CMPT 440 – Spring 2019: Quantum Finite Automata

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Introduction

Quantum finite automata is the combination between quantum mechanics and finite automata. It was first introduced by Moor and Crutchfield [1] and Kondacs and Watrous [2]. There are two different types of quantum finite automata 1-way and 2-way QFAs. 1-way QFAs recognize all languages as regular languages and tend to have less states than normal DFAs. 2-Way QFAs recognize non-regular languages and also tend to have less states than normal DFAs. It can be described as:

$T = (M = (Q, \Sigma, \delta, q_0, q_{acc}, q_{rej}))$.

Q is a finite set of states, Σ is an input alphabet, δ is transition function, q_0 is starting state,q_{acc} Q is accepting state, q_{rej} is reject state.[4]

Quantum finite automata is used as a model of quantum computing. Quantum computing in general has the ability to drastically reduce the space taken by normal finite automata.

An Example

Here is an example of a 1-way QFA.

$$V_a(|q_0>) = \frac{1}{2}|q_0) + \frac{1}{2}|q_1) + \frac{1}{\sqrt{2}}|q_{rej}), \tag{1}$$

$$V_a(|q_1>) = \frac{1}{2}|q_0) + \frac{1}{2}|q_1) - \frac{1}{\sqrt{2}}|q_{rej}), \tag{2}$$

$$V_\$(|q_0>) = |q_{rej}), V_\$(|q_1>) = |q_{acc}) \tag{3}$$

This QFA starts at state space is Q=$q_0, q_1, q_{acc}, q_{rej}$ the accepting states $Q_{acc} = q_{acc}$ and the rejecting states are $Q_{rej} = q_{rej}$[3]. Since QFAs are able to read any regular languages this allows any language to be the input.

References

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[1] C. Moore and J. P. Crutcheld. Quantum automata and quantum grammars. Theoretical Computer Science, 237(1-2):275306, 2000.

[2] A. Kondacs and J. Watrous. On the power of quantum nite state automata. Proceedingsof FOCS97, pages 6675.

[3] Ambainis, Andris Freivalds, Rusins. (1998). 1-way quantum finite automata: Strengths, weaknesses and generalizations. Proc. 39th FOCS.

[4] Nayak, Tanistha, and Tirtharaj Dash. Quantum Finite Automata, Quantum Pushdown Automata Quantum Turing Machine: A Study. vol. 3, Quantum Finite Automata, Quantum Pushdown Automata Quantum Turing Machine: A Study.