Meccano hexagons

https://github.com/heptagons/meccano/hexa

1 Meccano hexagons

1.1 Regular diagonals

A meccano hexagon can be build easily attaching sufficient equilateral triangles as small as one unit side. Also joining six bars to form a perimeter and using two more rods as **regular diagonals**, which means both diagonals are aligned along the triangular grid. Regular diagonals join opposite hexagon sides.

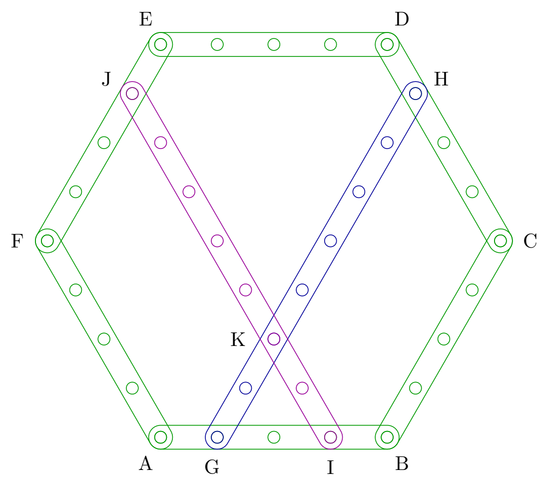

Figure 1: Hexagon of size 4 with two **regular diagonals** of size 7.

Consider figure 1. Start with rod $\overline{AB}$ and add two rods $\overline{GH}$ and $\overline{IJ}$ to form a triangle with three bolts at points G, I and K. At this moment, perimeter points A, B, H and G are rigid.

Then add perimeter rods $\overline{BC}$ and $\overline{CD}$ with a bolt at H. In the same way add perimeter rods $\overline{AF}$ and $\overline{EF}$ with a bolt at J. At this moment everything is rigid. Finally add rod $\overline{DE}$ with bolts at D and E.

1.2 Irregular diagonals

While regular diagonals are aligned along the triangular grid, **irregular diagonals** don't. Irregular diagonals join two adjacent hexagon sides, making (rigid) irregular triangles.

Consider figure 2. Start with an equilateral triangle ABC of side $\overline{AB}$. Test one by one the irregular diagonals from the point A to the points D_1, D_2..., D_n which are over the rod $\overline{BC}$. Define the tree variables

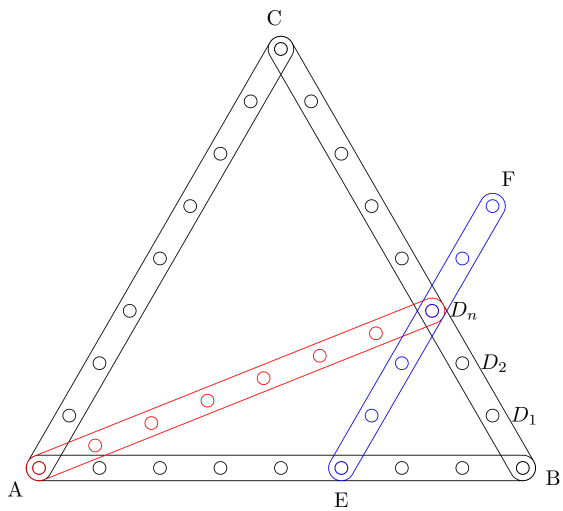

Figure 2: The red rod is an irregular diagonal, with integer length and joining two adjacent hexagon sides $\overline{AE}$ and $\overline{EF}$.

to use:

$$a = \overline{AB}$$
$$b = \overline{BD_n}$$
$$d = \overline{AD_n}$$

Acording to the cosines law and knowing the $\angle EBD_n = 60°$, calculate d:

$$\begin{aligned} d &= \sqrt{a^2 + b^2 - 2ab\cos\frac{\pi}{3}} \\ &= \sqrt{a^2 + b^2 - ab} \\ &= \sqrt{(a-b)^2 + ab} \end{aligned}$$

Reject any non-integer diagonal d, since any meccano rod length should be an integer. For the valid diagonal such as $\overline{AD_n}$, locate a point E over the rod $\overline{AB}$ such so the distance $\overline{BE}$ equals the distance $\overline{BD_n}$. From the point E create a new rod $\overline{EF}$ passing over the point D_n (blue rod in the figure). Finally we got a valid **irregular diagonal** d for the pair of adjacent hexagon sides $\overline{AE}$ and $\overline{EF}$.

1.3 Irregular diagonals program

We need a program to iterate over integer a, then over integer b to test whether d value is an integer too. Next golang program find the diagonals. We iterate from $a = 1$ to a given maximum (line 2). Then we iterate from $b = 1$ to $b <= a/2$ (line 3), to avoid repeating symmetric values. In order to reject repetitions by scaling we check for greatest commond divisor of a and b to be 1 (line 4). Then we calculate the diagonal using the formula $d^2 = (a-b)^2 + ab$ (line 5) and report only the case when the diagonal is a square number (line 8).

```
func triangle_diagonals(max int) {
  for a := 1; a < max; a++ {
    for b := 1; b <= a/2; b++ {
      if gcd(a, b) == 1 {
        diag := (a-b)*(a-b) + a*b
        cd := math.Sqrt(float64(diag))
        d := int(cd)
        if cd == float64(d) {
          num := float64(diag + a*a - b*b)
          den := 2.0 * cd * float64(a)
          angle := 180*math.Acos(num/den)/math.Pi
          fmt.Printf("a=%3d b=%3d d=%3d angle=%8.4f\n", a, b, d, angle)
        }
      }
    }
  }
}
func gcd(a, b int) int { // greatest common divisor
  if b == 0 {
    return a
  }
  return gcd(b, a % b)
}
```

1.4 Irregular diagonals results

The program found 13 distinct irregular diagonals sides $a <= 100$. Next table show the results including the angle EAD_n needed for the latex drawing scripts.

```
a=  8 b=  3 d=  7 angle= 21.7868
a= 15 b=  7 d= 13 angle= 27.7958
a= 21 b=  5 d= 19 angle= 13.1736
a= 35 b= 11 d= 31 angle= 17.8966
a= 40 b=  7 d= 37 angle=  9.4300
a= 48 b= 13 d= 43 angle= 15.1782
a= 55 b= 16 d= 49 angle= 16.4264
a= 65 b=  9 d= 61 angle=  7.3410
a= 77 b= 32 d= 67 angle= 24.4327
a= 80 b= 17 d= 73 angle= 11.6351
a= 91 b= 40 d= 79 angle= 26.0078
a= 96 b= 11 d= 91 angle=  6.0090
a= 99 b= 19 d= 91 angle= 10.4174
```

1.5 Examples of result A

Result A reports $a = 8$, $b = 3$ and $d = 7$, so the diagonal is of length 7 and the minimum hexagon size is $a - b = 5$. Figure 3 shows the smallest hexagon with irregular diagonals. In figure 4, the side is incremented to 6 and in figure 4 the side is incremented to 7 so all hexagon's rods are of the same length. Finally in figure 6, the size is incremented to 8 and we see two hexagons at the same time of sizes 7 and 8.

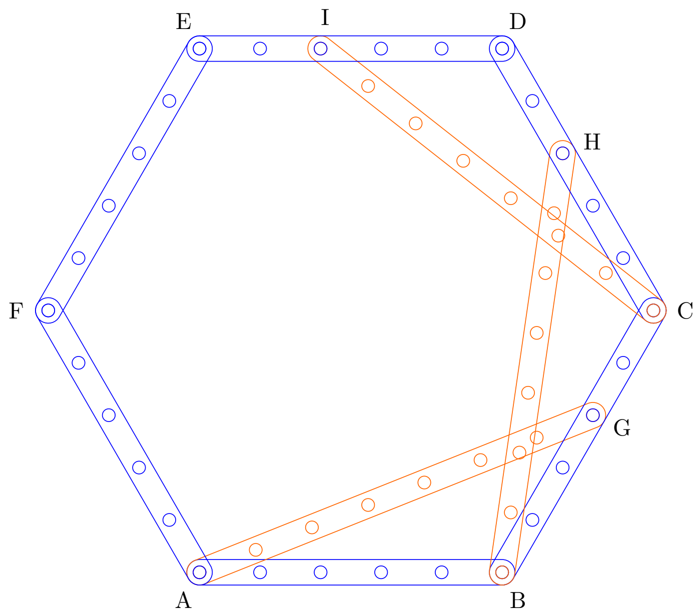

Figure 3: Hexagon side lenght:5, diagonal length:7.

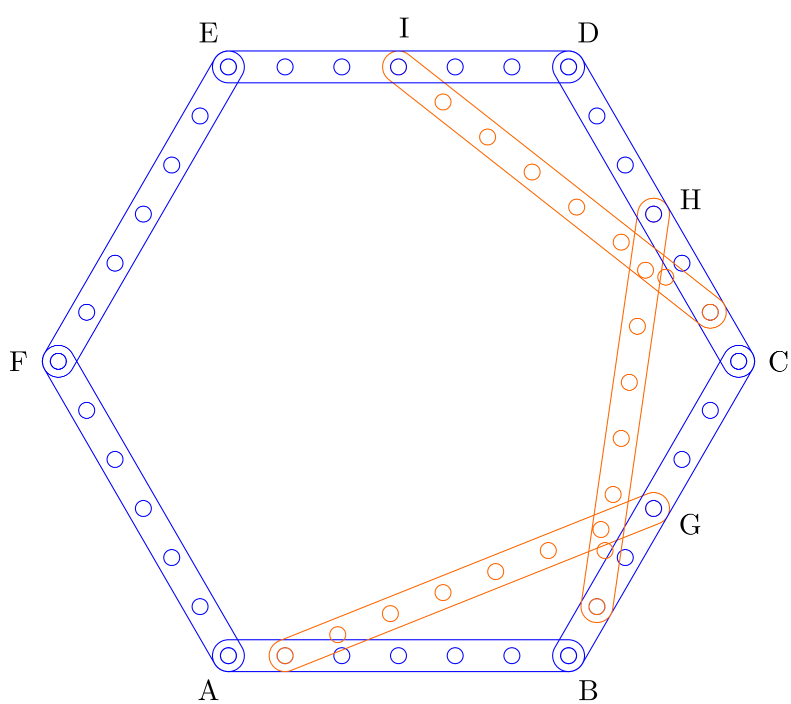

Figure 4: Hexagon side length:$5 + 1 = 6$, diagonal length:7.

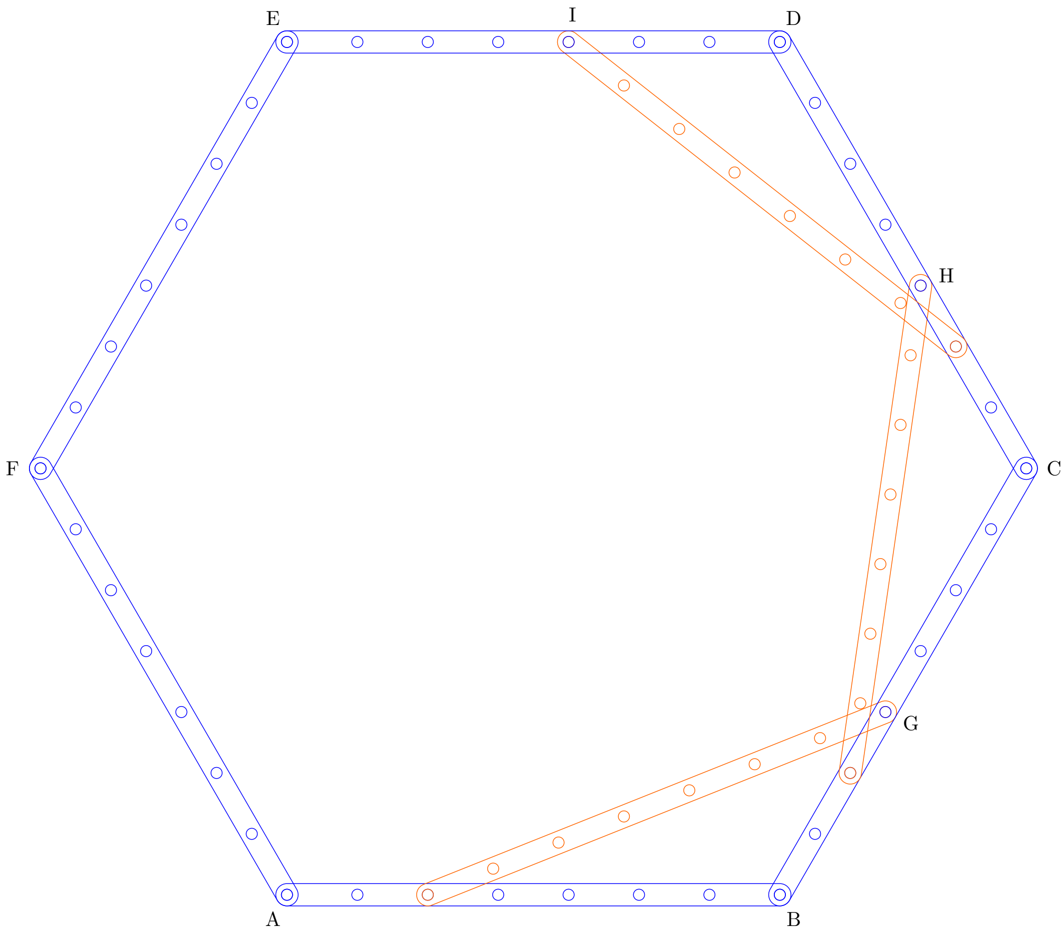

Figure 5: Hexagon sides $5 + 2 = 7$, diagonals 7.

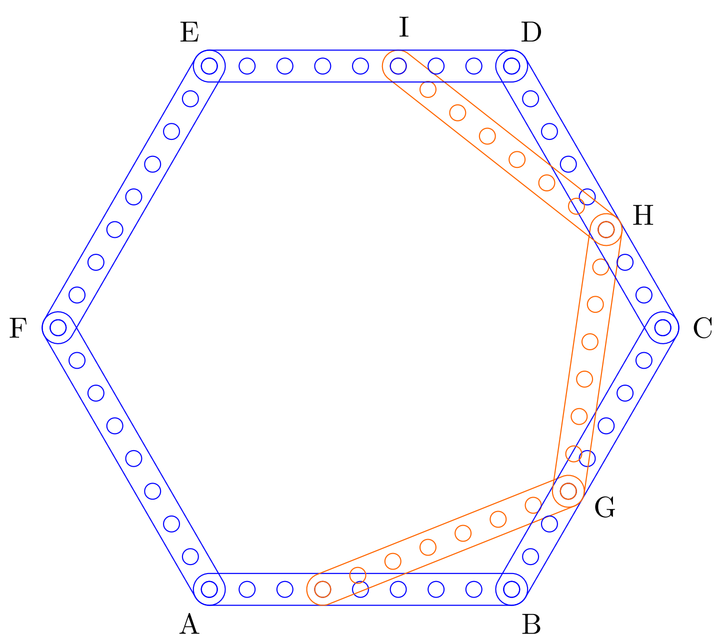

Figure 6: Hexagon sides $5 + 3 = 8$, diagonals 7.

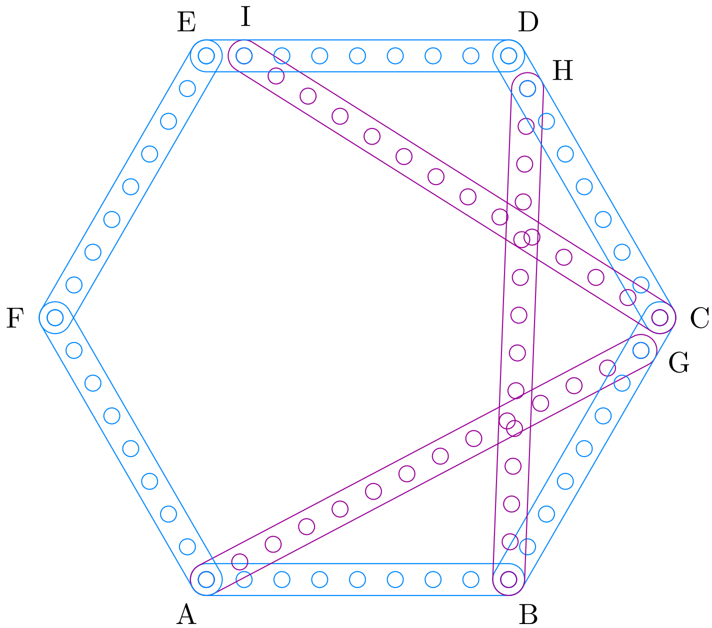

Figure 7: Hexagon sides 8, diagonals 13.

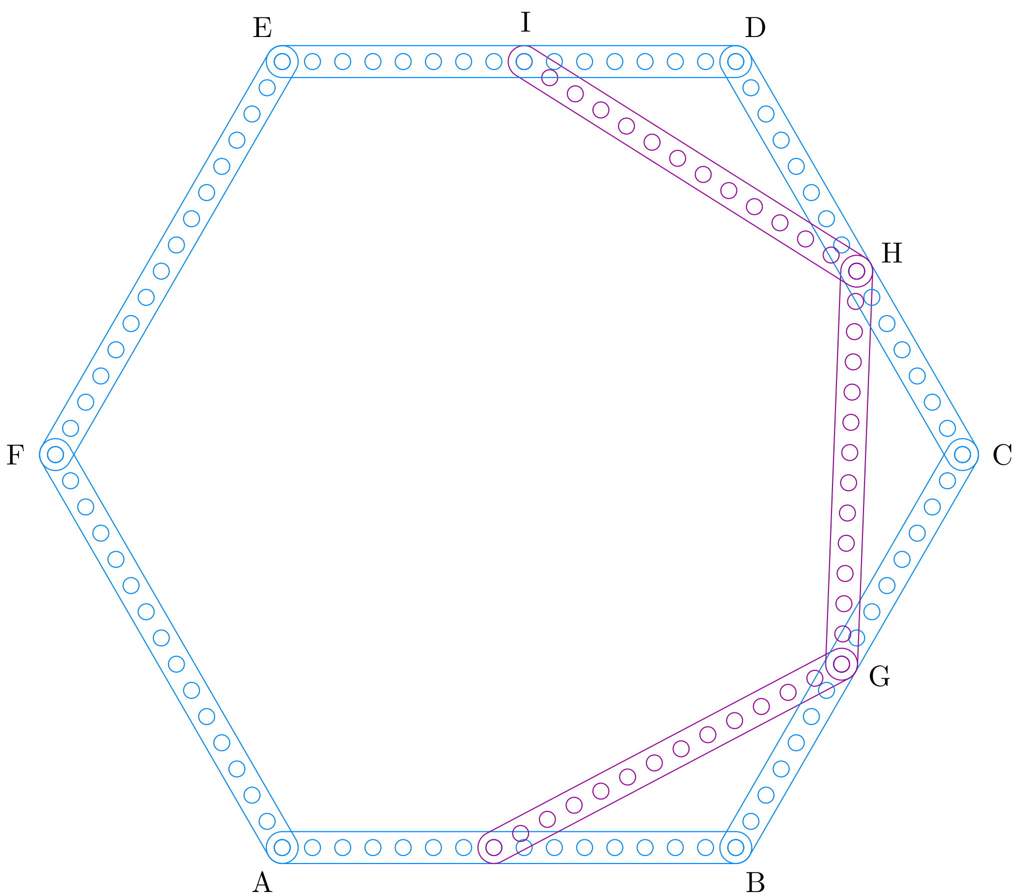

Figure 8: Hexagon sides $8 + 7 = 15$, diagonals 13.

1.6 Examples of result B

Result B reports $a = 15$, $b = 7$ and $d = 13$, so the diagonal is of length 13 and the minimum hexagon size is $a - b = 8$. Figure 7 shows the smallest hexagon with irregular diagonal 13 and figure 8 extends the side from 8 to 15 and we see two hexagons at the same time of sizes 13 and 15.