AC and DC Hall Effect experiment on a 300μm Si doped wafer

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Abstract

The Hall Effect is a phenomena of solid materials that has been in use since the turn of the century. In this experiment we used the AC and DC hall effect to see the properties of a Si doped semiconductor sample including: the type of semiconductor, carrier mobility and density and the resistivity of the sample. It was determined the sample was an n-type emiconductor, with a temperature dependent carrier density with values: $3.08x10^{20} \pm 0.09x10^{20}$ at 80K, $3.1x10^{20} \pm 0.3x10^{20}$ at 100K, $3.06x10^{20} \pm 0.07x10^{20}$ at 120K, $2.83x10^{20} \pm 0.04x10^{20}$ at 140K, $2.8x10^{20} \pm 0.1x10^{20}$ at 160K, $3.5x10^{20} \pm 0.1x10^{20}$ at 180K, $3.12x10^{20} \pm 0.07x10^{20}$ at 200K, $3.8x10^{20} \pm 0.2x10^{20}$ at 220K, $3.8x10^{20} \pm 0.2x10^{20}$ at 240K and $3.9x10^{20} \pm 0.2x10^{20}$ at 260K. With temperature dependent values of: $1.44x10^{-2} \pm 0.05x10^{-2}$ at 80K, $1.5x10^{-2} \pm 0.1x10^{-2}$ at 100K, $1.49x10^{-2} \pm 0.03x10^{-2}$ at 120K, $1.63x10^{-2} \pm 0.03x10^{-2}$ at 140K, $1.7x10^{-2} \pm 0.07x10^{-2}$ at 160K, $1.47x10^{-2} \pm 0.06x10^{-2}$ at 180K, $1.77x10^{-2} \pm 0.04x10^{-2}$ at 200K, $1.57x10^{-2} \pm 0.07x10^{-2}$ at 220K, $1.68x10^{-2} \pm 0.09x10^{-2}$ at 240K and $1.69x10^{-2} \pm 0.07x10^{-2}$ at 260K.

I Introduction

The hall Effect was discovered by Edwin H. Hall in 1879, who at the time was studying Rowland at Johns Hopkins University. During those days no one knew of the electron and by extension how it was that conduction actually happened. Due to this it took almost 50 years until the Hall Effect was fully understood with the formulation of quantum mechanics. The results that were gotten from the experiment were generally not very well understood [1].
However, when quantum mechanics was formulated and the Hall Effect became fully understood it began to be employed in the study of semiconductors. It was here that it fullfilled its promise in the study of the concentration and sign of charge carriers [1]. Both of which are to be found in this experiment.
There are four different types of materials: insulators and superconductors, semiconductors and conductors. The difference between all of these are the resistivity and conductivity values for them. Insulators and superconductors behave in a very similar fashion in that they allow almost no current to pass through them, they have near infinite resistivities and near zero conductivity values. Conductors are the polar opposite of them having very low resistivity values and extremely high conductivity. Semiconductors are the middle ground where the two meet having finite resistivity and conductivity values.
Semiconductors are an integral part in our daily lives since they have been in wide use in electronics such as: computers, cell phones and LED bulbs. The Hall Effect has given a far reaching insight into how and why it is that the properties listed above display such distinct behaviors.
For the experiment a large electromagnet with a magnetic field range of 0 T to 0.6678 T was used to create a homogenous strong magnetic field. To receive the signals a lock-in amplifier was used for the AC Hall Voltage measurements and a simple multi-meter was used to measure the DC Hall Voltage measurements. The temperature of the sample could also be controlled via a PID temperature controller that allowed to heat up the sample and it was cooled down through the use of Liquid Nitrogen (LN). The sample was housed in a cryostat chamber containing a very dilute Helium gas concentration to be a thermal contact between the inner chamber and the LN.
The sample was a Silicon doped semiconductor which we put at cryogenic temperatures and in a high intensity magnetic field to perform the measurements.

II Theory

From electricity and magnetism we know that charged particles passing through a magnetic field display the following force relation,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1}$$

Where, q is the charge of the particle, $\vec{E}$ is the electric field in the space of the particle, $\vec{v}$ is its

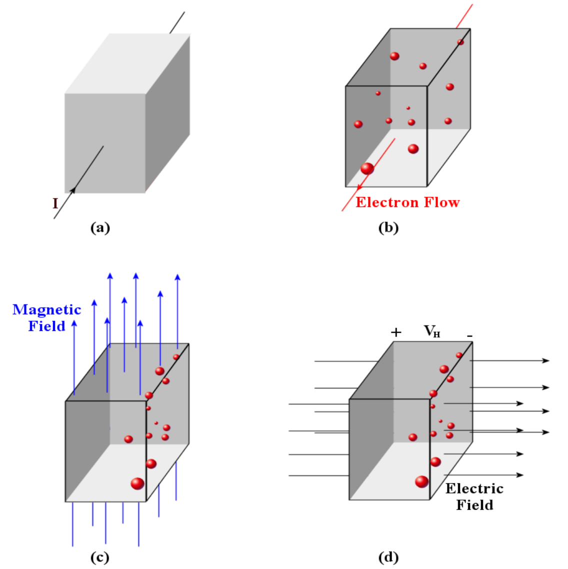

Figure 1: a). Representation of the general setup wiyh the sample and a wire providing current through the sample. b). A general representation of the flow of electrons in the sample. c). The general shift in the position of the electrons as they pass through a magnetic field. d). Electric field that is produced when the electrons distribute themselves in such a manner [2].

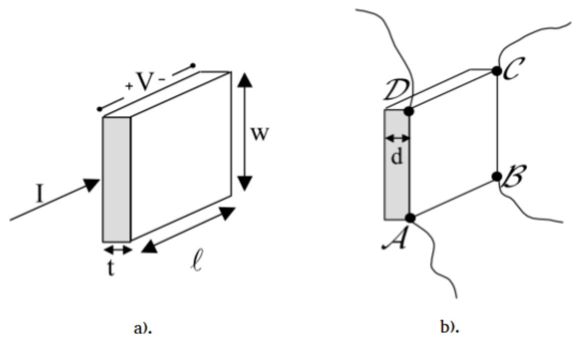

Figure 2: a). Current passing through a sample of dimension l,w,t. b). Sample of depth d with four contacts placed on the corners of the sample.

velocity and $\vec{B}$ is the magnetic field. Usually, we can consider the contribution of the electric field to the force to be zero so we can write equation 1 as,

$$\vec{F} = q\vec{v} \times \vec{B} \tag{2}$$

So that is to say that the charge carriers will receive a force that is perpendicular to their motion and as a cause will begin to curve. If we now think about a solid material through which we pass some current it would look like figure 1b. Where the electrons are free to move through the entire sample and have nothing directing their movement other than the electric current that is passing through the sample. For a pictorial representation of what happend when we apply a strong magnetic field perpendicular to the sample refer to figures 1c,d. This makes sense with equation 2 and what we generally know of charged particles passing through a perpendicular magnetic field. Looking at figure 1d the reason why it does in fact produce an electric field inside of the sample is due to the electrons that have curved to a side of the sample and since they can't conduct very easily through the a air they accumulate at the boundary of the sample toward which they curve creating a gradient of electrons and lack of electrons (positive charge) which gives rise to a potential difference through the sample and therefore an electric field through the sample.
This perpendicular magnetic field can be calculated with equation 3.

$$\vec{E}_t = \frac{-\vec{J} \times \vec{B}}{nq}[1] \tag{3}$$

Where, $\vec{J}$ is the current density, $\vec{B}$ is the magnetic field passing through the sample,n is the number carriers and q is the charge of the carriers.
Equation 3 is mainly useful were qe to know all of the parameters we need which, for this experiment, we need to find. So we take a different approach in our equation derivations starting with how to calculate the resistivity of a semiconductor sample.
When we have a sample with a current passing through it of certain current density, j, We can show that the electric field is related in the following way,

$$\vec{E} = \rho\vec{j}[3] \tag{4}$$

Where, ρ is the resistivity of the sample. By multiplying both sides of equation 3 by the length of the sample, l, we can get a relation between the voltage and the resistivity.

$$V = \rho \frac{l}{tw} I[3] \quad (5)$$

Where, l,t,w are the dimensions shown of figure 2b and I is the total current through the sample. By then applying Ohm's law, V=IR, we can get to the relation of resistance with resistivity.

$$R = \rho \frac{l}{wt} = R_s \frac{l}{w}[3] \quad (6)$$

Where, $R_s = \rho/t$. R_s is the sheet resistance of the sample and it is only dependent on the thickness of the sample as per the equation. Notice that if the sample is perfectly square, l=w, $R_s = R$ [3]. By performing a two contact resistivity measurement we can receive values for the resistivity of the sample. However, as it will be more apparent later these values that we get do have a certain amount of error and a more accurate method can be employed. This method is the va der Pauw method which was developed in 1958 by Leo J. van der Pauw.

To use this method the following conditions must be met [4].

a The contacts are at the circumference of the sample.

b The contacts are sufficiently small.

c The sample is homogeneous in thickness.

d The surface of the sample is singly connected,i.e.,the sample does not have isolated holes.

Now, the first condition assumes that the sample is a disc, but this method can work for any shape like a square. The derivation of the method can be found in [3] Now, the first condition assumes that the sample is a disc, but this method can work for any shape like a square. The derivation of the method can be found in [3]. In figure 2b we see that the contacts are placed in the corners. They do not have to necessarily be on the corners. They can be anywhere along the edge of the sample. According to van der Pauw's paper the resistivity of a sample using the four contact method can be calculated with the following equation.

$$\rho = \frac{\pi d}{\ln 2} \frac{R_{AB,CD} + R_{BC,DA}}{2} f \left(\frac{R_{AB,CD}}{R_{BC,DA}} \right) [4] \quad (7)$$

Where, d is the thickness of the sample (t in the case of fig. 2b), $R_{AB,CD}$ and $R_{BC,AD}$ represent current passing through the point and making the voltage measurement respectively. That is to say they are along the perpendicular current directions as can be inferred from figure 2b. The function f is a function that behaves in similar fashion to that which is shown on figure 3. Where it only satisfies the relation.

$$\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}} = f \operatorname{arccosh} \frac{\exp \ln 2/f}{2} [4] \quad (8)$$

Which can only be solved numerically to solve for f. Van der Pauw does provide another equation to approximate f where the ratio of $R_{AB,CD}/R_{BC,DA}$ are almost equal.

$$f \approx 1 - \left(\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}} \right)^2 \frac{\ln 2}{2} - \left(\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}} \right)^4 \left\{ \frac{(ln2)^2}{4} - \frac{(\ln 2)^2}{12} \right\} [4] \quad (9)$$

There is another method that can be used in which we use a table which is provided to us. However, to reduce errors from approximating to the second decimal point the f value is calculated for every ratio and was compared to the given f values. They agreed to the second decimal and was believed to be a better approximation than those given in the table.

In general, we can represent the resistivity and conductivity of a sample of comparable size as a tensor meaning that the electric field and the current density may not necessarily be parallel to one another. Such matrices are the following.

$$\rho = \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{bmatrix}$$

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{yy} \end{bmatrix}$$

Where, the x and y subscripts represent the sheet coordinate directions of the sample ignoring the z. Back in equation 3 we showed the relation between the current density and the magnetic field to find the electric field that is produced by the Hall Effect. Here, a different form of it will be shown using vectors and tensors.

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{bmatrix} \begin{bmatrix} j_x \\ 0 \end{bmatrix} [3] \quad (10)$$

Where, $j_y = 0$ since there is no input current along the y direction of the sample only along the x direction. When equation **??** is solved for the electric

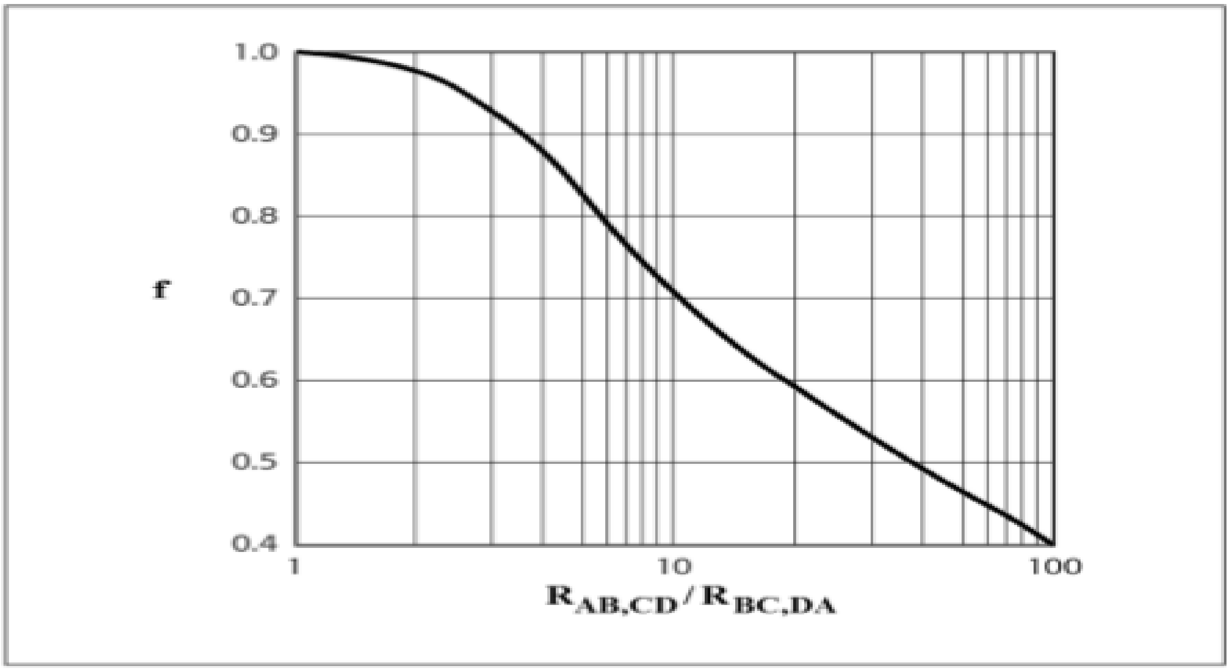

Figure 3: Plot of the f function from equation 7 [3].

field components we can get the following equations.

$$\begin{aligned} E_x &= \rho_{xx} j_x \\ E_y &= -\rho_{xy} j_x [3] \end{aligned} \tag{11}$$

Where, if we transform equation **??** as we did with equation 5 we get a relation between V_Hall and the xy resistance of the sample. Keep in mind that it is the Hall voltage as the field in the y direction is a direct cause of the magnetic filed that is along the z direction.

$$V_{Hall} = R_{xy} I [3] \tag{12}$$

Where, R_{xy} is given by the equation.

$$R_{xy} = \frac{R_H}{t} B [3] \tag{13}$$

References

[1] Purcell, E. M.; Morin, D. J. (2013). Electricity and Magnetism. New York: *Cambridge University Press*, 314-317.

[2] (2016,December 24).Van der Pauw method. *Wikipedia*. https://en.wikipedia.org/wiki/Van_der_Pauw_method

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[4] van der Pauw, L. J. (1958). A method of measuring specific resistivity and Hall Effect of discs of arbitrary shape. *Philips Research reports, 13*, 1-9.