Optical Pumping of Rb^{85} and Rb^{87}

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Abstract

Optical Pumping is a spectroscopical method that was developed in the 1950's and has been a very accurate method to determine spectroscopical properties of certain materials. In this experiment the following were determined: the individual g-factors, nuclear spins, cross sectional area and ratio of the periods. For Rb^{85} the g-factor and nuclear spin were found to be: 0.3260 ± 0.0005 and 2.571 respectively. For Rb^{87} they were found to be: 0.482 ± 0.001 and 1.576 respectively. The cross-sectional area and ratio of the periods were found to be: $1.8 \times 10^{-16} \pm 0.3 \times 10^{-16}$ and 1.44 ± 0.05 respectively.

I   Introduction

Optical Pumping is a spectroscopical method developed in 1950 by Alfred Kastler, whom received the Nobel Prize in physics in 1966 for his discovery. This method is one in which photons are utilized to create population differences of electronic excited and ground states. So the meaning and general concept is in the name itself.

Under startdard conditions the population difference required to carry out experiments is not possible because from statistical mechanics at thermal equilibrium we have an equal number of electrons that rise and fall from excitation levels. Due to this, they tend to cancel each others effects and no net population differences can be detected. This is also the basis of lasers where, a population difference needs to be created so that photons can be spontaneously emmitted by the lasing medium.

For this experiment the equipment that is being used is provided by TeachSpin and consists of an Radio Frequency (RF) discharge lamp, Interference Filter, Polarizers, Quarter Wave plate, absorption cell, optical detector, three sets of magnetic coils in a Helmholtz configuration and a RF magnetic coil. The sample is a Rubidium glass bulb that contains neon gas with a pressure of approximately 0.04 atm pressure. The presence of the neon gas is important as its spherical symmetry will reduce the interactions between the Rubidium atoms and the outside environment. They will act as a buffer gas.

Optical Pumping is a process in which has had much applicability in solid state and liquid state physics. However, we will only be dealing with a gas since at the solid and liquid phases the interactions between the neighboring atoms increases thus broadening the energy levels [1].

II   Theory

(i)   Structure of alkali atoms

In the experiment described in this paper we will be studying the absorption and emission from Rubidium isotopes (85 and 87) which are alkali atoms. As such the electronic structure of Rubidium is as such,

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^1 04s^2 4p^6 5s$$

Where we can show the shorthand version as,

$$[Kr]5s$$

Figure 3: *Coupling of angular momentum in an electron [3].*

Where, the superscripts show the number of electrons contained in each of the electronic shells. Since the only valence electron is in the 5s shell we can consider the atom to bev consisting of only one electron. This electron much like with other electrons can be described by means of the total angular momentum of the electron $\vec{J}$ where it is made up of components $\vec{S}$ and $\vec{L}$. Which, represent the spin angular momentum and orbital angular momentum respectively. The vectors are shown on figure 3

Since, these components are vectors we can represent the total angular momentum as such,
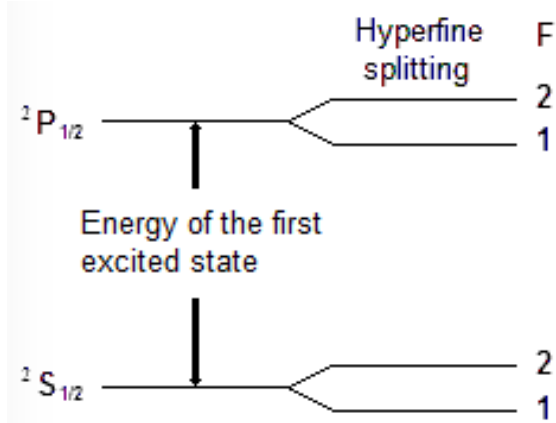
$$\vec{J} = \vec{S} + \vec{L} \tag{1}$$

Figure 1: *Hyperfine splitting energy diagram for I = 3/2 particle [3]*

Figure 2: *Hyperfine coupling in an alkali atom [3]*

Where, for an alkali atom in the ground state the value of $\vec{L}$ is zero from the quantum numbers associated with the orbital shell and the value of $\vec{S}$ will be 1/2. So, this gives rise to a total angular momentum of 1/2.

As it will later become apparent, we will display some of the energy levels in the notation $^{2s+1}L_J$. Where, all of the values are those taken from equation 1. To represent the ground state of an alkali atom we can write it with the formatting $^2S_{1/2}$. Since the value of $\vec{L}$ is zero by convention we write it as the letter S. Where it to be 1 we would write in P and so on.

If the electron were to be in a P state it would be able to have an angular momentum of $\vec{L} \pm \vec{S}$ and would take on the representations $^2P_{1/2}$ and $^2P_{3/2}$. Due to the difference in angular momentum the energy levels would have different energies. This arises due to the spin-orbit coupling of the angular momentum vectors [4].
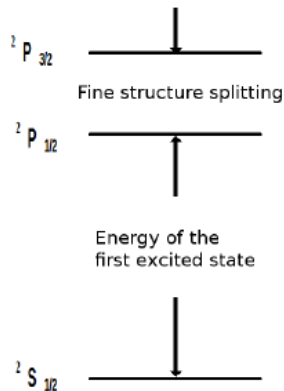

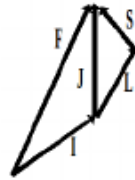Figure 4: *Pictoral representation of the energy difference in the fine structure. Not to scale [3].*

So far, we have been ignoring the effects of the nucleus to make the theory easier. However, this will not go on any longer. We will now consider the hyperfine splitting of the electron angular momenta. This arises from the spin-spin coupling where for the fine structure we had the spin-orbit coupling. The spin-spin coupling is due to the magnetic dipole moment of both the proton and electron. Now, it should be noted, that the magnetic dipole moment of the proton is much smaller than that compared to the electron. The dipole momenta can be given as the following.

$$\vec{\mu_p} = \frac{g_p e}{2m_p} \vec{S_p} \qquad \vec{\mu_e} = -\frac{e}{m_e} \vec{S_e} [4] \qquad (2)$$

Where, the p and e represent the proton and electron values respectively, g_p and g_e are the respective g-factors of value 5.59 and 2.00 and $\vec{S}$ represents the respective angular spin. Using derivations outlined in reference [4] we can get to an expression for the difference in energy of the two states with different angular momenta. The equation is as such.

$$\Delta E = \frac{4g_p \hbar}{3m_p m_e^2 c^2 r^4} [4] \qquad (3)$$

Where, c is the speed of light and r is the radius of the atom. In the reference they are making an example to a hydrogen atom with r equal to a (Bohr's atomic radius). This derivation can still be generalized to our use as we are dealing with an atom that can be approximated to be one that is like a hydrogen atom. Where we can also make the relation with the frequency as such.

$$\nu = \frac{\Delta E}{h} [4] \qquad (4)$$

2

Where the Hamiltonian is as such.

$$H = ha\vec{I} \cdot \vec{J}[3] \qquad (5)$$

Usually the energy difference between these two levels is very small and one can cause transitions in the hyperfine structure with an RF wave. A pictoral representation of the coupling of the magnetic dipole moment spins is shown on figure 2. Where a rough enegy separation schematic is given in figure 1. As is clearly seen the energy that is required to jump from one energy level to the next is muche greater than that required to make the transition in the hyperfine structure. Later we will discuss the implications of this with respect to optical pumping.

However next we will show another form of splitting where all of the degenerate energy levels of the electronic energy level diagram are split.

(ii) Interaction of an alkali atom with a magnetic field

This interaction is better known as the Zeeman effect and the general premise behind this effect is that by applying a magnetic field to an atom we can further split the degenerate energy levels according to their angular momenta.

In the Zeeman effect there are three main regions that are to be considered: weak field, intermediate and Strong field effects. For the purposes of this experiment we will only deal with the weak field effect as the strong field can require magnetic fields around 10T which is only achievable with extremely expensive equipment.

The Zeeman effect is an effect which can break the spin-orbit coupling of an electron creating a difference in energy between the different orbitals of an atom. The weak field Zeeman effect is named as such because the energy splitting from the Zeeman effect is very small comparatively and as such the Hyperfine splittings dominate with the Zeemna effect becoming the pertubation [4]. The hamiltonian of this effect is as shown.

$$H = ha\vec{I} \cdot \vec{J} - \frac{\mu_J}{J}\vec{J} \cdot \vec{B} - \frac{\mu_I}{I}\vec{I} \cdot B[3] \qquad (6)$$

Where, μ_J is the electronic dipole moment and μ_I is the nuclear magnetic dipole moment.

Figure 5 shows the energy levels from the weak field Zeeman effect. The energy levels are splitting into $2F + 1$ levels. Where F is the angular momentum of the atom, M is the projection of F onto the direction of the magnetic field. In figure 3 we show two levels for the ground state and the first excited

state. For this experiment we are only considering the first excited state $^2P_{1/2}$. The reason why we only consider this and not the $^2P_{3/2}$ is because at $\vec{J}$ values of 1/2 we can calculate the energy levels in closed form from quantum mechanics with the Breit-Rabi equation.

Since the electron can be considered to be a moving charge with charge 1.6e-19 Coulombs. This magnetic dipole moment has a value that is equal to the Bohr Magneton, μ_B. Ignoring the effects of the nucleus we can then represent the magnetic energy as.

$$U = \frac{M(\vec{L} + 2\vec{S}) \cdot \vec{J}}{J^2}\mu_B B = g_J \mu_B M B[3] \qquad (7)$$

Where, g_J is the Lande g-factor which describes the change in the magnetic moment of an electron bound in an atom, B is the magnetic field and M is the atomic spin component along the magnetic field direction. The g-factor is given by,

$$g_J = \frac{(\vec{L} + 2\vec{S}) \cdot \vec{J}}{J^2}$$
$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}[3] \qquad (8)$$

To show the interaction energy of an electron we simply need to add a negative sign to equation 7 from the negative charge of the electron. In the case of rubidium where we have a J and S value of 1/2 we find that the g-factor is equal to 2.00232. Now, if we were to include the iteraction of the nucleus to find the interaction energy the equation could be represented as follows.

$$U = -g_F \mu_B M B[3] \qquad (9)$$

Where, g_F can be shown to be the following.

$$g_F = g_J \frac{F(F+1) + J(J+1) - I(I-1)}{2F(F+1)}[3] \qquad (10)$$

Where, F represents the total angular momentum of the atom, I is the total nuclear spin angular momentum and J is the total electronic angular momentum. This quantity is highly dependent on the atom that is being used and as such there is no one set value as with the Lande g-factor.

the above equations are only applicable when the interaction energy with the magnetic field is very small and depends linearly. If the dependence is quadratic, which will be studied in this experiment, we must apply the Breit-Rabi equation which can be derived by diagonalizing the Hamiltonian in
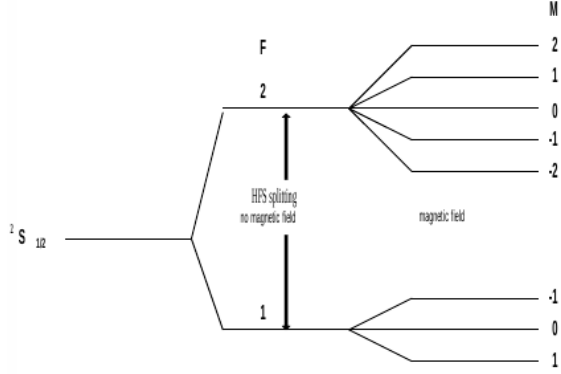
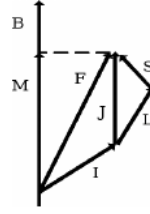Figure 5: *Energy levels of an alkali atom of the $^2S_{1/2}$ state in a weak magnetic field [3]*

Figure 6: *Zeeman effect in an alkali atom [3]*

equartion 6. The result of such is the following.

$$W(F,M) = -\frac{\Delta W}{2(2I+1)} - \frac{\mu_I}{I}MB \pm ...$$
$$\frac{\Delta W}{2}\left[1 + \frac{4M}{2I+I}x + x^2\right]^{1/2} \text{ [3]}$$

$$(11)$$

References

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[4] Griffiths, D J (2005). Introduction to Quantum Mechanics. Upper Saddle river, New Jersey: *Pearson Prentice Hall.*