

Meccano heptagons

<https://github.com/heptagons/meccano/hepta>

Abstract

We construct meccano¹ regular heptagons. We use seven equal strips to build the polygon perimeter and then we attach **internal diagonals** to make the polygon regular and rigid. Using some heptagonal identities we deduce three variables a , b and c corresponding to the heptagon side, an internal point of the side and a internal diagonal length. Then we find a closed formula $c = f(a, b)$ and use a program to iterate over a and b to get c integer which produces a solution.

1 Meccano heptagons

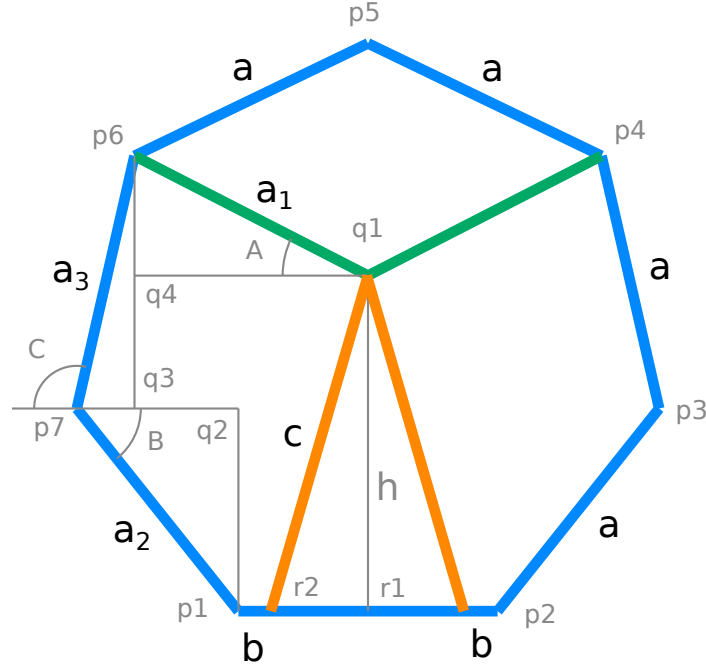


Figure 1: A meccano regular heptagon layout. First we define two integers a and b where $a > 2b$. We look for a third integer c to make the heptagon.

Consider the regular heptagon in figure 1. By inspection we identify three angles A , B and C :

$$A = \frac{\pi}{7}, B = \frac{2\pi}{7}, C = \frac{4\pi}{7}$$

Then we find the sines of the angles, noticing that the regular heptagon side is $a = a_1 = a_2 = a_3$:

¹ Meccano mathematics by 't Hooft

$$\sin A = \frac{\overline{p_6 q_4}}{a_1}, \sin B = \frac{\overline{p_1 q_2}}{a_2}, \sin C = \frac{\overline{p_6 q_3}}{a_3}$$

From the figure the height h corresponds to:

$$\begin{aligned} h &= \overline{p_1 q_2} + \overline{p_6 q_3} - \overline{p_6 q_4} \\ &= a_2 \sin B + a_3 \sin C - a_1 \sin A \\ &= a(-\sin A + \sin B + \sin C) \end{aligned}$$

According to *heptagonal triangles*²

$$\begin{aligned} \sin A - \sin B - \sin C &= -\frac{\sqrt{7}}{2} \\ \frac{h}{a} &= \frac{\sqrt{7}}{2} \\ h &= \frac{\sqrt{7}a}{2} \end{aligned}$$

Finally we get the c length as a function of lengths a and b :

$$\begin{aligned} c^2 &= h^2 + \overline{r_1 r_2}^2 \\ &= \left(\frac{\sqrt{7}a}{2}\right)^2 + \left(\frac{a-2b}{2}\right)^2 \\ &= \frac{7a^2}{4} + \frac{a^2 - 4ab + 4b^2}{4} + \\ &= \frac{8a^2 - 4ab + 4b^2}{4} \\ &= 2a^2 - ab + b^2 \end{aligned}$$

1.1 Heptagons search

A valid meccano heptagon needs to have the three lengths a , b and c as integers. With a software routine we look for c to be integer by incrementing the values of a and b , where $b < \lceil a/2 \rceil$.

1.2 Code

Following code function **Diagonals** (line 9) find several integer diagonals c in function of variables a and b . We iterate over $2 \leq a \leq \max$ (line 14), then we iterate over $1 \leq b < \lceil a/2 \rceil$ (line 17). Then we check c is and integer using the previous formula (lines 18 and 19). We store a solution not counting repetitions by scaling (line 20).

```

1 package hepta
2
3 import (
4     "math"
5 
```

²https://en.wikipedia.org/wiki/Heptagonal_triangle

```

6  "github.com/heptagons/meccano"
7  )
8
9  func Diagonals(max int) *meccano.Sols {
10
11     sols := &meccano.Sols{}
12
13     bMax, aa := 0, 0
14     for a := 2; a <= max; a++ {
15         bMax = int(math.Ceil(float64(a) / 2))
16         aa = 2*a*a
17         for b := 1; b < bMax; b++ {
18             f := float64(aa - a*b + b*b)
19             if c := int(math.Sqrt(f)); math.Pow(float64(c), 2) == f {
20                 sols.Add(a, b, c)
21             }
22         }
23     }
24     return sols
25 }

```

1.3 Results

Heptagons solutions for $a \leq 500$:

1	1	a=	3	b=	1	c=	4
2	2	a=	8	b=	1	c=	11
3	3	a=	33	b=	2	c=	46
4	4	a=	40	b=	17	c=	53
5	5	a=	55	b=	14	c=	74
6	6	a=	65	b=	31	c=	86
7	7	a=	85	b=	14	c=	116
8	8	a=	91	b=	2	c=	128
9	9	a=	95	b=	1	c=	134
10	10	a=	96	b=	47	c=	127
11	11	a=	105	b=	23	c=	142
12	12	a=	119	b=	46	c=	158
13	13	a=	120	b=	23	c=	163
14	14	a=	133	b=	62	c=	176
15	15	a=	144	b=	41	c=	193
16	16	a=	153	b=	7	c=	214
17	17	a=	161	b=	34	c=	218
18	18	a=	171	b=	34	c=	232
19	19	a=	175	b=	17	c=	242
20	20	a=	176	b=	79	c=	233
21	21	a=	207	b=	94	c=	274
22	22	a=	208	b=	47	c=	281
23	23	a=	225	b=	98	c=	298
24	24	a=	240	b=	7	c=	337
25	25	a=	240	b=	89	c=	319
26	26	a=	253	b=	47	c=	344
27	27	a=	261	b=	62	c=	352
28	28	a=	264	b=	1	c=	373
29	29	a=	275	b=	82	c=	368
30	30	a=	279	b=	41	c=	382
31	31	a=	297	b=	119	c=	394
32	32	a=	312	b=	73	c=	421
33	33	a=	319	b=	158	c=	422
34	34	a=	320	b=	79	c=	431
35	35	a=	325	b=	23	c=	452
36	36	a=	341	b=	142	c=	452
37	37	a=	351	b=	62	c=	478
38	38	a=	360	b=	31	c=	499
39	39	a=	377	b=	103	c=	506
40	40	a=	403	b=	146	c=	536
41	41	a=	407	b=	73	c=	554
42	42	a=	408	b=	167	c=	541
43	43	a=	429	b=	46	c=	592
44	44	a=	429	b=	191	c=	568
45	45	a=	435	b=	34	c=	604
46	46	a=	448	b=	137	c=	599
47	47	a=	451	b=	17	c=	632
48	48	a=	456	b=	119	c=	613
49	49	a=	465	b=	79	c=	634
50	50	a=	480	b=	17	c=	673
51	51	a=	481	b=	194	c=	638

1.4 Smallest examples

Figures 2 and 3 show the first two heptagons.

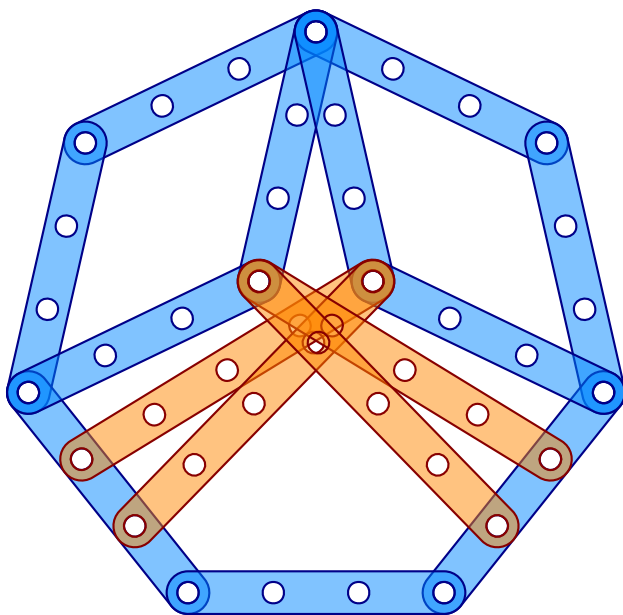


Figure 2: The first meccano heptagon with values $a = 3$, $b = 1$ and $c = 4$.

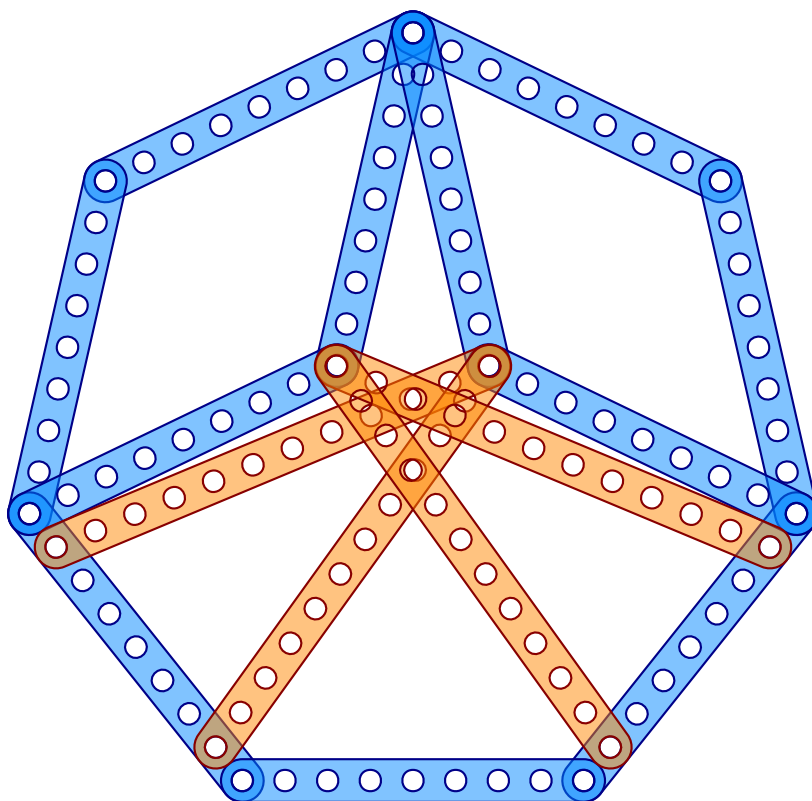


Figure 3: The second meccano heptagon with values $a = 8$, $b = 1$ and $c = 11$.