Meccano frames

https://github.com/heptagons/meccano/frames

Abstract

Meccano frames are groups of meccano [1] strips intended to be a base to build diverse meccano larger objects.

1 Triangle with extensions

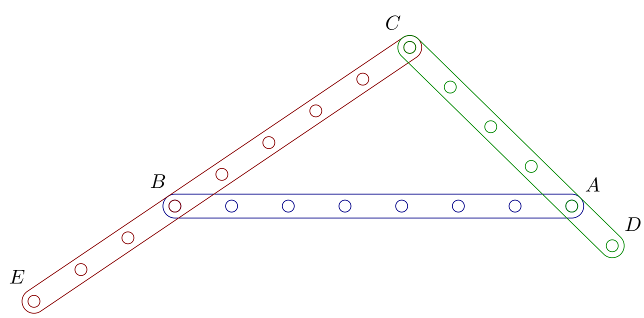

Figure 1: Triangle frame

2 Algebraic distance not right

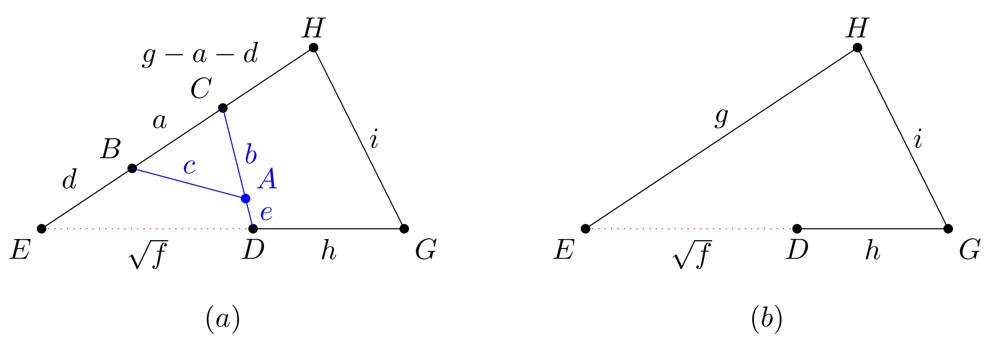

Figure 2: The five strips intented to form an algebraic distance $\sqrt{f}+h$.

From figure 2 (a) we know $\sqrt{f}$ distance between nodes E and D is produced by the three strips frame $a+d$, $b+e$ and c. Using the law of cosines we calculate the angle $\theta = \angle CED$ in terms of $\sqrt{f}$:

[1] Meccano mathematics by ‘t Hooft

$$\begin{aligned}
\cos\theta &= \frac{(a+d)^2 + (\sqrt{f})^2 - (b+e)^2}{2(a+d)\sqrt{f}} \\
&= \frac{((a+d)^2 + f - (b+e)^2)\sqrt{f}}{2(a+d)f} && (1) \\
&= \frac{m\sqrt{f}}{n} && (2) \\
m &= (a+d)^2 + f - (b+e)^2 && (3) \\
n &= 2(a+d)f && (4)
\end{aligned}$$

From figure 2 (a) we notice two sets of points are collinear: $\{E, B, C, H\}$ and $\{E, D, G\}$. Using the law of cosines we calculate the angle $\theta = \angle HEG$ in terms of distances g, h, i:

$$\begin{aligned}
\cos\theta &= \frac{g^2 + (\sqrt{f}+h)^2 - i^2}{2g(\sqrt{f}+h)} \\
&= \frac{g^2 + f + 2\sqrt{f}h + h^2 - i^2}{2g(\sqrt{f}+h)} \\
&= \frac{g^2 + f + h^2 - i^2 + 2\sqrt{f}h}{2g(\sqrt{f}+h)} && (5)
\end{aligned}$$

We multiply both numerator and denominator by $\sqrt{f} - h$ to eliminate the surd from denominator:

$$\begin{aligned}
\cos\theta &= \frac{(f + g^2 + h^2 - i^2)(\sqrt{f} - h) + 2\sqrt{f}h(\sqrt{f} - h)}{2g(\sqrt{f}+h)(\sqrt{f}-h)} \\
&= \frac{(f + g^2 + h^2 - i^2)(\sqrt{f} - h) + 2fh - 2\sqrt{f}h^2}{2g(f - h^2)} \\
&= \frac{-h(f + g^2 + h^2 - i^2 - 2f) + (f + g^2 + h^2 - i^2 - 2h^2)\sqrt{f}}{2g(f - h^2)} \\
&= \frac{h(f - g^2 - h^2 + i^2) + (f + g^2 - h^2 - i^2)\sqrt{f}}{2g(f - h^2)} \\
&= \frac{o + p\sqrt{f}}{q} && (6) \\
o &= h(f - g^2 - h^2 + i^2) && (7) \\
p &= f + g^2 - h^2 - i^2 && (8) \\
q &= 2g(f - h^2) && (9)
\end{aligned}$$

We compare both cosines equations 2 and 6:

$$\frac{m\sqrt{f}}{n} = \frac{o + p\sqrt{f}}{q} \tag{10}$$

Since all variables are integers we need two conditions. First o should be zero. And second $\frac{m}{n} = \frac{p}{q}$.

For condition 1, we force o to be zero:

$$\begin{aligned}
o &= 0 \\
h(f - g^2 - h^2 + i^2) &= 0 \\
f &= g^2 + h^2 - i^2 && (11)
\end{aligned}$$

For condition2, we force m, n, p, q as:

$$\begin{aligned}
\frac{m}{n} &= \frac{p}{q} \\
\frac{(a+d)^2 + f - (b+e)^2}{2(a+d)f} &= \frac{f + g^2 - h^2 - i^2}{2g(f - h^2)}
\end{aligned} \tag{12}$$

We replace the value of f of last equation RHS with the value of equation 11 of condition 1:

$$\begin{aligned}
\frac{(a+d)^2 - (b+e)^2 + f}{(a+d)f} &= \frac{f + g^2 - h^2 - i^2}{g(f - h^2)} \\
&= \frac{g^2 + h^2 - i^2 + g^2 - h^2 - i^2}{g(g^2 + h^2 - i^2 - h^2)} \\
&= \frac{2(g^2 - i^2)}{g(g^2 - i^2)} \\
&= \frac{2}{g} \\
((a+d)^2 - (b+e)^2 + f)g &= 2(a+d)f
\end{aligned} \tag{13}$$