Meccano hexagons gallery

https://github.com/heptagons/meccano/hexa/gallery

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Abstract

We build meccano [1] rigid regular hexagons from sides 4 to 24. The perimeter has six equal strips and for their rigidity we add some internal strips as diagonals under three conditions: Will remain totally inside the perimeter, don't overlap with any other and must not be parallel to any external strip. With algebra and then sofware, we produce `hexagonal triplets` to make triangles having an internal angle of 120° used as the base of the internal strips.

1 Hexagonal triplets

We look strips that can make rigid two consecutive internal sides the regular hexagon. Figure 1 show the first four cases found. From any figure we have the internal hexagon angle is $\theta \equiv \angle GBC = 2\pi/3$. First we define the hexagon side as $p \equiv \overline{BC}$. From the triangle $\triangle GBC$ we define the other two sides as $b \equiv \overline{GB}$ and $c = \overline{GC}$. By the law of cosines we know that:

$$\begin{aligned} c &= \sqrt{b^2 + p^2 - 2bp\cos\theta} \\ &= \sqrt{b^2 + p^2 - 2bp\left(-\frac{1}{2}\right)} \\ &= \sqrt{b^2 + p^2 + bp} \end{aligned} \tag{1}$$

Then we defie $a \equiv p + b$ and we get:

$$c = \sqrt{a^2 + b^2 - ab} \quad \texttt{where } c > a > b \tag{2}$$

a	b	c	p
8	3	7	5
15	7	13	8
21	5	19	16
35	11	31	24
40	7	37	33
48	13	43	35

Table 1: Hexagonal triplets $c > p > b$ as the sides of a triangle with an internal angle $2\pi/3$.

A software iterates first $0 < a < max$ and then $1 < b < a$ and record all c that is an integer. The first cases of such triangles with sides $c > p > b$ are shown table 1 and we call them `Hexagonal triplets`.

Figure 1 shows hexagons of sizes $p = \{5, 8, 16, 24\}$ with perimeter strips in orange made rigid adding three internal green strips of lenght $c = \{7, 13, 19, 31\}$. In the figure we have also an equilateral triangle

[1] Meccano mathematics by 't Hooft

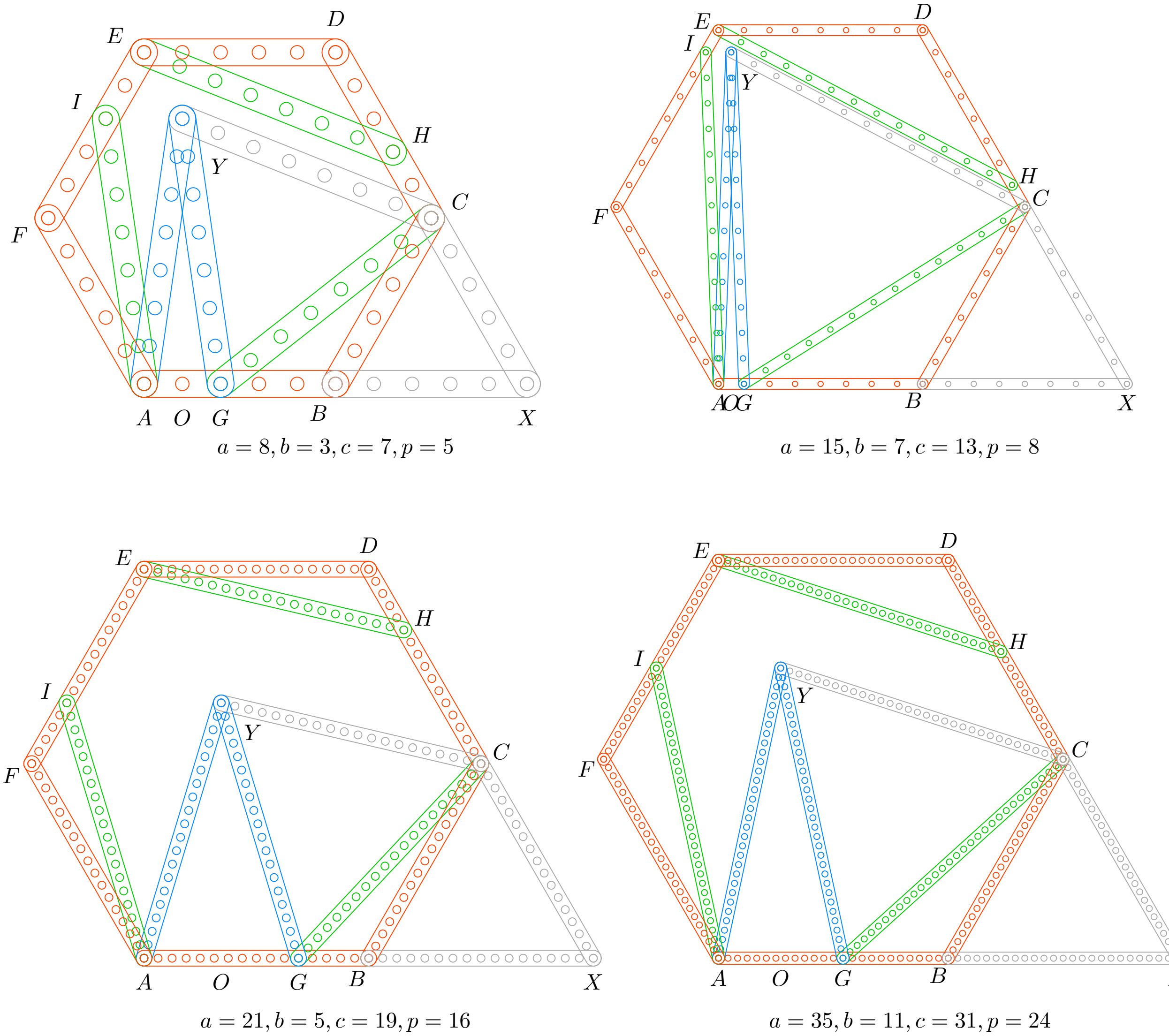

Figure 1: First four cases where internal strip $c = \overline{GC}$ is an integer and makes rigid two consecutive regular hexagon sides $p = \overline{AB} = \overline{BC}$. Our software inspect two integers $a > b$ and looks for $c = \sqrt{a^2 + b^2 - ab}$ to be an integer. In the figures $b = \overline{GB}$ and $a = \overline{GX} = p + b$.

$\triangle GCY$ and an isoscelles triangle $\triangle AGY$. The base of the isoscelles triangle is $x \equiv \overline{AG} = \overline{AB} - \overline{GB} = p - b$ and the equals sides are $\overline{AY} = \overline{GY} = c$. So we can calculate the height $y \equiv \overline{OY}$ substituting c using equation 1:

$$
\begin{aligned}
y &= \sqrt{(\overline{GY})^2 - (\overline{AO})^2} \\
&= \sqrt{c^2 - \left(\frac{p-b}{2}\right)^2} \\
&= \sqrt{b^2 + p^2 + bp - \left(\frac{p-b}{2}\right)^2} = \frac{(p+b)\sqrt{3}}{2} = \frac{a\sqrt{3}}{2} \qquad (3)
\end{aligned}
$$

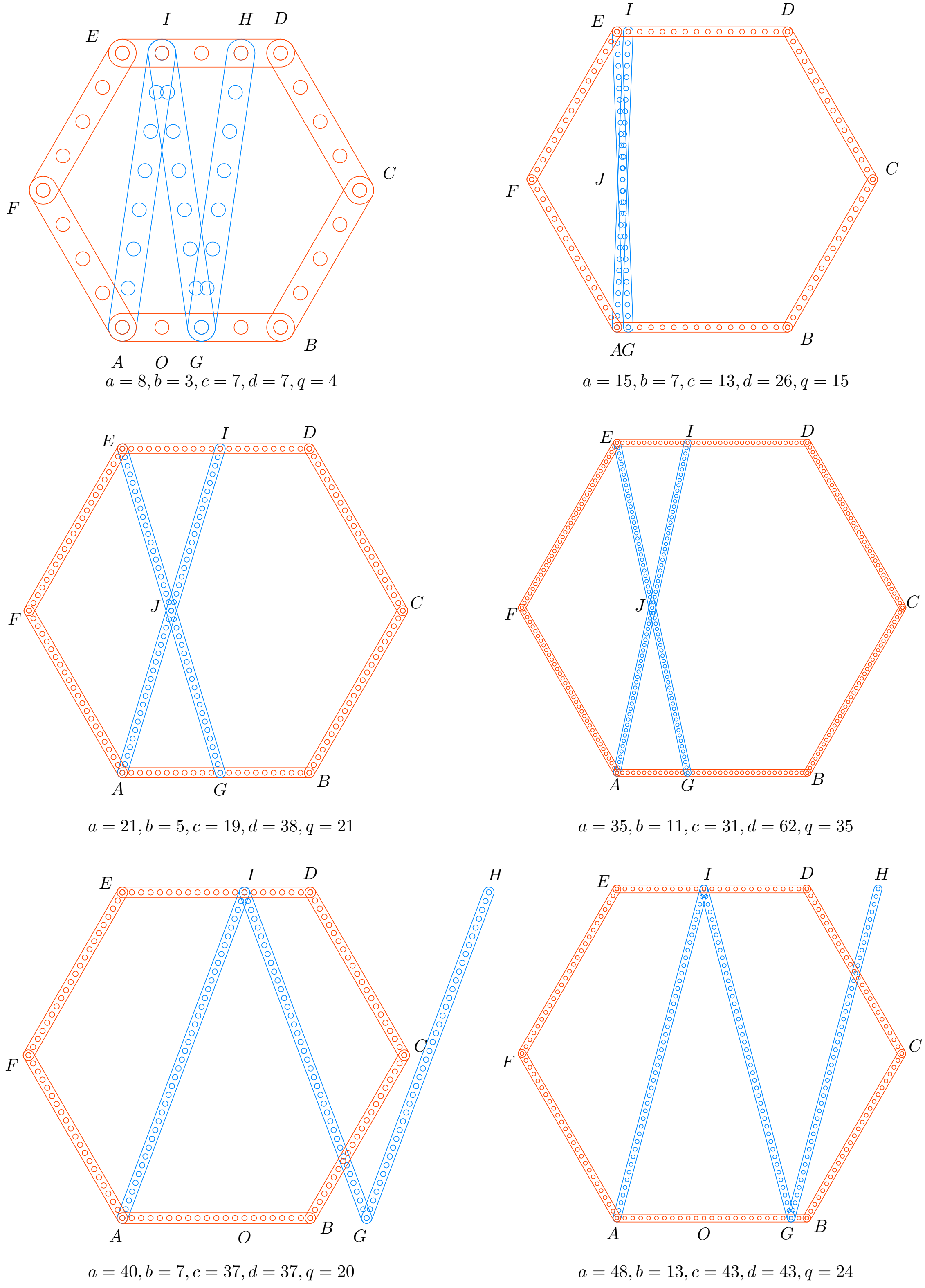

Figure 2: First six cases of integral distances c. When the distance $p - b = \overline{AG}$ is even, we use the strips $d = c$ to join opposites sides of hexagons of side $q = a/2$. When is odd, we use the strips $d = 2c$ to join opposites sides of hexagons of side $q = a$.

We know $\dfrac{a\sqrt{3}}{2}$ is the height of the regular hexagon of side $\dfrac{a}{2}$ so we can use the blue strips to connect opposite sides. Figure 2 show the smaller hexagons that have integer strips connecting opposites sides.

Through the gallery we will use the green and blue strips and their scaled copies as the internal diagonals which make some rigid regular hexagons from size 4 to 24. We prioritize minimum number of strips and bolts and the largest strips sizes as possible.

2 Hexagons of size $s < 10$

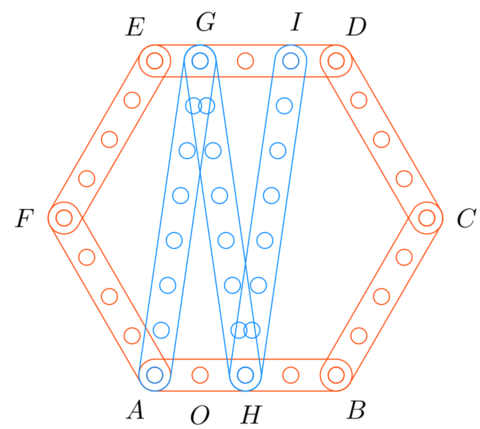

Figure 3: Hexagon of size $s = 4$. Diagonals $c = \overline{GH} = \overline{HI} = \overline{IG} = 7$.

Figure 3 show a regular hexagon of size 4 with three diagonals of size 7. The distance $\overline{OG} = \sqrt{(\overline{AG})^2 - (\overline{AO})^2} = \sqrt{7^2 - 1^2} = 4\sqrt{3}$.

3 Hexagons of size 13

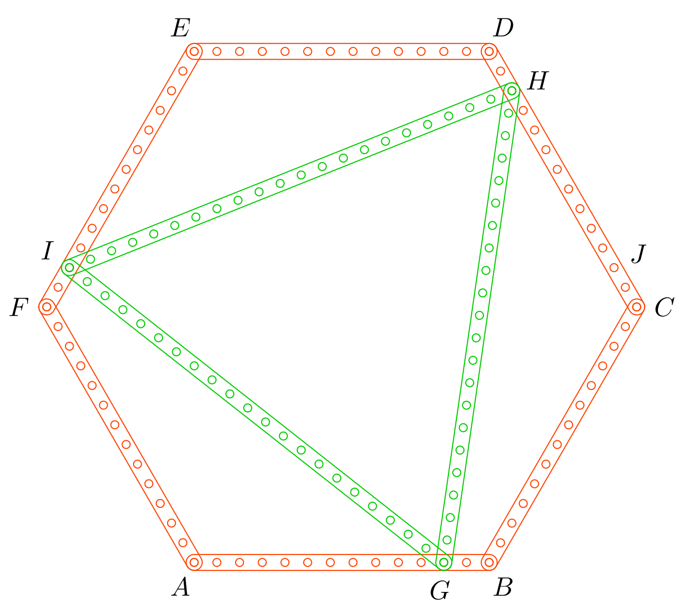

Figure 4: Hexagon of size $s = 13$. Diagonals $c = \overline{GH} = \overline{HI} = \overline{IG} = 21$.

Figure 4 show hexagon of size $s = 13$. First we detect an offset $o \equiv \overline{DH}$ which we use to calculate the sides p', b', c' of triangle $\triangle GJH$:

$$\begin{aligned} o &\equiv \overline{DH} = \overline{CJ} = 2 \\ p' &\equiv \overline{GJ} = \overline{BC} + o = 13 + 2 = 15 \\ b' &\equiv \overline{JH} = \overline{CD} - 2o = 13 - 2(2) = 9 \\ c' &\equiv \overline{GH} = 21 \end{aligned} \tag{4}$$

We confirm triplet c', p', b' is three times ($n = 3$) valid hexagonal triplet $c = 7, p = 5, b = 3$. This case has an equilateral triangle $\triangle GHI$ inside the hexagon because is a special of the general case when:

$$\begin{aligned} nb &= s - 2o \\ np &= s + o \\ nc &= \sqrt{(nb)^2 + (np)^2 - (nb)(np)} \\ &= \sqrt{(s - 2o)^2 + (s + o)^2 + (s - 2o)(s + o)} \\ &= \sqrt{3s^2 - 3so + 3o^2} \end{aligned} \tag{5}$$

First terms of this case is shown in the table 2

s	o	c	p	b
13	2	21	15	9
23	1	39	24	21
37	11	57	48	15
59	13	93	72	33
73	26	111	99	21
83	22	129	105	39
94	23	147	117	48

Table 2: Equilateral triangles side c inside regular hexagons side s.

4 Hexagons of size ≥ 20

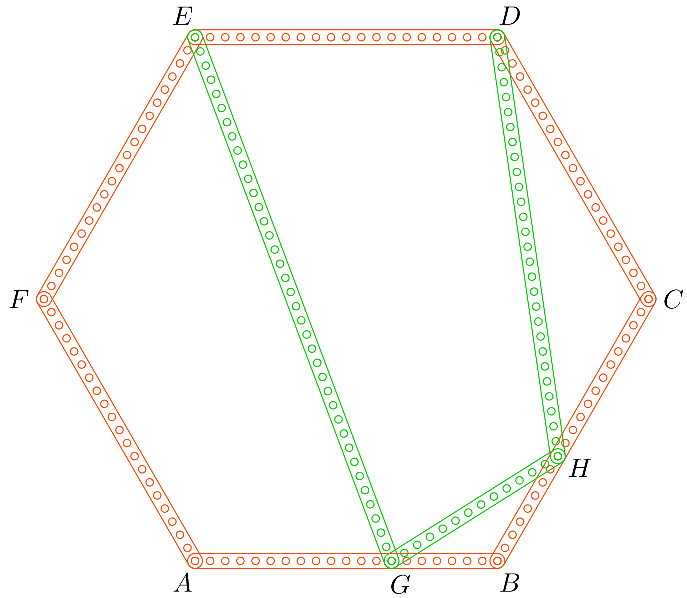

Figure 5: Hexagon of size 20. The three diagonals needs only two extra bolts at vertices G and H. $\overline{GH} = 13$, $\overline{HD} = 28$ and $\overline{EG} = 37$.

Figure 5 show regular hexagon $ABCDEF$ of size 20. Triangle $\triangle GBH$ sides are $c_1 = 13, p_1 = 8, b_1 = 7$. Triangle $\triangle HCD$ sides are $c_2 = 28, p_2 = 20, b_2 = 12$ which is triplet $(7, 5, 3)$ multiplied by 4. Right triangle $\triangle EAG$ has side $\overline{AE} = \sqrt{(\overline{EG})^2 - (\overline{AG})^2} = \sqrt{37^2 - 13^2} = 20\sqrt{3}$.

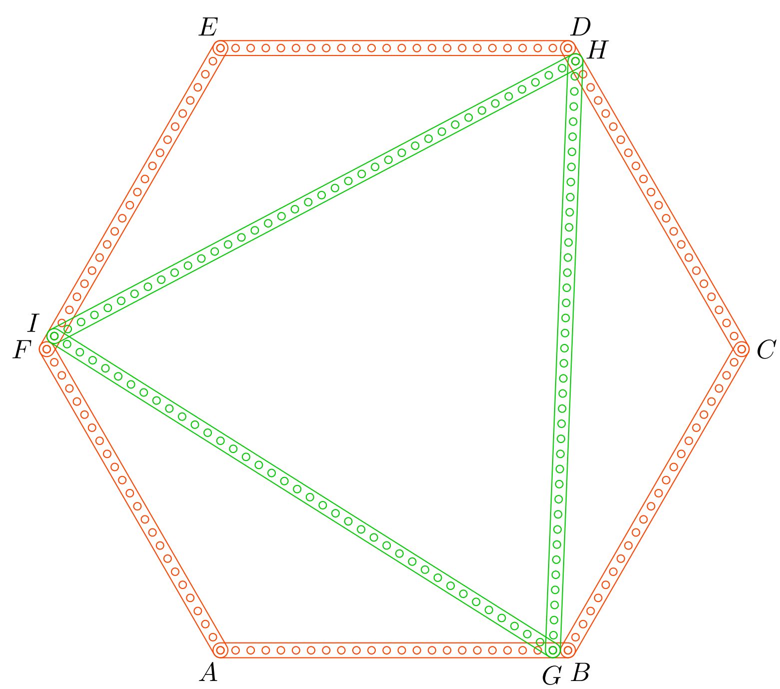

Figure 6: Hexagon of size $s = 23$. Diagonals $c = \overline{GH} = \overline{HI} = \overline{IG} = 39$.

Figure 6 show a regular hexagon of size $s = 23$. Is the second hexagon having an equilateral triangle of size $c = 39$ inside as is explained in table 2.

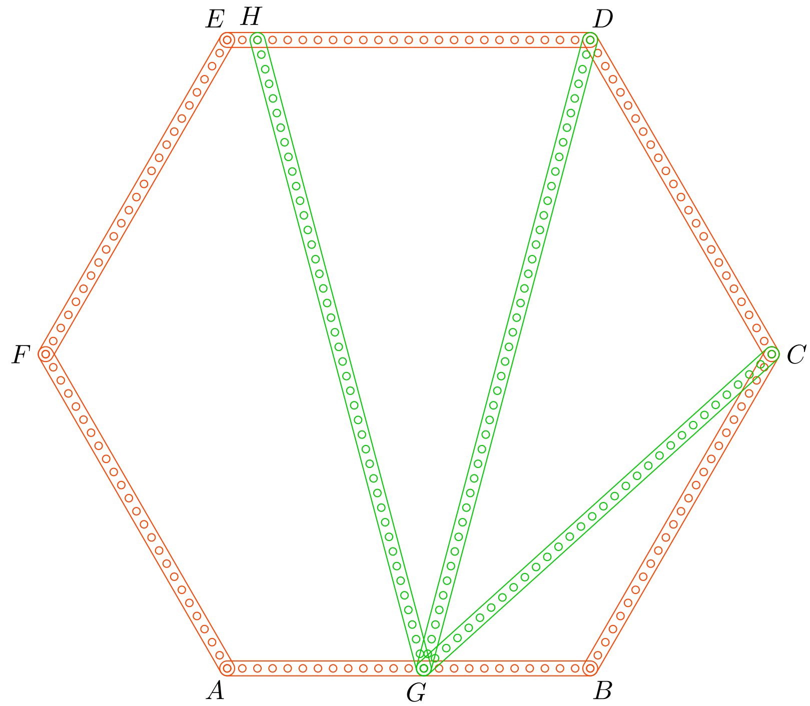

Figure 7: Hexagon of size $s = 24$. Needs only two extra bolts at vertices G and H. Diagonal $\overline{GC} = 31$ and diagonals $\overline{GD} = \overline{GH} = 43$. Segments $\overline{GB} = 11$ and $\overline{DH} = 22$.

Figure 7 show a regular hexagon of size $s = 23$. Triangle $\triangle GBC$ has sides $c_1 = 31, p_1 = 24, b_1 = 11$ which is the fourth triplet of table 1. Triangle $\triangle DHG$ is the isoscelles triangle shown in figure 2 case $a = 48$.