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①

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LECTURE - 41

WHEN WILL THE DOT PRODUCT BE ZERO?

The dot product can be zero,

→ If the angle between the two vectors is  $90^\circ$  i.e., if one or both of the vectors is zero vector.

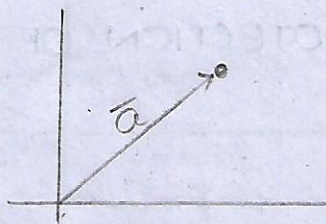
→ If the points are on the opposite axis.

UNIT VECTOR:

A unit vector is a vector of length 1 and sometimes called as direction vector.

→ It is denoted as  $\hat{a}$

$$\Rightarrow \hat{a} = \frac{\vec{a}}{\|\vec{a}\|}$$



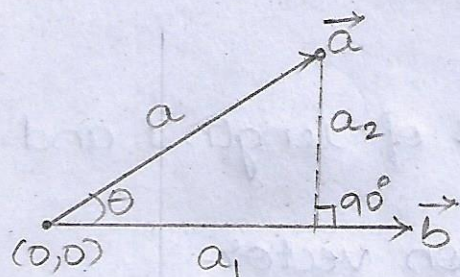


## ② PROJECTION OF A VECTOR:

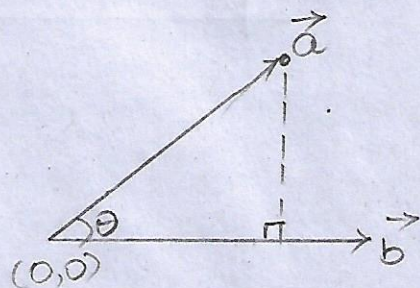
The vector projection is the vector produced when one vector is resolved into two component vectors i.e.,

one is  $\parallel$  to the other vector and one is  $\perp$  to the other vector.

→ The parallel vector is the vector projection.



## PROJECTION OF ONE VECTOR ON ANOTHER VECTOR:



Projection of  $\vec{a}$  on  $\vec{b}$

is represented as

$$\text{Proj}_{\vec{b}} \vec{a}$$

$$\Rightarrow \cos \theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{d}{\|\vec{a}\|}$$



Proj <sub>$\vec{b}$</sub>   $\vec{a}$  is denoted as d. <sup>(3)</sup>

$$\Rightarrow d = \|\vec{a}\| \cdot \cos \theta_{a,b} \rightarrow \text{MAGNITUDE}$$

$$\rightarrow \text{Proj}_{\vec{b}} \vec{a} = \text{MAGNITUDE} \times \text{DIRECTION}$$

$$= d \times \frac{\vec{b}}{\|\vec{b}\|}$$

$$= \{ \|\vec{a}\| \cdot \cos \theta \} \cdot \frac{\vec{b}}{\|\vec{b}\|}$$

LINE EQUATION:

A equation between two variables, that gives a straight line when plotted.

$$\Rightarrow Y = mX + C$$

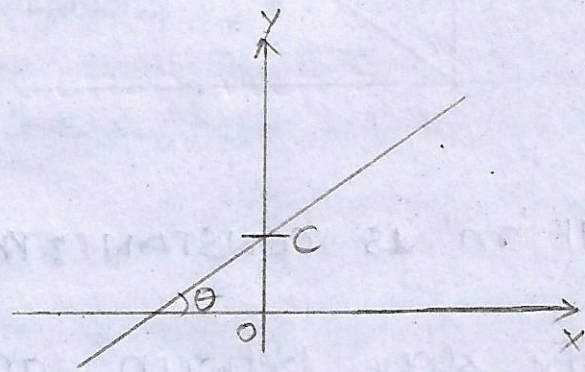
where

$m$  = slope of line eq<sup>n</sup>.

$C$  = intercept of  $y$ .

$\theta$  = angle between the line and  $x$ -axis

$\theta \rightarrow$  Tangent angle.



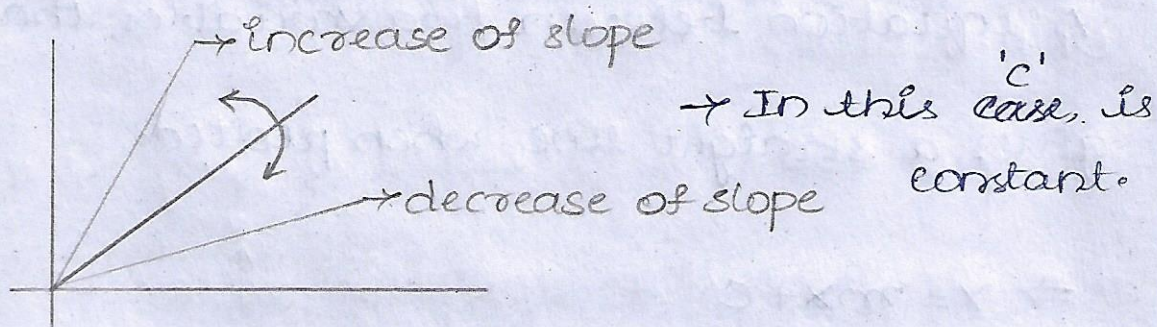


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WHAT HAPPENS WHEN WE CHANGE SLOPE OF LINE?

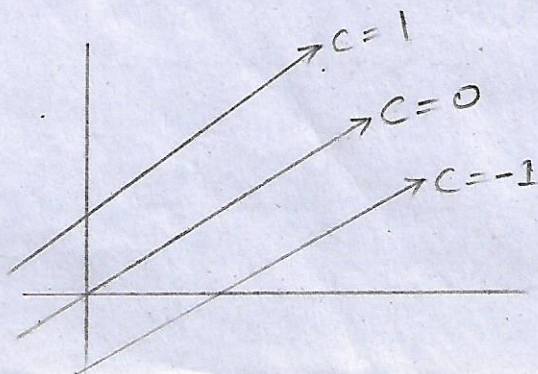
When a line shifts in such a way that it maintains the same steepness (almost a vertical slope) as the original line, but moves up or down, or to the right or left.

→ the  $y$ -intercept changes while the slope remained the same.



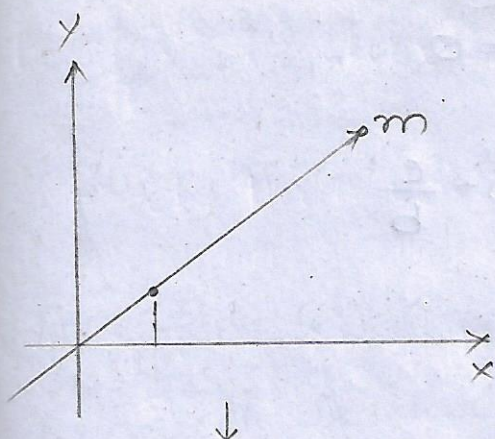
IF ' $m$ ' IS CONSTANT? THE

The slope between each pair of points is same for any given line.





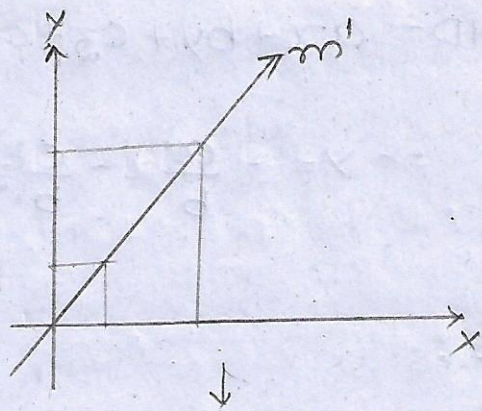
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As  $x \uparrow$ ,  $y \uparrow$



This property is used  
in Linear Regression.



As  $x \uparrow$ ,  $y \uparrow$  rapidly

In the above graphs, the intercept is same  
i.e., constant ( $\neq 0$ ) and the slope is changed.

We know the line eq<sup>n</sup> for 2D is  $y = mx + c$ .

The general form of line eq<sup>n</sup> is

$$ax + by + c = 0 \Rightarrow y = -\frac{a}{b}x - \frac{c}{b}$$

Intercept =  $-c/b$ , slope =  $-a/b$ .

$$\rightarrow 3D - ax + by + cz + d = 0$$

$$\Rightarrow y = \underbrace{\left(-\frac{a}{b}\right)}_{\substack{\downarrow \\ \text{slope} \\ \text{with 'x' axis}}} x - \underbrace{\left(\frac{c}{b}\right)}_{\substack{\downarrow \\ \text{slope with 'z' axis}}} z - \underbrace{\left(\frac{d}{b}\right)}_{\substack{\downarrow \\ \text{Intercept with} \\ \text{'y' axis}}}$$



$$\textcircled{6} \rightarrow 4D - ax + by + cz + dw + e = 0$$

$$\Rightarrow y = -\frac{a}{b}x - \frac{c}{b}z - \frac{d}{b}w - \frac{e}{b}$$

NOTE:

If 'n' dimensions hyperplanes are present, then we get 'n-1' slopes and <sup>will have</sup> exactly one intercept.

HYPERPLANE:

It is a subspace whose dimension is one less than that of its ambient space.

↳ If there are more number of dimensions<sup>(d)</sup>

then  $x_1, x_2, \dots, x_d$ ; then the

coefficients  $\rightarrow w_1, w_2, \dots, w_d$

$$\Rightarrow 2D \rightarrow w_1x_1 + w_2x_2 + w_0 = 0$$

where  $w_0$  is the final term and it is constant.



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$$\Rightarrow 3D - w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0.$$

$$\Rightarrow 4D - w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + w_0 = 0$$

For the above equations, the dot product is written as

$$w = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow w_1 x_1 + w_2 x_2 + w_0 = 0 \rightarrow [w_1 \ w_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_0 = 0$$

$$\Pi_2 \Rightarrow w^T \cdot x = 0.$$

where  $\Pi$  represents hyper plane

2 represents the dimensions.

$$w = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\Rightarrow w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0.$$

$$\Rightarrow [w_0 \ w_1 \ w_2 \ w_3] \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Pi_3 \Rightarrow w^T \cdot x = 0$$