

# MATH MODELS OF MIXING-III

## Mathematical Models of Mixing With Applications of Viscosity, Load Capacities and Mix Rates

$$\int \frac{d\eta}{\eta} = k \int dT$$

$$x_r(t) = \frac{\sum_{i=1}^{r-1} P_i + P(t)}{\sum_{i=1}^n (P_i + V_i)} = \frac{\sum_{i=1}^{r-1} P_i + \frac{rW_0(1 - e^{-rt})}{(1-r)}}{\sum_{i=1}^n (P_i + V_i)}$$

$$y = 2W_0kt$$

$$\frac{dy}{dt} = 2W_0k$$

### A Flushed Pigment Dispersion Process Model

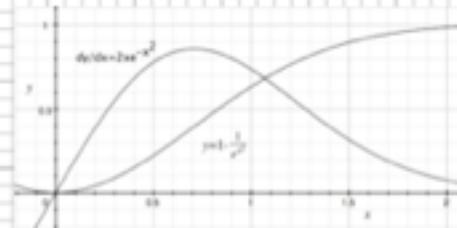
$$w(t) = W_0(1 - e^{-rt})$$

$$w'(t) = W_0 - W_0e^{-rt}$$

$$y = 1 - \frac{1}{e^{rt}}$$

$$y = 1 - e^{-rt}$$

$$P_o = r - \left( \sum_{i=1}^n P_i + V_i \right)$$



$$w'(t) \frac{d^2 w(t)}{dt^2} = \frac{d}{dt} \{ 2W_0k t e^{-rt} \}$$

$$\frac{d}{dt} wv = u \frac{dv}{dt} + v \frac{du}{dt}$$

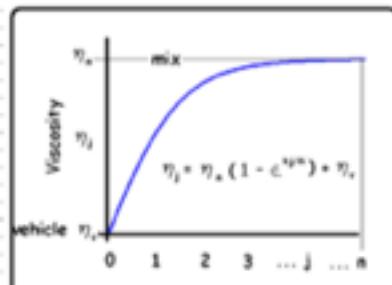
**Herbert Norman Sr.**

**Thornton, Colorado**

**July 9, 2010**

$$w'(t) = \frac{dw(t)}{dt} = 2W_0k t e^{-rt}$$

$$y = 2k x e^{-rt}$$



$$P_r(t) = \frac{rW_0(1 - e^{-rt})}{(1-r)}$$

$$\frac{d\eta(x)}{dx} = k\eta(x)$$

$$\frac{d\eta(x)}{\eta(x)} = kdx$$

$$\int \frac{d\eta(x)}{\eta(x)} = k \int dx$$

$$\ln \eta(x) = kx + c$$

$$\eta(x) = e^{kx} e^c$$

$$\eta(x) = C e^{kx}$$

# TABLE OF CONTENT

ABSTRACT .....	6
CHAPTER 1	
INTRODUCTION .....	12
Statement of the Problem .....	13
Purpose of the Study .....	14
Importance of the Study .....	14
Scope of the Study .....	15
Rationale of the Study .....	16
Definition of Terms .....	17
Overview of the Study .....	19
Viscosity .....	24
Dispersions .....	26
CHAPTER 2 REVIEW OF RELATED LITERATURE .....	28
Solvents, Oils, Resins & Driers .....	29
Introduction to Paint Chemistry .....	31
Viscosity & Flow Measurement .....	34
Paint Flow & Pigment Dispersion .....	37
Printing & Litho Inks .....	41
Physical Chemistry (Suspensions) .....	42
Fluid Mechanics & Hydraulics .....	43
Chemical Engineering Calculations .....	46
Ordinary Differential Equations .....	47
Geometric Series & Engineering Math .....	53

# TABLE OF CONTENT

CHAPTER 3	METHODOLOGY .....	55
	Approach .....	56
	Data Gathering Method .....	56
	Database of Study .....	57
	Validity of Data .....	58
	Originality and Limitation of Data .....	58
	Summary .....	59
CHAPTER 4	DATA ANALYSIS I .....	60
	Load Process (Mono-Vehicle) .....	61
	Treatment-I Model A (Log – Exponential) .....	63
	Treatment-II Model B (Log – Exponential) .....	73
	Treatment-III Model C (Geometric Series) .....	82
	Treatment-III Model D (Geometric Series) .....	80
CHAPTER 5	DATA ANALYSIS II .....	101
	Load Process (Multi -Vehicle) .....	102
	Mixing Stage Analysis (Water Displacement) .....	107
	Process Mixing-Time Models .....	111
	Process Rates & Analysis .....	122
	Temperature, Time & Viscosity .....	127
	Summary and Conclusions .....	132
CHAPTER 6	DATA ANALYSIS III (Software Development) .....	133
	QBASIC and MS-EXCEL (Development Tools) .....	134
	MODEL Flowchart (General Outline) .....	137
	MODEL Input & System Constants .....	137
	MODEL Flush Distribution .....	139

# TABLE OF CONTENT

MODEL Mix-Time & Detail Rates .....	141
MODEL Raw Material Substitution .....	145
MODEL Worksheet Comparison .....	147
CHAPTER 7 DATA ANALYSIS VI (Applications) .....	148
Resin Solutions .....	149
Temperature and Viscosity .....	150
Temperature, Viscosity & Vehicle Substitution .....	153
Sigma-Blade Mixer Applications .....	162
 BIBLIOGRAPHY .....	168
APPENDICES .....	ABCDE
Flush Formulae .....	A1
BASIC Program Code (Model-A0) .....	A6
BASIC Program Code (Model-A1) .....	A8
BASIC Program Code (Model-A3) .....	A10
BASIC Program Code (Model-AN) .....	A12
BASIC Program Code (Model-B0) .....	A14
BASIC Program Code (Model-C0) .....	A16
BASIC Program Code (Model-D0) .....	A18
BASIC Program Reports .....	A21
Flush Formulae Derivations .....	B
MathCAD Models .....	C
VBA - EXCEL Program Code .....	D
VBA - EXCEL Worksheet Reports .....	E

# **ABSTRACT**

## **CONTENT**

**Treatise Description**

**Model Descriptions**

**Mixer Process Description**

# **ABSTRACT**

## **Mathematical Models of Mixing With Applications of Viscosity, Load Capacities and Mix Rates**

**By**

**Herbert Norman Sr.**

This is a mathematical algorithm that approximates the total number of mixing stages [ $n$ ] required to process optimum amounts of reactants (varnish and aqueous pigment) in a mixing vessel of fixed capacity [B] and model the dynamics of mixing which occurs between stages. In many mixing procedures, the reactant amounts are calculated in increments [i] by algorithms to insure efficient use of the mixer's capacity, while attempting to adhere to a uniform viscosity function [ $\eta_i$ ] for the product.

The viscosity function defines how the paste will thicken over several unit-flushing stages [ j ], where j is distributed from 1 to n;  $1 \leq j \leq n$ . The distributions can be defined by mathematical functions or can be manually induced after being determined experimentally in a laboratory environment. In each stage of mixing, at least one of the added reactants is a calculated charge of vehicle (resin, solvent or varnish) or a charge of organic pigment presscake. The presscake has the physical properties of pigment suspended in water.

The mixing dynamics are comprised of two reactants (presscake and vehicle) which will form a slurry of suspended water, pigment and varnish. The resin and solvent (varnish) particles are more attracted to the pigment particles than the water, thus **wetting** the pigment and displacing the water in an environment where the vehicle-to-pigment ratio is greater than one. After the displaced water clears, it can be extracted from the system by means of pour-off and vacuum. The complete process is known as **flushing**.

The objective of this treatise is to expand the development of general mathematical capacity load models by creating several dynamic mixing simulations of the opti-

## **ABSTRACT**

mized flushing procedures. Given the input parameters, the models calculate flushed output parameters and the results of the mixing action of the pigment dispersing into the vehicle. This thesis develops three models that predict the weight of the products at the end of the mixing stages with relative times required for dispersion. These models are referred to as Treatments I, II and III. All three of the models produce feasible outputs, some of which were verified by processes used on actual manufacturing work orders. There will be new models developed in this paper which will simulate the flushing dynamics that occur between mixing stages. Since all of these simulations are math models, the procedures will be programmed on a computer. In this thesis, all source code for programs is provided and written in QuickBasic. Procedures and analysis will also be modeled in MathCad and MS-Excel worksheets.

The idea is a gift from God and a labor of love. It has been a wonderful journey since its inception in 1976. I am very grateful and appreciative to all of the people and my teachers who encouraged me to pursue this project. Even though a major part of the flushing manufacturing industry has relocated to Asia and Europe, my fascination with this technology remains and I am urged me on to study more about its applications. It seems as though it has taken on a new life through math modeling and computer science. And sincere thanks to my parents who ignited the fire within me to explore, learn and share discoveries with others. I miss them so very much.

Permission and authorization is hereby granted to the readers of this document by the author to copy, enhance in its entirety or in part, for the purposes of mathematical education. Your feedback and comments are encouraged. Please direct them to Herb Norman Sr.

Email Addresses:

[HerbNorman1@aol.com](mailto:HerbNorman1@aol.com)

[HerbNorman1@msn.com](mailto:HerbNorman1@msn.com)

# ABSTRACT

**Treatment -I (Model A)** requires initial amounts of pigment and vehicle to be charged to the mixer. The model calculates the amounts of pigment and vehicle charges that are required for each mixing stage so that the sum of the increment charges will equal the optimized total charge. In other words, this model distributes the total charge to agree with an empirical viscosity distribution. Optimization is the primary focus of this treatment while adhering to the viscosity distribution and holding the mixer capacity constant. The calculated capacity,  $[B_i]$ , is an output parameter and will be listed at each mixing stage to compare to the constant capacity,  $B$ . The input Allowance parameter,  $[E_0]$ , is an estimated % of the constant capacity. Mathematically,  $[E_0]$  is equal to the water displacement in the final mixing stage.

<u>INPUT DATA</u>	<u>OUTPUT DATA</u>
Capacity Constant $[B]$	Calculated Capacity at stage $[i] = B_i$
Initial Pigment Charge $\sum P_i$	Number of mixing stages $[n]$
Initial Vehicle Charge $\sum V_i$	% Vehicle after last stage $[x_n]$
Relative Viscosity of the Pigment $[\eta_p]$	System Viscosity Constant $[k_n]$
Relative Viscosity of the Vehicle $[\eta_v]$	Viscosity Distribution $[\eta_i]$
% Solids of Presscake $[r]$	% Pigment per stage $[x_p(i)]$
Viscosity Distribution Function $[\eta_i]$	% Vehicle per stage $[x_v(i)]$
Allowance $[E_0]$	Pigment Charge per stage $[P_i]$
	Vehicle Charge per stage $[V_i]$
	Water Displacement per stage $[w_d(i)]$
	Total Pigment Charge $\sum_{i=1}^n P_i$
	Total Vehicle Charge $\sum_{i=1}^n V_i$

## ABSTRACT

**Treatment - II (Model B)** requires  $[x_p]$ , the % pigment in the total mix, as an input parameter. This parameter along with the capacity,  $[B]$ , is used to calculate the initial pigment and vehicle charges, which are required as input parameters in Treatment-I. The remaining steps of the procedure and the objectives are identical to Treatment-I. The model uses the mixer's capacity,  $[B]$  along with the viscosity distribution as the critical input parameters to optimize the loading of each mixing stage  $[i]$ , and optimize the yield.

The total amount of pigment,  $\sum_{i=1}^n P_i$  and vehicle,  $\sum_{i=1}^n V_i$  required to charge the mixer is an output parameter in this procedure.

### INPUT DATA

Mixer Capacity (**B**)  
% Pigment after last stage ( $x_p$ )  
Relative Viscosity of the Pigment ( $\eta_p$ )  
Relative Viscosity of the Vehicle ( $\eta_v$ )  
% Solids of Presscake (**r**)  
Viscosity Distribution Function  $\eta_i$

Allowance  $E_0$

### OUTPUT DATA

Number of mixing stages (**n**)  
% Vehicle after last stage  $x_v$   
System Viscosity Constant ( $k_v$ )  
Viscosity Distribution  $\eta_v(i)$   
% Pigment per stage  $x_p(i)$   
% Vehicle per stage  $x_v(i)$   
Pigment Charge per stage  $P_i$   
Vehicle Charge per stage  $V_i$   
Water Displacement per stage  $w_d(i)$

Total Pigment Charge  $\sum_{i=1}^n P_i$

Total Vehicle Charge  $\sum_{i=1}^n V_i$

Calculated Capacity at stage (i),  $B_i$

# ABSTRACT

Treatment - III (Models C & D) uses the input parameter, Total Pigment Charge  $\sum_{i=1}^n P_i$ ,

to create the pigment distribution,  $P_i$ . In this model, the pigment distribution is a **geometric progression**, whose sum is equal to the input total pigment charge,  $\sum_{i=1}^n P_i$ . The

number of terms in the geometric progression, (**n**), is treated as the number of mixing stages in the flush procedure. The viscosity distribution is an output parameter based on the actual % pigment,  $x_p(i)$ , calculated at each incremental stage (**i**). The mixer capacity, **B**, is held constant through out the procedure. The calculated capacity  $B_i$ , is an output parameter and will be listed at each mixing stage to compare to the constant capacity, **B**. In this treatment, the allowance, **E<sub>0</sub>**, is not required.

## INPUT DATA

Total Pigment Charge	$\sum_{i=1}^n P_i$	Number of mixing stages [ <b>n</b> ]
Total Vehicle Charge	$\sum_{i=1}^n V_i$	% Vehicle after last stage [ <b>x<sub>v</sub></b> ]
Relative Viscosity of the Pigment [ $\eta_p$ ]	System Viscosity Constant [ <b>(k<sub>v</sub>)</b> ]	
Relative Viscosity of the Vehicle [ $\eta_v$ ]	Viscosity Distribution $\eta_i$	
% Solids of Presscake [ <b>r</b> ]	% Pigment per stage [ $x_p(i)$ ]	
Pigment Distribution Function $\eta_v$	% Vehicle per stage [ $x_v(i)$ ]	
<i>In a Geometric Progression model, Capacity <b>B<sub>i</sub></b> is Constant for all stages. [ <math>1 &lt; i &lt; n</math> ]</i>	Pigment Charge per stage [ $P_i$ ]	
	Vehicle Charge per stage [ $V_i$ ]	
	Water Displacement per stage [ $w_d(i)$ ]	

$$\text{Total Pigment Charge } \sum_{i=1}^n P_i$$

$$\text{Total Vehicle Charge } \sum_{i=1}^n V_i$$

# ABSTRACT

## Observed Process Reaction Per Mixing Stage (All Treatments)

A given amount of presscake, PW, is mixed with a given amount of vehicle, V, to produce a paste, PV (wetted pigment) and displaced water, W.



PW .....	Aqueous Pigment (Presscake)
W .....	Displaced Water
V .....	Resin or Resin Solution
PV = P+V .....	Pigment <b>wetting</b> (Paste)
P .....	Pigment (Non Aqueous)

Given a mixer of **bulk capacity (B)**, several **mixing stages (i = 1, 2, 3, ... n)** of **aqueous pigment (PW)** and **vehicle (V)** are charged to the mixer in calculated amounts such that the charge (**PW + V**) in any given stage (*i*), plus the paste or wetted pigment that has already been mixed in prior stages, will always equal or be less than the bulk capacity (**B**).

**Formula #2:** Before Mixing

$$B \geq \sum_{i=1} (P+V)_{1,2,3,\dots,i-1} + PW_i + V_i$$

**Formula #3:** After Mixing

$$B \geq \sum_{i=1} (P+V)_{1,2,3,\dots,i-1} + PV_i + W_i$$

The discharge of water, ( $W_i$ ), after any stage [i] of mixing creates the net capacity for the next stage of additives,  $(P_{i+1} + V_{i+1})$ .

# **CHAPTER - 1**

## **INTRODUCTION**

### **CONTENT**

**Statement of the Problem**

**Purpose of the Study**

**Importance of the Study**

**Scope of the Study**

**Rationale of the Study**

**Definition of Terms**

**Overview of the Study**

**Viscosity - Description**

**Dispersions - Description**

# **CHAPTER - 1**

## **INTRODUCTION**

### **Statement of the Problem:**

Most of the written references on pigment dispersion focus on the chemistry of organic colorants and the physical chemical properties of the mixes and suspensions. The flushing process has progressed over the years from grinding in a mixing vessel to movement through conduits to complex helical mixing chambers.

The former method involves adding aqueous pigment (presscake) and oil based vehicles into a sigma-blade mixing vessel over several stages. The mixing displaces the water from the pigment-presscake and encapsulates the pigment particles with the oil-based vehicles. The water is poured off of the pigment dispersion and the cycle is repeated until the vessel is filled to near capacity or until the target yield value of the mix is reached. The process is called flushing and it is as much of an art as it is a science. Process operators modify the procedures much like a cook uses a recipe as a general guideline. Very few processes are identical. Some pigment and organic ink manufacturers still use this process.

Quantifying the flush process as much as possible is the primary focus of this project. By using the above general description of the flushing process, models will be created to simulate the procedure. These models will use bulk load capacity, mixing time and viscosity as the major constraints to produce the number of mixing stages that are required to optimize the quantities of presscake and vehicle.

There are an infinite number of ways to load the ratio of vehicle to pigment charges for each addition. The ratio used for each charge, is usually determined by experimental methods in a laboratory environment. One of the project objectives in is to

# **CHAPTER - 1**

## **INTRODUCTION**

create models and methodologies that simulate this experimental process. By using the bulk load capacity and vehicle viscosity as input parameters, these models will estimate the required quantities of vehicle and pigment needed at each mixing stage. The expansion of this thesis will also generate relative mix times required to disperse the reactants at each stage of charging.

### **Purpose of the Study:**

The purpose of this project is to show how the models are created and used to predict and analyze the viscosities of resin solutions and pigment dispersions prior to and during actual mixing. The models are mathematical functions, which show how temperature, concentrations and other parameters relate to the flow of end mixed product.

Further development of these models will show how mathematical logic can be used to simulate and analyze complex mixing procedures using relative viscosity and mixing capacity. These models will simulate the paint flow and pigment dispersion dynamics used in industry.

### **Importance of the Study:**

The procedure for getting projects from concept to production works much the same as it did decades ago, except for the upgrades in plant, lab and computer equipment. Hopefully the system is more productive and efficient. The need for analysis still remains and is even more important. The experience of the technologist is just as important now, if not more so. The procedure that is run in the lab is a model of expected

# **CHAPTER - 1**

## **INTRODUCTION**

results in a production environment. The skill set of the technologist, the quality of the lab equipment used and the quality of the analysis of the results, will determine how well the lab results correlates to the production application.

### **Scope of the Study:**

Good models will yield plausible results, which can save time and resources in development and production. If it is useful, it can be a valuable tool. As previously mentioned, there are numerous models that are theoretically possible, but the models selected have been chosen because of their close correlation to the observed manufacturing process. This selection process will be explored in more detailed later in this thesis. The models developed in this project have been created with the aide of tools such as Math Cad, Microsoft Excel spreadsheets and QBasic programs. These developed applications will be listed in the appendices.

This project will refer to calculated or relative values of viscosity (poise). In no way is it intended for these values to be interpreted as the absolute viscosity nor the coefficient of viscosity of the dispersion. At best, the calculated viscosities and yield values are intended to estimate and quantify the relative thickness of paints, pigments, resins and solutions with respect to each other. And the relative mixing times are at best, a relative indicator that correlates the viscosities of the reactants with the time that it takes to mix them,

In this project, the bulk load capacity is the maximum pounds required to optimize the mixer and produce the desired output. The unit of measure used for the

# **CHAPTER - 1**

## **INTRODUCTION**

amounts of vehicle and pigment to be charged to the mixing vessel will also be pounds unless otherwise stated.

The treatment of the models uses mathematics, which range from Summation Algebra to Linear First Order Differential Equations. Most of the mathematical expressions will be derived from logical statements, much like postulates and proofs that are used in algebra and geometry. These proofs and derivations will also be detailed in the appendices.

### **The Rationale of the Study:**

A few years ago, the typical industrial coatings development group consisted of several gifted and creative people with many years of rheological and analytical backgrounds. Their expertise ranged from high school graduates who are curious and gifted in the graphic arts to Ph.D.'s. in Engineering, Physics and Chemistry. It has been my privilege to work with some of these individuals in the pigment manufacturing and finished ink industry. At that time, microcomputer technology was being introduced into the color and coatings industry. A typical pigment design problem would have required a senior technical person to outline or sketch a flush color procedure and assign it to a junior technician or engineer to work on. The technologist would review the lab procedure, make the final calculated adjustments and gather the materials needed to complete the lab procedure. Upon completion of the lab work, the technologist reviews the results, completes the analysis and returns the document to the senior technologist.

# **CHAPTER - 1**

## **INTRODUCTION**

The primary objective of the methodologies and models that are developed in this project is to emphasize their importance and improve the quality and productivity with regard to analysis and project management in a laboratory environment.

More specifically, this project will show how these models are used to estimate viscosities of resin solutions. The models are comprised of mathematical functions, which show how temperature, concentrations and mixer capacity affect the flow of resin and pigment dispersions.

Further development of these models will show how mathematical logic is used to analyze complex mixing procedures, relative viscosities and mixing capacities. These models simulate the paint flow and pigment dispersion dynamics that are currently used in the industry.

### **Definition of Terms:**

Apparent Viscosity	Term used to indicate that the viscosity is that of a non-Newtonian liquid. The adjective apparent is not meant to imply that the viscosity is an illusory value, but rather that the viscosity pertains to only one shear rate condition
Binding	Building up the body with respect to viscosity
Bulk Capacity	The maximum load in pounds a flush mixer will handle.
Colloid	Dispersions of small particles of one material in another.
Dilatant	Property of certain pigment dispersion systems which causes them to exhibit an abnormally high resistance to flow when the force which causes them to flow is suddenly increased.
Dispersion	Same as wetting.
Encapsulate	The movement of wetted particles into the body of the liquid or suspension.
Flocculation	The action of dispersed particles coming back together and forming clusters. As a result, the body builds up thus causing a higher viscosity or yield value.
Fluidity	The reciprocal of Newtonian viscosity. Unit of measure is (Rhe)

# **CHAPTER - 1**

## **INTRODUCTION**

Flushing	In the flushing process the moist cakes from the filter press are introduced into a jacketed kneading type mixer together with the calculated quantity of vehicle. During subsequent mixing, the oil or vehicle displaces the water by preferential wetting, the separated water being drawn off periodically; the final traces of water being removed, when necessary, by heat and partial vacuum. The batch is then sometimes given several grinds through a roller mill to complete the process.
Grinding	The mechanical breakup and separation of the particle clusters to isolated primary particles.
Newtonian Liquid	True liquid: A liquid in which the rate of flow is directly proportional to the applied force
Oil Absorption	The minimum amount of oil or varnish required to "wet" completely a unit weight of pigment of dry color. Raw linseed oil is the reference vehicle in the plant industry, while litho varnish of about twelve poises viscosity (#0 varnish) is the testing vehicle more commonly used in the printing ink industry.
Pigment	The solid portion of printing inks which impart the characteristics of color, opacity, and to a certain part of the printing ink that is visible to the eye when viewing printed matter.
Plastic	A viscous liquid which exhibits Plastic Flow. A liquid that has yield value in addition to viscosity, and a definite finite force must first be applied to the material to overcome the static effect of the yield value before the material may be made to flow.
Presscakes	The moist cakes from the filter press are used in the flush process
Pseudoplastic	Material that has variable fluidity and no yield value.
Rheology	The science of plastic flow
Thixotropy	Characteristic of false body or high yield value at rest. Applied agitation breaks down the false body to near newtonian flow, but will return to high yield upon standing.
Viscosity	The proportionality constant between a shearing force per unit area (F/A) and velocity gradient (dv/dx).
Wetting	Wetting refers to the displacement of gases (such as air) or other contaminants (such as water) that are absorbed on the surface of the pigment particle with subsequent attachment of the wetting medium to the pigment surface
Yield Value	The permanent property of an ink that is a measure of its inherent rigidity. It refers to a certain minimum shear stress that must be exceeded before flow takes place

### **Overview of the study:**

# CHAPTER - 1

## INTRODUCTION

Elementary science and basic chemistry taught us that matter existed in one of three states; solid, liquid or gas. As we grew older, we learned that substances exist in physical states, which are none of these three basic states, but fall somewhere in between. Smoke, molasses, varnish and paint are examples. P. W. Atkins, Physical Chemistry (1982), p. 842, a college textbook, defines a colloid as "... dispersions of small particles of one material in another."

This project will focus on the methodology and model development to approximate the flow and general rheological parameters combined with the load capacities of the mixing vessel using aqueous displacement. Herbert J. Wolfe, Printing and Litho Inks, (1967), p. 90, describes aqueous displacement (flushing), "In the flushing process the moist cakes from the filter press are introduced into a jacketed kneading type mixer together with the calculated quantity of vehicle. During subsequent mixing, the oil or vehicle displaces the water by preferential wetting, the separated water being drawn off periodically; the final traces of water being removed, when necessary, by heat and partial vacuum. The batch is then sometimes given several grinds through a roller mill to complete the process."

The references to "**calculated quantities of vehicle**" and "**dynamic mixing**" are the primary focus of this effort. Given a quantity of pigment paste, there are an infinite number of given quantities of vehicle that can be mixed with the paste, such that the ratio of vehicle to pigment solids is greater than one. The definition of wetting, according to Temple C. Patton, Paint Flow and Pigment Dispersion, 1<sup>st</sup> edition, (1963), p. 217, "Wetting refers to the displacement of gases (such as air) or other contaminants

# CHAPTER - 1

## INTRODUCTION

(such as water) that are absorbed on the surface of the pigment particle with subsequent attachment of the wetting medium to the pigment surface.”

This mixing process is repeated until a mass of flushed pigment, suspended in vehicles (oils, varnishes and resin). The relative viscosity of the end product is usually greater than the viscosity or yield value of the first mixing stage. ***The first and early mixing stages are usually where wetting takes place. Vehicle to pigment ratio is at its highest values during wetting, to maximize the dispersion and encapsulation of the pigment particles. Because wetting is followed by a series of grinding and binding stages, the vehicle to pigment ratio is gradually decreased. Sometimes vehicles of higher relative viscosities are used in these later stages in order to build the body of the mix.***

The relative viscosity increases sharply in the early stages and levels off as the number of mixing stages approaches the final stage ( $n$ ). A function that will model the building of the incremental viscosities, ( $\eta_i$ ), over the stages, ( $1 \leq i \leq n$ ), could be an exponential function  $(1 - e^x)$  or a logarithmic function,  $(a \ln x)$ . Refer to Figure 1.01 below.

# CHAPTER - 1

## INTRODUCTION

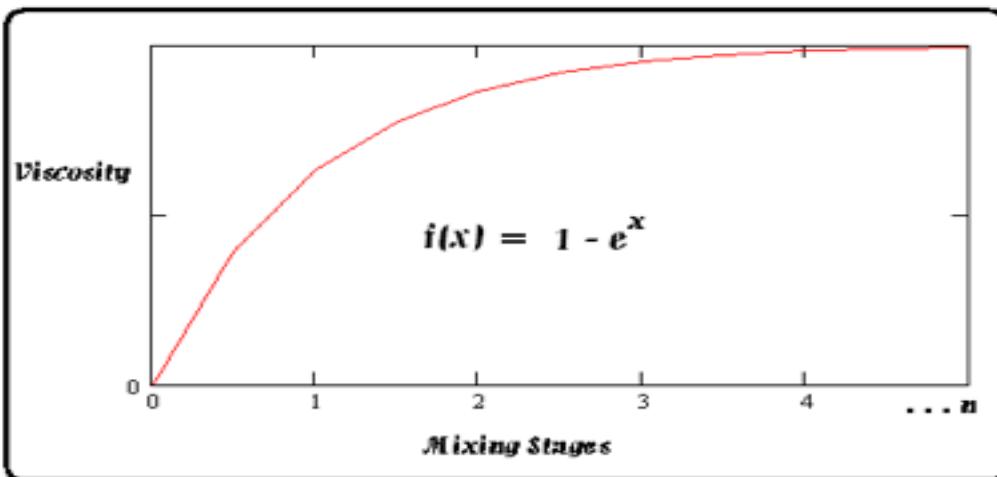


Figure 1.01a

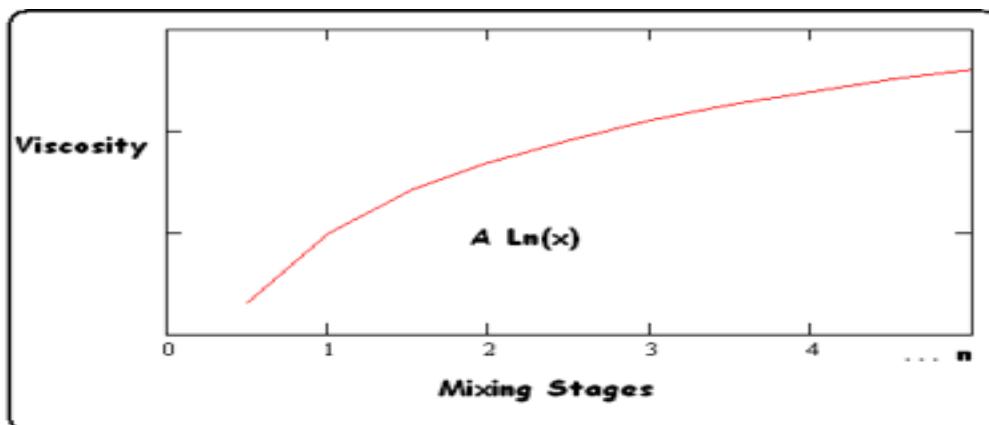


Figure 1.01b

In theory, there is no limit to the number of mixing stages that could be used, but in reality, mixing capacity and the capacity (quantity of materials) to be mixed, is one the key parameters, which implies a logical end point to stop the process.

Given a beaker and a spatula as the mixing utility, the capacity (**B**), of the beaker and the ability to apply shear to the mixture of paste and vehicles, tends to identify the some of the practical limits of the process. The contents of the beaker and the energy

# CHAPTER - 1

## INTRODUCTION

required to mix the vehicle and displace the water, should not exceed the beaker volume of the mixing unit and cause overflow. Once the water is squeezed from the sticky mass of wetted pigment, the water is discarded.

If the beaker volume is optimized prior to mixing, the new volume for the next addition is equal to the volume of water discarded. This mixing cycle is repeated until the working capacity of the mixer is reached and there is no more room to mix without overflow. The number of mixing stages ( $n$ ) required to flush ( $P$ ) amount of pigment is also determined experimentally and is one of the parameters that will be used in this project. For the sake of symbolic variables, ( $PW$ ) will be assigned to aqueous pigment paste, since it is composed of pigment, ( $P$ ), and water, ( $W$ ). The variable assigned to vehicle is ( $V$ ). The colloidal suspension or pigment dispersion is assigned the variable ( $PV$ ). Refer to Figure 1.02 below.

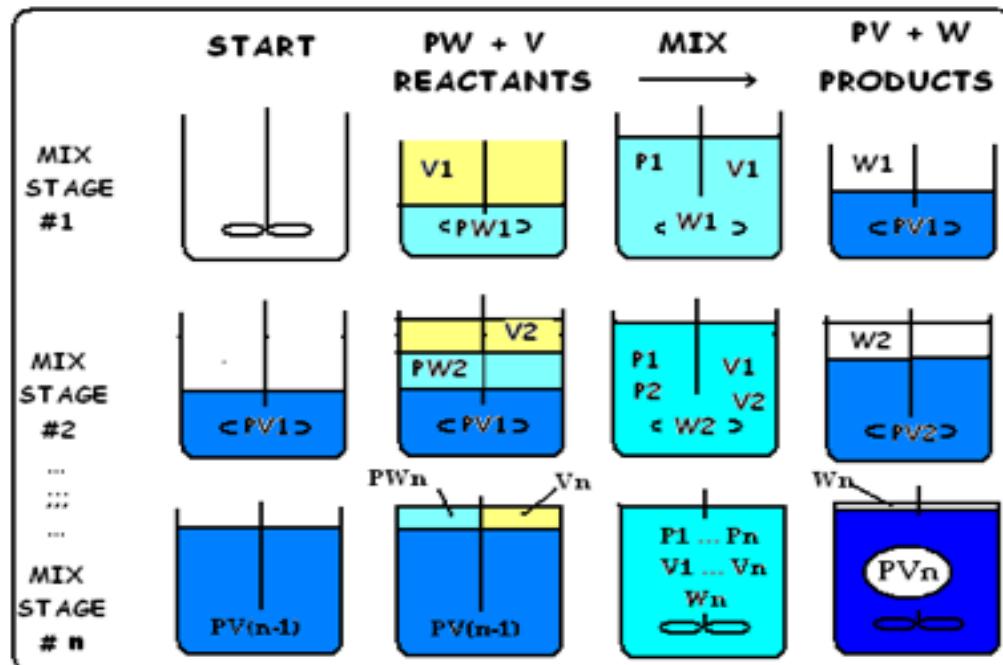


Figure 1.02 (Flush Sequence)

# CHAPTER - 1

## INTRODUCTION

The general mixing reaction equation is expressed as follows:

$$\text{Formula #1: } \mathbf{PW + V = PV + W}$$

Given a mixer of capacity (**B**), several increments (**n**) of aqueous pigment (**PW**), and vehicle (**V**), are charged to the mixer in amounts such that the incremental charge (**PW + V**), will not overflow the mixer vessel. At the end of each mixing stage, the water (**W**), becomes insoluble in the mixture (**PV + W**), and is discharged from the vessel leaving only a sticky mass of pigment dispersed in the vehicle (**PV**).

$$\text{Formula #2: Before Mixing: } B \geq \sum_{i=1}^{i-1} (P + V)_i + PW_i + V_i$$

$$\text{Formula #3: After Mixing: } B \geq \sum_{i=1}^{i-1} (P + V)_i + PV_i + W_i$$

In the mixing scenario given above, (**n**), the number of mixing stages required, has a direct relationship with the total pigment charge, (**P**), water displacement,  $P(1/r-1)$ , and total vehicle amount, (**V**). The number of mix stages, (**n**), varies indirectly with the final % vehicle ( $x_v$ ) and the working capacity of the mixer, (**B**). An empirical expression,

$$n = \left( \frac{\sum \frac{P}{r} + \sum V}{x_v B} \right)$$

will serve as an algorithm to estimate the parameter (**n**).

# **CHAPTER - 1**

## **INTRODUCTION**

### **Viscosity:**

The difficulty of mastering rheology, the science of flow and deformation, is best summarized by T. C. Patton's, Paint Flow and Pigment Dispersion, 2<sup>nd</sup> edition, (1979), p. 1, "Unfortunately, flow phenomena can become exceedingly complex. Even such a simple action as stirring paint in a can with a spatula involves a flow pattern that challenges exact mathematical analysis. However, simplifications and reasonable approximations can be introduced into coating rheology that permit the development of highly useful mathematical expressions. These in turn allow the ink or paint engineer to proceed with confidence in controlling and predicting the flow performance of inks or paint coatings."

Viscosity is defined as the opposition to fluidity. Water passes through a funnel quickly; boiled oil slowly, while treacle would pass through very slowly. An explanation for such varied rates of liquid movement is as follows. When a liquid is caused to move, a resistance to the motion, is set up between adjacent layers of the liquid, just as when a block of wood is dragged along the floor. In the latter case, friction arises between the two solid surfaces; in the case of a liquid, friction arises between moving surfaces within it. This internal friction is called viscosity. The frictional force, which opposes motion is felt when one moves a hand through a tub of water. All liquids show a resistance to flow. Although forces applied externally, affect the rate of liquid flow, viscosity is concerned only with the internal frictional effect.

If two layers of a liquid are moving at different speeds the faster moving layer experiences resistance to its motion, while the slower moving layer experiences a force which increases its velocity. The coefficient of viscosity is defined as the force in dynes

# CHAPTER - 1

## INTRODUCTION

required per square centimeter to maintain a difference in velocity of 1 cm/sec between two parallel layers of the fluid, which are ( $\Delta d$ ), 1 cm apart. This is best represented in the following expression from James F. Shackelford's, Introduction to Materials Science

for Engineers, (1985), p.329,  $\eta = \frac{f\Delta d}{\Delta v a}$ , where ( $\eta$ ) is the coefficient of viscosity in poise,

" $a$ " is the area in  $\text{cm}^2$ , ( $\Delta v$ ) is change in velocity in  $\text{cm/sec}$  and ( $f$ ) is the applied force

in dynes.  $\eta = \frac{\text{stress}}{\text{rate of shear}}$

The liquids for whose rate of flow varies directly with the applied force ( $f$ ), are called **Newtonian Liquids**. However, **Non-Newtonian** flow is observed when the dispersed molecules are elongated, when there are strong attractions between them or when dissolved or suspended matter is present, as in resin and paint solutions. Most paint and pigment solutions show Non-Newtonian viscosity to some degree.

### **Newtonian (Simple Flow):**

An ideal liquid having a constant viscosity at any given temperature for low to moderate shear rates

# **CHAPTER - 1**

## **INTRODUCTION**

### **Non-Newtonian (Plastic Flow):**

Flow with a yield value. This is a minimum shear stress value that must be exceeded before flow will take place. Below yield value, the substance has elastic properties. (Pigment-Resin-Solvent Solutions)

### **Non-Newtonian (Pseudoplastic Flow):**

A hybrid flow, which simulates plastic flow at moderate to high shear rates, and Newtonian flow at low shear rates. (Paint and Ink Solutions)

### **Non-Newtonian (Dilatant Flow):**

Viscosity is reduced as shear stress is increased. This type of solution gets thicker on increased agitation. (Rare Paint Systems)

### **Non-Newtonian (Thixotropic Flow):**

Much like Pseudoplastic flow, but more complex and plasticized. In general, thixotropic breakdown (loss of viscosity) is fostered by an increase in the shear stress, by prolonging the shear time. When the shear stress is removed, recovery of thixotropic viscosity ensues as thixotropic structure is again built up throughout the paint system.

### **Dispersions:**

As a vehicle is incorporated with pigment by a mixing action, a good dispersion initially displays significant resistance to sudden pressure, turning dull in appearance. With further vehicle addition, the mixture reaches a point where it coalesces into a

# **CHAPTER - 1**

## **INTRODUCTION**

smooth glossy mass. A small additional increment of vehicle converts the mass into a mobile dilatant dispersion.

Physically, this dispersion is characterized by deflocculated particles, fully separated by a minimum of dispersion vehicle to give a relatively closely packed system. If the shear stress applied to this dispersion is low, sufficient time is allowed for the particles to slip and slide around each other without contact. This action results in a minimum viscosity resistance. If the shear stress is high, then adjacent particles will ram through the mix barrier and separate themselves to establish solid-to-solid contact. Without the lubrication afforded by the intervening dispersion vehicle, major viscous resistance is exhibited.

Besides mixing the vehicle with the pigment particles, there is another phenomena taking place which affects the body and consistency of the dispersion. This action is absorption. The amount of absorption that takes place depends on the interactive properties of the surface of the pigment and the properties of the vehicle. The absorption causes some puffiness about the surface of the pigment particles and thus the same is observed on a larger sampled mass. This puffiness causes a slight build up in viscosity of the dispersion and also contributes to the flocculation.

The above properties will provide the basic resource for constructing the logic and math models to simulate the flushing process.

# **CHAPTER - 2**

## **REVIEW OF RELATED LITERATURE**

### **CONTENT**

**Solvents, Oils, Resins & Driers**

**Introduction to Paint Chemistry**

**Viscosity & Flow Measurement**

**Paint Flow & Pigment Dispersion**

**Printing & Lithographic Inks**

**Physical Chemistry - Suspensions**

**Fluid Mechanics & Hydraulics**

**Chemical Engineering Calculations**

**Ordinary Differential Equations**

**Geometric Series & Engineering Math**

# **CHAPTER - 2**

## **REVIEW OF RELATED LITERATURE**

### **Paint Technology Manuals**

#### **PART TWO – Solvents, Oils, Resins and Driers**

**Published on behalf of The Oil & Colour Chemists' Association – 1961**

This manual covers the chemistry and physical chemical characteristics of oils and resins. The book was very popular with technologists in the coatings industry because it covered the chemical derivations and practical applications with regard to paint manufacturing and ink making. Regarding this project, it was a very useful resource for information on resins and solvents.

Sometimes the technologist encounters significant chemical reactions when mixing certain resin solutions such as driers. Without taking into account the basic chemistry of solvents and resins, one might assume that just mixing some oil with resin, a varnish like substance will result. And by adding more oil or solvent to the mix, one would expect the result to be a thinner solution, which should flow more easily. But what if there is a reaction with the oxygen in the air, solvent and the resin and the mix begins to thicken. This is what happens when a drier is created.

Coatings of all resin solutions have a tendency to dry because of a chemical process called oxidation. But what categorizes a resin solution as a drier is the relative rate of drying, resin concentration and sometimes temperature.

## **CHAPTER - 2**

### **REVIEW OF RELATED LITERATURE**

"Paints have been made for centuries by mixing pigments such as red lead, white lead and umber with drying oils, and it became obvious that these paints dried faster than the raw oils. Eventually it was discovered that oils stored in the presence of lead or manganese compounds, e.g. red lead or manganese dioxide, or better still if heated in the presence of these compounds so as to produce oil-soluble products, developed improved drying properties; this formed the basis of the production of boiled linseed oil; one of the foundations of paint formulation.  
" (Atherton, 1961, p. 31)

Drying is just one of the many challenges that a coating technologist will encounter. Because of various degrees of chemical reactions, there are numerous levels of compatibility of solvents, resins and pigments. Today these dispersions are classified into solvent and resin systems.

This project employs non-drying dispersions, which will allow wetting to take place without rapid oxidation and aggregation. The mixing methodology assumes ideal systems of resins and solvents. This manual on resins and solvents gave me a great appreciation on the complexity and sophistication of the behavior of pigment and resin dispersions. There is much room for further development of this mixing model using non-idealistic resin solutions as vehicles. The ink chemistry and physics involved in the actual rheology of pigment dispersions go far beyond the level of mathematics used in this project.

## **CHAPTER - 2**

# **REVIEW OF RELATED LITERATURE**

### **Introduction to Paint Chemistry**

**By G. P. A. Turner - 1967**

Turner's treatment of paint chemistry is somewhat of a general treatment of the physics and chemistry of paint. It reviews the inorganic and organic systems of paint chemistry. Turner also incorporates some of the information that was previously covered on oils, solvents, resins and driers. This book on paint chemistry is more of a general textbook on the manufacturing and production of paint. It covers general atomic theory as it relates to molecular bonding of compounds used in paint. He discusses viscosity of suspensions and colloids. There is an introduction to substrates and color theory, where the science of polymer coating is explained quite clearly.

The chapter on pigmentation describes dough mixers, which are used by many pigment manufacturers to produce distributions.

"A fourth type of mill is the *heavy duty or ,pug' mixer*, in which roughly S-shaped blades revolve in opposite directions and at different speeds in adjacent troughs. A stiff paste is required. Several alternative mills are available, which the reader may discover elsewhere." (Turner, 1967, p. 119)

The dough mixers mentioned above is an accurate description of the mixing vessels used in processing flush-color dispersions. The S-shaped blades are called sigma blades. The paint mixing procedure is much like the flush procedure. The primary stages and their functions are described as follows.

"It is obvious from the mention of stiff pastes that the whole paint is not charged into the mill. In fact, the paint maker aims to put in the maximum amount of pigment of

## **CHAPTER - 2**

### **REVIEW OF RELATED LITERATURE**

pigment and the minimum amount of varnish to get the largest possible paint yield from his mill. This mixture forms the grinding or *first stage*. When the dispersion is complete (after a period varying from 10 minutes to 48 hours according to the materials and machinery involved), the consistency is reduced with further resin solution or solvent, so that the mill can be emptied as cleanly as possible. This is the 'let-down' or *second stage* and may take up two hours. The *third* or final *stage* (carried out in a mixing tank) consists of the completion of the formula by addition of the remaining ingredients. A break-down of a possible ball mill formula looks like this:

<u>%</u>	
<b>Pigment</b>	<b>10.0</b> <b>Stage I (grinding)</b> , Then add:
<b>Resin</b>	<b>1.0</b>
<b>Solvent</b>	<b>3.0</b>
<b>Resin</b>	<b>1.0</b> <b>Stage II (let down)</b> , Empty mill – then add:
<b>Solvent</b>	<b>3.0</b>
<b>Resin</b>	<b>29.0</b> <b>Stage III (completion of formula)</b>
<b>Solvent</b>	<b>51.5</b>
<b>Additives</b>	<b>1.5</b>
<b>TOTAL</b>	<b>100.0 %</b>

The exact composition of Stage I is found by experiment, to give the minimum grinding time and the most stable and complete dispersion. Stages II and III also require

## **CHAPTER - 2**

### **REVIEW OF RELATED LITERATURE**

care, as hasty additions in an incorrect order can cause the pigment to re-aggregate (flocculate).

The amount of pigment in the formula is that required for the appropriate colour, hiding power, gloss, consistency and durability. As a rough guide, the amount might vary from one third of the binder weight to an equal weight (for a glossy pastel shade)." (Turner, 1967, p. 119)

The above procedure is very much like the total flush procedure. This project is focused on the referenced grinding Stage I where the pigment is introduced into the system. In flushing, several stages are required to introduce all of the pigment into the system. The first stage of the series of grinding stages is called the wetting stage. In the wetting stage of flushing, usually the largest charge of pigment and vehicle is introduced to the mixer, where the vehicle charge is greater than the pigment. The purpose for the vehicle-to-pigment ratio being greater than one is to allow for the encapsulation of the pigment particles and maximum displacement of water. A low viscosity, due to the large amount of vehicle present, generally characterizes the wetting stage.

The subsequent stages are grinding stages, where the rest of the pigment is charged to the mixer in lesser amounts. The vehicle-to-pigment ratio for these stages is usually less than one. A graph of the viscosity of the dispersion with respect to the number of stages, usually looks similar to an exponential growth function. Refer to Figures 1.01a and 1.01b.

The 10.0% of pigment in the total paint dispersion shown above is 71.4% of Stage I. In flush procedures after the last stage of pigment charge, the percent pigment is usually in the range of 50% to 60%.

## **CHAPTER - 2**

# **REVIEW OF RELATED LITERATURE**

### **Ferranti Instrument Manual**

#### **The Measurement and Control of Viscosity**

#### **And Related Flow Properties**

**McKennell, R., Ferranti Ltd., Moston & Mancheser (1960)**

The Ferranti Instrument Manual was written to give the ink technician an overview of the complexity of measuring viscosity. The manual lists four major types of viscometers and some examples of each. Several types of non-Newtonian fluids are discussed. Different types of non-Newtonian measurements are exemplified and matched with the best type of viscometer. There are suggestions and examples of experimental techniques for measuring various types of non-Newtonian substances for experimental purposes as well as calibration.

A brief overview is given of how viscometers generate automatic flow-curve recordings and the curves are analyzed. Basic viscosity formulae are listed and discussed. Specific flow problems and suggested solutions are discussed.

# **CHAPTER - 2**

## **REVIEW OF RELATED LITERATURE**

The list of the four major types of viscometers is listed below. It is taken from the table of contents of the manual.

### **Types of Viscometer:**

#### **I. Capillary Viscometers**

**Absolute viscometers**

**Relative viscometers**

#### **II. Falling Body Viscometers**

**The Falling Sphere Viscometers**

**The Rolling Sphere Viscometers**

**The Falling Coaxial Cylinder Viscometer**

**The Band Viscometer**

#### **III. Rotational Viscometers**

**Coaxial Cylinder Viscometer**

**Cone-plate Viscometers**

#### **IV. Vibration Viscometers**

Besides being a great source for viscosity terminology, the section on special flow problems, the suggestion of using the function of percent solids content against apparent viscosity, is a major corner stone of the methodology of this project.

## CHAPTER - 2

### REVIEW OF RELATED LITERATURE

**The percentage solids content of slurries** and similar suspensions may be rapidly determined by constructing a curve of percent solids content against apparent viscosity. A suitable shear rate must be chosen and adopted as standard and equilibrium apparent viscosity readings taken on a number of slurries of known percentage solids content. Determination can be made in a fraction of the time required using conventional gravimetric techniques, with an accuracy which is acceptable for many applications. (Ferranti and McKennell, 1955, "Liquid Flow Problems and Their Solution": Reprint from Chemical Product)

Figure 2.01 is a report that was generated from one of the computer programs written by the author for this project. It shows how percent content is compared to apparent viscosity can be used as part of the flush dispersion analysis.

Total Pigment Charge ... (P) = 1350	Total vehicle Charge (V) = 1200				
Mixer Capacity ..... (B) = 3000	% Solids of Pigment (r) = .2				
Calculated Series Ratio (R <sub>f</sub> ) = 1.449	Pigment viscosity (N <sub>p</sub> ) = 240000				
Number of Mixing Stages (n) = 5	Vehicle viscosity t (N <sub>v</sub> ) = 100				
j	Pigment	Vehicle	Cum Pigment	Cum Vehicle	Pour-off
1	495.9365	520.3176	495.9365	520.3176	1983.746
2	342.2612	272.44	838.1977	792.7576	1369.045
3	236.2051	188.0192	1074.403	980.7768	944.8204
4	163.0125	129.7579	1237.415	1110.535	652.05
5	112.5	89.55006	1349.915	1200.085	450
j	Pigment	Vehicle	% Pigment	% Vehicle	Viscosity
1	495.9365	520.3176	.1080441	.8919559	231.8418
2	342.2612	272.44	.2006736	.7993264	476.7612
3	236.2051	188.0192	.3348936	.6651064	1355.177
4	163.0125	129.7579	.5293785	.4706215	6157.445
5	112.5	89.55006	.5294118	.4705882	6159.04

Figure 2.01  
A Report From Model-C

## **CHAPTER - 2**

# **REVIEW OF RELATED LITERATURE**

### **Paint Flow and Pigment Dispersion**

#### **Patton, T. C., 1st edition (1963) & 2<sup>nd</sup> edition (1979)**

Patton takes more of a mathematical approach to explore the dynamic properties of resin and pigment dispersions. Both editions provide a practical and comprehensive overview of rheological aspects of paint and coatings technology. The second edition includes expanded material on pigment-binder geometry, the theoretical aspects of dispersion; and a more detailed breakdown of grinding equipment.

The sections that are most referenced for this project are the ones which elaborate on viscosity, the effects of temperature and resin concentration on viscosity and pigment dispersion theory.

#### **Viscosity**

The treatment of viscosity theory is the same as the other resources. Patton uses tables which lists various substances and their viscosities to help the reader better understand the concept of flow. He also uses tables to show how well viscosity formulae correlate to actual experimental data. The models in this project will also use tables. I considered this illustrative technique to be very effective especially in showing results for analysis.

## CHAPTER - 2

### REVIEW OF RELATED LITERATURE

**Table 1-1**

	<b>Shear Stress (dynes/cm<sup>2</sup>)</b>	<b>Shear Rate (sec<sup>-1</sup>)</b>	<b>Viscosity (poises)</b>
<b>Emulsion</b>	280	7	40
	500	29	17
	625	72	9
<b>Vinyl Plastisol</b>	710	36	20
	1430	58	25
	2130	77	28

(Patton, 1963, 1<sup>st</sup> edition, p. 9, Table 1-1)

#### **Temperature and Viscosity**

Patton refers to a formula based on experimental data of temperature and related viscosity of a liquid.

“It has been found experimentally that for any given viscosity  $\eta$  the change in viscosity  $d\eta$  produced by a change in temperature  $dT$  is substantially the same for most liquids. Furthermore, the function  $f(\eta)$  of Eq. 1 depends primarily on the magnitude of the viscosity only (it does not depend appreciably on the nature of the liquid).”

$$\frac{d\eta}{dT} = f(\eta) \quad \text{Eq. 1} \quad (\text{Patton, 1979, 2<sup>nd</sup> edition, p. 91})$$

“Of the many equations that have been proposed for relating viscosity to temperature, one appears to represent the viscosity/temperature relationship most accurately. It is commonly referred to as Andrade’s equation (Eq. 2).

$$\eta = A(10^{B/T}) \quad \text{Eq. 2}$$

## CHAPTER - 2

# REVIEW OF RELATED LITERATURE

Equation 2 can be expressed alternatively in logarithmic form as Eq. 3.

$$\log \eta = \log A + \frac{B}{T} \quad \text{Eq. 3}$$

Temperature T must be expressed in absolute units ( $K = 273 + C$  or  $R = 460 + F$ ), and A and B are constants for the liquid in question.

If subscripts 1 and 2 are used to denote the conditions for two different temperatures, it can be readily shown (by subtraction) that the two conditions are related by Eq. 4."

$$\log \frac{\eta_1}{\eta_2} = B \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \quad \text{Eq. 4 (Patton, 1979, 2<sup>nd</sup> edition, p. 93)}$$

The following table shows how temperature effects the viscosity of linseed oil and also how well the above equations fit actual experimental data. This table will also serve as a resource to measure the accuracy of the formulae and models which will be developed in this project.

**Table 4-2: Comparison of Viscosity Values for Linseed Oil by Eqs. 2, 3 & 4 with Experimentally Determined Values**

<u>Temperature</u>	Viscosity Values (poises)			
	Exp.	Eq. 2	Eq. 3	Eq. 4
(F) 50	0.60	0.56	0.61	0.59
86	0.33	0.33	(0.33 Used in computation)	
122	0.18	0.20	0.18	0.19
194	0.071	0.071	(0.071 Used in computation)	
302	0.029	0.015	0.023	0.019

(Patton, 1979, 1st edition, p. 85, Table 4-2)

## CHAPTER - 2

# REVIEW OF RELATED LITERATURE

### Resin Concentration

“A common viscosity problem calls for calculating the change in a solution viscosity produced by a change in resin concentration. Such a change may be due to addition of let-down thinner, or it may occur as a result of blending together two compatible resin solutions.” (Patton, 1979, 1st edition, p. 88)

### Equations Relating Viscosity to Resin Concentration

“The simplest expression and possibly a fully adequate one for most purposes for relating solution viscosity to resin concentration takes the form of Eq. 9, where x is the fractional content of nonvolatile resin in the resin solution and A and B are constants.

$$\eta = A(10^{Bx}) \quad \text{or} \quad \log \eta = \log A + Bx \quad \text{Eq. 9}$$

To evaluate the constants A and B, solution viscosities at two different resin concentrations must be known. Once A and B are determined, a viscosity for any third resin concentration is obtained by straightforward substitution in Eq. 9.” (Patton, 1979, 1st edition, p. 88)

The data in the above tables will be referenced in later chapters to illustrate how other methodologies compare for accuracy and use in several models. Patton uses logarithms to the base-10 in the above methodologies. The formulae that will be used in the development of the flush models will use logarithms to the base-e or natural logs.

## **CHAPTER - 2**

# **REVIEW OF RELATED LITERATURE**

### **Printing and Litho Inks**

**Wolfe, H. J. (1967)**

Printing and Litho Inks blends the history and art of ink making with the world of science and technology. It is the source of many ink-making terms that are used in this project. Flushing is described as follows.

“As is well known, the kneading type of mixer also is employed in the “flushing” of pulp colors, i.e., the production of pigment-in-oil pastes directly from pigment-in-water pastes, by introducing the water-pulp color and the varnish into the mixer and agitating until the varnish has displaced the water. Steam-jacketed mixers are generally employed for this purpose. Air-tight covers also may be fitted to these mixers so that vacuum may be employed to remove the water from the pulp more rapidly.” (Wolfe, 1967, p. 445)

The viscosity of the dispersion, relative particle size and oil absorption of the pigment are very important characteristics that help determine the point at which to stop mixing. Wolfe provides tables and details about the properties of the different classes and types of pigment in dry color state. These dry properties directly relate to the flush procedure (pigment suspended in water) because the resulting dispersion after displacing the water (flushing), will have the same properties as if it were mixed dry. If the pigment particle size and the vehicles are the same, then the end result should be the same. The only difference will be the grinding methods. Regarding pigments, resins and solvents, Wolfe lists standard testing procedures, test equipment and test methods used in the printing ink industry. For example:

## **CHAPTER - 2**

### **REVIEW OF RELATED LITERATURE**

The term “oil absorption” as used in the dry color and printing ink industries refers to the minimum amount of oil or varnish required to “wet” completely a unit weight of pigment of dry color. Raw linseed oil is the reference vehicle in the plant industry, while litho varnish of about twelve poises viscosity (#0 varnish) is the testing vehicle more commonly used in the printing ink industry. (Wolfe, 1967, p. 472)

Viscosity is without a doubt, the most important characteristic of a printing ink vehicle, since it determines the length, tack and fluidity of the vehicle; which in turn in a large measure, determines the working qualities of the resulting inks. Although listed separately, the properties of viscosity are directly related to those of oil absorption and particle size. These three topics are key to the flush models developed in this project.

### **Physical Chemistry**

**Atkins, P. W. (1982)**

The concepts of suspensions and viscosity are covered quite extensively in physical chemistry. To get more clarification on these terms, Atkins’ “Physical Chemistry” is a good resource. For example:

“A major characteristic of liquids is their ability to flow. Highly viscous liquids, such as glass and molten polymers, flow very slowly because their large molecules get entangled. Mobile liquids like benzene have low viscosities. Water has a higher viscosity than benzene because its molecules bond together more strongly and this hinders the flow.

## **CHAPTER - 2**

### **REVIEW OF RELATED LITERATURE**

We can expect viscosities to decrease with increasing temperatures because the molecules then move more energetically and can escape from their neighbors more easily. (Atkins, 1982, p.18)

Regarding the relationship of viscosity to particle size as stated above in Wolfe, Atkins confirms this as follows:

The presence of macromolecules affects the viscosity of the medium, and so its measurement can be expected to give information about size and shape. The effect is large even at low concentrations, because the big molecules affect the surrounding fluid's flow over a long range." (Atkins, 1982, p.825)

### **Fluid Mechanics and Hydraulics**

#### **Giles, R. V. (June, 1962)**

Fluid mechanics and hydraulics explained the viscosity in such an abstract manner that it was somewhat limited as a resource in this project. However, the viscosity units of measure were clearly explained and came in very handy when the property of density was introduced.

##### **Absolute or Dynamic Viscosity ( $\mu$ )**

Viscosity of a fluid is that property which determines the amount of its resistance to a shearing force. Viscosity is due primarily to interaction between fluid molecules. (Poise, lb-sec/ft<sup>2</sup>)

##### **Kinematic Viscosity ( $\nu$ )**

Kinematic coefficient of viscosity is defined as the ratio of absolute viscosity to that of mass density ( $\rho$ ). (Stokes, ft<sup>2</sup>/sec =  $\mu/\rho$ )

**(Giles, 1962, p.3)**

## CHAPTER - 2

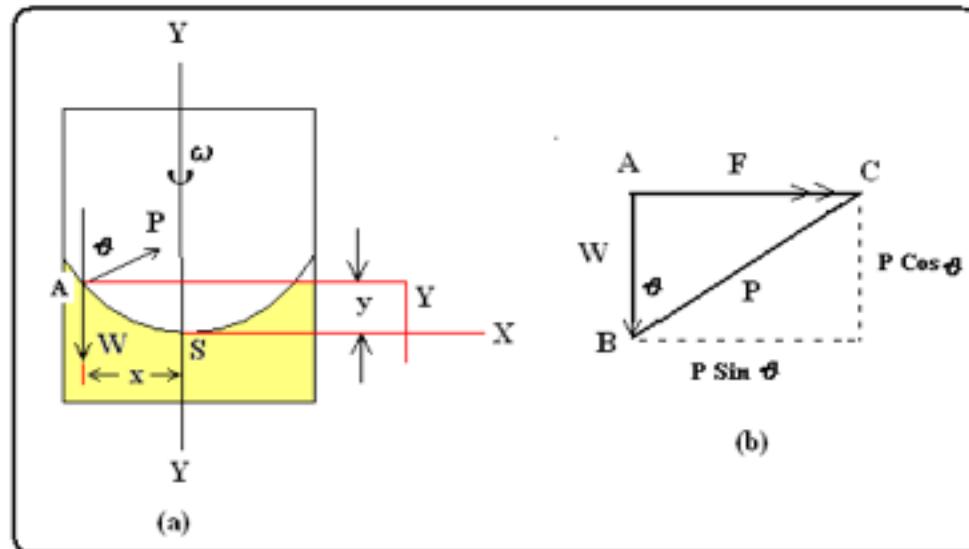
# REVIEW OF RELATED LITERATURE

### Rotation of Fluid Masses – Open Vessels

The form of the free surface of the liquid in a rotating vessel is that of a paraboloid of revolution. Any vertical plane through the axis of rotation which cuts the fluid will produce a parabola. The equation of the parabola is,  $y = \frac{\omega^2}{2g} x^2$

where  $x$  and  $y$  are coordinates, in feet, of any point in the surface measured from the vertex in the axis of revolution and  $\omega$  is the constant angular velocity in rad/sec. Proof of this equation is given in Problem 7. (Giles, 1962, p.42)

**Problem 7.** An open vessel partly filled with a liquid rotates about a vertical axis at constant angular velocity. Determine the equation of the free surface of the liquid after it has acquired the same angular velocity as the vessel.



## CHAPTER - 2

# REVIEW OF RELATED LITERATURE

**Solution:** Fig. (a) represents a section through the rotating vessel, and any particle **A** is at a distance  $x$  from the axis of rotation. Forces acting on mass **A** are the weight **W** vertically downward and **P** which is normal to the surface of the liquid since no friction is acting. The acceleration of mass **A** is  $x\omega^2$ , directed toward the axis of rotation. The direction of the resultant of forces **W** and **P** must be in the direction of this acceleration, as shown in Fig. (b).

$$\text{From Newton's second law, } F_x = Ma_x \text{ or (1)} \quad P \sin \theta = \frac{W}{g} x \omega^2$$

$$\text{From } \sum Y = 0 \quad (2) \quad P \cos \theta = W$$

$$\text{Dividing (1) by (2),} \quad (3) \quad \tan \theta = \frac{x \omega^2}{g}$$

Now  $\vartheta$  is also the angle between the X-axis and a tangent drawn to the curve of **A** in Fig. (a). The slope of this tangent is  $\tan \theta$  or  $\frac{dy}{dx}$ .

Substituting in (3) above,

$$\frac{dy}{dx} = \frac{x \omega^2}{g} \quad \text{from which, by integration,} \quad y = \frac{\omega^2}{2g} x^2 + C_1$$

To evaluate the constant of integration, **C**<sub>1</sub>: When  $x = 0$ ,  $y = 0$  and  $C_1 = 0$ .

(Giles, 1962, p.42, Problem 7)

## **CHAPTER - 2**

# **REVIEW OF RELATED LITERATURE**

### **Manual of Chemical Engineering Calculations & Shortcuts**

#### **New Analysis Provides Formula to Solve Mixing Problems**

**Brothman, A, Wollan, G, & Feldman, S. (1947)**

Universally used by the process industries, mixing operations have been the subject of considerable study and research for several years. Despite these efforts, mixing has remained an empirical art with little foundation of scientific analysis as found in other important unit operations. A new approach based on a study of kinetics and on the concept that mixing is essentially an operation of three-dimensional shuffling, has resulted in a formula for solving practical problems.

“Mixing is that unit operation in which energy is applied to a mass of material for the purpose of altering the initial particle arrangement so as to effect a more desirable particle arrangement. While the object of this treatment is usually to blend two or more materials into a more homogenous mixture, it may also serve to promote accompanying reactions, or it may support other unit operations such as heat transfer.”

**(Brothman, Wollan and Feldman, 1947, p.175)**

The above referenced chapter is about applications of analytical methods, which brings forth a new relationship between mixing time and mixing completion. Based on the theory of probability and resulting from a study of mixing kinetics, the derived expression and its implications may well lead the way to closer and more reliable correlation of mechanical design and functional performance of mixers. The concepts and mixing methodologies are explained in shuffling operation, blending, turbulence and liquid

## CHAPTER - 2

# REVIEW OF RELATED LITERATURE

mixing. Most of the concepts and methodologies referenced in this book included the mixing time.

In this project the time of mixing will have a direct correlation to some of the algorithms that will be used to estimate number of mixing stages and the dynamic mixing times required. The dynamic mixing concepts are related to continuous functions and the load models are mathematically discrete.

### Advanced Engineering Mathematics

Kreyszig, E. (August 1988)

#### Modeling Physical Applications

Differential equations are of great importance in engineering, because many physical laws and relations appear mathematically in the form of differential equations. Referring to T. C. Patton's expression (Eq. 1), which describes the physical relationship of viscosity ( $\eta$ ) and temperature ( $T$ ).

$$\frac{d\eta}{dT} = f(\eta) \quad \text{Eq. 1 (Patton, 1979, 2<sup>nd</sup> edition, p. 91)}$$

Although Patton uses the differential expression (Eq. 1), to describe the relationship between viscosity and temperature, the development of the formulae that are used in Andrade's equations (Eq. 2, Eq. 3, and Eq. 4), is not shown.

$$\eta = A(10^{B/T}) \quad \text{Eq. 2}$$

$$\log \eta = \log A + \frac{B}{T} \quad \text{Eq. 3}$$

## CHAPTER - 2

### REVIEW OF RELATED LITERATURE

$$\log \frac{\eta_1}{\eta_2} = B \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \quad \text{Eq. 4}$$

(Patton, 1979, 2<sup>nd</sup> edition, p. 93)

Kreyszig describes the development process in a detailed step by step example of a radioactive decay problem below.

#### EXAMPLE 5. Radioactivity, exponential decay

*Experiments show that a radioactive substance decomposes at a rate proportional to the amount present. Starting with a given amount of substance, say, 2 grams, at a certain time, say,  $t = 0$ , what can be said about the amount available at a later time?*

**Solution.** 1<sup>st</sup> Step. Setting up a mathematical model (a differential equation) of the physical process.

We denote by  $y(t)$  the amount of substance still present at time  $t$ . the rate of change is  **$dy/dt$** . According to the physical law governing the process of radiation,  **$dy/dt$**  is proportional to  $y$ .

$$(9) \quad \frac{dy}{dt} = ky$$

Here  **$k$**  is a definite physical constant whose numerical value is known for various radioactive substances. (For example, in the case of radium  $^{88}\text{Ra}^{226}$  we have  $k \sim -1.4 \times 10^{-11} \text{ sec}^{-1}$ .) Clearly, since the amount of substance is positive and decreases with time,  **$dy/dt$**  is negative, and so is  **$k$** . We see that the physical process under consideration is described mathematically by an ordinary differential equation of the first order. Hence this equation is the mathematical model of that process. **Whenever a physical law involves a rate of change of a function, such as velocity,**

## CHAPTER - 2

# REVIEW OF RELATED LITERATURE

***acceleration, etc., it will lead to a differential equation. For this reason differential equations occur frequently in physics and engineering.***

**2<sup>nd</sup> Step. Solving the differential equation.** At this early stage of our discussion no systematic method for solving (9) is at our disposal. However, (9) tells us that if there is a solution  $y(t)$ , its derivative must be proportional to  $y$ . From calculus we remember that exponential functions have this property. In fact the function  $e^{kt}$  or more generally

$$(10) \quad y(t) = ce^{kt}$$

where  $c$  is any constant, is a solution of (9) for all  $t$ , as can readily be verified by substituting (10) into (9). [We shall see later (in Sec. 1.2) that (10) includes ***all*** solutions of (9); that is (9) does not have singular solutions.]

**3<sup>rd</sup> Step. Determination of a particular solution.** It is clear that our physical process has a unique behavior. Hence we can expect that by using further given information we shall be able to select a definite numerical value of  $c$  in (10) so that the resulting particular solution will describe the process properly. The amount of substance  $y(t)$  still present at time  $t$  will depend on the initial amount of substance given. This amount is ***2 grams*** at  $t = 0$ . Hence we have to specify the value of  $c$  so that  $y = 2$  when  $t = 0$ . This condition is called an ***initial condition***, since it refers to the initial state of the physical system. By inserting this condition

$$(11) \quad y(0) = 2$$

## CHAPTER - 2

# REVIEW OF RELATED LITERATURE

in (10) we obtain

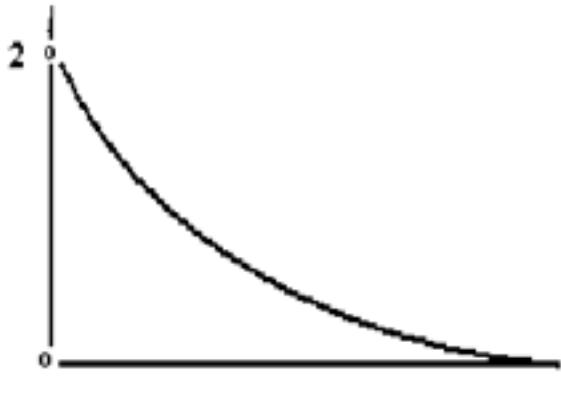


Fig. 5. Radioactivity (exponential decay)

$$y(0) = ce^0 = 2 \quad \text{or} \quad c = 2$$

If we use this value of c, then the solution (10) takes the particular form

$$(12) \quad y(t) = 2e^{kt}$$

This particular solution of (9) characterizes the amount of substance still present at any time  $t \geq 0$ . The physical constant k is negative, and  $y(t)$  decreases, as shown in

Fig. 5 above.

**4<sup>th</sup> Step. Checking.** From (12) we have

$$\frac{dy}{dt} = 2ke^{kt} = ky \quad \text{and} \quad y(0) = 2e^0 = 2$$

## CHAPTER - 2

### REVIEW OF RELATED LITERATURE

We see that the function (12) satisfies the equation (9) as well as the initial condition (11). ***The student should never forget to carry out this important final step, which shows whether the function is (or is not) the solution of the problem.***

(Kreyszig, 1988, p.8)

Based on the Kreyszig modeling example, (Eq. 1),  $\frac{d\eta}{dT} = f(\eta)$ , has the following solution.

$$\frac{d\eta}{dT} \propto \eta$$

$$\frac{d\eta}{\eta} = k dT$$

$$\int \frac{d\eta}{\eta} = k \int dT$$

$$\ln \eta = kT + c$$

$$\eta = e^{kT+c}$$

$$\eta = Ce^{kT} \quad \text{given; } C = e^c$$

Since molecular motion approaches zero, at absolute zero at ***T=0°K***,  $\eta_0$ .

(Temperature in degrees Kelvin)

$$\eta = e^{k(0)+c}$$

$$\eta_0 = e^c = C \quad \text{at } T = 0^\circ K$$

$$\eta = \eta_0 e^{kT} \qquad \qquad \qquad \text{Eq. 5a}$$

Note: Refer to T.C. Patton's experimental data Table 4-2a listed below.

Convert degrees Fahrenheit (F), to absolute, degrees Kelvin (K) and add the additional

columns (K) and Eq. 5a.  ${}^oC = 273 + \frac{5}{9}({}^oF - 32)$

## CHAPTER - 2

### REVIEW OF RELATED LITERATURE

The viscosity and temperature data from the table (Used in computation) was plugged into the author's new model equation, Eq. 5a, to create a pair of simultaneous equations;  $.33 \text{ poise}@303^{\circ}K$  and  $.071 \text{ poise}@363^{\circ}K$

$$.33 = \eta_o e^{k(303)} \quad \text{and} \quad 0.071 = \eta_o e^{k(363)}$$

When this pair of simultaneous equations are solved for the constants,  $k$  and  $\eta_o$ , their calculated values are;  $k = -0.02561$  and  $\eta_o = 773.05$ .

$\eta = 773.05e^{-0.02561(T)}$ , is used to calculate the data in column Eq. 5a.

**Table 4-2a: Comparison of Viscosity Values for Linseed Oil by Eqs. 2, 3, 4 & 5a with Experimentally Determined Values**

<b>Viscosity Values (poises)</b>						
<b>Calculated</b>						
<b>Temperature</b>						
(F)	(K)	Exp.	Eq. 2	Eq. 3	Eq. 4	Eq. 5a
50	283	0.60	0.56	0.61	0.59	0.55
<b>86</b>	<b>303</b>	<b>0.33</b>	<b>0.33</b>	<b>(0.33 Used in computation)</b>		
122	323	0.18	0.20	0.18	0.19	0.198
<b>194</b>	<b>363</b>	<b>0.071</b>	<b>0.071</b>	<b>(0.071 Used in computation)</b>		
302	423	0.029	0.015	0.023	0.019	0.015

## CHAPTER - 2

# REVIEW OF RELATED LITERATURE

After comparing the calculations of Eq. 2 and Eq. 5a, the conclusion is that the same methodology was used to solve Patton's differential equation. The exception is that T. C. Patton used logarithms to the **base "10,"** where this project uses natural logs or logarithms to the **base "e."**

Since the calculus is much more straightforward, this project will also be using the natural logarithmic methodology for solutions in the flush models to show the relationship between dispersion concentrations and viscosity distributions.

### Engineering Mathematics

Stroud, K.A., 5<sup>th</sup> edition (2001)

One of the models, Model-C, that will be created is based on the geometric progression. This project found Stroud to be an excellent source for reviewing the concepts relating to the Geometric Series. The applications of geometric progression that first came to mind were problems of finance, like compound interest.

Model-C uses the sequences of the pigment concentrations after each break as a geometric series. If the pigment concentrations are geometric in nature, then their pigment charges are geometric. A geometric model will allow the progression elements to be summed by formula. The very first illustration that Stroud uses in his chapter, Series 1, "Geometric series (geometric progression), denoted by GP," problem 11:

## CHAPTER - 2

# REVIEW OF RELATED LITERATURE

An example of a GP is the series:

$$1 + 3 + 9 + 27 + 81 + \dots \text{ etc.}$$

Here you can see that any term can be written from the previous term by multiplying it by a constant factor 3. This constant factor is called the **common ratio** and is found by selecting any term and dividing it by the previous one:

$$\text{e.g. } 27 \div 9 = 3 ; 9 \div 3 = 3 ; \dots \text{ etc.}$$

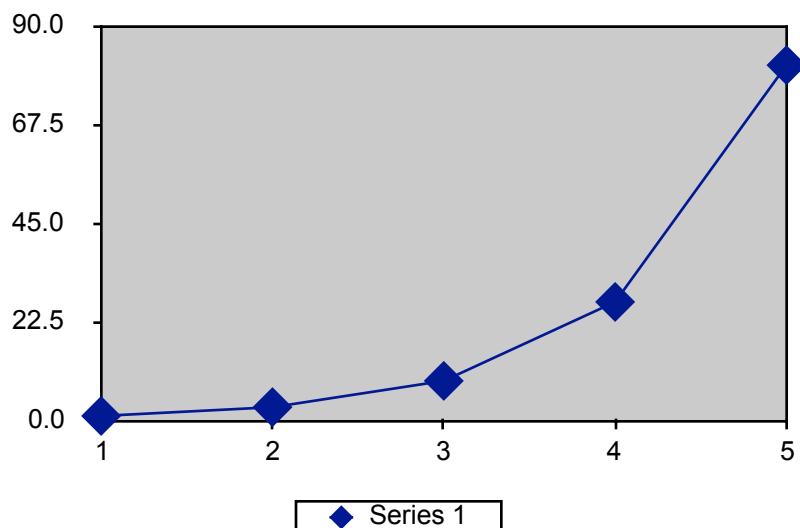
A GP therefore has the form:

$$a + ar + ar^2 + ar^3 + \dots \text{ etc.}$$

where, a = first term, r = **common ratio**.

(Stroud, 2001, p. 752)

When the author saw this example, he thought of the flush distribution of pigment charges. The graph of the geometric sequence 1, 3 ,9, 27, 81, is shown below.



To the author, it looks like the upside-down version of the exponential graph in chapter 1. Therefore he felt that he should be able to manipulate the sequence, mathematically and produce the exponential pattern. This is what led to the creation of “Model-C, The Geometric Series.”

# **CHAPTER - 3**

## **METHODOLOGY**

### **CONTENT**

**Approach**

**Data Gathering Method**

**Database of Study**

**Validity of Data**

**Originality and Limitation of Data**

**Summary**

# **CHAPTER 3**

## **METHODOLOGY**

### **Approach**

The initial idea which led to the development of this project was conceived while serving in the capacity of a formulator whose duties were to prepare the work orders and process the procedures in a pigment manufacturing plant. The senior technologist was the person who initiated the process by preparing the small research batches in the laboratory and in the plant. Upon completion of the development stage, the plant formulator would scale and convert the procedure up so that it could be processed on a work order in a production size sigma-blade mixer.

After preparing numerous plant procedures for flush dispersions, a common pattern was noticed about all of the procedures and that was the pigment and vehicle charge distribution. As the mixing stages progressed from the initial stage to the final stage, both the pigment and the vehicle charges were always decreasing in amount. It was also noticed that the viscosity of the dispersions seemed to be increasing in an exponential growth curve pattern. It was also noticed that when the development procedures were scaled up to production capacities, adjustments had to be made because of a non-linear relationship between the material distribution and the production mixer capacities.

### **Data Gathering Method**

The need to make the connection between material distribution and the mixer capacity sparked an interest and curiosity, which leads to an in-depth journey into the research of pigment and paint dispersions. The subject matter is a scientific and mathematical excursion into the world of measurement of viscosity and its applications.

## CHAPTER 3

# METHODOLOGY

Creating mathematical models to simulate the flush procedure was the best way try to produce the same patterns that kept showing up in the plant work orders. The methodology by which the models were created can best be described as mathematical. Most of the math focused on the dynamics of exponential functions and their applications. The viscosity applications required the use of first order differential equations and math relating to logarithmic functions. The mixer capacity applications involved summation algebra.

The methodology of the flush models had to be created with pure mathematics and then modeled into programs. Once the models were coded into program logic and math scripts, it was easier to experiment with the parameters and to analyze their inter-relationships.

### **Database of Study**

Most of the research literature is focused on dispersions and the viscosity of resin solutions. Temple C. Patton's "Paint Flow and Pigment Dispersion" was the primary resource for this project because of his mathematical treatment of the subject.

The mathematics that is used in this project is a result of training and coatings technology over the years. Some of the advanced math required some review in the area of applied differential equations. The mathematics of finance is a great resource for reviewing applications of the geometric series.

MathCAD, MS Excel and Basic Programming are very useful tools in creating and testing the models. Programming experience are very good resources for producing quick results with minimum effort.

# **CHAPTER 3**

## **METHODOLOGY**

### **Validity of Data**

The resource literature contains tables of experimental data that serves as a target for the models to reproduce. The primary strategy of the project is to use logical empirical modeling to reproduce the experimental results. The validity of the output from the models will rely entirely on analysis. Hopefully, a good model will produce outputs that closely mimic the experimental data.

# **CHAPTER 3**

## **METHODOLOGY**

### **Originality and Limitations of Data**

There is very little literature found that can verify that this approach to creating flush load capacity models has been attempted. The concept is quite simple in nature, but because of the complexity of the technology related to viscosity measurement and fluid dynamics, it gets quite involved mathematically. The model output is empirical, but its objective is to serve as a tool for the ink technologist when analyzing flush procedures.

After the models were completed, they only opened the door to more questions. The first models only address the load capacity of the mixing stages as a discrete function. There is so much more to be learned from the mixing dynamics that take place between the stages. The expansion of this project will use simplified units of measure to obtain its objective. However, there is much potential to advance the scope of this project into the areas of mixing energy and manufacturing cost analysis.

### **Summary of Chapter 3**

Most of the methods and techniques used in creating the models for this project are simple and straightforward. The output of the models can only be analyzed and compared to data that is documented within the resource literature.

The core of the creation of the models lies within the chapters that show the steps in the longhand mathematical development of the logical functions and relationships from which the models are built. The proof development will be shown in the appendix.

# **CHAPTER - 4**

## **DATA ANALYSIS - I (MONO-VEHICLE)**

### **CONTENT**

**Load Process (Mono-Vehicle)**

**Treatment-I: Model A (Log - Exponential)**

**Treatment-II: Model B (Log - Exponential)**

**Treatment-III: Model C (Geometric Series)**

**Treatment-III: Model D (Geometric Series)**

**Treatment-II: Model BN (Log - Exponential)**

# CHAPTER 4

## DATA ANALYSIS - I

### Load Process Reaction Per Mixing Stage (Mono-Vehicle)

A given amount of presscake, PW, is mixed with a given amount of vehicle, V, to produce a paste, PV (wetted pigment) and displaced water, W.

Appendix A1-Formula #23:  $PW + V \longrightarrow PV + W$

PW .....	Aqueous Pigment (Presscake)
W .....	Displaced Water
V .....	Resin or Resin Solution
PV = P+V .....	Pigment <b>wetting</b>
P .....	Pigment (Non Aqueous)

The time required to break down the components into a slurry of water (W), pigment (P), vehicle (V) and form a paste of the wetted pigment (PV) is proportional to the amount of water displaced (W), relative viscosity of the components ( $\eta$ ), temperature and oil absorption of the pigment. Given a mixer of **bulk capacity (B)**, several **mixing stages (i = 1, 2, 3, ... n)** of **aqueous pigment (PW)** and **vehicle (V)** are charged to the mixer in calculated amounts such that the charge **(PW + V)** in any given stage (*i*), plus the paste or wetted pigment that has already been mixed in prior stages, will always equal or be less than the bulk capacity (**B**).

Appendix A1-Formula #24: Before Mixing  $B \geq \sum_{i=1}^n (P+V)_{1,2,3,\dots,i-1} + PV_i + V_i$

Formula #25: After Mixing  $B \geq \sum_{i=1}^n (P+V)_{1,2,3,\dots,i-1} + PV_i + W_i$

The discharge of water, ( $W_i$ ), after any stage of mixing creates the net capacity for the next stage of additives, ( $P_{i+1} + V_{i+1}$ ).

# CHAPTER 4

## DATA ANALYSIS - I

$$W_i = PW_{i+1} + V_{i+1}$$

$$W_{i+1} = PW_{i+2} + V_{i+2}$$

$$W_{i+2} = PW_{i+3} + V_{i+3}$$

...

$$W_{\infty-1} = PW_{\infty} + V_{\infty} + W_{\infty} \quad \text{given} \quad 1 \leq i < \infty$$

Theoretically, this process could go on forever;  $i \rightarrow \infty$ , but a point is rapidly approached where a decision must be made to end the process. This final stage is designated as the ***n***th or last stage (***n***). So the final expression that shows ***W<sub>n</sub>*** is;  $W_{n-1} = PW_n + V_n + W_n$  given:  $1 \leq i \leq n$

The function or algorithm which approximates the number of stages required to mix a total amount of pigment,  $\sum_{i=1}^n P_i$ , having a solids contents of (r), with a total amount of ve-

hicle or resin solution,  $\sum_{i=1}^n V_i$ , into a mixer vessel of bulk capacity (B), is **Formula #8**;

$n = \frac{(1/r)(\sum_{i=1}^n P_i + \sum_{i=1}^n V_i)}{x_{pn} B}$ . The ratio of total pigment charge to total charge is designated as

**Formula #11, *x<sub>pn</sub>*.**  $x_{pn} = \frac{\sum_{i=1}^n P_i}{\sum_{i=1}^n (P_i + V_i)}$ . This ratio also is indirectly related to the number of

stages (***n***) in **Formula #8** above, which is required to completely mix the pigment with the resin solution and displace all of the water.

**Treatment – I: MODEL (A)** requires initial amounts of pigment and vehicle (non-optimized) to be charged to the mixer. This model calculates the pigment and vehicle charges that are required for each mixing stage so that the sum of the increment

## CHAPTER 4

### DATA ANALYSIS - I

charges equal the optimized total charge and distributes the total charge to agree with the given viscosity distribution. Optimization is the primary focus of this treatment while adhering to a given viscosity distribution and holding the mixer capacity constant. The calculated capacity,  $B_{(i)}$ , is an output parameter and will be listed at each mixing stage to compare to the constant capacity,  $B$ . The input parameter,  $E_0$  (Allowance), is the estimated % of the constant capacity. Theoretically,  $E_0$  is equal to the water displacement in the final mixing stage.

#### INPUT DATA

Capacity Constant <b>B</b>	Calculated Capacity at stage (i): $B_{(i)}$ .
Initial Pigment Charge $\sum P_i$	Number of mixing stages ( <b>n</b> )
Initial Vehicle Charge $\sum V_i$	% Vehicle after last stage ( $x_n$ )
Relative Viscosity of the Pigment ( $\eta_p$ )	System Viscosity Constant ( $k_v$ )
Relative Viscosity of the Vehicle ( $\eta_v$ )	Viscosity Distribution ( $\eta_i$ )
% Solids of Presscake ( <b>r</b> )	% Pigment per stage $x_{p(i)}$
Viscosity Distribution Function <b>f(i)</b>	% Vehicle per stage $x_{v(i)}$
Allowance <b>E<sub>0</sub></b>	Pigment Charge per stage $P_{(i)}$
	Vehicle Charge per stage $V_{(i)}$
	Water Displacement per stage $w_{d(i)}$
	Total Pigment Charge $\sum P_i$
	Total Vehicle Charge $\sum V_i$

## CHAPTER 4 DATA ANALYSIS - I

**Example Problem MODEL-A:** Given the non-optimized charge:

Pigment Charge: .....  $\sum P_i$  = 1350 lbs

Vehicle Charge: .....  $\sum V_i$  = 1200 lbs

Relative Viscosity of the Pigment: ..... ( $\eta_p$ ) = 240,000 poise

Relative Viscosity of the Vehicle: ..... ( $\eta_v$ ) = 100 poise

% Solids of Presscake: ..... (r) = 20%: (0.20)

Mixer Capacity: ..... B = 3000 lbs

Capacity Allowance  $E_0$  = 15%: .....  $Eff\% = 85\%$

### Refer to Formula #22

Optimize the flush procedure to be mixed in a vessel of fixed Capacity (B).

- ❖ Using the non optimized data as INPUT, the following procedure lists the steps required to calculate the OUTPUT distributions and parameters which maximizes the yields after each loading stage (i).

### Solution Steps MODEL-A:

#### Step #1

Reference Appendix A1-Formula #11:

Calculate the % pigment ( $x_p$ ) in total charge:

- This is the pigment content of the completed flushed paste mix after all of the water has been removed.

$$x_p = \frac{\sum_{i=1}^n P_i}{(\% Eff)(B)} = \frac{\sum_{i=1}^n P_i}{\sum_{i=1}^n (P_i + V_i)}$$

# CHAPTER 4

## DATA ANALYSIS - I

### Step #1 (Continued)

$$x_p = \frac{1350}{(1350 + 1200)} \quad \boxed{x_p = 0.5294}$$

Calculate the % vehicle ( $x_v$ ) in the total charge

Reference Appendix A1-Formula #14:

- This is the vehicle content of the completed flushed paste mix after all of the water has been removed.

$$x_v + x_p = 1; \quad x_v = 1 - x_p \quad x_v = 1 - 0.5294 \quad \boxed{x_v = 0.4706}$$

### Step #2

Reference Appendix A1-Formula #8:

- Calculate the number of complete mixing stages (n), required to displace all of the water in the total charge of presscake.

**Formula #8**

$$n = \frac{(1 - E_0) \frac{x_p}{r} + x_v}{x_{v_n}} = \frac{(0.85)(\frac{0.5294}{0.20}) + 0.4706}{0.4706}$$

$$n = 5.7810 \quad \boxed{\text{Round } 5.7810 \text{ up to } n = 6}$$

Stage counter  $i$ , (1 to n);       $1 \leq i \leq n$

# CHAPTER 4

## DATA ANALYSIS - I

### Step #3

Reference Appendix A2-Formula #15:

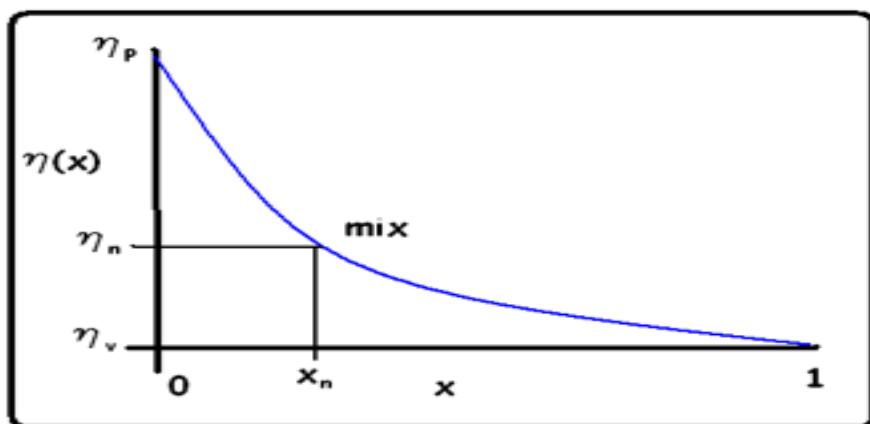
Calculate the Relative Viscosity Constant for the mix ( $k_v$ )

This is one of the arbitrary constants used in the differential equation

$\left( \frac{d\eta(x)}{dx} = k\eta(x) \right)$ , which describes how the viscosity moves from the relative pigment viscosity ( $\eta_p$ ) to the relative vehicle viscosity ( $\eta_v$ ). As vehicle is added to

the system, the initial viscosity ( $\eta_p$ ) is reduced to the end viscosity, ( $\eta_n$ ), which is

equal to the relative viscosity of the mix at  $x_n = x_v$



# CHAPTER 4

## DATA ANALYSIS - I

$$\begin{array}{lll}
 \frac{d\eta(x)}{dx} = k\eta(x) & given\_x = 0 & given\_x = 1 \\
 \frac{d\eta(x)}{\eta(x)} = kdx & \eta(x) = Ce^{kx} & \eta(x) = \eta_p e^{kx} \\
 \int \frac{d\eta(x)}{\eta(x)} = k \int dx & \eta(0) = Ce^{k(0)} & \eta(1) = \eta_p e^{k(1)} \\
 \ln \eta(x) = kx + c & \eta(0) = C = \eta_p & \eta_v = \eta_p e^k \\
 \eta(x) = e^{kx} e^c & \eta(x) = \eta_p e^{kx} & \ln \frac{\eta_v}{\eta_p} = k \\
 \eta(x) = Ce^{kx} & & 
 \end{array}$$

given:  
 $\eta(x) = \eta_p e^{kx}$   
 $- 0 \leq x \leq 1$

$$k_v = \ln \left( \frac{\eta_v}{\eta} \right) \quad k_v = \ln \left( \frac{100}{240000} \right) \quad \boxed{k_v = -7.7832}$$

### Step #4

Reference Appendix A2-Formula #16

Calculate the Relative End Viscosity ( $\eta_n$ )

$$\eta_n = \eta_p e^{k_v x_v} \quad \text{Viscosity of the mix}$$

$$\eta_n = 240000 e^{(-7.7832)(0.4706)} \quad \boxed{\eta_n = 6158.7}$$

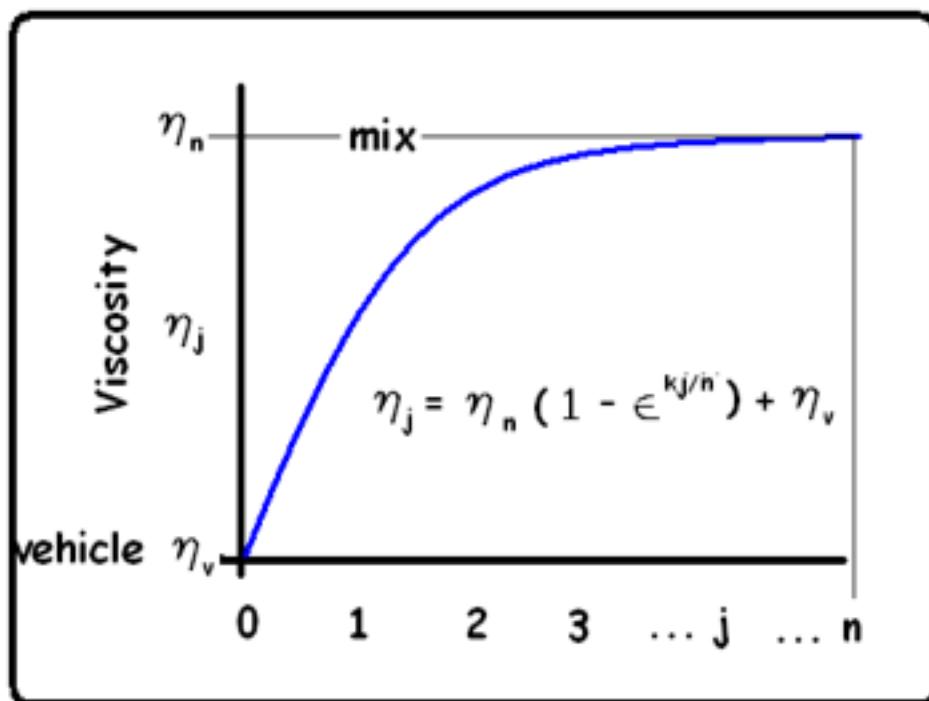
## CHAPTER 4 DATA ANALYSIS - I

### Step #5 & #6

Relative Viscosity Distribution Equation w/r to Mixing Stages:  $0 \leq j \leq n$

The Relative Viscosity ( $\eta_j$ ) increases from Vehicle Viscosity ( $\eta_v$ ) to the Relative Mix Viscosity ( $\eta_n$ ) as illustrated in the figure below.

The growth function is expressed:  $\eta_j = \eta_n (1 - e^{-kj/n}) + \eta_v$  as the mix progresses from stage (0), initial condition Vehicle Viscosity ( $\eta_v$ ) to the end Relative Mix Viscosity ( $\eta_n$ ):  $0 \leq j \leq n$



# CHAPTER 4

## DATA ANALYSIS - I

### Step #5 & #6 (Continued)

#### Calculate the Mix Viscosity Distribution Constant ( $k$ )

$$\eta_j = \eta_n (1 - e^{k \left( \frac{j}{n} \right)}) + \eta_v$$

$$\eta_j = \eta_n - \eta_n e^{k \left( \frac{j}{n} \right)} + \eta_v$$

*Given – j = n :*

$$\eta_n = \eta_n - \eta_n e^{k \left( \frac{n}{n} \right)} + \eta_v$$

$$\eta_n = \eta_n - \eta_n e^{k(1)} + \eta_v$$

$$\eta_n e^{k_{vn}} = \eta_n - \eta_n + \eta_v$$

$$\eta_n e^k = \eta_v$$

$$\eta_n e^k = \eta_v$$

$$e^k = \frac{\eta_v}{\eta_n}$$

$$k = \ln \left( \frac{\eta_v}{\eta_n} \right)$$

$$k_{vn} = \ln \left( \frac{\eta_v}{\eta_n} \right)$$

The function becomes:  $\eta_j = \eta_n \left( 1 - e^{k_{vn} \left( \frac{j}{n} \right)} \right) + \eta_v$ , by ( $k \circledast k_{vn}$ )

Range of  $(\eta_j)$  is:  $(\eta_v \leq \eta_j \leq \eta_n)$ : Domain is:  $0 \leq j \leq 6$ :  $n = 6$

The Viscosity of the Mix:  $\eta_n = 6158.7$  at % vehicle:  $x_v = 0.4706$

The Vehicle Viscosity:  $\eta_v = 100$

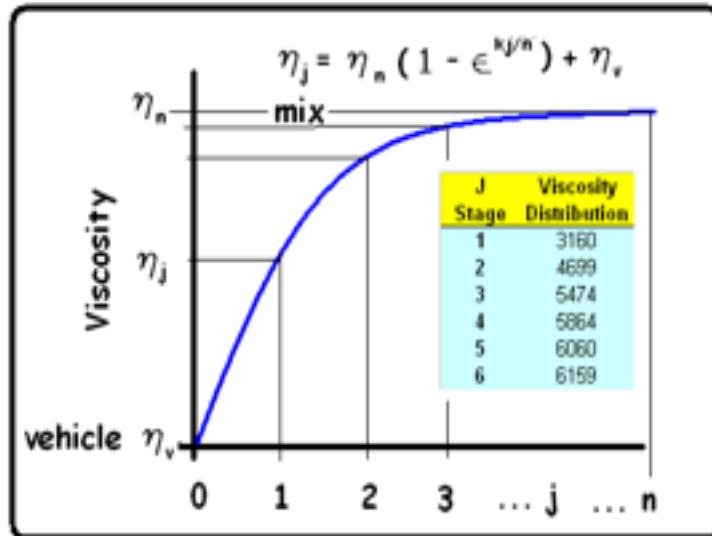
Mix Viscosity Distribution Constant:  $k_{vn} = \ln \left( \frac{\eta_v}{\eta_n} \right) = -4.1204$

Reference Appendix A2-Formula #17 & #18

# CHAPTER 4

## DATA ANALYSIS - I

The Viscosity Distribution:  $\eta_j = \eta_n \left( 1 - e^{-k_{vn} \frac{j}{n}} \right) + \eta_v$



### Step #7

Reference Appendix A3-Formula #26 & Viscosity Distribution ( $\eta_j$ )

Calculate the % Vehicle Distribution ( $x_{v_i}$ ) and % Pigment Distribution ( $x_{p_i}$ )

$$\ln\left(\frac{\eta_i}{\eta_p}\right)$$

Using Formula #26  $x_{v_i} = \frac{\ln\left(\frac{\eta_i}{\eta_p}\right)}{k_v}$  % Vehicle Distribution given ( $1 \leq i \leq n$ )

$$\text{Given: } x_v + x_p = 1 \quad x_p = 1 - x_v$$

I Stage	Viscosity Distribution	Mix % Vehicle	Mix % Pigment
1	3160	0.5564	0.4436
2	4699	0.5054	0.4946
3	5474	0.4857	0.5143
4	5864	0.4769	0.5231
5	6060	0.4727	0.5273
6	6159	0.4706	0.5294

# CHAPTER 4

## DATA ANALYSIS - I

### Step #8

Reference Appendix A2-Formula #19

#### Calculate the Pigment Distribution ( $P_i$ )

$$\text{Using Formula #19} \quad P_i = \frac{Bx_{p_i} - \sum_{i=1}^{i-1} P_i}{\frac{1}{r} + x_{v_i} (1 - \frac{1}{r})} \quad \text{Pigment Distribution for } 1 \leq i \leq n$$

$\eta_i$	$x_{v_i}$	$x_{p_i}$	$P_i$
<b>i</b>		<b>Viscosity</b>	<b>Mix</b>
Stage		Distribution	% Vehicle
1		3160	0.5564
2		4699	0.5054
3		5474	0.4857
4		5864	0.4769
5		6060	0.4727
6		6159	0.4706
			Pigment Charge
			<b>479.69</b>
			<b>337.16</b>
			<b>237.46</b>
			<b>166.53</b>
			<b>116.15</b>
			<b>80.58</b>

### Step #9

Reference Appendix A1-Formula #6

#### Calculate the Water Displacement Distribution ( $W_i$ )

$$\text{Using Formula #6} \quad W_i = \frac{P_i}{r} - P_i \quad \text{Water displacement distribution } 1 \leq i \leq n$$

$\eta_i$	$x_{v_i}$	$x_{p_i}$	$P_i$	$W_i$
<b>i</b>		<b>Viscosity</b>	<b>Mix</b>	<b>Pigment</b>
Stage		Distribution	% Vehicle	Charge
1		3160	0.5564	479.69
2		4699	0.5054	337.16
3		5474	0.4857	237.46
4		5864	0.4769	166.53
5		6060	0.4727	116.15
6		6159	0.4706	80.58
				Water Displaced
				<b>1918.76</b>
				<b>1348.63</b>
				<b>949.84</b>
				<b>666.13</b>
				<b>464.58</b>
				<b>322.32</b>

# CHAPTER 4

## DATA ANALYSIS - I

### Step #10

Reference Appendix A3-Formula #20      **Calculate Vehicle Distribution ( $V_i$ )**

Using Formula #20     $V_i = B - \sum_{i=1}^{i-1} (P_i + V_i) - \frac{P_i}{r}$     Vehicle distribution     $1 \leq i \leq n$

$$\eta_i \quad x_{v_i} \quad x_{p_i} \quad P_i \quad W_i \quad V_i$$

i Stage	Viscosity Distribu- tion	Mix % Vehicle	Mix % Pigment	Pigment Charge	Water Dis- placed	Vehicle Charge
1	3160	0.5564	0.4436	479.69	1918.76	<b>601.55</b>
2	4699	0.5054	0.4946	337.16	1348.63	<b>232.97</b>
3	5474	0.4857	0.5143	237.46	949.84	<b>161.33</b>
4	5864	0.4769	0.5231	166.53	666.13	<b>117.18</b>
5	6060	0.4727	0.5273	116.15	464.58	<b>85.4</b>
6	6159	0.4706	0.5294	80.58	322.32	<b>61.69</b>

*Small discrepancies between these reports are due to rounding errors.*

**Treatment – II- MODEL-B** requires ( $x_p$ ), the % pigment in the total mix, as an input parameter. This parameter along with the capacity, B, is used to calculate the initial pigment and vehicle charges, which are required as input parameters in Treatment-I. The remaining steps of the procedure and the objectives are identical to Treatment-I. The model uses the mixer's capacity along with the viscosity distribution as the critical input parameters to optimize the loading of each mixing stage and optimize the yield. The total amount of pigment and vehicle required to charge the mixer is an output parameter in this procedure.

### INPUT DATA

Mixer Capacity (**B**)

### OUTPUT DATA

Number of mixing stages (**n**)

## CHAPTER 4

### DATA ANALYSIS - I

% Pigment after last stage ( $x_p$ )	% Vehicle after last stage ( $x_n$ )
Relative Viscosity of the Pigment ( $\eta_p$ )	System Viscosity Constant ( $k_v$ )
Relative Viscosity of the Vehicle ( $\eta_v$ )	Viscosity Distribution ( $\eta_i$ )
% Solids of Presscake ( $r$ )	% Pigment per stage $x_{p(i)}$
Viscosity Distribution Function $f(i)$	% Vehicle per stage $x_{v(i)}$
Allowance $E_0$	Pigment Charge per stage $P_{(i)}$
	Vehicle Charge per stage $V_{(i)}$
	Water Displacement per stage $w_{d(i)}$
	Total Pigment Charge $\sum P_i$
	Total Vehicle Charge $\sum V_i$
	Calculated Capacity at stage (i) $B_{(i)}$

# CHAPTER 4

## DATA ANALYSIS - I

### Example Problem MODEL-B:

Given:

Pigment Content of the mix:  $(x_p) = 0.5294$

Relative Viscosity of the Pigment:  $(\eta_p) = 240,000 \text{ poise}$

Relative Viscosity of the Vehicle:  $(\eta_v) = 100 \text{ poise}$

% Solids of Presscake:  $(r) = 20\%: (0.20)$

Capacity Allowance:  $E_0 = 15\%: (Eff\% = 85\%)$

Refer to Formula #22)

Optimize the flush procedure to be mixed in a vessel of Capacity,  $B = 3000 \text{ lbs}$ .

Given the same input parameters of MODEL-A with the exception being  $(x_p)$ , which is a calculated output parameter, the output of MODEL-B is expected to be the same as the optimized output of MODEL-A.

### Solution Steps MODEL-B:

#### Step #1

Calculate the % vehicle ( $x_v$ ) in the total charge.

$$x_v + x_p = 1; \quad x_v = 1 - x_p \quad x_v = 1 - 0.5294 \quad x_v = 0.4706$$

#### Step #2

Using Formula #8: Estimate the number of required mixing stages ( $n$ )

$$\text{Formula #8: } n = \frac{(1 - E_0) \frac{x_p}{r} + x_v}{x_{v_n}} = \frac{(0.85)(\frac{0.5294}{0.20}) + 0.4706}{0.4706}$$

$$n = 5.7810$$

$$\boxed{\text{Round } 5.7810 \text{ up to } n = 6}$$

Stage counter  $i$ , ( $1$  to  $n$ );  $1 \leq i \leq n$

# CHAPTER 4

## DATA ANALYSIS - I

### Step #3

Using Formula #15  $k_v = \ln\left(\frac{\eta_v}{\eta_p}\right)$  Viscosity Constant for the mix

$$k_v = \ln\left(\frac{100}{240000}\right) = -7.7832$$

### Step #4

Using Formula #16  $\eta_n = \eta_p e^{k_v x_v}$  Viscosity of the mix

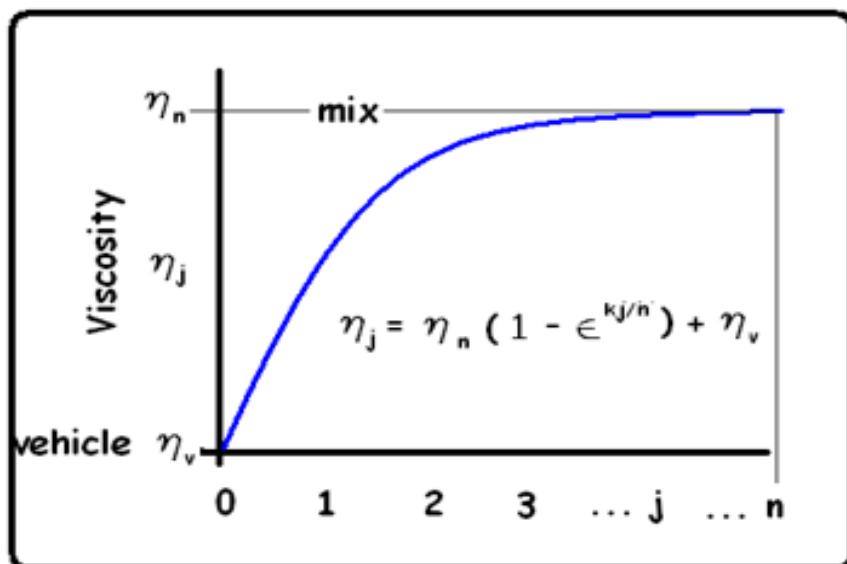
$$\eta_n = 240000 e^{(-7.7832)(0.4706)} = 6158.626$$

### Step #5 & #6

Relative Viscosity Distribution Equation w/r to Mixing Stages:  $0 \leq j \leq n$

The Relative Viscosity ( $\eta_j$ ) increases from Vehicle Viscosity ( $\eta_v$ ) to the Relative Mix Viscosity ( $\eta_n$ ) as illustrated in the figure below.

The growth function is expressed:  $\eta_j = \eta_n (1 - e^{-k(j/n)}) + \eta_v$  as the mix progresses from stage (0), initial condition Vehicle Viscosity ( $\eta_v$ ) to the end Relative Mix Viscosity ( $\eta_n$ ):  $0 \leq j \leq n$



# CHAPTER 4

## DATA ANALYSIS - I

**Calculate the Mix Viscosity Distribution Constant ( $k$ )**

$$\begin{array}{lll} \eta_j = \eta_n \left(1 - e^{k \left(\frac{j}{n}\right)}\right) + \eta_v & Given \_ j = n: & \eta_n e^k = \eta_v \\ \eta_j = \eta_n - \eta_n e^{k \left(\frac{j}{n}\right)} + \eta_v & \eta_n = \eta_n - \eta_n e^{k \left(\frac{n}{n}\right)} + \eta_v & e^k = \frac{\eta_v}{\eta_n} \\ Given \_ j = 0: & \eta_n = \eta_n - \eta_n e^{k(1)} + \eta_v & k = \ln \left( \frac{\eta_v}{\eta_n} \right) \\ \eta_0 = \eta_n - \eta_n e^{k(0)} + \eta_v & \eta_n e^{k_{vn}} = \eta_n - \eta_n + \eta_v & \\ \eta_0 = \eta_n - \eta_n + \eta_v & \eta_n e^k = \eta_v & k_{vn} = \ln \left( \frac{\eta_v}{\eta_n} \right) \\ \eta_0 = \eta_v & & \end{array}$$

The function becomes:  $\eta_j = \eta_n \left(1 - e^{k_{vn} \left(\frac{j}{n}\right)}\right) + \eta_v$ , by ( $k \otimes k_{vn}$ )

Range of  $(\eta_j)$  is:  $(\eta_v \leq \eta_j \leq \eta_n)$ : Domain is:  $0 \leq j \leq 6$ :  $n = 6$

The Viscosity of the Mix:  $\eta_n = 6158.7$  at % vehicle:  $x_v = 0.4706$

The Vehicle Viscosity:  $\eta_v = 100$

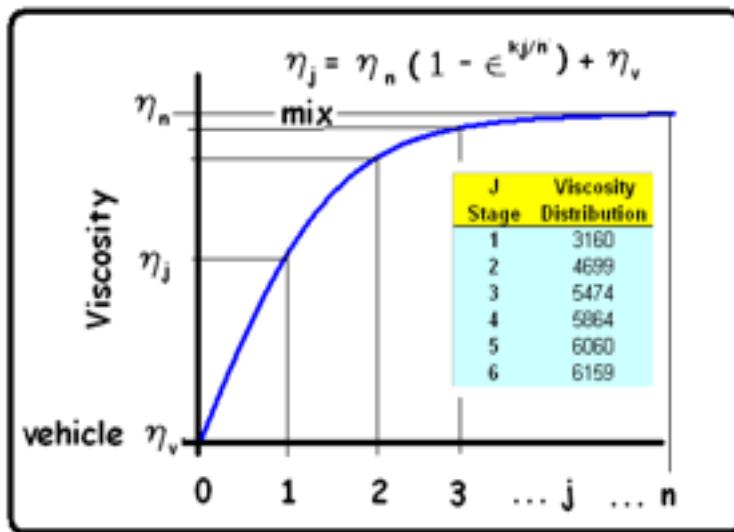
Mix Viscosity Distribution Constant:  $k_{vn} = \ln \left( \frac{\eta_v}{\eta_n} \right) = -4.1204$

Reference Appendix A2-Formula #17 & #18

The Viscosity Distribution:  $\eta_j = \eta_n \left(1 - e^{k_{vn} \left(\frac{j}{n}\right)}\right) + \eta_v$

# CHAPTER 4

## DATA ANALYSIS - I



### Step #7

Reference Appendix A3-Formula #26 & Viscosity Distribution ( $\eta_j$ )

Calculate the % Vehicle Distribution ( $x_{v_i}$ ) & % Pigment Distribution ( $x_{p_i}$ )

$$\ln\left(\frac{\eta_i}{\eta_p}\right)$$

Using Formula #26  $x_{v_i} = \frac{\ln\left(\frac{\eta_i}{\eta_p}\right)}{k_v}$  % Vehicle Distribution given ( $1 \leq i \leq n$ )

Given:  $x_v + x_p = 1$

$$x_p = 1 - x_v$$

I Stage	Viscosity Distribution	Mix % Vehicle	Mix % Pigment
1	3160	0.5564	0.4436
2	4699	0.5054	0.4946
3	5474	0.4857	0.5143
4	5864	0.4769	0.5231
5	6060	0.4727	0.5273
6	6159	0.4706	0.5294

### Step #8

# CHAPTER 4

## DATA ANALYSIS - I

Reference Appendix A2-Formula #19

**Calculate the Pigment Distribution ( $P_i$ )**

$$\text{Using Formula #19 } P_i = \frac{Bx_{p_i} - \sum_{i=1}^{i-1} P_i}{\frac{1}{r} + x_{v_i}(1-\frac{1}{r})} \quad \text{Pigment Distribution for } 1 \leq i \leq n$$

$\eta_i$	$x_{v_i}$	$x_{p_i}$	$P_i$	
i	Viscosity	Mix	Mix	Pigment
Stage	Distribution	% Vehicle	% Pigment	Charge
1	3160	0.5564	0.4436	<b>479.69</b>
2	4699	0.5054	0.4946	<b>337.16</b>
3	5474	0.4857	0.5143	<b>237.46</b>
4	5864	0.4769	0.5231	<b>166.53</b>
5	6060	0.4727	0.5273	<b>116.15</b>
6	6159	0.4706	0.5294	<b>80.58</b>

### Step #9

Reference Appendix A1-Formula #6

**Calculate the Water Displacement Distribution ( $W_i$ )**

$$\text{Using Formula #6 } W_i = \frac{P_i}{r} - P_i \quad \text{Water displacement distribution } 1 \leq i \leq n$$

$\eta_i$	$x_{v_i}$	$x_{p_i}$	$P_i$	$W_i$	
i	Viscosity	Mix	Mix	Pigment	Water Displaced
Stage	Distribution	% Vehicle	% Pigment	Charge	
1	3160	0.5564	0.4436	479.69	<b>1918.76</b>
2	4699	0.5054	0.4946	337.16	<b>1348.63</b>
3	5474	0.4857	0.5143	237.46	<b>949.84</b>
4	5864	0.4769	0.5231	166.53	<b>666.13</b>
5	6060	0.4727	0.5273	116.15	<b>464.58</b>
6	6159	0.4706	0.5294	80.58	<b>322.32</b>

# CHAPTER 4

## DATA ANALYSIS - I

### Step #10

Reference Appendix A3-Formula #20

**Calculate the Vehicle Distribution ( $V_i$ )**

$$\text{Using Formula #20} \quad V_i = B - \sum_{i=1}^{i-1} (P_i + V_i) - \frac{P_i}{r} \quad \text{Vehicle distribution } 1 \leq i \leq n$$

$\eta_i$	$x_{v_i}$	$x_{p_i}$	$P_i$	$W_i$	$V_i$	
I Stage	Viscosity Distribu- tion	Mix % Vehicle	Mix % Pigment	Pigment Charge	Water Dis- placed	Vehicle Charge
1	3160	0.5564	0.4436	479.69	1918.76	<b>601.55</b>
2	4699	0.5054	0.4946	337.16	1348.63	<b>232.97</b>
3	5474	0.4857	0.5143	237.46	949.84	<b>161.33</b>
4	5864	0.4769	0.5231	166.53	666.13	<b>117.18</b>
5	6060	0.4727	0.5273	116.15	464.58	<b>85.4</b>
6	6159	0.4706	0.5294	80.58	322.32	<b>61.69</b>

Refer to the model, MathCad MODEL\_B in the appendix (A16)

Refer to BASIC Program Reports: MODEL-A2 in the appendix (A11)

### BASIC PROGRAM REPORTS

```

Mixer Capacity <B> ..... B =? 3000
x Pigment Charge <xp> ..... xp =? .5294
x Solids of Pigment <r> ..... r =? .20
Vehicle Viscosity <nv> ..... nv =? 100
Pigment Viscosity <np> ..... np =? 2400000
Prior Residual <W> ..... W =? 0
W Vehicle Content [xv(0)] ... xv(0) =? 0
# Viscosity x Pgmt      Pigment      Vehicle      Water
-----  -----  -----
1 3159.519  .4436      480        600        1920
2 4699.112  .4946      337        235        1348
3 5473.856  .5143      237        163        948
4 5863.719  .5231      167        113        668
5 6059.903  .5273      116        88         464
6 6158.625  .5294      81         59         324

Pigment Charge <INPUT> 1349.97
Vehicle Charge <INPUT> 1200.03
Sum of Pigment Charge 1418
Sum of Vehicle Charges 1258
Sum of Water Displacement 5672
Original n .... n0 5.631024

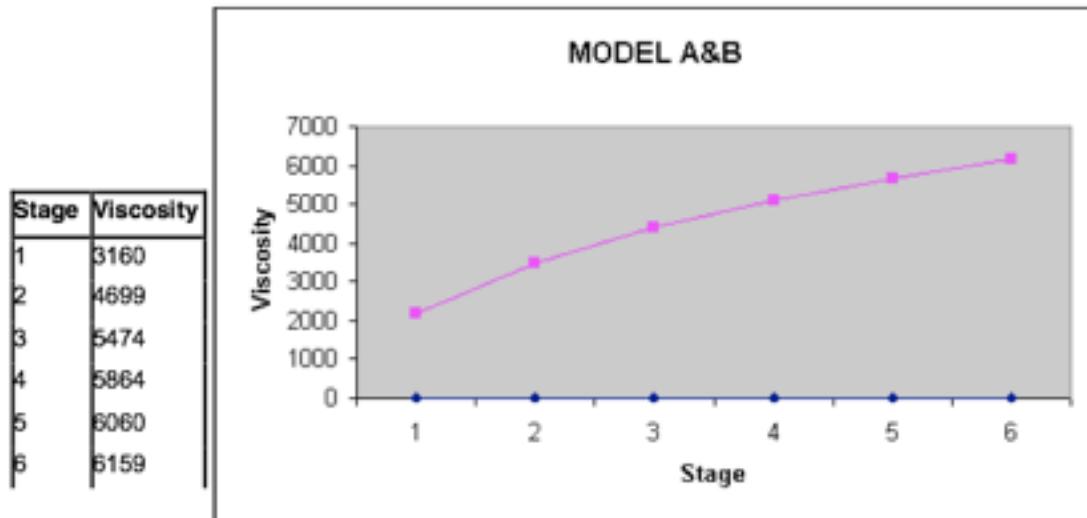
```

### REPORT-MODEL-B

# CHAPTER 4

## DATA ANALYSIS - I

Graph Relative Viscosity vs Stage



MODELS A & B follow mathematical theory growth curve shown in Chapter-1.

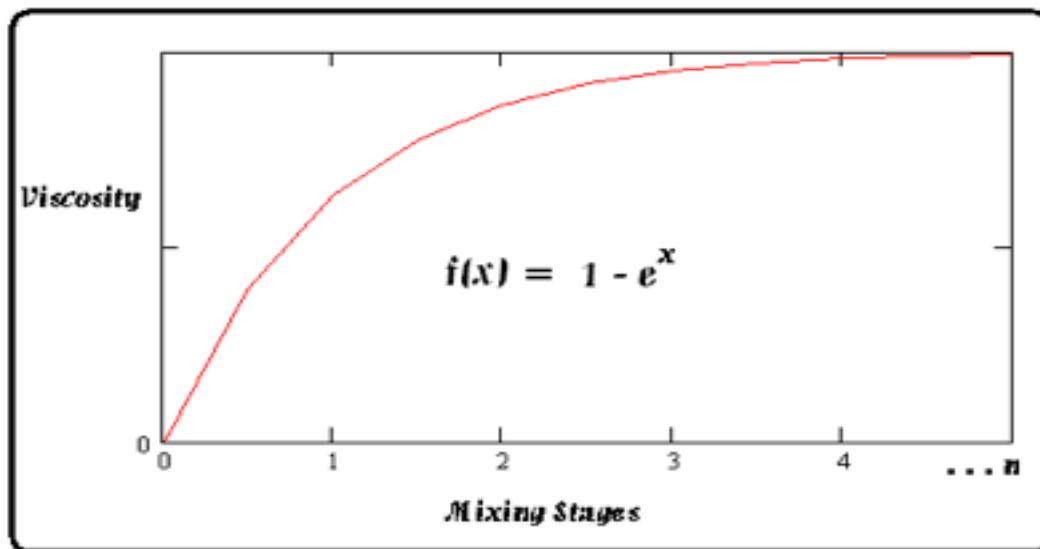


Figure 1.01a (Chapter -1)

## CHAPTER 4

### DATA ANALYSIS - I

#### Analysis of Vehicle-to-Pigment Ratio:

Stage	Viscosity	% Pigment	Pigment	Vehicle	Water	Cum V-P Ratio
1	3160	0.4436	479.69	601.55	1918.76	1.25
2	4699	0.4946	337.16	232.97	1348.63	1.02
3	5474	0.5143	237.46	161.33	949.84	0.94
4	5864	0.5231	166.53	117.18	666.13	0.91
5	6060	0.5273	116.15	85.4	464.58	0.9
6	6159	0.5294	80.58	61.69	322.32	0.89

This model's first stage has a vehicle-to-pigment ratio of 1.25, which is near the low end of the flush process target range ( $1.3 \leq V / P \_ Ratio \leq 1.8$ ). An excess of vehicle is required in the early stages for proper pigment wetting and oil absorption. The following stages are grinding stages ( $V / P \_ Ratio < 1.0$ ) where the pigment is dispersed and the relative viscosity increases toward the target viscosity,  $[\eta_n]$

$$\eta_n : (\eta_n = 6159 \text{ @ } \%Pigment = 0.5294).$$

Given the above output results and analysis for MODEL-A & B, the algorithms (A & B) seem to produce output distributions that appear to be near expectation w/r to V/P Ratios for flush parameters.

## CHAPTER 4

### DATA ANALYSIS - I

Treatment – III-MODEL-C uses the input parameter, Total Pigment Charge,  $\sum P_i$ , to create the pigment distribution,  $P_i$ . In this model, the pigment distribution is a **geometric progression**, whose sum always equal the input total pigment charge,  $\sum P_i$ . The number of terms in the geometric progression, ( $n$ ), is treated as the number of mixing stages in the flush procedure. The viscosity distribution is an output parameter based on the actual % pigment,  $x_{p_i}$ , calculated at each incremental stage (i). The mixer capacity,  $B$ , is held constant through out the procedure. The calculated capacity,  $B_i$ , is an output parm and will be listed at each mixing stage to compare to the constant capacity,  $B$ .

#### INPUT DATA

Total Pigment Charge       $\sum P_i$

Total Vehicle Charge       $\sum V_i$

Relative Viscosity of the Pigment ( $\eta_p$ )

Relative Viscosity of the Vehicle ( $\eta_v$ )

% Solids of Presscake ( $r$ )

Pigment Distribution Function  $f(i)$

*In a Geometric Progression model,*

**Capacity  $B_i$  is Constant for all**

**stages. ( $1 < i < n$ )**

#### OUTPUT DATA

Number of mixing stages ( $n$ )

% Vehicle after last stage ( $x_n$ )

System Viscosity Constant ( $k_v$ )

Viscosity Distribution  $\eta_i$

% Pigment per stage  $x_{p_i}$

% Vehicle per stage  $x_{v_i}$

Pigment Charge per stage  $P_i$

Vehicle Charge per stage  $V_i$

Water Displacement per stage  $w_{d_i}$

Total Pigment Charge  $\sum P_i$

Total Vehicle Charge  $\sum V_i$

# CHAPTER 4

## DATA ANALYSIS - I

### Example Problem MODEL-C1 (Non-Optimized - Geometric Series):

**Given:**

Pigment Charge:  $\sum P_i = 1350 \text{ lbs}$

Vehicle Charge:  $\sum V_i = 1200 \text{ lbs}$

Pigment Viscosity:  $(\eta_p = 240000 \text{ poise})$

Vehicle Viscosity  $\eta_v = 100 \text{ poise}$

% Solids of Presscake ( $r = 20\%$ ):

Capacity Allowance:  $E_0 = 15\%$

Generate a procedure to be flushed in a vessel of capacity,  $B = 3000 \text{ lbs}$

### Solution Steps MODEL-C1 (Non-Optimized - Geometric Series):

#### **Step #1**

Using Formula #11: Calculate the % pigment ( $x_p$ ) in total charge

$$\text{Formula #11: } x_p = \frac{\sum_{i=1}^n P_i}{\sum_{i=1}^n (P_i + V_i)}$$

$$x_p = \frac{\sum_{i=1}^n P_i}{\sum_{i=1}^n (P_i + V_i)} = \frac{1350}{(1350 + 1200)} \quad x_p = 0.5294$$

$$x_v + x_p = 1; \quad x_v = 1 - x_p \quad x_v = 1 - 0.5294 \quad x_v = 0.4706$$

#### **Step #2**

Using Formula #8: Estimate the number of required mixing stages (n)

$$\text{Formula #8} \quad n = \frac{\frac{\sum_{i=1}^n P_i}{r} + \frac{\sum_{i=1}^n V_i}{x_{p_n} B}}{\frac{1350}{0.2} + 1200} = \frac{1648.2}{1648.2}$$

$$n = 5.0057$$

Round (5.0057) to  $n = 5$  Stage counter  $i$ , (1 to n);  $1 \leq i \leq n$

# CHAPTER 4

## DATA ANALYSIS - I

**Step #3**      **Pigment charge in last stage (n):  $P_n$**

$$\text{Using Formula #21: } P_n = r \frac{B - \left( \sum_{i=1}^n P_i + V_i \right)}{1 - r}, \text{ Final (n}_{\text{th}}\text{) Pigment Charge}$$

**Given:**  $B = 3000$ ,  $\left( \sum_{i=1}^n P_i + V_i \right) = 2550$  **and**  $r = 0.20$

**Then**  $P_n = (.20) \frac{3000 - (2550)}{.80}$

$P_n = 112.5$

**Step #4**

Given the Geometric Series,  $a, aR, aR^2, aR^3, \dots, aR^n$ :

The sum of this series function is:  $S = \frac{a(R^n - 1)}{R - 1}$

Substitute the variables,  $a = P_n$ , (first element in the series) Substitute the variables,  $aR$ , (second element in the series) Substitute the variables,  $aR^2$ , (third element in the series), ... ;

After substitution:  $a, aR, aR^2, aR^3, \dots, aR^n$

Sum of the series,  $S = \sum_{i=1}^n P_i$  becomes  $\sum_{i=1}^n P_i = \frac{P_n(R^n - 1)}{R - 1}$

To reverse the order of the series:  $aR^n, aR^{n-1}, aR^{n-2}, \dots, aR, a$

Use the formula,  $\sum_{i=1}^n P_i = \frac{P_n(R^n - 1)}{R - 1} - P_n$ .

# CHAPTER 4

## DATA ANALYSIS - I

An **iterative algorithm** in MathCad, is used to solve for the series ratio, (R) in the Non-Optimized C1 example below. Using the MathCad Root Solver function, (R) is found to be;  $R \approx 1.449$ ; given  $N = 5.0057$  and rounded to  $n = 5$ .

The Pigment Distribution Series is generated by:

$$P_{n-i+1} = P_n R^{i-1}; \text{ given } (1 \leq i \leq n), n = 5 \text{ and } P_n = 112.5$$

$$P_{5-i+1} = (112.5)(1.449)^{i-1}$$

$$\sum_{i=1}^5 P_i = 1350$$

generates

$P_5 = 112.5$
$P_4 = 163.01$
$P_3 = 236.21$
$P_2 = 342.26$
$P_1 = 495.94$

$P_1 = 495.9$
$P_2 = 342.3$
$P_3 = 236.2$
$P_4 = 163.0$
$P_5 = 112.5$

or

### Step #5 Water displacement distribution

$$\text{Using Formula #6 } W_i = \frac{P_i}{r} - P_i$$

Given:  $(1 \leq i \leq n)$ ,  $n = 5$ ,  $r = .20$  and pigment distribution  $P_i$

$$W_i = \frac{P_i}{0.2} - P_i$$

generates

$$\sum_{i=1}^5 W_i = 5400$$

$W_1 = 1984$
$W_2 = 1369$
$W_3 = 945$
$W_4 = 652$
$W_5 = 450$

# CHAPTER 4

## DATA ANALYSIS - I

### Step #6 The Vehicle Distribution ( $V_i$ )

Using,  $V_1 = B - P_1 - W_1$ :

$$V_1 = 3000 - 495.94 - 1983.75$$

$$\boxed{V_1 = 520.31}$$

Using,  $V_2 = B - V_1 - P_1 - \frac{P_2}{r}$ :

$$V_2 = 3000 - 520.31 - 495.94 - \frac{342.26}{0.20}$$

$$\boxed{V_2 = 272.45}$$

Using Formula #20  $V_i = B - \sum_{i=1}^{i-1} (P_i + V_i) - \frac{P_i}{r}$

Given:  $(3 \leq i \leq n)$ ,  $n = 5$ ,  $r = .20$  and pigment distribution  $P_i$

Given:  $(1 \leq i \leq 2)$   $\boxed{V_1 = 520.31}$  and  $\boxed{V_2 = 272.45}$

$$V_i = B - \sum_{i=1}^{i-1} (P_i + V_i) - \frac{P_i}{r}$$

**generates**

$$\sum_{i=1}^5 V_i = 1200.09$$

$$\boxed{V_1 = 520.3}$$

$$\boxed{V_2 = 272.4}$$

$$\boxed{V_3 = 188.0}$$

$$\boxed{V_4 = 129.8}$$

$$\boxed{V_5 = 89.6}$$

### Step #7 Calculate the Mix Viscosity Constant ( $k_v$ )

Using Formula #15  $k_v = \ln\left(\frac{\eta_v}{\eta_p}\right)$

Given Relative Vehicle Viscosity:  $\eta_v = 100$  \_ poise

Given Relative Pigment Viscosity:  $\eta_p = 240000$  \_ poise

$$k_v = \ln\left(\frac{100}{240000}\right)$$

$$\boxed{k_v = -7.7832}$$

# CHAPTER 4

## DATA ANALYSIS - I

### Step #8 Calculate Viscosity of the Mix $\eta_n$

Using Formula #16  $\eta_n = \eta_p e^{k_v x_v}$

Given Relative Pigment Viscosity:  $\eta_p = 240000 \text{ poise}$

Given the Mix Viscosity Constant:  $k_v = -7.7832$

% vehicle in total charge:  $x_v = 0.4706$

$$\eta_n = 240000 e^{(-7.7832)(0.4706)} \quad \boxed{\eta_n = 6159.19} \text{ Step #9 \%}$$

**Pigment Distribution  $(x_{p_i})$**

**% Vehicle Distribution  $(x_{v_i})$**

$$\text{Using } x_{p_i} = \frac{\sum_{i=1}^i P_i}{\sum_{i=1}^i (P_i + V_i)} \quad \text{and,} \quad x_{v_i} = 1 - x_{p_i}$$

Given: **Pigment Distribution  $(P_i)$  & Vehicle Distribution  $(V_i)$**

$$x_{p_1} = 0.4880$$

$$x_{v_1} = 0.5120$$

$$x_{p_2} = 0.5139$$

$$x_{v_2} = 0.4861$$

$$x_{p_i} = \frac{\sum_{i=1}^i P_i}{\sum_{i=1}^i (P_i + V_i)} \Rightarrow \begin{aligned} x_{p_3} &= 0.5228 & x_{v_i} = 1 - x_{p_i} \Rightarrow x_{v_3} &= 0.4772 \\ x_{p_4} &= 0.5270 & x_{v_4} &= 0.4730 \\ x_{p_5} &= 0.5294 & x_{v_5} &= 0.4706 \end{aligned}$$

# CHAPTER 4

## DATA ANALYSIS - I

### Step #9      Relative Viscosity Distribution ( $\eta_i$ )

Using  $\eta_i = \eta_p e^{k_v x_{v_i}}$

Given Relative Pigment Viscosity:  $\eta_p = 240000 \text{ - poise}$

Given the Mix Viscosity Constant:  $k_v = -7.7832$

% Vehicle Distribution ( $x_{v_i}$ )

$$\eta_1 = 232$$

$$\eta_2 = 477$$

$$\eta_i = \eta_p e^{k_v x_{v_i}} \Rightarrow \eta_3 = 1355$$

$$\eta_4 = 6157$$

$$\eta_5 = 6159$$

Refer to the model, MathCad MODEL\_C1 (Non-Optimized) in the appendix (A12)

total Pigment Charge ... $\langle P \rangle = 1350$	Total Vehicle Charge $\langle U \rangle = 1200$					
Mixer Capacity ..... $\langle B \rangle = 3000$	$\times$ Solids of Pigment $\langle r \rangle = .2$					
Calculated Series Ratio $\langle R_i \rangle = 1.449$	Pigment Viscosity $\langle \eta_p \rangle = 240000$					
Number of Mixing Stages $\langle n \rangle = 5$	Vehicle Viscosity $\langle \eta_v \rangle = 100$					
<hr/>						
j	Pigment	Vehicle	Cum Pigment	Cum Vehicle	Utr-Dsp	Cum Utr
1	495.9365	520.3176	495.9365	520.3176	1983.747	1983
2	342.2612	272.44	838.1977	792.7576	1369.046	3352
3	236.2051	188.0192	1074.403	980.7768	944.8215	4296
4	163.0125	129.7579	1237.415	1110.535	652.0511	4948
5	112.5	89.55006	1349.915	1200.085	450.0011	5398
<hr/>						
j	Pigment	Vehicle	$\times$ Pigment	$\times$ Vehicle	Viscosity	
1	495.9365	520.3176	.4880044	.5119956	231.8418	
2	342.2612	272.44	.5139305	.4860695	476.7612	
3	236.2051	188.0192	.5227781	.4772219	1355.177	
4	163.0125	129.7579	.5270194	.4729806	6157.445	
5	112.5	89.55006	.5293785	.4706215	6159.04	

RE-

**PORT MODEL C1.BAS (Non Optimized)**

# CHAPTER 4

## DATA ANALYSIS - I

### Analysis of Vehicle-to-Pigment Ratio:

Model-C1

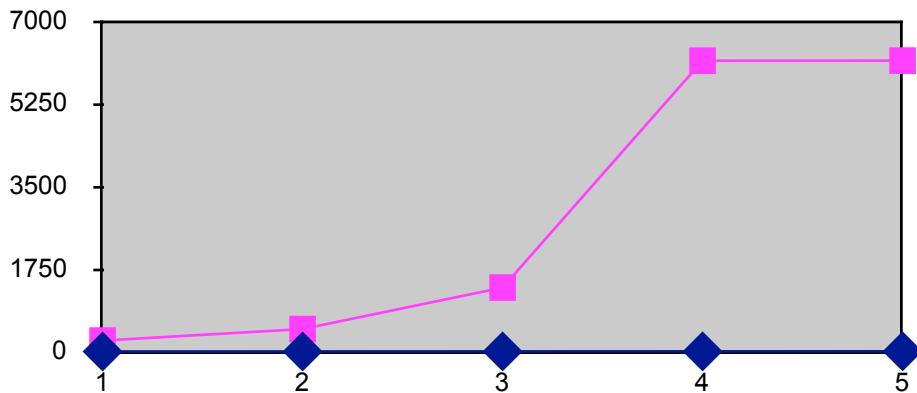
Stage	Viscosity	% Pigment	Pigment	Vehicle	Water	Cum V-P Ratio
1	232	0.488	496	520	1984	1.04
2	477	0.5139	342	272	1369	0.95
3	1355	0.5228	236	188	945	0.91
4	6157	0.527	163	130	652	0.9
5	6159	0.5294	113	86	450	0.89

A low vehicle-to-pigment ratio in the first stage implies inadequate wetting which usually results in a very short period for that mixing stage. The pigment dispersion is usually poor which results in large pigment particles. This condition is also characterized by subsequent stages with vehicle-pigment near 1.0.

Total Pigment Charge ... $\langle P \rangle$ = 1358	Total Vehicle Charge $\langle V \rangle$ = 1200				
Mixer Capacity ..... $\langle B \rangle$ = 3000	$\times$ Solids of Pigment $\langle r \rangle$ = .2				
Calculated Series Ratio $\langle R_i \rangle$ = 1.449	Pigment Viscosity $\langle N_p \rangle$ = 240000				
Number of Mixing Stages $\langle n \rangle$ = 5	Vehicle Viscosity $\langle N_v \rangle$ = 100				
j Pigment	Vehicle	Cum Pigment	Cum Vehicle	Wtr-Dsp	Cum Wtr
-- -----	-----	-----	-----	-----	-----
1 495.9365	520.3176	495.9365	520.3176	1983.747	1983
2 342.2612	272.44	838.1977	792.7576	1369.046	3352
3 236.2051	188.0192	1074.403	980.7768	944.8215	4296
4 163.0125	129.7579	1237.415	1110.535	652.0511	4948
5 112.5	89.55006	1349.915	1200.085	450.0011	5398
j Pigment	Vehicle	$\times$ Pigment	$\times$ Vehicle	Viscosity	
-- -----	-----	-----	-----	-----	
1 495.9365	520.3176	.4880044	.5119956	231.8418	
2 342.2612	272.44	.5139305	.4860695	476.7612	
3 236.2051	188.0192	.5227781	.4772219	1355.177	
4 163.0125	129.7579	.5278194	.4729806	6157.445	
5 112.5	89.55006	.5293785	.47086215	6159.04	

### REPORT MODEL C1.BAS (Non Optimized)

## CHAPTER 4 DATA ANALYSIS - I



The above graphic characteristic is a slow rise from the low viscosities to the end mix viscosity with respect to sequenced mixing stages. Too sharp of an increase implies a low vehicle-to-pigment ratio and aggregation or large pigment clusters result. In summary, trends that are drastic deviations from the exponential growth pattern indicated below. The slow rise in viscosity is an indicator of abnormality.

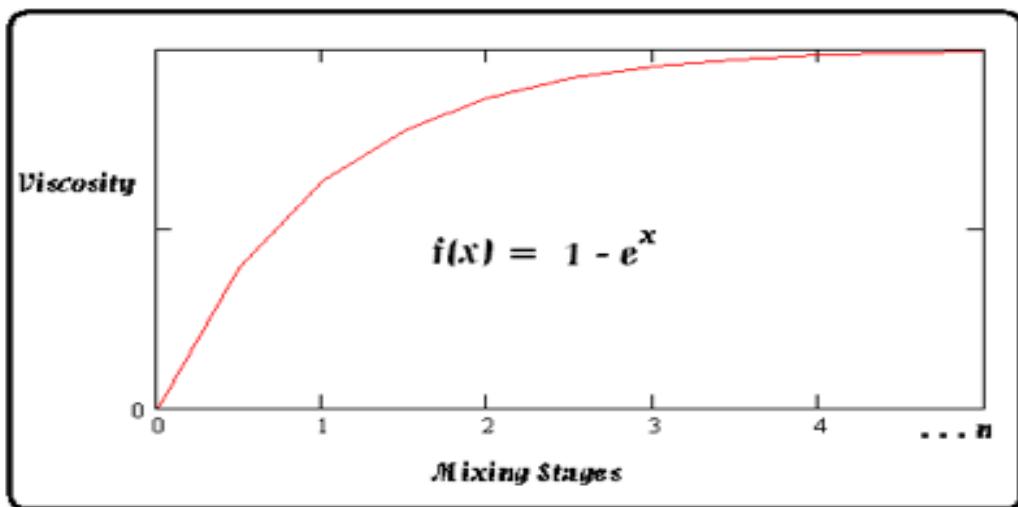


Figure 1.01a (Chapter - 1)

# CHAPTER 4

## DATA ANALYSIS - I

### Example Problem MODEL-C2 (Optimized):

Using the output of MODEL A2 as the INPUT charge to this MODEL-C,

$$\sum P_i = 1393.1953 \text{ lbs.}$$

$$\sum V_i = 1238.3959 \text{ lbs.}$$

Relative Viscosity of the Pigment ( $\eta_p$ ) = 240,000 poise

Relative Viscosity of the Vehicle ( $\eta_v$ ) = 100 poise

% Solids of Presscake (r) = 20%: (0.20)

Capacity Allowance  $E_0$  = 15%: ( $Eff\% = 85\%$       **Refer to Formula #22**).

Optimize the flush procedure to be mixed in a vessel of Capacity, B = 3000 lbs.

### Solution Steps MODEL-C2 (Optimized):

#### **Step #1**

Using Formula #11: Calculate the % pigment ( $x_p$ ) in total charge

$$\text{Formula #11: } x_p = \frac{\sum_{i=1}^n P_i}{\sum_{i=1}^n (P_i + V_i)}$$

$$x_p = \frac{\sum_{i=1}^n P_i}{\sum_{i=1}^n (P_i + V_i)} = \frac{1393}{(1393 + 1239)} \quad \boxed{x_p = 0.5294}$$

$$x_v + x_p = 1; \quad x_v = 1 - x_p \quad x_v = 1 - 0.5294 \quad \boxed{x_v = 0.4706}$$

# CHAPTER 4

## DATA ANALYSIS - I

### Step #2

Using Formula #8: Estimate the number of required mixing stages (n)

$$\text{Formula #8} \quad n = \frac{\sum_{i=1}^n P_i}{r} + \frac{\sum_{i=1}^n V_i}{x_{p_n} B} = \frac{\frac{1393}{0.2} + 1239}{1648.2}$$

$$n = 4.6742$$

Round (4.6742) to n = 5    Stage counter i, (1 to n);     $1 \leq i \leq n$

### Step #3      Pigment charge in last stage (n): $P_n$

$$\text{Using Formula #21: } P_n = r \frac{B - \left( \sum_{i=1}^n P_i + V_i \right)}{1 - r}, \text{ Final (n}_{\text{th}}\text{) Pigment Charge}$$

$$\text{Given: } B = 3000, \left( \sum_{i=1}^n P_i + V_i \right) = 2632 \text{ and } r = 0.20$$

$$\text{Then } P_n = (.20) \frac{3000 - (2632)}{.80}$$

$$P_n = 92$$

### Step #4

Given the Geometric Series,  $a, aR, aR^2, aR^3, \dots, aR^n$ :

$$\text{The sum of this series function is: } S = \frac{a(R^n - 1)}{R - 1}$$

Substitute the variables,  $a = P_n$ , (first element in the series) Substitute the variables,  $aR$ , (second element in the series) Substitute the variables,  $aR^2$ , (third element in the series), ... ;

After substitution:  $a, aR, aR^2, aR^3, \dots, aR^n$

# CHAPTER 4

## DATA ANALYSIS - I

**Step #4 (Continued):**

$$\text{Sum of the series, } S = \sum_{i=1}^n P_i \text{ becomes } \sum_{i=1}^n P_i = \frac{P_n(R^n - 1)}{R - 1}$$

To reverse the order of the series:  $aR^n, aR^{n-1}, aR^{n-2}, \dots, aR, a$

$$\text{Use the formula, } \sum_{i=1}^n P_i = \frac{P_n(R^n - 1)}{R - 1} - P_n.$$

An **iterative algorithm** in MathCad, is used to solve for the series ratio, (R) in the Non-Optimized C1 example below. Using the MathCad Root Solver function, (R) is found to be;  $R \approx 1.5759$ ; given  $N = 5.1656$  and rounded to  $n = 5$ .

The Pigment Distribution Series is generated by:

$$P_{n-i+1} = P_n R^{i-1}; \text{ given } (1 \leq i \leq n), n = 5 \text{ and } P_n = 92$$

$$P_{5-i+1} = (112.5)(1.5759)^{i-1}$$

$$\sum_{i=1}^5 P_i = 1393$$

generates

$P_5 = 92.0$
$P_4 = 145.0$
$P_3 = 228.5$
$P_2 = 360.1$
$P_1 = 567.4$

$P_1 = 567.4$
$P_2 = 360.1$
$P_3 = 228.5$
$P_4 = 145.0$
$P_5 = 92.0$

# CHAPTER 4

## DATA ANALYSIS - I

### Step #5 Water displacement distribution

Using Formula #6  $W_i = \frac{P_i}{r} - P_i$

Given:  $(1 \leq i \leq n)$ ,  $n = 5$ ,  $r = .20$  and pigment distribution  $P_i$

generates

$$\boxed{\begin{array}{l} W_1 = 2269 \\ W_2 = 1440 \\ W_3 = 914 \\ W_4 = 580 \\ W_5 = 368 \end{array}} \quad W_i = \frac{P_i}{0.2} - P_i \quad \sum_{i=1}^5 W_i = 5571$$

### Step #6 The Vehicle Distribution ( $V_i$ )

Using,  $V_1 = B - (P_1 + W_1)$ :  $V_1 = 3000 - (567.41 + 2269)$

$$\boxed{V_1 = 162.95}$$

Using,  $V_2 = B - P_1 - V_1 - \frac{P_2}{r}$ :  $V_2 = 3000 - 567.41 - 162.95 - \frac{360.06}{0.20}$

$$\boxed{V_2 = 469.37}$$

Using Formula #20  $V_i = B - \sum_{i=1}^{i-1} (P_i + V_i) - \frac{P_i}{r}$

Given:  $(3 \leq i \leq n)$ ,  $n = 5$ ,  $r = .20$  and pigment distribution  $P_i$

Given:  $(1 \leq i \leq 2)$   $\boxed{V_1 = 162.95}$  and  $\boxed{V_2 = 469.37}$

# CHAPTER 4

## DATA ANALYSIS - I

### Step #6 (Continued)

$$V_i = B - \sum_{i=1}^{i-1} (P_i + V_i) - \frac{P_i}{r}$$

generates

$$\sum_{i=1}^5 V_i = 1239.0$$

$V_1 = 162.9$
$V_2 = 469.4$
$V_3 = 297.8$
$V_4 = 189.0$
$V_5 = 119.9$

### Step #7 Calculate the Mix Viscosity Constant ( $k_v$ )

Using Formula #15  $k_v = \ln\left(\frac{\eta_v}{\eta_p}\right)$

Given Relative Vehicle Viscosity:  $\eta_v = 100 \text{ - poise}$

Given Relative Pigment Viscosity:  $\eta_p = 240000 \text{ - poise}$

$$k_v = \ln\left(\frac{100}{240000}\right) \quad k_v = -7.7832$$

### Step #8 Calculate Viscosity of the Mix $\eta_n$

Using Formula #16  $\eta_n = \eta_p e^{k_v x_v}$

Given Relative Pigment Viscosity:  $\eta_p = 240000 \text{ - poise}$

Given the Mix Viscosity Constant:  $k_v = -7.7832$

% vehicle in total charge:  $x_v = 0.4706$

$$\eta_n = 240000 e^{(-7.7832)(0.4706)} \quad \eta_n = 6159.19$$

# CHAPTER 4

## DATA ANALYSIS - I

### Step #9

Using Formula #15  $k_v = \ln(\frac{\eta_v}{\eta_p})$  Viscosity Constant for the mix

$$k_v = \ln(\frac{100}{240000}) = -7.7832$$

### Step #10 % Pigment Distribution ( $x_{p_i}$ ) & % Vehicle Distribution ( $x_{v_i}$ )

$$\text{Using } x_{p_i} = \frac{\sum_{i=1}^i P_i}{\sum_{i=1}^i (P_i + V_i)} \quad \text{and,} \quad x_{v_i} = 1 - x_{p_i}$$

Given: Pigment Distribution ( $P_i$ ) & Vehicle Distribution ( $V_i$ )

$$x_{p_i} = \frac{\sum_{i=1}^i P_i}{\sum_{i=1}^i (P_i + V_i)} \Rightarrow \begin{aligned} x_{p_1} &= 0.7769 & x_{v_1} &= 0.2231 \\ x_{p_2} &= 0.5946 & x_{v_2} &= 0.4054 \\ x_{p_3} &= 0.5541 & x_{v_3} &= 0.4459 \\ x_{p_4} &= 0.5376 & x_{v_4} &= 0.4624 \\ x_{p_5} &= 0.5292 & x_{v_5} &= 0.4708 \end{aligned}$$

### Step #11 Relative Viscosity Distribution ( $\eta_i$ )

$$\text{Using } \eta_i = \eta_p e^{k_v x_{v_i}}$$

Given Relative Pigment Viscosity:  $\boxed{\eta_p = 240000 \text{ - poise}}$

Given the Mix Viscosity Constant:  $\boxed{k_v = -7.7832}$

% Vehicle Distribution ( $x_{v_i}$ )

$$\eta_i = \eta_p e^{k_v x_{v_i}} \Rightarrow \begin{aligned} \eta_1 &= 202 \\ \eta_2 &= 396 \\ \eta_3 &= 1149 \\ \eta_4 &= 6150 \\ \eta_5 &= 6151 \end{aligned}$$

Refer to the model, MathCad MODEL\_C2 (Optimized) in the Appendix (A12)

# CHAPTER 4

## DATA ANALYSIS - I

### Analysis of Vehicle-to-Pigment Ratio:

**Model -**

**C2**

Stage	Viscosity	% Pigment	Pigment	Vehicle	Water	V-P Ratio
1	202	0.7769	567	163	2270	0.28
2	396	0.5946	360	469	1440	0.68
3	1149	0.5541	228	297	914	0.8
4	6150	0.5376	145	189	580	0.86
5	6152	0.5292	92	120	368	0.89

This model's vehicle-to-pigment ratios (0.3 to 0.9) are the inverse of what is expected of an ideal flush procedure (1.7 to 0.8). The V/P Ratios of the C2 Model implies insufficient wetting which leads to high agglomeration in the early stages. The following grinding appear to be insufficient to breakdown the pigment clusters. The optimized input increased both the total pigment and vehicle charges by 3% each. The C2 distribution models an attempt to disperse more pigment over the same number of stages at the same capacity. The output of MODEL-C2 shown below, is quite predictable.

Total Pigment Charge ... $\langle P \rangle$ = 1393	Total Vehicle Charge $\langle V \rangle$ = 1239					
Mixer Capacity ..... $\langle B \rangle$ = 3000	$\times$ Solids of Pigment $\langle r \rangle$ = .2					
Calculated Series Ratio $\langle R_i \rangle$ = 1.3693	Pigment Viscosity $\langle N_p \rangle$ = 240000					
Number of Mixing Stages $\langle n \rangle$ = 6	Vehicle Viscosity $\langle N_v \rangle$ = 100					
<b>j</b>	<b>Pigment</b>	<b>Vehicle</b>	<b>Cum Pigment</b>	<b>Cum Vehicle</b>	<b>Wtr-Dsp</b>	<b>Cum Wtr</b>
1	442.8747	785.6264	442.8747	785.6264	1771.499	1771
2	323.4315	154.3415	766.3062	939.9679	1293.726	3064
3	236.2021	112.7156	1002.508	1052.683	944.8083	4008
4	172.4984	82.31625	1175.887	1135	689.9937	4697
5	125.9756	60.11553	1300.982	1195.115	503.9025	5200
6	92	43.90247	1392.982	1239.018	368.0001	5568
<b>j</b>	<b>Pigment</b>	<b>Vehicle</b>	<b><math>\times</math> Pigment</b>	<b><math>\times</math> Vehicle</b>	<b>Viscosity</b>	
1	442.8747	785.6264	.3605001	.6395	190.5109	
2	323.4315	154.3415	.4491108	.5508891	317.2981	
3	236.2021	112.7156	.4877931	.5122069	637.9797	
4	172.4984	82.31625	.5086595	.4913405	1660.293	
5	125.9756	60.11553	.5212065	.4787935	6151.221	
6	92	43.90247	.5292486	.4707514	6151.543	

**REPORT MODEL-C2.BAS (Optimized)**

# CHAPTER 4

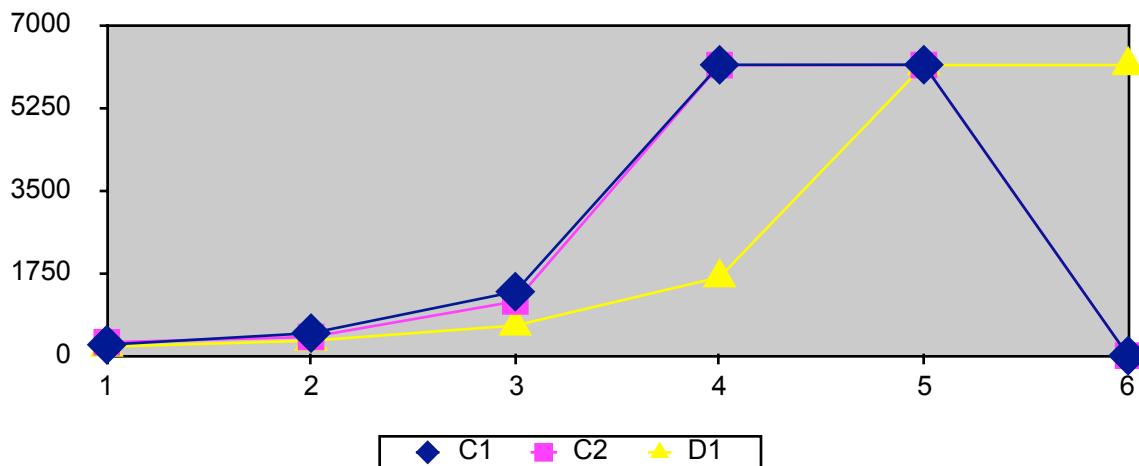
## DATA ANALYSIS - I

**Treatment – III-MODEL-C L:** Given the above output results and analysis for MODEL-C2 & MODEL-C1, an algorithm (D1) using the optimized input dispersed over 6 stages seems to be the next logical model to design. Refer to the results below MODEL-D1.

Total Pigment Charge ... $\langle P \rangle$ = 1393	Total Vehicle Charge $\langle U \rangle$ = 1239					
Mixer Capacity ..... $\langle B \rangle$ = 3000	$\times$ Solids of Pigment $\langle r \rangle$ = .2					
Calculated Series Ratio $\langle R_i \rangle$ = 1.3693	Pigment Viscosity $\langle N_p \rangle$ = 240000					
Number of Mixing Stages $\langle n \rangle$ = 6	Vehicle Viscosity $\langle N_u \rangle$ = 100					
<b>J</b>	<b>Pigment</b>	<b>Vehicle</b>	<b>Cum Pigment</b>	<b>Cum Vehicle</b>	<b>Wtr-Dsp</b>	<b>Cum Wtr</b>
1	442.8747	785.6264	442.8747	785.6264	1771.5	1771
2	323.4315	154.3415	766.3062	939.9679	1293.727	3064
3	236.2021	112.7156	1002.508	1052.683	944.8893	4088
4	172.4984	82.31625	1175.007	1135	689.9947	4697
5	125.9756	60.11553	1300.982	1195.115	503.9035	5200
6	92	43.90247	1392.982	1239.018	368.0011	5568
<b>J</b>	<b>Pigment</b>	<b>Vehicle</b>	<b><math>\times</math> Pigment</b>	<b><math>\times</math> Vehicle</b>	<b>Viscosity</b>	<b>U/P Ratio</b>
1	442.8747	785.6264	.36050001	.6395	190.5109	1.773925
2	323.4315	154.3415	.4491108	.5508891	317.2901	1.226622
3	236.2021	112.7156	.4877931	.5122069	637.9797	1.05005
4	172.4984	82.31625	.5086595	.4913405	1660.293	.9659517
5	125.9756	60.11553	.5212065	.4787935	6151.221	.9186253
6	92	43.90247	.5292486	.4787514	6151.543	.8894712

### MODEL D1.BAS

This model's vehicle-to-pigment ratios (1.8 to 0.9) appear to yield results that model the expected ideal flush procedure (1.7 to 0.8). The V/P Ratios of the D1 Model imply sufficient wetting in the first two stages, followed by grinding stages which gradually transition the mix from wetting phase to grinding as shown in the figure below.



## **CHAPTER 4**

### **DATA ANALYSIS - I**

As mentioned in the abstract, the initial objective of this project is to develop general mathematical models, which will simulate optimized flushing procedures. Given a minimum of input parameters, the models should calculate estimates of flush output parameters. The Importance, Scope and Rationale of the Study in Chapter 1, references the need for productive tools which can assist the technologist with design and analysis of a flush procedure.

The programs that were created for this phase of the project have used Bulk Loading quantities required to optimize the incremental mixing stages. Models A and B use logarithmic & exponential functions to generate the viscosity distributions. Models C and D uses the geometrical & infinite series to generate viscosity distributions. Models A, B & C calculate and use the number of mixing stages required for the procedure. Only Model-D allows a manual input of all the critical parameters, including (n) the number of mixing stages, but the distribution is based on the geometrical/infinite series. The missing program, MODEL-AN will be created to allow input of (n), the number of mixing stages, using the logarithmic & exponential functions to generate the viscosity distributions.

#### **Models A, B & BN**

- Logarithmically dispersed
- Optimizes the input reactants (Output Quantity > Input Quantity)
- Model-BN: Manual input of (n) Number of Stages

#### **Models C & D**

- Geometrically dispersed
- Input Non-Optimized (Input Quantity = Output Quantity)
- Model-D: Manual input of (n) Number of Stages

## **CHAPTER 4**

### **DATA ANALYSIS - I**

This completes the first phase of the project (ANALYSIS-I) which focuses on capacity, stage charge and viscosity distribution. The next phase (ANALYSIS-II) will merge the concepts of mixing time, temperature and substitutions of vehicle blends of various viscosities including resins.

# **CHAPTER - 5**

## **DATA ANALYSIS - II (MULTI-VEHICLE)**

### **CONTENT**

**Load Process (Multi-Vehicle Substitution)**

**Mixing Stage Analysis (Water Displacement)**

**Process Mixing-Time Models**

**Process Rates & Analysis**

**Temperature, Viscosity & Mechanics**

# CHAPTER 5

## DATA ANALYSIS - II

### Load Process Reaction Using Two or More Vehicles

The previous chapters used examples consisting of single pigment and one vehicle charges. Most pigment dispersions are comprised of blends of pigments with several different vehicles. Excluding the effect of temperature and evaporation on the vehicle charge, the following section shows models can be created to simulate a blend of 2 or more different vehicles. The respective distributions that are generated by the Models (A, B, C, D), will be unchanged if the viscosity of the vehicle blend is equal to the Input relative vehicle viscosity used in the models.

Referring back to Chapter 2 – “Equations Relating Viscosity to Resin Concentration” and “Modeling Physical Applications” directly apply to this phase of the project. Given the Models (A-D) output vehicle distribution, ( $V_i$ ) the vehicle quantities are matched with a blend of alternative vehicles whose blend viscosity is equal to the model input viscosity ( $\eta_v$ )

The blend function:  $\eta_v = \eta_A e^{k_{AB} x_B}$       Solute (B) and Solvent (B)

Blend % Solvent ( $x_B$ ) in decimal

Viscosity constant ( $k_B$ ):  $k_B = \ln\left(\frac{\eta_B}{\eta_A}\right)$

**EXAMPLE 5a:** Refer to Report MODEL-A1

	$\eta_i$	$x_{v_i}$	$x_{p_i}$	$P_i$	$W_i$	$V_i$
I Stage	Viscosity Distribution	Mix % Vehicle	Mix % Pigment	Pigment Charge	Water Displaced	Vehicle Charge
1	3160	0.5564	0.4436	479.69	1918.76	<b>601.55</b>
2	4699	0.5054	0.4946	337.16	1348.63	<b>232.97</b>
3	5474	0.4857	0.5143	237.46	949.84	<b>161.33</b>
4	5864	0.4769	0.5231	166.53	666.13	<b>117.18</b>
5	6060	0.4727	0.5273	116.15	464.58	<b>85.4</b>
6	6159	0.4706	0.5294	80.58	322.32	<b>61.69</b>

Substitute Resin & Solvent in Stage #1: Varnish & Solvent in Stage #4

# CHAPTER 5

## DATA ANALYSIS - II

### EXAMPLE 5a: (Continued)

Given: Viscosity ( $\eta_v = 100 p$ )

Stage #1 - Alternative Vehicles

Total Vehicle Stage Charge: ( $V_1 = 601.55$ )

A) Resin X Viscosity: ( $\eta_A = 66300 p$ )

B) Solvent Y Viscosity: ( $\eta_B = 0.056 p$ )

Viscosity constant ( $k_{AB}$ )

$$\eta_v = 100$$

$$k_{AB} = \ln\left(\frac{\eta_B}{\eta_A}\right)$$

$$100 = \eta_A e^{k_{AB} x_B}$$

$$k_{AB} = \ln\left(\frac{0.056}{66300}\right) = \ln(e^{k_{AB} x_B})$$

$$\ln(0.001508) = k_{AB} x_B$$

$$k_{AB} = -13.9843$$

$$\frac{-6.496775}{-13.9843} = x_B$$

$$0.4646 = x_B$$

B) Solvent Y Charge:

$$Solvent = V_1(x_B)$$

$$Solvent = 601.55(0.4646)$$

$$Solvent = 279.5$$

A) Resin X Charge:

$$Resin = V_1 - Solvent$$

$$Resin = 601.55 - 279.5$$

$$Resin = 322.1$$

# CHAPTER 5

## DATA ANALYSIS - II

### EXAMPLE 5a: (Continued)

Given: Viscosity ( $\eta_v = 100 p$ )

Stage #4 - Alternative Vehicles

Total Vehicle: ( $V_4 = 117.18$ )

A) Varnish X Viscosity: ( $\eta_A = 5500 p$ )

B) Solvent Y Viscosity: ( $\eta_B = 0.056 p$ )

Viscosity constant ( $k_{AB}$ )

$$\eta_v = 100$$

$$k_{AB} = \ln\left(\frac{\eta_B}{\eta_A}\right)$$

$$k_{AB} = \ln\left(\frac{0.056}{5500}\right)$$

$$100 = \eta_A e^{k_{AB} x_B}$$

$$\ln\left(\frac{100}{5500}\right) = \ln(e^{k_{AB} x_B})$$

$$\ln(0.01818) = k_{AB} x_B$$

$$\frac{-4.00733}{-11.4949} = x_B$$

$$0.3486 = x_B$$

B) Solvent Y Charge:

$$Solvent = V_1(x_B)$$

$$Solvent = 117.18(0.3486)$$

$$Solvent = 40.8$$

A) Varnish X Charge:

$$Varnish = V_4 - Solvent$$

$$Varnish = 117.18 - 40.8$$

$$Varnish = 76.4$$

# CHAPTER 5

## DATA ANALYSIS - II

Compare Flush Procedures: (Mono vs. Multi-Vehicle)

### Mono-Vehicle

#### Procedure

Stage	Pigment	Charge	Vehicle	Charge
1	P-001	479.7	Vehicle	601.6
2	P-001	337.2	Vehicle	233.2
3	P-001	237.5	Vehicle	161.3
4	P-001	166.5	Vehicle	117.2
5	P-001	116.2	Vehicle	85.4
6	P-001	80.6	Vehicle	61.7
Total		1417.7		1260.4

### Multi -Vehicle

#### Procedure

Ref: Model - A1

Stage	Pigment	Charge	Vehicle	Charge
1	P-001	479.7	Resin	322.1
			Solvent	279.5
2	P-001	337.2	Vehicle	233.2
3	P-001	237.5	Vehicle	161.3
4	P-001	166.5	Varnish	76.4
			Solvent	40.8
5	P-001	116.2	Vehicle	85.4
6	P-001	80.6	Vehicle	61.7
Total		1417.7		1260.4

## **CHAPTER 5**

### **DATA ANALYSIS - II**

**EXAMPLE 5a** with reference to Report MODEL-A1 and the calculations shown above is an illustration of how to substitute a blends of multi-vehicle components for singular vehicle additions. Mixing Stages #1 & #4 show how blends of heavier varnish and solvent is substituted for respective single additions in a mono-vehicle procedure. The addition mix viscosity and the pigment charge are held constant while the amounts heavy varnish and solvent are computed.

This vehicle substitution could have been the choice of combinations of several resins, solvents and varnishes of various viscosities. The choice and combinations are endless. Many ink makers and ink Manufacturers use various substitution techniques ranging from experienced guesses to higher level mathematical computations which are advanced beyond the algebra used in this illustration. The practical application of multi-vehicle substitution range from raw material availability to material cost analysis in manufacturing, production and research environments.

The purpose of this dissertation is to develop some simple mathematical models that simulate some of the complexities of dynamic mixing. A more thorough and in-depth development of this subject is encouraged and may require scientific and mathematical disciplines beyond those used in this paper. These efforts are out of mathematical curiosity and a deep fascination with merging the art and science of ink making industry with some of the rapidly growing computer technology of our modern age, Analyzing and creating math models of manufacturing processes is a great avenue for Applied Mathematics on many levels of education.

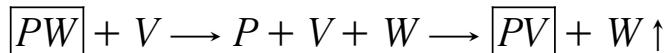
# CHAPTER 5

## DATA ANALYSIS - II

### Mixing Stage Analysis (Water Displacement)

The previous chapters discuss the load parameters and the total water displacement for the mixing stage. This section will develop models to describe the rate of water release during the flushing process. The focus will be on using the input & output mixing parameters of a stage to estimate the intermediate slurry mixing rates and mixing times.

#### Appendix A3-Formula #23



*Physical mix of presscake and vehicle (**PW+V**), mixing to a slurry (**P+V+W**), to produce a product paste of wetted pigment, (**PV**), with displaced water, (**W**)*

The water displacement distribution (**W<sub>d</sub>**), can be very helpful to develop a model which will help simplify some of the complex dynamics that occurs during the flushing process. As water is released into the mix, the pigment is being encapsulated by the vehicle. The following is an example of a 1st stage 20% pigment presscake particle (**PW**), mixed with vehicle to a paste of 44.36% pigment content. The calculations and two dimension figures below is an attempt to empirically illustrate how the 1st stage wetting process replaces 4 parts water with 1.25 parts vehicle. The displace water (**W<sub>↑</sub>**) is poured off upon completion of the end stage.

$$\boxed{PW}$$

$$r = 0.2$$

$$\frac{P}{P+W} = \frac{1}{5}$$

$$\frac{P}{P+W} = \frac{1}{1+4}$$

$$\boxed{PV}$$

$$x_p = 0.4436$$

$$\frac{P}{P+V} \approx \frac{1}{2.25}$$

$$\frac{P}{P+V} \approx \frac{1}{1+\boxed{1.25}}$$

$$\frac{P}{P+V} \approx \frac{1}{1+\boxed{\frac{5}{4}}} \Rightarrow \boxed{P_4V_5}$$

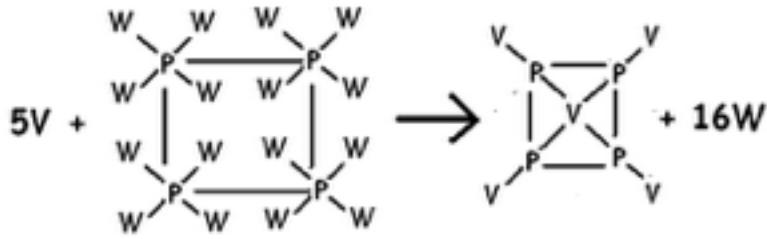
$$\boxed{PW} + V \longrightarrow \boxed{P_yV_x} + nW \uparrow$$

$$PW_4 + \frac{5}{4}V \longrightarrow PV_{\frac{5}{4}} + 4W \uparrow$$

$$P_4W_{16} + 5V \longrightarrow P_4V_5 + 16W \uparrow$$

# CHAPTER 5

## DATA ANALYSIS - II



The 2nd stage input is the wetted  $P_4V_5 + [PW_4] + V$ . The end of the mixing stage the % pigment content is 49.46%. The cumulative V to P ratio is 1.02. The empirical ratio x/y to approximate the V to P ratio of 1.02, for stage #2 is x = 1 & y = 1.

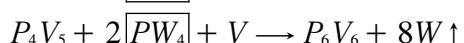
*End Mixed Paste:*  $[PV]$

$$x_p = 0.4946$$

$$\frac{P}{P+V} = \frac{1}{2.02}$$

$$\frac{P}{P+V} = \frac{1}{1 + \boxed{1.02}}$$

$$\frac{P}{P+V} \approx \frac{1}{1 + \boxed{\frac{1}{1}}} \Rightarrow [P_1V_1]$$



The 3rd stage input is the wetted  $P_6V_6 + [PW_4] + V$ . The end of the mixing stage the % pigment content is 51.43%. The cumulative V to P ratio is 0.94. The empirical ratio x/y to approximate the V to P ratio of 0.94, for stage #3 is x = 15 & y = 16.

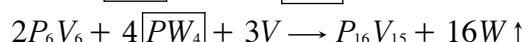
*End Mixed Paste:*  $[PV]$

$$x_p = 0.5143$$

$$\frac{P}{P+V} = \frac{1}{1.94}$$

$$\frac{P}{P+V} = \frac{1}{1 + \boxed{0.94}}$$

$$\frac{P}{P+V} \approx \frac{1}{1 + \boxed{\frac{15}{16}}} \Rightarrow [P_{16}V_{15}]$$



## CHAPTER 5

### DATA ANALYSIS - II

The 4th stage input is the wetted  $P_{16}V_{15} + \boxed{PW_4} + V$ . The end of the mixing stage the % pigment content is 52.31%. The cumulative V to P ratio is 0.91. The empirical ratio x/y to approximate the V to P ratio of 0.91, for stage #3 is x = 10 & y = 11.

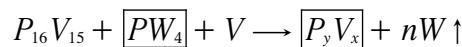
*End Mixed Paste:*  $\boxed{PV}$

$$x_p = 0.5231$$

$$\frac{P}{P+V} = \frac{1}{1.91}$$

$$\frac{P}{P+V} = \frac{1}{1 + \boxed{0.91}}$$

$$\frac{P}{P+V} \approx \frac{1}{1 + \boxed{\frac{10}{11}}} \Rightarrow \boxed{P_{11}V_{10}}$$



The 5th stage input is the wetted  $P_{11}V_{10} + \boxed{PW_4} + V$ . The end of the mixing stage the % pigment content is 52.73%. The cumulative V to P ratio is 0.90. The empirical ratio x/y to approximate the V to P ratio of 0.90, for stage #3 is x = 9 & y = 10.

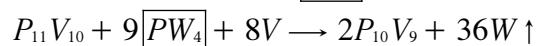
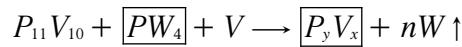
*End Mixed Paste:*  $\boxed{PV}$

$$x_p = 0.5273$$

$$\frac{P}{P+V} = \frac{1}{1.90}$$

$$\frac{P}{P+V} = \frac{1}{1 + \boxed{0.90}}$$

$$\frac{P}{P+V} \approx \frac{1}{1 + \boxed{\frac{9}{10}}} \Rightarrow \boxed{P_{10}V_9}$$



The 6th stage input is the wetted  $P_{10}V_9 + \boxed{PW_4} + V$ . The end of the mixing stage the % pigment content is 52.94%. The cumulative V to P ratio is 0.89. The empirical ratio x/y to approximate the V to P ratio of 0.89, for stage #3 is x = 8 & y = 9.

## CHAPTER 5

### DATA ANALYSIS - II

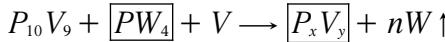
*End Mixed Paste:*  $\boxed{PV}$

$$x_p = 0.5294$$

$$\frac{P}{P+V} = \frac{1}{1.89}$$

$$\frac{P}{P+V} = \frac{1}{1 + \boxed{0.89}}$$

$$\frac{P}{P+V} \approx \frac{1}{1 + \boxed{\frac{8}{9}}} \Rightarrow \boxed{P_9 V_8}$$



At the end of the mixing process (6th stage), the % pigment content is 54.94%. By using the same type calculations as the 1st stage, the resulting empirical model implies there will be about 8 vehicle particles for every 9 pigment particles.

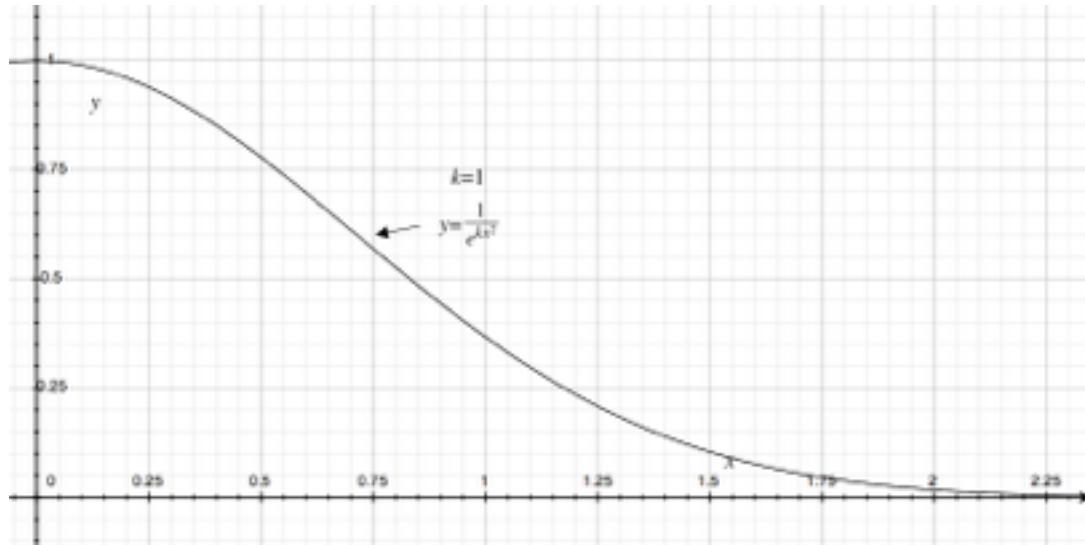
There are many choices of particle configurations that can be used, but the configuration used here are only intended to imply ideal wetting. The actual particle configurations are clusters of complex non-uniform agglomerations.



The beginning of the mixing stage, the rate of water displacement is observed to be near zero and gradually increases to maximum rate. After the majority of the water has been displaced by the vehicle, the rate of displacement slows to approach zero at the end of the mixing stage. Because of the agglomeration and non-uniformity of the wetted pigment particles, some of the water is trapped and is difficult to completely displace by the mixing action. This description is characteristic of the right or positive side of the standard empirical bell-curve (Probability Error Function) given  $x \geq 0$  &  $k = 1$ .

# CHAPTER 5

## DATA ANALYSIS - II



### Process Mixing Time Models

Y-Axis represents the proportion of the process remaining to be completed in the mix stage [i]. The Y-Axis or [w(t)] can also be used to model the amount of water remaining to be displaced in the mixing process with respect to the X-Axis, the mixing time (t).

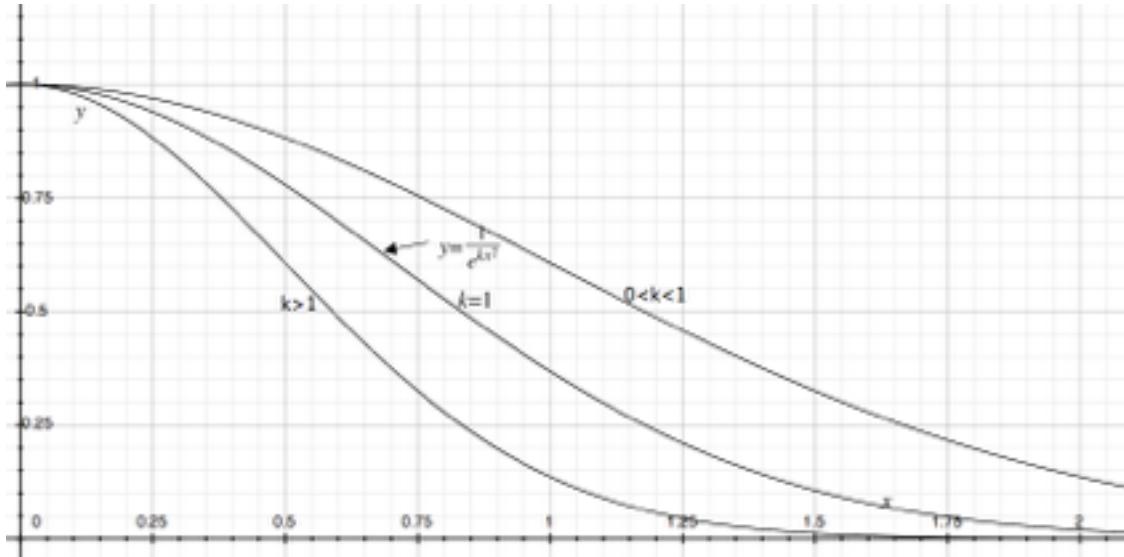
Given the Error Function,  $\left[ y = \frac{1}{e^{kt^2}} \right]$ , the mixing time constant for the stage is (k).

The standard or normal error function is expressed when ( $k = 1$ ). When ( $k > 1$ ), implies that mixing time is relatively shorter than standard and the paste is being formed at a faster rate than standard. Keep in mind that the error function  $\left[ y = \frac{1}{e^{kt^2}} \right]$  models the amount remaining. Therefore the amount that is displaced is the complementary expression,  $\left[ y = 1 - \frac{1}{e^{kt^2}} \right]$ . During this treatment, the amount of water displaced and the amount of water remaining will be frequently referenced.

# CHAPTER 5

## DATA ANALYSIS - II

When ( $k < 1$ ), mixing time is relatively longer than standard and the paste is being formed at a slower rate than standard. Refer to the figure below.



As water is displaced, the paste is formed. The Y-Axis (the remainder of the process - stage #1) is expressed as:  $\left[1 - \frac{w(t)}{W_D}\right]$  where  $[w(t)]$  is a water displacement function. **Refer to Appendix A1 Formula #6.**  $\left[W_i = \frac{P_i}{r} - P_i\right]$  to transform  $[W_i \rightarrow w_i(t)]$ .

Development of Water Displacement with respect to time  $[w(t)]$

Development of Pigment Wetted with respect to time  $[P(t)]$

$$\text{Given: } \left[W_i = \frac{P_i}{r} - P_i\right] \text{ and } \left[y = \frac{1}{e^{kt^2}}\right]; \quad 1 - \frac{w_i(t)}{W_D} = \frac{1}{e^{kt^2}}$$

$$\begin{aligned} W_i(t) &= \frac{P_i(t)}{r} - P_i(t) & 1 - \frac{w_i(t)}{W_D} &= \frac{1}{e^{kt^2}} \\ W_i(t) &= \frac{P_i(t)}{r} - \frac{rP_i(t)}{r} & 1 - \frac{1}{e^{kt^2}} &= \frac{w_i(t)}{W_D} & \frac{P_i(t)}{r}(1-r) &= W_D(1 - e^{-kt^2}) \\ W_i(t) &= \frac{P_i(t)}{r}(1-r) & 1 - e^{-kt^2} &= \frac{w_i(t)}{W_D} & P_i(t) &= \frac{rW_D(1 - e^{-kt^2})}{(1-r)} \\ w_i(t) &= \frac{P_i(t)}{r}(1-r) \mid w_i = W_i & W_D(1 - e^{-kt^2}) &= w_i(t) \end{aligned}$$

$w_i(t) = W_D(1 - e^{-kt^2})$

$P_i(t) = \frac{rW_D(1 - e^{-kt^2})}{(1-r)}$

## CHAPTER 5

### DATA ANALYSIS - II

As the paste enters into its final phase of formation, the displaced water starts to clear and adhere to the mixer blades. The mass is moist and spongy due to trapped water within and on the surface of the paste. The clear water is poured off and the mass is allowed to undergo a few additional minutes of mixing to squeeze out trapped water. Due to the grinding and shearing action of the mixer blades the temperature of the mass is elevated as the paste thickens. In a manufacturing environment the water start to clear to a slight amber tint as the water content of the paste approaches 3 - 5%.

The moisture content is defined as the % water remaining in the wetted paste as sampled from the mixer after mixing time (t). The mix constant (k) is calculated from a first stage sample taken part way through paste formation.

#### EXAMPLE 5b:

Given DATA-1, the lab sample analysis of the paste, use the Probability Function to estimate the time of 95% completion ( $T_{95}$ ) for the mixing Stage #1. The process has been mixing for 1.75 hours & the water content is at 40%.

$$\text{Error Function} \\ y = \frac{1}{e^{kx^2}} = e^{-kx^2}$$

$$\text{Water Displacement Function} \\ w(t) = W_D(1 - e^{-kt^2})$$

$$\text{Pigment Wetting Function} \\ P_i(t) = \frac{rW_D(1 - e^{-kt^2})}{(1 - r)}$$

#### DATA-1

Process Stage:	#1
Bulk Capacity (B):	3000 lbs
Total Water Displacement:	$W_D = 1918.78 \text{ lbs}$
Pigment Content:	$r = .2 \text{ or } 20\%$
Process Time:	$t = 1.75 \text{ hrs}$
Paste Water Content:	40%
Temperature:	$25^\circ \text{C} / 77^\circ \text{F}$

# CHAPTER 5

## DATA ANALYSIS - II

Estimate y: 
$$y \approx \frac{Water \%}{W_D} = \frac{B(Water \%)}{W_D} = \frac{3000(.4)}{1918.78} = .6254$$

Using  $y = 0.6254$  and  $\left[ y = \frac{1}{e^{kt^2}} \right]$

Calculate the Amount of Water Displaced [w(t)] with respect to time:

$$0.6254 = e^{-k(1.75)^2}$$

$$\ln(0.6254) = -1.75^2 k$$

$$\frac{\ln(0.6254)}{-1.75^2} = k$$

$$\frac{-0.4694}{-3.0625} = k$$

$$k = 0.1533$$

$$w_i(t) = W_D(1 - e^{-kt^2})$$

$$w_i(1.75) = 1918.78(1 - e^{-0.1533(1.75^2)})$$

$$w_i(1.75) = 1918.78(1 - e^{-0.4695})$$

$$w_i(1.75) = 1918.78(1 - 0.6253)$$

$$w_i(1.75) = 1918.78(0.3747)$$

$$w_i(1.75) = 718.94$$

Water Remaining = (Water %)(W<sub>D</sub>) = (0.6254)(1918.78) = 1200.01 lbs.

Water Remaining = W<sub>D</sub> - w<sub>i</sub>(t) = (1918.78 - 718.94) = 1199.84

Derivation of the function of Wetted Pigment formation [P(t)] with respect to time:

$$P_i(t) = \frac{rW_D(1 - e^{-kt^2})}{(1 - r)}$$

$$P_i(1.75) = \frac{(0.2)(1918.76)(1 - 0.6254)}{(1 - 0.2)}$$

$$P_i(1.75) = \frac{(0.2)(718.77)}{(1 - 0.2)}$$

$$P_i(1.75) = 179.69$$

$$P_i(t) = \frac{rw_i(t)}{(1 - r)}$$

## CHAPTER 5

### DATA ANALYSIS - II

Time required to process stage #1 to 95% completion [T<sub>95</sub>] is estimated 4.42 hrs.

$$\left[ y = \frac{1}{e^{kt^2}} \right] \text{ Given } k = 0.1533$$

$$.05 = e^{-kt^2}$$

$$.05 = e^{-0.1533t^2}$$

$$\ln(0.05) = -0.1533t^2$$

Using

$$\frac{\ln(0.05)}{-0.1533} = t^2$$

$$19.5416 = t^2$$

$$4.42 = t$$

Four hours is a typical time for mixing the 1st stage in a Baker-Perkins 500 gallon Mixer. Therefore the model estimates appear to be reasonable. Referring to the previous EXAMPLE-5a

	$\eta_i$	$x_{v_i}$	$x_{p_i}$	$P_i$	$W_i$	$V_i$
I	Viscosity Distribution	Mix	Mix	Pigment	Water Displaced	Vehicle Charge
Stage		% Vehicle	% Pigment	Charge		
1	3160	0.5564	0.4436	479.69	1918.76	<b>601.55</b>
2	4699	0.5054	0.4946	337.16	1348.63	<b>232.97</b>
3	5474	0.4857	0.5143	237.46	949.84	<b>161.33</b>
4	5864	0.4769	0.5231	166.53	666.13	<b>117.18</b>
5	6060	0.4727	0.5273	116.15	464.58	<b>85.4</b>
6	6159	0.4706	0.5294	80.58	322.32	<b>61.69</b>

Stage #1 would displace an estimated 1918.76 lbs of water if allowed to mix to completion. But mixing to 95% completion would displace 1823 lbs of water, leaving about a residual of 96 lbs added to the displacement in the next stage. These estimations are quite normal for the wetting stages of mixing.

# CHAPTER 5

## DATA ANALYSIS - II

### Mixer-Time Methodology (A): $k$ & $W_{D1}$ constant

The times for the remaining mixing stages can be calculated as follows:

$$k = 0.1533 \text{ and } W_D = 1918.78 \text{ is constant for all stages, } 1 \leq i \leq 6$$

Use the Water Distribution  $[W_i]$  from the Table above in EXAMPLE-5a

Substitute  $[W_i]$  for  $w_i(t)$  in Water Displacement Function and solve for  $[t]$ .

$$w_i(t) = W_D(1 - e^{-kt^2})$$

$$\frac{w_i(t)}{W_D} = 1 - e^{-kt^2}$$

$$e^{-kt^2} = 1 - \frac{w_i(t)}{W_D}$$

$$-kt^2 = \ln\left(1 - \frac{w_i(t)}{W_D}\right)$$

$$t^2 = \left(\frac{1}{-k}\right) \ln\left(1 - \frac{w_i(t)}{W_D}\right)$$

$$t = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{w_i(t)}{W_D}\right)}$$

$$t = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{w_i(t)}{W_D}\right)}$$

$$T_{95i} = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{W_i}{W_D}\right)}$$

Given:  $1 \leq i \leq 6$

$$R_i = W_{D_i}(1 - 0.95)$$

Total time for stage #1 to mix to 95%  $T_{95\%} = 4.42 \text{ hrs}$  Residual:  $R_1 = (1918.78)(0.05)$   
 $R_1 = 95.94 \text{ lbs}$

**W<sub>d2</sub>** Water to displace in stage #2:  $w_2(t) = 95.94 + 1348.63 = 1444.57 \text{ lbs}$

$$T_{952} = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{W_2}{W_D}\right)}$$

$$T_{952} = \sqrt{\left(\frac{1}{-0.1533}\right) \ln\left(1 - \frac{1444.57}{1918.78}\right)}$$

Stage #2:

$$T_{952} = \sqrt{\left(\frac{1}{-0.1533}\right) \ln\left(1 - \frac{1444.57}{1918.78}\right)}$$

$$T_{952} = \sqrt{(-6.5232) \ln(0.2471)}$$

$$T_{952} = \sqrt{(-6.5232) \ln(0.2471)}$$

$$T_{952} = 3.02$$

$$R_i = W_{D_i}(1 - 0.95)$$

$$\text{Residual } R_2 = (1444.57)(0.05)$$

$$R_2 = 72.22 \text{ lbs}$$

# CHAPTER 5

## DATA ANALYSIS - II

**W<sub>d3</sub>** Water to displace in stage #3:  $w_3(t) = 72.23 + 949.84 = 1022.07 \text{ lbs}$

$$T_{95_3} = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{W_3}{W_D}\right)}$$

$$R_i = W_{D_i}(1 - 0.95)$$

Stage #3:  $T_{95_3} = \sqrt{(-6.5232) \ln\left(1 - \frac{1022.07}{1918.78}\right)}$  Residual  $R_3 = (1022.07)(0.05)$

$$T_{95_3} = \sqrt{(-6.5232) \ln(0.4671)}$$

$$R_3 = 51.10 \text{ lbs}$$

$$\boxed{T_{95_3} = 2.23 \text{ hrs}}$$

**W<sub>d4</sub>** Water to displace in stage #4:  $w_4(t) = 51.10 + 666.13 = 717.23 \text{ lbs}$

$$T_{95_4} = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{W_4}{W_D}\right)}$$

$$R_i = W_{D_i}(1 - 0.95)$$

Stage #4:  $T_{95_4} = \sqrt{(-6.5232) \ln\left(1 - \frac{717.23}{1918.78}\right)}$  Residual  $R_4 = (717.23)(0.05)$

$$T_{95_4} = \sqrt{(-6.5232) \ln(0.6262)}$$

$$R_4 = 35.86 \text{ lbs}$$

$$\boxed{T_{95_4} = 1.75 \text{ hrs}}$$

**W<sub>d5</sub>** Water to displace in stage #5:  $w_5(t) = 35.86 + 464.58 = 500.44 \text{ lbs}$

$$T_{95_5} = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{W_5}{W_D}\right)}$$

$$R_i = W_{D_i}(1 - 0.95)$$

Stage #5:  $T_{95_5} = \sqrt{(-6.5232) \ln\left(1 - \frac{500.44}{1918.78}\right)}$  Residual  $R_5 = (500.44)(0.05)$

$$T_{95_5} = \sqrt{(-6.5232) \ln(0.7392)}$$

$$R_5 = 25.02 \text{ lbs}$$

$$\boxed{T_{95_5} = 1.40 \text{ hrs}}$$

**W<sub>d6</sub>** Water to displace in stage #6:  $w_6(t) = 25.02 + 322.32 = 347.34 \text{ lbs}$

$$T_{95_6} = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{W_6}{W_D}\right)}$$

$$R_i = W_{D_i}(1 - 0.95)$$

Stage #6:  $T_{95_6} = \sqrt{(-6.5232) \ln\left(1 - \frac{347.34}{1918.78}\right)}$  Residual  $R_6 = (347.34)(0.05)$

$$T_{95_6} = \sqrt{(-6.5232) \ln(0.8190)}$$

$$R_6 = 17.37 \text{ lbs}$$

$$\boxed{T_{95_6} = 1.14 \text{ hrs}}$$

Stage	#1	#2	#3	#4	#5	#6
-------	----	----	----	----	----	----

**Total Time (hrs):**  $4.42 + 3.02 + 2.23 + 1.75 + 1.40 + 1.14 = 13.96 \text{ hrs}$

**Total Process of all 6 Stages:**  $\sum_{i=1}^6 T_{95_i} = 13.96 \text{ hrs}$

## CHAPTER 5

### DATA ANALYSIS - II

This calculation process uses Stage #1 mixing time constant **[k]** and arbitrary constant for water displacement **[W<sub>D1</sub>]** for all stages. This assumes the characteristics of subsequent stages are the same as the first. Given the capacity **[B]** is decreased by the cumulative amount of paste formed at the end of each stage, I would expect the mixing time constant **[k]** to also change after the completion of each stage. Since the displaced water is removed upon completion of each stage, the arbitrary constant **[W<sub>D</sub>]** will become the Total Water Displaced **[W<sub>Di</sub>]** for that particular mixing stage **[i]**. If the distribution model is uniform, the expectation is that **[k]** will increase as the process progresses. The most accurate way to calculate the mixing times for the remaining stages is to sample each stage after the paste has disassociated and started to reform. Repeat the same calculation process used in Stage #1. Compute the **[k<sub>i</sub>]** for that mixing stage and use the Probability Error Function to estimate the 95% mixing time.

Since this is all about estimation and modeling, the next methodology Mixer-Time Methodology (B) will increase **[k<sub>1</sub>]** by a factor of **[i]** as the stage count increases. There is no actual sampling data to verify how **[k]** changes with respect to **[i]**, so Mixer-Time Methodologies (A & B) are pure conjecture. However, the purpose of these models are to verify with some degree of confidence that the value **[k]** has a direct correlation to the stage and the Mix-Times. As the stage **[i → n]** approaches the final mixing stage, the incremental 95% Mix-Times **T<sub>95</sub>** appear to decrease in both methodologies (A & B) even though the Total Mix-Times are different.

# CHAPTER 5

## DATA ANALYSIS - II

**Mixer-Time Methodology (B):  $k_i = (k)(i)$  &  $W_D = W_{Di}$**

The times for the remaining mixing stages can be calculated as follows:

$$k_i = (k)(i) = 0.1533(i) \text{ and } W_D = W_{Di} \text{ for each subsequent stage, } 1 \leq i \leq 6$$

Use the Water Distribution [W<sub>i</sub>] from the Table above in EXAMPLE-5a

Substitute [W<sub>i</sub>] for w<sub>i</sub>(t) in Water Displacement Function and solve for [t].

$$w_i(t) = W_D(1 - e^{-kt^2})$$

$$\frac{w_i(t)}{W_D} = 1 - e^{-kt^2}$$

$$e^{-kt^2} = 1 - \frac{w_i(t)}{W_D}$$

$$-kt^2 = \ln\left(1 - \frac{w_i(t)}{W_D}\right)$$

$$t^2 = \left(\frac{1}{-k}\right) \ln\left(1 - \frac{w_i(t)}{W_D}\right)$$

$$t = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{w_i(t)}{W_D}\right)}$$

$$t = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{w_i(t)}{W_D}\right)}$$

$$T_{95i} = \sqrt{\left(\frac{1}{-k}\right) \ln\left(1 - \frac{W_i}{W_D}\right)}$$

Given:  $1 \leq i \leq 6$

Time for

$$R_i = W_{Di}(1 - 0.95)$$

stage #1 to mix to 95%

$$T_{95\%} = 4.42 \text{ hrs}$$

$$\text{Residual } R_1 = (1918.78)(0.05)$$

$$R_1 = 95.94 \text{ lbs}$$

**W<sub>d2</sub>** Water to displace in stage #2:  $w_2(t) = 95.94 + 1348.63 = 1444.57 \text{ lbs}$

The total water to be displaced is the denominator:  $W_D = w_2(t) = 1444.57 \text{ lbs.}$

The numerator [**w(t)**] will always be **95% of W<sub>D</sub>**:  $w(t) = 95\%(W_D)$

So where [i = 2] & [k = 0.1533], the T<sub>95</sub> expression becomes:

$$T_{95i} = \sqrt{\left(\frac{1}{-ik}\right) \ln\left(1 - \frac{w_i(t)}{W_D}\right)}$$

$$T_{952} = \sqrt{\left(\frac{1}{-2k}\right) \ln\left(1 - \frac{(0.95)(W_D)}{W_D}\right)}$$

$$R_i = W_{Di}(1 - 0.95)$$

$$\text{Stage #2: } T_{952} = \sqrt{\left(\frac{1}{-2k}\right) \ln(1 - .95)} \quad \text{Residual } R_2 = (1444.57)(0.05)$$

$$T_{952} = \sqrt{\left(\frac{1}{-2(0.1533)}\right) \ln(0.05)}$$

$$R_2 = 72.22 \text{ lbs}$$

$$T_{952} = \sqrt{9.7708}$$

$$T_{952} = 3.13 \text{ hrs}$$

**W<sub>d3</sub>** Water to displace in stage #3:  $w_3(t) = 72.23 + 949.84 = 1022.07 \text{ lbs}$

# CHAPTER 5

## DATA ANALYSIS - II

$$\begin{array}{lll}
 T_{95_3} = \sqrt{\left(\frac{\ln(0.05)}{-3k}\right)} & & R_i = W_{D_i}(1 - 0.95) \\
 \text{Stage #3: } T_{95_3} = \sqrt{\left(\frac{-2.9957}{-0.4599}\right)} & \text{Residual } R_3 = (1022.07)(0.05) \\
 T_{95_3} = \sqrt{6.5139} & & R_3 = 51.10 \text{ lbs} \\
 \boxed{T_{95_3} = 2.55 \text{ hrs}} & &
 \end{array}$$

$$\begin{array}{lll}
 \mathbf{W_{d4}} \text{ Water to displace in stage #4: } w_4(t) = 51.10 + 666.13 = 717.23 \text{ lbs} & & \\
 \text{Stage #4: } T_{95_4} = \sqrt{\left(\frac{\ln(0.05)}{-4k}\right)} & & R_i = W_{D_i}(1 - 0.95) \\
 T_{95_4} = \sqrt{\left(\frac{-2.9957}{-0.6132}\right)} & \text{Residual } R_4 = (717.23)(0.05) \\
 T_{95_4} = \sqrt{4.8853} & & R_4 = 35.86 \text{ lbs} \\
 \boxed{T_{95_4} = 2.21 \text{ hrs}} & &
 \end{array}$$

$$\begin{array}{lll}
 \mathbf{W_{d5}} \text{ Water to displace in stage #5: } w_5(t) = 35.86 + 464.58 = 500.44 \text{ lbs} & & \\
 \text{Stage #5: } T_{95_5} = \sqrt{\left(\frac{\ln(0.05)}{-5k}\right)} & & R_i = W_{D_i}(1 - 0.95) \\
 T_{95_5} = \sqrt{\left(\frac{-2.9957}{-0.7665}\right)} & \text{Residual } R_5 = (500.44)(0.05) \\
 T_{95_5} = \sqrt{3.9083} & & R_5 = 25.02 \text{ lbs} \\
 \boxed{T_{95_5} = 1.98 \text{ hrs}} & &
 \end{array}$$

$$\begin{array}{lll}
 \mathbf{W_{d6}} \text{ Water to displace in stage #6: } w_6(t) = 25.02 + 322.32 = 347.34 \text{ lbs} & & \\
 \text{Stage #6: } T_{95_6} = \sqrt{\left(\frac{\ln(0.05)}{-5k}\right)} & & R_i = W_{D_i}(1 - 0.95) \\
 T_{95_6} = \sqrt{\left(\frac{-2.9957}{-0.9198}\right)} & \text{Residual } R_6 = (347.34)(0.05) \\
 T_{95_6} = \sqrt{3.2569} & & R_6 = 17.37 \text{ lbs} \\
 \boxed{T_{95_6} = 1.80 \text{ hrs}} & &
 \end{array}$$

Stage	#1	#2	#3	#4	#5	#6
<b>Total Time (hrs):</b>	4.42	+ 3.13	+ 2.55	+ 2.21	+ 1.98	+ 1.80 = 16.09 hrs
<b>Total Process of all 6 Stages:</b>	$\sum_{i=1}^6 T_{95_i}$	= 16.09 hrs				

As in the previous calculation process, Mixer-Time Methodology (B) computes the mixing time constant [k] that is used in Stage #1 and the subsequent stages. This

## CHAPTER 5

### DATA ANALYSIS - II

procedure assumes the mixing time constant [k] used in subsequent stages are related to the [k] of the first by product of stage counter [i] and [k]. Given the expression,

$1 - \frac{w(t)}{W_{Di}}$ , the ratio,  $\frac{w(t)}{W_{Di}}$  will always be equal to .95. Thus  $\ln\left(1 - \frac{w(t)}{W_D}\right)$  will also remain

constant [-2.99573]. This simplifies the calculation to  $T_{95_i} = \sqrt{\left(\frac{1}{-ik}\right)\ln(0.05)}$  where [.05]

represents [1 - %Completion], which is held constant throughout the entire calculation procedure. For this procedure, the Residual Water is required to compute the Mix-Time, because it is assumed that only 5% of the total water to be displace will be remaining after time  $T_{95_i}$ .

As stated previously, the Mixer-Time Methodologies (A & B) are conjecture, but the mix-Time Estimates for the stages imply a direct relationship of [ $k_i$ ] with respect to the stage counter [i]. All that can be concluded is that each stage could have a different [k]. As an experienced ink technician, I tend to favor the later model (B), which directly related [k<sub>i</sub>] with net capacity, viscosity, pigment particle concentration and mechanical shearing. Given uniform distributions, [k<sub>i</sub>] appears to increase as the stage counter increases. Mixer -Time Methodologies (A) & (B) both generally correlate to observations of flush work orders and procedures with regard to pigment distributions, mix-times and relative viscosity trends.

# CHAPTER 5

## DATA ANALYSIS - II

### Process Rates & Analysis

Rate Analysis is all about the slopes of the functions developed in the previous sections. The first derivative of the (Water Remaining) model generates the function which describes how the dependent variable is changing with respect to the independent variable. On a 2-dimensional [x, y] graph, the 1st derivative is expressed as  $\frac{dy}{dx}$ . If the 2-dimensions are [t, w(t)], the 1st derivative is  $\frac{dw(t)}{dt} = w'(t)$ .

Compare the graphs of the Water Functions and 1st derivatives below.

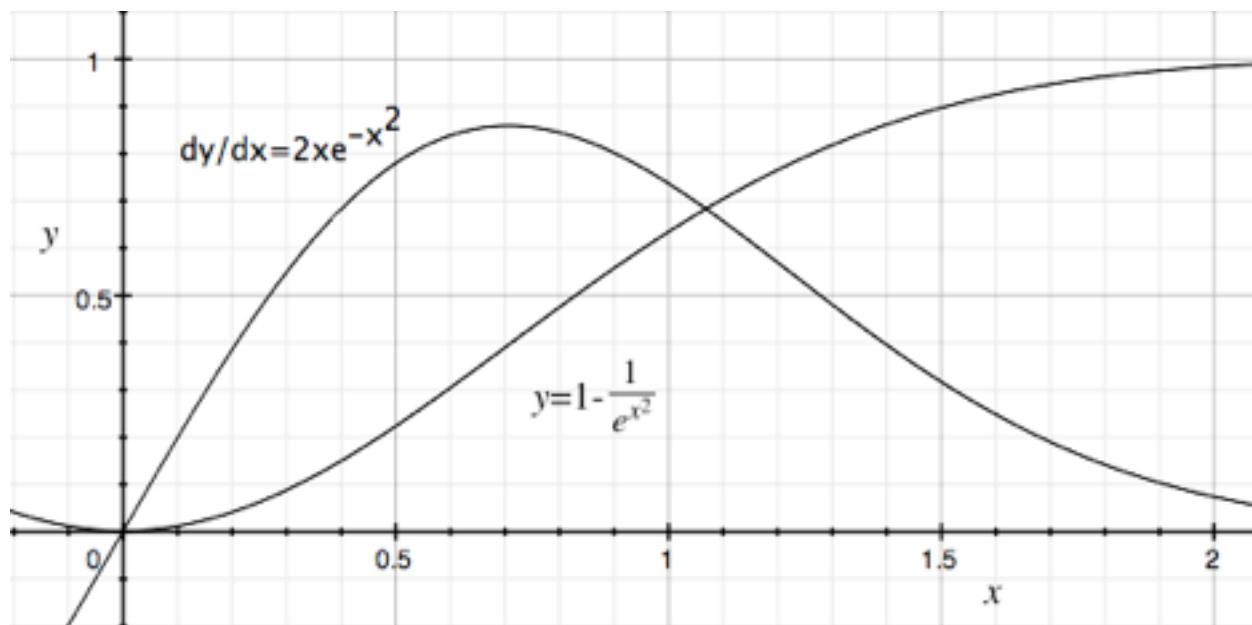
The Y-Axis represents the proportion of water and the X-Axis is the time axis

Water Displacement Function:      k=1

$$\begin{aligned}w(t) &= W_D(1 - e^{-kt^2}) \\w(t) &= W_D - W_D e^{-kt^2} \\y &= 1 - \frac{1}{e^{kt^2}} \\y &= 1 - e^{-kt^2}\end{aligned}$$

Displacement Rate Function:

$$\begin{aligned}w'(t) &= \frac{dw(t)}{dt} = 2W_D k t e^{-kt^2} \\y &= 2kxe^{-kt^2}\end{aligned}$$



# CHAPTER 5

## DATA ANALYSIS - II

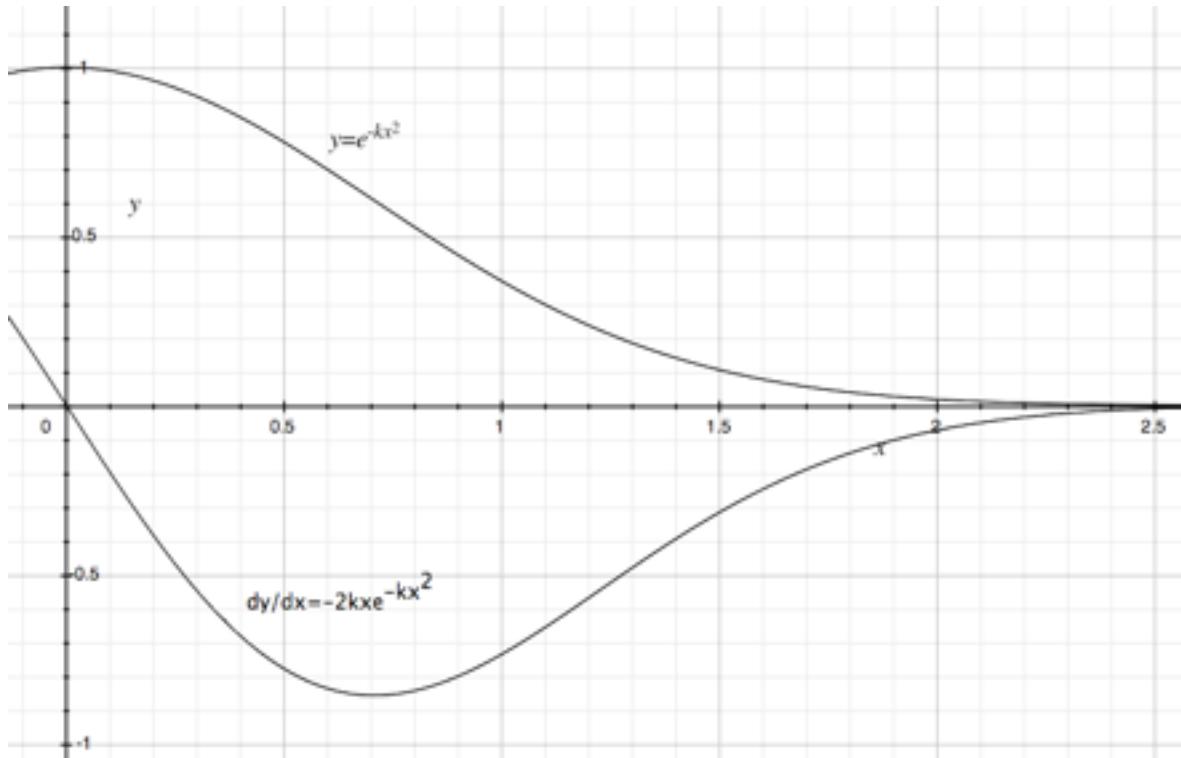
Water Remaining Function:

$$k=1$$

Water Remaining Rate Function:

$$\boxed{w(t) = W_D e^{-kt^2}} \\ y = e^{-kx^2}$$

$$\boxed{w'(t) = -2W_D kte^{-kt^2}} \\ y = -2kxe^{-kx^2}$$



**Referring to EXAMPLE 5b**

**Find the Reaction Rates after  $t = 1.75$  hrs of mixing (sampling in Stage #1)**

Stage #1	Calculated	$k = 0.1533$
		$W_D = 1918.78 \text{ lbs}$

**Given the Water Displacement Function:**

$$\boxed{\begin{aligned} w(t) &= W_D(1 - e^{-kt^2}) \\ w(t) &= W_D - W_D e^{-kt^2} \\ w(t) &= 1918.76 - 1918.76e^{-0.1533t^2} \end{aligned}}$$

# CHAPTER 5

## DATA ANALYSIS - II

**Given the Water Displacement Rate Function:**

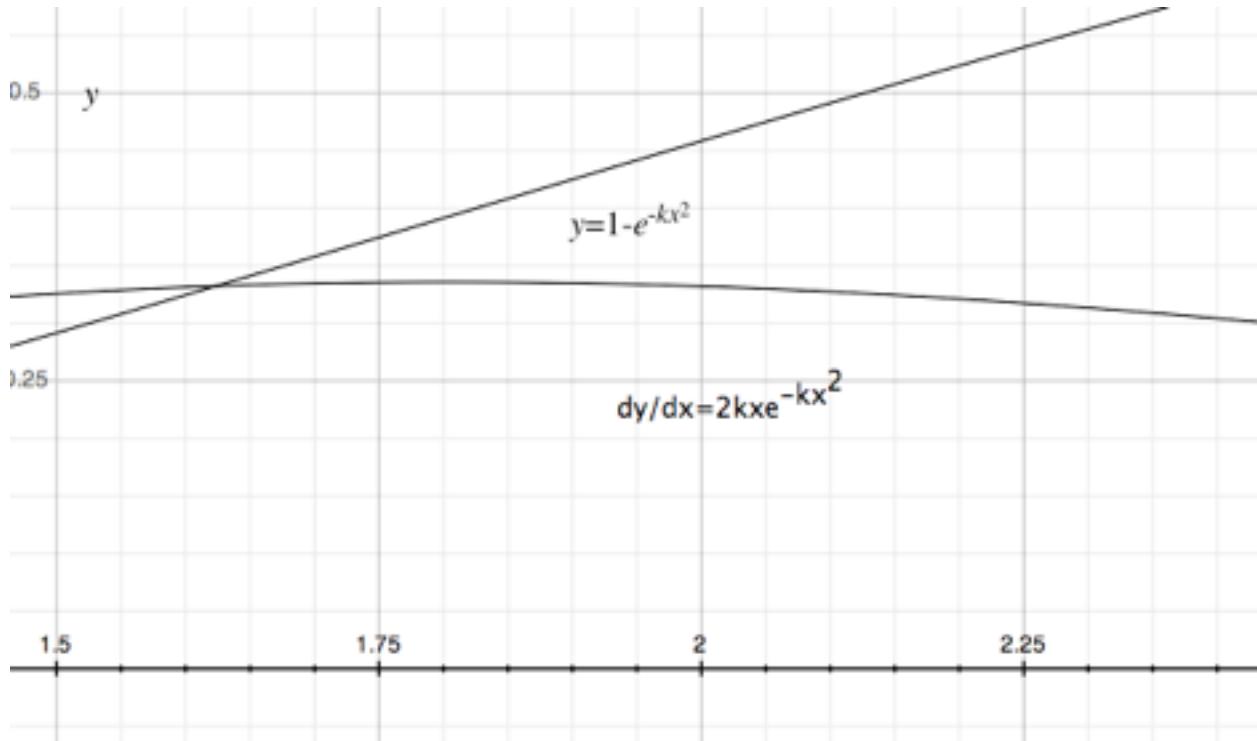
$$\frac{dw(t)}{dt} = w'(t) = 2W_D k t e^{-kt^2}$$

$$w'(t) = 3837.52(0.1533)t e^{-0.1533t^2}$$

$$w'(t) = 588.29t e^{-0.1533t^2}$$

**Reaction Rate:**  $w'(t) = 588.29t e^{-0.1533t^2}$  @  $t = 1.75 \text{ hrs}$

**Reaction Rate:**  $w'(1.75) = 643.78 \text{ lbs/hr} = 10.73 \text{ lbs/min}$



The maximum rate of displacement is can be calculated by setting the 2nd derivative of the rate function equal to zero and solving for [t]. In other words, finding where the rate function has a slope equal to zero.

Given the rate function:

$$w'(t) = 2W_D k t e^{-kt^2}$$

2nd Derivative:

$$w''(t) \frac{d^2w(t)}{dt^2} = \frac{d}{dt}(2W_D k t e^{-kt^2})$$

$$\frac{d}{dt} uv = u \frac{dv}{dt} + v \frac{du}{dt}$$

$$u = 2W_D k t$$

$$\frac{du}{dt} = 2W_D k$$

$$v = e^{-kt^2}$$

$$\frac{dv}{dt} = -2k t e^{-kt^2}$$

# CHAPTER 5

## DATA ANALYSIS - II

$$w''(t) = \frac{d^2w(t)}{dt^2} = u \frac{dv}{dt} + v \frac{du}{dt}$$

$$\begin{aligned} w''(t) &= 2W_Dkt \frac{dv}{dt} + e^{-kt^2} \frac{du}{dt} \\ w''(t) &= 2W_Dkt(-2kte^{-kt^2}) + e^{-kt^2}(2W_Dk) \\ w''(t) &= -4W_Dk^2t^2e^{-kt^2} + 2W_Dke^{-kt^2} \\ w''(t) &= 2W_Dke^{-kt^2}(-2kt^2 + 1) \\ w''(t) &= 2W_Dke^{-kt^2}(1 - 2kt^2) \end{aligned}$$

Maximum Rate:  $w''(t) = 0$

$$0 = 2W_Dke^{-kt^2}(1 - 2kt^2)$$

Root #1:

$$\begin{cases} 0 = 2W_Dke^{-kt^2} \\ 0 = \frac{1}{e^{kt^2}} \\ t \rightarrow \infty \end{cases}$$

Root #2:

$$\begin{cases} 0 = 1 - 2kt^2 \\ 2kt^2 = 1 \\ t^2 = \frac{1}{2k} \\ t = \pm\sqrt{\frac{1}{2k}} \\ t = \pm\frac{1}{\sqrt{2k}} \end{cases}$$

Given:  $\begin{cases} k = 0.1533 \\ W_D = 1918.78 \text{ lbs} \end{cases}$

As [t] approaches infinity, the rate slope nears zero.

Root #2:  $t = \pm\frac{1}{\sqrt{2k}}$   $t = 1.806 \text{ hrs}$  (Time of Max Rate)

At  $t = 1.806 \text{ hrs}$  (Time of Max Rate): Calculate Max Rate:  $w'(t) = 2W_Dkte^{-kt^2}$

$$\begin{aligned} w'(t) &= 2(1918.76)(0.1533)(1.806)e^{-(0.1533)(1.806)^2} \\ w'(t) &= 1062.45e^{-0.5} \end{aligned}$$

$$w'(t) = \frac{dw(t)}{dt} = 644.41 \text{ lbs/hr}$$

The maximum rate [644.41 lbs/hr] is achieved at [t = 1.806], shortly after sampling in the 1st Stage at [t = 1.75 hrs] when the rate of displacement was [643.78 lbs/hr]. At [T<sub>95</sub> = 4.42 hrs] the displacement rate has decreased to [130.12 lbs/hr] or [2.17 lbs/min].

## CHAPTER 5

### DATA ANALYSIS - II

Pigment Wetting Function is:  $P(t) = \frac{r}{1-r}w(t)$

Pigment Wetting Rate is:

$$\frac{dP(t)}{dt} = \frac{r}{1-r} \left[ \frac{dw(t)}{dt} \right]$$

$$\frac{dP(t)}{dt} = \frac{r}{1-r} [2W_D kte^{-kt^2}]$$

Therefore, the pigment wetting expression is the same as the water displacement expression except for the constant factor  $\frac{r}{1-r}$  which happens to be the same ratio as the initial vehicle to presscake ratio at ( $t = 0$ ).

$$\frac{\sum V}{(\sum P + W_D)} = \frac{r}{1-r} = .25 = \frac{600}{480 + 1918.76} \mid r = .2$$

Given: Stage #1       $[k = 0.1533]$

$[W_D = 1918.76]$        $[t = 1.806]$

$$w(t) = W_D(1 - e^{-kt^2})$$

$$P(t) = \frac{r}{1-r}w(t)$$

$$\frac{dP(t)}{dt} = \frac{r}{1-r} \left[ \frac{dw(t)}{dt} \right]$$

$$\frac{dP(t)}{dt} = \frac{r}{1-r} [2W_D kte^{-kt^2}]$$

$$\frac{dP(t)}{dt} = \frac{r}{1-r} [588.29te^{-kt^2}]$$

$$\frac{dP(t)}{dt} = \frac{r}{1-r} [588.29te^{-kt^2}]$$

$$\frac{dP(1.806)}{dt} = 0.25[1062.45e^{-0.5}]$$

$$\frac{dP(1.806)}{dt} = 265.61e^{-0.5}$$

$$\frac{dP(1.806)}{dt} = 161.10 \text{ lbs/hr}$$

$$\frac{dP(1.806)}{dt} = 2.69 \text{ lbs/min}$$

In summary, the water displacement functions discussed in this project fall into two general categories: Water Remaining and Water Displacement. The Water Remaining is a decreasing function while the water displaced is an increasing function. The pigment wetting references the Water Displacement function. The rates of the water functions are first derivatives of Water Remaining and Water Displacement functions.

## **CHAPTER 5**

### **DATA ANALYSIS - II**

#### **Temperature, Viscosity and Mixing Mechanics**

Temperature effects the mixing in several ways. Elevated temperatures reduce viscosity as described in chapter 2. The constant mechanical rotation of the sigma blades in the mixer and the kinetic shearing action also elevates the temperature as a result of friction and particle bombardment. Part of this effect can be explained with the Brownian motion model. In addition to the shearing action of the sigma blades in the mixing bowl, the clearance between the blades and the mixing bowl surface generate friction. This action is also directly proportional to the shear and contributes to the elevation of temperature in the mix.

Observations tend to imply that the higher the temperature of the mix, the lower the viscosity. However, the lower the viscosity of the mix, the slower the rate of reaction and water displacement. The dynamics are made even more complex by taking into account the effect of evaporation of solvents. Evaporation tends to increase the temperature of the mix, thus causing an increase in viscosity as solvent leaves the system. Higher temperatures tend to reduce the mechanical energy requirements that are needed to mix the pastes and achieve even distribution. This effect will minimize aggregation.

The thermal and mechanical effects of displaced water have positive and negative impacts on the mixing process. The more water displaced, the greater the effects of de-agglomeration of the pigment particles. Displaced water also acts as a coolant or temperature moderator to keep viscosity change stable. However, the displaced water also requires mechanical energy which takes away from the applied energy of the constant rotating sigma blades. As the mixing progresses through the process, there is

## **CHAPTER 5**

### **DATA ANALYSIS - II**

less water remaining in each stage. And as you may expect, the mixing paste temperature increases through the mixing stages as less water is displaced.

The judgement and skill set of the mixer operator is an important factor in the mixing process. Much like a chef in a restaurant who prepares a meal from a recipe, the mixer operator uses the work order as guide to flush a batch of organic pigment. There will be times when he must add or extract heat to the mixer manually to accelerate, moderate or stabilize the process. The condition of the raw materials (presscake, varnish or solvent) may require in-process adjustments to the procedure. Environmental or machine conditions may necessitate procedure modifications during the mixing process. These dynamics, although very important to the successful completion of a quality product, will not be emulated or modeled in this thesis.

These are just a few of the many parameters which effect the mixing dynamics of pastes with respect to viscosity. The intent of this treatment is to find parameters which can be used as the independent variables to be the input into an algorithm that will serve as a common thread to estimate the behavior of the product paste. My choices are narrowed to time and energy. Unfortunately, I was not able to collect data on the power and energy used by the mixers during the process. However I was able to acquire some mix time and temperature data for some processes. The rest of this treatment will develop models to estimate the dependent variables (viscosity and temperature) using time as the independent base variable. It is also my intent to explore the relationship between viscosity and energy as applied to the flushing process,

#### **Time and Temperature Relationship**

Chapter 2 details the inverse relationship between temperature and viscosity. As temperature increases, viscosity decreases. The viscosity at the end of the mixing

## CHAPTER 5

### DATA ANALYSIS - II

stages compared to time appears to be a direct relationship. As the mixing time progresses, the temperature of the mass paste increases. Because of the friction and particle bombardment that is generated from the mixing action, heat is generated within the mass of the stiffening paste. The increase in temperature slows the rate of viscosity build up as the water is displaced and the paste is formed. However, the viscosity build up from the particle formation (mechanical flushing) is much grater than the lowering of viscosity due to increasing temperature. The injection of heat into the mixing also increases the flow thus easing the task of particle distribution and minimizing aggregation of the paste.

#### **EXAMPLE 5a (Report MODEL-A1) With Temperature Data**

i	Viscosity	Mix	Pigment	Water	Vehicle	TEMP	TEMP	$\Delta t$ Mix	Cum Mix
Stage	Distribution	% Pigment	Charge	Displaced	Charge	°F	°C	Hrs	Hrs
1	3160	0.4436	479.69	1918.76	601.55	65 - 75	18.3 - 23.9	4.42	4.42
2	4699	0.4946	337.16	1348.63	232.97	75 - 83	23.9 - 28.3	3.02	7.44
3	5474	0.5143	237.46	949.84	161.33	83 - 87	28.3 - 30.6	2.23	9.67
4	5864	0.5231	166.53	666.13	117.18	87 - 95	30.6 - 35.0	1.75	11.42
5	6060	0.5273	116.15	464.58	85.4	95 - 105	35.0 - 40.6	1.40	12.82
6	6159	0.5294	80.58	322.32	61.69	105 - 110	40.6 - 43.3	1.14	13.96

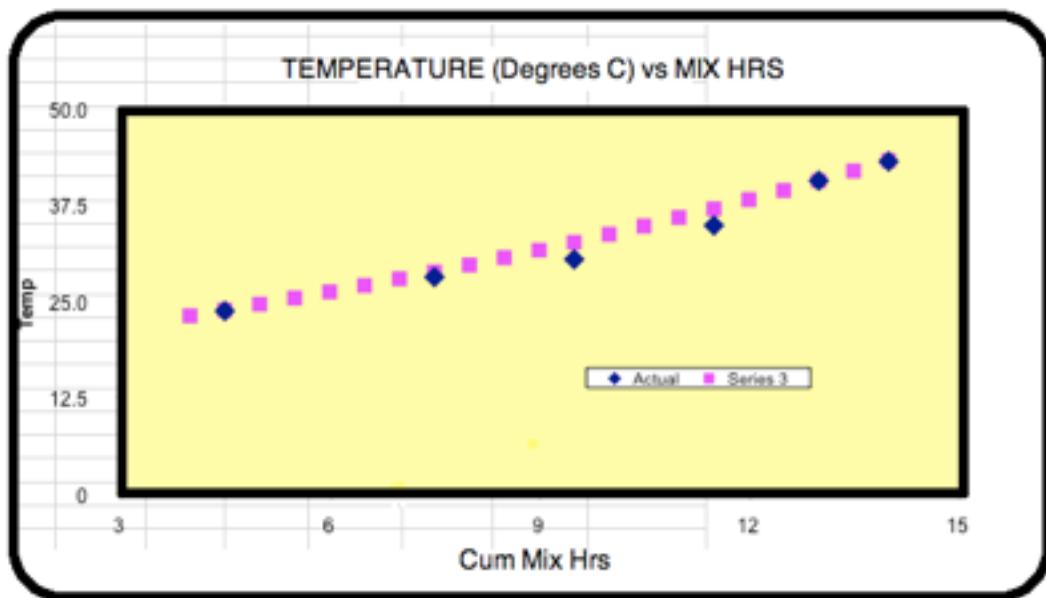
The chart shown above displays temperature data related to the respective mixing stages. There is a positive correlation between temperature, viscosity, % pigmrnt and cumulative hours of mixing. In other words, as the mixing progresses through its stages, the product paste is thickening and getting warmer. The rising temperature is an indicator of the increase in kinetic energy that the paste is undergoing while mixing.

Even though both viscosity and temperature are rising in the same direction during the flushing process, there is a point where the viscosity could peak and start to drop while the temperature could continue to increase. These temperatures are very high

## CHAPTER 5

### DATA ANALYSIS - II

and past the solvent boiling point when this behavior occurs. The batch would encounter severe challenges at these high temperatures (greater than 200° C). Problems with solvent evaporation and pigment heat capacity would also be encountered. At very high temperatures, synthetic resins begin to harden and become discolored. The process could literally cook and transform into a hard plastic amorphous state and lock up the sigma blades of the mixer. However, flushing is usually done in environments less than 55° C (130° F).



To formulate the estimate function  $T(t)$  from the table in EXAMPLE 5a: 
$$T = T_o e^{kt}$$

$$43.3 = T_o e^{k(13.96)}$$

$$23.9 = T_o e^{k(4.42)}$$

$$T_o = 23.9 e^{-4.42k}$$

Given

$$T_o = 23.9 e^{-4.42k}$$

$$T_o = 23.9 e^{-4.42(0.0623)}$$

$$\begin{aligned} t &= 4.42 \text{ hrs} & T &= 23.9 {}^{\circ}\text{C} \\ t &= 13.96 \text{ hrs} & T &= 43.3 {}^{\circ}\text{C} \end{aligned}$$

$$43.3 = 23.9 e^{-4.42k} e^{k(13.96)}$$

$$T_o = 18.15$$

$$1.812 = e^{9.54k}$$

$$T(t) = 18.15 e^{0.0623t}$$

$$\ln(1.812) = 9.54k$$

$$Est = T(t)$$

$$k = 0.0623$$

## CHAPTER 5 DATA ANALYSIS - II

This estimation is an empirical model that predicts the temperature of the paste mass through the flush process by way of hours of mix time. Refer to the following table.

Cum Mix Hrs	ACTUAL °C	Est T(t)
4.42	23.9	23.9
7.44	28.3	28.9
9.67	30.6	33.2
11.42	35.0	37.0
12.82	40.6	40.3
13.96	43.3	43.3

### Time and % Pigment Relationship [x<sub>p</sub>(t)] and % Vehicle [x<sub>v</sub>(t)]

The % pigment charge in any given stage (i) is the ratio of the cumulative pigment charge to the cumulative sum of pigment and vehicle. Chapter 4 details the Formula #11 - Appendix (A):

$$x_{P_i} = \frac{\sum_{j=1}^i P_j}{\sum_{j=1}^i (P_j + V_j)} = \frac{\sum_{j=1}^{i-1} P_j + P_i}{\sum_{j=1}^i (P_j + V_j)}$$

Given P<sub>i</sub> is the pigment charge at stage (i).

Since P<sub>i</sub> can be expressed as a function of time; P<sub>i</sub>(t) in formula #31:

$$P_i(t) = \frac{rW_D(1 - e^{-kt^2})}{(1 - r)}$$

$$x_{P_i}(t) = \frac{\sum_{j=1}^{i-1} P_j + P_i(t)}{\sum_{j=1}^i (P_j + V_j)} = \frac{\sum_{j=1}^{i-1} P_j + \frac{rW_D(1 - e^{-kt^2})}{(1 - r)}}{\sum_{j=1}^i (P_j + V_j)}$$

Pigment Wetting Rate is:

$$\frac{dP(t)}{dt} = \frac{r}{1-r} \left[ \frac{dw(t)}{dt} \right]$$

$$\frac{dP(t)}{dt} = \frac{r}{1-r} [2W_D kte^{-kt^2}]$$

# CHAPTER 5

## DATA ANALYSIS - II

Therefore the expressions for % Vehicle [x<sub>v</sub>(t)] are as follows:

$$x_{V_i}(t) = 1 - x_{P_i}(t)$$

$$x_{V_i}(t) = 1 - \frac{\sum_{j=1}^{i-1} P_j + \frac{rW_D(1 - e^{-kt^2})}{(1-r)}}{\sum_{j=1}^i (P_j + V_j)}$$

Given:  $T_{95_{i-1}} < t_i < T_{95_i}$

$$T_{95_i} = \sqrt{\left(\frac{1}{k}\right) \ln\left(1 - \frac{W_i}{W_D}\right)}$$

### Time and Relative Viscosity [ $\eta(t)$ ]

The Relative Viscosity is changing with respect to time (Mix Hours) as the pigment charge [P<sub>i</sub>] is being wetted by the vehicle charge [V<sub>i</sub>] in stage (i). In the previous section, the % pigment and time relationship [x<sub>p</sub>(t)] was developed. This section illustrates the relationship between the relative viscosity and time using much of the same logic.

Refer to Appendix (A) Formula #16 for Relative Viscosity:

$$\eta_i = \eta_p e^{k_v x_v}$$

$$\eta_i(t) = \eta_p e^{k_v x_v(t)}$$

$$\eta_i(t) = \eta_p e^{k_v [1 - x_p(t)]}$$

$$\eta_i(t) = \eta_p e^{k_v \left[ 1 - \frac{\sum_{j=1}^{i-1} P_j + \frac{rW_D(1 - e^{-kt^2})}{(1-r)}}{\sum_{j=1}^i (P_j + V_j)} \right]}$$

Given:

$$x_{V_i}(t) = 1 - x_{P_i}(t)$$

$$x_{V_i}(t) = 1 - \frac{\sum_{j=1}^{i-1} P_j + \frac{rW_D(1 - e^{-kt^2})}{(1-r)}}{\sum_{j=1}^i (P_j + V_j)}$$

### Summary and Conclusions

The objective of **ANALYSIS-II** was to integrate some of the models with concepts of mixing time, temperature changes and substitutions of vehicles. It also introduced methodologies of blending resins of vehicles of various viscosities and the effect on mixing rates, stage charge water and viscosity distributions. This completes the second phase of the project (**ANALYSIS-II**).

The objective of the next phase (**ANALYSIS-III**) is to show how methodologies and software tools are developed for more detailed analysis. As each phase of the project is completed, I will include corrections, enhancements and clarifications to previous chapters of this life-long project.

# **CHAPTER - 6**

## **DATA ANALYSIS - III**

### **(SOFTWARE DEVELOPMENT)**

#### **CONTENT**

**QBASIC & MS-EXCEL (Development Tools)**

**MODEL FLOWCHART (General Outline)**

**MODEL INPUT & SYSTEM CONSTANTS**

**MODEL FLUSH DISTRIBUTION**

**MODEL MIX-TIME & DETAIL RATES**

**MODEL RAW MATERIAL SUBSTITUTION**

**MODEL WORKSHEET DESCRIPTION**

# CHAPTER 6

## SOFTWARE DEVELOPMENT

### QBASIC and MS-EXCEL (Development Tools)

In the early stages of this project, QBASIC Programs were developed to assist with the analysis. As computer technology improved over the years, the project emulation models also improved. Software such as MathCad and MS-Excel became available and were used to enhance the math models. The following sections will show how QBASIC, MS-Excel Worksheets and VBA Macros were used in this development process.

QBASIC was the first programming tool used for development of the early models in this project. Because of its easy-to-learn script rules, QBASIC contained all of the mathematical functions that were being used in the model development. The logic flow was also very straight forward looked very much like pseudocode and was easy to follow. QBASIC is also a very result oriented programming language.

The first models written in earlier versions of BASIC (BASIC-A and GW-BASIC) are omitted from this treatment because the author felt they would only be of historical significance to the project. The later QBASIC version is included for several reasons: First, because of its English-like key words and commands, the script code is very transparent and easy to follow. Another feature of QBASIC is the Text Copy-Paste compatibility which allows it to be easily transferred to other modern systems.

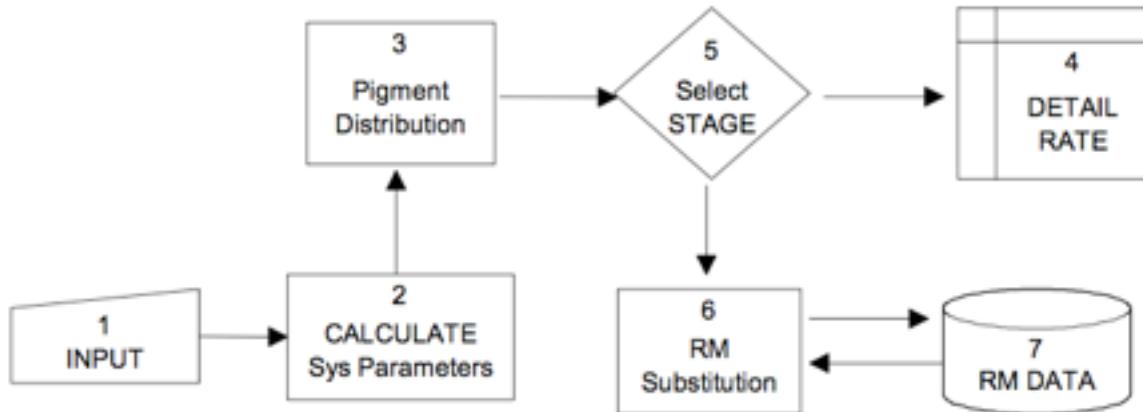
The Row-Column format of the programmed reports led to a natural progression to Excel Worksheets. The built-in mathematical functions and charting capabilities provide an excellent programming environment for creating the mixing models and reports for analysis. MS-Excel also has a feature that enhances the development environment and has built-in VBA (Visual Basic Application) Macro capabilities. The compatibility of MS-Excel and QBASIC allows many QBASIC scripts to be copied directly from its environment into the macro with minimum modification of the code.

The path of this project's evolution was from observations of the manufacturing process (Flush Dispersion) to mathematical model development using BASIC programs, MathCad and MS-Excel models. The progression was anything but smooth and organized. There were plenty of setbacks, restarts, back and forth "do-overs," periods of frustration, and those classic 2:00 AM epiphanies ("Ah Ha" moments).

# CHAPTER 6

## SOFTWARE DEVELOPMENT

The following section is an attempt to illustrate the transition of each model from math formulation through QBASIC Programming into MS-EXCEL Macro VBA.



**MANUAL MODEL FLOWCHART  
(ALL MODELS)**

### **#1 INPUT**

The initial batch parameters are entered: Total Pigment and Vehicle Charge; Relative Viscosities for Pigment and Vehicle; % Solids of Presscake; Mixer Bulk Capacity; Process Stage # for Detail, Sample Process Time; Sample Water Content %; Mix Estimate to Completion; Stage # for Raw Material Substitution; RM Item Codes.

### **#2 Calculate System Parameters**

Calculate the system constants and parameters: Number of Stages; % Mix of Pigment and Vehicle; System Viscosity Constant; Relative Viscosity of the End Mix; The Water Distribution Constants; Mix-Time Constants; Total Mix-Times per Method (A & B)

### **#3 Pigment Distribution**

Output of Distributions, Charts, Tables and Graphs: Vehicle, Pigment, Water & Mix-Time Table of Distribution by Stage (ROW) and Data (COLUMN); CHART/GRAFH of Relative Viscosity vs Mix Stage #

# **CHAPTER 6**

## **SOFTWARE DEVELOPMENT**

### **#4 Detail Rates**

Output Rates of Water and Pigment Distributions, Charts, Tables and Graphs: Mix-Time Estimates; Table of Water Displacement Distribution by elapsed mix-time; Table of Water Displacement Rate by elapsed mix-time; Pigment Wetting and Paste Formation Rates by elapsed mix-time; CHART/GRAF of Charge Units (Lbs) vs Mix-Time (Hrs).

### **#5 Select Stage**

Select Stage [i] for Detail Rates and Raw Material Substitution. Because there are two separate routines (Detail Rates) and (Raw Material Substitution), each has its separate INPUT CELL in the spreadsheet so the functions can remain independent of each other.

### **#6 RM Substitution**

Select Raw Material to substitute in the Viscosity Distribution: A separate worksheet contains a 3-Column list of Raw Material (Item Codes), Item Descriptions and their Related Viscosities. This worksheet (DATA\_01) functions as a database which is sorted by Item Code and accessed via the INPUT (Item Code) in the PGM-Model Worksheets.

### **#7 RM Data (DATA-01)**

Separate worksheet containing the referenced list of RM Item Codes and related values.

### **Notes on EXCEL**

The Excel function (VLOOKUP) allows the user to Input an Item Code which will match a cell in the sorted Item Code column of the DATA\_01. The function is designed to let the programmer define the array of the target table being searched (DATA-01), which must be at least 2 columns. If a match is found in the search column at a given row, the function allows the programmer to define which column to retrieve data from. EXCEL also allows for optional CONDITIONING, i.e. (VLOOKUP) function can be conditioned to retrieve a default value when there is no match found. There is also flexibility built into the (VLOOKUP) which defines the search as “EXACT” or “APPROXIMATE.”

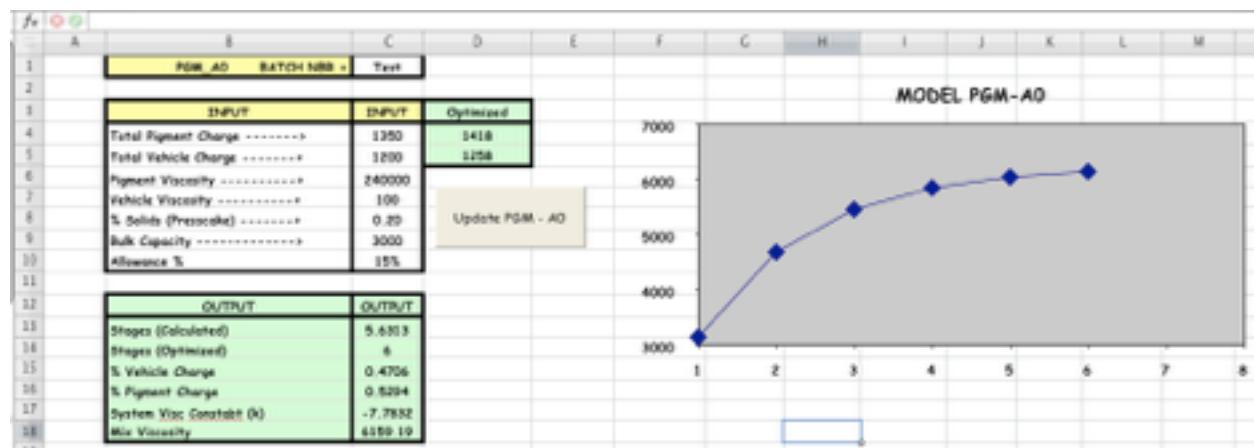
# CHAPTER 6

## SOFTWARE DEVELOPMENT

### EXCEL MODEL: PGM-A0 (QBASIC MODEL MDL-0)

The first EXCEL Model in the series of Treatment-I is (PGM-A0). The referenced QBASIC Program is (MDL\_A0.BAS). The objective of this model is to emulate the procedure outlined in Chapter 4; Data Analysis-I (Page 64). The previous chapters show development of mathematical formulae and expressions that are used to emulate the process. This chapter will reference the formulated procedure and show how it is transformed into an EXCEL Worksheet application.

#### Part #1: Input & System Constants:



$$\begin{aligned} \text{Total Pigment Charge } e \sum_{i=1}^n P_i &= 1350 \\ \text{Total Vehicle Charge } e \sum_{i=1}^n V_i &= 1200 \\ \text{Pigment Viscosity } \eta_p &= 240000 \end{aligned}$$

$$\begin{aligned} \text{Vehicle Viscosity } \eta_v &= 100 \\ \% \text{ Solids } r &= 20\% (0.20) \\ \text{Bulk Capacity } B &= 3000 \end{aligned}$$

$$\begin{aligned} x_p &= \frac{\sum_{i=1}^n P_i}{\sum_{i=1}^n (P_i + V_i)} = 0.5294 \\ x_v &= (1 - x_p) = 0.4706 \end{aligned}$$

$$x_{p_n} = x_p$$

$$n = \frac{(1/r) \left( \sum_{i=1}^n (P_i + V_i) \right)}{x_{p_n}} = 5.6313$$

$$1 \leq i \leq n$$

$$\begin{aligned} k_v &= \ln(\eta_v / \eta_p) = -7.7832 \\ \eta_n &= \eta_p e^{k_v x_v} = 6159.19 \end{aligned}$$

$x_p$  = % Pigment

$x_{pn}$  = % Pigment in Mix

$k_v$  = System Viscosity Constant

$x_v$  = % Vehicle

$n$  = Nbr of mix stages

$\eta_n$  = Mix Viscosity

# CHAPTER 6

## SOFTWARE DEVELOPMENT

**VBA CODE : Worksheet Code executed by Button “Update PGM-A0”**

**Note: REM is remarks and comments within the VBA code**

### Clear previous data from cells

EXCEL\_MACROS: MDL-A0

```
Sub Mixer_PGM_A0()
    Worksheets("PGM_A0").Activate
    Rem CLEAR CONTENTS
    Worksheets("PGM_A0").Range("C13:C18").ClearContents
    Worksheets("PGM_A0").Range("C21:S35").ClearContents
    Worksheets("PGM_A0").Range("A28:A35").ClearContents
```

### Assign Input to VBA Variables

Rem Input Parameters

```
Rem =====
nb$ = Cells(1, "C"): Rem INPUT "Batch Nbr (NB$) ..... NB$ ="; nb$
P = Cells(4, "C"): Rem INPUT "Total Pigment Charge (P) ..... P ="; P
V = Cells(5, "C"): Rem INPUT "Total Vehicle Charge (V) ..... V ="; V
np = Cells(6, "C"): Rem INPUT "Pigment Viscosity (np) ..... np ="; no
nv = Cells(7, "C"): Rem INPUT "Vehicle Viscosity (nv) ..... nv ="; nv
r = Cells(8, "C"): Rem INPUT "% Solids of Pigment (r) ..... r ="; r
B = Cells(9, "C"): Rem INPUT "Mixer Capacity (B) ..... B ="; B
Allowance_pct = Cells(10, "C")
tp = Cells(23, "A"): Rem INPUT "Process Time ..... tp
Cells(29, "A") = 1: b2 = 1: Rem INPUT "2nd Water Distribution Constant .. b2=1
wp = Cells(24, "A"): Rem INPUT "Water Content % ..... wp
Mix_pct = Cells(27, "A"): Rem INPUT "Mix to % Completion .... Mix_pct
```

### Calculate VBA System Constants and OUTPUT: Area C13: C18

Rem Calculate Constants

```
Rem =====
kv = Log(nv / np): Cells(17, "C") = kv
xv = V / (P + V): xp = (1 - xv): Cells(15, "C") = xv: Cells(16, "C") = xp
nmix = np * Exp(kv * xv): Cells(18, "C") = mix
n = Int((P / r + V) / (xv * B) + 0.5): Cells(14, "C") = n
n0 = (P / r + V) / (xv * B): Cells(13, "C") = n0
a = Log(nv / nmix)
```

# CHAPTER 6

## SOFTWARE DEVELOPMENT

### Flush Distribution Output

	C	D	E	F	G	H	I	J	K	L	M	N
20	Stage (i)	Viscosity	% Paint	% Veh	Pigment	Vehicle	Wtr Disp	Cum Paint	Cum Veh	P(i)=V(i)	Cum Chrg	V/P Ratio
21	1	3159.85	0.4437	0.5563	480	600	1920	480	600	1080	1080	1.25
22	2	4699.58	0.4947	0.5053	337	235	1348	817	835	572	1652	1.02
23	3	5474.38	0.5143	0.4857	237	163	948	1054	998	400	2052	0.95
24	4	5864.27	0.5231	0.4769	167	113	668	1221	1111	280	2332	0.91
25	5	6060.45	0.5273	0.4727	116	88	464	1337	1199	204	2536	0.90
26	6	6159.19	0.5294	0.4706	81	59	324	1418	1258	140	2676	0.89
27												

The Viscosity Distribution:  $\eta_j = \eta_n \left( 1 - e^{-k_{vn} \left( \frac{j}{n} \right)} \right) + \eta_v$

Calculate the % Vehicle Distribution ( $x_{v_i}$ ) and % Pigment Distribution ( $x_{p_i}$ )

$$\ln \left( \frac{\eta_i}{\eta_p} \right)$$

Using Formula #26  $x_{v_i} = \frac{\ln \left( \frac{\eta_i}{\eta_p} \right)}{k_v}$  % Vehicle Distribution given ( $1 \leq i \leq n$ )

$$\text{Given: } x_v + x_p = 1 \quad x_p = 1 - x_v$$

Calculate the Pigment Distribution ( $P_i$ )

$$P_i = \frac{Bx_{p_i} - \sum_{i=1}^{i-1} P_i}{\frac{1}{r} + x_{v_i} \left( 1 - \frac{1}{r} \right)}$$

Using Formula #19 Pigment Distribution for  $1 \leq i \leq n$

Calculate Vehicle Distribution ( $V_i$ )

$$\text{Using Formula #20 } V_i = B - \sum_{i=1}^{i-1} (P_i + V_i) - \frac{P_i}{r} \quad \text{Vehicle distribution } 1 \leq i \leq n$$

Calculate the Water Displacement Distribution ( $W_i$ )  $1 \leq i \leq n$

$$\text{Using Formula #6 } W_i = \frac{P_i}{r} - P_i \quad \text{Water displacement distribution}$$

$$\% \text{Water displacement: } \% W_i = \frac{R_{i-1} + W_i}{R_{i-1} + W_i + \sum_{i=1}^n (P_i + V_i)}$$

# CHAPTER 6

## SOFTWARE DEVELOPMENT

### Calculation of Flush Distribution (VBA ): Area A22:A27

```
Rem Calculate Viscosity Distribution n(j)
Rem =====
For j = 1 To n
    Rem   n(j) = nmix * (1 - Exp(a * j / n)) + nv
    Cells(20 + j, "C") = j: Rem Stage (j)
    Cells(20 + j, "D") = nmix * (1 - Exp(a * j / n)) + nv: Rem Viscosity=n(j)

    Rem   xv(j) = INT(((LOG(n(j) / np)) / kv) * 10000 + .5) / 10000
    Cells(20 + j, "F") = Int(((Log(Cells(20 + j, "D")) / np)) / kv) * 10000 + 0.5) / 10000: Rem
    %Veh=xv(j)
    Cells(20 + j, "E") = 1 - Cells(20 + j, "F"): Rem %Pgmt=1-xv(j)

    If j = 1 Then
        Cells(20 + j, "G") = Int(B * Cells(20 + j, "E") / ((1 / r) + Cells(20 + j, "F") * (1 - 1 / r)) +
        0.5)
        Cells(20 + j, "H") = B - Cells(20 + j, "G") / r
    End If

    If j > 1 Then
        K1 = K1 + Cells(20 + j - 1, "G"): K2 = K2 + Cells(20 + j - 1, "H")
        Cells(20 + j, "G") = Int((B * Cells(20 + j, "E") - K1) / ((1 / r) + Cells(20 + j, "F") * (1 - 1 / r)) +
        + 0.5): Rem Pigment
        Cells(20 + j, "H") = Int(B - (K1 + K2) - Cells(20 + j, "G") / r + 0.5): Rem Vehicle
    End If

    Cells(20 + j, "I") = Cells(20 + j, "G") * (1 / r - 1): Rem Water Displacement wd(j)
    SumP = SumP + Cells(20 + j, "G"): SumV = SumV + Cells(20 + j, "H"): SumW = SumW +
    Cells(20 + j, "I")
    Cells(20 + j, "J") = SumP:
    Cells(20 + j, "K") = SumV:
    Cells(20 + j, "L") = Cells(20 + j, "G") + Cells(20 + j, "H")
    Cells(20 + j, "M") = SumP + SumV
    Cells(20 + j, "N") = SumV / SumP
Next j

Cells(4, "D") = SumP: Cells(5, "D") = SumV
```

# CHAPTER 6

## SOFTWARE DEVELOPMENT

### INPUT for Flush Distribution and OUTPUT for Mix-Time Calculations

The screenshot shows two tables in an Excel spreadsheet.

**Table 1: Flush Distribution INPUT DATA (Mix Rate)**

19	
20	
21	<b>Flush Distribution</b>
22	<b>INPUT DATA (Mix Rate)</b>
23	= Process Stage #
24	1
25	= Process Time hrs (tp)
26	1.75
27	= Water Content % (wp)
28	40.00%
29	= Temp deg (C)
30	25.0
31	= Temp deg (F)
32	77.0
33	= Mix to % Completion
34	95.0%

**Table 2: Output Data for Mix-Time Calculations**

C	D	E	F	G	H	I
19	20	21	22	23	24	25
Stage (i)	Water Residual	% Water Displaced	Mix Time Est	Mix Time Method-A	Mix Time Method-B	
1	96.00	64.00%	3.16	4.42	4.42	
2	72.20	46.64%	1.57	3.01	3.12	
3	51.01	33.21%	1.24	2.22	2.55	
4	35.95	23.57%	0.97	1.75	2.21	
5	25.00	16.47%	0.71	1.40	1.98	
6	17.45	11.54%	0.48	1.14	1.80	
27						

The screenshot shows a table with calculated mix-time parameters and totals.

A	B
28	0.1535 = Water Dist Const (kw)
29	1 = 2nd Water Dist Const (b)
30	1.83 = Mix Time Const (tx)
31	1.73 = X-Val Const for % Complete
32	<b>Mix Time Totals (Hours)</b>
33	8.13 = Mix Time Estimate
34	13.95 = Mix Time Method (A)
35	16.08 = Mix Time Method (B)

Refer to Chapter 5 (DATA ANALYSIS II)

Page 116: Calculation of Mix-Times

### VBA Mix-Time Calculations

```

Rem Calculate Mix Time Parameters
Rem =====
kw = Log(B * wp / Cells(21, "I")) / -tp ^ 2: Cells(28, "A") = kw
tx = tp / Sqr(-Log(wp)): Cells(30, "A") = tx
x_val = Sqr(-Log(1 - Mix_pct)): Cells(31, "A") = x_val
Wtr_1 = Cells(21, "I"): Wtr_Residual = 0: WR = 0: Wtr_pct = 0
T_est = x_val * tx: T_Init = T_est: Sum_T_est = 0
TA_est = Sqr((1 / -kw) * Log(1 - Mix_pct)): Sum_TA_est = 0
TB_est = 0: Sum_TB_est = 0

```

# CHAPTER 6

## SOFTWARE DEVELOPMENT

```
For j = 1 To n
    Wtr_Residual = (1 - Mix_pct) * (Wtr_Residual + Cells(20 + j, "I")): Cells(20 + j, "O") =
    Wtr_Residual
    Wtr_pct = (WR + Cells(20 + j, "I")) / (WR + Cells(20 + j, "I") + Cells(20 + j, "M")): Cells(20 +
    j, "P") = Wtr_pct
    TB_est = Sqr((1 / (-kw * j)) * Log(1 - Cells(27, "A"))): Cells(20 + j, "S") = TB_est
    If j = 1 Then
        Cells(20 + j, "Q") = T_est
        Cells(20 + j, "R") = TA_est
    End If
    If j > 1 Then
        T_est = T_Init - Sqr(tx ^ 2 * Log(Wtr_pct) * (-1)): Cells(20 + j, "Q") = T_est
        TA_est = Sqr((1 / -kw) * Log(1 - (Cells(20 + j, "I") + Cells(19 + j, "O")) / Wtr_1)): Cells(20 +
        j, "R") = TA_est
    End If
    Sum_T_est = Sum_T_est + T_est
    Sum_TA_est = Sum_TA_est + TA_est
    Sum_TB_est = Sum_TB_est + TB_est
    WR = Wtr_Residual
Next j

Cells(33, "A") = Sum_T_est
Cells(34, "A") = Sum_TA_est
Cells(35, "A") = Sum_TB_est

End Sub
```

### Detail Mix-Time Rate Calculations (User Select Stage #)

### EXCEL WORKSHEET FUNCTIONS (INPUT @ CELL (A43))

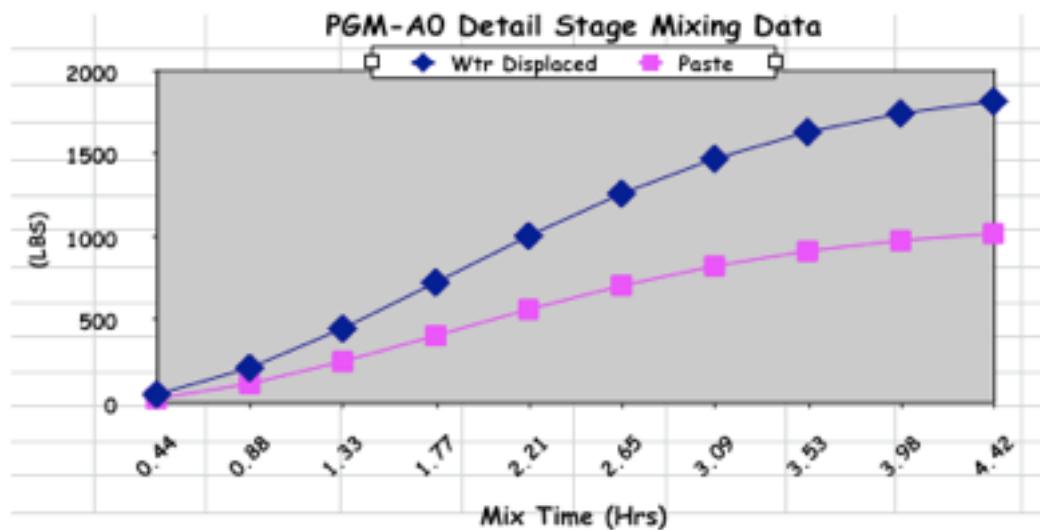
A	B
41	Detailed Mixing Process Data
42	Mass Units = # (lbs, kg, etc)
43 1	--> Select Stage (INPUT)
44 4.42	--> Stage Mix Time (hrs)
45 1920	--> Stage Water Displacement
46 0.1535	--> Stage Constant (Method A)

# CHAPTER 6

## SOFTWARE DEVELOPMENT

### OUTPUT Detail Mix-Time Rate - Select Stage #1 (CELL (A43)

C	D	E	F	G	H	I	
41	Est						
42	Mix Time	Wtr Displaced	Rate Wtr Displaced	Pigment Wetting	Rate-Paste Wetting	Paste Formation	Rate-Paste Formtn'
43	t (hrs)	w(t) #	w'(t) #/hr	P(t) #	P'(t) #/hr	PV(t) #	PV'(t) #/hr
44	0.44	56.67	252.69	14.17	63.17	31.87	142.14
45	0.88	216.82	461.94	54.21	115.48	121.96	259.84
46	1.33	453.75	596.52	113.44	149.13	255.23	335.54
47	1.77	731.12	644.90	182.78	161.22	411.26	362.75
48	2.21	1012.09	615.61	253.02	153.90	569.30	346.28
49	2.65	1266.97	531.34	316.74	132.84	712.67	298.88
50	3.09	1477.62	419.94	369.40	104.99	831.16	236.22
51	3.53	1637.74	306.21	409.44	76.55	921.23	172.25
52	3.98	1750.38	207.02	437.60	51.75	984.59	116.45
53	4.42	1824.00	130.19	456.00	32.55	1026.00	73.23



The Estimate Mix-Time (4.42 Hrs) at CELL A44 for the selected stage is split into ten (10) equal intervals: Column C44:C53. Each row of the table estimates the amount of water displaced, rate of water displaced, pigment wetting, rate of pigment wetting, paste formation and paste formation rate.

# CHAPTER 6

## SOFTWARE DEVELOPMENT

**Process Time at Select Stage i(t):**      **Column CELLS C44: C53**

**Water Displacement w(t):**      **Column CELLS D44: D53**

$$w_i(t) = W_D(1 - e^{-kt^2})$$

**Rate of Water Displacement w'(t):**      **Column CELLS E44: E53**

$$w'(t) = 2W_D kte^{-kt^2}$$

**Pigment Wetting P<sub>i</sub>(t):**      **Column CELLS F44: F53**

$$P_i(t) = \frac{rW_D(1 - e^{-kt^2})}{(1 - r)}$$

**Rate of Pigment Wetting P<sub>i</sub>'(t):**      **Column CELLS G44: G53**

$$\begin{aligned} \frac{dP(t)}{dt} &= \frac{r}{1-r} \left[ \frac{dw(t)}{dt} \right] \\ \frac{dP(t)}{dt} &= \frac{r}{1-r} [2W_D kte^{-kt^2}] \end{aligned}$$

**Paste Formation PV<sub>i</sub>(t):**      **Column CELLS H44: H53**

$$\begin{aligned} PV(t) &= \frac{P_i(t)(P_i + V_i)}{P_i} \\ P_i(t) + V_i(t) &= \frac{P_i(t)(P_i + V_i)}{P_i} \end{aligned}$$

**Rate of Paste Formation PV<sub>i</sub>(t):**      **Column CELLS I44: I53**

$$\begin{aligned} PV'(t) &= \frac{P'_i(t)PV_i(t)}{P_i(t)} \\ \frac{d[P_i(t) + V_i(t)]}{dt} &= \frac{P'_i(t)PV_i(t)}{P_i(t)} \end{aligned}$$

# CHAPTER 6

## SOFTWARE DEVELOPMENT

### Raw Material Substitution (User Select Stage # & RM Item Codes)

The vehicle used in the process has a designated viscosity  $[\eta_v]$ . Cell C39 is the INPUT for the Stage # where the substitution will be made. The Cells C37: C38 contain the INPUT Raw Material Item Codes which the system uses as search parameters in the database worksheet DATA\_01 for a match. If a match is found, the viscosities are retrieved into the corresponding OUTPUT Cells (D37: D38). The charges and the percents are calculated and displayed in Cells E37: F38. The bottom row in cells D39 and E39 display the original values in the selected stage.

### EXCEL WORKSHEET FUNCTIONS (INPUT @ CELL (C37: C39))

The screenshot shows an Excel spreadsheet with a table of data. The columns are labeled A through F. Row 36 contains a blank cell. Rows 37, 38, and 39 contain data. Row 37 has a yellow background and contains the text "Input the Item Codes ----->" followed by two dropdown menus: "X-K1387" and "S-535". Row 38 has a yellow background and contains the text "Vehicle Substitution ----->" followed by a dropdown menu: "0.0560". Row 39 has a yellow background and contains the text "for Stage # ----->" followed by a dropdown menu: "2". Columns C, D, E, and F have yellow headers: "ITEM", "Viscosity", "Charge", and "Pct %". The data rows (37, 38, 39) have green backgrounds. The table is enclosed in a black rounded rectangle.

A	B	C	D	E	F
36					
37	Input the Item Codes ----->	X-K1387	66300.00	125.82	53.54%
38	Vehicle Substitution ----->	S-535	0.0560	109.18	46.46%
39	for Stage # ----->	2	100.00	235.00	100.00%

Refer to Chapter-5; Data Analysis-II; Page 102 - “Load Process Reaction Using Two or More Vehicles.” This example shows how two raw material items (K1387 Resin & #535 solvent) are combined in a calculated proportion to equal the viscosity of the theoretical vehicle used in the procedure. The routine in the EXCEL Worksheet area (A36: F39) is described as follows:

Given the theoretical vehicle viscosity  $[\eta_v = 100p]$ , find the substitute proportions and charges if the raw material items are K1387 Resin & #535 solvent.

$$\begin{aligned} \text{Viscosity of the Vehicle Blend: } \eta_v &= 100 \\ \text{Viscosity of K1387 resin} &= \eta_a = 66300 \\ \text{Viscosity of } \#535 \text{ solvent} &= \eta_b = 0.056 \\ \text{Blend Constant: } k &= \ln\left(\frac{\eta_b}{\eta_a}\right) \\ k &= -13.9843 \end{aligned}$$

$$\begin{aligned} \eta_v &= \eta_a e^{kx_b} \quad \text{Given } x_b = \text{proportion of solvent} \\ 100 &= 66300 e^{-13.9843 x_b} \\ \ln\left(\frac{\eta_v}{\eta_a}\right) &= kx_b \quad x_b = \frac{\ln\left(\frac{\eta_v}{\eta_a}\right)}{k} \\ x_b &= \frac{-6.49677}{-13.9843} \quad x_b = 0.4646 \end{aligned}$$

# CHAPTER 6

## SOFTWARE DEVELOPMENT

Reversing the order of variable assignment results in the complement proportion.

Viscosity of the Vehicle Blend:  $\eta_v = 100$   
 Viscosity of #535 solvent =  $\eta_a = 0.056$   
 Viscosity of K1387 resin =  $\eta_b = 66300$   
 Blend Constant:  $k = \ln\left(\frac{\eta_b}{\eta_a}\right)$   
 $k = 13.9843$

$$\eta_v = \eta_a e^{kx_b} \text{ Given } x_b = \text{proportion of resin}$$

$$100 = 0.056 e^{13.9843 x_b}$$

$$\ln\left(\frac{\eta_v}{\eta_a}\right) = kx_b \quad x_b = \frac{\ln\left(\frac{\eta_v}{\eta_a}\right)}{k}$$

$$x_b = \frac{7.48757}{13.9843} \quad x_b = 0.5354$$

Resin Charge = $(x_{\text{Resin}})V_i$
Resin Charge = $(x_{\text{Solvent}})V_i$

Resin Charge = $(x_{\text{Resin}})V_i$
Resin Charge = $(x_{\text{Solvent}})V_i$
$(x_{\text{Resin}}) + (x_{\text{Solvent}}) = 1$

	A	B	C
1	CODE	DESCRIPTION	VISCOSITY
2	P-PGMT	General Organic Pigment	240000
3	S-470	#470 Oil	0.033
4	S-500	#500 Oil	0.0573
5	S-5300	#5300 Oil	0.405
6	S-535	#535 Oil	0.056
7	S-GEN	General Industrial Solvent	0.00944
8	S-HYD47	Hydrocarbon Solvent #47	0.0385
9	S-HYD52	Hydrocarbon Solvent #52	0.058
10	S-TDA	Trydecyl Alcohol Ethoxylate	0.161
11	V-0	Litho #0 Regular	115
12	V-00	Litho #00 Regular	70
13	V-000	Litho #000 Regular	20
14	V-1	Litho #000 Regular	204
15	V-2	Litho #000 Regular	325
16	V-3	Litho #000 Regular	458
17	V-4	Litho #000 Regular	808
18	V-5	Litho #000 Regular	1746
19	V-6	Litho #000 Regular	2263
20	V-7	Litho #000 Regular	48000
21	X-454K	Pentaerythritol Ester of Rosin #454	8000000
22	X-858K	Pentaerythritol Ester of Rosin #858	2450000
23	X-K1387	Modified Phenolic Resin #1387	66300
24	X-K444	Pentaerythritol Ester of Rosin #454	670000000
25	X-P6140	Pico 6140	46500
26	X-Pent K	Pentaerythritol Ester of Rosin	7400000
27			

# CHAPTER 6

## SOFTWARE DEVELOPMENT

### MODEL Worksheet Comparison

MODEL	INPUT	OPTIMIZE	DESCRIPTION
PGM-A0	Pigment Charge Vehicle Charge Vehicle Visc. Pigment Visc. % Solids P/C Bulk Capacity	YES	Exponential Model Calculates the number of stages Calculates % Pigment of Mix Optimizes initial input charges Viscosity & Pigment Distribution Mix-Time Estimates Detail Process Rates by stage
PGM-AN	Pigment Charge Vehicle Charge Nbr of Stages Vehicle Visc. Pigment Visc. % Solids P/C Bulk Capacity	YES	Exponential Model Calculates % Pigment of Mix Optimizes initial input charges Viscosity & Pigment Distribution Mix-Time Estimates Detail Process Rates by stage
PGM-B0	% Pgmt of Mix Vehicle Visc. Pigment Visc. % Solids P/C Bulk Capacity	YES	Exponential Model Calculates initial input charges Viscosity & Pigment Distribution Mix-Time Estimates Detail Process Rates by stage
PGM-C0	Pigment Charge Vehicle Charge Vehicle Visc. Pigment Visc. % Solids P/C Bulk Capacity	NO	Geometric Series Model Calculates the number of stages Calculates % Pigment of Mix Distributes input geometrically Viscosity & Pigment Distribution Mix-Time Estimates Detail Process Rates by stage
PGM-D0	Pigment Charge Vehicle Charge Nbr of Stages Vehicle Visc. Pigment Visc. % Solids P/C Bulk Capacity	NO	Geometric Series Model Calculates % Pigment of Mix Distributes input geometrically Viscosity & Pigment Distribution Mix-Time Estimates Detail Process Rates by stage

# **CHAPTER - 7**

## **DATA ANALYSIS - IV**

### **(APPLICATIONS)**

#### **CONTENT**

**Resin Solutions**

**Temperature and Viscosity**

**Temperature & Vehicle Substitution**

**Sigma-Blade Mixer Applications**

# CHAPTER 7

## APPLICATIONS

### RESIN SOLUTIONS

#### Problem #1A

Neglecting the effect of temperature, calculate the RM-Item proportions.

Refer to TABLE (RM-Item Codes)

Given:

Resin Solution Relative Viscosity  $\eta = 850 \text{ p}$

Resin, X-454K @ Relative Viscosity  $\eta_K = 8 \times 10^6 \text{ p}$

Solvent S-470 @ Relative Viscosity  $\eta_S = 3.3 \times 10^{-2} \text{ p}$

$$\begin{aligned}\eta_V &= \eta_K e^{kx_s} \quad | \quad k = \ln\left(\frac{\eta_S}{\eta_K}\right) \\ \eta_V &= \text{Resin Solution Viscosity} \\ \eta_K &= \text{Resin Relative Viscosity} \\ \eta_S &= \text{Solvent Relative Viscosity} \\ x_s &= \text{Solvent Proportion}\end{aligned}$$

$$\begin{aligned}k &= \ln\left(\frac{\eta_S}{\eta_K}\right) \\ k &= -19.3062 \\ 850 &= 8 \times 10^6 e^{-19.3062(x_s)} \\ -19.3062(x_s) &= \ln\left(\frac{850}{8 \times 10^6}\right) \\ x_s &= 0.4739 \quad x_K = 0.5261\end{aligned}$$

The solvent proportion = 47.39% The resin proportion = 52.61%

# CHAPTER 7

## APPLICATIONS

### Problem #1B

The lab test results of the resin solution in Problem #1A is listed below. Calculate the function that estimates relative viscosity between 75° F (23.9° C) and 110° F (43.3° C). Then estimate the relative viscosity at 87° F (30.6° C).

Rel Viscosity	Temp ° F	Temp ° C	Temp ° K
850	75	23.9	297.0
600	110	43.3	316.5

Refer to Chapter 2 “Review of Related Literature”, Pages (51-52) for formula derivation.

$$\eta(T) = \eta_0 e^{k(T)} \quad | T = {}^{\circ}\text{Kelvin}$$

Using simultaneous equations:

$$\begin{aligned} 850 &= \eta_0 e^{k(297.0)} \\ 600 &= \eta_0 e^{k(316.5)} \\ 850e^{-k(297.0)} &= \eta_0 \\ 600e^{-k(316.5)} &= \eta_0 \end{aligned}$$

$$\begin{aligned} \eta_0 &= \eta_0 \\ 850e^{-k(297.0)} &= 600e^{-k(316.5)} \\ \ln\left(\frac{850}{600}\right) &= 297.0k - 316.5k \\ k &= -0.01786 \end{aligned}$$

$$\begin{aligned} \eta_0 &= 850e^{-k(297.0)} \\ \eta_0 &= 850e^{(-0.01786)(297.0)} \\ \eta_0 &= 171136 \end{aligned}$$

$$\begin{aligned} \eta(T) &= \eta_0 e^{k(T)} \quad | T = {}^{\circ}\text{Kelvin} \\ \eta(303.7{}^{\circ}\text{K}) &= 171136e^{(-0.01786)(303.7)} \quad | T = 303.7{}^{\circ}\text{K} \quad (87{}^{\circ}\text{F}) \\ \eta(303.7{}^{\circ}\text{K}) &= 1711367e^{(-5.4241)} \\ \eta(303.7{}^{\circ}\text{K}, 87{}^{\circ}\text{F}) &= 755p \end{aligned}$$

# CHAPTER 7

## APPLICATIONS

For this solution, a 46% increase in Fahrenheit temperature results in a 29% reduction in the relative viscosity (poise). Temperature - viscosity relationship is key in other flush applications.

### **Problem #1C**

Vehicle-A has a Temperature - Viscosity response listed below. The flush model is using a theoretical vehicle of 100 p. Calculate the vehicle temperature ( $^{\circ}\text{K}$ ,  $^{\circ}\text{C}$ ,  $^{\circ}\text{F}$ ) needed to process at 100 p.

Rel Viscosity	Temp $^{\circ}\text{F}$	Temp $^{\circ}\text{C}$	Temp $^{\circ}\text{K}$
200	72	22.2	295.4
80	90	32.2	305.4

$$\eta(T) = \eta_0 e^{k(T)} \quad | \quad T = {}^{\circ}\text{Kelvin}$$

Using simultaneous equations:

$$\begin{aligned} 200 &= \eta_0 e^{k(295.4)} \\ 80 &= \eta_0 e^{k(305.4)} \\ 200e^{-k(295.4)} &= \eta_0 \\ 80e^{-k(305.4)} &= \eta_0 \end{aligned}$$

$$\begin{aligned} \eta_0 &= \eta_0 \\ 200e^{-k(295.4)} &= 80e^{-k(305.4)} \\ \ln\left(\frac{200}{80}\right) &= 295.4k - 305.4k \\ k &= -0.09163 \end{aligned}$$

$$\begin{aligned} \eta_0 &= 200e^{-k(295.4)} \\ \eta_0 &= 200e^{(0.09163)(295.4)} \\ \eta_0 &= 1.138 \times 10^{14} \end{aligned}$$

## CHAPTER 7

# APPLICATIONS

$$\begin{aligned}\eta(T) &= \eta_0 e^{k(T)} \quad | T = {}^{\circ} Kelvin \\ \eta(T) &= 1.138 \times 10^{14} e^{(-0.09163)T} \quad | T = {}^{\circ} Kelvin \\ 100 &= 1.138 \times 10^{14} e^{(-0.09163)T} \\ \ln\left(\frac{100}{1.138 \times 10^{14}}\right) &= (-0.09163)T \\ (302.96 {}^{\circ}K, 85.7 {}^{\circ}F, 29.8 {}^{\circ}C) &= T\end{aligned}$$

Processing Temperature of Vehicle-A should be (303<sup>o</sup>K, 29.8<sup>o</sup>C, 85.7<sup>o</sup>F) to equal 100p.

All solution systems respond differently to temperature changes because of their chemical and physical properties. (65<sup>o</sup> - 120<sup>o</sup> F) is a typical temperature range for most flush process. The temperature boundary points make excellent points to setup the Temperature-Viscosity system constants for Raw Material Resin Solutions. The system constants could consist of the two arbitrary constants [ $\eta_0$ ,  $k$ ] appended to each record. Refer to [ $\eta_0$ , TEMP-0] & [ $k$ , TEMP-K] which correlate to [ $\eta_0$ ,  $k$ ] respectively.

# CHAPTER 7

## APPLICATIONS

A	B	C	D	E	
1	CODE	DESCRIPTION	VISCOSITY	TEMP-0	TEMP-K
2	P-PGMT	General Organic Pigment	240000		
3	S-470	#470 Oil	0.033		
4	S-500	#500 Oil	0.0573		
5	S-5300	#5300 Oil	0.405		
6	S-535	#535 Oil	0.056		
7	S-GEN	General Industrial Solvent	0.00944		
8	S-HYD47	Hydrocarbon Solvent #47	0.0385		
9	S-HYD52	Hydrocarbon Solvent #52	0.058		
10	S-TDA	Tridecyl Alcohol Ethoxylate	0.161		
11	V-0	Litho #0 Regular	115		
12	V-00	Litho #00 Regular	70		
13	V-000	Litho #000 Regular	20		
14	V-1	Litho #000 Regular	204		
15	V-2	Litho #000 Regular	325		
16	V-3	Litho #000 Regular	468		
17	V-4	Litho #000 Regular	808		
18	V-5	Litho #000 Regular	1746		
19	V-6	Litho #000 Regular	2263		
20	V-7	Litho #000 Regular	48000		
21	V-EXP200	Experimental Resin Solution #200	200	1.138E+11	-0.09163
22	V-EXP850	Experimental Resin Solution #850	850	171136	-0.01786
23	X-454K	Pentaerythritol Ester of Rosin #454	8000000		
24	X-858K	Pentaerythritol Ester of Rosin #858	2450000		
25	X-K1387	Modified Phenolic Resin #1387	66300		
26	X-K444	Pentaerythritol Ester of Rosin #454	670000000		
27	X-P6140	Pico 6140	46500		
28	X-Pent K	Pentaerythritol Ester of Rosin	7400000		
29					

## CHAPTER 7

# APPLICATIONS

Resin content is very important in pigment dispersions. Resin and heavy bodied varnishes enhance the gloss of the finished ink. However, resin and heavy varnishes make poor wetting agents in the early mixing stages. Most formulations start with lighter varnishes in the early stages and gradually increase the viscosity of the vehicles as the paste body builds. Increasing temperature decreases the viscosity of most varnishes and resin solutions as illustrated in Problems #1B and #1C. The lower viscosity vehicles flow easily and make good wetting agents. Increasing the temperature is another way to reduce the viscosity of the vehicle without adding solvents. Solvents reduce the resin proportion and also reduce the ink gloss and reflectivity. While the math may look good on paper the variance is quite difficult to control because of so many variables. Quality control and consistency in RM resins and varnishes is critical. Small changes in variance can produce challenges in the manufacturing process.

A large variance in vehicle response has always been a high priority challenge for ink technologists. Recent advances in computer technology and quantitative methods have increased the opportunity for exploration and creativity in solving these problems.

As we explore more applications in this chapter the intent is to stimulate more discussion and creativity using problem solving methods with math models. Math and

## **CHAPTER 7**

## **APPLICATIONS**

statistics are very powerful tools which enhance the abilities of the ink technologist in designing and analyzing ink and color systems.

# CHAPTER 7

## APPLICATIONS

### Problem #1D

Low viscosity vehicles are used in the early mixing stages as wetting agents. Medium viscosity vehicles are used to build the body of the paste in the intermediate stages. And the heavier vehicles are added in the later stages. Using temperature and the following vehicle viscosities, design three stages of RM (Raw Material) proportions to meet the work order specifications listed below.

Process viscosity of each stage is 100p.

System solvent (S-535) can be used in stages #3 & #6

Stage #1 Max Temperature is 90° F:      Vehicle-A (V-1: Litho #1 Reg)

Stage #3 Max Temperature is 100° F:      Vehicle-B (V-3: Litho #3 Reg)

Stage #6 Max Temperature is 120° F:      Vehicle-C (V-7: Litho #7 Reg)

### Raw Material Data

CODE	DESCRIPTION	VISCOSITY	TEMP-0	TEMP-k
S-535	#535 Oil	0.056		
V-1	Litho #1 Regular	204	5.13904E+14	-0.09577
V-3	Litho #3 Regular	458	1.06816E+13	-0.08037
V-7	Litho #7 Regular	48000	4.88517E+25	-0.16285

# CHAPTER 7

## APPLICATIONS

### Problem #1D Solution

Stage #1 (Wetting) uses heat to reduce V-1 viscosity from 204 p to 100 p. Given

Max Temperature = 90° F

Relative viscosity of V-1 = 100p @ 90.4° F

$$\begin{aligned}\eta(T) &= \eta_0 e^{k(T)} \mid T = {}^{\circ}\text{Kelvin} \\ 100 &= 5.139 \times 10^{14} e^{-0.09577(T)} \\ T &= \frac{\ln\left(\frac{100}{5.139 \times 10^{14}}\right)}{-0.09577} \\ T &= 305.6 (32.5{}^{\circ}\text{C}, 90.4{}^{\circ}\text{F})\end{aligned}$$

Stage #3 (Intermediate) has Max Temperature = 100° F

Temperature reduces V-3 viscosity from 458 p to 149.9 p

$$\begin{aligned}\eta(T) &= \eta_0 e^{k(T)} \mid T = {}^{\circ}\text{Kelvin} \\ \eta(T) &= 1.068 \times 10^{13} e^{-0.08037(319.9)} \\ \eta(T) &= 149.9\end{aligned}$$

Additional solvent (S-535: #535 Oil) is needed to reduce viscosity to 100 p

$$\begin{aligned}\eta &= \eta_v e^{k_s x_s} \mid k_s = \ln\left(\frac{\eta_s}{\eta_v}\right) \text{ and } x_s = \text{Solvent Proportion} \\ k_s &= \ln\left(\frac{\eta_s}{\eta_v}\right) = \ln\left(\frac{0.056}{149.9}\right) \\ k_s &= -7.8924 \\ x_s &= \frac{\ln\left(\frac{\eta}{\eta_v}\right)}{k_s} = \frac{\ln\left(\frac{100}{149.9}\right)}{-7.8924} \\ x_s &= .0513 (5.1\%) \quad \boxed{\begin{array}{l}\eta = \text{Target Viscosity}(100p) \\ \eta_s = \text{Solvent Viscosity} \\ \eta_v = \text{Function Viscosity} = [\eta(T)] \\ k_s = \text{System Constant}\end{array}}\end{aligned}$$

At 100° F, the vehicle mix of (94.9% V-3 and 5.1% #535 Oil) has an expected relative viscosity of 100 p.

# CHAPTER 7

## APPLICATIONS

Stage #6 (Binding Stage) has Max Temperature = 120° F (48.9° C, 322.0° K)

Temperature reduces V-7 viscosity from 48000 p to 823.1 p

$$\begin{aligned}\eta(T) &= \eta_0 e^{k(T)} \mid T = {}^{\circ}K \text{ Kelvin} \\ \eta(T) &= 4.8851 \times 10^{25} e^{-0.16285(322.0)} \\ \eta(T) &= 823.1\end{aligned}$$

Additional solvent (S-535: #535 Oil) is needed to reduce viscosity to 100 p

$$\begin{aligned}\eta &= \eta_v e^{k_s x_s} \mid k_s = \ln\left(\frac{\eta_s}{\eta_v}\right) \text{ and } x_s = \text{Solvent Proportion} \\ k_s &= \ln\left(\frac{\eta_s}{\eta_v}\right) = \ln\left(\frac{0.056}{823.1}\right) \\ k_s &= -9.5955 \\ x_s &= \frac{\ln\left(\frac{\eta}{\eta_v}\right)}{k_s} = \frac{\ln\left(\frac{100}{823.1}\right)}{-9.5955} \\ x_s &= .2197 (22.0\%) \end{aligned}$$

$$\begin{aligned}\eta &= \text{Target Viscosity}(100p) \\ \eta_s &= \text{Solvent Viscosity} \\ \eta_v &= \text{Function Viscosity} = [\eta(T)] \\ k_s &= \text{System Constant}\end{aligned}$$

At 120° F, the vehicle mix of (78.0% V-7 and 22.0% #535 Oil) has an expected relative viscosity of 100 p.

The substitution feature in the Excel Math Models, vehicle substitution routine can be expanded to include the effects of temperature. As mentioned earlier, temperature is only one of the controls that the technologists and flusher operators use to manipulate the dispersion process. The viscosity in the Excel models serve as a guide for the stages in the process. The base viscosity is the Target Viscosity referenced in the above calculations as temperature becomes the independent variable. The following example is an application of vehicle substitution using Model PGM-A0 and Problem #1D.

# CHAPTER 7

## APPLICATIONS

### Problem #1D Solution

#### Vehicle Substitution in Stages #1, #3 and #6

#### MODEL PGM-A0 (Before Substitution)

Stage (j)	Viscosity	% Pgmt	% Veh	Pigment	Vehicle
1	3159.85	0.4437	0.5563	480	600
2	4699.58	0.4947	0.5053	337	235
3	5474.38	0.5143	0.4857	237	163
4	5864.27	0.5231	0.4769	167	113
5	6060.46	0.5273	0.4727	116	88
6	6159.19	0.5294	0.4706	81	59

#### MODEL PGM-A0 (After Substitution)

Problem#	PGM-A0
Stage (j)	RM-Code
1	V-1
	600
	90
	3159.85
	0.4437
	0.5563
	480
	600
2	
3	V-3
	154.6
	5-535
	8.4
	100
	5474.38
	0.5143
	0.4857
	237
	163
4	
5	
6	V-7
	46
	5-535
	13
	120
	5864.27
	0.5231
	0.4769
	167
	113
	6060.46
	0.5273
	0.4727
	116
	88
	6159.19
	0.5294
	0.4706
	81
	59

# CHAPTER 7

## APPLICATIONS

### Problem #2A

How does an increase in the base vehicle viscosity (from 100p to 200p) effect the flush distribution? The Total Pigment & Vehicle Charge is unchanged. (P = 1350; V = 1200)

#### MODEL-A0 - Analysis: Vehicle Base Viscosity (200p vs. 100p)

##### INPUT

Total Charge	2550	2550	No Change
Viscosity	100p	200p	+100% @ 75° F

##### OUTPUT

Optimize Pigment	1418	1413	-0.4% (Not Significant)
Optimize Vehicle	1258	1259	+0.0% (Not Significant)
Stages (Calc)	5.6313	5.6313	No Change (Formula)
Sys Visc Const (k)	-7.7832	-7.0901	
Mix Viscosity	6159.19	8534.63	+ 38.6% (Significant)
Wtr Dist Const (kw)	0.1535	0.1494	
Mix-Time Est	8.13 hrs	8.14 hrs	+0.0% (Not Significant)
Mix-Time (A)	13.95 hrs	14.23 hrs	+ 2.0% (Not Significant)
Mix-Time (B)	16.08 hrs	16.30 hrs	+ 1.4% (Not Significant)

Doubling the vehicle viscosity increases the Mix Viscosity by 38.6%. The results of a One-Way Anova @ .05 alpha test imply a significant difference between the

## CHAPTER 7 APPLICATIONS

means. The F-Test of the variances failed to detect a significant difference. The Mix is comprised of 47.06% vehicle. The Total Mix-Times show a very slight increase of 1 - 2% which is statistically insignificant using the 1-Way Anova @ .05 alpha Test. The model reports provide a quick visual of the tables where the results can be compared and analyzed.

**Compare Viscosity Distribution**

Viscosity	T = 75 deg F	T = 75 deg F
Stage (j)	100p	200p
1	3159.85	4169.09
2	4699.58	6292.33
3	5474.38	7428.14
4	5864.27	8035.73
5	6060.46	8360.76
6	6159.19	8534.63

The temperature of 75° F is relative and is intended to imply thermal kinetics are not to be considered in this example. As earlier indicated, relative temperature at the completion of each stage is expected to increase as a direct result of friction and particle bombardment (Boltzmann's Constant) as the paste relative viscosity increases.

# CHAPTER 7

## APPLICATIONS

### Problem #2B

Compare the relative temperature distributions for the 100p & 200p base vehicles.

Discuss how the temperature distribution for the 200p base vehicle system shown below could change. Refer to Chapter 1: Page 24 (Definition of Viscosity).

**EXAMPLE 5a (Report MODEL-A1)  
With Temperature Data 100p Base Vehicle**

i	$\eta$	$X_p$	$P_i$	$W_D$	$V_i$	TEMP	TEMP	$\Delta t$ Mix	Cum Mix
Stage	Distribution	% Pigment	Pigment Charge	Water Displaced	Vehicle Charge	°F	°C	Hrs	Hrs
1	3160	0.4436	479.69	1918.76	601.55	65 - 75	18.3 - 23.9	4.42	4.42
2	4699	0.4946	337.16	1348.63	232.97	75 - 83	23.9 - 28.3	3.02	7.44
3	5474	0.5143	237.46	949.84	161.33	83 - 87	28.3 - 30.6	2.23	9.67
4	5864	0.5231	166.53	666.13	117.18	87 - 95	30.6 - 35.0	1.75	11.42
5	6060	0.5273	116.15	464.58	85.4	95 - 105	35.0 - 40.6	1.40	12.82
6	6159	0.5294	80.58	322.32	61.69	105 - 110	40.6 - 43.3	1.14	13.96

**With Temperature Data 200p Base Vehicle**

Stage (j)	Viscosity	% Pgmt	Wtr Disp	Vehicle	Temp F	Temp C	Mix (A)	Cum Mix
1	4169.09	0.4284	1896	630	65 - 75	18.3 - 23.9	4.48	4.48
2	6292.33	0.4864	1336	226	75 - 83	23.9 - 28.3	3.07	7.55
3	7428.14	0.5098	948	151	83 - 87	28.3 - 30.6	2.27	9.82
4	8035.73	0.5209	672	108	87 - 95	30.6 - 35.0	1.79	11.61
5	8360.76	0.5265	472	82	95 - 105	35.0 - 40.6	1.45	13.06
6	8534.63	0.5294	328	62	105 - 110	40.6 - 43.3	1.18	14.23

The coefficient of viscosity is defined as the force in dynes required per square centimeter to maintain a difference in velocity of 1 cm/sec between two parallel layers of the fluid, which are ( $\Delta d$ ), 1 cm apart. This is best represented in the following expression

## CHAPTER 7

### APPLICATIONS

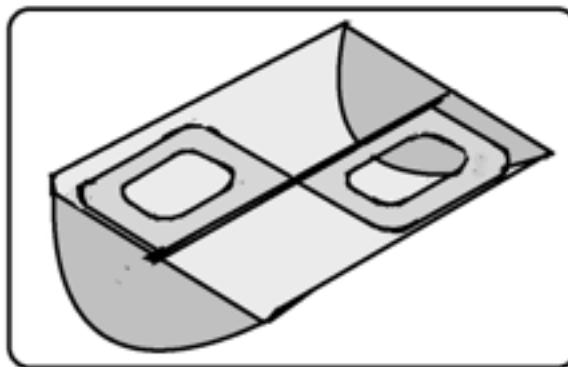
from James F. Shackelford's, Introduction to Materials Science for Engineers, (1985), p.

329,  $\eta = \frac{f\Delta d}{\Delta v a}$ , where ( $\eta$ ) is the coefficient of viscosity in poise, "a" is the area in  $\text{cm}^2$ ,

( $\Delta v$ ) is change in velocity in  $\text{cm/sec}$  and ( $f$ ) is the applied force in dynes.

The area [A] that is sheared by the edge of the sigma-blades and the rate of shear is constant. The force [F] needed to maintain a constant RPM must increase as the water is displaced and the paste thickens. Because of the thicker paste, the resistance to shear increases, thus a higher viscosity. The direct relationship between viscosity and Force is illustrated in the algebraic expression below. Because of the friction and shearing action, the temperature of the paste also increases. As illustrated in the previous problems in this section, an increase in temperature also reduces the viscosity.

$$\eta = \frac{\text{shear stress}}{\text{shear rate}}$$
$$\eta = \frac{(F/A)}{\text{shear rate}}$$
$$\eta(\text{poise}) = \frac{(F(\text{dynes})/A(\text{cm}^2))}{\text{shear rate}(\text{sec}^{-1})}$$



# CHAPTER 7

## APPLICATIONS

### Baker-Perkins Dual Sigma Blade Mixer Specifications:

500 Gallon Capacity

Rotation Speed +/- 3%                          Angular Shear = 130°

Torque =    300 HP

$$HP = \frac{2\pi RF(rps)}{550} \quad R = \text{ft}, \quad F = \text{lbs}, \quad rps = \text{rotations/sec}$$

$$F = \frac{\eta Av}{kx} \quad F(\text{lbs}) = \frac{F(\text{dynes})}{(454)(980)}$$

$\eta$  = Viscosity\_(poise)

$A$  = Shear\_Area\_(sq - cm)                           $k = 1$  \_\_\_\_ Vertical(Sharp\_Blade)

$v$  = Velocity\_(cm / sec)                                   $k = 1.5$  \_\_\_\_ Rounded\_Blade

$kx$  = Gap\_Thickness\_(cm)                                   $k = 2$  \_\_\_\_ Horizontal(Parallel\_Blade)

Gap Thickness (x)	1.0 in	2.54 cm
Front Blade =	25.5 rpm	0.4250 rps
Rear Blade =	14.5 rpm	0.2417 rps
Blade Length (L) =	65.5 in	166.37 cm
Blade Radius (R) =	14.0 in	35.56 cm
Shell Circumference (C) =	87.96 in	223.43 cm
Shell Area (360°) $2\pi RL$ =	5761.68 sq-in	37172.06 sq-cm
Shear Area (130°) =	2080.61 sq-in	13423.24 sq-cm

$$\text{Shear\_Area} = 2\pi RL \left( \frac{130^\circ}{360^\circ} \right)$$

Front Blade Velocity =	37.38 in/sec	94.95 cm/sec
Front Blade Power =	191.25 HP	
Real Blade Velocity =	21.26 in/sec	53.99 cm/sec
Rear Blade Power =	108.75 HP	

# CHAPTER 7

## APPLICATIONS

### Viscosity, Film Thickness & Frictional Force

The viscosity is directly proportional to the product of film thickness and frictional forces. Viscosity is also inversely proportional to the product of the area covered by the film and the velocity of the moving plate applicator.

$$\eta \propto \frac{Fx}{Av} \quad \eta = k \frac{Fx}{Av}$$

The constant of proportionality (k), is directly related to the type of blade applicator used to apply the film. As indicated in the Baker-Perkins Dual Sigma Blade Mixer Specifications, the contact edge of the sigma blade is rounded and implies a proportionality constant ( $k = 1.5$ ). There are two sigma mixing blades of equal radii ( $R = 14.0$  in; 35.6 cm). Each of the mixing blades are shearing over equal bowl areas of  $130^\circ$ , but rotating at different speeds over an area ( $A = 2080.6$  sq-in; 13423.2 sq-cm).

The total torque power delivered to the mixer is distributed proportionally between the two rotating mixer blades. Given a total torque power of 300 hp distributed over the two mixer (191.25 hp Front at 25.5 rpm) and (108.75 hp Rear at 14.5 rpm).

Referencing classical newtonian physics, the work (ft-lbs) required to overcome the drag produced by the viscosity is expressed as follows:

$$Work(ft-lbs) = Force \times Distance \quad Work(ft-lbs) = Force(lbs) \times 2\pi R \times (rps)$$

$$HP = \frac{Work(ft-lbs)}{550} \quad Force(lbs) = \frac{Force(dynes)}{(980)(454)}$$

**HP = Horsepower    R = Blade Radius (ft)                  rps = rotations per second**

### Problem #3A

Given the above parameters, calculate the frictional force (drag) and the power required to overcome this drag on both the front and rear blades for a slurry viscosity of 21.7 poise and paste viscosity of 3179.2 poise.

# CHAPTER 7

## APPLICATIONS

A	B	C	D	E
1	SLURRY	Charge (Lbs)	Viscosity	%
2	P1 =	480	240000	16.00%
3	V1 =	600	100	20.00% -7.7832
4	W1 =	1920	0.01	64.00% -16.9936
5	Slurry =	3000	0.96	
6	Paste =	1080	3179.19	
7				
8				
9	Mixer Specs	English	CGS	
10	Blade-Radius R	in, cm	14	35.56
11	Length (L)	in, cm	65.5	166.37
12	Thickness (X)	in, cm	1	2.54
13	Area Shell-360	sq-in, sq-cm	5761.68	37172.06
14	Area Shell-130	sq-in, sq-cm	2080.61	13423.24
15	Power	Max HP	300.00	
16	Power-Front	Max HP	191.25	
17	Power-Rear	Max HP	108.75	
18				
19	Front-Blade Velocity	RPM, RPS in/sec, cm/sec	25.5 37.38	0.425 94.96
21	Power-Slurry	HP, % Max	0.05	0.0%
22	Power-Paste	HP, % Max	162.49	54.2%
23	Total Power	HP, % Max	162.54	54.2%
24	Force (Slurry)	Lbs, Dynes)	0.72	320108
25	Force (Paste)	Lbs, Dynes)	2390.55	1063602064
26				
27				
28	Rear-Blade Velocity	RPM, RPS in/sec, cm/sec	14.5 21.26	0.242 54.00
30	Power-Slurry	HP, % Max	0.02	0.0%
31	Power-Paste	HP, % Max	52.54	17.5%
32	Total Power	HP, % Max	52.56	17.5%
33	Force (Slurry)	Lbs, Dynes)	0.41	182022
34	Force (Paste)	Lbs, Dynes)	1359.33	604793330
35				

# CHAPTER 7

## APPLICATIONS

### Problem #3A - Calculations

**Force on Front Blade** is calculated as follows:

$$F = \frac{\eta Av}{kx}$$

$$F(\text{lbs}) = \frac{F(\text{dynes})}{(454)(980)}$$

$\eta$  = Viscosity\_(poise)

$A$  = Shear\_Area\_(sq-cm)

$v$  = Velocity\_(cm/sec)

$kx$  = Gap\_Thickness\_(cm)

$k = 1.5$  \_\_\_\_\_ Rounded Blade

$x$  = Gap Thickness

$$\text{Front Blade} \quad F_1 = \frac{\eta_1 Av}{1.5x}$$

Given Front & Rear Bowls of same dimensions & area 130 degree shell

Refer to Baker-Perkins 500 gallon Specifications

Bowl Mixing Area (2080.61 sq-in) (13,423.24 sq-cm)

Blade Radius (14 in) (35.56 cm)

Front Blade Speed (37.38 in/sec) (94.96 cm/sec) (0.425 rotations/sec)

### SLURRY

$$F_1 = \frac{\eta_1 Av}{1.5x} = \frac{(21.7p)(13,423.24\text{cm}^2)(94.96\text{ cm/sec})}{(1.5)(2.54\text{cm})}$$

$$F_1 = 7,250,208 \text{ dynes} \quad F_1 = 16.30 \text{ lbs}$$

$$HP = \frac{2\pi(\text{Radius ft})(\text{Force lbs})(\text{Blade Velocity } \frac{\text{rotatoins}}{\text{sec}})}{550}$$

$$HP = \frac{2\pi(\frac{14}{12})(16.3)(0.425)}{550} = 1.11 \text{ hp (front)}$$

### PASTE

$$F_1 = \frac{\eta_1 Av}{1.5x} = \frac{(3179.19p)(13,423.24\text{cm}^2)(94.96\text{ cm/sec})}{(1.5)(2.54\text{cm})}$$

$$F_1 = 1.064 \times 10^9 \text{ dynes} \quad F_1 = 2390.55 \text{ lbs}$$

$$HP = \frac{2\pi(\text{Radius ft})(\text{Force lbs})(\text{Blade Velocity } \frac{\text{rotatoins}}{\text{sec}})}{550}$$

$$HP = \frac{2\pi(\frac{14}{12})(2390.55)(0.425)}{550} = 162.49 \text{ hp (front)}$$

# CHAPTER 7

## APPLICATIONS

Rear Blade Speed (21.26 in/sec) (54.00 cm/sec) (0.242 rotations/sec)

### SLURRY

$$F_3 = \frac{\eta_1 Av}{1.5x} = \frac{(21.7p)(13,423.24\text{cm}^2)(54.00 \frac{\text{cm}}{\text{sec}})}{(1.5)(2.54\text{cm})}$$

$$F_2 = 4,122,668 \text{ dynes} \quad F_2 = 9.27 \text{ lbs}$$

$$HP = \frac{2\pi(\text{Radius ft})(\text{Force lbs})(\text{Blade Velocity } \frac{\text{rotatoins}}{\text{sec}})}{550}$$

$$HP = \frac{2\pi(\frac{14}{12})(9.27)(0.242)}{550} = 0.36 \text{ hp (rear)}$$

### PASTE

$$F_2 = \frac{\eta_2 Av}{1.5x} = \frac{(3179.19p)(13,423.24\text{cm}^2)(54.00 \frac{\text{cm}}{\text{sec}})}{(1.5)(2.54\text{cm})}$$

$$F_2 = 6.048 \times 10^8 \text{ dynes} \quad F_2 = 1359.33 \text{ lbs}$$

$$HP = \frac{2\pi(\text{Radius ft})(\text{Force lbs})(\text{Blade Velocity } \frac{\text{rotatoins}}{\text{sec}})}{550}$$

$$HP = \frac{2\pi(\frac{14}{12})(1359.33)(0.242)}{550} = 52.54 \text{ hp (rear)}$$

SLURRY (21.67p)

PASTE (3179.19p)

FRONT	1.11 HP	162.49 HP
REAR	0.36 HP	52.54 HP
TOTAL	1.47 HP	215.03 HP

# **CHAPTER 7**

## **APPLICATIONS**

### **BIBLIOGRAPHY**

**Atkins, P. W. (1982) 2<sup>nd</sup> edition**, Physical Chemistry, W. H. Freeman and Company, The Nature of Matter: Orientation and Background (p 18), The Structure and Properties of Macromolecules - Viscosity (p 825)

**Brothman, A, Wollan, G, & Feldman, S. (1947)**, Manual of Chemical Engineering Calculations and Shortcuts, New Analysis Provides Formula to Solve Mixing Problems (p. 175-184)

**Giles R. V. (1962) 2<sup>nd</sup> edition**, Fluid Mechanics and Hydraulics, Schaum's Outline Series, McGraw-Hill Book Co., Properties of Fluids (p 3), Translation and Rotation of Liquid Masses (p. 42- 49)

**Kreyszig, E. (August 1988) 6<sup>th</sup> edition**, Advanced Engineering Mathematics, Differential Equations of the First Order (Part A), John Wiley & Sons

**Levenspiel, Octave (1962)**, Chemical Reaction Engineering, Chapter 10 - Mixing of Fluids (p. 309-331), John Wiley & Sons

**McKennell, R., Ferranti Ltd., Moston & Mancheser (1960)**, Ferranti Instrument Manual, The Measurement and Control of Viscosity, And Related Flow Properties, "Liquid Flow Problems and Their Solution": 1955 Reprint from Chemical Product

**Paint Technology Manuals (1961)**, PART TWO – Solvents, Oils, Resins and Driers Published on behalf of The Oil & Colour Chemists' Association – (Atherton, 1961, p. 31)

**Patton, T. C., 1<sup>st</sup> edition (1963) & 2<sup>nd</sup> edition (1979)**, Paint Flow and Pigment Dispersion, A Rheological Approach to Coating and Ink Technology, John Wiley & Sons

## **CHAPTER 7**

## **APPLICATIONS**

**Stroud, K.A., 5<sup>th</sup> edition (2001)**, Engineering Mathematics, Geometric Series (Geometric Progression), Series 1-2 (p 747-802), Industrial Press, Inc.

**Stroud, K.A., Booth, Dexter J. (2005)**, Differential Equations, First Order Differential Equations (Program 1: p 1-38), Industrial Press, Inc.

**Turner, G. P. A. (1967)**, Introduction to Paint Chemistry, Pigmentation (p. 119)

**Wolfe, Herbert Jay, Macnair-Dorland Co. (1967)**, Printing and Litho Inks, In cooperation with a Group of Specialists in the Printing Ink, Pigment, Resin, Vehicle and Allied Industries