

# Meccano pentagons gallery

<https://github.com/heptagons/meccano/penta/gallery>

## Abstract

We show constructions of meccano rigid regular pentagons from side 12 to 3. We restrict all internal strips, we call diagonals, to remain inside the pentagon's perimeter and that don't overlap. Several programs found the solutions and we show some alternatives and prove the claimed values are exact.

## 1 Pentagons of size 12

A program found that side 12 is the smallest pentagon that can be made rigid with a rhombus and two strips as diagonals so need only 4 strips as diagonals. We show two cases.

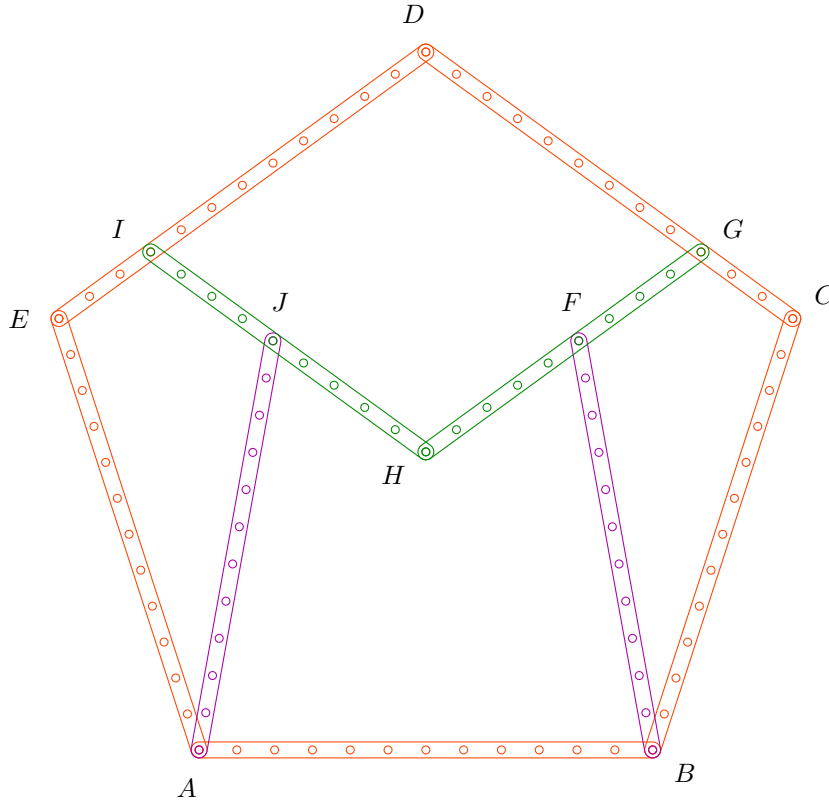


Figure 1: Pentagon size 12 (case a).

Figure 1 show a regular pentagon  $A, B, C, D, E$  of side 12 with a rhombus  $D, I, H, G$  of side 9. We prove strips  $AJ, BF$  are correct. First we calculate the abscissas going through vertices  $A, E, I, J$  subtracting

when we move to the left and adding when we move to the right:

$$AJ_x = AE_x + EI_x + IJ_x \quad (1)$$

$$\begin{aligned} &= -\overline{AE} \cos\left(\frac{2\pi}{5}\right) + \overline{EI} \cos\left(\frac{\pi}{5}\right) + \overline{IJ} \cos\left(\frac{\pi}{5}\right) \\ &= -12 \left(\frac{\sqrt{5}-1}{4}\right) + 3 \left(\frac{1+\sqrt{5}}{4}\right) + 4 \left(\frac{1+\sqrt{5}}{4}\right) = \frac{19-5\sqrt{5}}{4} \end{aligned} \quad (2)$$

Then we calculate the ordinates going to the same order of vertices adding when we go up and subtracting when we go down:

$$AJ_y = -AE_y + EI_y + IJ_y \quad (3)$$

$$\begin{aligned} &= \overline{AE} \sin\left(\frac{2\pi}{5}\right) + \overline{EI} \sin\left(\frac{\pi}{5}\right) - \overline{IJ} \sin\left(\frac{\pi}{5}\right) \\ &= 12 \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right) + 3 \left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right) - 4 \left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right) \\ &= \frac{12\sqrt{10+2\sqrt{5}} - \sqrt{10-2\sqrt{5}}}{4} = \frac{\sqrt{1450+190\sqrt{5}}}{4} \end{aligned} \quad (4)$$

Finally we calculate the distance  $\overline{AJ}$  which coincides with strip size 11:

$$\overline{AJ} = \sqrt{(AJ_x)^2 + (AJ_y)^2} \quad (5)$$

$$\begin{aligned} &= \sqrt{\left(\frac{19-5\sqrt{5}}{4}\right)^2 + \frac{1450+190\sqrt{5}}{16}} \\ &= \sqrt{\frac{486-190\sqrt{5}}{16} + \frac{1450+190\sqrt{5}}{16}} = \sqrt{121} = 11 \end{aligned} \quad (6)$$

Figure 2 show a regular pentagon  $A, B, C, D, E$  of size 12 with a rhombus  $D, I, H, G$  of size 12. We prove strips  $GH, IJ$  are correct. First we calculate the abscissas going through vertices  $G, A, E, H$  subtracting when we move to the left and adding when we move to the right:

$$GH_x = -GA_x - AE_x + EH_x \quad (7)$$

$$\begin{aligned} &= -\overline{GA} - \overline{AE} \cos\left(\frac{2\pi}{5}\right) + \overline{EH} \cos\left(\frac{\pi}{5}\right) \\ &= -4 - 12 \left(\frac{\sqrt{5}-1}{4}\right) + 3 \left(\frac{1+\sqrt{5}}{4}\right) = \frac{-1-9\sqrt{5}}{4} \end{aligned} \quad (8)$$

Then we calculate the ordinates going to the same order of vertices adding when we go up and subtracting when we go down:

$$GH_y = AG_y + AE_y - EH_y \quad (9)$$

$$\begin{aligned} &= 0 + \overline{AE} \sin\left(\frac{2\pi}{5}\right) - \overline{EH} \sin\left(\frac{\pi}{5}\right) \\ &= 12 \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right) - 3 \left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right) \\ &= \frac{12\sqrt{10+2\sqrt{5}} - 3\sqrt{10-2\sqrt{5}}}{4} = \frac{\sqrt{1530-18\sqrt{5}}}{4} \end{aligned} \quad (10)$$

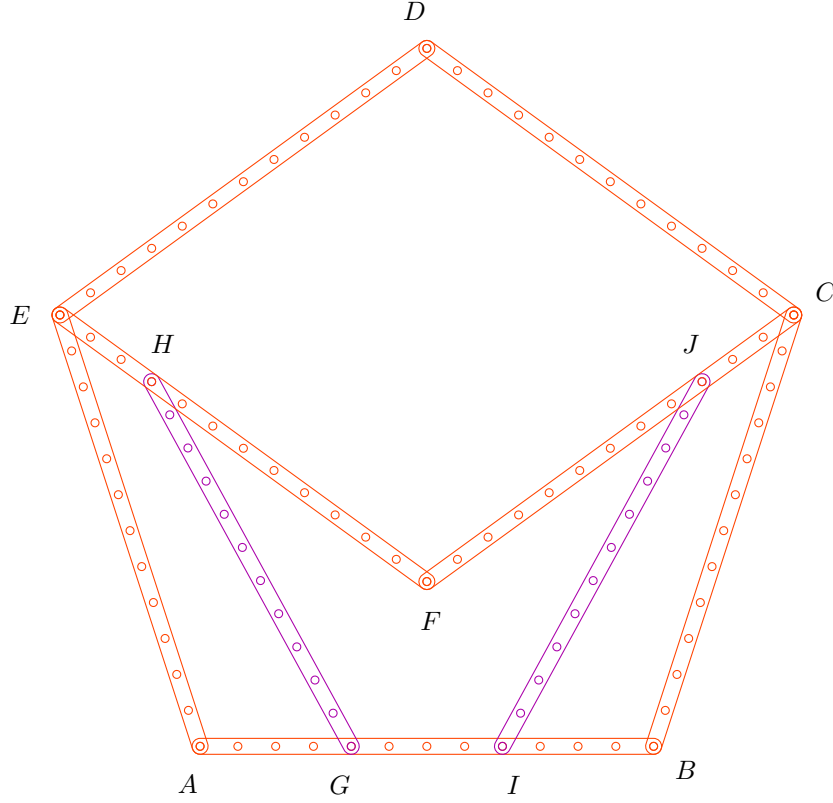


Figure 2: Pentagon size 12 case (b).

Finally we calculate the distance  $\overline{GH}$  which coincides with strip size 11:

$$\overline{GH} = \sqrt{(GH_x)^2 + (GH_y)^2} \quad (11)$$

$$= \sqrt{\left(\frac{-1 - 9\sqrt{5}}{4}\right)^2 + \frac{1530 - 18\sqrt{5}}{16}}$$

$$= \sqrt{\frac{406 + 18\sqrt{5}}{16} + \frac{1530 - 18\sqrt{5}}{16}} = \sqrt{121} = 11 \quad (12)$$

## 2 Pentagon of size 11