Meccano hexagons gallery

https://github.com/heptagons/meccano/hexa/gallery

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Abstract

We build meccano [1] rigid regular heptagons.

1 Heptagon internals

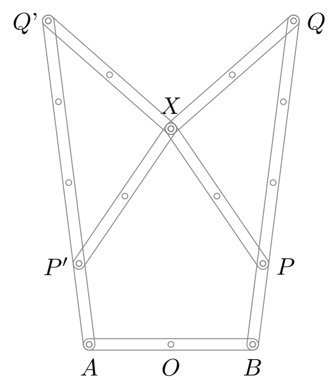

Figure 1: Cluster with fixed vertices $A = (-1, 0)$, $B = (+1, 0)$, $X = (0, \sqrt{7})$ useful to build a regular heptagon.

Figure 1 show a cluster to make a triangle $\triangle ABX$ useful to make a regular heptagon. With the law of cosines we calculate first the angle $\theta \equiv \angle PQX$ and then the distance $\overline{BX}$:

$$\cos\theta = \frac{(\overline{PQ})^2 + (\overline{QX})^2 - (\overline{XP})^2}{2(\overline{PQ})(\overline{QX})} = \frac{3^2 + 2^2 - 2^2}{2(3)(2)} = \frac{3}{4} \tag{1}$$

$$\overline{BX} = \sqrt{(\overline{BQ})^2 + (\overline{QX})^2 - 2(\overline{BQ})(\overline{PQ})\cos\theta} = \sqrt{4^2 + 2^2 - 2(4)(2)\left(\frac{3}{4}\right)} = 2\sqrt{2} \tag{2}$$

Since $\overline{AX} = \overline{BX}$ triangle $\triangle ABX$ is isoscelles and we calculate $\overline{OX}$:

$$\overline{OX} = \sqrt{(\overline{BX})^2 - (\overline{OB})^2} = \sqrt{(2\sqrt{2})^2 - 1^2} = \sqrt{7} \tag{3}$$

Figure 2 show equilateral heptagons $ABCDEFG$ connected to the cluster of figure 1. The heptagon has two internal strips $\overline{DX}$ and $\overline{FX}$ and vertices A, B and X are fixed. We move the strips symmetrically. In figure (a) we have the maximum extension to the top limited by fact that vertices B and D_0 cannot be separated more since both are collinear with vertice C_0, being the angle $\beta_0 = \pi/2$. From (a) we reach state (b) reducing β until $\alpha_1 = \beta_1 = \delta_1 = 3\pi/7$ the regular heptagon. From (b) we reach state (c) reducing β until $\beta = \pi/2$. In figure (d) we reach the maximum extension to the bottom limited by the fact that vertices X and C_3 cannot be separated more since both are collinear with vertice D_3, being the angle $\beta_3 = acos(3/4)$.

[1] Meccano mathematics by ‘t Hooft

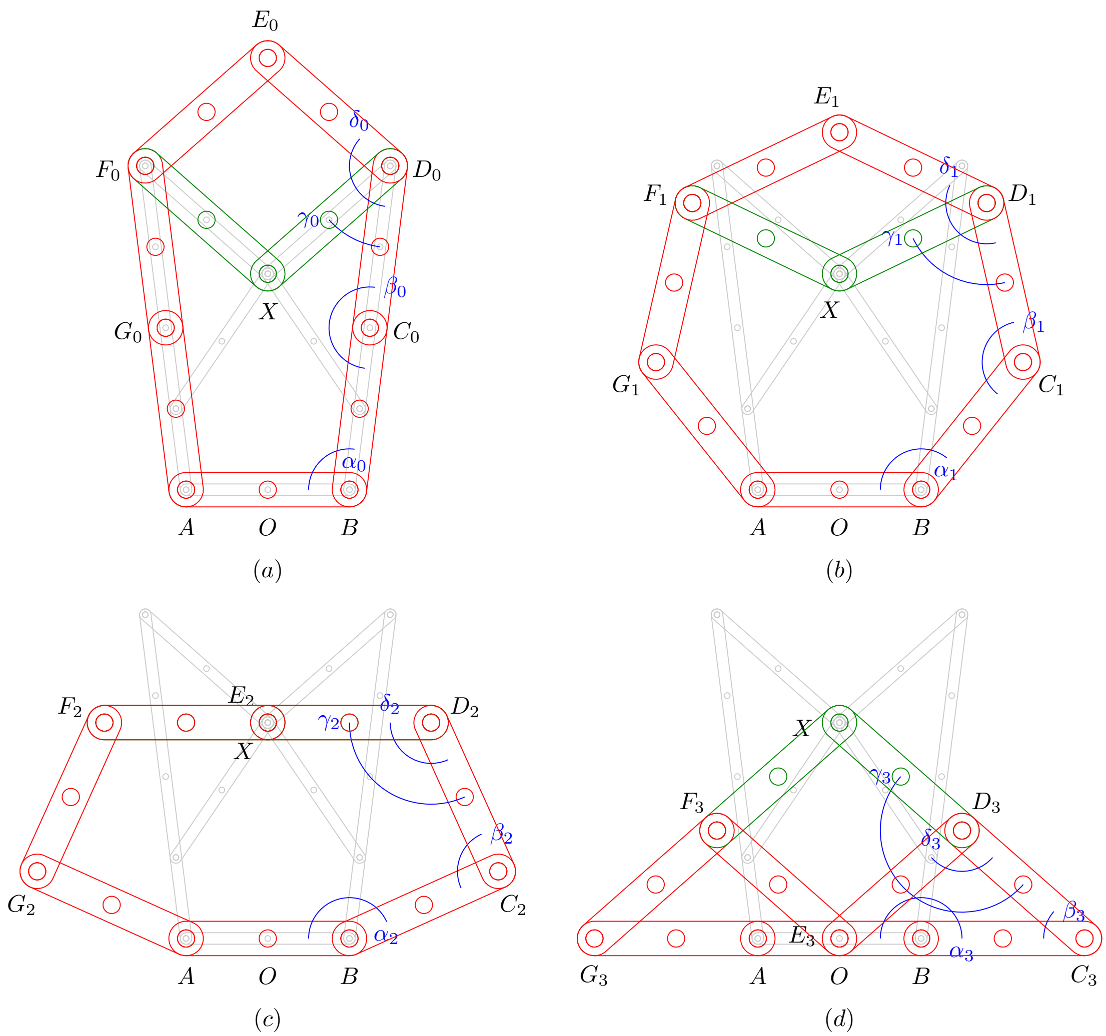

Figure 2: Equilateral heptagons connected to rigid triangle $\triangle ABX$.