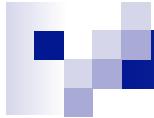


***CMSC 409:***  
***Artificial Intelligence***

<http://www.people.vcu.edu/~mmanic/>

**Virginia Commonwealth University,  
Fall 2015,  
Dr. Milos Manic  
([misko@vcu.edu](mailto:misko@vcu.edu))**

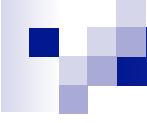


# ***CMSC 409: Artificial Intelligence***

**Session # 20,21**

## **Topics for today**

- Announcements
- Previous session review
- Fuzzy Logic
  - *Introduction, membership degree*
  - *Some of the key researchers*
- Fuzzy controller
  - *Zadeh min-max controller*
  - *Fuzzification*
  - *Fuzzy inference engine (rule table)*
  - *Defuzzification*
- Zadeh fuzzy controller
  - *Design example, inputs, outputs (singletons)*

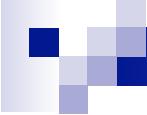


# ***CMSC 409: Artificial Intelligence***

## **Session # 20,21**

### **Announcements**

- Blackboard
  - Slides, class paper instructions and template uploaded
- Assignments, update
  - project 3 deadline yesterday (Nov. 4)
- Paper
  - *The third draft - due Nov. 9, 2015*
  - *In addition to previous draft, it should contain a technique (or selection thereof), you plan on using to solve the selected problem (check out the class paper instructions for the 3<sup>rd</sup> draft)*
- Subject line and signature
  - *Please use specified in syllabus*



## □ Fuzzy Logic

- *Introduction, membership degree*
- *AND, OR, NOT*
- *Entropy*
- *Subsethood*
- *Composition*
  - *Min-Max; Product-Sum; Product-Max;*

- *Fuzzy Associative Memories (FAM)*
  - *correlation product (max of products)*
  - *correlation minimum encoding (max of min)*

# Fuzzy systems

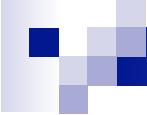
- Developed by Lofti Zadeh
- <http://www.cs.berkeley.edu/~zadeh/>
- Over 15 honorary doctorates
- His work was cited in over 30,000 publications



- Boolean Logic
- Multivalued Logic
- Fuzzy Logic



- Hans-Jurgen Zimmerman



# IJCNN 07

- **James C. Bezdek, University of West Florida in Pensacola**
  - Fuzzy c-means (FCM)
  - 2007 IEEE Frank Rosenblatt Award Recipient
  - [http://www.ieee.org/portal/pages/about/awards/bios/2007\\_Bios/2007Rosenblatt-Bezdek.html](http://www.ieee.org/portal/pages/about/awards/bios/2007_Bios/2007Rosenblatt-Bezdek.html)

*(Taken from IEEE Portal Pages)*

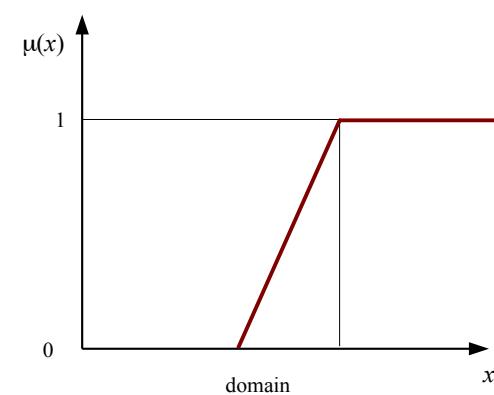
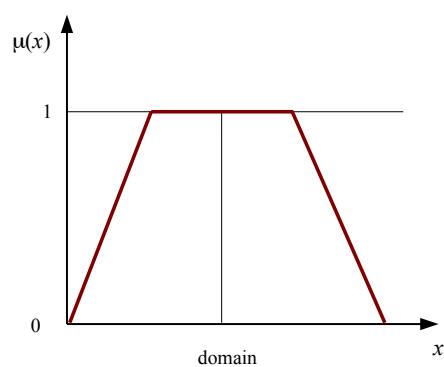
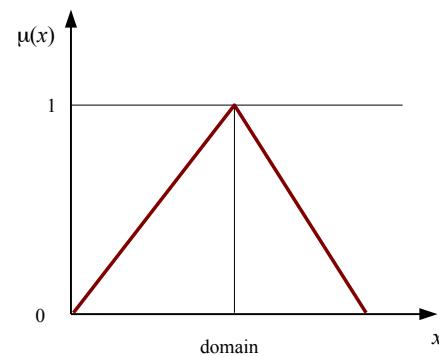
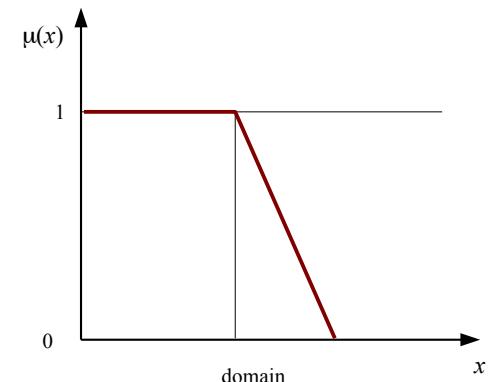
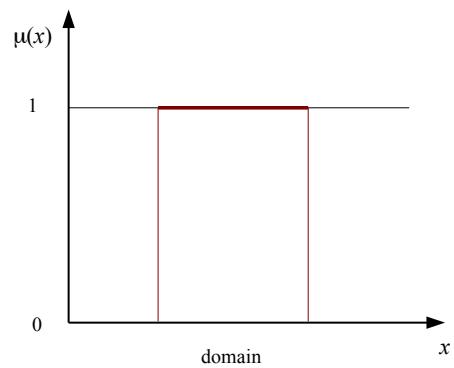
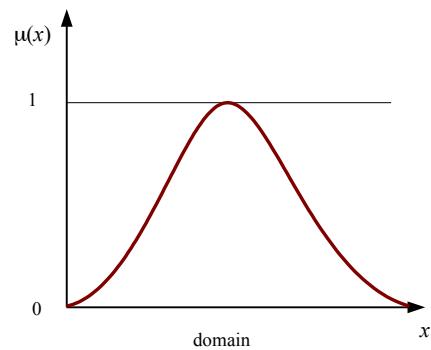


“FCM is considered one of the most important discoveries in fuzzy pattern recognition and related areas and the clustering algorithm of choice for most practitioners in fuzzy exploratory data analysis. The original model has inspired many applications in related areas of pattern recognition and image processing.”



# Briefly on Fuzzy Logic (FL)

## Data representation using fuzzy sets



‘Bell’ shaped, triangular, trapezoidal, shouldered fuzzy sets

# Fuzzy systems

- Inputs can be any value from 0 to 1.
- The basic fuzzy principle is similar to Boolean logic.
- Min and max operators are used instead of AND and OR, respectively.
- The NOT operator also becomes  $1 - \#$ .

$A \cap B \cap C \Rightarrow \min\{A, B, C\}$  – smallest value of A, B or C

$A \cup B \cup C \Rightarrow \max\{A, B, C\}$  – largest value of A, B or C

$\bar{A} \Rightarrow 1 - A$  – one minus A      **complement**

**Boolean**

**Fuzzy**

$A \cap B$

0	0	0
0	1	0
1	0	0
1	1	1

$A \cup B$

0	0	0
0	1	1
1	0	1
1	1	1

$A \cap B$

0.2	0.3
0.2	0.8
0.7	0.3
0.7	0.8

*conjunction*

$A \cup B$

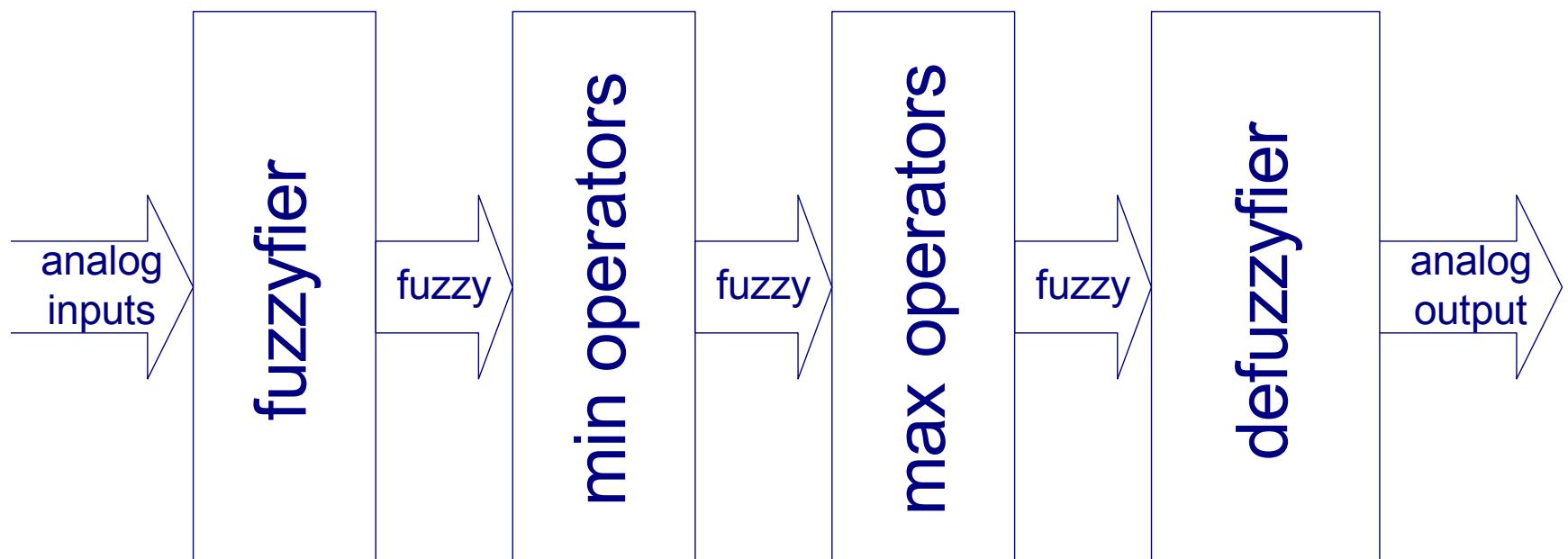
0.2	0.3	0.3
0.2	0.8	0.8
0.7	0.3	0.7
0.7	0.8	0.8

*disjunction*

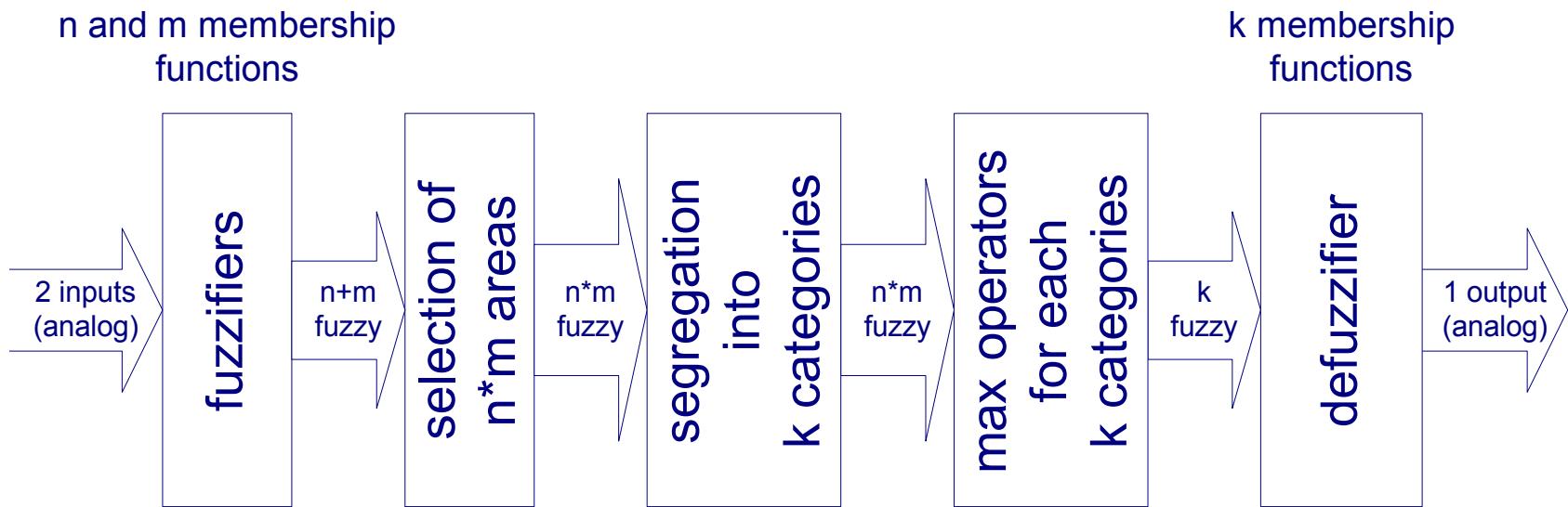
- *Fuzzy controller*
  - *Zadeh min-max controller*
- *Fuzzification*
- *Fuzzy inference engine*
- *Defuzzification*

# Fuzzy Systems

Block diagram of Zadeh fuzzy controller



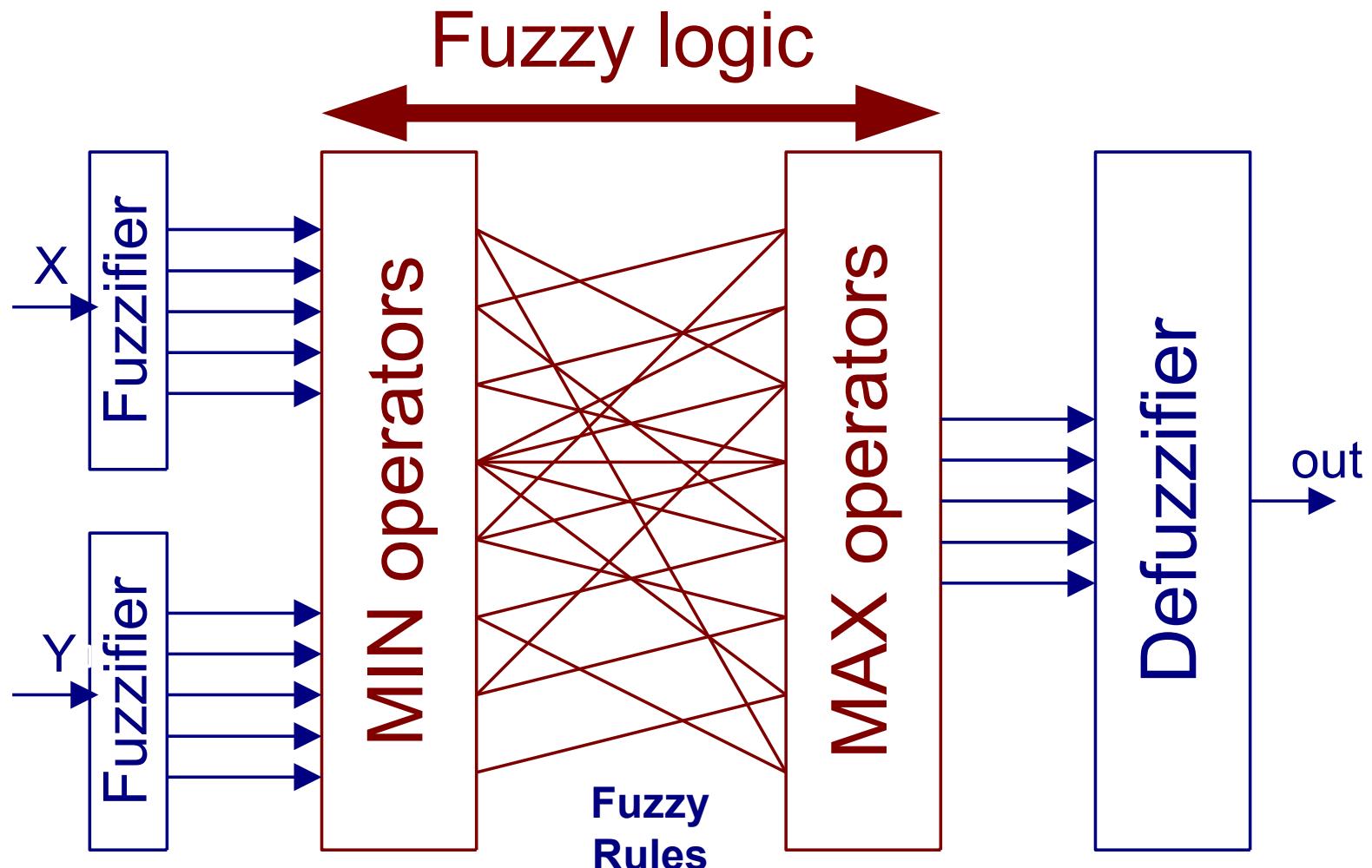
# Fuzzy Systems



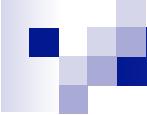
*Block diagram for Zadeh fuzzy controller*

# Fuzzy Systems

Block diagram of Zadeh fuzzy controller



- *Fuzzy controller*
  - *Zadeh min-max controller*
- *Fuzzification*
- *Fuzzy inference engine*
- *Defuzzification*



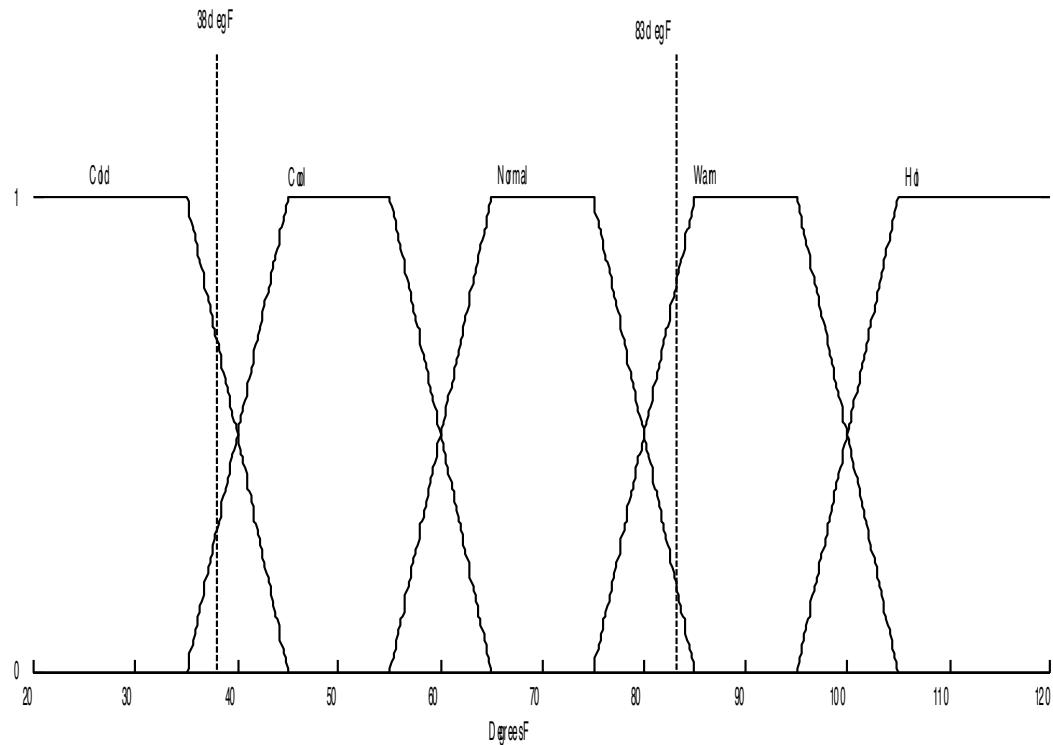
# Fuzzy Systems

## *fuzzification*

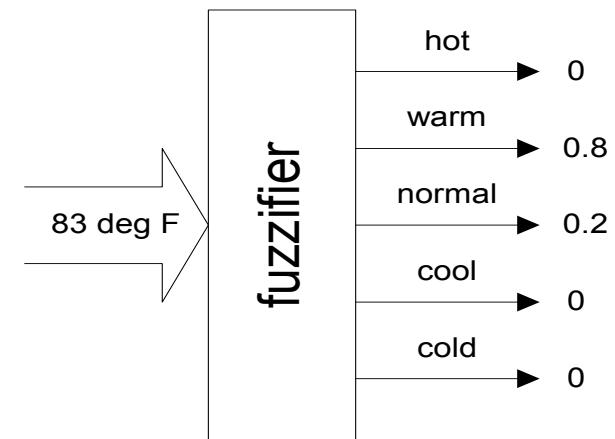
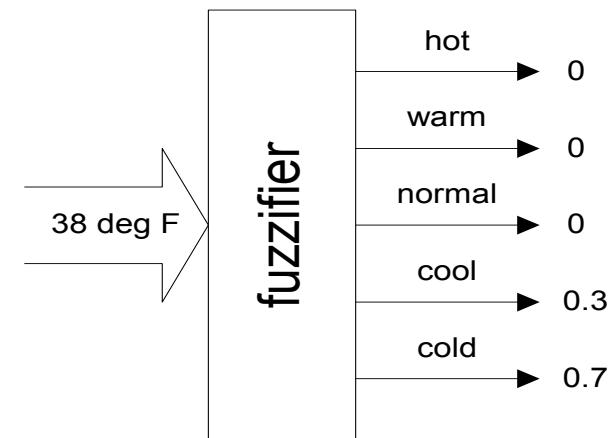
- There are three major types of membership functions
  - Gaussian, Triangular and Trapezoidal
- Three basic membership function rules
  1. Each point of an input should belong to at least one membership function (preferable 2 or more)
  2. The sum of two overlapping functions should not be greater than 1.
  3. For higher accuracy, more membership functions can be used, but this can lead to system instability and will require a larger fuzzy table.

# Fuzzy Systems

## *fuzzification*



trapezoidal membership function

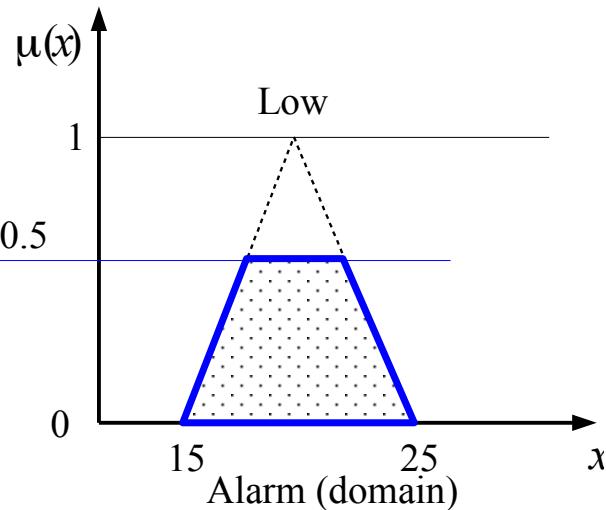
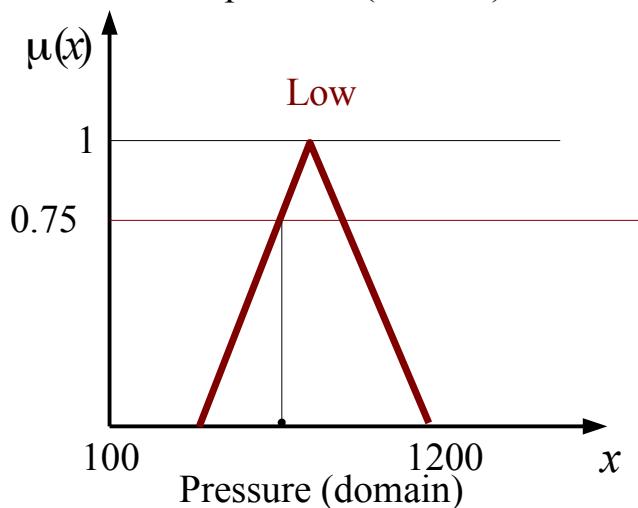
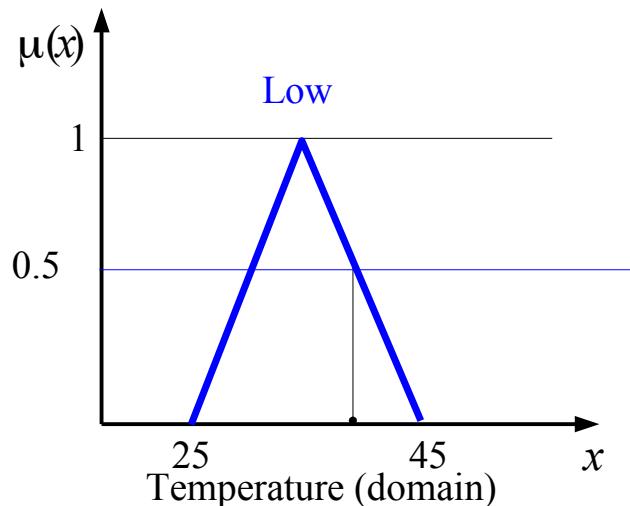


fuzzification results

- *Fuzzy controller*
  - *Zadeh min-max controller*
- *Fuzzification*
- *Fuzzy inference engine*
- *Defuzzification*

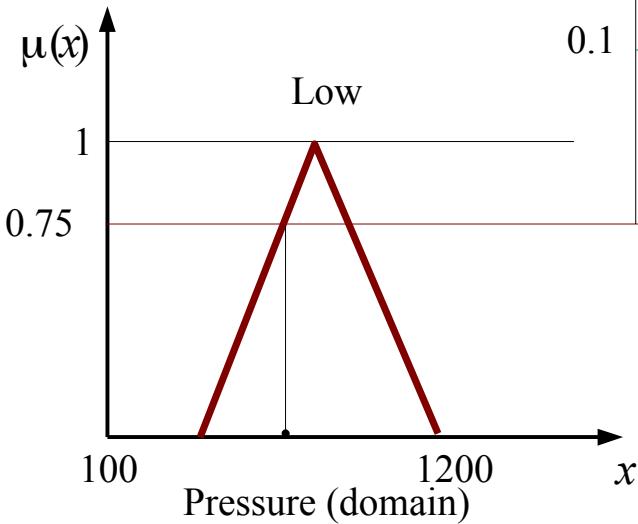
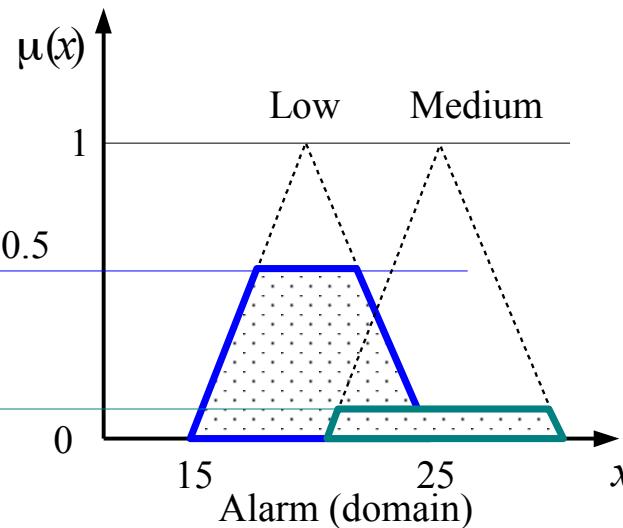
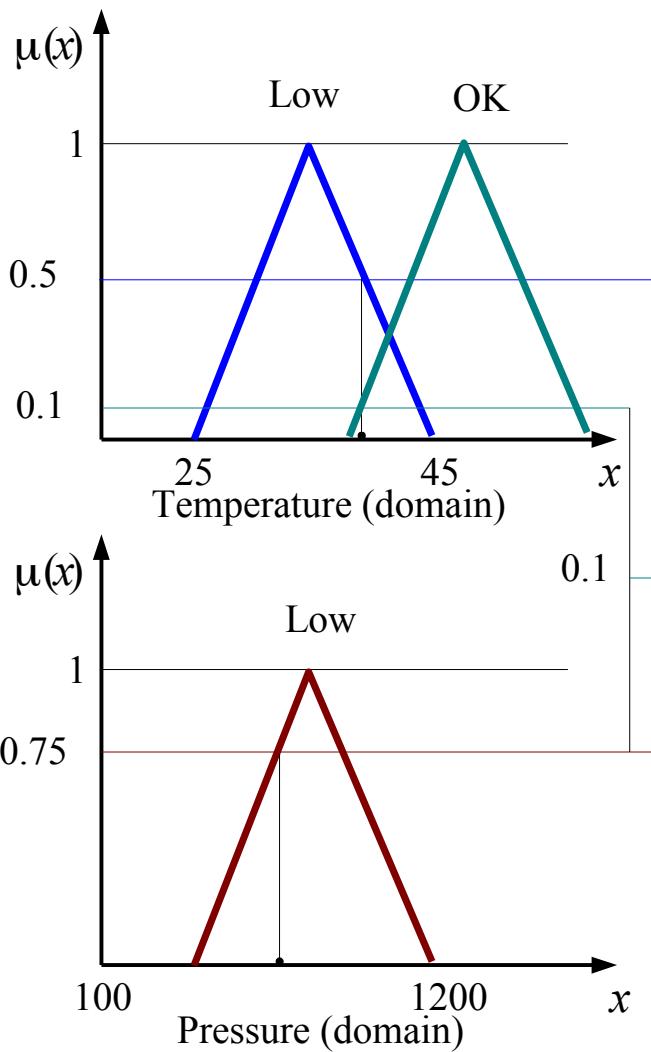
# Briefly on Fuzzy Logic (FL)

Fuzzy sets connected by a Zadeh AND operator



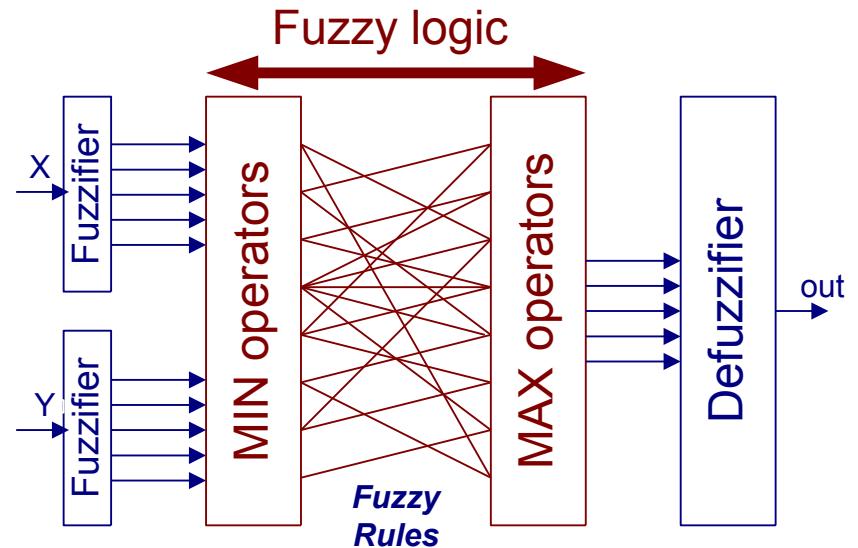
# Briefly on Fuzzy Logic (FL)

Zadeh min-max rule



# Fuzzy Systems - Rule Evaluation - Zadeh fuzzy tables

Controller takes temperature reading from two inputs



	Input 2				
Input 1	→				
↓	cold	cool	normal	warm	hot
cold	A	A	A	B	A
cool	A	A	B	C	B
normal	A	B	C	C	C
warm	A	B	C	D	D
hot	B	C	D	E	E

# Fuzzy Systems - Rule Evaluation - Zadeh fuzzy tables

	Input 2				
Input 1	→				
↓	0 cold	0 cool	0.2 normal	0.8 warm	0 hot
0.7 cold	A	A	A	B	A
0.3 cool	A	A	B	C	B
0 normal	A	B	C	C	C
0 warm	A	B	C	D	D
0 hot	B	C	D	E	E

	Input 2				
Input 1	→				
↓	0 cold	0 cool	0.2 normal	0.8 warm	0 hot
0.7 cold	0	0	0.2 A	0.7 B	0
0.3 cool	0	0	0.2 B	0.3 C	0
0 normal	0	0	0	0	0
0 warm	0	0	0	0	0
0 hot	0	0	0	0	0

MIN

# Fuzzy Systems - Rule Evaluation - Zadeh fuzzy tables

	Input 2				
Input 1	→				
↓	0 cold	0 cool	0.2 normal	0.8 warm	0 hot
0.7 cold	0	0	0.2 A	0.7 B	0
0.3 cool	0	0	0.2 B	0.3 C	0
0 normal	0	0	0	0	0
0 warm	0	0	0	0	0
0 hot	0	0	0	0	0

MAX

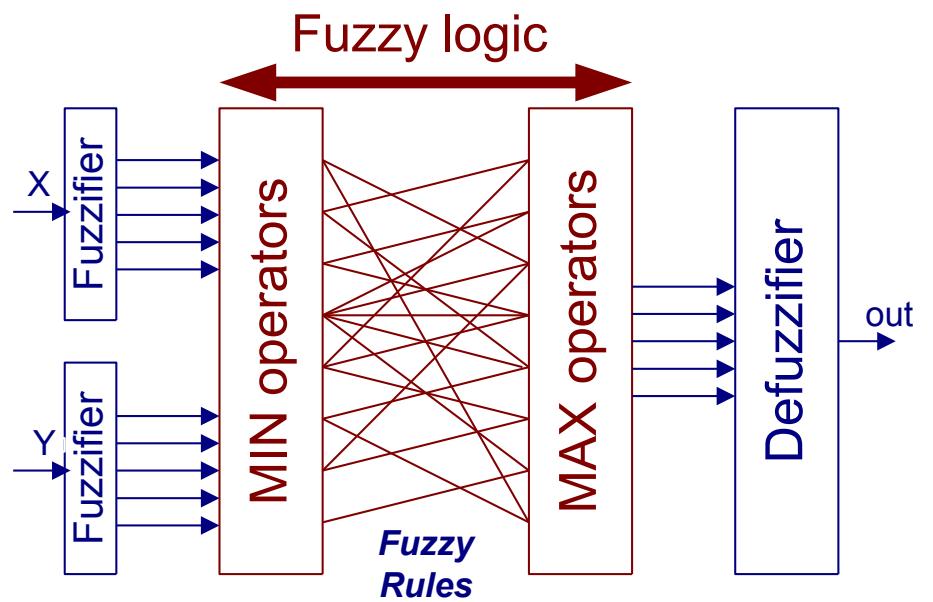
*max of all A cells = 0.2*

*max of all B cells = 0.7*

*max of all C cells = 0.3*

*max of all D cells = 0*

*max of all E cells = 0*



- *Fuzzy controller*
  - *Zadeh min-max controller*
- *Fuzzification*
- *Fuzzy inference engine*
- *Defuzzification*

# Fuzzy Systems - Defuzzification

- The equation to describe the defuzzification process.
  - $n$  – Number of membership functions
  - $z_k$  – Fuzzy output variables
  - $c_k$  – analog values from table
- Outputs:
  - Zadeh

$$\text{Output} = \frac{\sum_{k=1}^n z_k c_k}{\sum_{k=1}^n z_k}$$

$$\text{Output} = \frac{0.2 * A + 0.7 * B + 0.3 * C}{0.2 + 0.7 + 0.3}$$

- Takagi-Sugeno

$$\text{Output} = \frac{0.2 * O3 + 0.7 * O4 + 0.2 * O8 + 0.3 * O9}{0.2 + 0.7 + 0.3 + 0.2}$$

# Fuzzy Systems - Defuzzification

- The equation to describe the defuzzification process.

- n – Number of membership functions
- $z_k$  – Fuzzy output variables
- $c_k$  – analog values from table

$$\text{Output} = \frac{\sum_{k=1}^n z_k c_k}{\sum_{k=1}^n z_k}$$

$$\text{Output} = \frac{0.2 * A + 0.7 * B + 0.3 * C}{0.2 + 0.7 + 0.3}$$

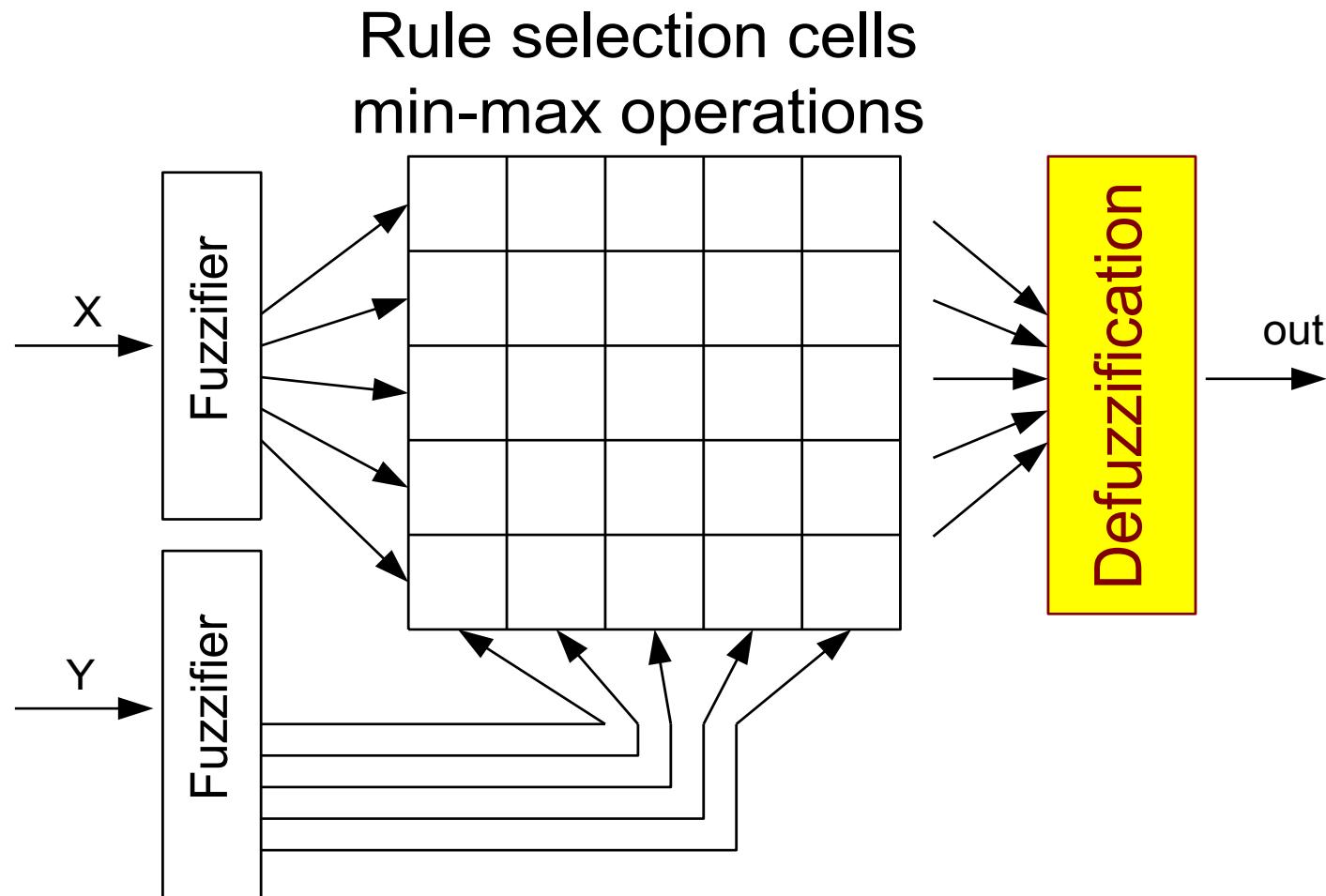
Output data for  
*singleton* encoding

A	-12
B	-3
C	8
D	20
E	35

$$\text{Output} = \frac{0.2 * (-12) + 0.7 * (-3) + 0.3 * (8)}{0.2 + 0.7 + 0.3} = \frac{-2.1}{1.2} = 1.75$$

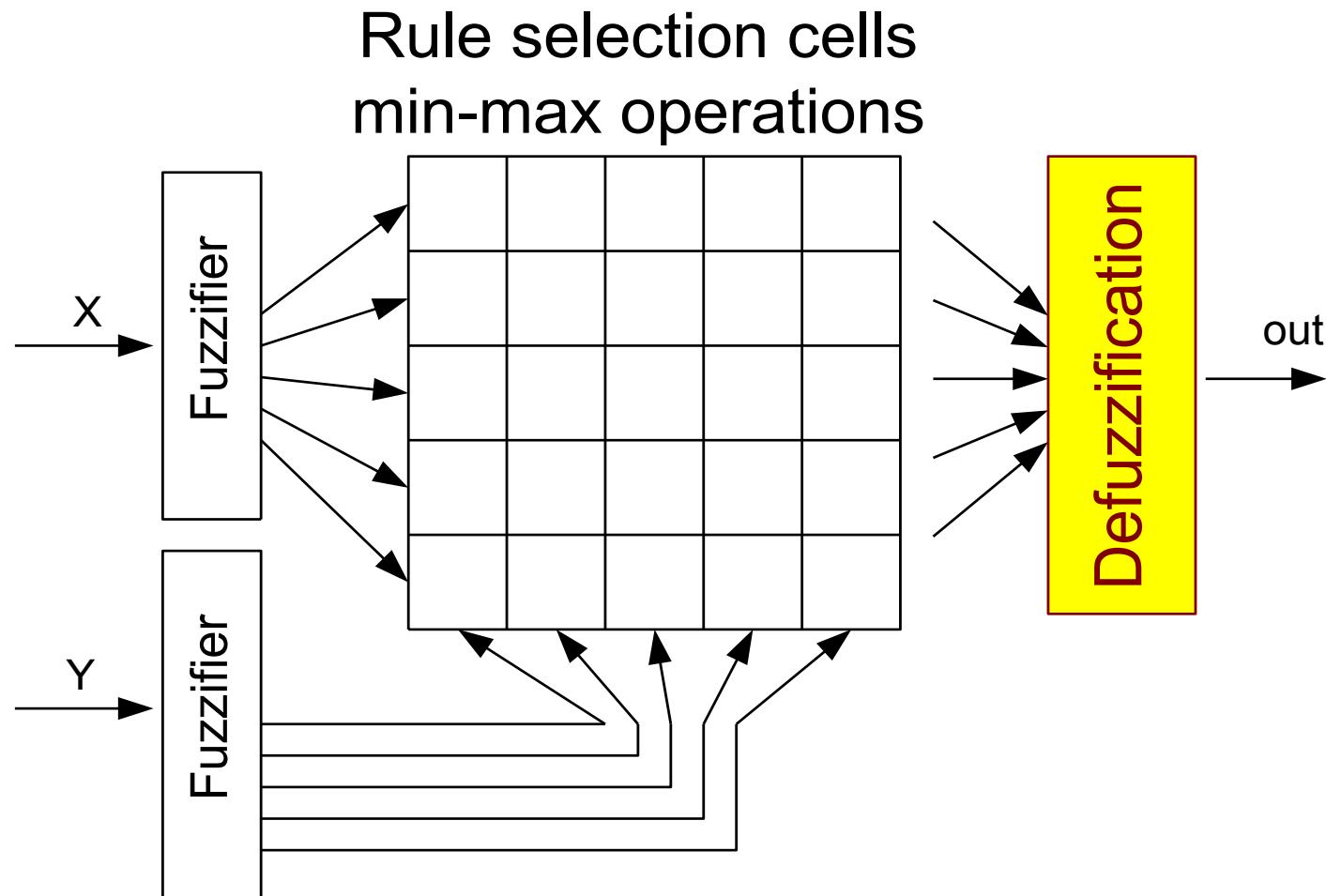
# Fuzzy Systems

Block diagram of Zadeh fuzzy controller

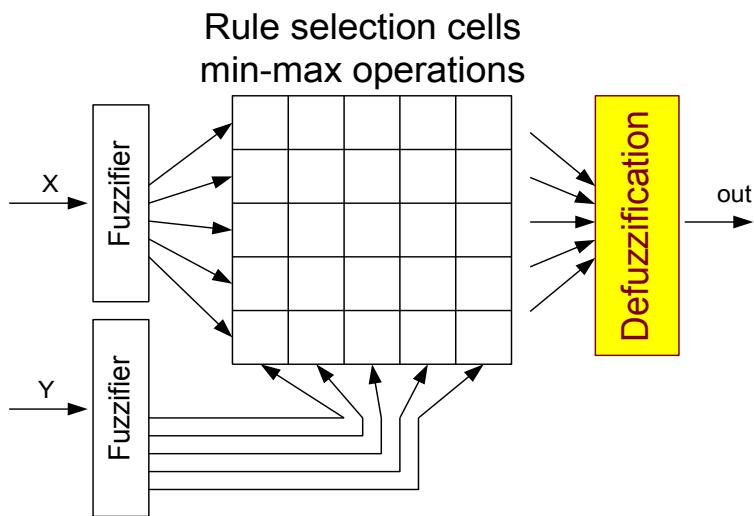


# Fuzzy Systems

Block diagram of Zadeh fuzzy controller



# Zadeh Fuzzy Tables



	Input 2				
Input 1	→				
↓	cold	cool	normal	warm	hot
cold	A	A	A	B	A
cool	A	A	B	C	B
normal	A	B	C	C	C
warm	A	B	C	D	D
hot	B	C	D	E	E

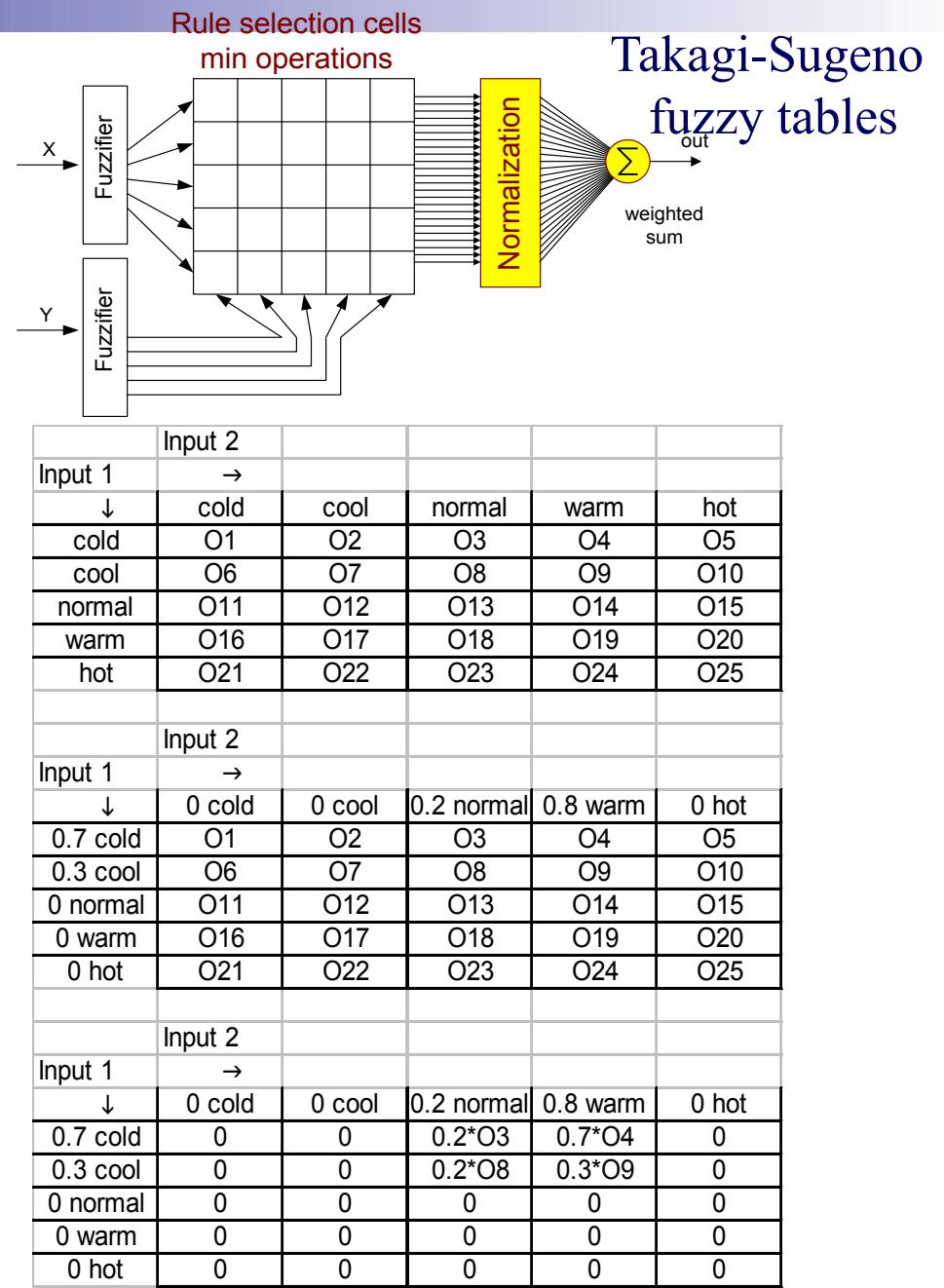
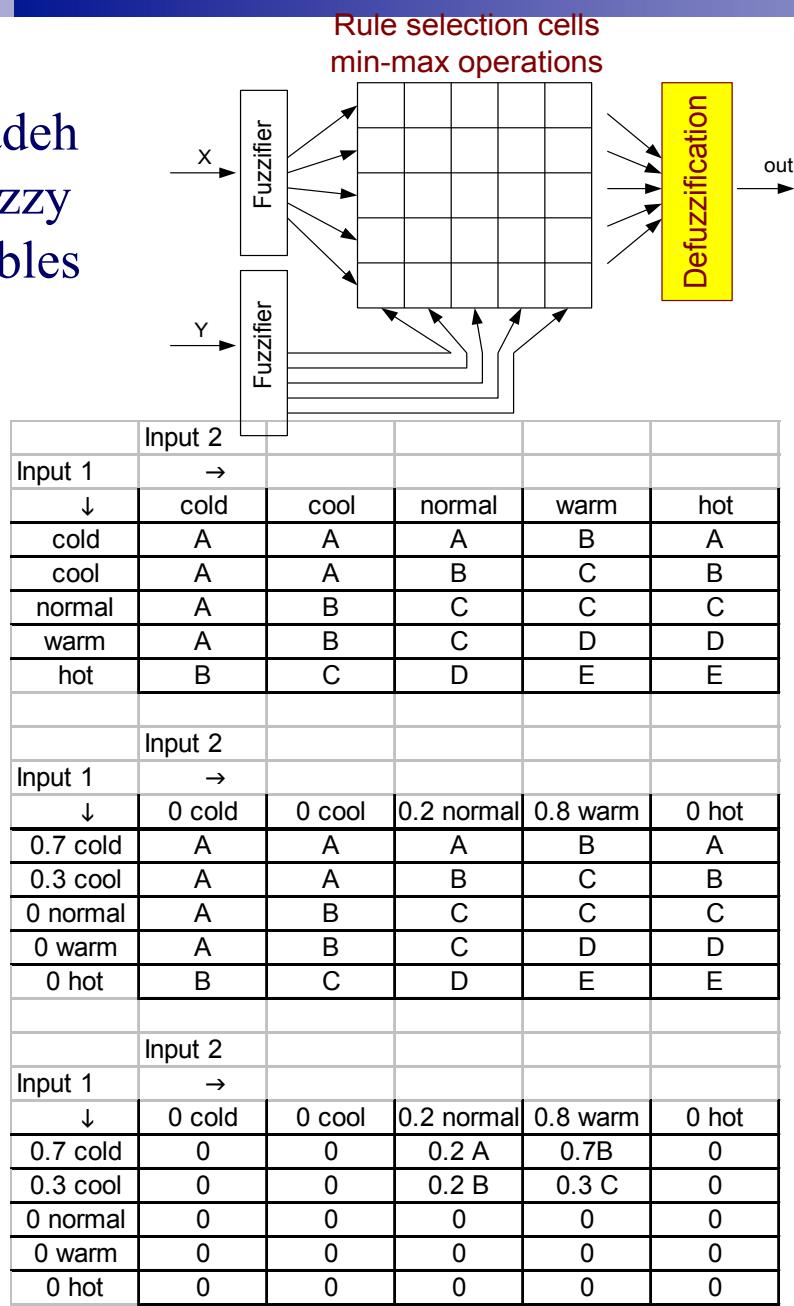
  

	Input 2				
Input 1	→				
↓	0 cold	0 cool	0.2 normal	0.8 warm	0 hot
0.7 cold	A	A	A	B	A
0.3 cool	A	A	B	C	B
0 normal	A	B	C	C	C
0 warm	A	B	C	D	D
0 hot	B	C	D	E	E

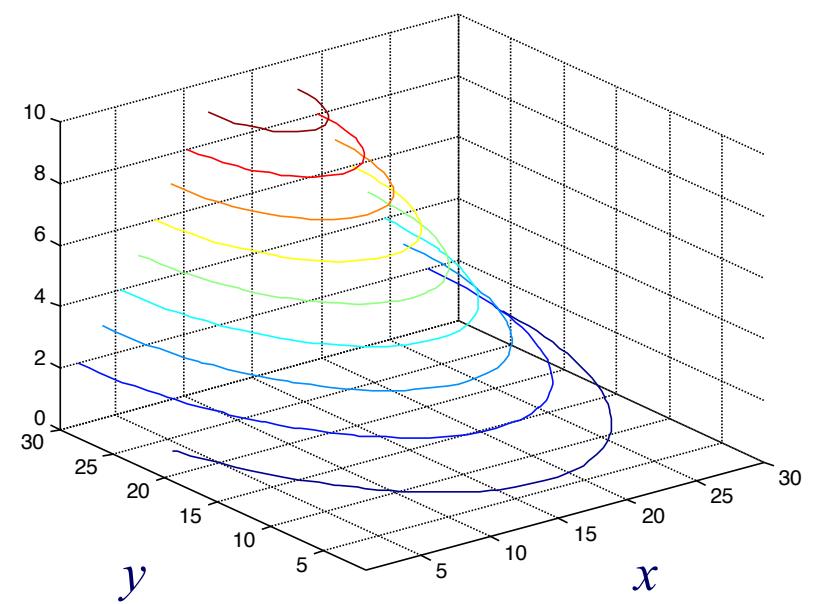
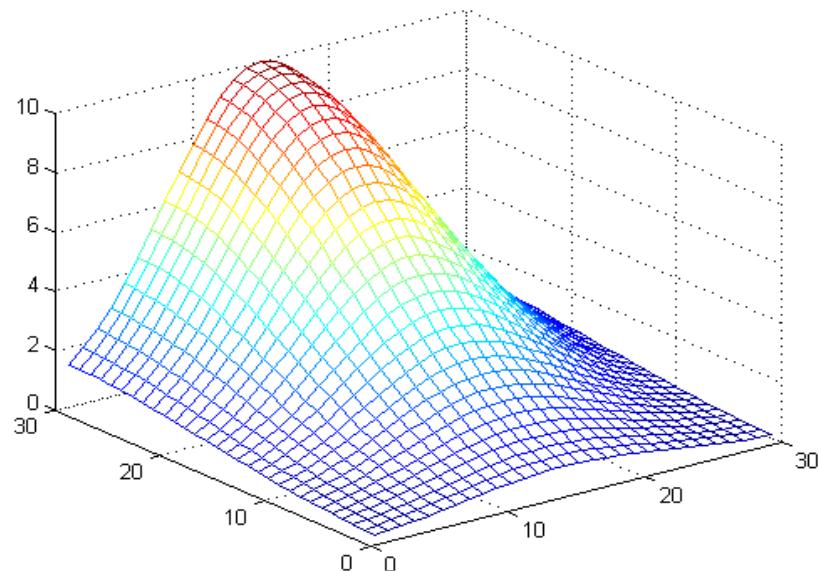
	Input 2				
Input 1	→				
↓	0 cold	0 cool	0.2 normal	0.8 warm	0 hot
0.7 cold	0	0	0.2 A	0.7B	0
0.3 cool	0	0	0.2 B	0.3 C	0
0 normal	0	0	0	0	0
0 warm	0	0	0	0	0
0 hot	0	0	0	0	0

## Zadeh fuzzy tables

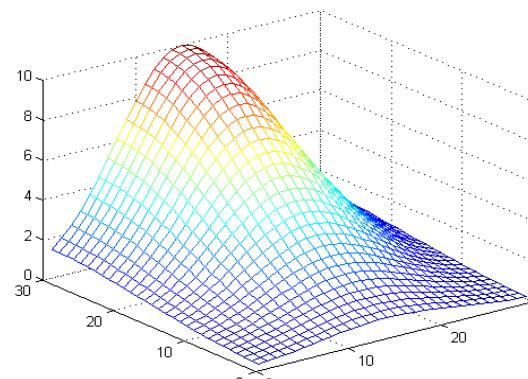


- *Zadeh fuzzy controller*
  - *design example*
  - *inputs, outputs (singletons)*
- *Fuzzy systems, various implementations*

# Design Example of Zadeh Fuzzy Controller

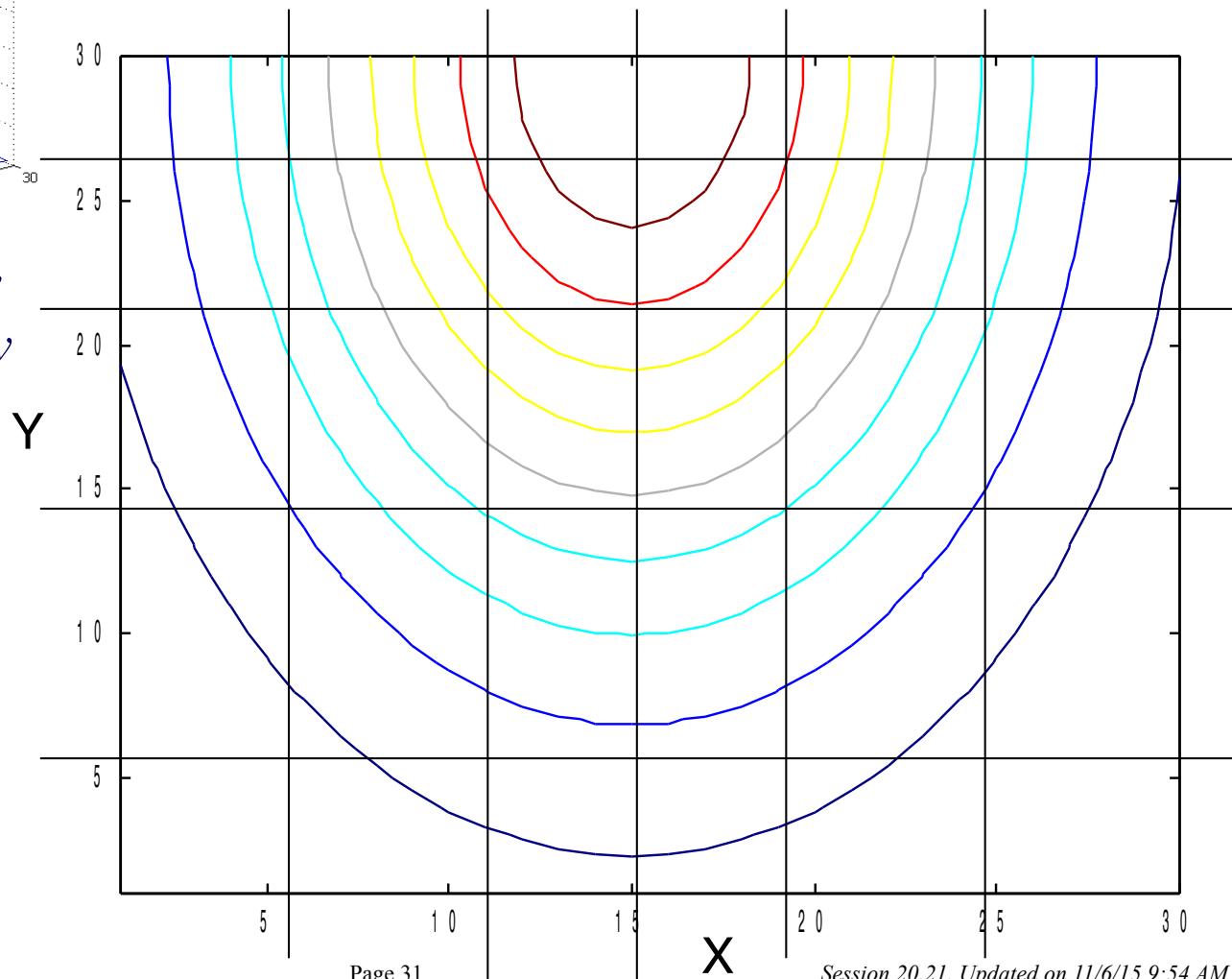


# Design Example of Zadeh Fuzzy Controller

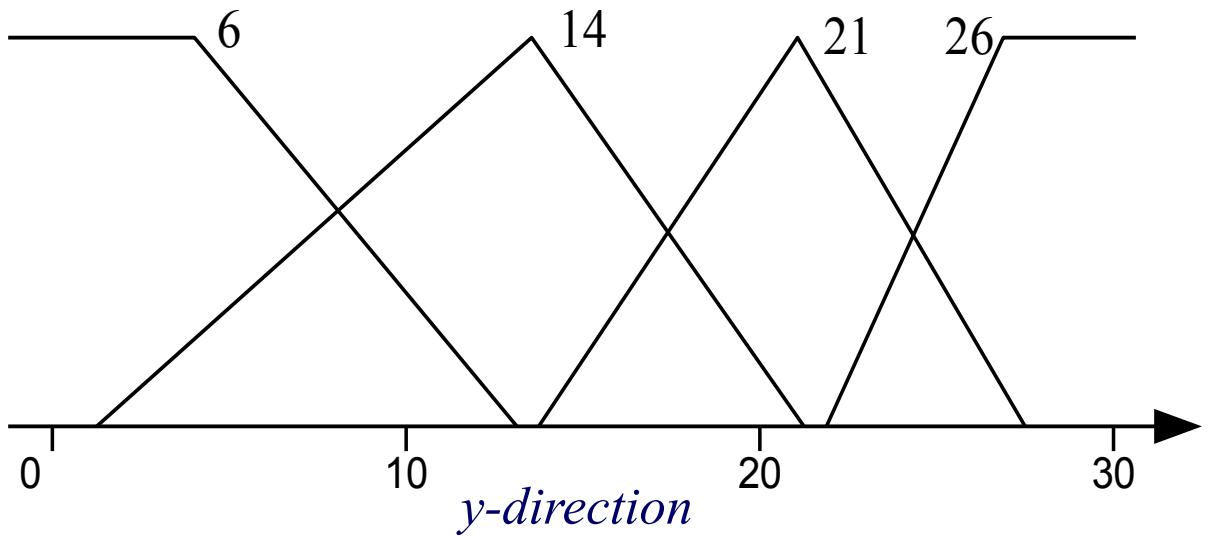
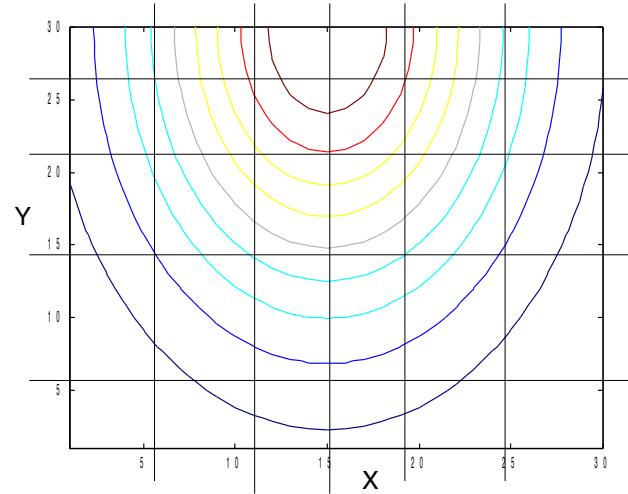
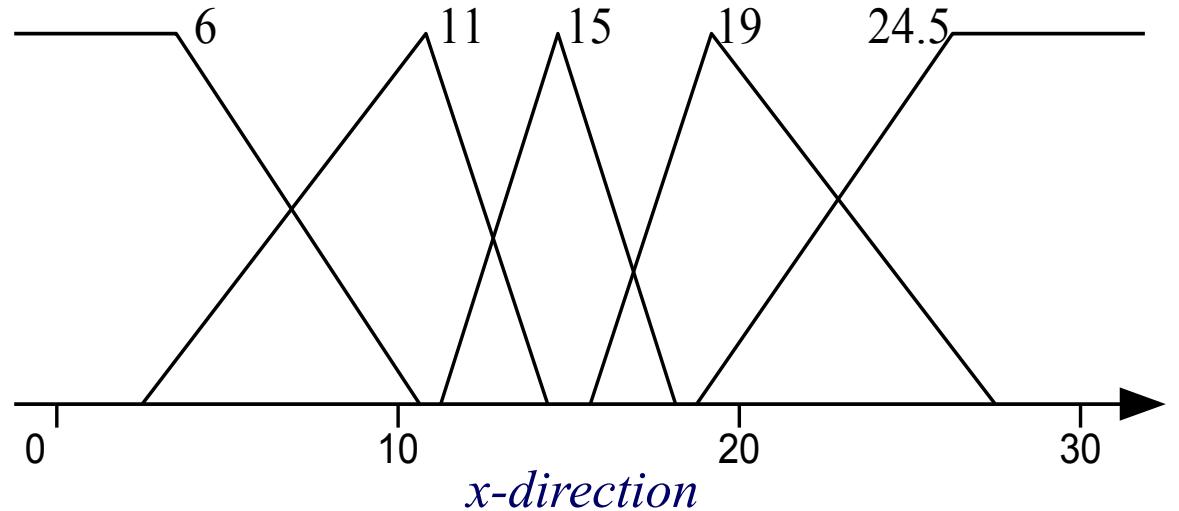
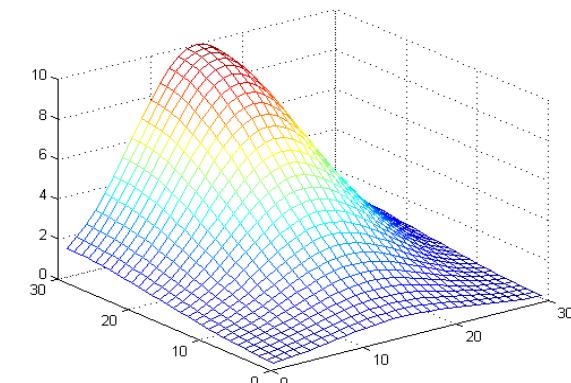


*higher density*

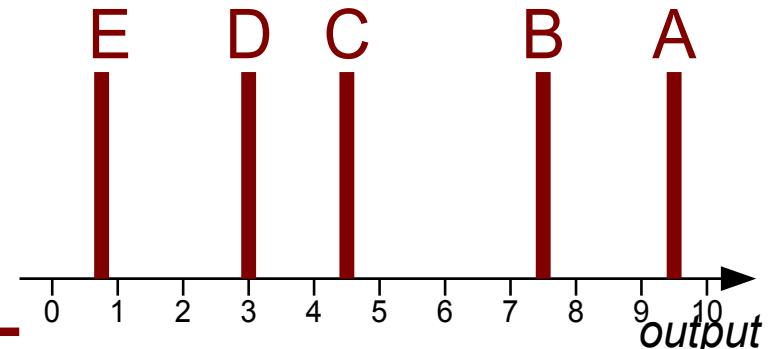
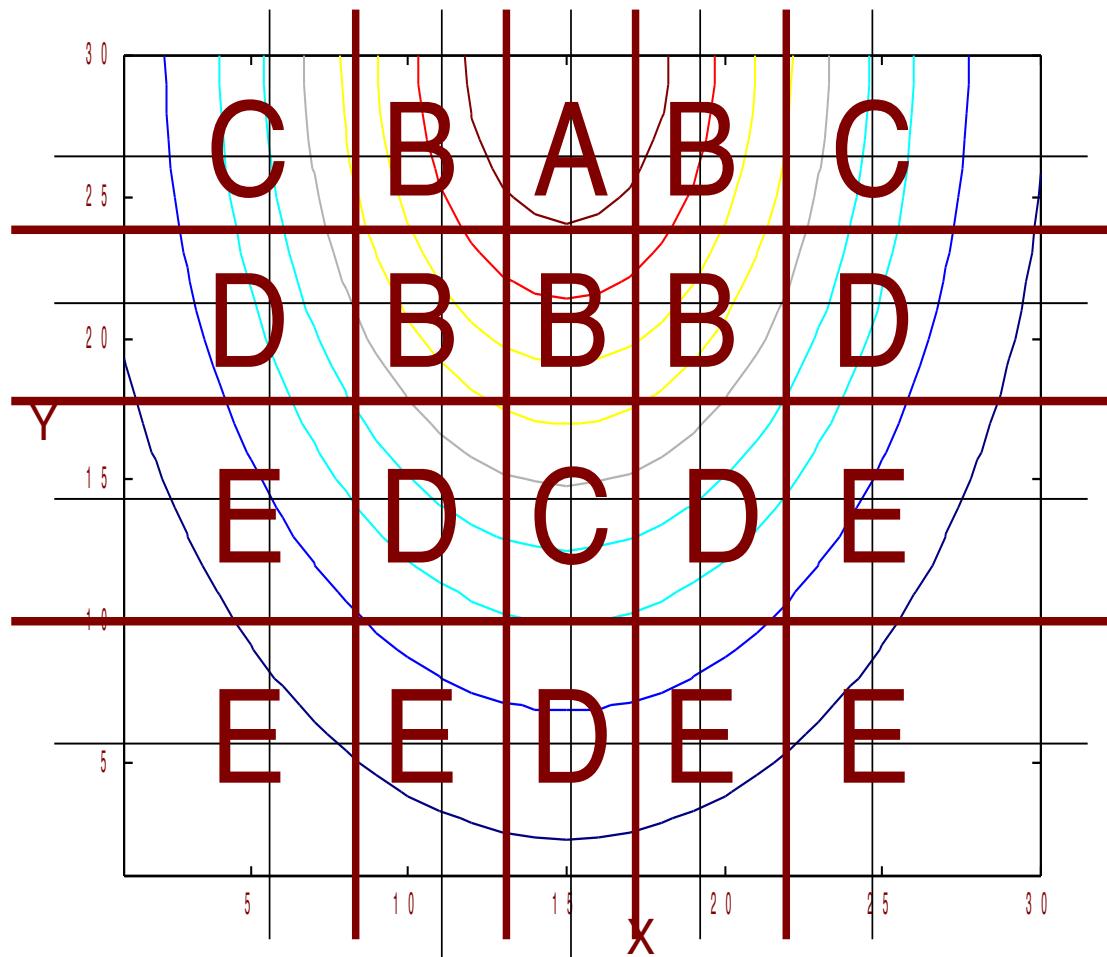
*How many rows & columns do you need to define a table?*



# Design Example of Zadeh Fuzzy Controller

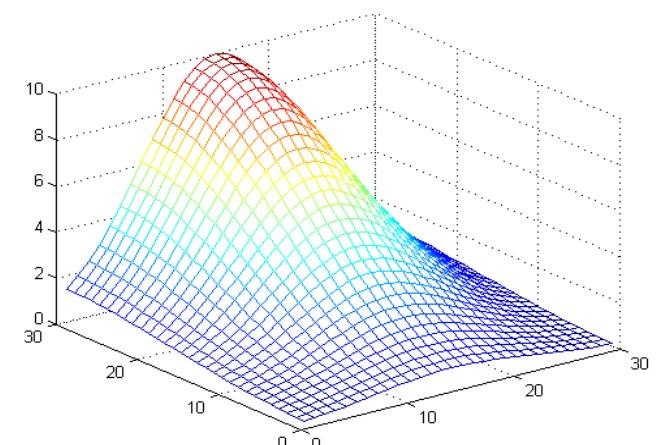


# Design Example of Zadeh Fuzzy Controller



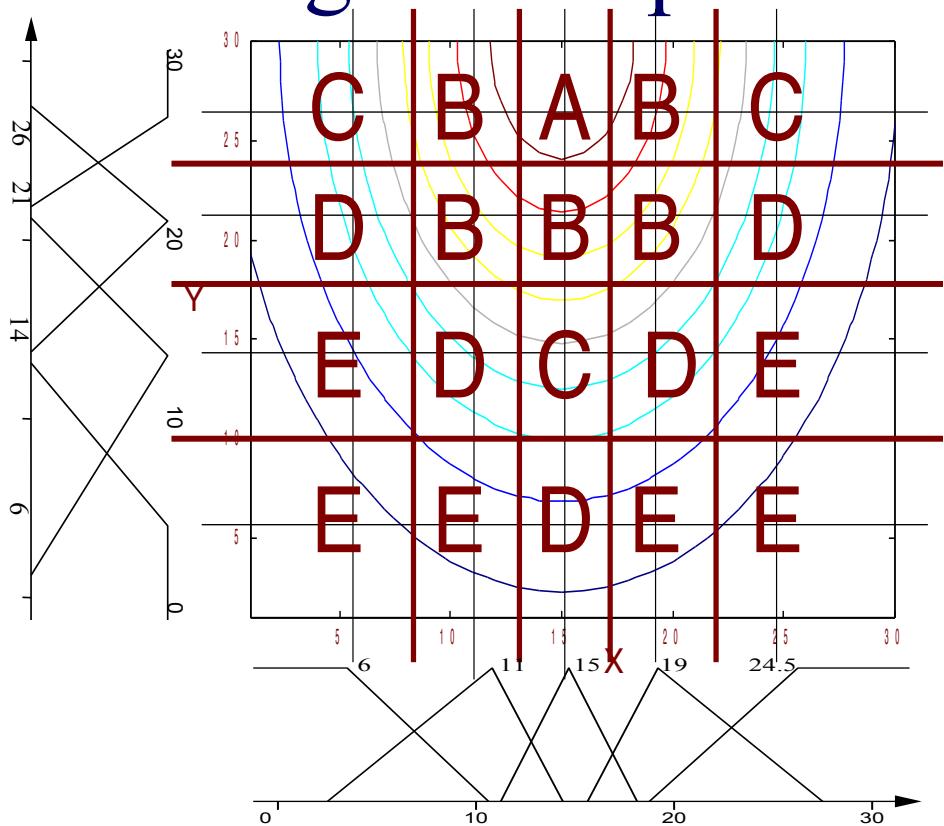
outputs (*singletons*)

5 levels on the output

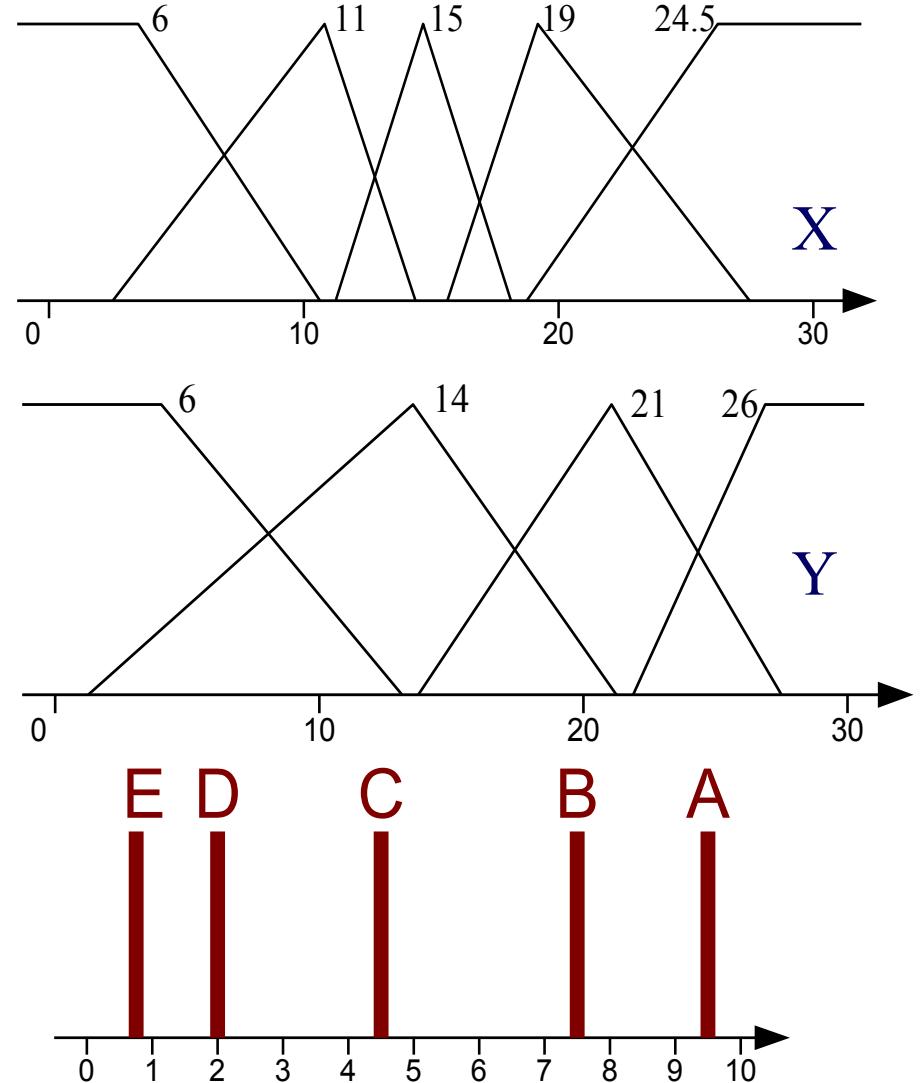


*note the different dimensions or natures of I/O parameters...*

# Design Example of Zadeh Fuzzy Controller

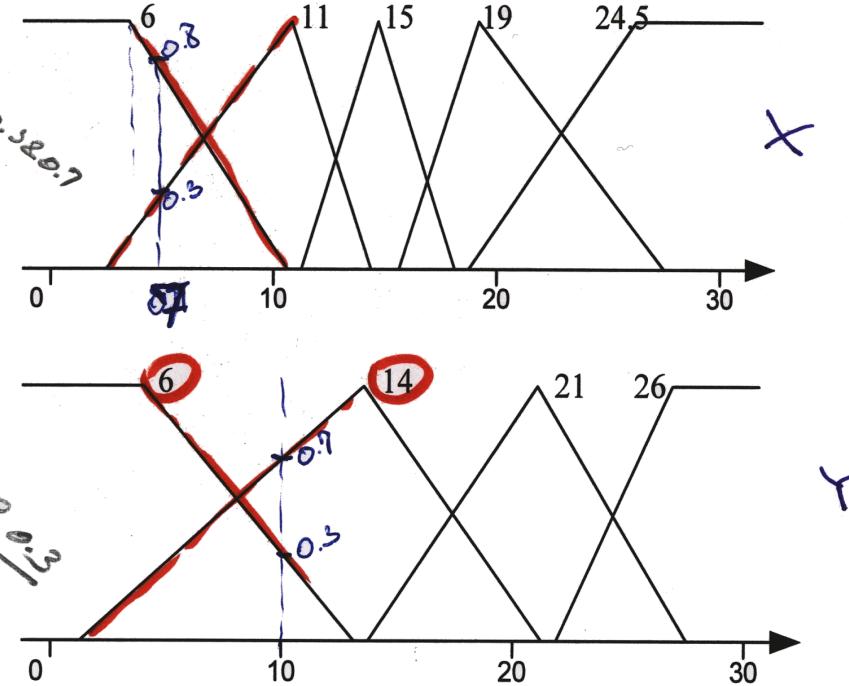
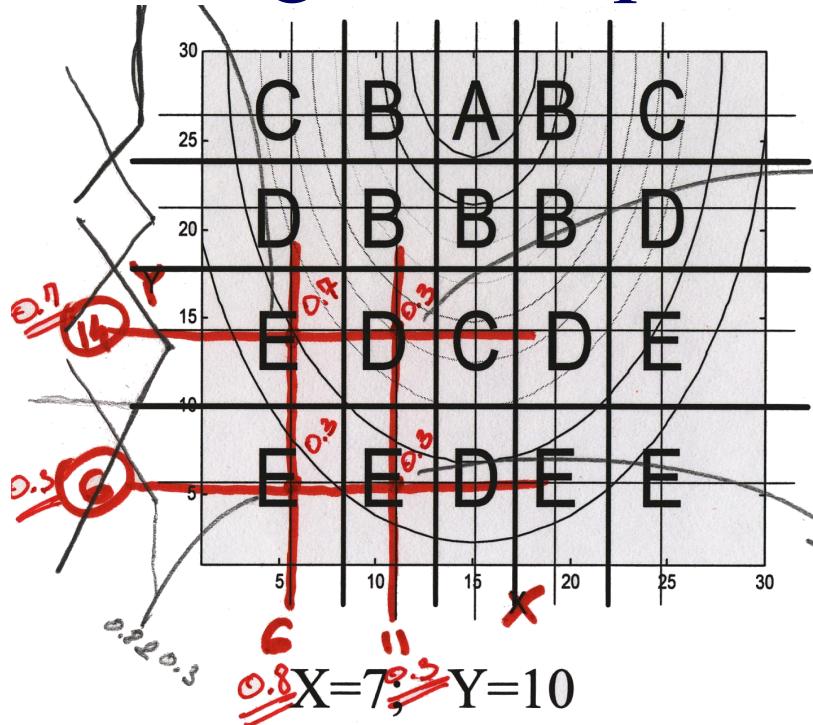


X=7; Y=10;



E=0.7; D=3; C=3.5; B=7.5; A=9.5;  
Session 20,21, Updated on 11/6/15 9:54 AM

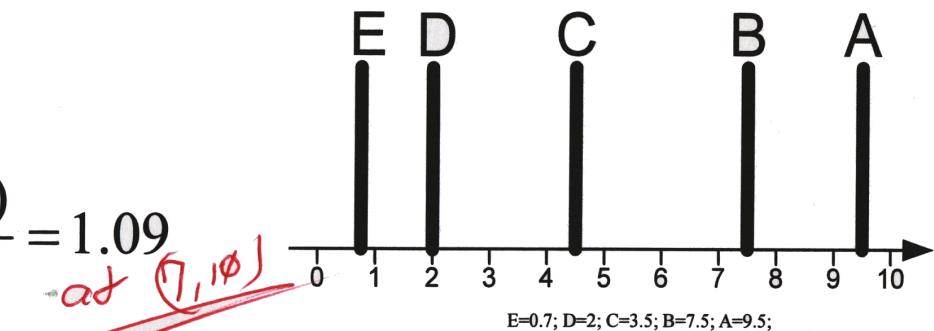
# Design Example of Zadeh Fuzzy Controller



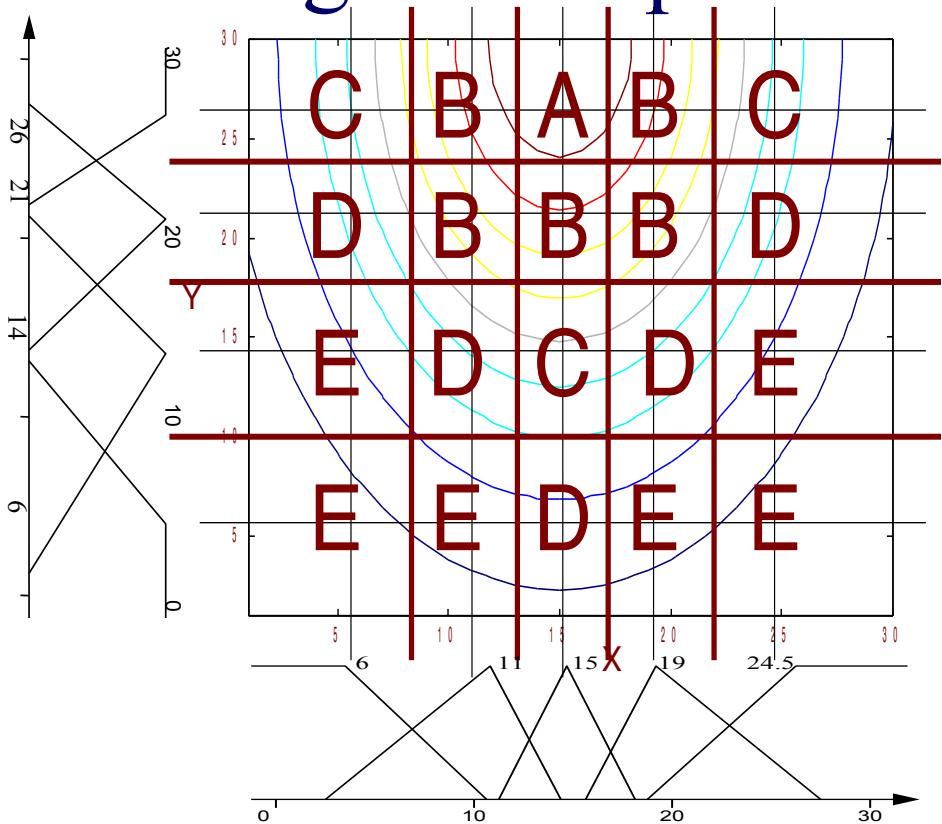
$$\max(E)=0.7;$$

$$\max(D)=0.3;$$

$$out = \frac{0.7 \cdot 0.7(E) + 0.3 \cdot 2(D)}{0.7 + 0.3} = 1.09$$



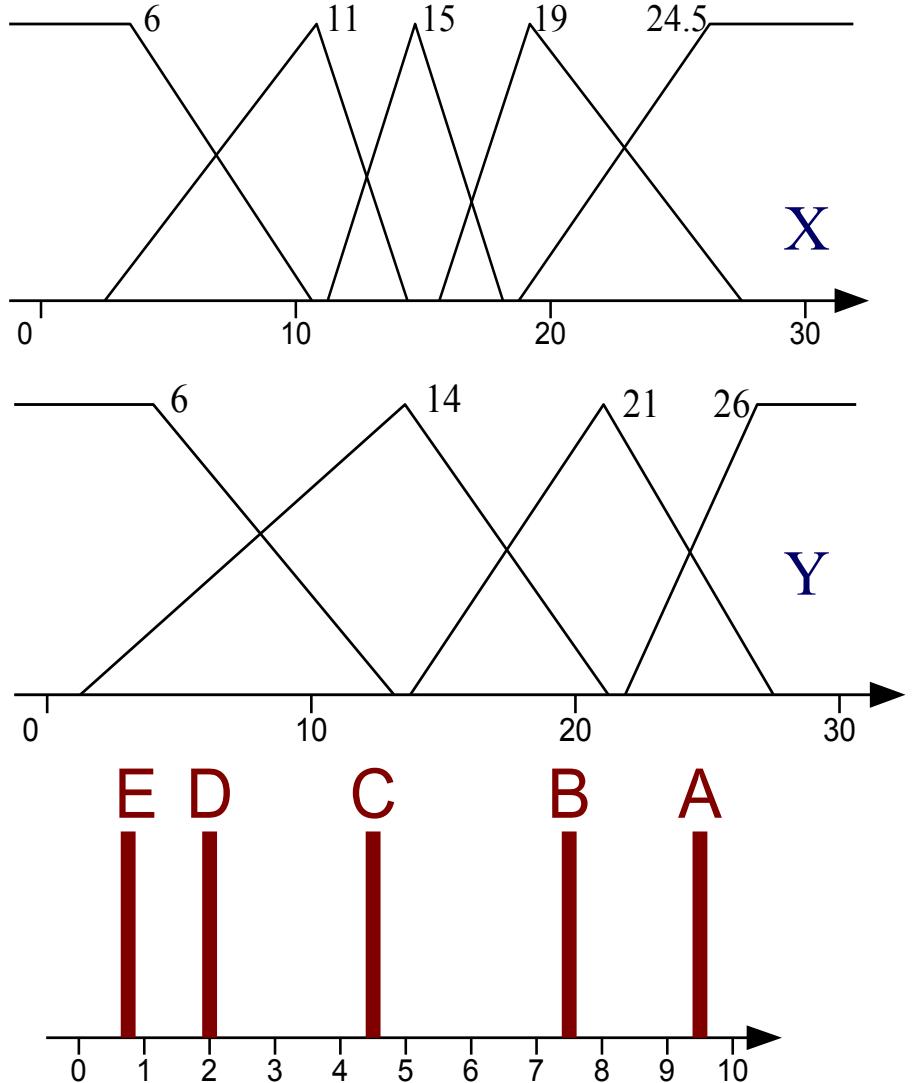
# Design Example of Zadeh Fuzzy Controller



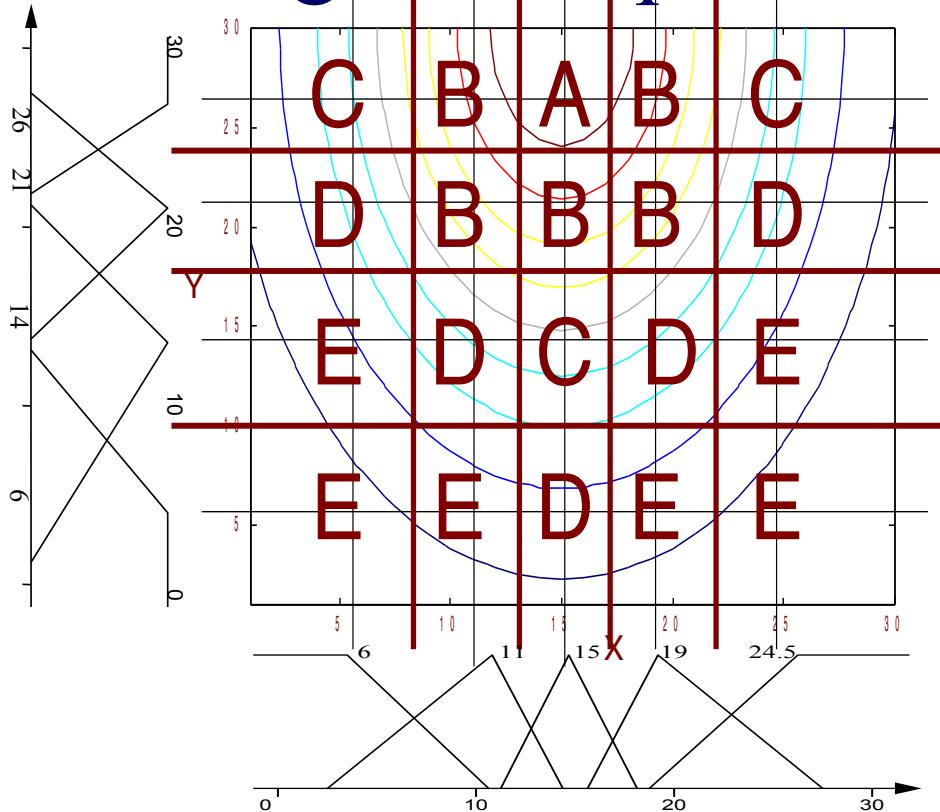
$$\max(E)=0.7; \\ \max(D)=0.3;$$

$$X=7; \quad Y=10;$$

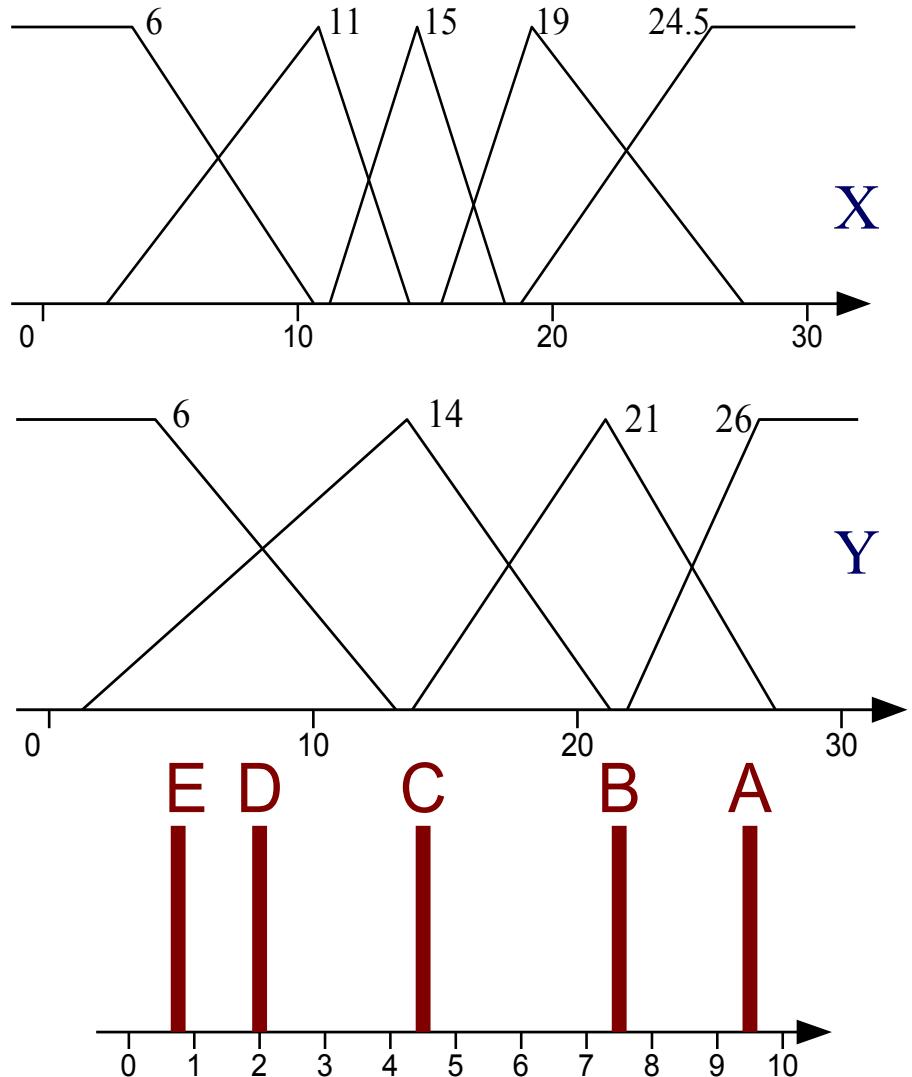
$$\text{out} = \frac{0.7 \cdot 0.7_{(E)} + 0.3 \cdot 2_{(D)}}{0.7 + 0.3} = 1.09$$



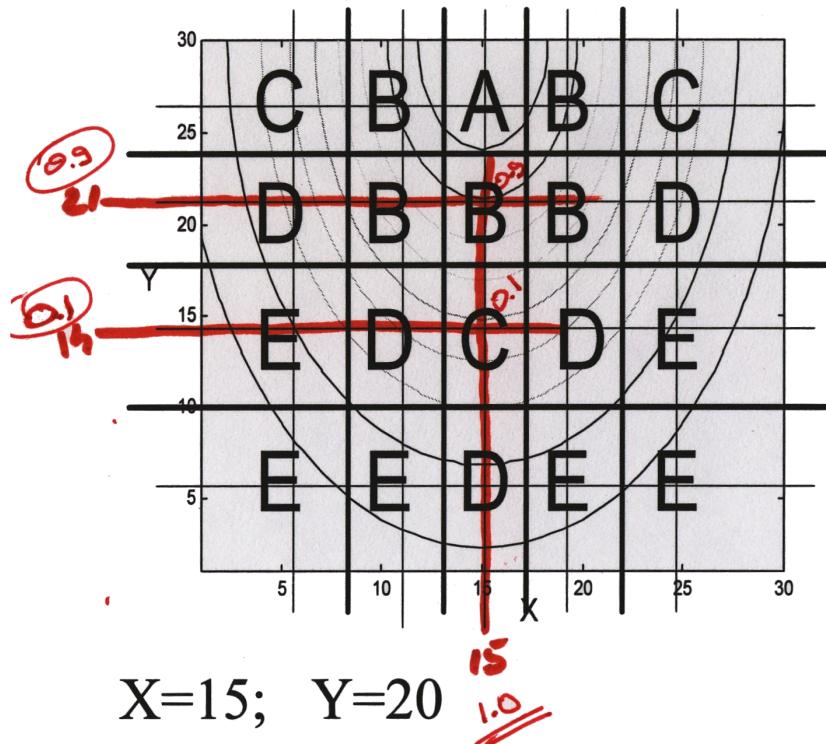
# Design Example of Zadeh Fuzzy Controller



X=15; Y=20;



# Design Example of Zadeh Fuzzy Controller

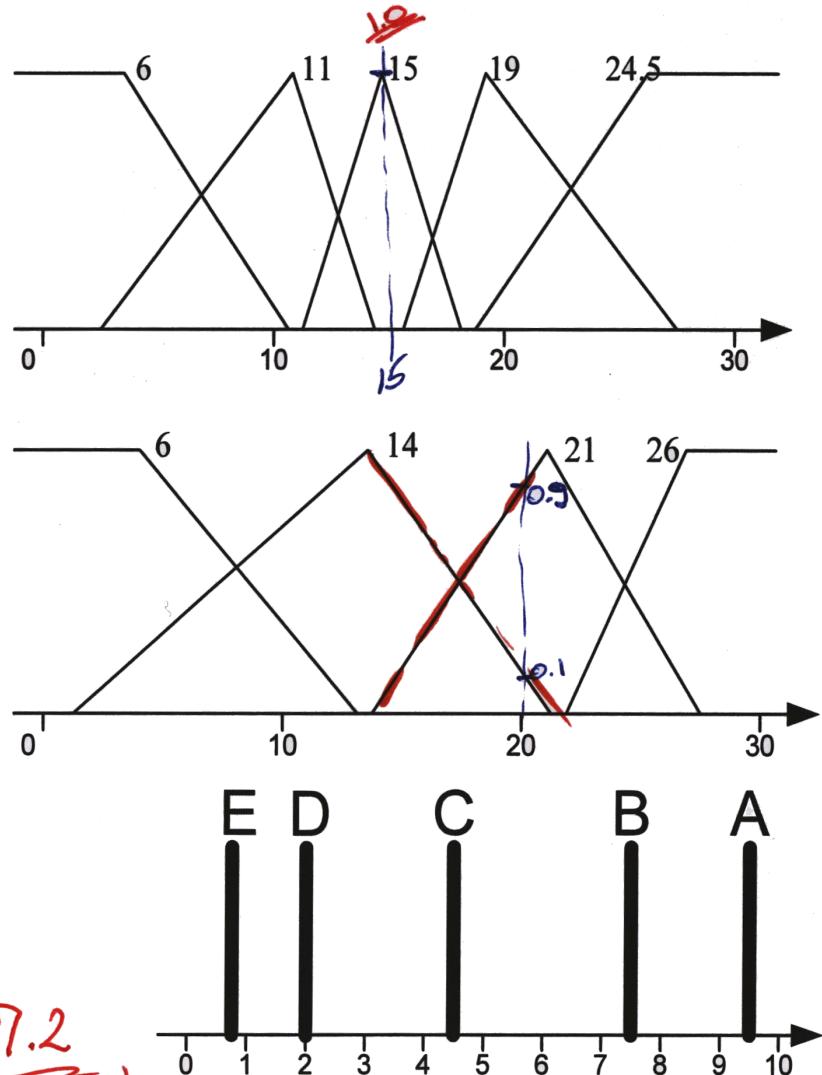


$$\max(C) = 0.1$$

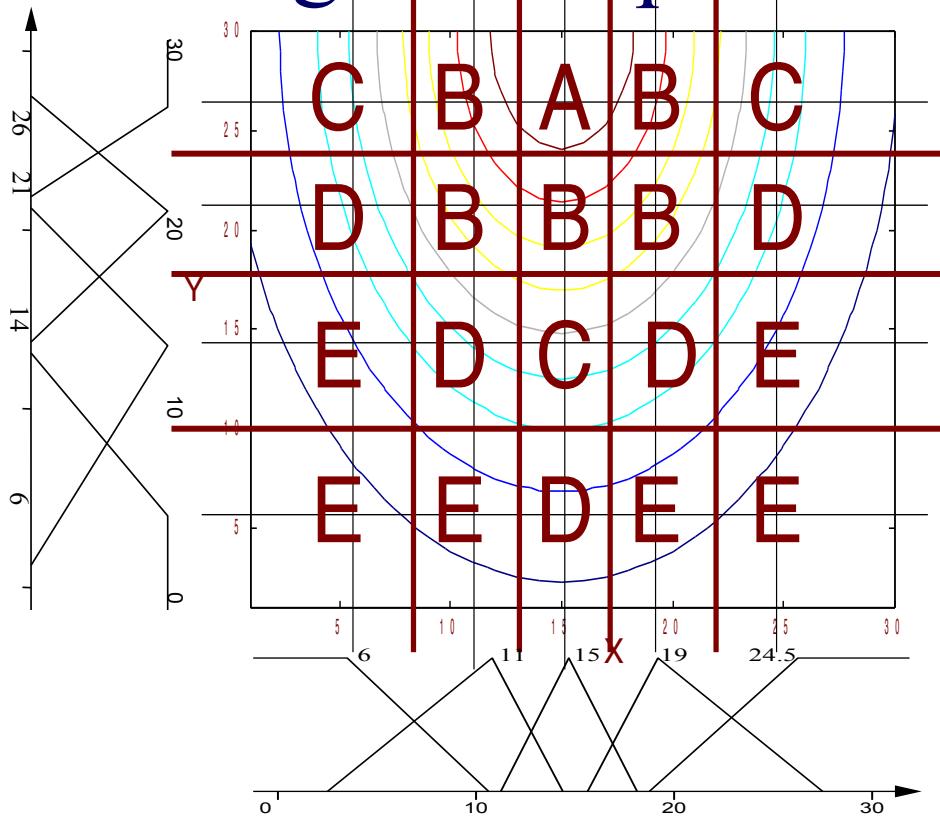
$$\max(B) = 0.9$$

$$out = \frac{0.9 \cdot 7.5(B) + 0.1 \cdot 4.5(C)}{0.9 + 0.1} = 7.2$$

~~at (15, 20)~~

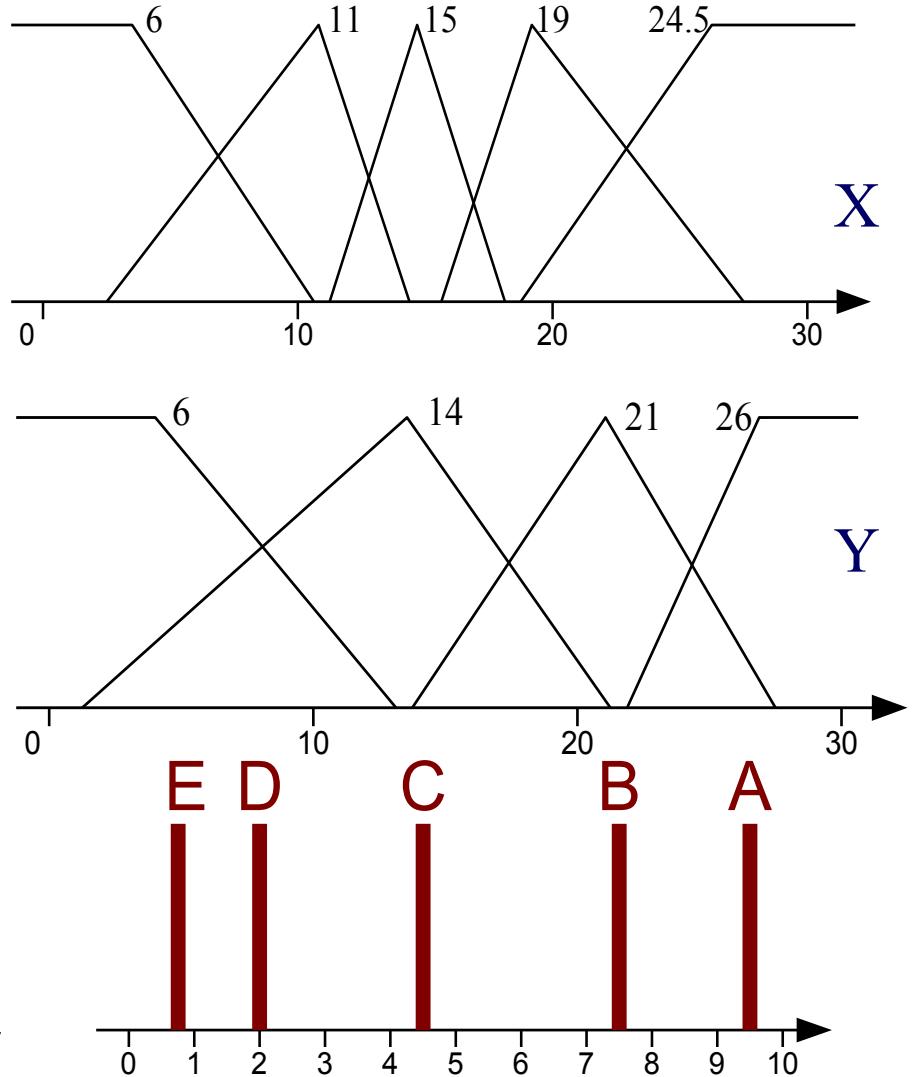


# Design Example of Zadeh Fuzzy Controller



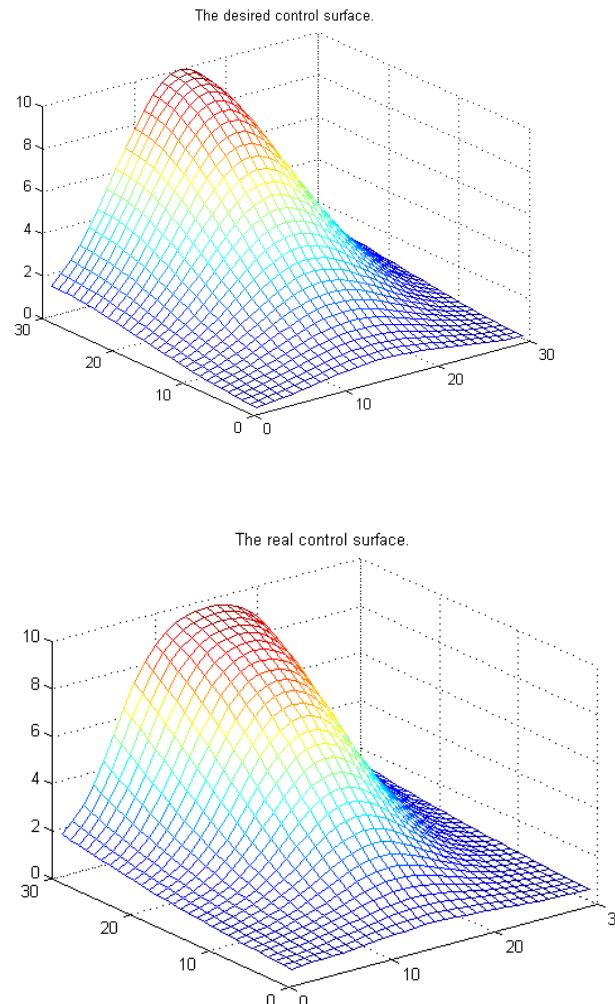
$X=15; \quad Y=20; \quad \max(C)=0.1;$   
 $\max(B)=0.9;$

$$\text{out} = \frac{0.9 \cdot 0.75_{(B)} + 0.1 \cdot 4.5_{(C)}}{0.9 + 0.1} = 7.2$$

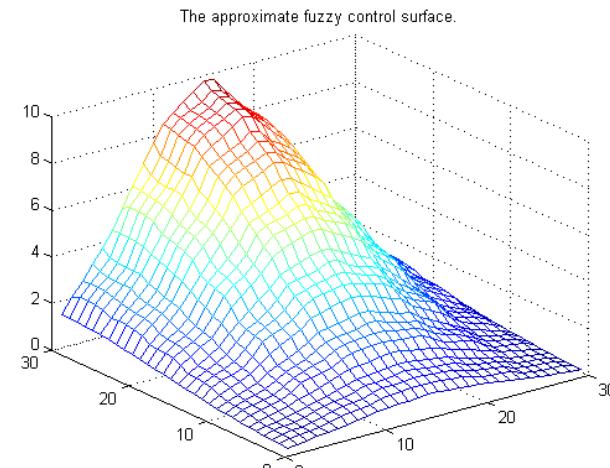


# Design Examples of Fuzzy Controller

## Required surface



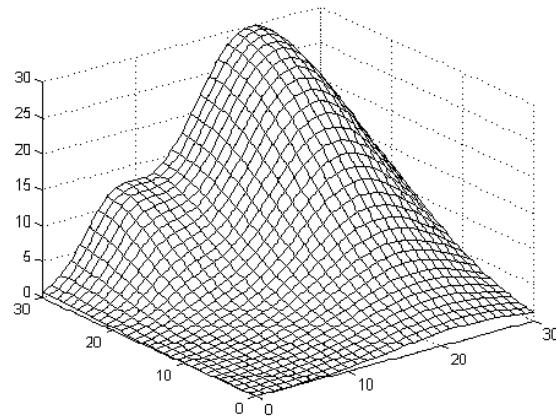
Sugeno,  
7\*7 triangular memb. functions



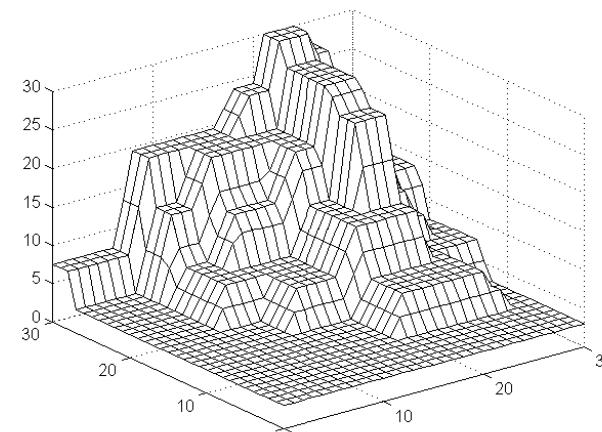
- 14 input values stored
- 49 output values stored
- 63 values together

Neural network, two hidden neurons,  
9 weight values stored.

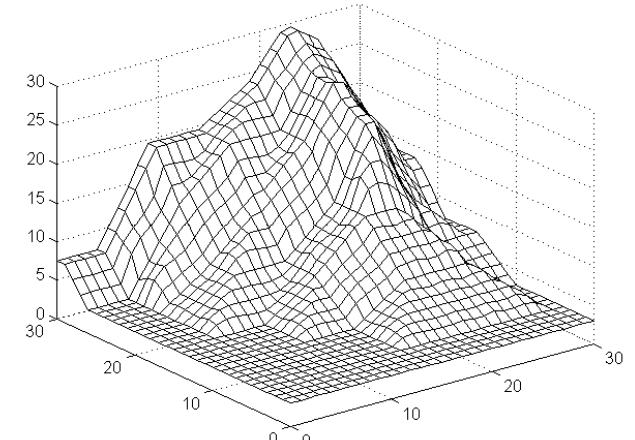
# Various Implementations of Fuzzy Systems



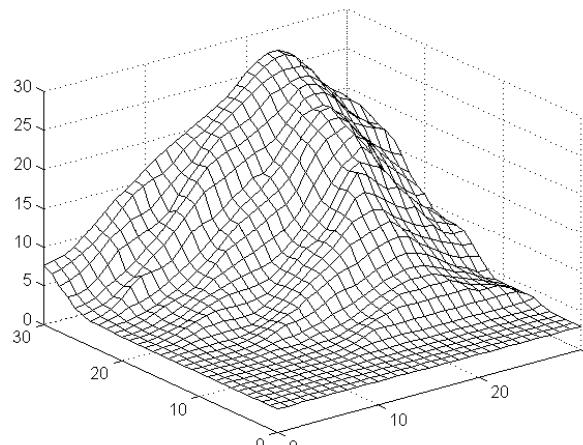
Required  
control surface



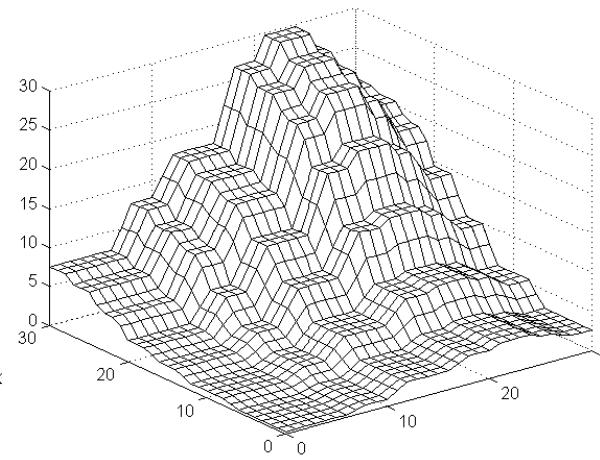
Zadeh,  
trapezoidal memb. functions



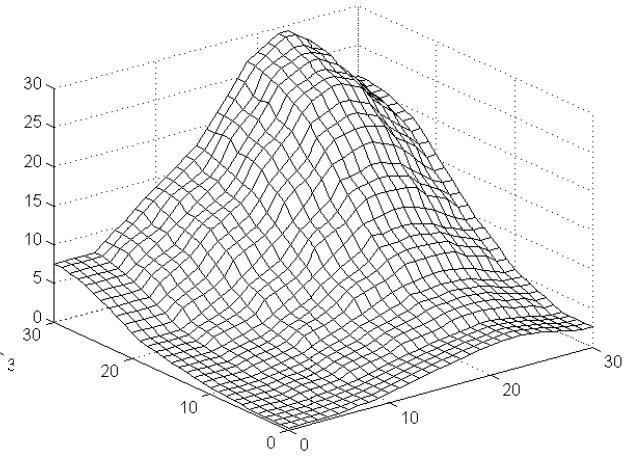
Zadeh,  
triangular memb. functions



Zadeh,  
Gaussian memb. functions



Takagi-Sugeno,  
trapezoidal memb. functions



Takagi-Sugeno,  
triangular memb. functions