AC and DC Hall Effect experiment on a 300μm Si doped wafer

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April 10, 2017

Abstract

The Hall Effect is a phenomena of solid materials that has been in use since the turn of the century. In this experiment we used the AC and DC hall effect to see the properties of a Si doped semiconductor sample including: the type of semiconductor, carrier mobility and density and the resistivity of the sample. It was determined the sample was an n-type emiconductor, with a temperature dependent carrier density with values: $3.08x10^{20} \pm 0.09x10^{20}$ at 80K, $3.1x10^{20} \pm 0.3x10^{20}$ at 100K, $3.06x10^{20} \pm 0.07x10^{20}$ at 120K, $2.83x10^{20} \pm 0.04x10^{20}$ at 140K, $2.8x10^{20} \pm 0.1x10^{20}$ at 160K, $3.5x10^{20} \pm 0.1x10^{20}$ at 180K, $3.12x10^{20} \pm 0.07x10^{20}$ at 200K, $3.8x10^{20} \pm 0.2x10^{20}$ at 220K, $3.8x10^{20} \pm 0.2x10^{20}$ at 240K and $3.9x10^{20} \pm 0.2x10^{20}$ at 260K. With temperature dependent values of: $1.44x10^{-2} \pm 0.05x10^{-2}$ at 80K, $1.5x10^{-2} \pm 0.1x10^{-2}$ at 100K, $1.49x10^{-2} \pm 0.03x10^{-2}$ at 120K, $1.63x10^{-2} \pm 0.03x10^{-2}$ at 140K, $1.7x10^{-2} \pm 0.07x10^{-2}$ at 160K, $1.47x10^{-2} \pm 0.06x10^{-2}$ at 180K, $1.77x10^{-2} \pm 0.04x10^{-2}$ at 200K, $1.57x10^{-2} \pm 0.07x10^{-2}$ at 220K, $1.68x10^{-2} \pm 0.09x10^{-2}$ at 240K and $1.69x10^{-2} \pm 0.07x10^{-2}$ at 260K.

I Introduction

The hall Effect was discovered by Edwin H. Hall in 1879, who at the time was studying under Rowland at Johns Hopkins University. During those days no one knew of the electron and by extension how it was that conduction actually happened. Due to this it took almost 50 years until the Hall Effect was fully understood with the formulation of quantum mechanics. The results that were gotten from the experiment were generally not very well understood [1].

However, when quantum mechanics was formulated and the Hall Effect became fully understood it began to be employed in the study of semiconductors. It was here that it fullfilled its promise in the study of the concentration and sign of charge carriers [1]. Both of which are to be found in this experiment.

There are four different types of materials: insulators and superconductors, semiconductors and conductors. The difference between all of these are the resistivity and conductivity values for them. Insulators and superconductors behave in a very similar fashion in that they allow almost no current to pass through them, they have near infinite resistivities and near zero conductivity values. Conductors are the polar opposite of them having very low resistivity values and extremely high conductivity. Semiconductors are the middle ground where the two meet having finite resistivity and conductivity values.

Semiconductors are an integral part in our daily lives since they have been in wide use in electronics such as: computers, cell phones and LED bulbs. The Hall Effect has given a far reaching insight into how and why it is that the properties listed above display such distinct behaviors.

For the experiment a large electromagnet with a magnetic field range of 0 T to 0.6678 T was used to create a homogenous strong magnetic field. To receive the signals a lock-in amplifier was used for the AC Hall Voltage measurements and a simple multi-meter was used to measure the DC Hall Voltage measurements. The temperature of the sample could also be controlled via a PID temperature controller that allowed to heat up the sample and it was cooled down through the use of Liquid Nitrogen (LN). The sample was housed in a cryostat chamber containing a very dilute Helium gas concentration to be a thermal contact between the inner chamber and the LN.

The sample was a Silicon doped semiconductor which we put at cryogenic temperatures and in a high intensity magnetic field to perform the measurements.

II Theory

From electricity and magnetism we know that charged particles passing through a magnetic field display the following force relation,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1}$$

Where, q is the charge of the particle, $\vec{E}$ is the electric field in the space of the particle, $\vec{v}$ is its

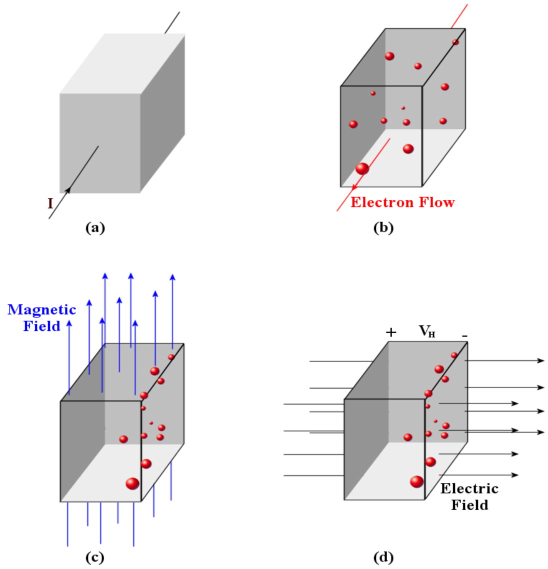


Figure 1: a). Representation of the general setup wiyh the sample and a wire providing current through the sample. b). A general representation of the flow of electrons in the sample. c). The general shift in the position of the electrons as they pass through a magnetic field. d). Electric field that is produced when the electrons distribute themselves in such a manner [2].

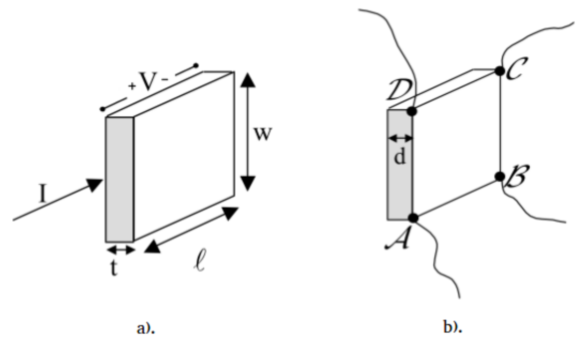


Figure 2: a). Current passing through a sample of dimension l,w,t. b). Sample of depth d with four contacts placed on the corners of the sample.

velocity and $\vec{B}$ is the magnetic field. Usually, we can consider the contribution of the electric field to the force to be zero so we can write equation 1 as,

$$\vec{F} = q\vec{v} \times \vec{B} \tag{2}$$

So that is to say that the charge carriers will receive a force that is perpendicular to their motion and as a cause will begin to curve. If we now think about a solid material through which we pass some current it would look like figure 1b. Where the electrons are free to move through the entire sample and have nothing directing their movement other than the electric current that is passing through the sample. For a pictorial representation of what happend when we apply a strong magnetic field perpendicular to the sample refer to figures 1c,d. This makes sense with equation 2 and what we generally know of charged particles passing through a perpendicular magnetic field. Looking at figure 1d the reason why it does in fact produce an electric field inside of the sample is due to the electrons that have curved to a side of the sample and since they can't conduct very easily through the a air they accumulate at the boundary of the sample toward which they curve creating a gradient of electrons and lack of electrons (positive charge) which gives rise to a potential difference through the sample and therefore an electric field through the sample.
This perpendicular magnetic field can be calculated with equation 3.

$$\vec{E}_t = \frac{-\vec{J} \times \vec{B}}{nq}[1] \tag{3}$$

Where, $\vec{J}$ is the current density, $\vec{B}$ is the magnetic field passing through the sample,n is the number carriers and q is the charge of the carriers.
Equation 3 is mainly useful were qe to know all of the parameters we need which, for this experiment, we need to find. So we take a different approach in our equation derivations starting with how to calculate the resistivity of a semiconductor sample.
When we have a sample with a current passing through it of certain current density, j, We can show that the electric field is related in the following way,

$$\vec{E} = \rho\vec{j}[3] \tag{4}$$

Where, ρ is the resistivity of the sample. By multiplying both sides of equation 3 by the length of the sample, l, we can get a relation between the voltage and the resistivity.

$$V = \rho \frac{l}{tw} I [3] \tag{5}$$

Where, l,t,w are the dimensions shown of figure 2b and I is the total current through the sample. By then applying Ohm's law, V=IR, we can get to the relation of resistance with resistivity.

$$R = \rho \frac{l}{wt} = R_s \frac{l}{w} [3] \tag{6}$$

Where, $R_s = \rho/t$. R_s is the sheet resistance of the sample and it is only dependent on the thickness of the sample as per the equation. Notice that if the sample is perfectly square, l=w, $R_s = R$ [3]. By performing a two contact resistivity measurement we can receive values for the resistivity of the sample. However, as it will be more apparent later these values that we get do have a certain amount of error and a more accurate method can be employed. This method is the va der Pauw method which was developed in 1958 by Leo J. van der Pauw.

To use this method the following conditions must be met [4].

a The contacts are at the circumference of the sample.

b The contacts are sufficiently small.

c The sample is homogeneous in thickness.

d The surface of the sample is singly connected,i.e.,the sample does not have isolated holes.

Now, the first condition assumes that the sample is a disc, but this method can work for any shape like a square. The derivation of the method can be found in [3] Now, the first condition assumes that the sample is a disc, but this method can work for any shape like a square. The derivation of the method can be found in [3]. In figure 2b we see that the contacts are placed in the corners. They do not have to necessarily be on the corners. They can be anywhere along the edge of the sample. According to van der Pauw's paper the resistivity of a sample using the four contact method can be calculated with the following equation.

$$\rho = \frac{\pi d}{\ln 2} \frac{R_{AB,CD} + R_{BC,DA}}{2} f \left(\frac{R_{AB,CD}}{R_{BC,DA}} \right) [4] \tag{7}$$

Where, d is the thickness of the sample (t in the case of fig. 2b), $R_{AB,CD}$ and $R_{BC,AD}$ represent current passing through the point and making the voltage measurement respectively. That is to say they are along the perpendicular current directions as can be inferred from figure 2b. The function f is a function that behaves in similar fashion to that which is shown on figure 3. Where it only satisfies the relation.

$$\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}} = f \operatorname{arccosh} \frac{\exp\left(\ln\left(2/f\right)\right)}{2} [4] \tag{8}$$

Which can only be solved numerically to solve for f. Van der Pauw does provide another equation to approximate f where the ratio of $R_{AB,CD}/R_{BC,DA}$ are almost equal.

$$f \approx 1 - \left(\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}} \right)^2 \frac{\ln 2}{2} - \left(\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}} \right)^4 \left\{ \frac{(ln2)^2}{4} - \frac{(\ln 2)^2}{12} \right\} [4] \tag{9}$$

There is another method that can be used in which we use a table which is provided to us. However, to reduce errors from approximating to the second decimal point the f value is calculated for every ratio and was compared to the given f values. They agreed to the second decimal and was believed to be a better approximation than those given in the table.

In general, we can represent the resistivity and conductivity of a sample of comparable size as a tensor meaning that the electric field and the current density may not necessarily be parallel to one another. Such matrices are the following.

$$\rho = \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{bmatrix}$$

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{yy} \end{bmatrix}$$

Where, the x and y subscripts represent the sheet coordinate directions of the sample ignoring the z. Back in equation 3 we showed the relation between the current density and the magnetic field to find the electric field that is produced by the Hall Effect. Here, a different form of it will be shown using vectors and tensors.

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{bmatrix} \begin{bmatrix} j_x \\ 0 \end{bmatrix} [3] \tag{10}$$

Where, $j_y = 0$ since there is no input current along the y direction of the sample only along the x direction. When equation 10 is solved for the electric

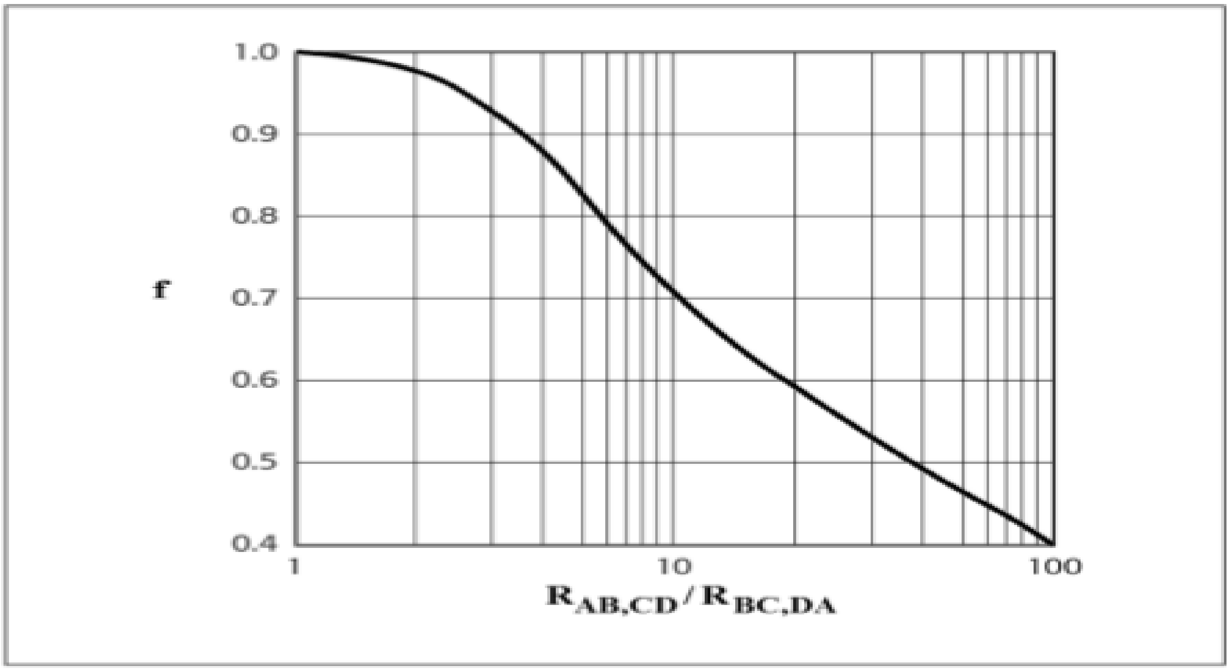


Figure 3: Plot of the f function from equation 6 [3].

field components we can get the following equations.

$$\begin{aligned} E_x &= \rho_{xx} j_x \\ E_y &= -\rho_{xy} j_x [3] \end{aligned} \tag{11}$$

Where, if we transform equation 11 as we did with equation 4 we get a relation between V_Hall and the xy resistance of the sample. Keep in mind that it is the Hall voltage as the field in the y direction is a direct cause of the magnetic filed that is along the z direction.

$$V_{Hall} = R_{xy} I [3] \tag{12}$$

Where, R_{xy} is given by the equation.

$$R_{xy} = \frac{R_H}{t} B [3] \tag{13}$$

Where, R_H is the Hall constant which depends on the carrier density and the charge of the carriers as shown.

$$R_H = \frac{1}{nq} [3] \tag{14}$$

Where, n is the number of carriers per unit volume and q is the charge of such carriers. Since, n is a integer value the sign of the Hall constant solely depends on the charge of the carriers. So, if the Hall constant is negative the charge carriers of the sample will be electrons and in the case of a positive value it will be holes (positive charge areas). In semiconductors this helps to identify what type of doping is used. If the semiconductor is n-type there is an excess of electrons where if it is p-type there is an excess of positively charged holes. Must be careful to not say protons as they do not flow when conducting electricity.

By combining equations 13 and 13 we can make the following relationship between the Hall constant and the magnetic field.

$$R_H = \frac{V_{Hall}}{B} \frac{t}{I} [3] \tag{15}$$

Where, we can also re-write the equation to make the relationship between the magnetic field and the carrier density in a sample.

$$n = \frac{B}{V_{Hall}} \frac{I}{qt} \tag{16}$$

Carrier mobility is defined as the ability of a charge carrier to pass through a sample material. The carrier mobility can be defined as the ratio between the drift velocity and the applied electric field. Which by transforming into the components of the electric field we can represent the carrier mobility as a function of the carrier density or the Hall constant.

$$\begin{aligned} \mu &= \frac{1}{\rho n q} \\ &= \frac{R_H}{\rho} \\ &= \frac{V_{Hall}}{B} \frac{t}{\rho q} [3] \end{aligned} \tag{17}$$

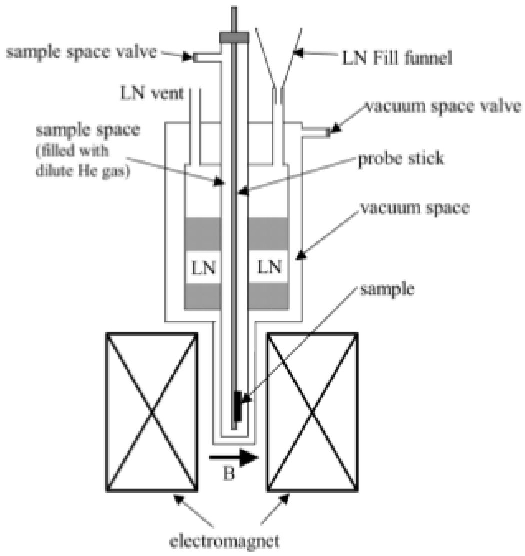


Figure 4: A general diagram of the cryostat that is to be used. There are three chambers in the cryostat [3].

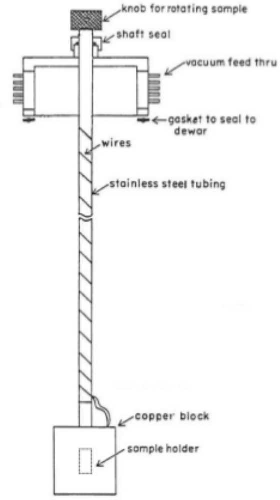


Figure 5: Schematic of the sample probe [3].

III Experimental Set-Up

For this experiment a combination of equipment was used to make the measurements including a cryostat which housed the sample probe containing the sample, a temperature controller, a lock-in amplifier, a DC source, a vacuum pump and an electromagnet.

(i) Cryostat

Since, it is necessary to make low temperature measurements in this experiment a three chamber cryostat is to be used. Where, the chambers are labeled on figure 4. The inner and outer chambers must be at a near vacuum which can be achieved with the provided vacuum pump. **NEVER add liquid nitrogen until the air in the cryostat is evacuated**. Not doing so could result in the air moisture condensing inside the cryostat when the liquid nitrogen is added. The inner chamber should be vacummed twice once to remove most of the air in the chamber and a second time by putting Helium gas in the chamber and proceeding to them vacuum the Helium gas out of the chamber. This ensures that most if not all of the air has been taken out of the chamber and only very dilute He gas is left in the chamber. The outer chamber should be vacuumed for 5 minutes as it is a much larger volume than the inner chamber. Should the air in the cryostat freeze the sample or the entire cryostat could be damaged [3].

Once this is done the liquid nitrogen can be added into the cryostat by use of the funnel. The cryostat has a max capacity of approximately 5 liters of liquid. Be careful when puring the liquid nitrogen as it will sputter when it comes into contact with the funnel. **Always wear the safety equipment provided including: cryo-gloves, goggles and the lab coat** [3].

(ii) Sample probe

The sample to be tested is mounted on a sample probe that will be placed in the cryostat by the instructor and won't need to be taken out. If there is ever a problem and it does need to be taken out of the cryostat the entire cryostat must reach room temperature before doing anything as breaking the seal in the inner chamber could damage the cryostat. The sample holder is shown on figure 5

The sample holder is actually a small box that is found at the end of the entire tube apparatus. On the sample holder the following components can be found [3].

a The suspension tube, which slides in the cap to vary the height of the specimen.

b A copper vessel around the specimen and heater, which helps to keep the temperature unifor and reduces thermal fluctuations.

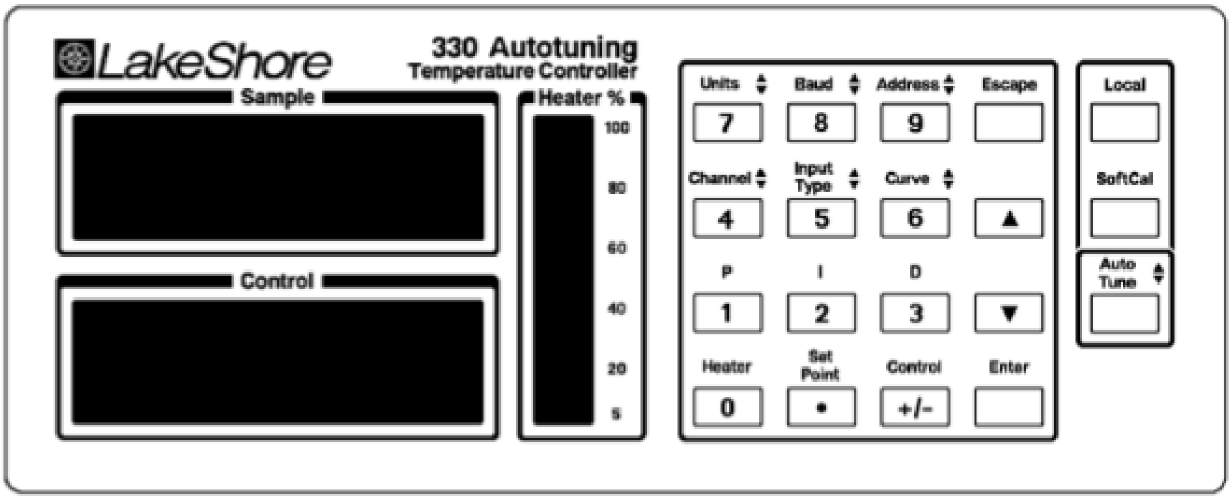


Figure 6: Front panel of the temperature controller with the labeled components [3].

c A temperature sensitive diode (DT471) that is used to measure the specimen temperature.

d A heater which balances the cooling by the cold exchange gas (helium) that will surround the can and specimen, and helps control the rate of change in temperature of the specimen.

e Sample holder which provides a flat mounting surface for the sample and the temperature sensitive diode.

The sample that is to be tested is a doped silicon wafer with a thickness of $300\mu m$. It is mounted on the sample holder with the temperature sensitive diode located behind the sample. Since, there is a relation between resistance and temperature the temperature is found by the measured voltage across the diode which is given a current in the μA range [3].

(iii) Temperature Controller

To control the temperature a Lakeshore 330 Autotuning temperature controller is used. There are three LCD screens on the controller the top screen displays the measured temperature of the diode, the bottom gives the temperature that is set for the controller to maintain and the vertical screen gives the amount of current that is passing through the diode as a percentage. The amount of current passing through the sample is set by the heater value which can be: Low, Medium or High. The amount of current that is passed through the diode at each setting is given by the following table.

Heater Range	Heater Current
HIGH	0 to 1 A
MEDIUM	0 to 0.3 A
LOW	0 to 0.1 A

Table 1: Heater current values at different set values [5]

The temperature controller utilizes a PID (Proportional-Integral-Derivative) feedback control loop to approach and maintain the temperature. The P, I, D parameters can be found on the main face of the Temperature Controller on figure 6. But, it is only necessary to leave them in the default values of 50, 25, 0 for the P, I, D respectively.

The heater setting can be changed by pressing the Heater button and cycling through the three differnt settings. The temperature change rate should not be set higher than 10 K/min in order to preserve the temperature equilibrium in the sample space. This can be checked and changed by holding the set point button until the screen changes to the rate. Using the keypad the rate can be set to whatever value is necessary.

(iv) Lock-in amplifier

A Stanford Research Systems Model SR810 Lock-in amplifier is used in making the AC measurements in this experiment. The general layout of the front panel is shown on figure 7. A lock-in amplifier is very useful in experiments that involve a lot of noisy data as it can reduce the noise from the data significantly and give a very clean signal.

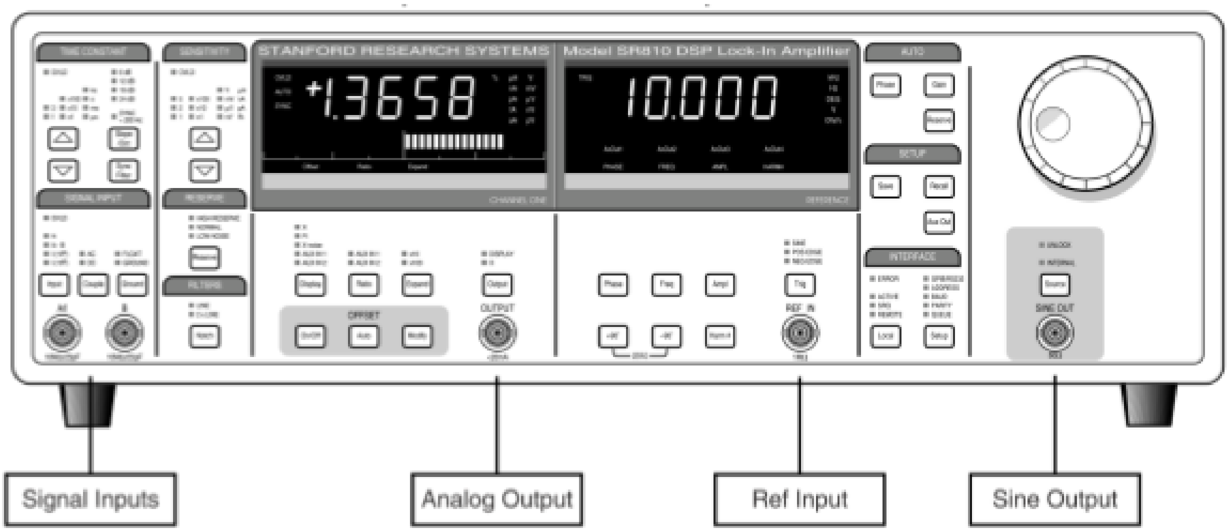


Figure 7: Front panel of the lock-in amplifier with the labeled components [3].

It can do so by multypling the input signal by the frequency of the reference signal and integrating it with respect to some time frame. The resulting DC signal has a frequency that is the same as that of the reference signal. The lock-in amplifier is very sensitive to different phases due to the orthogonality of sine and cosine functions [3].
For this experiment there will be a V_{rms} output of 1.000 V that will pass through a 1 $M\Omega$ resistor to make a 1 μA current signal. This signal will come from the Sine Output to a box with the resistor with two outputs that will connect to the wiring interface. The signal is received by the Signal Inputs. This will be an AC signal that will be transformed into a DC signal by the lock-in amplifier by the process described above. The following should be set to take measurements as well:

a Couple to AC

b Ground to Float

c Time Constant to 1 s (1x1 s) and 12 dB

d Display to R

The sensitivity should be set to a value larger than that which you are trying to measure. It is recommended to use a sensitivity of 10 mV. Remember to hit the Phase button under AUTO on the right of the panel.

(v) DC source

The DC source that is to be used is a Keithley 227 DC source. To perform the DC Hall Voltage measurements it will need to be set to output a 1 mA current. The knobs on the top of the device set the output current. Be mindful of where the light is as it is the decimal point. To produce a 1 mA current set the following to the said values [3].

a Set the units digit to 1 and all others to zero

b Set the rightmost knob to mA with 300 MAX COMP VOLT

Be sure to measure if the output is exactly 1 mA with a multi-meter. The first decimal knob may need to be played around with to get it as close to one as you can as it tends to be a bit loose and hard to set to exactly zero.

(vi) Vacuum pumping system

A single mechanical pump is used to vacuum both sections of the cryostat. Valves are provided on the respective spaces to give a good seal. When the pump is to be removed from the vacumming valves, make sure to close the valves to ensure that a vacuum is maintained at all times. If the pump is shut off before the valves are closed some air may be able to leak back into the vacuum spaces [3].

(vii) Electromagnet

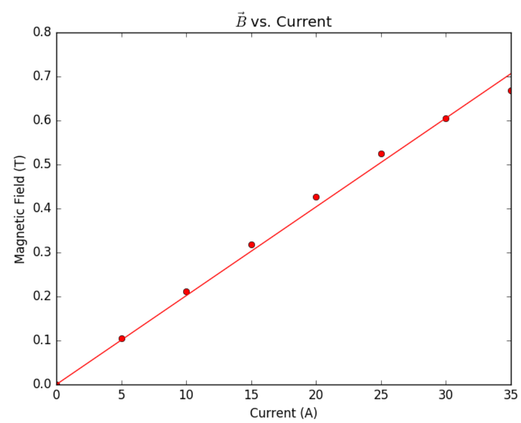
A water cooled electromagnet is used to produce the static magnetic field to make the Hall Effect measurements. There is a power supply unit that can be used to provide the current to the electromagnet. **Do not exceed 35 A on the power**

Figure 8: Front panel of the DC [3].

supply as it can overheat the magnet and permanently damage it. To be able to measure two different magnetic field directions a bar is provided at the top of the sample probe. Refer to the markings on this bar to see the direction of the magnetic field with respect to the plane of the sample. **Make sure that the bar is well aligned with the magnetic plates prior to making measurements as we assume that the magnetic field is perfectly perpendicular to the sample. Tiny deviations from this could result in significant differences as the cross product in equation 2 may not be perfect**. The table below shows the field strength in Tesla as a function of the electromagnet current.

Current (A)	Magnetic Field (T)
0	0
5	0.1056
10	0.2125
15	0.3179
20	0.4265
25	0.5258
30	0.6058
35	0.6678

Table 2: Magnetic field strength as a function of current. Measured experimentally [3].
When plotted this looks like the plot on the figure below.



IV Results and Discussion

All of the longitudal directions will be using the labeling as shown on figure 9.

(i) DC 2-contact resistance

In this part of the experiment we measured the resistance of the sample using an ohm-meter for the four possible longitudal directions.
The data that is gotten is shown below.

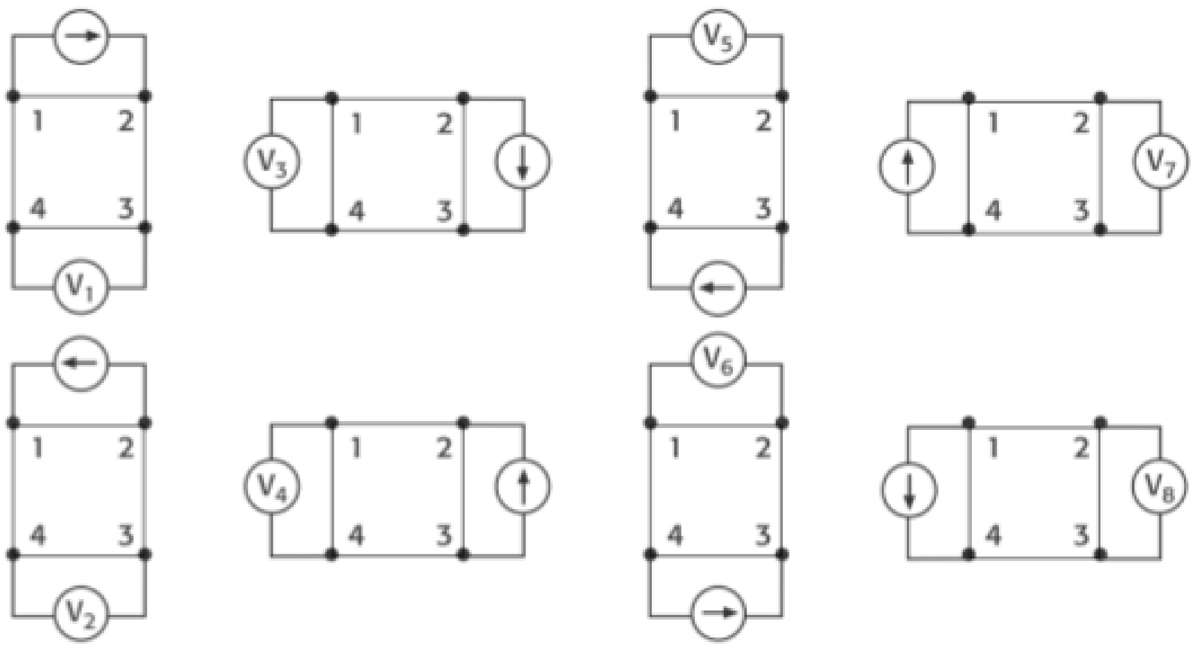


Figure 9: All 8 configurations for part ii in the experiment [3]

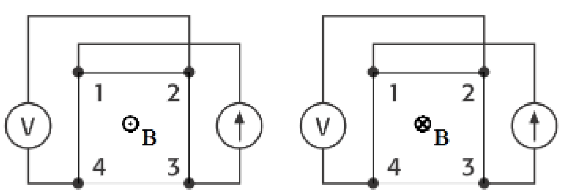


Figure 10: DC Hall measurement configuration for the room temperature measurement in part iii of the experiment [3].

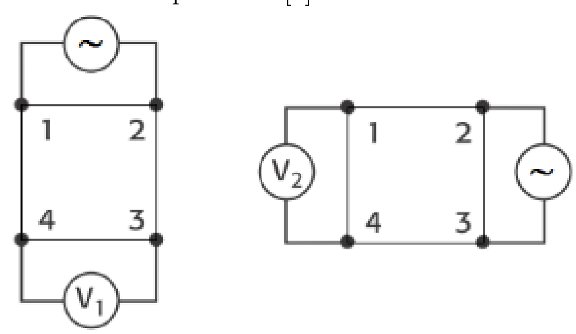


Figure 11: AC resistivity measurement configurations for part v of the experiment [3].

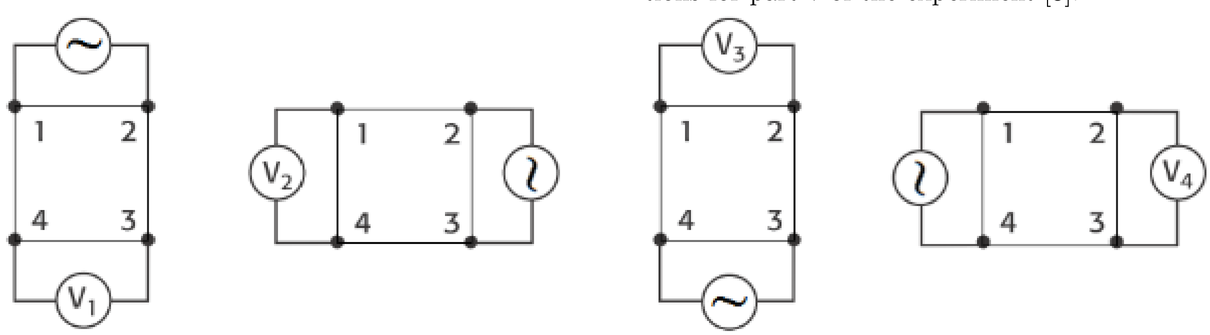


Figure 12: AC resistivity measurement configurations for part iv in the experiment [3].

Direction	Resistance ($k\Omega$)
1,2	4.77
2,3	2.57
3,4	6.62
4,1	7.02

Table 3: Data for part (i)

From this data it is not possible to calculate the resistivity as we have a the resistance of the sample through those points and according to equation 6 we need to know the other dimensions of the sample, for which, we did not.

(ii) DC 4-contact resistance

In this portion of the experiment we measured the voltage from passing current through two adjacent points and measuring the voltage through the other two. All 8 configurations that are used are shown on figure 9. The data for it is shown on the table below where the direction of the current flow is important as it is we are the DC source. The current value was set to be 0.99 mA.

Direction From $\rightarrow$ to	Absolute Voltage (mV)
1 $\rightarrow$ 2	1.169
2 $\rightarrow$ 1	1.179
2 $\rightarrow$ 3	0.3037
3 $\rightarrow$ 2	0.2955
3 $\rightarrow$ 4	1.069
4 $\rightarrow$ 3	0.839
4 $\rightarrow$ 1	0.3003
4 $\rightarrow$ 1	0.2821

Table 4: Flow of current and measured voltage for DC 4-contact resistance.

From these values the van der Pauw method can be applied to calculate the resistivity of the sample. Since the value of the measured voltage is expected to be the same in both directions (ie. from A to B and B to A), the average of the two values was taken and used in the calculations. The van der Pauw resistivity was calculated using equations 7 and 9 using C++. The results are shown in the following table where the current direction is no longer important.

Direction	Resistivity (Ωm)
1,2	0.877 $\pm$ 0.004
2,3	0.774 $\pm$ 0.004
3,4	0.764 $\pm$ 0.004
4,1	0.867 $\pm$ 0.004

Table 5: Results of the resistivity calculations using the van der Pauw method for the DC 4-contact resistance.

For the error analysis a constant error of 0.010 mV was assumed for all of the measurements and the error in the f value was ignored.

(iii) DC Hall Effect measurement - determining carrier type

For this part of the experiment the carrier type was determing by analyzing the slope of the line of data that was taken at different magnetic field values taken at room temperature. The configurations used are the ones showed on figure 10. A current value of 1.00 mA was passed through the sample. The data is shown below.

Magnetic Field (T)	V_{Hall} (V)
-0.6678	0.698
-0.6058	0.695
-0.5258	0.691
-0.4265	0.687
-0.3179	0.683
-0.2125	0.679
-0.1056	0.674
0.0000	0.6685
0.1056	0.663
0.2125	0.660
0.3179	0.657
0.4265	0.653
0.5258	0.649
0.6058	0.645
0.6678	0.642

Table 6: Data from measuring the Hall Voltage using a DC source.

When the data from table 6 is plotted we get the plot shown on figure 13. A linear fir was applied to see that there is a very linear trend in the Hall Voltage as a function if the magnetic field.

In order, to identify the type of carriers that are present in the semiconductor sample it is helpful to go back to equations 16 and 17, both of which, were set up in a very clever way to show that the sign of the slope is equal to the sign of the charge carriers. This comes about because, looking at equation 16 it was said that if the Hall constant is negative then we have negatively charged carriers. Which,

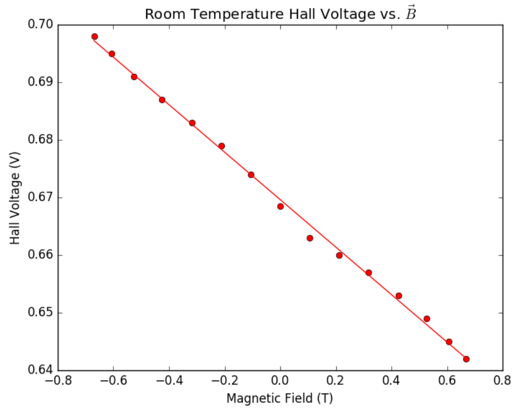


Figure 13: DC Room temperature Hall Voltage measurements as a function of magnetic field. A linear fit line is provided in the table with a slope of -0.0412 $\pm$ 0.0005 and a y intercept of 0.6696 $\pm$ 0.0002.

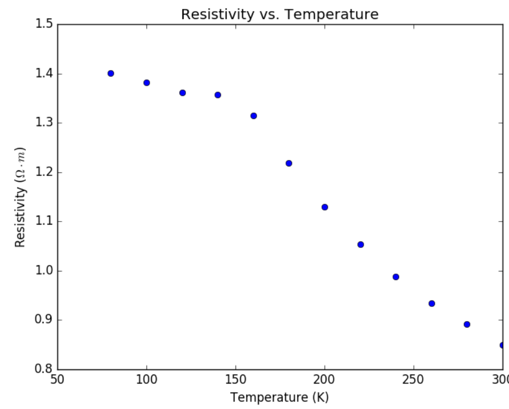


Figure 14: Plot of resistivity as a function of temperature. Resistivity calculated with the van der Pauw method.

we do as per the slope of the line that is given in the caption of figure 13 for which equation 16 can be re-written in the following way.

$$R_H = s\frac{t}{I} \tag{18}$$

Where, t and I are both positive and s is the slope from the data on table 6 and can be positive or negative.
In conclusion, the semiconductor sample is a n-type semiconductor with excess electrons.

(iv) AC 4-contact resistance

For this part of the experiment much like in part ii we are interested in finding the resistivity of the sample by measuring voltages through adjacent points on the sample and using the van der Pauw method on the data. The configurations that were used are shown in figure 12. The data shown below was taken in the order of the configurations.

Direction	Voltage (mV)
1,2	1.045
2,3	0.313
3,4	0.733
4,1	0.312

Table 7: Data from the measurement of the voltage by passing a AC source from the Lock-in amplifier.
When the van der Pauw method was applied to the data on table 7 we got the following resistivities.

Direction	Resistivity (Ωm)
1,2	0.823 $\pm$ 0.004
2,3	0.670 $\pm$ 0.004
3,4	0.669 $\pm$ 0.004
4,1	0.822 $\pm$ 0.004

Table 8: Results from applying the van der Pauw method on the data from table 7.
Where, we also assume the error in measurement to be 0.005 mV anf the f value to contribute an ignorable portion.
When comparing the two data sets from tables 5 and 8 we see that the results do differ and are not equal to each other within their uncertainties. We believe this to be because in the DC measurement there may have been noise from other sources which do not get cancelled and can contribute to a greater error degree. Also, the slight magnetization in the metallic plates of the electromagnet may skew the results by inducing a slight Hall voltage.
We believe that the AC measurements of the resistivities are more accurate than the DC due to noise from other DC sources that may arise in the sample.

(v) Resistivity versus Temperature

For this part of the experiment we wanted to find the relation between resistivity of the sample and the temperature. The configurations that were used to take the data were those shown of figure 11. The data that we took is shown in the table below. Where the first row gives through which points the current is being sent through and the second begins the data column labels.

Flow of current	Through 1,2	Thorugh 3,4
Temperature (K)	Voltage (mv)	Voltage (mV)
79.88	1.870	0.493
100.00	1.838	0.489
120.04	1.796	0.488
140.04	1.766	0.497
160.04	1.705	0.484
180.04	1.574	0.452
200.01	1.451	0.422
220.1	1.338	0.400
240.1	1.244	0.381
260.1	1.164	0.365
280.0	1.276	0.280
300.0	1.186	0.278

Table 9: Voltage data taken at various temperatures with an AC source from the Lock-in amplifier. Should be noted that the last two measurements for 280K and 300K were taken on a different day so this may account for the sharp change in the data.

Much like we have been doing in the previous parts of the experiment we use the van der Pauw method to get the resistivities at the various temperatures. When we do this we get the following data.

Temp.(K)	Resistivity (Ωm)
79.88	1.40 ± 0.01
100.00	1.38 ± 0.01
120.04	1.36 ± 0.01
140.04	1.36 ± 0.01
160.04	1.31 ± 0.01
180.04	1.22 ± 0.01
200.01	1.13 ± 0.01
220.1	1.05 ± 0.01
240.1	0.99 ± 0.01
260.1	0.93 ± 0.01
280.0	0.89 ± 0.01
300.0	0.85 ± 0.01

Table 10: Results of using the van der Pauw method on the data from table 9.

We assumed errors in our data to be 0.010 mV and the contribution from the f value error once again to be insignificant.

The data from table 10 is plotted on figure 14. Now the interesting thing about the plot is the slight plateau at the very beginning of the plot. This goes against what we are initially taught in electrodynamics, in which, the resistivity of a material goes up with increasing temperature. While that holds for most materials the same does not necessarily apply for semiconductors. The reason being, band gap energy. In semiconductors there is a certain band gap which is related to the spacings between orbital shells of the atom. So we also know that temperature can be represented as vibrational kinetic energy. So what does this all mean? In semiconductors there are two different shell levels: the valence and the conduction. Where the valence is the highest occupied shell by electrons and the conduction includes the excitation levels of electrons. At extremely low temperatures the electrons have a relatively low vibrational kinetic energy and from the Fermi-Dirac statistics formula.

$$f(E) = \frac{1}{\exp{(E - E_F)/kT} + 1} \tag{19}$$

Where, k is the boltzmann constant, E is the energy at which we are looking for the electron, E_F is the fermi energy and T is the temperature. We see that as T aproaches 0 and we are below the fermi level the probability of finding an electron in the valence band is 1 and 0 for the conduction band. As the temperature increases the function shifts from being a rigid step-function and smooths out at the ends until the probability of electrons in the conduction band is non-zero and there can be conduction of electrons and as the temperature increases the number of electrons that can be found on the conduction band decreases. Therefore, the resistivity decreases with temperature sice there will be more electrons readily available to conduct electricity.

The reason why it may plateau at a certain value may be due to the fact that the semiconductor is an n-type silicon sample, so not all of the electrons will be able to go to the valence band at low temperatures so some stay in the conduction band but this number is very small to those that are found in the valence.

(vi) AC Hall Voltage

In this final section of the experiment we found the Hall Voltages for different temperatures at different magnetic field strengths. The data that was taken for this part of the experiment will be included in the next section due to the amount of it. The data on figure 16 was fitted linearly with the equation y = ax + b. The data used was the average of the two configurations shown on 15. The parameter values of the fit are displayed on the table below. We only took data from 80 to 260 K.

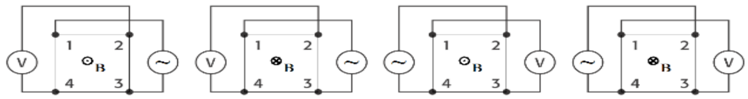


Figure 15: AC Hall Effect measurement configurations labeling for the data was cofiguration 1-4 from left to right [3]

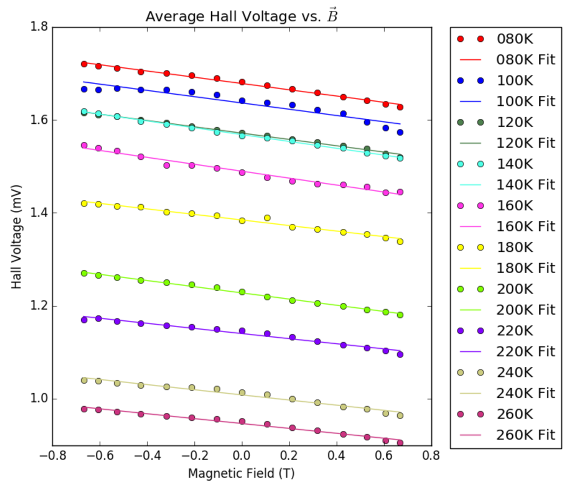


Figure 16: Plot of average Hall Voltages as a function of magnetic field. Refer to the legend to navigate through the different temperature values.

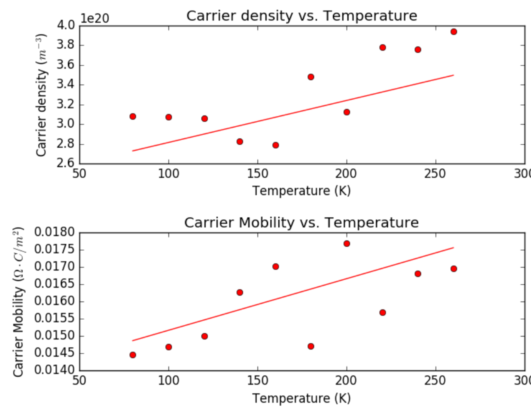


Figure 17: Plot of the Carrier density and mobility calculated from the slopes of figure 16. The fit lines are only trend lines intended to help see the general trend of the data.

Fit-line Temp. (K)	Parameter a	Parameter b
80	-0.067 ± 0.002	1.6784 ± 0.0009
100	-0.068 ± 0.006	1.637 ± 0.003
120	-0.068 ± 0.001	1.5716 ± 0.0007
140	-0.074 ± 0.001	1.5684 ± 0.0005
160	-0.075 ± 0.003	1.490 ± 0.001
180	-0.060 ± 0.002	1.385 ± 0.001
200	-0.067 ± 0.001	1.2277 ± 0.0006
220	-0.055 ± 0.003	1.140 ± 0.001
240	-0.055 ± 0.003	1.008 ± 0.001
260	-0.053 ± 0.002	0.9466 ± 0.0009

Table 11: Slopes and intercepts of the data in figure 16

With the slopes that are shown abpove we were then able to use equations 17 and **??** replacing V/B with the slope to calculate the carrier density and mobility. For the error in both we used simple error propagation in the slope to calculate the respective uncertanties. Both the fits and the calculations for this part were done solely in python. The data that we calculated then is shown below.

Temp.(K)	Density ($\times 10^{20} m^{-3}$)	Mobility ($\times 10^{-2} \Omega C/m^2$)
80	3.08 ± 0.09	1.44 ± 0.05
100	3.1 ± 0.3	1.50 ± 0.05
120	3.06 ± 0.07	1.50 ± 0.03
140	2.83 ± 0.04	1.63 ± 0.03
160	2.8 ± 0.1	1.70 ± 0.07
180	3.5 ± 0.1	1.47 ± 0.06
200	3.12 ± 0.07	1.77 ± 0.04
220	3.8 ± 0.2	1.57 ± 0.07
240	3.8 ± 0.2	1.68± 0.09
260	3.9 ± 0.2	1.70 ± 0.07

Table 12: Results of the carrier density and mobility using the slopes from table 11

The data on table 12 is plotted on figure 17. The plots do not look like they should and we believe that this was due to the equipment since most of our time in lab was spent troubleshooting the cause of large errors and fluctuations in the received values of the voltage into the Lock-in amplifier. However, when fitted with a general trend line it follows the expected behaviors that both should folllow an inverse trend to that seen in figure 14. So there should be a slight plateau at low temperatures and then it rises as the temperature goes up. From the explanation given for the resistivity decrease the carrier density is an increasing function.

(vii) Data for part vi

Here is all of the data from part iv which is arranged into three columns: Magnetic field, configurations 1,2 and configurations 3,4. Arranged in increasing temperature. Something to notice is that the third column has a positive slope where the 2 is negative when the data was plotted the magnetic field of the third column was negated to get a negative slope. This is due to the flow direction of the current and the definition of our axes were inverted. However, the data will be presented as it was taken.

Temperature = 80 K

Magnetic Field (T)	$V_{1,2}$ (mV)	$V_{3,4}$ (mV)
-0.6678	1.730	1.620
-0.6058	1.726	1.626
-0.5258	1.720	1.635
-0.4265	1.712	1.641
-0.3179	1.708	1.650
-0.2125	1.703	1.660
-0.1056	1.698	1.667
0.0000	1.691	1.674
0.1056	1.683	1.683
0.2125	1.675	1.688
0.3179	1.667	1.693
0.4265	1.657	1.697
0.5258	1.650	1.702
0.6058	1.642	1.707
0.6678	1.635	1.711

Temperature = 100 K

Magnetic Field (T)	$V_{1,2}$ (mV)	$V_{3,4}$ (mV)
-0.6678	1.671	1.567
-0.6058	1.667	1.575
-0.5258	1.675	1.592
-0.4265	1.678	1.609
-0.3179	1.671	1.617
-0.2125	1.667	1.626
-0.1056	1.662	1.631
0.0000	1.647	1.638
0.1056	1.644	1.646
0.2125	1.638	1.655
0.3179	1.628	1.659
0.4265	1.620	1.653
0.5258	1.599	1.663
0.6058	1.591	1.663
0.6678	1.581	1.662

Temperature = 120 K

Magnetic Field (T)	$V_{1,2}$ (mV)	$V_{3,4}$ (mV)
-0.6678	1.621	1.514
-0.6058	1.616	1.521
-0.5258	1.614	1.531
-0.4265	1.607	1.540
-0.3179	1.600	1.548
-0.2125	1.592	1.553
-0.1056	1.586	1.560
0.0000	1.578	1.566
0.1056	1.573	1.572
0.2125	1.564	1.580
0.3179	1.558	1.588
0.4265	1.550	1.595
0.5258	1.545	1.601
0.6058	1.534	1.605
0.6678	1.526	1.609

Temperature = 140 K

Magnetic Field (T)	$V_{1,2}$ (mV)	$V_{3,4}$ (mV)
-0.6678	1.637	1.521
-0.6058	1.630	1.526
-0.5258	1.623	1.534
-0.4265	1.612	1.544
-0.3179	1.608	1.550
-0.2125	1.600	1.562
-0.1056	1.592	1.567
0.0000	1.574	1.559
0.1056	1.557	1.557
0.2125	1.549	1.568
0.3179	1.542	1.575
0.4265	1.535	1.581
0.5258	1.525	1.592
0.6058	1.519	1.599
0.6678	1.514	1.600

Temperature = 160 K

Magnetic Field (T)	$V_{1,2}$ (mV)	$V_{3,4}$ (mV)
-0.6678	1.556	1.447
-0.6058	1.552	1.453
-0.5258	1.544	1.456
-0.4265	1.529	1.462
-0.3179	1.516	1.470
-0.2125	1.511	1.476
-0.1056	1.505	1.482
0.0000	1.493	1.482
0.1056	1.471	1.488
0.2125	1.461	1.494
0.3179	1.455	1.490
0.4265	1.460	1.513
0.5258	1.456	1.523
0.6058	1.436	1.528
0.6678	1.444	1.535

Temperature = 180 K

Magnetic Field (T)	$V_{1,2}$ (mV)	$V_{3,4}$ (mV)
-0.6678	1.426	1.333
-0.6058	1.420	1.344
-0.5258	1.415	1.347
-0.4265	1.405	1.350
-0.3179	1.396	1.358
-0.2125	1.393	1.364
-0.1056	1.390	1.369
0.0000	1.388	1.378
0.1056	1.410	1.398
0.2125	1.375	1.404
0.3179	1.373	1.409
0.4265	1.367	1.420
0.5258	1.363	1.415
0.6058	1.350	1.417
0.6678	1.346	1.416

Temperature = 200 K

Magnetic Field (T)	$V_{1,2}$ (mV)	$V_{3,4}$ (mV)
-0.6678	1.279	1.187
-0.6058	1.273	1.193
-0.5258	1.268	1.198
-0.4265	1.263	1.203
-0.3179	1.261	1.209
-0.2125	1.256	1.217
-0.1056	1.251	1.221
0.0000	1.234	1.226
0.1056	1.219	1.230
0.2125	1.207	1.236
0.3179	1.202	1.240
0.4265	1.195	1.246
0.5258	1.186	1.254
0.6058	1.182	1.258
0.6678	1.176	1.262

Temperature = 220 K

Magnetic Field (T)	$V_{1,2}$ (mV)	$V_{3,4}$ (mV)
-0.6678	1.172	1.096
-0.6058	1.174	1.100
-0.5258	1.169	1.107
-0.4265	1.164	1.114
-0.3179	1.161	1.122
-0.2125	1.156	1.130
-0.1056	1.154	1.139
0.0000	1.150	1.144
0.1056	1.141	1.146
0.2125	1.135	1.152
0.3179	1.126	1.156
0.4265	1.119	1.162
0.5258	1.112	1.164
0.6058	1.107	1.171
0.6678	1.097	1.168

Temperature = 240 K

Magnetic Field (T)	$V_{1,2}$ (mV)	$V_{3,4}$ (mV)
-0.6678	1.041	0.963
-0.6058	1.038	0.968
-0.5258	1.035	0.977
-0.4265	1.029	0.983
-0.3179	1.028	0.992
-0.2125	1.024	0.997
-0.1056	1.022	1.008
0.0000	1.015	1.014
0.1056	1.011	1.020
0.2125	1.002	1.025
0.3179	0.993	1.024
0.4265	0.984	1.029
0.5258	0.979	1.033
0.6058	0.971	1.038
0.6678	0.965	1.039

Temperature = 260 K

Magnetic Field (T)	$V_{1,2}$ (mV)	$V_{3,4}$ (mV)
-0.6678	0.979	0.906
-0.6058	0.976	0.910
-0.5258	0.972	0.917
-0.4265	0.968	0.923
-0.3179	0.965	0.931
-0.2125	0.961	0.937
-0.1056	0.958	0.945
0.0000	0.952	0.951
0.1056	0.946	0.955
0.2125	0.939	0.959
0.3179	0.932	0.962
0.4265	0.926	0.968
0.5258	0.919	0.971
0.6058	0.911	0.976
0.6678	0.906	0.978

V Conclusion

We were able to find all that we were looking for and found what we expected with the temperature dependence of the carrier mobility and density along with the resistivity. The Hall Effect has been a method that has been very helpful in the study of materials by allowing us to gain more knowledge about what it is that actually moves the electricity and how temperature affects it. // After the birth of quantum mechanics the Hall Effect became more easily explained since the electron and other subatomic particles were coming to light and it allowed scientists to add to it. One of the advancements is the Quantum Hall Effect (QHE) from which cam Fractional QHE and Integral QHE with the same goal to gain a deeper understanding of the materials that we work with and interact with on our day to day lives [6].

References

[1] Purcell, E. M.; Morin, D. J. (2013). Electricity and Magnetism. New York: *Cambridge University Press*, 314-317.

[2] (2016,December 24).Van der Pauw method. *Wikipedia*. https://en.wikipedia.org/wiki/Van_der_Pauw_method

[3] UB 2015 Lab Manual. Hall Effect

[4] Van der Pauw, L. J. (1958). A method of measuring specific resistivity and Hall Effect of discs of arbitrary shape. *Philips Research reports, 13*, 1-9.

[5] LakeShore (2000). User's Manual Model 330 Autotuning Temperature Controller.

[6] Lynn, J. W. (1990). The Quantum Hall Effect. 175 Fifth Avenue, New York, New York: *Springer-Verlag New York*.