

Meccano octagons

<https://github.com/heptagons/meccano/octa>

Abstract

We construct meccano¹ regular octagons using eight equal strips to build the a perimeter and attaching **internal diagonals** to make the polygon regular and rigid. The attached diagonals form angles of 135° to fix consecutive sides. We prepare a three variables formula $z = f(x, y)$ and catalog solutions when the variables are integers.

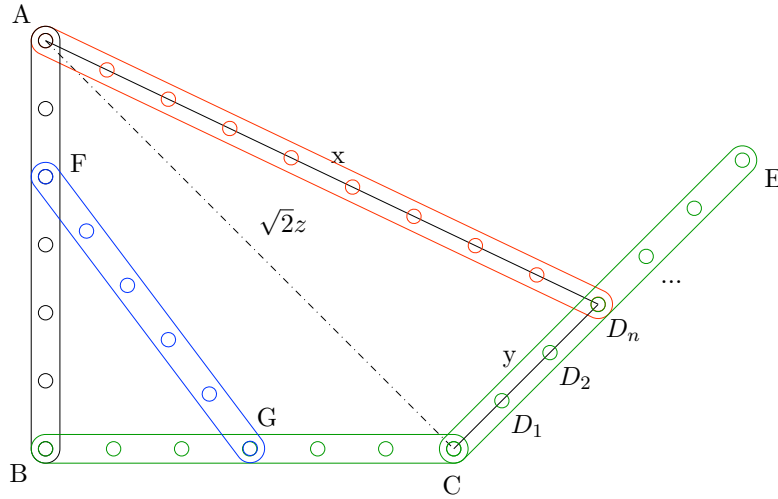


Figure 1: Construction of a 135° angle with meccano strips.

1 Meccano regular octagons

We build meccano regular octagons forming rigid strips with internal angles of 135° . Figure 1 show the internal diagonals construction. First, we build two triangles with adjacent angles adding 135° . Consider triangle ABC with $\angle BCA = 45^\circ$ and triangle ACD_n with $\angle ACD_n = 90^\circ$, so $\angle BCE = 135^\circ$. Lets define:

$$\begin{aligned} x &= \overline{AD_n} \\ y &= \overline{CD_n} \\ z &= \overline{AB} = \overline{BC} = \overline{CE} \end{aligned}$$

The internal diagonal we are looking for is the integer x (shown in red) and the two adjacent octagon sides are $z = \overline{BC} = \overline{CE}$ (shown in green). The two angles common hypotenuse is shown as a dashed line with a length of $\overline{AC} = \sqrt{2}z$ so:

$$\begin{aligned} (\sqrt{2}z)^2 &= x^2 - y^2 \\ 2z^2 &= x^2 - y^2 \\ z &= \sqrt{\frac{x^2 - y^2}{2}} \end{aligned}$$

¹ Meccano mathematics by 't Hooft

We run a program using the above formula to iterate over integers pair $x > y$ and expecting to find z as an integer too.

1.1 Program

Next go-lang listing program finds valid octagons diagonals a and sides $\max(b, c)$. First we iterate diagonals from 1 to a given maximum (line 2). Then we iterate over integer b from 1 to a (line 3). We calculate $2c^2$ and check if its even (lines 4, 5) and then we check if c is an integer (line 8). To prevent repetitions by scaling we check the greatest common divisor to be 1 (line 9) and print a valid result where the octagon size is the maximum value of b or c . (line 11).

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1 func Angles135(max int) {
2     for x := 1; x < max; x++ {
3         for y := 1; y < x; y++ {
4             if zz := x*x - y*y; zz % 2 == 0 {
5                 f := math.Sqrt(float64(zz / 2))
6                 if z := int(f); f == float64(z) {
7                     if meccano.Gcd(z, meccano.Gcd(x, y)) == 1 {
8                         a := int(math.Max(float64(y), f))
9                         fmt.Printf("a=%3d x=%3d y=%3d z=%3d\n", a, x, y, z)
10                    }
11                }
12            }
13        }
14    }
15 }

```

1.2 Results

First results for $x < 200$ are shown in the next listing. The octagon size is $a = \max(y, z)$.

1	a= 2 x= 3 y= 1 z= 2	23	a= 97 x= 99 y= 97 z= 14
2	a= 7 x= 9 y= 7 z= 4	24	a= 89 x=107 y= 89 z= 42
3	a= 7 x= 11 y= 7 z= 6	25	a= 72 x=113 y= 49 z= 72
4	a= 12 x= 17 y= 1 z= 12	26	a= 84 x=121 y= 23 z= 84
5	a= 17 x= 19 y= 17 z= 6	27	a= 73 x=123 y= 73 z= 70
6	a= 23 x= 27 y= 23 z= 10	28	a=119 x=123 y=119 z= 22
7	a= 20 x= 33 y= 17 z= 20	29	a=113 x=129 y=113 z= 44
8	a= 31 x= 33 y= 31 z= 8	30	a=127 x=129 y=127 z= 16
9	a= 24 x= 41 y= 23 z= 24	31	a= 90 x=131 y= 31 z= 90
10	a= 30 x= 43 y= 7 z= 30	32	a=119 x=137 y=119 z= 48
11	a= 47 x= 51 y= 47 z= 14	33	a=103 x=139 y=103 z= 66
12	a= 49 x= 51 y= 49 z= 10	34	a= 89 x=153 y= 89 z= 88
13	a= 40 x= 57 y= 7 z= 40	35	a=103 x=153 y=103 z= 80
14	a= 41 x= 57 y= 41 z= 28	36	a=161 x=163 y=161 z= 18
15	a= 41 x= 59 y= 41 z= 30	37	a=110 x=171 y= 71 z=110
16	a= 42 x= 67 y= 31 z= 42	38	a=167 x=171 y=167 z= 26
17	a= 71 x= 73 y= 71 z= 12	39	a=112 x=177 y= 79 z=112
18	a= 56 x= 81 y= 17 z= 56	40	a=161 x=177 y=161 z= 52
19	a= 79 x= 83 y= 79 z= 18	41	a=126 x=179 y= 17 z=126
20	a= 73 x= 89 y= 73 z= 36	42	a=137 x=187 y=137 z= 90
21	a= 60 x= 97 y= 47 z= 60	43	a=151 x=187 y=151 z= 78
22	a= 70 x= 99 y= 1 z= 70	44	a=132 x=193 y= 49 z=132

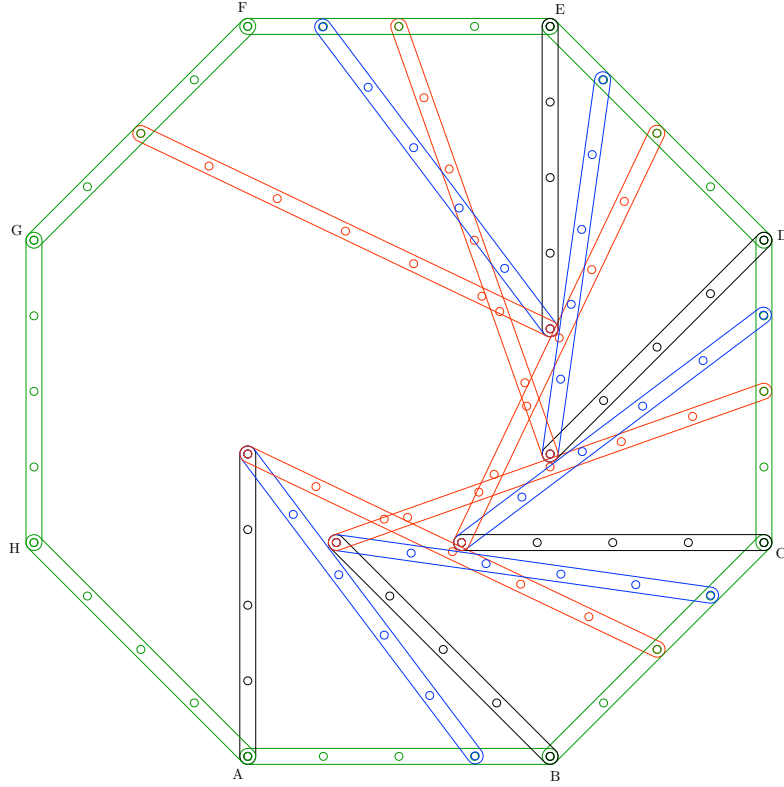


Figure 2: The smallest octagon with diagonals $x = 6$ and sides $z = 4$. In order to prevent bolts collisions we need to use strips with holes well separated. Four 135° units are needed to fix six consecutive rigid sides. This complex figure is simplified in next figure.

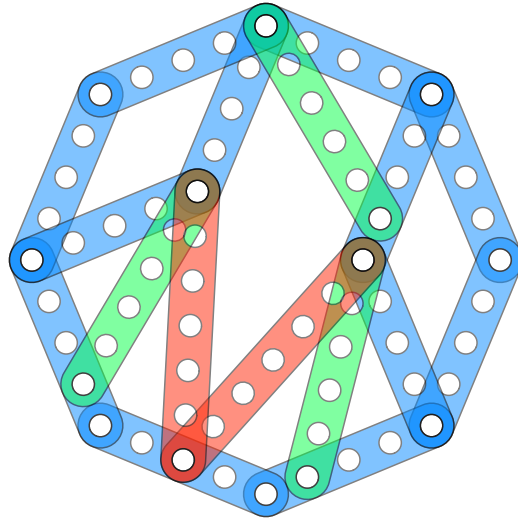


Figure 3: The smallest octagon again with diagonals $x = 6$ and sides $z = 4$ but with fewer extra strips. This construction is of the size of fig 2 but we put only two adjacent x red bars face to face and complete the rest with two $z = 4$ in blue and one more pythagorean green strip of size 5.

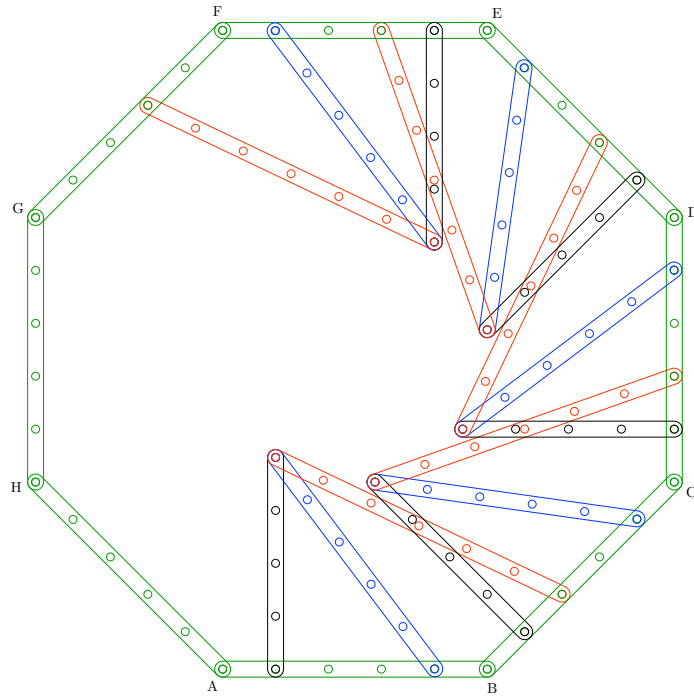


Figure 4: Octagon with diagonals $x = 6$ and sides $z + 1 = 5$.

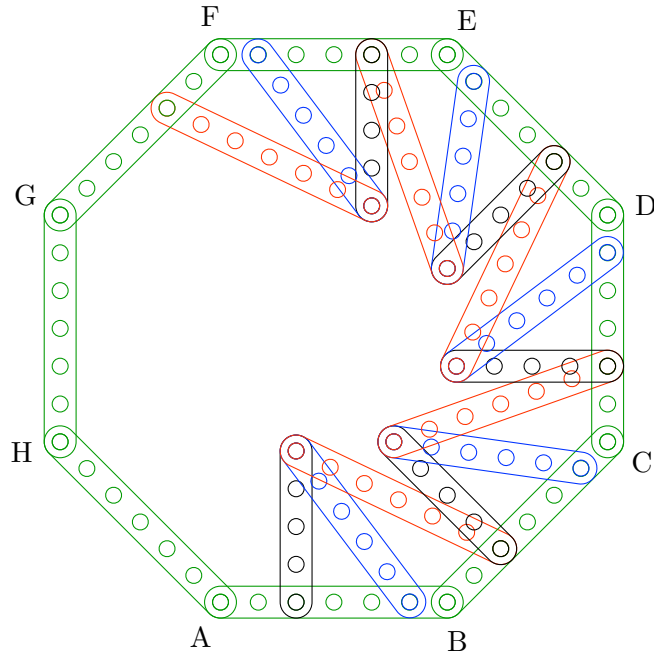


Figure 5: Octagon with diagonals $x = 6$ and sides $z + 2 = 6$.

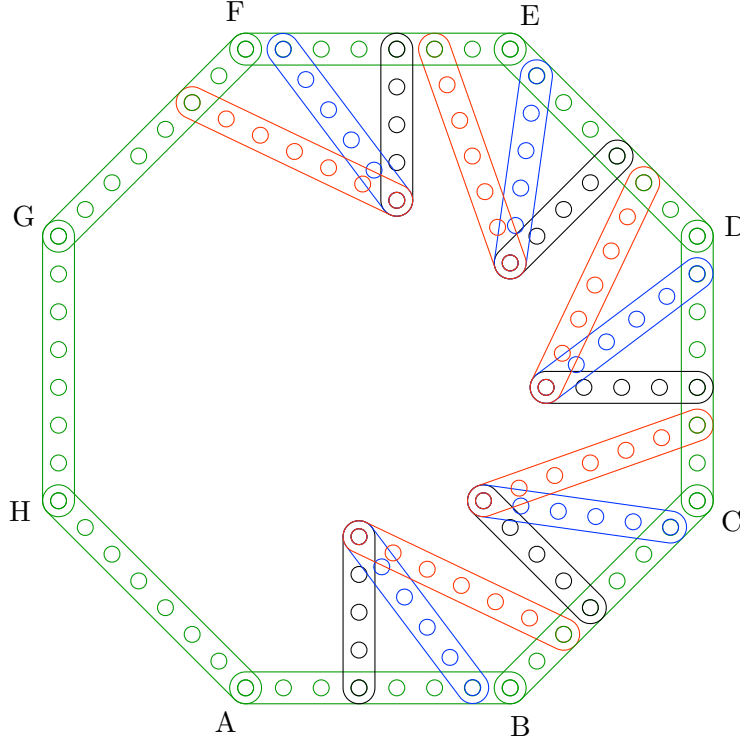


Figure 6: Octagon with diagonals $x = 6$ and sides $z + 3 = 7$.

1.3 Examples with diagonal $x = 6$

We can't use the first result:

$$a = 2, x = 3, y = 1, z = 2$$

because the octagon size is too small, is only $z = 2$. At least the octagon size should be 4 because the smallest right angle can be made with the pythagorean triplet 3, 4, 5. To make rigid the angle of 90° of triangle ABC in figure 1 we need a rod of size 5 at least, as shown the points F and G . Multiplying the first result by 2 we get:

$$a = 4, x = 6, y = 2, z = 4$$

and this will be the smallest octagon since $z = 4$ can hold a 3 – 4 – 5 triplet. Figure 2 is the smallest octagon; we need to scale the bars in order the bolts don't collapse with others, the complexity of bars can be simplified symmetrically as is shown in figure 3. In figure 4 we increase the side from 4 to 5 but keeping the same diagonal of 6. In figure 5 we increase the side to 6 and in figure 6 the side is increased to 7.

1.4 Examples with diagonal $x = 9$

With the second result:

$$a = 7, x = 9, y = 7, z = 4$$

we can form a second group of octagons. Figure 7 shows the smallest octagon with diagonals 9 and sides 7. Figures 8, 9, 10 and 11 show octagons with diagonals 9 with sides of 8, 9, 10 and 11.

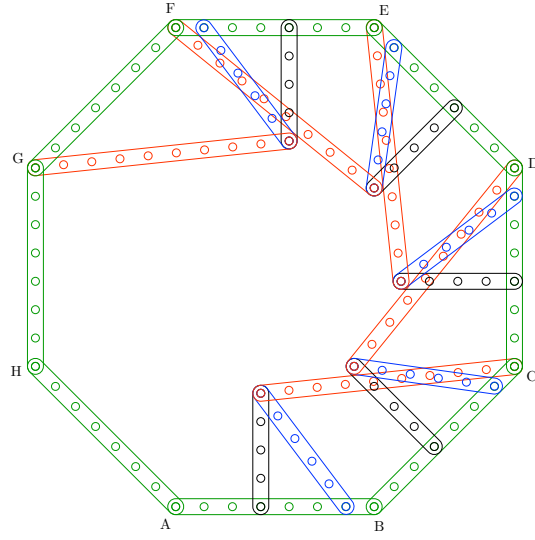


Figure 7: Octagon with diagonal $x = 9$ and side $y = 7$.

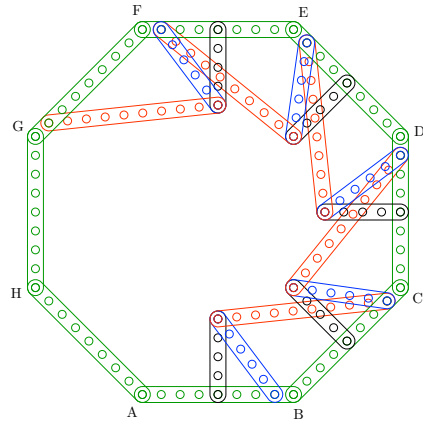


Figure 8: Octagon with diagonal $x = 9$ and side $y + 1 = 8$.

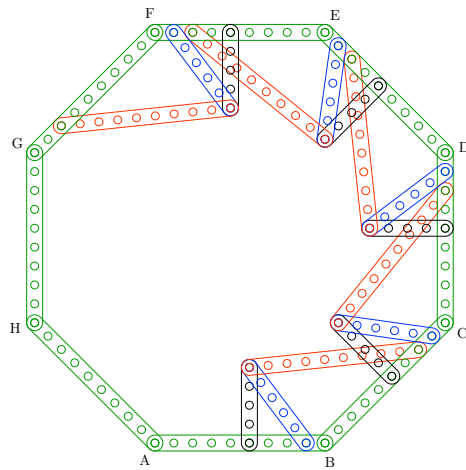


Figure 9: Octagon with diagonal $x = 9$ and side $y + 2 = 9$.

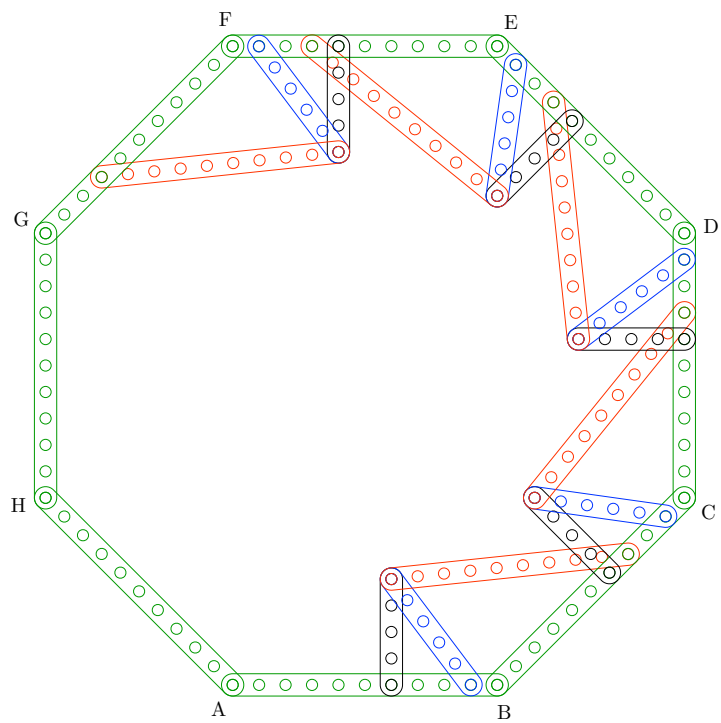


Figure 10: Octagon with diagonal $x = 9$ and side $y + 3 = 10$.

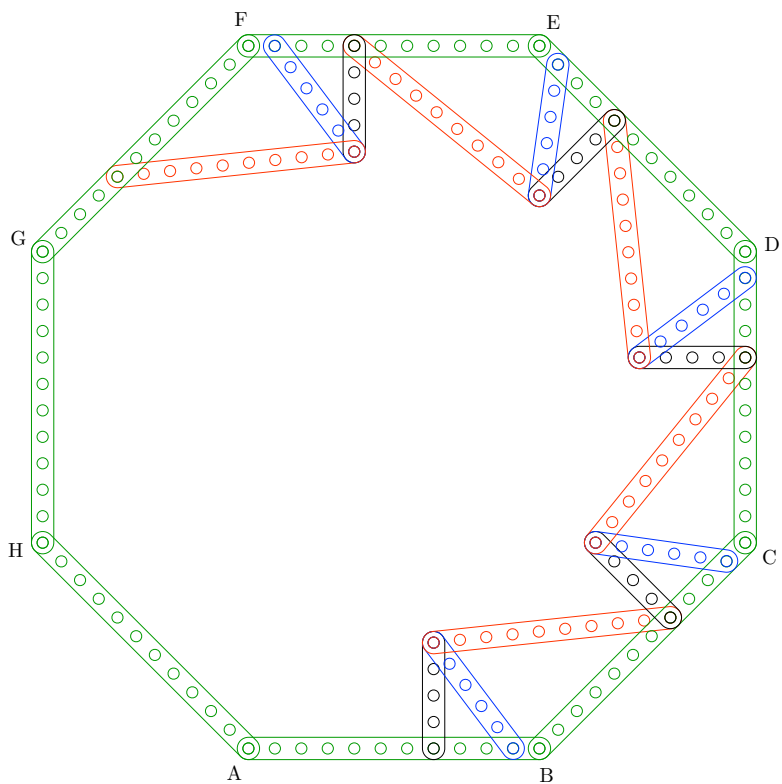


Figure 11: Octagon with diagonal $x = 9$ and side $y + 4 = 11$.

1.5 Example with double diagonals $x = 6, x = 9$

By comparing figures 6 and 10 both with sides= $y = 7$ we can make use of two diagonals at the same time and omit the rod FG of figure 1 used until now to make the 90° angle. Figure 12 show a octagon angle with two diagonals. In other words, for two results or their scaling, when both have the same y , we can use the two diagonals an omit the 5 rod.

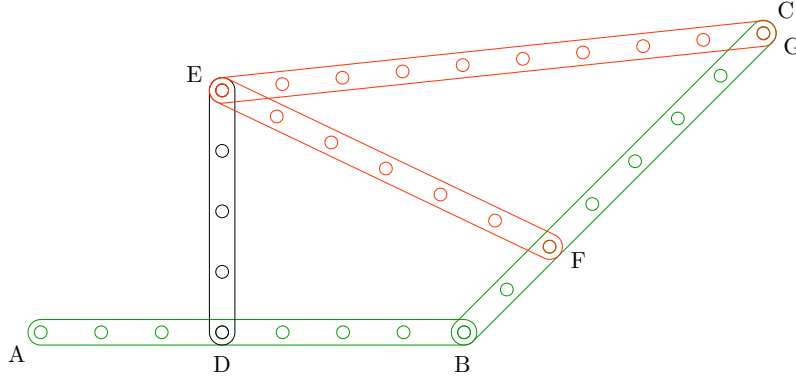


Figure 12: Octagon angle ABC fixed with two diagonals. The union of rods \overline{BG} , \overline{EF} and \overline{EG} is rigid. Adding two rods \overline{AB} and \overline{DE} remains rigid.