# CMSC 409: Artificial Intelligence

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Fall 2015,
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# *CMSC 409: Artificial Intelligence*

## Session # 11

## Topics for today

- Announcements
- Previous session review
- Classification as competitive learning
  - *Kohonen Networks*
  - *Derivation*
  - *Steps*
  - *Example*
  - *Problems & remedies*

# CMSC 409: *Artificial Intelligence*

## Session # 11

- Blackboard
    - Slides, class paper instructions and template uploaded

- Project #2
    - Deadline Oct. 9, 2015
- Paper
    - *The second draft - due* Oct. 19, 2015
    - *Literature review and updated problem description (check out the class paper instructions for the 2nd draft)*
- Subject line and signature
    - *Please use specified in syllabus*

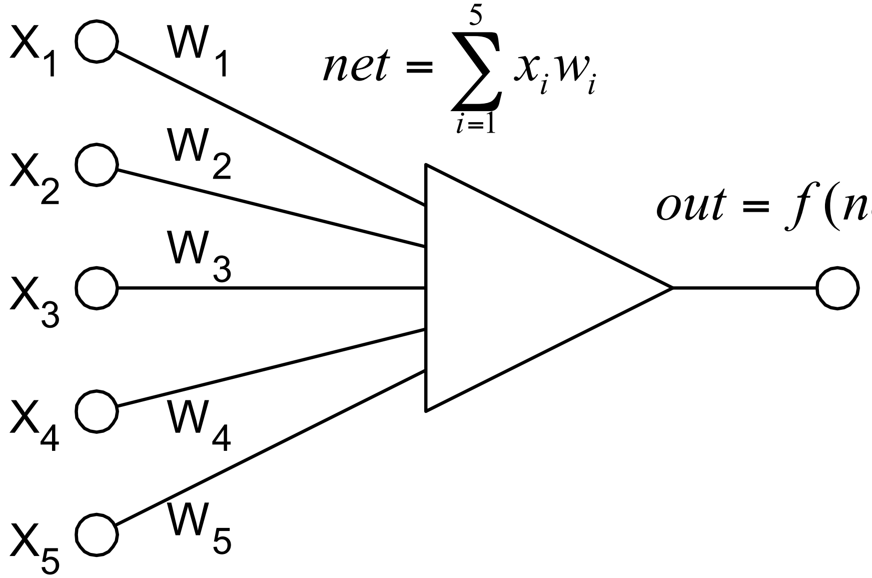
# Classification

# Classification as competitive learning

- *Kohonen Networks*
- *Derivation*
- *Steps*
- *Example*
- *Problems & remedies*

Kohonen, T. (1988) Self-Organization and Associative Memory, 2nd Ed. New York, Springer-Verlag.
Kohonen, T. (1982) Self-organized formation of topologically correct feature maps. Biological Cybernetics, 43:59-69.
Kohonen, T. (1990) The Self-Organizing Map. Proceedings of the IEEE, 78:1464-1480.
Kohonen, T. (1995) Self-Organizing Maps. Springer, Berlin.
Kohonen, T., Oja, E., Simula, O., Visa, A., and Kangas, J. (1996). Engineering applications of the self-organizing map. Proceedings of the IEEE, 84:1358-1384.

# Kohonen Network




$$net = \sum_{i=1}^{5} x_i w_i$$

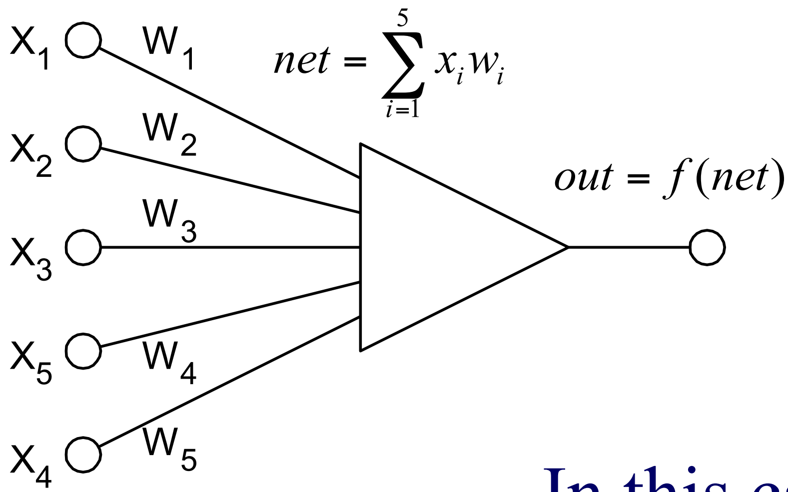
$$net = \sum_{i=1}^{5} x_i w_i = \mathbf{X}\mathbf{W}^T$$

Remember:

*If inputs are binaries, for example* ***X****=[1, -1, 1, -1, -1] then the maximum net value is when weights are identical to the input pattern:*

***W****=[1, -1, 1, -1, -1]*

# Kohonen Network




Also….

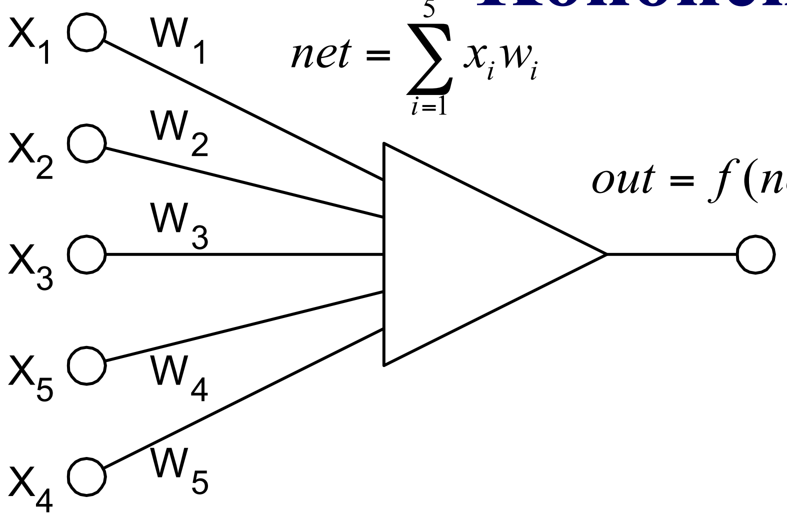
In this case *net* = 5.

For binary weights and patterns *net* value can be found using equation:

$$net = \sum_{i=1}^{n} x_i w_i = \mathbf{X}\mathbf{W}^T = n - 2HD$$

where *n* is the number of inputs and *HD* is the Hamming distance between input vector **X** and weight vector **W**.

# Kohonen Network




$$net = \sum_{i=1}^{5} x_i w_i$$

$$out = f(net)$$

*This concept can be extended to weights and patterns with analog values as long as both lengths of the weight vector and input pattern vectors are the same.*

The Euclidean distance between weight vector **W** and input vector **X** is:

$$\|\mathbf{W}-\mathbf{X}\| = \sqrt{(w_1 - x_1)^2 + (w_2 - x_2)^2 + \cdots + (w_n - x_n)^2}$$

Also can be written as:

$$\|\mathbf{W}-\mathbf{X}\| = \sqrt{\sum_{i=1}^{n}(w_i - x_i)^2} = \sqrt{\sum_{i=1}^{n}(w_i w_i - 2w_i x_i + x_i x_i)}$$

$$\|\mathbf{W}-\mathbf{X}\| = \sqrt{\mathbf{W}\mathbf{W}^T - 2\mathbf{W}\mathbf{X}^T + \mathbf{X}\mathbf{X}^T}$$ *(matrix form)*

# Kohonen Network

$$\|\mathbf{W}-\mathbf{X}\| = \sqrt{\mathbf{W}\mathbf{W}^T - 2\mathbf{W}\mathbf{X}^T + \mathbf{X}\mathbf{X}^T}$$

Now, if the lengths of both the weight and input vectors are normalized to value of one:

$$\|\mathbf{X}\| = 1 \qquad \text{and} \qquad \|\mathbf{W}\| = 1$$

then the equation simplifies to:

$$\boxed{\|\mathbf{W}-\mathbf{X}\| = \sqrt{2 - 2\mathbf{W}\mathbf{X}^T}}$$

NOTE: the maximum value of net value (net=1), is when **W** and **X** are identical…

# Kohonen Networks

- *Derivation*
- *Steps*
- *Example*
- *Problems & remedies*

# Kohonen Network

## *(unsupervised training process)*

1. All patterns are normalized (the lengths of the pattern vectors are normalized to unity).

2. Weights are chosen randomly for all neurons

$$
\begin{cases}
z_1 = \dfrac{x_1}{\sqrt{\sum_{i=1}^{n} x_i^2}} \\
\quad \cdots \\
z_n = \dfrac{x_n}{\sqrt{\sum_{i=1}^{n} x_i^2}}
\end{cases}
$$

# Kohonen Network

*(unsupervised training process)*

3. Lengths of the weight vectors are normalized to unity.

4. A pattern is applied to an input and *net* values are calculated for all neurons

$$net = \sum_{i=1}^{5} z_i v_i = \mathbf{ZV}^T$$

$$\begin{cases} v_1 = \dfrac{w_1}{\sqrt{\sum_{i=1}^{n} w_i^2}} \\ \quad \cdots\cdots \\ v_n = \dfrac{w_n}{\sqrt{\sum_{i=1}^{n} w_i^2}} \end{cases}$$

# Kohonen Network
## *(unsupervised training process)*

5. A winning neuron is chosen (neuron with largest *net* value).

6. Weights for the winner $k$ are modified using a weighted average:

$$\boxed{\mathbf{W}_k = \mathbf{V}_k + \alpha \mathbf{Z}}$$

*where:*
$\alpha$ - is the learning constant,
$k$ – index of winning neuron

weights of other neurons are not modified.

# Kohonen Network

*(unsupervised training process)*

7. Weights for the winning neuron are normalized.

8. Another pattern is applied (go to step 4.).

$$\begin{cases} v_1 = \dfrac{w_1}{\sqrt{\sum_{i=1}^{n} w_i^2}} \\ \cdots\cdot \\ v_n = \dfrac{w_n}{\sqrt{\sum_{i=1}^{n} w_i^2}} \end{cases}$$

# Kohonen Network
## *(unsupervised training process)*

During pattern applications **some neurons** are frequent winners and other never take part in the process. The latter ones are eliminated and the **number of recognized clusters** is equal to the **number of surviving neurons**.

NOTE: *number of clusters might not be known upfront. You can start with larger network and eliminate neurons as you go. However, there is a danger of misclassification in this case.*

**Things to keep in mind:**

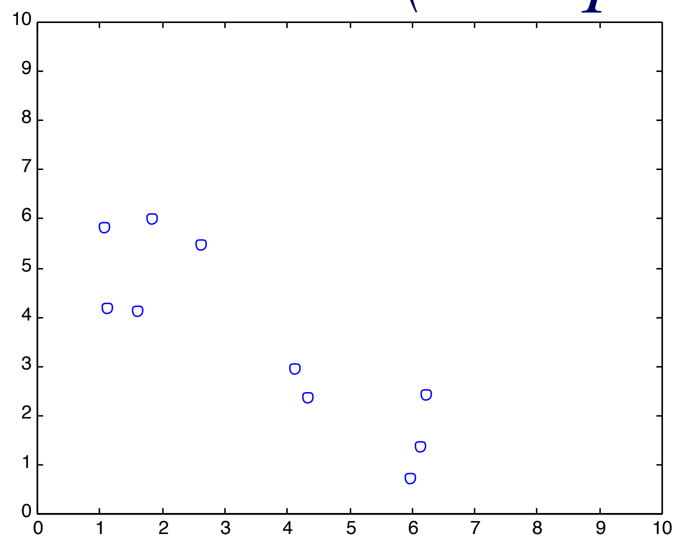
• The classification is strongly dependent on the initial set of randomly chosen weights, and order of updating.

• During the normalization process important information about the length of input patterns is lost.

# Kohonen Networks

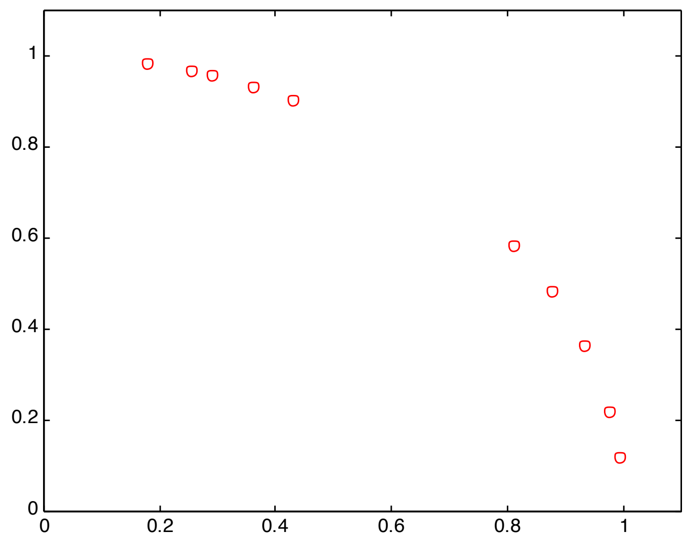
- *Derivation*
- *Steps*
- *Example*
- *Problems & remedies*

# Kohonen Network
## *(unsupervised training process)*




Original patterns

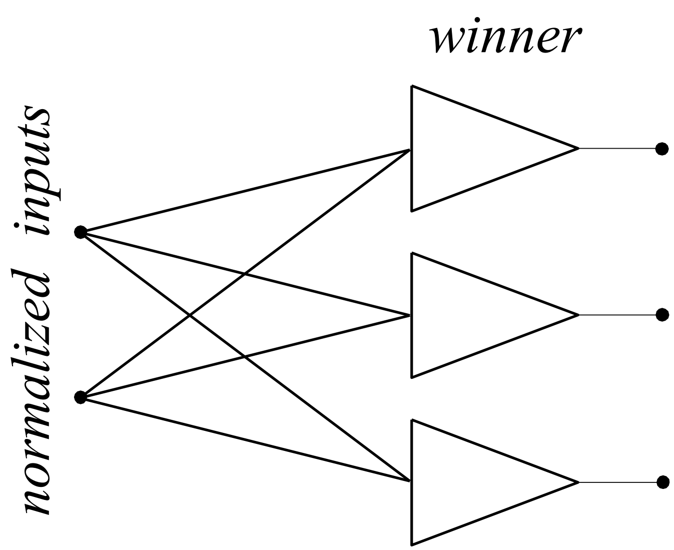
| | |
|---|---|
| 5.9630 | 0.7258 |
| 4.1168 | 2.9694 |
| 1.8184 | 6.0148 |
| 6.2139 | 2.4288 |
| 6.1290 | 1.3876 |
| 1.0562 | 5.8288 |
| 4.3185 | 2.3792 |
| 2.6108 | 5.4870 |
| 1.5999 | 4.1317 |
| 1.1046 | 4.1969 |




Normalized patterns

| | |
|---|---|
| 0.9927 | 0.1208 |
| 0.8110 | 0.5850 |
| 0.2894 | 0.9572 |
| 0.9314 | 0.3640 |
| 0.9753 | 0.2208 |
| 0.1783 | 0.9840 |
| 0.8759 | 0.4825 |
| 0.4296 | 0.9030 |
| 0.3611 | 0.9325 |
| 0.2545 | 0.9671 |

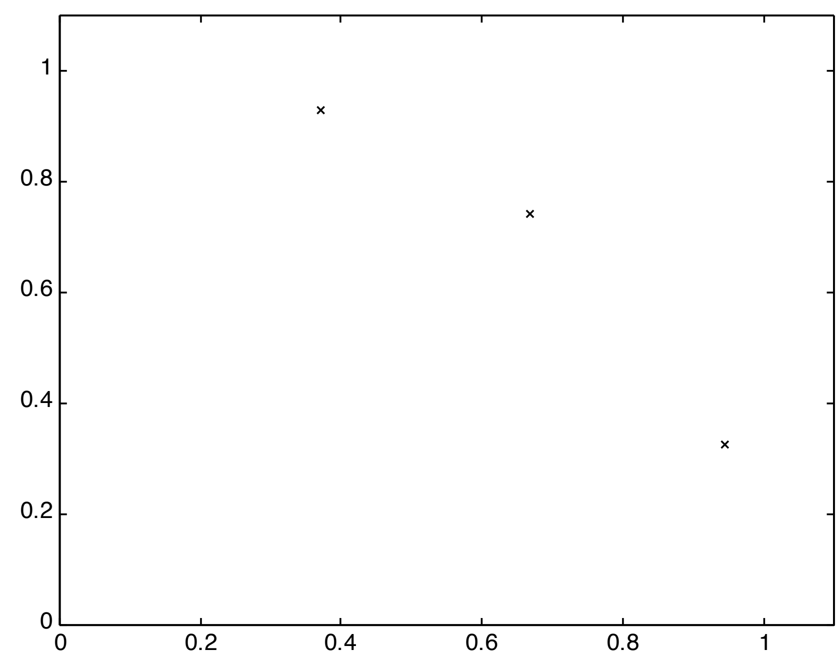
# Kohonen Network
## *(unsupervised training process)*



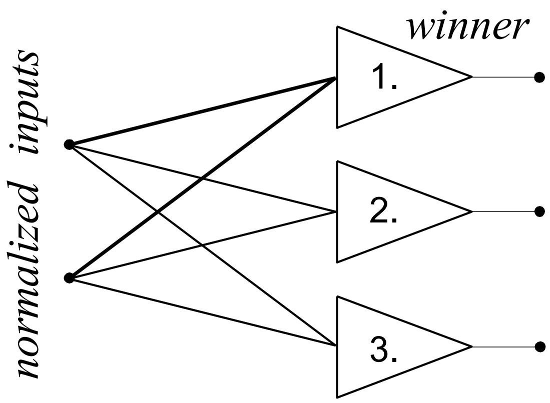

*randomly chosen number of neurons…*

$$\begin{bmatrix} .9459 & .3243 \\ .669 & .7433 \\ .3714 & .9285 \end{bmatrix}$$

normalized initial weights



# Kohonen Network
## *(unsupervised training process)*




applying the first pattern $Z_1 = \begin{bmatrix} 0.9927 & 0.1208 \end{bmatrix}$

*initial weights* net

$$\begin{bmatrix} .9459 & .3243 \\ .6690 & .7433 \\ .3714 & .9285 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.9782 \\ 0.7539 \\ 0.4809 \end{bmatrix}$$

From here, neuron #1 is the winner. Therefore, weights for neuron #1 are updated and normalized:

$$\left(\frac{1.2437}{\sqrt{1.2437^2 + 0.3606^2}}\right) = 0.96044$$

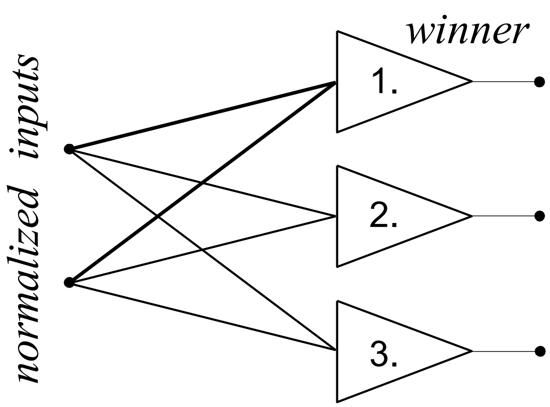
$$\mathbf{W}_k = \mathbf{V}_k + \alpha \mathbf{Z} = (0.9459 \quad 0.3243) + \text{alpha}\ (0.9927 \quad 0.1208) =$$

$$= (0.9459 \quad 0.3243) + 0.3\ (0.9927 \quad 0.1208) = (1.2437 \quad 0.3606) \xRightarrow{\textit{normalization}} (0.9605 \quad 0.2784)$$

$$\begin{bmatrix} .9459 & .3243 \\ .6690 & .7433 \\ .3714 & .9285 \end{bmatrix} \xRightarrow{\text{weight update}} \begin{bmatrix} .9605 & .2784 \\ .6690 & .7433 \\ .3714 & .9285 \end{bmatrix}$$

*1st neuron updated*

# Kohonen Network
## *(unsupervised training process)*




applying the second pattern $\begin{bmatrix} 0.8110 & 0.5850 \end{bmatrix}$

*initial weights* net

$$\begin{bmatrix} .9605 & .2784 \\ .6690 & .7433 \\ .3714 & .9285 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.9418 \\ 0.9774 \\ 0.8444 \end{bmatrix}$$

From here, neuron #2 is the winner. Therefore weights for neuron #2 are updated and normalized:

$$\mathbf{W}_k = \mathbf{V}_k + \alpha\mathbf{Z} = (0.6690 \quad 0.7433) + \text{alpha}\,(0.8110 \quad 0.5850)$$

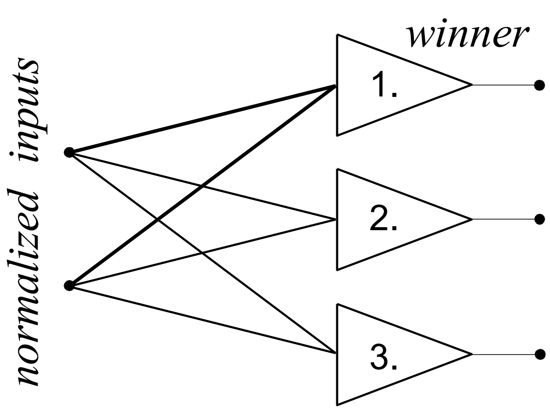
*normalization*

$$= (0.6690 \quad 0.7433) + 0.3\,(0.8110 \quad 0.5850) = (0.9123 \quad 0.9188) \Longrightarrow (0.7046 \quad 0.7096)$$

$$\begin{bmatrix} .9605 & .2784 \\ .6690 & .7433 \\ .3714 & .9285 \end{bmatrix} \underset{\text{weight update}}{\Longrightarrow} \begin{bmatrix} .9605 & .2784 \\ .7046 & .7096 \\ .3714 & .9285 \end{bmatrix}$$

*2nd neuron updated*

# Kohonen Network
## *(unsupervised training process)*




applying the third pattern $Z_3 = \begin{bmatrix} 0.2894 & 0.9572 \end{bmatrix}$

*initial weights* net

$$\begin{bmatrix} .9605 & .2784 \\ .7046 & .7096 \\ .3714 & .9285 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.5445 \\ 0.8832 \\ 0.9962 \end{bmatrix}$$

From here, neuron #3 is the winner. Therefore weights for neuron #3 are updated and normalized:

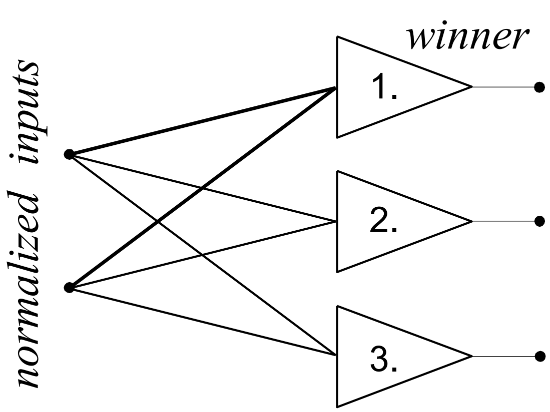
$$\mathbf{W}_k = \mathbf{V}_k + \alpha\mathbf{Z} = (0.3714 \quad 0.9285) + \text{alpha}\,(0.2894 \quad 0.9572)$$

$$= (0.3714 \quad 0.9285) + 0.3\,(0.2894 \quad 0.9572) = (0.4582 \quad 1.2156) \overset{\textit{normalization}}{\Longrightarrow} (0.3527 \quad 0.9357)$$

$$\begin{bmatrix} .9605 & .2784 \\ .7046 & .7096 \\ .3714 & .9285 \end{bmatrix} \xrightarrow{\text{weight update}} \begin{bmatrix} .9605 & .2784 \\ .7046 & .7096 \\ .3527 & .9357 \end{bmatrix}$$

*3rd neuron updated*

# Kohonen Network
## *(unsupervised training process)*




applying the fourth pattern $Z_4 = \begin{bmatrix} 0.9314 & 0.3640 \end{bmatrix}$

*initial weights* net

$$\begin{bmatrix} .9605 & .2784 \\ .7046 & .7096 \\ .3527 & .9357 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.996 \\ 0.915 \\ 0.669 \end{bmatrix}$$

From here, neuron #1 is the winner. Therefore weights for neuron #3 are updated and normalized:

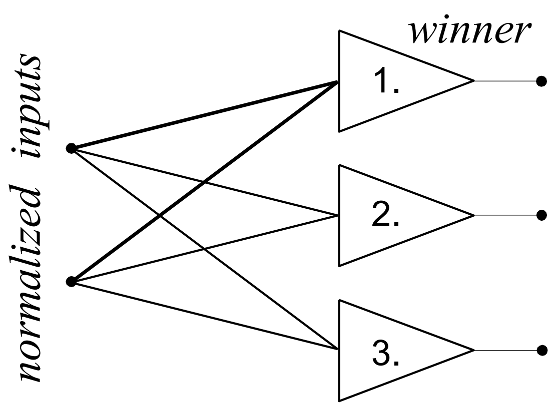
$$\mathbf{W}_k = \mathbf{V}_k + \alpha \mathbf{Z} = (0.9605 \quad 0.2784) + \text{alpha} \; (0.9314 \quad 0.3640)$$

$$= (0.9605 \quad 0.2784) + 0.3 \; (0.9314 \quad 0.3640) = (1.2399 \quad 0.3876) \overset{\textit{normalization}}{\Longrightarrow} (0.9544 \quad 0.2983)$$

$$\begin{bmatrix} .9605 & .2784 \\ .7046 & .7096 \\ .3527 & .9357 \end{bmatrix} \underset{\text{weight update}}{\Longrightarrow} \begin{bmatrix} .9544 & .2983 \\ .7046 & .7096 \\ .3527 & .9357 \end{bmatrix}$$

*1st neuron updated*

# Kohonen Network
## *(unsupervised training process)*




Process continues for all patterns

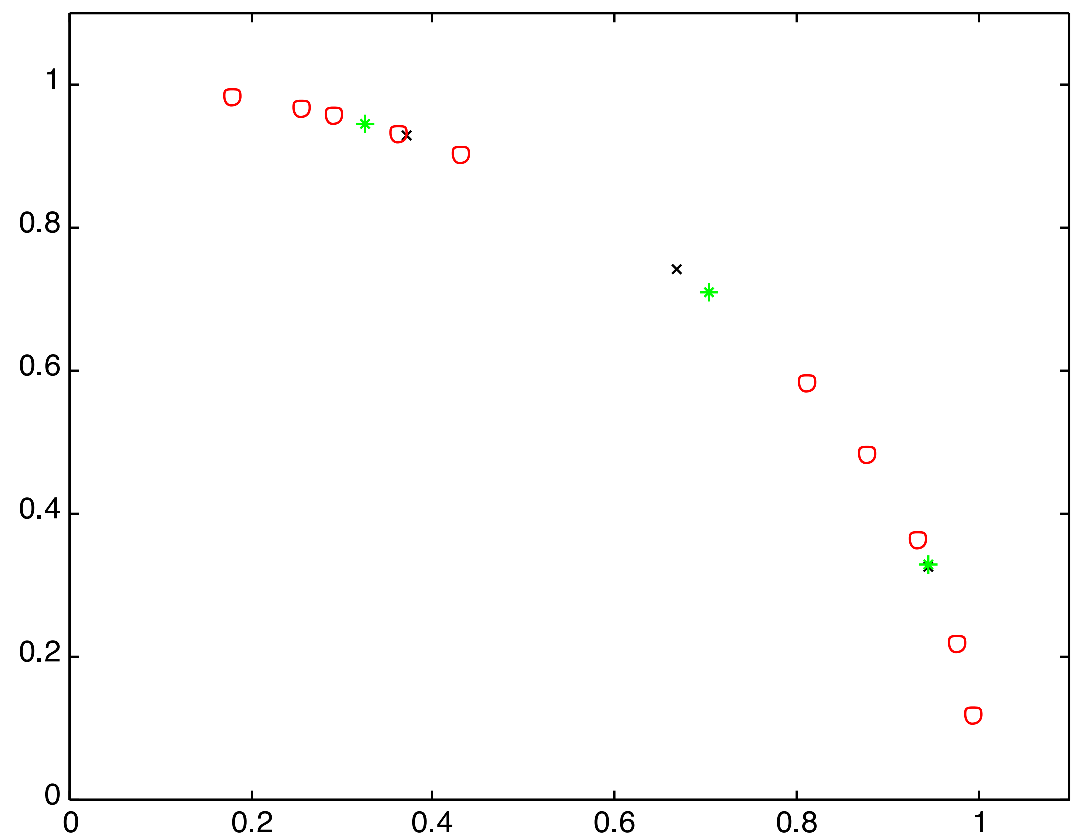
weights after the first iteration

$$\begin{bmatrix} .9605 & .2784 \\ .6690 & .7433 \\ .3714 & .9285 \end{bmatrix}$$

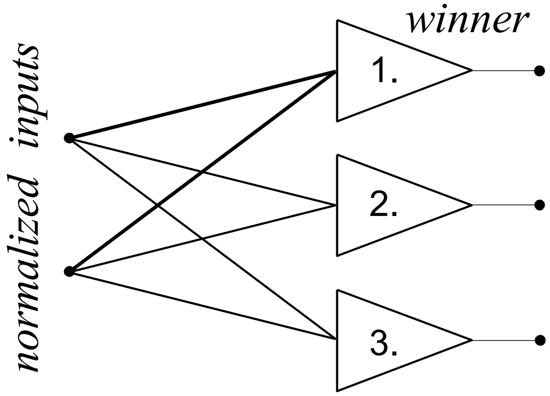
o - normalized patterns

x - initial weights

* - current weights



# Kohonen Network
## *(unsupervised training process)*



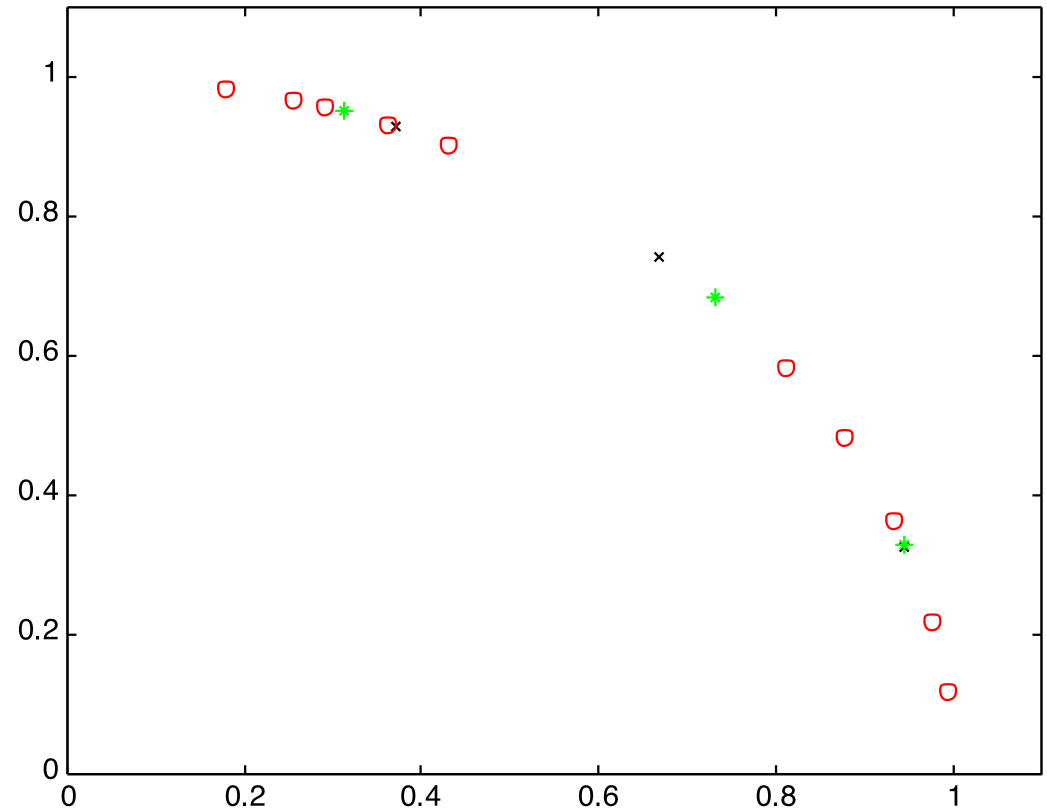

weights after the second iteration

| | |
|---|---|
| 0.9439 | 0.3302 |
| 0.7309 | 0.6825 |
| 0.3119 | 0.9501 |

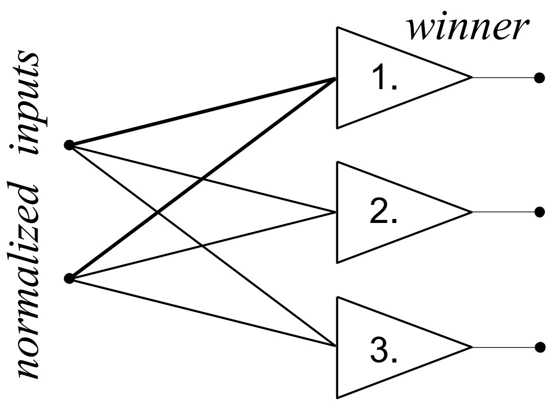
o - normalized patterns

x - initial weights

* - current weights



# Kohonen Network
## *(unsupervised training process)*




weights after the 30 iterations

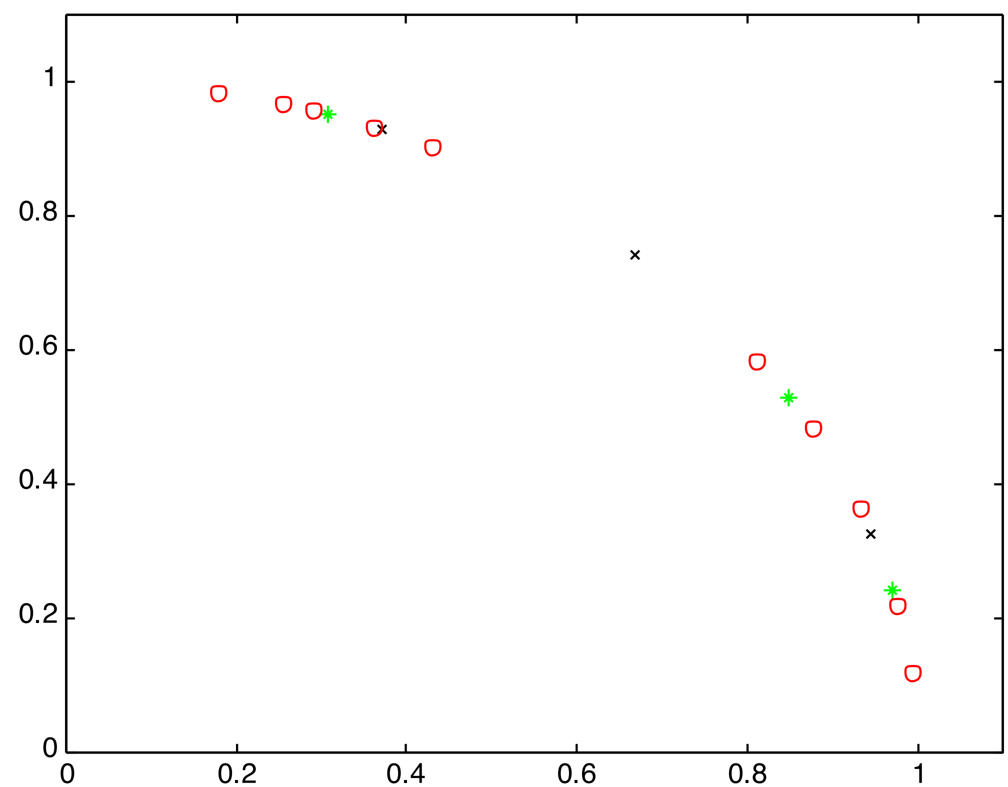
| | |
|---|---|
| 0.9699 | 0.2435 |
| 0.8492 | 0.5281 |
| 0.3072 | 0.9516 |

o - normalized patterns

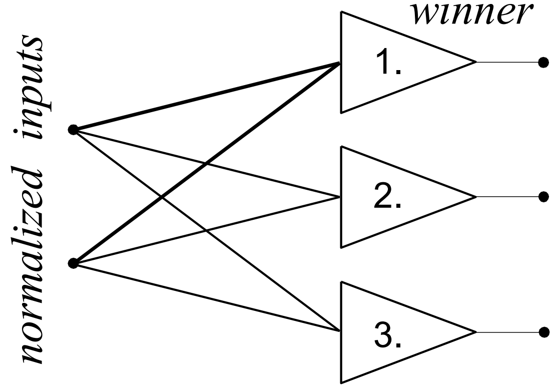
x - initial weights

* - current weights

weights are representing center of clusters

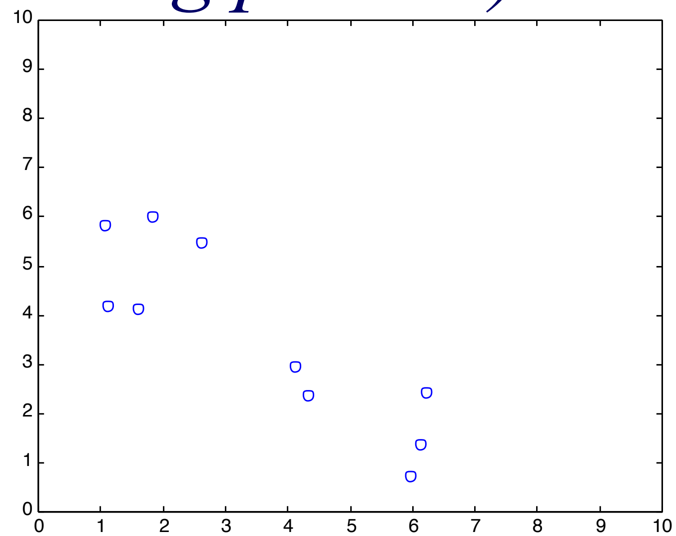


# Kohonen Network
## *(unsupervised training process)*



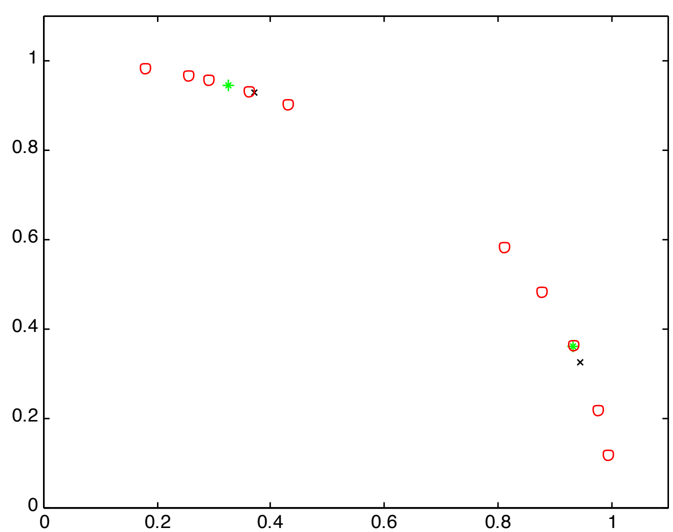

Initial weights

$$\begin{bmatrix} .9605 & .2784 \\ .7046 & .7096 \\ .3527 & .9357 \end{bmatrix}$$



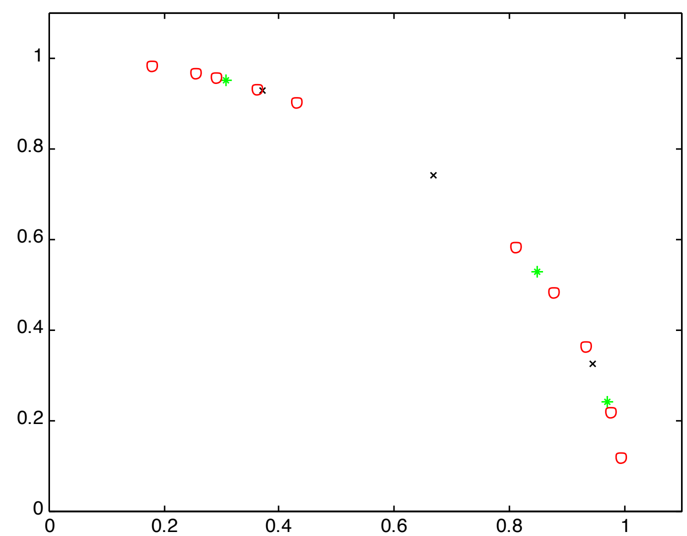
Patterns (initially)

after one iteration



Patterns (after one iterat.)

after 30 iterations



Patterns (after 30 iterat.)

# Kohonen Networks

- *Derivation*
- *Steps*
- *Example*
- *Problems & remedies*

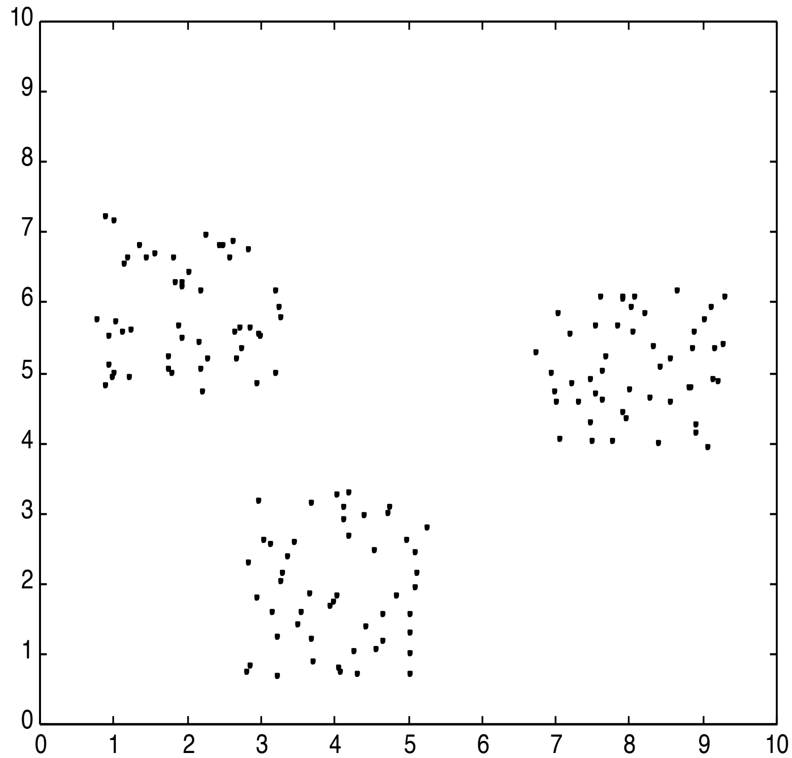
# Kohonen Networks
## *Problems & Remedies*

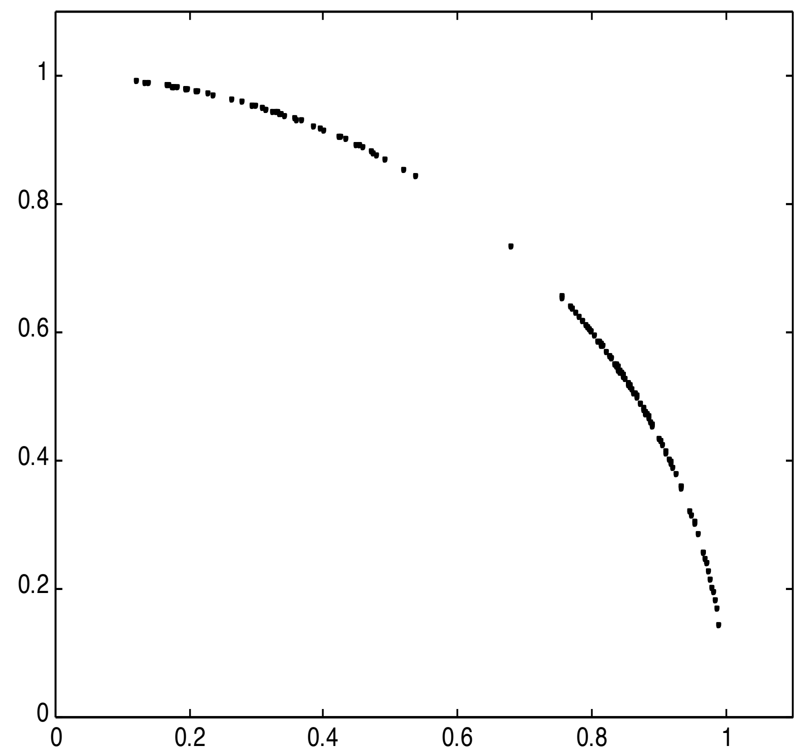
1. Important information about the length of the vector is lost during the normalization process

2. Clustering depends on:
   a) Order patterns are applied
   b) Number of initial neurons
   c) Initial weights

# Kohonen Networks
## *Problems & Remedies*

Important information about length of the vector is lost during the normalization process



# Kohonen Networks
## *Problems & Remedies*

Problem:

*Important information about length of the vector is lost during the normalization process.*

Possible remedies:

• The problem can be solved by increasing a dimension by one and usage of vector angles as variables. Lengths are the same.
This approach (used by Kohonen) leads to complex trigonometric computations

• Other way to approach the problem is to project patterns into hypersphere of higher dimensionality. This way all patterns have the same length but important information is not lost.