

CS151B/EE116C – Solutions to Homework #3

The delay Calculation: The delay = the gate delay + the max delay among inputs

Assumption:

The sum in 1 bit full adder is implemented by 2 input-XOR gates: $\text{Sum} = (a \text{ xor } b) \text{ xor } C$

1. $G_0 = A_0 * B_0 = 2T$ (the delay of 2input –AND gate) + $\max(0T, 0T) = 2T$

2. $P_0 = A_0 \text{ xor } B_0 = 2T + \max(0T, 0T) = 2T$

-> All G_i and P_i are $2T$

3. $G_\alpha = (G_0 * P_1 * P_2 * P_3) + (G_1 * P_2 * P_3) + (G_2 * P_3) + (G_3)$
 $= 5T$ (the delay of 4input-OR gate) + $\max(5T+2T, 3T+2T, 2T+2T, 2T) = 5T + 7T = 12T$

4. $P_\alpha = P_0 * P_1 * P_2 * P_3 = 5T + \max(2T, 2T, 2T, 2T) = 7T$

-> All G_i and P_i of the 2-level CLG in the 16bit-HCLA are $12T$ and $7T$ respectively.

5. $C_{12} = G_y + G_\beta * P_y + G_\alpha * P_\beta * P_y + C_0 * P_\alpha * P_\beta * P_y$
 $= 5T$ (4 input-OR gate) + $\max(12T, 2T+\max(12T, 7T), 3T+\max(12T, 7T, 7T) + 5T+\max(0T, 7T, 7T, 7T))$
 $= 5T + 15T = 20T$

6. C_{15} corresponds to the C_3 in the third 4bit-CLA (CLAY) in the 16bit-HCLA
 $C_{15} = G_2 + G_1 * P_2 + G_0 * P_1 * P_2 + \text{C12} * P_0 * P_1 * P_2$ since C_{12} is the $C_{in}(C_0)$ in this CLA.
 $= 5T$ (4 input-OR) + $\max(2T, 2T+2T, 3T+2T, 5T+\text{C12}) = 5T + 25T = 30T$

7. $C_{16} = G_\delta + G_y * P_\delta + G_\beta * P_\delta * P_y + G_\alpha * P_\delta * P_\beta * P_y + C_0 * P_\delta * P_\beta * P_y * P_\alpha$
 $= 7T + \max(12T, 2T+\max(12T, 7T), 3T+\max(12T, 7T, 7T), 5T+\max(12T, 7T, 7T, 7T) + 7T + \max(0T, 7T, 7T, 7T, 7T))$
 $= 7T + \max(12T, 14T, 15T, 17T, 14T) = 7T + 17T = 24T$

8. S_{15} corresponds to the S_3 in the bottom 4bit-CLA in 16bit-HCLA
 $S_{15} = (a \text{ xor } b) \text{ xor } C_{15} = 2T + \max(2T+2T, 30T) = 32T$

9. C_{20} corresponds to the carry out (C_4) of top 4bit-CLA in the 16bit-CSA.
 So, $C_{20}(C_4) = G_3 + G_2 * P_3 + G_1 * P_3 * P_2 + G_0 * P_3 * P_2 * P_1 + \text{C16} * P_3 * P_2 * P_1 * P_0$, but C_{16} 's delay is $0T$ in the CSA
 $= 7T + \max(2T, 2T+2T, 3T+2T, 5T+2T, 7T+2T) = 7T+9T = 16T$

10. S_{19} corresponds to the S_3 of the top 4bit-CLA in the 16bit-CSA.
 To calculate the delay of S_{19} , the delay of C_{19} is required.
 C_{19} also corresponds to the C_3 of the top 4bit-CLA in the 16bit-CSA.
 So, $C_{19}(C_3) = G_2 + G_1 * P_2 + G_0 * P_2 * P_1 + \text{C16} * P_2 * P_1 * P_0$, but C_{16} 's delay is $0T$ in the CSA
 $= 5T + \max(2T, 2T+2T, 3T+2T, 5T+2T) = 5T+7T = 12T$
 $S_{19} = (a \text{ xor } b) \text{ xor } C_{19} = 2T + \max(2T+2T, 12T) = 14T$

11. C_{24} corresponds to the carry out (C_4) of 2nd top 4bit-CLA in the 16bit-CSA.
 So, $C_{24}(C_4) = G_3 + G_2 * P_3 + G_1 * P_3 * P_2 + G_0 * P_3 * P_2 * P_1 + C_0 * P_3 * P_2 * P_1 * P_0$
 $= G_3 + G_2 * P_3 + G_1 * P_3 * P_2 + G_0 * P_3 * P_2 * P_1 + \text{C20} * P_3 * P_2 * P_1 * P_0$ since C_{20} is C_0 in this CLA

$$= 7T + \max(2T, 2T+2T, 3T+2T, 5T+2T, 7T+16T) = 7T+23T = 30T$$

12. C31 corresponds to the C3 in the bottom 4bit-CLA in the 16bit-CSA

To calculate the delay of C31, the delay of C28 (Cin) is required.

C28 corresponds to the carry out (C4) of third 4bit-CLA in the 16bit-CSA.

$$C28 = G3 + G2 * P3 + G1 * P3 * P2 + G0 * P3 * P2 * P1 + C0 * P3 * P2 * P1 * P0$$

$$= G3 + G2 * P3 + G1 * P3 * P2 + G0 * P3 * P2 * P1 + C24 * P3 * P2 * P1 * P0, \text{ since the Cin of this CLA is C24}$$

$$= 7T + \max(2T, 2T+2T, 3T+2T, 5T+2T, 7T+30T) = 7T+37T = 44T$$

$$C31(C3) = G2 + G1 * P2 + G0 * P2 * P1 + C0 * P2 * P1 * P0 = G2 + G1 * P2 + G0 * P2 * P1 + C28 * P2 * P1 * P0$$

$$= 5T + \max(2T, 2T+2T, 3T+2T, 5T+44T) = 5T+49T = 54T$$

13. C32(after MUX) = 4T (the delay of MUX) + max (C32s, selection bit(C16))

C32 corresponds to the carry out of the bottom 4bit-CLA in the 16bit-CSA.

$$C32 = G3 + G2 * P3 + G1 * P3 * P2 + G0 * P3 * P2 * P1 + C0 * P3 * P2 * P1 * P0$$

$$= G3 + G2 * P3 + G1 * P3 * P2 + G0 * P3 * P2 * P1 + C28 * P3 * P2 * P1 * P0$$

$$= 7T + \max(2T, 2T+2T, 3T+2T, 5T+2T, 7T+44T) = 7T+51T = 58T, \text{ C28 is 44T from the solution 12.}$$

Since C16 is 24T from the solution 7.

$$C32(\text{after MUX}) = 4T + \max(58T, 58T, 22T) = 62T$$

14. S31(after MUX) = 4T (the delay of MUX) + max(S31s, C16)

$$S31 = (a \text{ xor } b) \text{ xor } C31 = 2T + \max(2T, 54T) = 56T$$

$$S31(\text{after MUX}) = 4T + \max(56T, 56T, 24T) = 60T$$