Meccano hexagons gallery

https://github.com/heptagons/meccano/hexa/gallery

2023/12/27

Abstract

We build meccano [1] rigid regular heptagons.

1 Heptagon internals

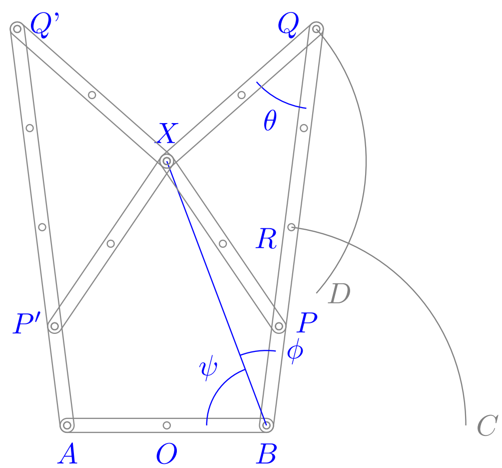

Figure 1: Cluster of four strips with fixed vertices A at $(-1, 0)$, B at $(+1, 0)$ and X at $(0, \sqrt{7})$.

Figure 1 show a cluster to make a triangle $\triangle ABX$ useful to make a regular heptagon. With the law of cosines we calculate first the angle $\theta \equiv \angle PQX$ and then the distance $\overline{BX}$:

$$\cos\theta = \frac{(\overline{PQ})^2 + (\overline{QX})^2 - (\overline{XP})^2}{2(\overline{PQ})(\overline{QX})} = \frac{3^2 + 2^2 - 2^2}{2(3)(2)} = \frac{3}{4} \tag{1}$$

$$\overline{BX} = \sqrt{(\overline{BQ})^2 + (\overline{QX})^2 - 2(\overline{BQ})(\overline{PQ})\cos\theta} = \sqrt{4^2 + 2^2 - 2(4)(2)\left(\frac{3}{4}\right)} = 2\sqrt{2} \tag{2}$$

Since $\overline{AX} = \overline{BX}$ then triangle $\triangle ABX$ is isoscelles and we calculate $\overline{OX}$ as simple as:

$$\overline{OX} = \sqrt{(\overline{BX})^2 - (\overline{OB})^2} = \sqrt{(2\sqrt{2})^2 - 1^2} = \sqrt{7} \tag{3}$$

Also with the law of cosines we calculate angles $\phi = \angle QBX$ and $\psi = \angle OBX$:

$$\cos\phi = \frac{(\overline{BP})^2 + (\overline{BX})^2 - (\overline{PX})^2}{2(\overline{BP})(\overline{BX})} = \frac{1^2 + 8 - 2^2}{2(2\sqrt{2})(1)} = \frac{5\sqrt{2}}{8} \tag{4}$$

$$\cos\psi = \frac{\overline{BX}}{\overline{OB}} = \frac{2\sqrt{2}}{1} = 2\sqrt{2} \tag{5}$$

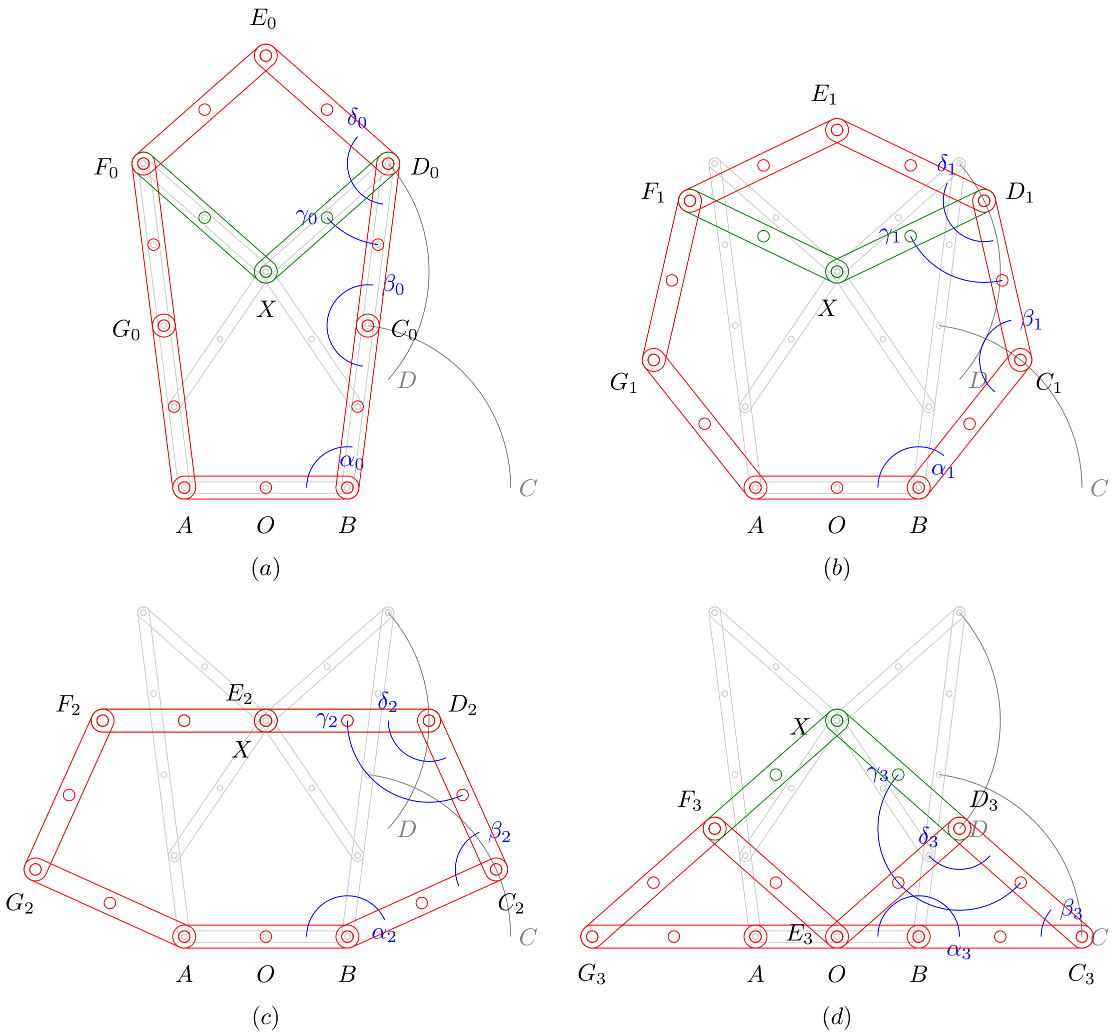

Figure 2: Equilateral heptagons connected to rigid triangle $\triangle ABX$ in four positions. In red we have the perimeter $\overline{ABCDEF}$ in green two strips $\overline{DXF}$. In (a) we have the heptagon as the tallest degenerated pentagon, in (b) we have the regular heptagon, in c we have a degenerated hexagon and in (d) the shortest degenerated pentagon.

Figure 2 show equilateral heptagons $ABCDEFG$ connected to the cluster of figure 1. Vertices A and B are fixed while vertices C,D,E and F move. Is important to note that vertice C positions C_0, C_1, C_2, C_3 follow the circular curve C and vertice D positions $D_0, D_1, D_2, D3$ follow the circular curve D.

The heptagon has two internal strips $\overline{DX}$ and $\overline{FX}$ connected to vertices D and F and to fixed vertex X.

We move the strips symmetrically. In figure (a) we have the maximum extension to the top limited by fact that vertices B and D_0 cannot be separated more since both are collinear with vertice C_0, being the angle $\beta_0 = \pi/2$.

From (a) we reach state (b) reducing β until $\alpha_1 = \beta_1 = \delta_1 = 5\pi/7$ forming the regular heptagon.

From (b) we reach state (c) reducing β until $\beta = \pi/2$.

In figure (d) we reach the maximum extension to the bottom limited by the fact that vertices X and C_3 cannot be separated more since both are collinear with vertice D_3, being the angle $\beta_3 = \arccos(3/4)$.

Table 1 show the values of angles $\alpha, \beta, \gamma, \delta$ for the positions $0, 1, 2, 3$ corresponding to figures $(a), (b), (c), (d)$.

[1] Meccano mathematics by ‘t Hooft

$Position$	α	β	γ	δ
0	$\phi+\psi$	π	θ	$2(\pi-\phi-\psi)-\theta$
1	$\frac{5\pi}{7}$	$\frac{5\pi}{7}$	$\frac{3\pi}{7}$	$\frac{5\pi}{7}$
2	$\pi-\psi-\frac{\pi}{4}$	$\pi/2$	$\frac{\pi}{4}-\psi$	$\frac{\pi}{4}-\psi$
3	π	θ	π	$\phi+\psi$

Table 1: Heptagon internal angles of positions $0-4$ shown in figure 2. Where θ, ϕ and ψ were defined in equations 1, 4 and 5 and shown in figure 1.