AC and DC Hall Effect experiment on a 300μm Si doped wafer

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Abstract

The Hall Effect is a phenomena of solid materials that has been in use since the turn of the century. In this experiment we used the AC and DC hall effect to see the properties of a Si doped semiconductor sample including: the type of semiconductor, carrier mobility and density and the resistivity of the sample. It was determined the sample was an n-type emiconductor, with a temperature dependent carrier density with values: $3.08x10^{20} \pm 0.09x10^{20}$ at 80K, $3.1x10^{20} \pm 0.3x10^{20}$ at 100K, $3.06x10^{20} \pm 0.07x10^{20}$ at 120K, $2.83x10^{20} \pm 0.04x10^{20}$ at 140K, $2.8x10^{20} \pm 0.1x10^{20}$ at 160K, $3.5x10^{20} \pm 0.1x10^{20}$ at 180K, $3.12x10^{20} \pm 0.07x10^{20}$ at 200K, $3.8x10^{20} \pm 0.2x10^{20}$ at 220K, $3.8x10^{20} \pm 0.2x10^{20}$ at 240K and $3.9x10^{20} \pm 0.2x10^{20}$ at 260K. With temperature dependent values of: $1.44x10^{-2} \pm 0.05x10^{-2}$ at 80K, $1.5x10^{-2} \pm 0.1x10^{-2}$ at 100K, $1.49x10^{-2} \pm 0.03x10^{-2}$ at 120K, $1.63x10^{-2} \pm 0.03x10^{-2}$ at 140K, $1.7x10^{-2} \pm 0.07x10^{-2}$ at 160K, $1.47x10^{-2} \pm 0.06x10^{-2}$ at 180K, $1.77x10^{-2} \pm 0.04x10^{-2}$ at 200K, $1.57x10^{-2} \pm 0.07x10^{-2}$ at 220K, $1.68x10^{-2} \pm 0.09x10^{-2}$ at 240K and $1.69x10^{-2} \pm 0.07x10^{-2}$ at 260K.

I Introduction

The hall Effect was discovered by Edwin H. Hall in 1879, who at the time was studying Rowland at Johns Hopkins University. During those days no one knew of the electron and by extension how it was that conduction actually happened. Due to this it took almost 50 years until the Hall Effect was fully understood with the formulation of quantum mechanics. The results that were gotten from the experiment were generally not very well understood [1].

However, when quantum mechanics was formulated and the Hall Effect became fully understood it began to be employed in the study of semiconductors. It was here that it fullfilled its promise in the study of the concentration and sign of charge carriers [1]. Both of which are to be found in this experiment.

There are four different types of materials: insulators and superconductors, semiconductors and conductors. The difference between all of these are the resistivity and conductivity values for them. Insulators and superconductors behave in a very similar fashion in that they allow almost no current to pass through them, they have near infinite resistivities and near zero conductivity values. Conductors are the polar opposite of them having very low resistivity values and extremely high conductivity. Semiconductors are the middle ground where the two meet having finite resistivity and conductivity values.

Semiconductors are an integral part in our daily lives since they have been in wide use in electronics such as: computers, cell phones and LED bulbs. The Hall Effect has given a far reaching insight into how and why it is that the properties listed above display such distinct behaviors.

For the experiment a large electromagnet with a magnetic field range of 0 T to 0.6678 T was used to create a homogenous strong magnetic field. To receive the signals a lock-in amplifier was used for the AC Hall Voltage measurements and a simple multi-meter was used to measure the DC Hall Voltage measurements. The temperature of the sample could also be controlled via a PID temperature controller that allowed to heat up the sample and it was cooled down through the use of Liquid Nitrogen (LN). The sample was housed in a cryostat chamber containing a very dilute Helium gas concentration to be a thermal contact between the inner chamber and the LN.

The sample was a Silicon doped semiconductor which we put at cryogenic temperatures and in a high intensity magnetic field to perform the measurements.

II Theory

From electricity and magnetism we know that charged particles passing through a magnetic field display the following force relation,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1}$$

Where, q is the charge of the particle, $\vec{E}$ is the electric field in the space of the particle, $\vec{v}$ is its

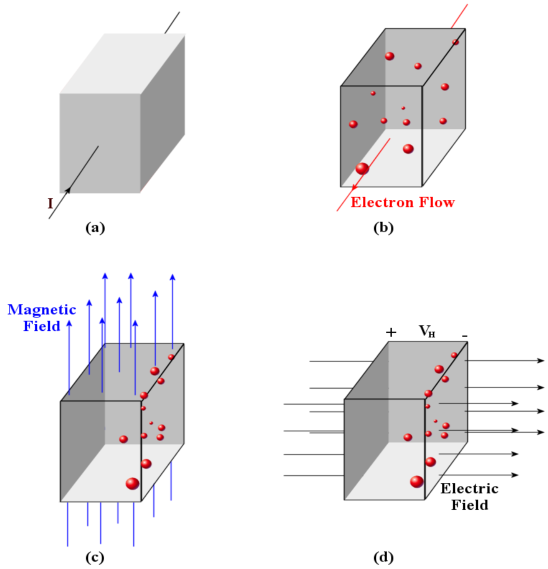

Figure 1: a). Representation of the general setup wiyh the sample and a wire providing current through the sample. b). A general representation of the flow of electrons in the sample. c). The general shift in the position of the electrons as they pass through a magnetic field. d). Electric field that is produced when the electrons distribute themselves in such a manner [2].

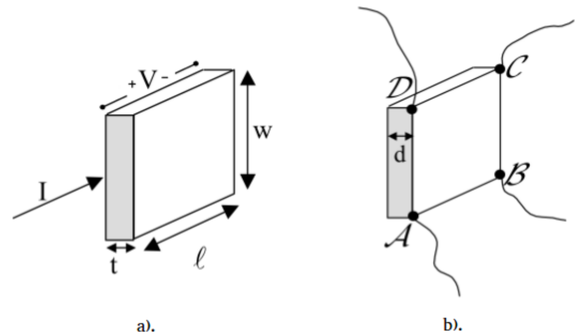

Figure 2: a). Current passing through a sample of dimension l,w,t. b). Sample of depth d with four contacts placed on the corners of the sample.

velocity and $\vec{B}$ is the magnetic field. Usually, we can consider the contribution of the electric field to the force to be zero so we can write equation 1 as,

$$\vec{F} = q\vec{v} \times \vec{B} \tag{2}$$

So that is to say that the charge carriers will receive a force that is perpendicular to their motion and as a cause will begin to curve. If we now think about a solid material through which we pass some current it would look like figure 1b. Where the electrons are free to move through the entire sample and have nothing directing their movement other than the electric current that is passing through the sample. For a pictorial representation of what happend when we apply a strong magnetic field perpendicular to the sample refer to figures 1c,d. This makes sense with equation 2 and what we generally know of charged particles passing through a perpendicular magnetic field. Looking at figure 1d the reason why it does in fact produce an electric field inside of the sample is due to the electrons that have curved to a side of the sample and since they can't conduct very easily through the a air they accumulate at the boundary of the sample toward which they curve creating a gradient of electrons and lack of electrons (positive charge) which gives rise to a potential difference through the sample and therefore an electric field through the sample.
This perpendicular magnetic field can be calculated with equation 3.

$$\vec{E}_t = \frac{-\vec{J} \times \vec{B}}{nq}[1] \tag{3}$$

Where, $\vec{J}$ is the current density, $\vec{B}$ is the magnetic field passing through the sample,n is the number carriers and q is the charge of the carriers.
Equation 3 is mainly useful were qe to know all of the parameters we need which, for this experiment, we need to find. So we take a different approach in our equation derivations starting with how to calculate the resistivity of a semiconductor sample.
When we have a sample with a current passing through it of certain current density, j, We can show that the electric field is related in the following way,

$$\vec{E} = \rho\vec{j}[3] \tag{4}$$

Where, ρ is the resistivity of the sample. By multiplying both sides of equation 3 by the length of the sample, l, we can get a relation between the voltage and the resistivity.

$$V = \rho \frac{l}{tw} I [3] \quad (5)$$

Where, l,t,w are the dimensions shown of figure 2b and I is the total current through the sample. By then applying Ohm's law, V=IR, we can get to the relation of resistance with resistivity.

$$R = \rho \frac{l}{wt} = R_s \frac{l}{w} [3] \quad (6)$$

Where, $R_s = \rho/t$. R_s is the sheet resistance of the sample and it is only dependent on the thickness of the sample as per the equation. Notice that if the sample is perfectly square, l=w, $R_s = R$ [3]. By performing a two contact resistivity measurement we can receive values for the resistivity of the sample. However, as it will be more apparent later these values that we get do have a certain amount of error and a more accurate method can be employed. This method is the va der Pauw method which was developed in 1958 by Leo J. van der Pauw.

To use this method the following conditions must be met [4].

a The contacts are at the circumference of the sample.

b The contacts are sufficiently small.

c The sample is homogeneous in thickness.

d The surface of the sample is singly connected,i.e.,the sample does not have isolated holes.

Now, the first condition assumes that the sample is a disc, but this method can work for any shape like a square. The derivation of the method can be found in [3] Now, the first condition assumes that the sample is a disc, but this method can work for any shape like a square. The derivation of the method can be found in [3]. In figure 2b we see that the contacts are placed in the corners. They do not have to necessarily be on the corners. They can be anywhere along the edge of the sample. According to van der Pauw's paper the resistivity of a sample using the four contact method can be calculated with the following equation.

$$\rho = \frac{\pi d}{\ln 2} \frac{R_{AB,CD} + R_{BC,DA}}{2} f \left(\frac{R_{AB,CD}}{R_{BC,DA}} \right) [4] \quad (7)$$

Where, d is the thickness of the sample (t in the case of fig. 2b), $R_{AB,CD}$ and $R_{BC,AD}$ represent current passing through the point and making the voltage measurement respectively. That is to say they are along the perpendicular current directions as can be inferred from figure 2b. The function f is a function that behaves in similar fashion to that which is shown on figure 3. Where it only satisfies the relation.

$$\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}} = f \operatorname{arccosh} \frac{\exp \ln 2/f}{2} [4] \quad (8)$$

Which can only be solved numerically to solve for f. Van der Pauw does provide another equation to approximate f where the ratio of $R_{AB,CD}/R_{BC,DA}$ are almost equal.

$$f \approx 1 - \left(\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}} \right)^2 \frac{\ln 2}{2} - \left(\frac{R_{AB,CD} - R_{BC,DA}}{R_{AB,CD} + R_{BC,DA}} \right)^4 \left\{ \frac{(ln2)^2}{4} - \frac{(\ln 2)^2}{12} \right\} [4] \quad (9)$$

There is another method that can be used in which we use a table which is provided to us. However, to reduce errors from approximating to the second decimal point the f value is calculated for every ratio and was compared to the given f values. They agreed to the second decimal and was believed to be a better approximation than those given in the table.

In general, we can represent the resistivity and conductivity of a sample of comparable size as a tensor meaning that the electric field and the current density may not necessarily be parallel to one another. Such matrices are the following.

$$\rho = \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{bmatrix}$$

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{yy} \end{bmatrix}$$

Where, the x and y subscripts represent the sheet coordinate directions of the sample ignoring the z. Back in equation 3 we showed the relation between the current density and the magnetic field to find the electric field that is produced by the Hall Effect. Here, a different form of it will be shown using vectors and tensors.

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{bmatrix} \begin{bmatrix} j_x \\ 0 \end{bmatrix} [3] \quad (10)$$

Where, $j_y = 0$ since there is no input current along the y direction of the sample only along the x direction. When equation 10 is solved for the electric

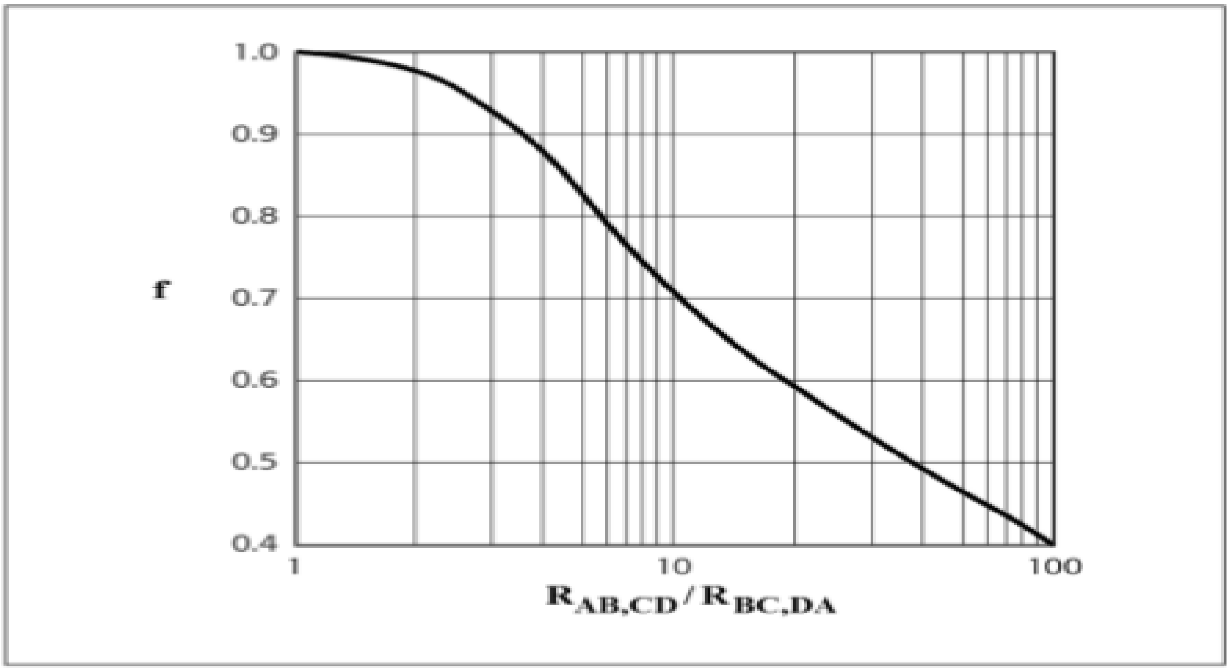

Figure 3: Plot of the f function from equation 6 [3].

field components we can get the following equations.

$$\begin{aligned} E_x &= \rho_{xx} j_x \\ E_y &= -\rho_{xy} j_x [3] \end{aligned} \tag{11}$$

Where, if we transform equation 11 as we did with equation 4 we get a relation between $V_H all$ and the xy resistance of the sample. Keep in mind that it is the Hall voltage as the field in the y direction is a direct cause of the magnetic filed that is along the z direction.

$$V_{Hall} = R_{xy} I [3] \tag{12}$$

Where, R_{xy} is given by the equation.

$$R_{xy} = \frac{R_H}{t} B [3] \tag{13}$$

Where, R_H is the Hall constant which depends on the carrier density and the charge of the carriers as shown.

$$R_H = \frac{1}{nq} [3] \tag{14}$$

Where, n is the number of carriers per unit volume and q is the charge of such carriers. Since, n is a integer value the sign of the Hall constant solely depends on the charge of the carriers. So, if the Hall constant is negative the charge carriers of the sample will be electrons and in the case of a positive value it will be holes (positive charge areas). In semiconductors this helps to identify what type of doping is used. If the semiconductor is n-type there is an excess of electrons where if it is p-type there is an excess of positively charged holes. Must be careful to not say protons as they do not flow when conducting electricity.

By combining equations 13 and 13 we can make the following relationship between the Hall constant and the magnetic field.

$$R_H = \frac{V_{Hall}}{B} \frac{t}{I} [3] \tag{15}$$

Where, we can also re-write the equation to make the relationship between the magnetic field and the carrier density in a sample.

$$n = \frac{B}{V_{Hall}} \frac{I}{qt} \tag{16}$$

Carrier mobility is defined as the ability of a charge carrier to pass through a sample material. The carrier mobility can be defined as the ratio between the drift velocity and the applied electric field. Which by transforming into the components of the electric field we can represent the carrier mobility as a function of the carrier density or the Hall constant.

$$\begin{aligned} \mu &= \frac{1}{\rho n q} \\ &= \frac{R_H}{\rho} \\ &= \frac{V_{Hall}}{B} \frac{t}{\rho q} [3] \end{aligned} \tag{17}$$

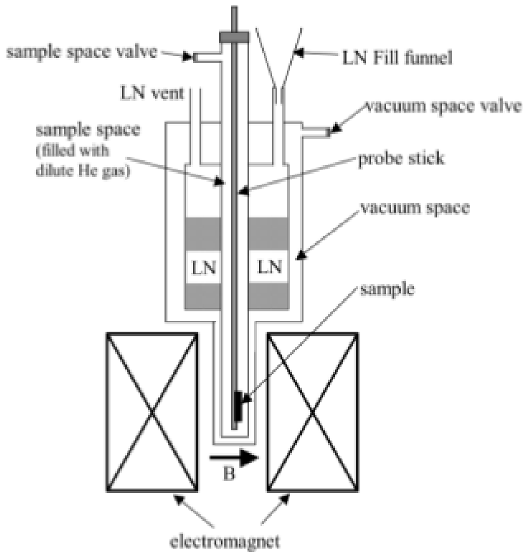

Figure 4: A general diagram of the cryostat that is to be used. There are three chambers in the cryostat [3].

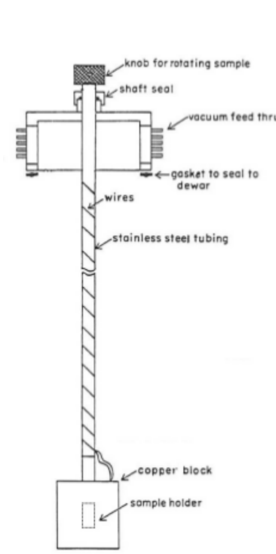

Figure 5: Schematic of the sample probe [3].

III Experimental Set-Up

For this experiment a combination of equipment was used to make the measurements including a cryostat which housed the sample probe containing the sample, a temperature controller, a lock-in amplifier, a DC source, a vacuum pump and an electromagnet.

(a) Cryostat

Since, it is necessary to make low temperature measurements in this experiment a three chamber cryostat is to be used. Where, the chambers are labeled on figure 4. The inner and outer chambers must be at a near vacuum which can be achieved with the provided vacuum pump. **NEVER add liquid nitrogen until the air in the cryostat is evacuated**. Not doing so could result in the air moisture condensing inside the cryostat when the liquid nitrogen is added. The inner chamber should be vacummed twice once to remove most of the air in the chamber and a second time by putting Helium gas in the chamber and proceeding to them vacuum the Helium gas out of the chamber. This ensures that most if not all of the air has been taken out of the chamber and only very dilute He gas is left in the chamber. The outer chamber should be vacuumed for 5 minutes as it is a much larger volume than the inner chamber. Should the air in the cryostat freeze the sample or the entire cryostat could be damaged [3].

Once this is done the liquid nitrogen can be added into the cryostat by use of the funnel. The cryostat has a max capacity of approximately 5 liters of liquid. Be careful when puring the liquid nitrogen as it will sputter when it comes into contact with the funnel. **Always wear the safety equipment provided including: cryo-gloves, goggles and the lab coat** [3].

(b) Sample probe

The sample to be tested is mounted on a sample probe that will be placed in the cryostat by the instructor and won't need to be taken out. If there is ever a problem and it does need to be taken out of the cryostat the entire cryostat must reach room temperature before doing anything as breaking the seal in the inner chamber could damage the cryostat. The sample holder is shown on figure 5

The sample holder is actually a small box that is found at the end of the entire tube apparatus. On the sample holder the following components can be found [3].

a The suspension tube, which slides in the cap to vary the height of the specimen.

b A copper vessel around the specimen and heater, which helps to keep the temperature unifor and reduces thermal fluctuations.

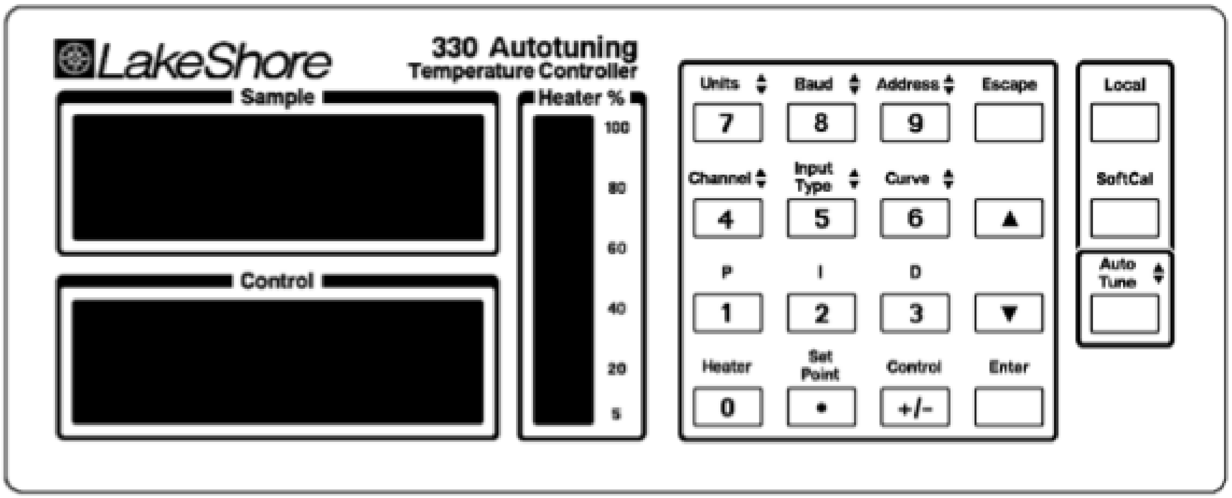

Figure 6: Front panel of the temperature controller with the labeled components [3].

c A temperature sensitive diode (DT471) that is used to measure the specimen temperature.

d A heater which balances the cooling by the cold exchange gas (helium) that will surround the can and specimen, and helps control the rate of change in temperature of the specimen.

e Sample holder which provides a flat mounting surface for the sample and the temperature sensitive diode.

The sample that is to be tested is a doped silicon wafer with a thickness of $300 \mu m$. It is mounted on the sample holder with the temperature sensitive diode located behind the sample. Since, there is a relation between resistance and temperature the temperature is found by the measured voltage across the diode which is given a current in the μA range [3].

(c) Temperature Controller

To control the temperature a Lakeshore 330 Autotuning temperature controller is used. There are three LCD screens on the controller the top screen displays the measured temperature of the diode, the bottom gives the temperature that is set for the controller to maintain and the vertical screen gives the amount of current that is passing through the diode as a percentage. The amount of current passing through the sample is set by the heater value which can be: Low, Medium or High. The amount of current that is passed through the diode at each setting is given by the following table.

Heater Range	Heater Current
HIGH	0 to 1 A
MEDIUM	0 to 0.3 A
LOW	0 to 0.1 A

Table 1: Heater current values at different set values [5]

The temperature controller utilizes a PID (Proportional-Integral-Derivative) feedback control loop to approach and maintain the temperature. The P, I, D parameters can be found on the main face of the Temperature Controller on figure 6. But, it is only necessary to leave them in the default values of 50, 25, 0 for the P, I, D respectively.

The heater setting can be changed by pressing the Heater button and cycling through the three differnt settings. The temperature change rate should not be set higher than 10 K/min in order to preserve the temperature equilibrium in the sample space. This can be checked and changed by holding the set point button until the screen changes to the rate. Using the keypad the rate can be set to whatever value is necessary.

(d) Lock-in amplifier

A Stanford Research Systems Model SR810 Lock-in amplifier is used in making the AC measurements in this experiment. The general layout of the front panel is shown on figure 7. A lock-in amplifier is very useful in experiments that involve a lot of noisy data as it can reduce the noise from the data significantly and give a very clean signal.

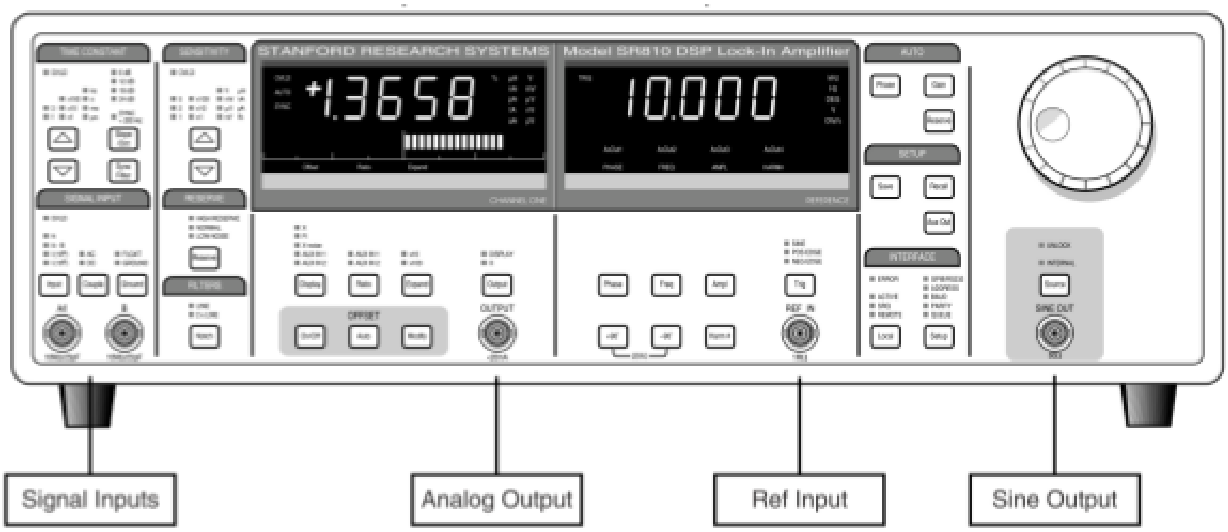

Figure 7: Front panel of the lock-in amplifier with the labeled components [3].

It can do so by multypling the input signal by the frequency of the reference signal and integrating it with respect to some time frame. The resulting DC signal has a frequency that is the same as that of the reference signal. The lock-in amplifier is very sensitive to different phases due to the orthogonality of sine and cosine functions [3].
For this experiment there will be a V_{rms} output of 1.000 V that will pass through a 1 $M\Omega$ resistor to make a 1 μA current signal. This signal will come from the Sine Output to a box with the resistor with two outputs that will connect to the wiring interface. The signal is received by the Signal Inputs. This will be an AC signal that will be transformed into a DC signal by the lock-in amplifier by the process described above. The following should be set to take measurements as well:

a Couple to AC

b Ground to Float

c Time Constant to 1 s (1x1 s) and 12 dB

d Display to R

The sensitivity should be set to a value larger than that which you are trying to measure. It is recommended to use a sensitivity of 10 mV. Remember to hit the Phase button under AUTO on the right of the panel.

(e) DC source

The DC source that is to be used is a Keithley 227 DC source. To perform the DC Hall Voltage measurements it will need to be set to output a 1 mA current. The knobs on the top of the device set the output current. Be mindful of where the light is as it is the decimal point. To produce a 1 mA current set the following to the said values [3].

a Set the units digit to 1 and all others to zero

b Set the rightmost knob to mA with 300 MAX COMP VOLT

Be sure to measure if the output is exactly 1 mA with a multi-meter. The first decimal knob may need to be played around with to get it as close to one as you can as it tends to be a bit loose and hard to set to exactly zero.

(f) Vacuum pumping system

A single mechanical pump is used to vacuum both sections of the cryostat. Valves are provided on the respective spaces to give a good seal. When the pump is to be removed from the vacumming valves, make sure to close the valves to ensure that a vacuum is maintained at all times. If the pump is shut off before the valves are closed some air may be able to leak back into the vacuum spaces [3].

(g) Electromagnet

A water cooled electromagnet is used to produce the static magnetic field to make the Hall Effect measurements. There is a power supply unit that can be used to provide the current to the electromagnet. **Do not exceed 35 A on the power**

Figure 8: Front panel of the DC [3].

supply as it can overheat the magnet and permanently damage it. To be able to measure two different magnetic field directions a bar is provided at the top of the sample probe. Refer to the markings on this bar to see the direction of the magnetic field with respect to the plane of the sample. **Make sure that the bar is well aligned with the magnetic plates prior to making measurements as we assume that the magnetic field is perfectly perpendicular to the sample. Tiny deviations from this could result in significant differences as the cross product in equation 2 may not be perfect**. The table below shows the field strength in Tesla as a function of the electromagnet current.

Current (A)	Magnetic Field (T)
0	0
5	0.1056
10	0.2125
15	0.3179
20	0.4265
25	0.5258
30	0.6058
35	0.6678

Table 2: Magnetic field strength as a function of current. Measured experimentally [3].

IV Results and Discussion

All of the longitudal directions are labeled with the same lettering as that in figure 2b.

(a) DC 2-contact resistance

In this part of the experiment we measured the resistance of the sample using an ohm-meter for the four possible longitudal directions.
The data that is gotten is shown below.

Direction	Resistance ($k\Omega$)
AB	4.77
BC	2.57
CD	6.62
EF	7.02

Table 3: Data for part (i)

From this data it is not possible to calculate the resistivity as we have a the resistance of the sample through those points and according to equation 6 we need to know the other dimensions of the sample, for which, we did not.

(b) DC 4-contact resistance

In this portion of the experiment we measured the voltage from passing current through two adjacent points and measuring the voltage through the other two. The data for it is shown on the table below where the direction of the current flow is important as it is we are the DC source. The current value was set to be 0.99 mA.

Direction From → to	Absolute Voltage (mV)
A → B	1.169
B → A	1.179
B → C	0.3037
C → B	0.2955
C → D	1.069
D → C	0.839
D → A	0.3003
A → D	0.2821

Table 4: Flow of current and measured voltage for DC 4-contact resistance.

From these values the van der Pauw method can be applied to calculate the resistivity of the sample. Since the value of the measured voltage is expected to be the same in both directions (ie. from A to B and B to A), the average of the two values was taken and used in the calculations. The van der Pauw resistivity was calculated using equations 7 and 9 using C++. The results are shown in the following table where the current direction is no longer important.

Direction	Resistivity (Ωm)
AB	0.877 ± 0.004
BC	0.774 ± 0.004
CD	0.764 ± 0.004
DA	0.867 ± 0.004

Table 5: Results of the resistivity calculations using the van der Pauw method for the DC 4-contact resistance.

For the error analysis a constant error of 0.010 mV was assumed for all of the measurements and the error in the f value was ignored.

(c) DC Hall Effect measurement - determining carrier type

For this part of the experiment the carrier type was determing by analyzing the slope of the line of data that was taken at different magnetic field values taken at room temperature. The data is shown below.

References

[1] Purcell, E. M.; Morin, D. J. (2013). Electricity and Magnetism. New York: *Cambridge University Press*, 314-317.

[2] (2016,December 24).Van der Pauw method. *Wikipedia*. https://en.wikipedia.org/wiki/Van_der_Pauw_method

[3] UB 2015 Lab Manual. Hall Effect

[4] Van der Pauw, L. J. (1958). A method of measuring specific resistivity and Hall Effect of discs of arbitrary shape. *Philips Research reports, 13*, 1-9.

[5] LakeShore (2000). User's Manual Model 330 Autotuning Temperature Controller.