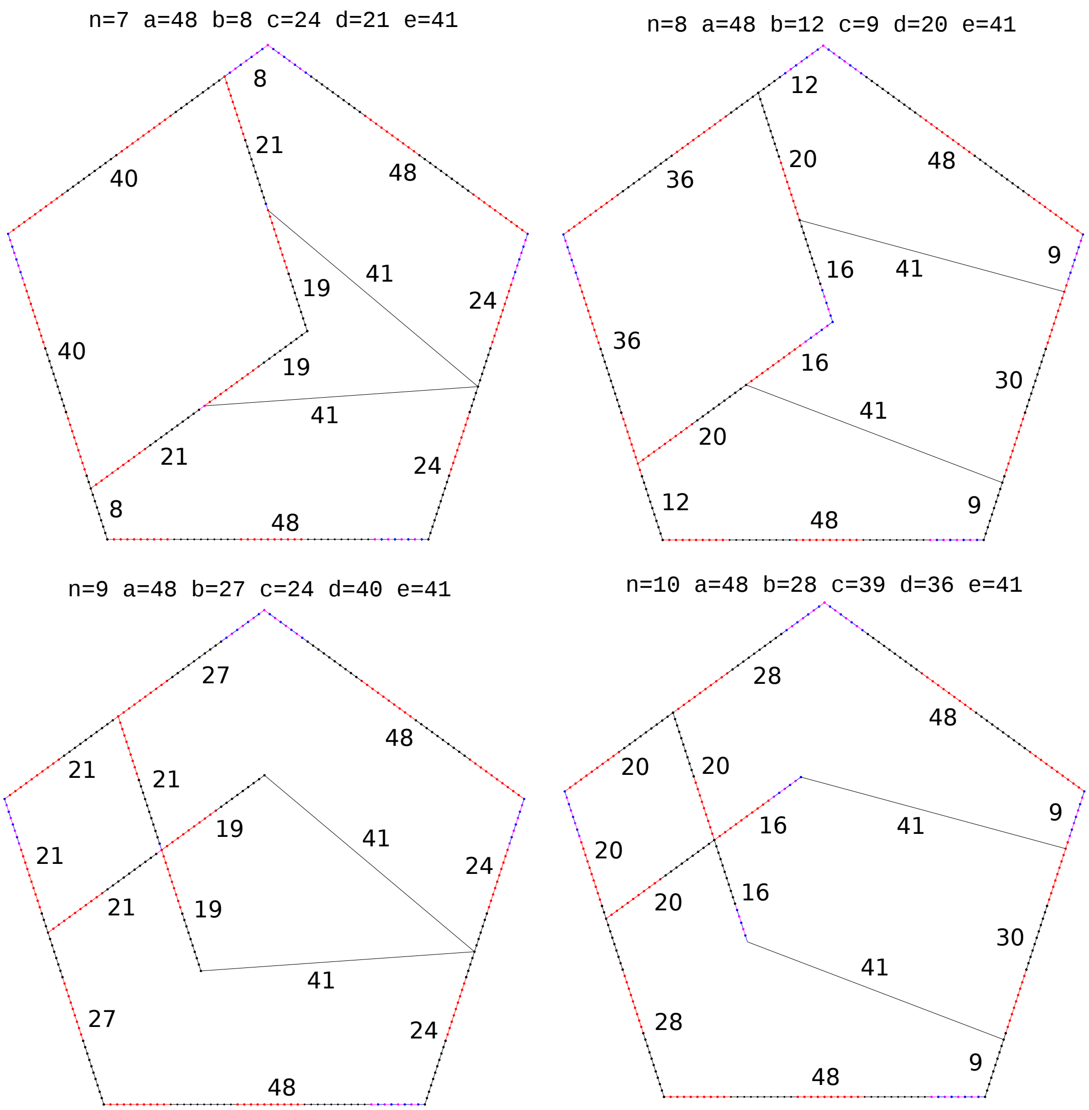

Figure 7: Pentagons of type 2 with $a = 48$