

# *Testing a 189 Year Old Theory of Mortality*

Bruce Carnes, Tarynn Witten

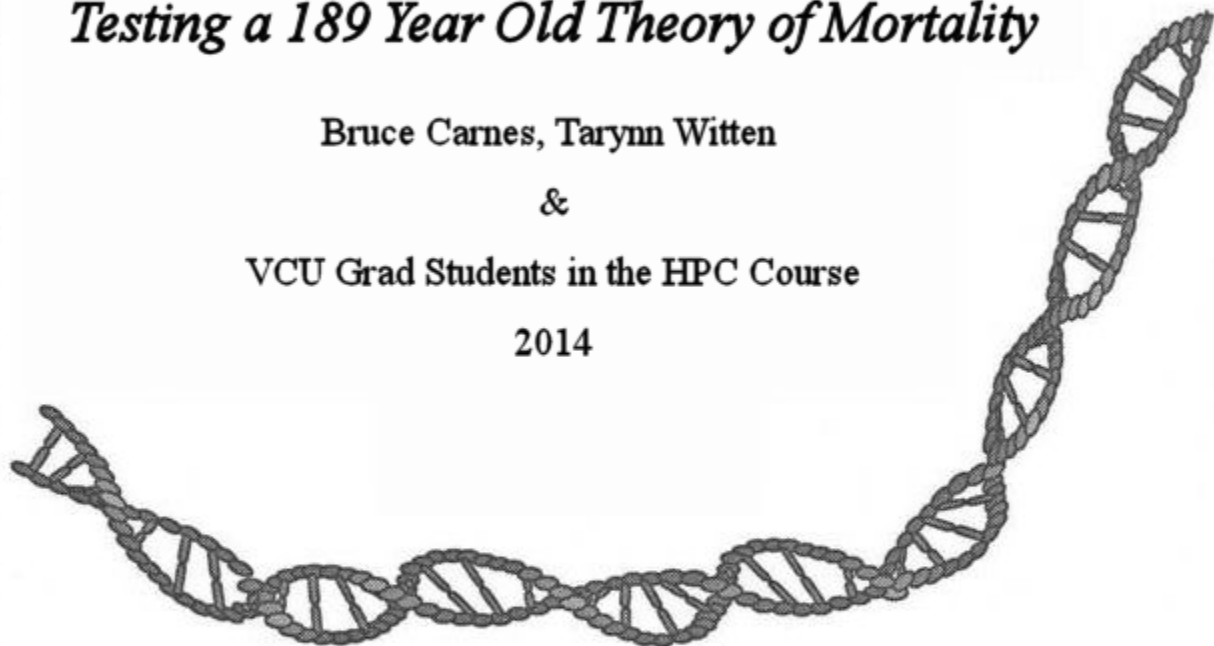
&

VCU Grad Students in the HPC Course

2014

$\hat{\lambda}$

t



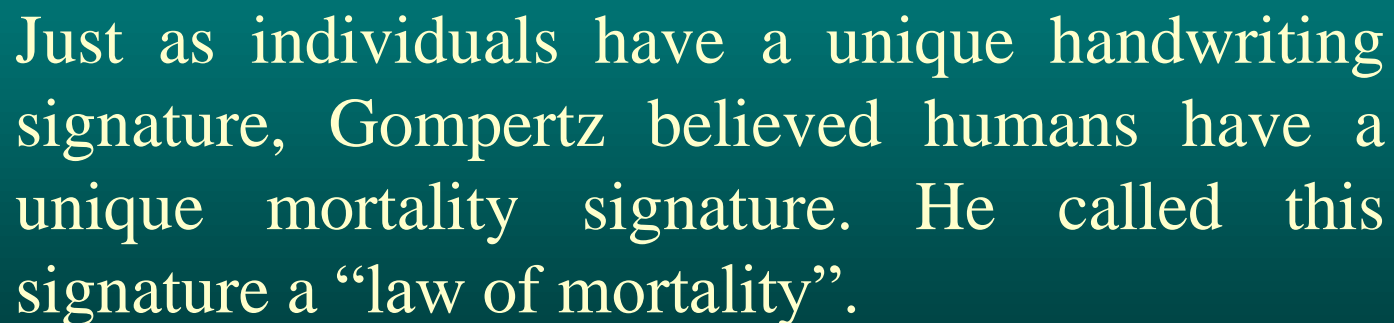
XXIV. *On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of Life Contingencies. In a Letter to FRANCIS BAILY, Esq. F.R.S. &c. By BENJAMIN GOMPERTZ, Esq. F.R.S.*

Read June 16, 1825.



$$\lambda(t) = \alpha \exp(\beta t)$$

1825



opposed such a suspicion to the truth, the following extracts from Mr. Gompertz's writings will testify:—"This equation between the number living and the age," he says in his first paper, "is deserving of attention, because it appears corroborated during a long portion of life by experience; as I derive the same equation from various published tables of mortality, during a long period of man's life, which experience therefore proves that the hypothesis approximates to the law of mortality during the same portion of life; and in fact the hypothesis itself was derived from an analysis of the experience here alluded to." And again, speaking of the

Gompertz  
1825 quote

where  $a + a' + a'' + \dots = A$ , and  $b + b' + b'' + \dots = B$ .

I do not profess to be able to separate the whole category of diseases into the two classes specified—viz., diseases depending for their intensity solely upon the gradual diminution of the vital power, and those which depend upon other causes, the nature of which we do not at present understand. I apprehend that medical science is not sufficiently advanced to render such a desideratum possible of attainment at present. I propose only at present to show that there are certain diseases—and those too of a well-defined and strictly homogeneous character—which follow Mr. Gompertz's law far more closely than the aggregate mortality from all diseases taken together. I shall then have given sufficient reason for the

Makeham's  
1865 quote

## Makeham's Improvement of the Gompertz Law

The two following tables are taken from the supplement before referred to. They give, first, the number of annual deaths (from all causes) to 1,000,000 living; and secondly, the number of annual deaths from certain specified causes to the same number living. The causes of death, as well as the ages for which they are given, have of course been selected as the most favourable exponents of the law of geometrical progression; but it will be observed that the former embrace all the principal vital organs of the body, and the latter include the whole of the period from early manhood to the confines of extreme old age.

*Male Life, 1851-60.*

Ages.	Total Force of Mortality.	PARTIAL FORCES OF MORTALITY.					
		Lungs.	Heart.	Kidneys.	Stomach and Liver.	Brain.	Sum of five preceding Columns.
25-34	9,574	772	514	174	464	638	2,562
35-44	12,481	1,524	1,002	292	890	1,180	4,888
45-54	17,956	3,092	1,898	471	1,664	1,990	9,115
55-64	30,855	6,616	4,130	937	3,032	4,097	18,812
65-74	65,332	13,416	8,714	2,453	4,837	9,831	39,251

Competing Risk Theory is Born (1865)

*Female Life, 1851-60.*

Ages.	Total Force of Mortality.	PARTIAL FORCES OF MORTALITY.					
		Lungs.	Heart.	Kidneys.	Stomach and Liver.	Brain.	Sum of five preceding Columns.
25-34	9,925	582	603	109	570	532	2,395
35-44	12,147	1,049	1,118	151	937	872	4,127
45-54	15,198	2,062	2,064	212	1,608	1,681	7,627
55-64	27,007	5,027	4,558	317	2,967	3,818	16,687
65-74	58,656	11,016	8,916	485	4,692	8,905	34,014

The modification which I have suggested, viz., there are certain partial forces of mortality (how many I do not pretend to say) which increase in intensity with the age in a constant geometrical ratio, while there are also certain other partial forces which do not so increase, may be tested by an examination of the six columns which follow that of the *total* force above referred to. The tendency to a geometrical progression is more or less apparent in all of them; the average rate of increase being such that the force of mortality somewhat more than doubles itself in 10 years.

It should be observed that, in addition to the diseases of the particular organs specified, other diseases of a kindred nature are also included under each of the above five partial forces. Possibly

Recognition of Incomplete Knowledge

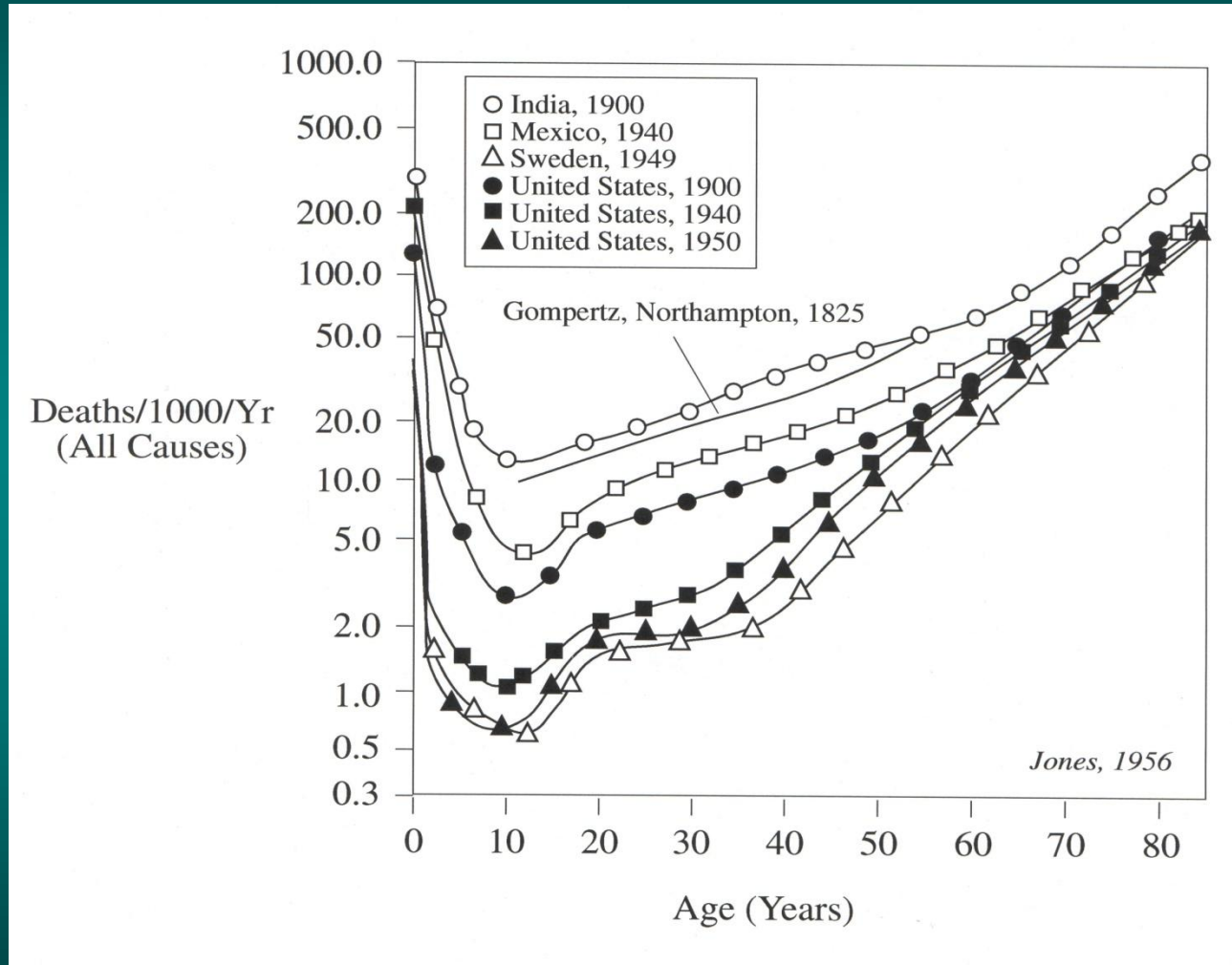


## *Cause-of-Death Matters*

In the preceding examination of the results of the Registrar-General's returns of deaths, I have confined myself to the object of proving that Gompertz's law is traced much more distinctly in the deaths arising from certain specified diseases, than in the deaths arising from all causes together. If I have succeeded in this object (and I think it can scarcely be denied that I *have* succeeded), I have justified the introduction of an additional term in the formula representing the total force of mortality ; but I have as yet brought forward nothing to show that such additional term is a constant in respect of the age, and varying only with the peculiar characteristics which distinguish different sets of observations from each other.

Makeham's binary partition:  
one force follows Gompertz's Law  
and the other is constant mortality  $\neq \text{fn}(\text{age})$

# *All Cause Mortality: Hard to see a Gompertz Law Here*



Hardin Jones. *Advances in Biological and Medical Physics* (1956)



# Constructing a Binary Mortality Partition

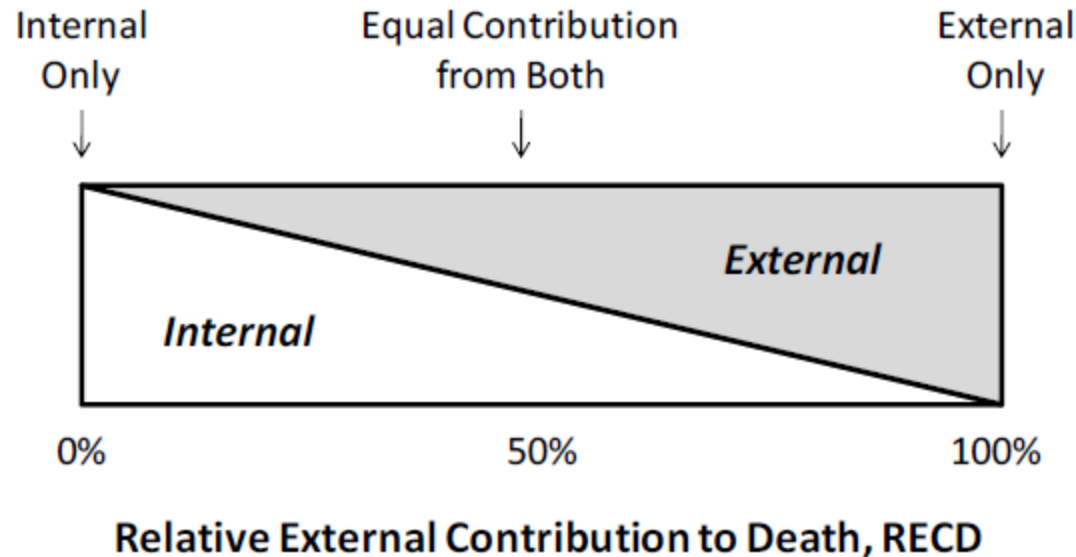
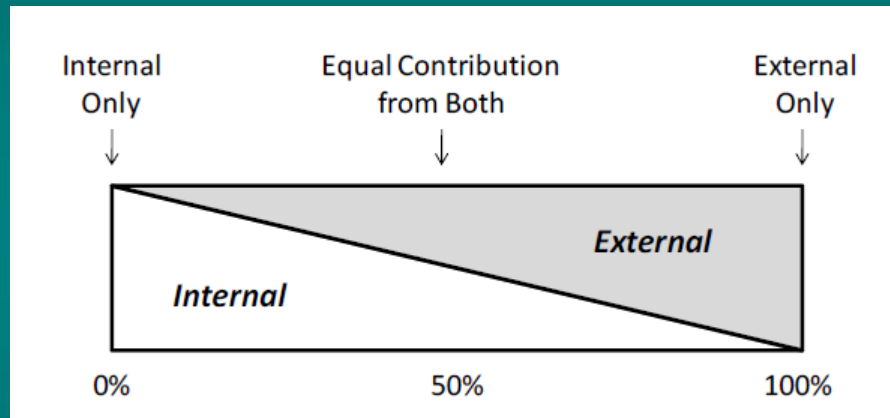


Figure 1. Relative contribution to death scale used by the omniscient Evaluator

- Cause-of-death is poorly understood
- Most people agree on extrinsic deaths
- Intrinsic = not Extrinsic

# *Big Debate*

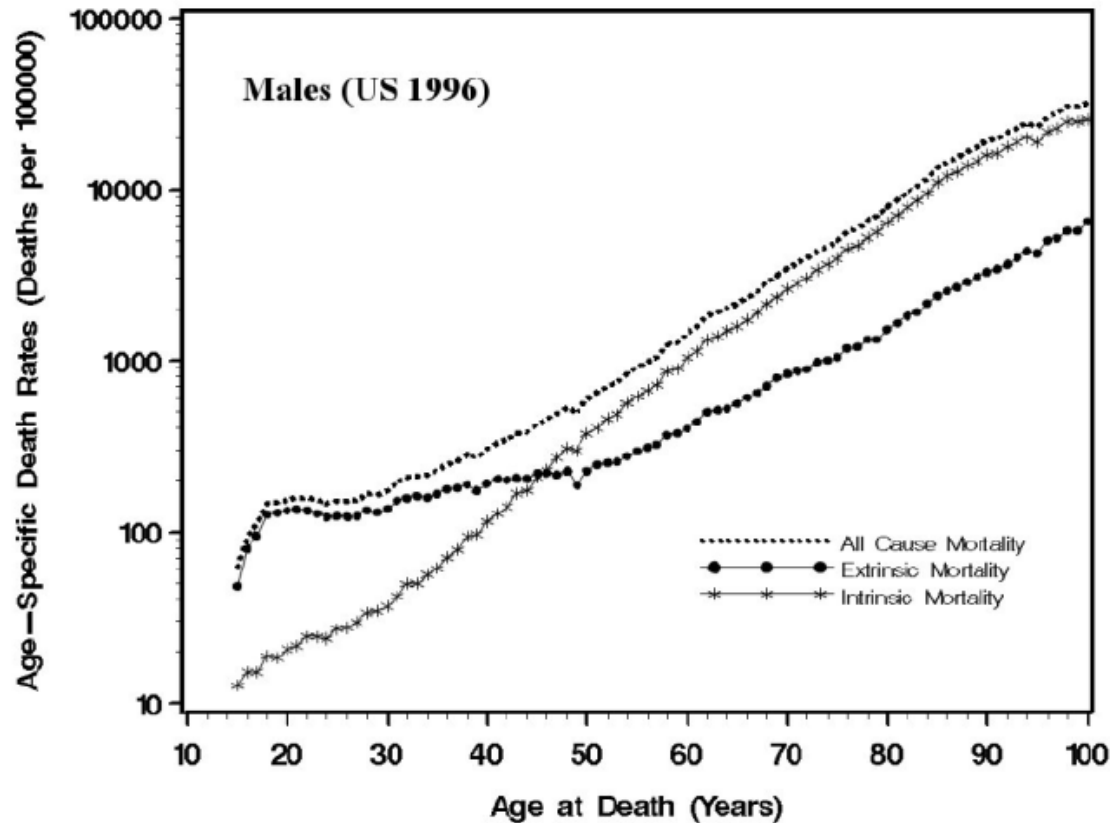
- Binary bins are simplistic and thus unrealistic
- We don't know enough to make assignments
- A Cause can involve both intrinsic & extrinsic



If you agree that accidents, homicide, suicide, infectious and parasitic disease, deaths linked to addiction and poor health decisions are extrinsic, then the rest are likely to be mainly intrinsic.

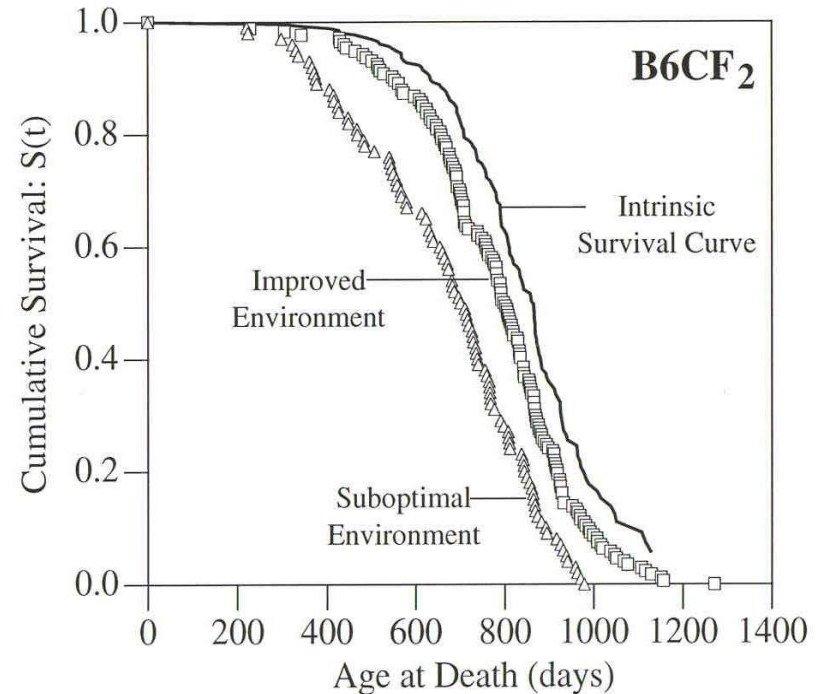
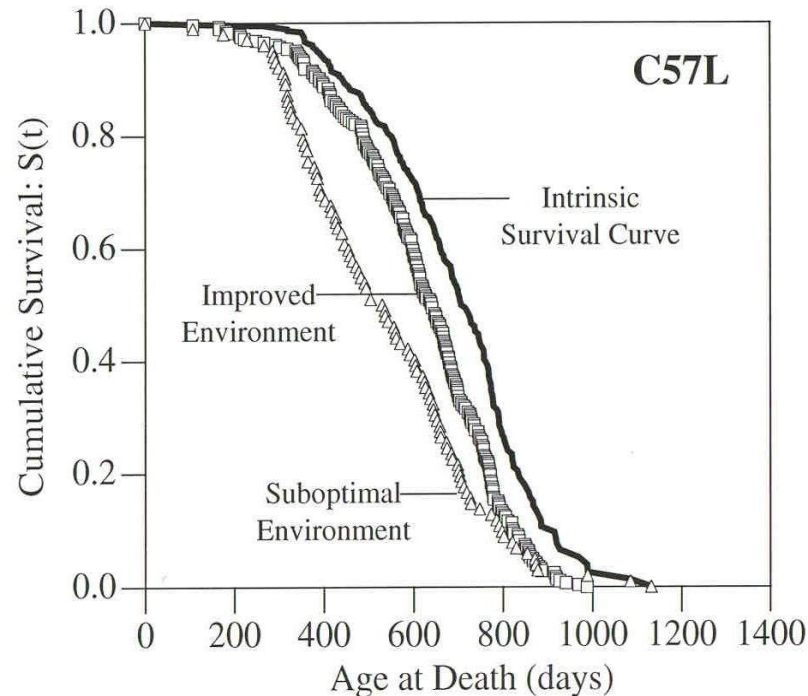
Intrinsic deaths are defined by a process of elimination.

Figure 2B. Age-specific death rates (ages 15 and older) for all-cause mortality and its intrinsic and extrinsic subgroups.



Intrinsic exhibits Gompertzian behavior, extrinsic does not. So what does all-cause mortality predict?

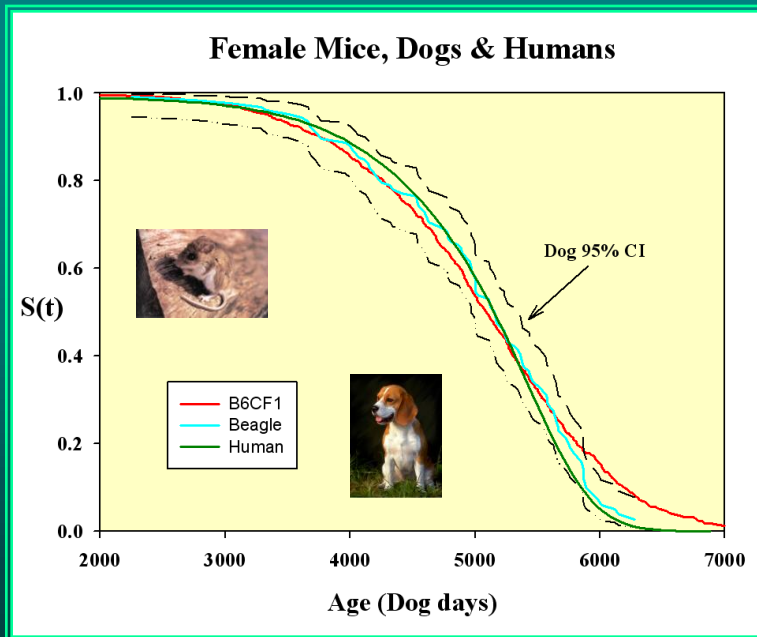
# Tests for an Intrinsic Mortality Signature (Mouse Strains – 1 inbred & 1 hybrid)



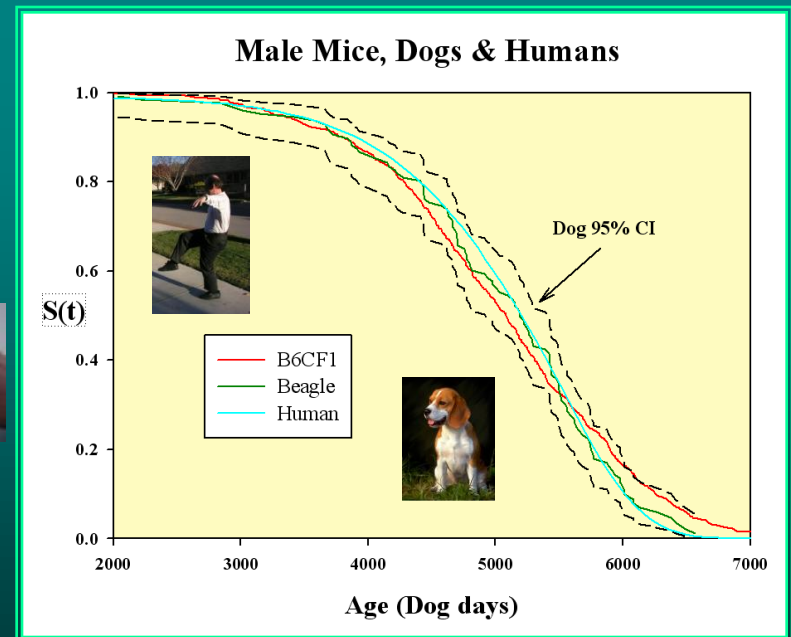
Carnes & Olshansky. 2001. Experimental Gerontology 36: 419-430.

# *Abridged Version of Interspecies Scaling Logic*

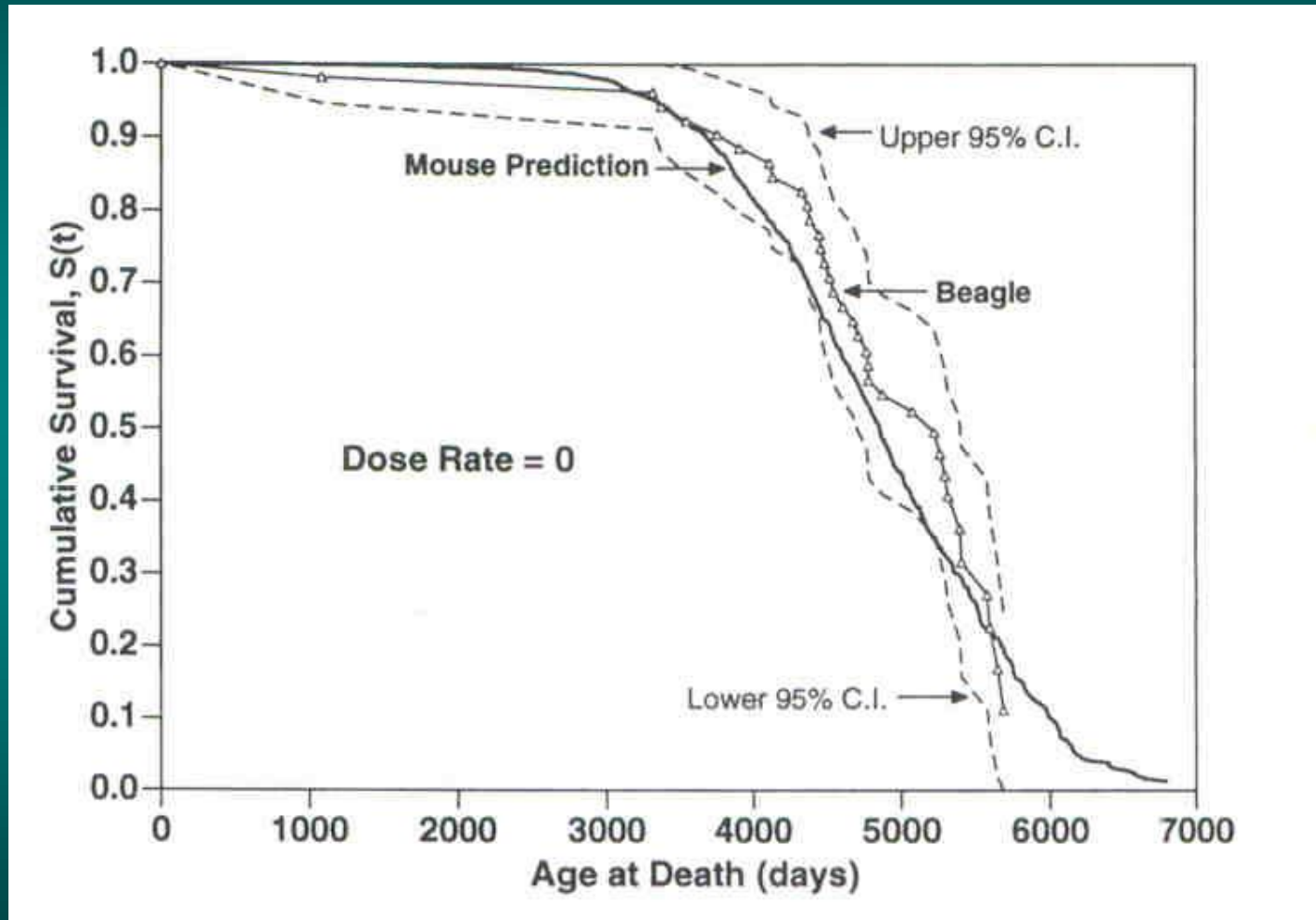
- We measure time on a human time scale
- Mammals share a common life history
- Conception, Growth, Development, Maturation, Reproduction, Nurturing and Grand-parenting
- These milestones occur on a biological time scale
- They also occur in the same order for all species



*stretch  
time*



# Interspecies Comparison of Intrinsic Mortality (Mouse to Dog)



Carnes, Olshansky, Grahn. 1998. Radiation Research 149: 487-492.



# Gompertz Mortality Acceleration

We are interested in how "mortality" changes as a function of age. To examine this behavior, we convert the interval based life table estimates of hazard rates into a parametric equation amenable to the calculus of physics. The equation we use is the traditional Gompertz hazard rate function - a speed or velocity metric:

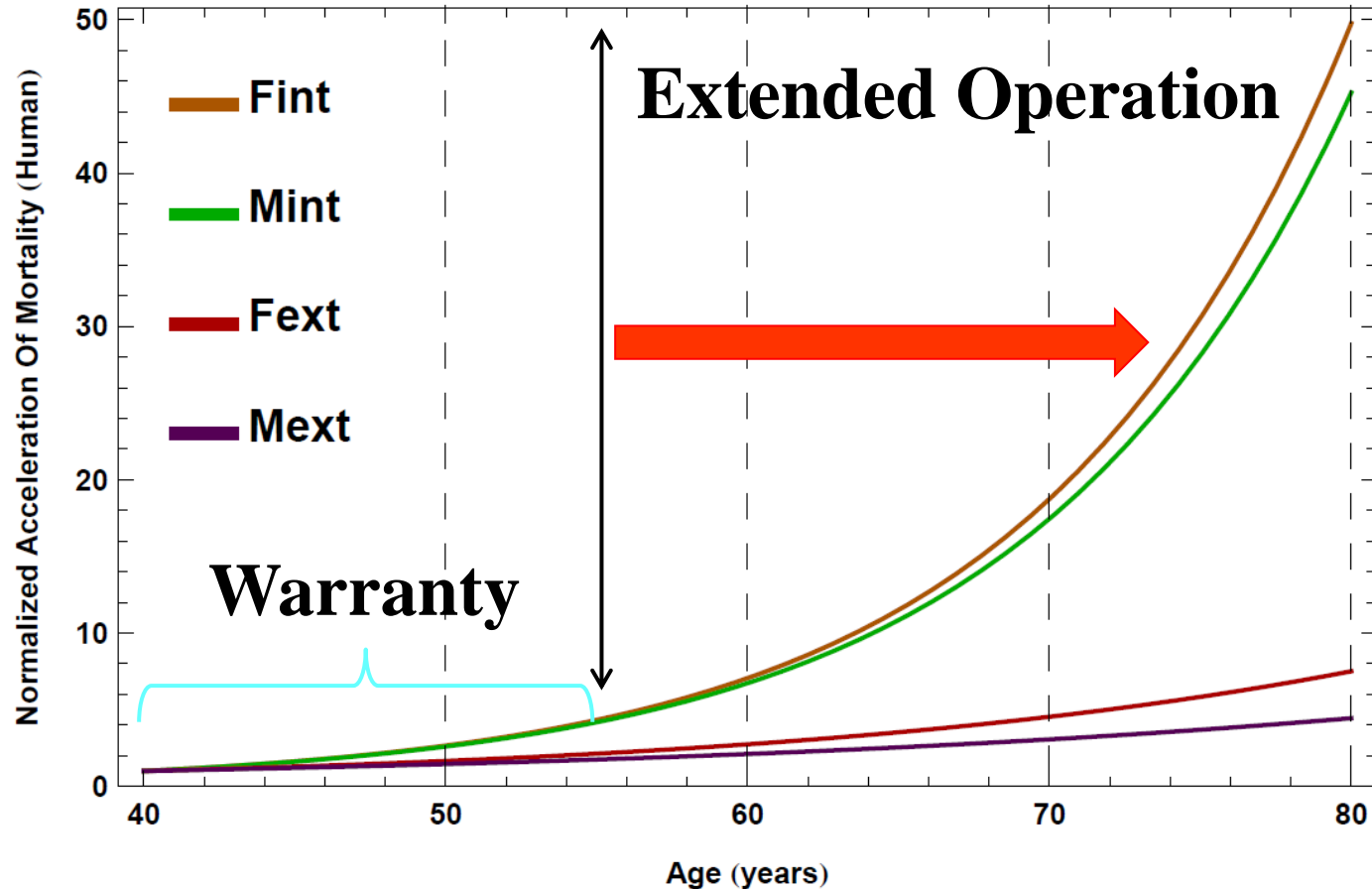
$$\lambda(t) = \alpha e^{\beta t}$$

The first derivative of the Gompertz velocity function produces a new function that measures what physicists call "acceleration" (20):

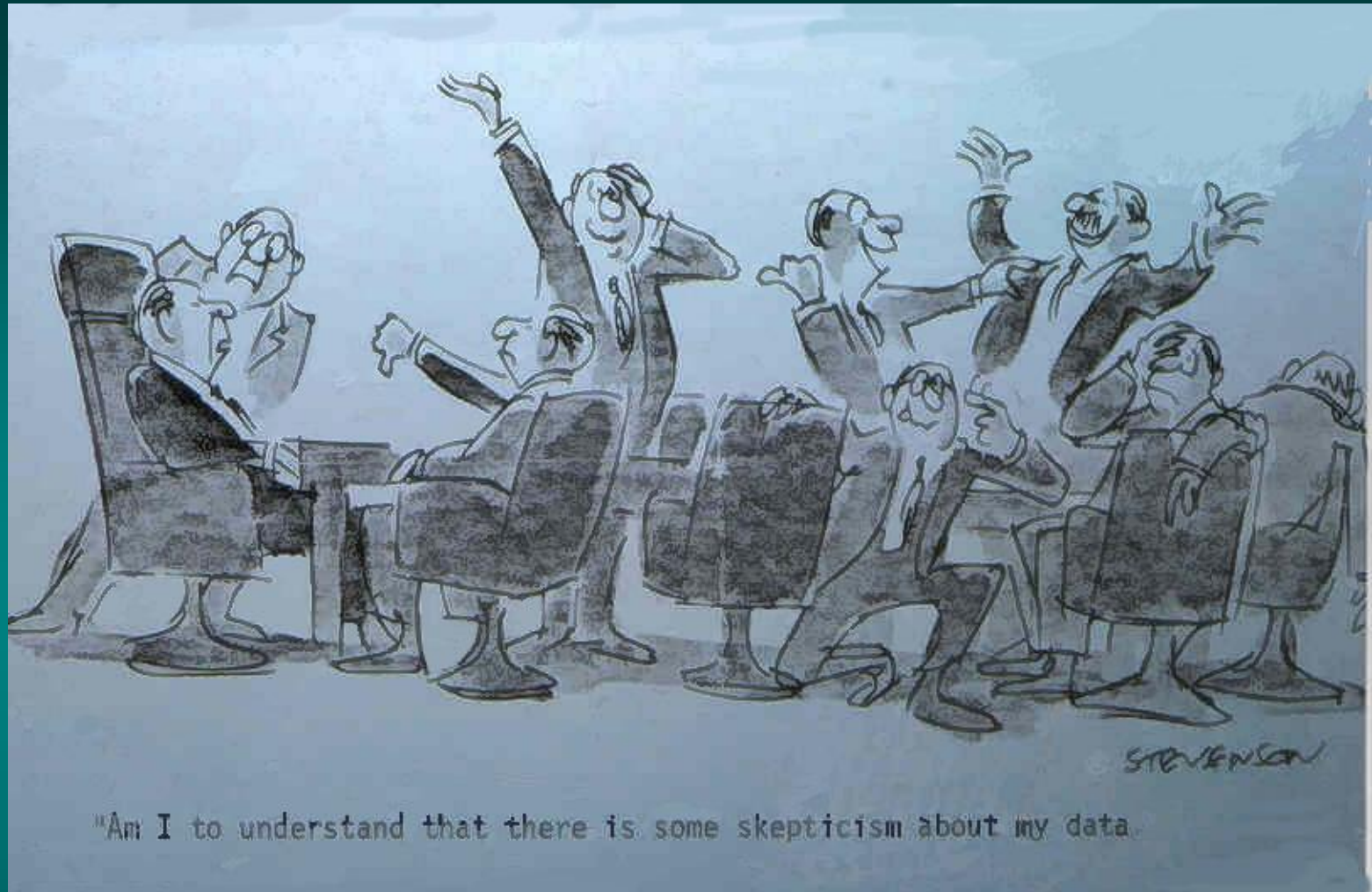
$$\frac{d}{dt} \lambda(t) = \alpha \beta e^{\beta t} = \beta \lambda(t)$$

In other words, the new function describes the "acceleration of mortality" that a Gompertz population experiences as it travels along its dying out trajectory on the age axis - expressed simply as the Gompertz slope ( $\beta$ ) times the hazard rate ( $\lambda(t)$ ).

# *The Transition to Accelerated Mortality*



Evidence in support of Makeham's predictions.



## **The Demographic Response to Biologically Motivated Mortality Partitions!**

# *A Modern Day Test of a 189 Year Old Hypothesis*

Hypothesis: Mortality comparisons using a biologically motivated partition of cause-of-death will reveal more similarity than comparisons based on any other binary partition.

Creating all possible binary mortality partitions was impossible until the advent of HPC.

## *Planning a Test*

### Logic:

- There is no pathology for animals in nature.
- Lab mice would work, but humans are sexier.
- Stronger impact if we use multiple countries.
- Even stronger if we use multiple time points.
- Earlier times have lower life expectancies.

W.H.O. has collected mortality data for decades and we know the W.H.O. director of mortality and morbidity.

# *Calculating Human Mortality Metrics*

Human data with age at death exist but is restricted to small study populations that are subsets and hence not representative of the larger population from which they came. That does not help us.

Instead, we must use life tables where number of deaths are known in intervals of age from 0 to 110. The number living at those ages come from the Census Bureau and is only surveyed once every 10 years and interpolated for intercensal years.

Finally, cause-of-death comes from death certificates which are viewed as barely useful but it is all we have.



# *Current Plan*

W.H.O. Data Used In Project				
Country	W.H.O ID	Life Expectancy	World Rank	Year
South Africa	1430	61	159	2009
Guatemala	2250	71.5	123	2008
Romania	4270	74	101	2010
Brazil	2070	76.5	56	2002
United States	2450	79.8	35	2010
Australia	5020	83	9.5	2010
Sweden	4290	83	9.5	2010

Wanted to compare old & new data but decided to stay within years using most recent ICD-10 coding. Selected countries with population data in the most complete format (Frmat = 0). Countries are geographically distributed with life expectancies ranging from 61 to 83.

# Defining Intrinsic & Extrinsic Mortality

```

081 IF (UCOD >= 'J00' AND UCOD < 'J40') THEN CAUSE = 'PIAD';
082 IF (UCOD >= 'J40' AND UCOD < 'J60') THEN CAUSE = 'PCLD';
083 IF (UCOD >= 'J60' AND UCOD < 'J80') THEN CAUSE = 'EXTL';
084 IF (UCOD >= 'J80' AND UCOD < 'K00') THEN CAUSE = 'POTH';
085 IF (UCOD >= 'K00' AND UCOD < 'L00') THEN CAUSE = 'DIGD';
086 IF (UCOD >= 'L00' AND UCOD < 'M00') THEN CAUSE = 'SKIN';
087 IF (UCOD >= 'M00' AND UCOD < 'N00') THEN CAUSE = 'MUSK';
088 IF (UCOD >= 'N00' AND UCOD < 'O00') THEN CAUSE = 'GNUR';
089 IF (UCOD >= 'O00' AND UCOD < 'Q00') THEN CAUSE = 'CBPN';
090 IF (UCOD >= 'Q00' AND UCOD < 'R00') THEN CAUSE = 'CGEN';
091
092 IF ((UCOD >= 'H00' AND UCOD < 'I00') OR
093     (UCOD >= 'R00' AND UCOD < 'S00') OR
094     (UCOD >= 'Z00' AND UCOD <= 'U99')) THEN CAUSE = 'REST';
095
096 IF (UCOD >= 'S00' AND UCOD < 'Z00') THEN CAUSE = 'EXTL';
097
098 * ICD CODE DEFINITIONS FOR EXTRINSIC & INTRINSIC CAUSES FROM CARNES RESEARCH;
099 EXTRIN = (UCOD <= 'B99' OR UCOD >= 'V01') OR (UCOD >= 'D50' AND UCOD <= 'D539') OR (UCOD >= 'D590' AND UCOD < 'D610')
100          OR (UCOD >= 'E40' AND UCOD <= 'E649') OR (UCOD >= 'F10' AND UCOD <= 'F19') OR (UCOD >= 'G00' AND UCOD <= 'G03')
101          OR (UCOD >= 'I00' AND UCOD <= 'I09') OR (UCOD >= 'J00' AND UCOD <= 'J98') OR (UCOD >= 'K04' AND UCOD <= 'K10')
102          OR (UCOD >= 'K700' AND UCOD <= 'K743') OR (UCOD >= 'L00' AND UCOD <= 'L08') OR (UCOD >= 'L55' AND UCOD <= 'L59')
103          OR (UCOD >= 'M00' AND UCOD <= 'M03') OR (UCOD >= 'N34' AND UCOD <= 'N390') OR (UCOD >= 'N70' AND UCOD <= 'N739')
104          OR (UCOD >= 'O00' AND UCOD <= 'O99');
105
106 INTRIN = (EXTRIN=0);
107

```

As you can see, ICD codes are organized around organ systems. This is what Makeham did; we simply have more information than he had.

International Statistical Classification of Diseases and Related Health Problems 10th Revision

Chapter	Blocks	Title
I	A00–B99	Certain infectious and parasitic diseases
II	C00–D48	Neoplasms
III	D50–D89	Diseases of the blood and blood-forming organs and certain disorders involving the immune mechanism
IV	E00–E90	Endocrine, nutritional and metabolic diseases
V	F00–F99	Mental and behavioural disorders
VI	G00–G99	Diseases of the nervous system
VII	H00–H59	Diseases of the eye and adnexa
VIII	H60–H95	Diseases of the ear and mastoid process
IX	I00–I99	Diseases of the circulatory system
X	J00–J99	Diseases of the respiratory system
XI	K00–K93	Diseases of the digestive system
XII	L00–L99	Diseases of the skin and subcutaneous tissue
XIII	M00–M99	Diseases of the musculoskeletal system and connective tissue
XIV	N00–N99	Diseases of the genitourinary system
XV	O00–O99	Pregnancy, childbirth and the puerperium
XVI	P00–P96	Certain conditions originating in the perinatal period
XVII	Q00–Q99	Congenital malformations, deformations and chromosomal abnormalities
XVIII	R00–R99	Symptoms, signs and abnormal clinical and laboratory findings, not elsewhere classified
XIX	S00–T98	Injury, poisoning and certain other consequences of external causes
XX	V01–Y98	External causes of morbidity and mortality
XXI	Z00–Z99	Factors influencing health status and contact with health services

# *Constructing a Life Table*

All life tables begin with two simple pieces of information.

$D_i$  = number of deaths in the interval derived from a country's death index.

$K_i$  = census estimate of the midyear population in age interval  $i$ , where  $i$  is typically a 1 year or 5 year age interval.

These are used to create the following metric

$$M_i = D_i / K_i = \text{Age Specific Rate}$$

## *Note*

Non-demographers will say  $M_i$  is bogus because a death rate is deaths per unit of time and population size is not a unit of time.

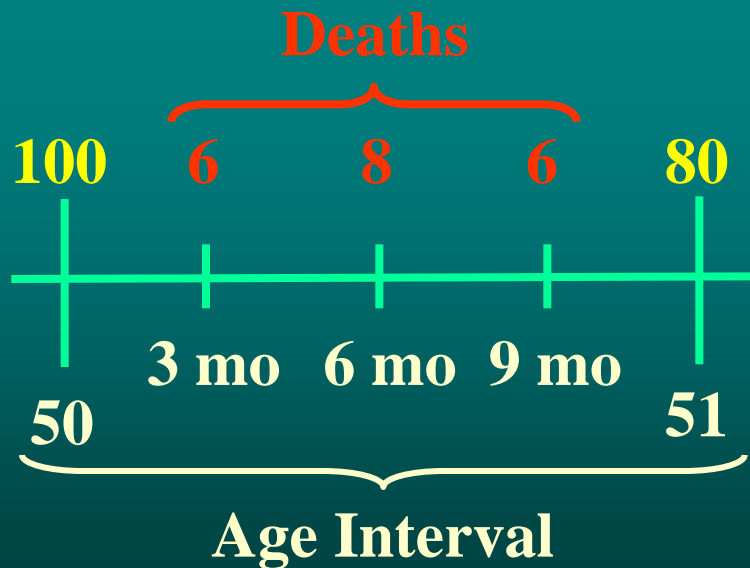
As will be seen,  
demographers have found  
a way around this problem.

# Anatomy of a Death Rate

$$m(t) = \frac{d}{\underbrace{\text{PYR}}_{\text{Person Years at Risk}}}$$

← NCHS (CDC)  
← Census Bureau

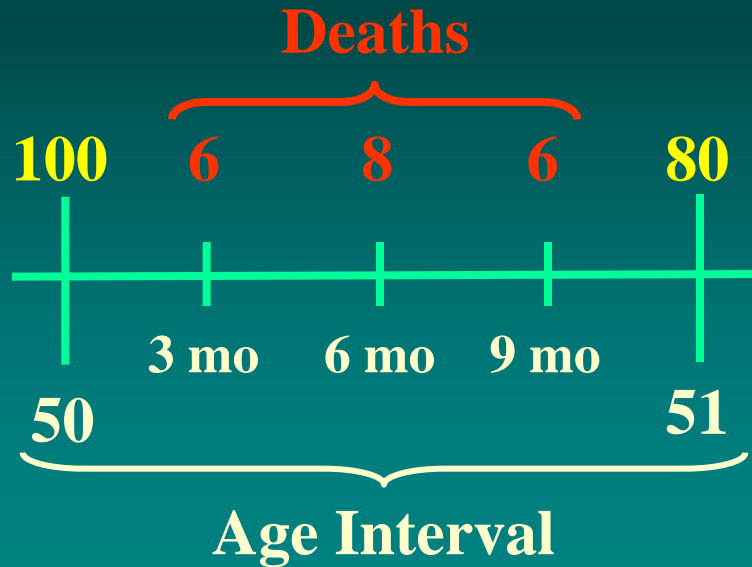
Person Years  
at  
Risk



Time	PYR
3 mo	18 mo
6 mo	48 mo
9 mo	54 mo
1 yr	80 yr
<b>Total</b>	<b>90 PYR</b>

$$m(50) = 20/90 = 0.2222$$

# Tricky Actuaries



## Assumption

Deaths are distributed  
uniformly or  
symmetrically  
across the interval.

- If true, 90 people would be alive at the middle of our interval.
- Our detailed calculation led to  $PYR = 90$ .

- • Mid-interval “population size” is a surrogate for “person years”!  

90 People

90 PYR

$$m(t) = \frac{d}{\text{Pop(July 1)}}$$

← NCHS (CDC)

← Census Bureau



We know:

$$q_i = \frac{d_i}{l_i} \quad \therefore d_i = q_i l_i \quad (5)$$

Therefore,

$$\begin{aligned} m_i &= \frac{q_i l_i}{w_i l_i - w_i (1 - f_i) q_i l_i} \\ &= \frac{q_i}{w_i - w_i (1 - f_i) q_i} \quad (6) \\ &= \frac{q_i}{w_i (1 - (1 - f_i) q_i)} \end{aligned}$$

and solving for  $q_i$ :

$$m_i w_i (1 - (1 - f_i) q_i) = q_i$$

$$q_i = m_i w_i - m_i w_i (1 - f_i) q_i$$

$$q_i + m_i w_i (1 - f_i) q_i = m_i w_i$$

$$q_i (1 + m_i w_i (1 - f_i)) = m_i w_i$$


$$q_i = \frac{w_i m_i}{1 + w_i (1 - f_i) m_i} \quad \left. \begin{array}{l} \text{Chiang, pg 73} \\ \text{J45, pg 299} \end{array} \right\} \quad (7)$$

$q_i$

All-Cause Mortality

the Basic Building  
Block  
of a  
Life Table

# Simple All-Cause Mortality Table

		Country	Year	Sex	AGE	W	DTHS	POP	Mtot	Qtot
1		4050	2006	1	0	1	125	33009	0.00378685	0.00377969
2		4050	2006	1	1	1	13	33220	0.00039133	0.00039125
3		4050	2006	1	2	1	4	33563	0.00011918	0.00011917
4		4050	2006	1	3	1	4	32900	0.00012158	0.00012157
5		4050	2006	1	4	1	3	33015	9.08678e-5	9.08637e-5
6		4050	2006	1	7	5	19	173340	0.00010961	0.00054791
7		4050	2006	1	12	5	24	180371	0.00013306	0.00066507
8		4050	2006	1	17	5	75	163583	0.00045848	0.00228979
9		4050	2006	1	22	5	105	148470	0.00070721	0.00352983
10		4050	2006	1	27	5	104	164047	0.00063396	0.00316481
11		4050	2006	1	32	5	164	191861	0.00085479	0.00426481
12		4050	2006	1	37	5	256	200952	0.00127394	0.00634946
13		4050	2006	1	42	5	438	212206	0.00206403	0.01026718
14		4050	2006	1	47	5	663	188678	0.00351392	0.01741661
15		4050	2006	1	52	5	1097	180980	0.00606144	0.02985481
16		4050	2006	1	57	5	1603	185992	0.00861865	0.04218432
17		4050	2006	1	62	5	2381	176235	0.01351037	0.06534477
18		4050	2006	1	67	5	2476	121592	0.02036318	0.09688376
19		4050	2006	1	72	5	3210	91237	0.03518309	0.1616933
20		4050	2006	1	77	5	4020	68411	0.05876248	0.25617823
21		4050	2006	1	82	5	4354	45495	0.09570282	0.38612983
22		4050	2006	1	87	5	3462	22402	0.15453977	0.55736227
23		4050	2006	1	92	5	1853	7260	0.25523416	0.77906243
24		4050	2006	1	97	5	560	1360	0.41176471	1.01449275
25		4050	2006	2	0	1	96	31444	0.00305305	0.00304839
26		4050	2006	2	1	1	4	31768	0.00012591	0.0001259

# Creating a Multiple Decrement Life Table (intrinsic & extrinsic $q(x)$ )

**Life Table**

AGE		Conditional Probability of Death
1	$q(x_1)$	$q(x_1) = q(x_{11}) + q(x_{12}) + \dots + q(x_{1N})$
2	$q(x_2)$	$q(x_2) = q(x_{21}) + q(x_{22}) + \dots + q(x_{2N})$
.		
.		
i	$q(x_i)$	$q(x_i) = q(x_{i1}) + q(x_{i2}) + \dots + q(x_{iN})$

$$q(x) = q(x_1) + q(x_2) + \dots + q(x_N) \quad [\text{All Cause} = \text{Sum of Specific Causes}]$$

$$q(x) = \sum q(x_I) + \sum q(x_E) \quad [\text{Collapsing Causes into Intrinsic \& Extrinsic}]$$

## References for Mortality Partitioning:

Carnes & Olshansky. 1997. A biologically motivated partitioning of mortality. *Experimental Gerontology* 32(6): 615-631.

Carnes, Holden, Olshansky, Witten and Siegel. 2006. Mortality partitions and their relevance to research on senescence. *Biogerontology* 7: 183-198.

$$q(x) = q(x_1) + q(x_2) + \dots + q(x_N) \quad [All\ Cause = Sum\ of\ Specific\ Causes]$$

$$q(x) = \sum q(x_I) + \sum q(x_E) \quad [Collapsing\ Causes\ into\ Intrinsic\ \&\ Extrinsic]$$

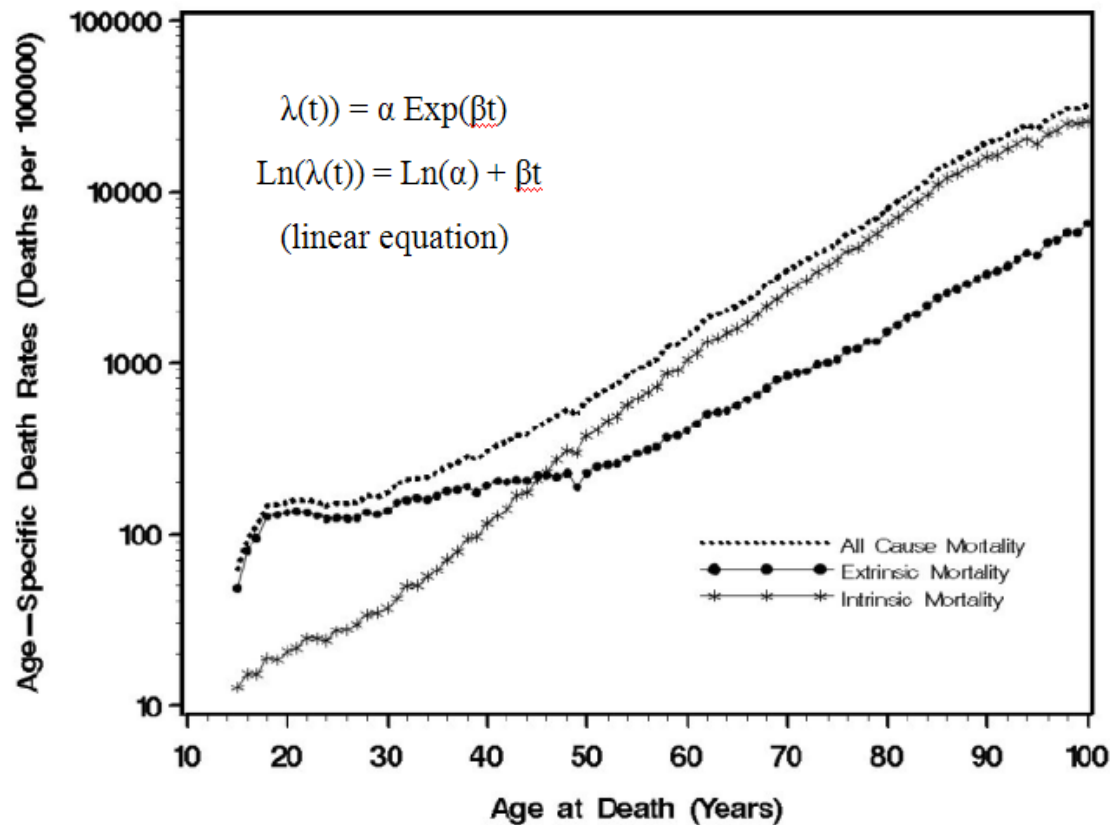
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098 * ICD CODE DEFINITIONS FOR EXTRINSIC & INTRINSIC CAUSES FROM CARNES RESEARCH;
099   EXTRIN = (UCOD <= 'B99' OR UCOD >= 'V01') OR (UCOD >= 'D50' AND UCOD <= 'D539') OR (UCOD >= 'D590' AND UCOD < 'D610')
100           OR (UCOD >= 'E40' AND UCOD <= 'E649') OR (UCOD >= 'F10' AND UCOD <= 'F19') OR (UCOD >= 'G00' AND UCOD <= 'G03')
101           OR (UCOD >= 'I00' AND UCOD <= 'I09') OR (UCOD >= 'J00' AND UCOD <= 'J98') OR (UCOD >= 'K04' AND UCOD <= 'K10')
102           OR (UCOD >= 'K700' AND UCOD <= 'K743') OR (UCOD >= 'L00' AND UCOD <= 'L08') OR (UCOD >= 'L55' AND UCOD <= 'L59')
103           OR (UCOD >= 'M00' AND UCOD <= 'M03') OR (UCOD >= 'N34' AND UCOD <= 'N390') OR (UCOD >= 'N70' AND UCOD <= 'N739')
104           OR (UCOD >= 'O00' AND UCOD <= 'O99');
105
106   INTRIN = (EXTRIN=0);
107

```

	Radix	QEXTRIN	QINTRIN	EDth	IDth
1	100000	0.00015119	0.0036285	15.1187578	362.850188
2	99622.0311	0.00009029	0.00030096	8.99481116	29.9827039
3	99583.0535	0	0.00011917	0	11.8674874
4	99571.1861	0	0.00012157	0	12.1051834
5	99559.0809	3.02879e-5	6.05758e-5	3.01543413	6.03086825
6	99550.0346	0.00014419	0.00040372	14.3536925	40.1903391
7	99495.4905	0.0002494	0.00041567	24.8144558	41.3574263
8	99429.3187	0.00158759	0.0007022	157.852744	69.819483
9	99201.6464	0.00268939	0.00084044	266.792117	83.3725366
10	98851.4818	0.00185628	0.00130853	183.49615	129.349745
11	98538.6359	0.00273052	0.00153429	269.061813	151.187114
12	98118.3869	0.0029019	0.00344756	284.729832	338.268775

Figure 2B. Age-specific death rates (ages 15 and older) for all-cause mortality and its intrinsic and extrinsic subgroups.



Intrinsic exhibits Gompertzian (i.e., linear) behavior, extrinsic and all-cause do not.

# All Possible Partitions

**Life Table**

AGE		Conditional Probability of Death
1	$q(x_1)$	$q(x_1) = q(x_{11}) + q(x_{12}) + \dots + q(x_{1N})$
2	$q(x_2)$	$q(x_2) = q(x_{21}) + q(x_{22}) + \dots + q(x_{2N})$
.		
.		
i	$q(x_i)$	$q(x_i) = q(x_{i1}) + q(x_{i2}) + \dots + q(x_{iN})$

$$q(x) = q(x_1) + q(x_2) + \dots + q(x_N) \quad [\text{Sum of Specific Causes}]$$

$$q(x) = \sum q(x_I) + \sum q(x_E) \quad [\text{Intrinsic \& Extrinsic Partitions}]$$

$$q(x) = \sum q(x_A) + \sum q(x_B) \quad [\text{Combinatoric Partitions}]$$

## HPC Challenge:

There are 35 death codes that capture every ICD code that exists. I want HPC to create every possible "unique" binary partition of these 35 codes. In other words, order within a partition does not matter. AB and C or BA and C or CA and B are considered duplicates.



# *Comparing Mortality Schedules*

- Functions of  $q(x)$  can be used to estimate density function, hazard function and survivorship curves.
- The hazard function and survivorship function are far and away the most common way to compare mortality schedules.
- However, a much simpler way exists to accomplish the same goal – compare mortality schedules.

$q(x)$  is used to estimate deaths at each age

Demographers create a hypothetical population called a Radix (typically) 100,000 at age 0. This enables them to report on deaths per 100,000 as a convenient way to compare different causes of death.

# Multiple Decrement Life Table

AGE	Radix	QEXTRIN	EDth	QINTRIN	IDth	Dth
0	100000	0.00015119	15.1187578	0.0036285	362.850188	377.968946
1	99622.0311	0.00009029	8.99481116	0.00030096	29.9827039	38.977515
2	99583.0535	0	0	0.00011917	11.8674874	11.8674874
3	99571.1861	0	0	0.00012157	12.1051834	12.1051834
4	99559.0809	3.02879e-5	3.01543413	6.05758e-5	6.03086825	9.04630238
7	99550.0346	0.00014419	14.3536925	0.00040372	40.1903391	54.5440316
12	99495.4905	0.0002494	24.8144558	0.00041567	41.3574263	66.1718821
17	99429.3187	0.00158759	157.852744	0.0007022	69.819483	227.672227
22	99201.6464	0.00268939	266.792117	0.00084044	83.3725366	350.164654
27	98851.4818	0.00185628	183.49615	0.00130853	129.349745	312.845895
32	98538.6359	0.00273052	269.061813	0.00153429	151.187114	420.248927
37	98118.3869	0.0029019	284.729832	0.00344756	338.268775	622.998608
42	97495.3883	0.00398498	388.517073	0.0062822	612.485738	1001.00281
47	96494.3855	0.00512253	494.295735	0.01229408	1186.30976	1680.6055
52	94813.78	0.00683095	647.668598	0.02302385	2182.97862	2830.64722
57	91983.1328	0.00631581	580.947631	0.03586852	3299.29842	3880.24605
62	88102.8868	0.00798628	703.614135	0.05735849	5053.4486	5757.06273
67	82345.824	0.00829538	683.089742	0.08858838	7294.88291	7977.97265
72	74367.8514	0.01037658	771.683896	0.15131672	11253.0991	12024.783
77	62343.0683	0.01357362	846.221279	0.24260461	15124.7155	15970.9368
82	46372.1315	0.02181625	1011.66587	0.36431359	16893.9975	17905.6634
87	28466.4681	0.03074991	875.341374	0.52661236	14990.7939	15866.1353
92	12600.3328	0.06180366	778.746659	0.71725878	9037.69932	9816.44598
97	2783.88686	0.09963768	277.380032	0.91485507	2546.85302	2824.23305

You can see the Radix diminish at each age as Death,  $q(x)$ , takes it's toll.

In the end, you have 2 columns of death, Intrinsic & Extrinsic in this case.

Those columns will be used to compare mortality between our 7 countries.

# What Are We Comparing?

For each binary partition created, each of the 7 countries will have 2 partitions. We also have our invariant “intrinsic” partition. So, 15 death columns all told.

We have three things to check out:

- Will partitions emerge that create similarity between the 7 countries?
- Will that similarity be superior to our intrinsic base line that defines optimum similarity?
- If so, can the ICD codes responsible for the improvement be identified?

Output from 1 Run of the HPC Program									
AGE	C1		.....	C7			Intrinsic		
	P1	P2		P1	P2		C1	.....	C7
0	d011	d012		d031	d032		d01		d07
1	d111	d112		d131	d132		d11		d17
2	d211	d212		d231	d232		d21		d27
3	d311	d312		d331	d332		d31		d37
4	d411	d412		d431	d432		d41		d47
.							.		.
i	di11	di12		di71	di72		di1		di7

**Output:** age-specific deaths for 7 countries, 2 mortality partitions per country plus the age-specific intrinsic deaths used as a baseline for comparison.

**1<sup>st</sup> Test:** Compare P1 to P2 within each country and compute similarity metric.

Save the best similarity Chi-square.

**2<sup>nd</sup> Test:** Compare P1 deaths across countries and compute similarity metric.

**3<sup>rd</sup> Test:** Compare P2 deaths across countries and compute similarity metric.

Save the best similarity Chi-square from Test 2 & 3 combined.

**3<sup>rd</sup> Test:** Compare Intrinsic partition across countries and compute similarity metric.

Store the 3 Chi-square statistics, their probability and the rank of the 3 metrics: W = within, A = across and I = intrinsic result where Rank = 1 for the smallest Chi-square and so on.

Fisher's Exact Test or something similar will be used to generate our similarity metric which is expressed as a Chi-square and the probability of it being exceeded. Identical death distributions produce a Chi-square = 0. Thus, our expectation/hypothesis is that Intrinsic Chi-square will be smaller than those derived from other binary partitions (i.e., our non-theoretical HPC generated partitions).