Meccano four frame

https://github.com/heptagons/meccano/frames/four

Abstract

Four frame is a group of four rigid meccano [1] strips.

1 Four frame

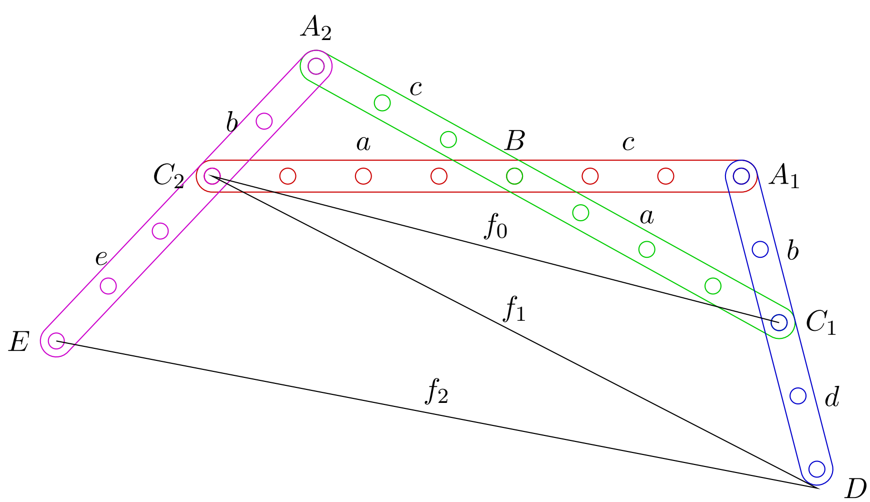

Figure 1: Four frame.

Figure 1 show the four-strips frame. First we calculate f_0 with the law of cosines:

$$\beta \equiv \angle A_1BC_1 \tag{1}$$

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac} \tag{2}$$

$$\pi - \beta = \angle C_1BC_2 \tag{3}$$

$$\begin{aligned} f_0^2 &= a^2 + a^2 - 2aa\cos(\pi-\beta) \\ &= 2a^2(1+\cos\beta) \\ &= 2a^2\frac{2ac+a^2+c^2-b^2}{2ac} = \frac{a(2ac+a^2+c^2-b^2)}{c} \\ &= \frac{a((a+c)^2-b^2)}{c} = \frac{a(a+c+b)(a+c-b)}{c} \end{aligned} \tag{4}$$

$$f_0 = \frac{\sqrt{ac(a+b+c)(a-b+c)}}{c} \tag{5}$$

[1] Meccano mathematics by ‘t Hooft

To simplify we define $m = c(a+c+b)(a+c-b)$ so we have:

$$m \equiv c(a+b+c)(a-b+c) \tag{6}$$

$$f_0 = \frac{\sqrt{am}}{c} \tag{7}$$

We first found $\cos\theta$ in function of f_0 also with the law of cosines:

$$\theta \equiv \angle A_1C_1C_2 \tag{8}$$

$$\begin{aligned}
\cos\theta &= \frac{b^2+f_0^2-(a+c)^2}{2bf_0} && (9)\\
&= \frac{f_0^2+(b^2-(a+c)^2)}{2bf_0}\\
&= \frac{f_0^2+(b+a+c)(b-a-c)}{2bf_0}\\
&= \frac{f_0^2-(a+b+c)(a-b+c)}{2bf_0}\\
&= \frac{f_0^2-\frac{m}{c}}{2bf_0}\\
&= \frac{f_0^2-\frac{f_0^2c^2}{ac}}{2bf_0}\\
&= \frac{f_0-\frac{f_0c}{a}}{2b}\\
&= \frac{(a-c)f_0}{2ab} && (10)
\end{aligned}$$

Now we calculate f_1:

$$\begin{aligned}
\pi-\theta &\equiv \angle C_1C_2D && (11)\\
f_1^2 &= f_0^2+d^2-2f_0d\cos(\pi-\theta)\\
&= f_0^2+d^2+2f_0d\cos\theta\\
&= f_0^2+d^2+2f_0d\frac{(a-c)f_0}{2ab}\\
&= f_0^2+d^2+\frac{d(a-c)}{ab}f_0^2\\
&= d^2+\frac{ab+d(a-c)}{ab}f_0^2\\
&= d^2+\frac{ab+d(a-c)}{ab}\left(\frac{am}{c^2}\right)\\
&= d^2+\frac{m(ab+d(a-c))}{bc^2}\\
f_1 &= \frac{\sqrt{b^2c^2d^2+bm(ab+d(a-c))}}{bc} && (12)
\end{aligned}$$