Flexible Modeling of Transition Processes via Bayesian Spline Rate Models

with Application to Estimating and Projecting Modern Contraceptive Prevalence

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- **This presentation:** We propose a new type of model, called *B-spline Transition Models*, for flexibly estimating indicators that follow transitions.

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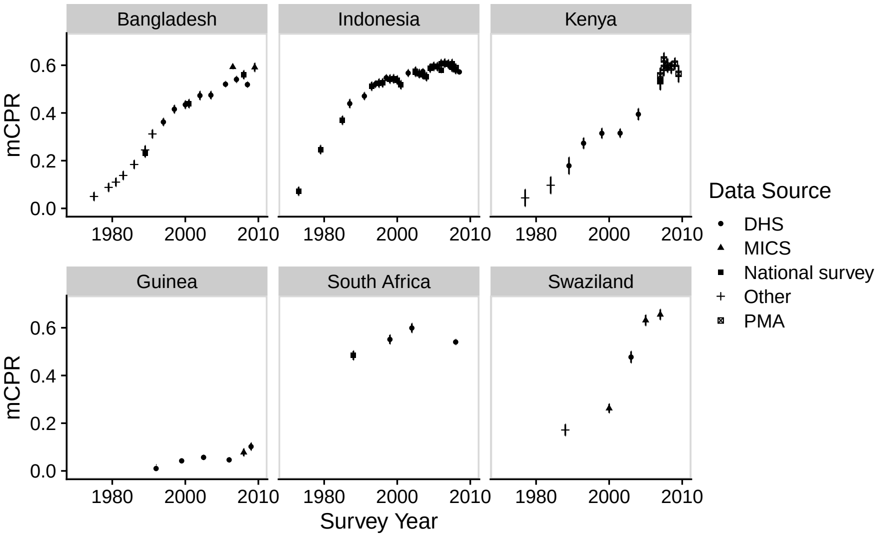
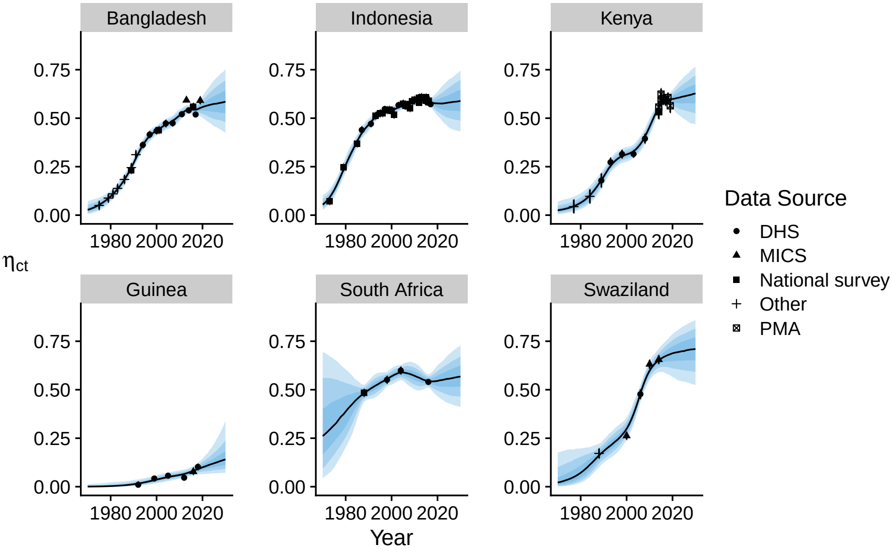
- Goal: estimate and project mCPR in countries from 1970-2030.
- Dataset aggregated by United Nations Population Division (UNPD) from surveys conducted by governments or international organizations.

Raw Data



Bangladesh
Indonesia
Kenya
Guinea
South Africa
Swaziland
mCPR
0.0
0.2
0.4
0.6
1980
2000
2010
Survey Year
Data Source
DHS
MICS
National survey
Other
PMA

Example Fits



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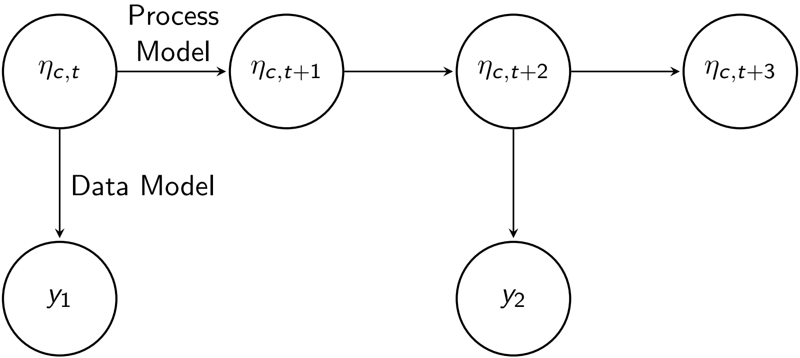
- *Process model* describes evolution of $\eta_{c,t}$.
- *Data model* describes relationship between y_i and $\eta_{c[i],t[i]}$.

Modeling Framework



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- The systematic component has the following form:

$$g_3(t, \boldsymbol{\eta}_{c,s\neq t}, \boldsymbol{\alpha}_c) = \begin{cases} \Omega_c, & t = t_c^*, \\ g_1(\eta_{c,t-1}) + f(\eta_{c,t-1}, P_c, \boldsymbol{\beta}_c), & t > t_c^*, \\ g_1(\eta_{c,t+1}) - f(\eta_{c,t+1}, P_c, \boldsymbol{\beta}_c), & t < t_c^*, \end{cases}$$

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- The function f is called the *transition function*.

B-spline Transition Model

- Define a transition function f_b as:

$$f_b(\eta_{c,t}, P_c, \boldsymbol{\beta}_c) = \sum_{j=1}^{J} \underbrace{h_j(\beta_{c,j})}_{\text{coefficient}} \cdot \underbrace{B_j(\eta_{c,t}/P_c)}_{\text{basis function}},$$

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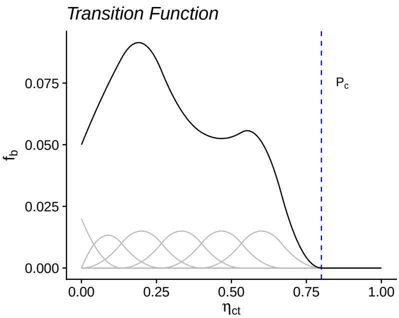
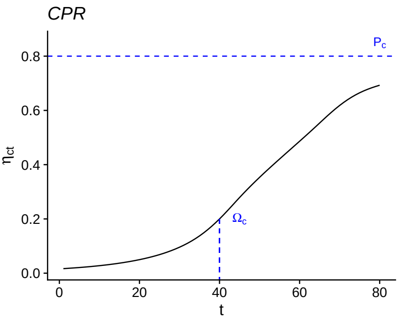
- Flexibility of f_b can be tuned through the spline degree and number and positioning of knots.

Example B-spline Transition Function





Smoothing component

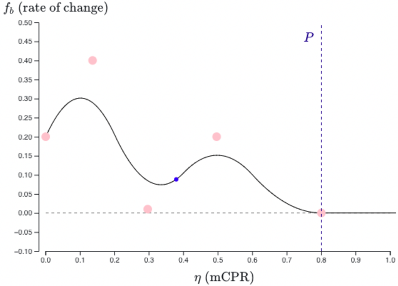
- Recall the process model has two components:

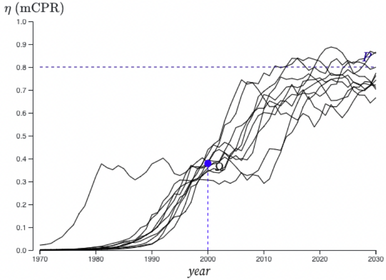
$$g_1(\eta_{c,t}) = \underbrace{g_3(t, \boldsymbol{\eta}_{c,s\neq t}, \boldsymbol{\alpha}_c)}_{\text{systematic}} + \underbrace{\epsilon_{c,t}}_{\text{smoothing}} .$$

- Smoothing component: AR(1) process of the form

$$\epsilon_{c,t} | \epsilon_{c,t-1}, \tau, \rho \sim N(\rho * \epsilon_{c,t-1}, \tau^2)$$

Smoothing component



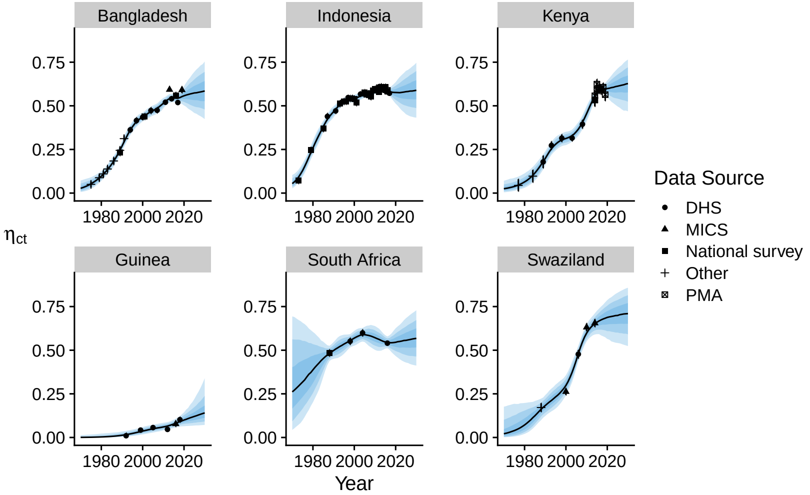


Data Model: connection to observed data

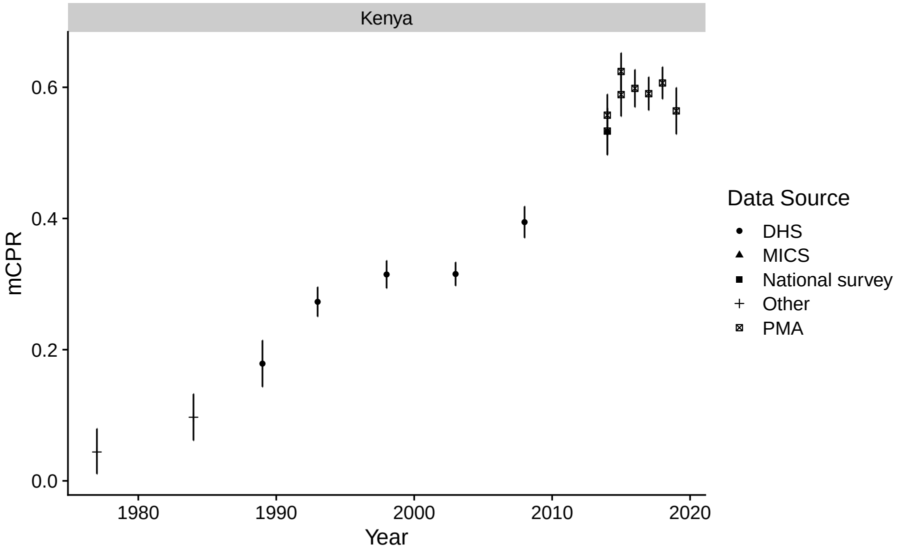
- Let y_i, $i = 1, \ldots, n$ be the observed mCPR for country $c[i]$ and year $y[i]$ from data source $d[i]$.
- For each observation we have an estimate s_i^2 of the sampling error.
- We also expect each data source to have additional non-sampling error $\sigma^2_{d[i]}$.
- Truncated normal data model:

$$y_i | \eta_{c[i],t[i]}, \sigma^2_{d[i]} \sim N_{(0,1)} \left(\eta_{c[i],t[i]}, s_i^2 + \sigma^2_{d[i]} \right).$$

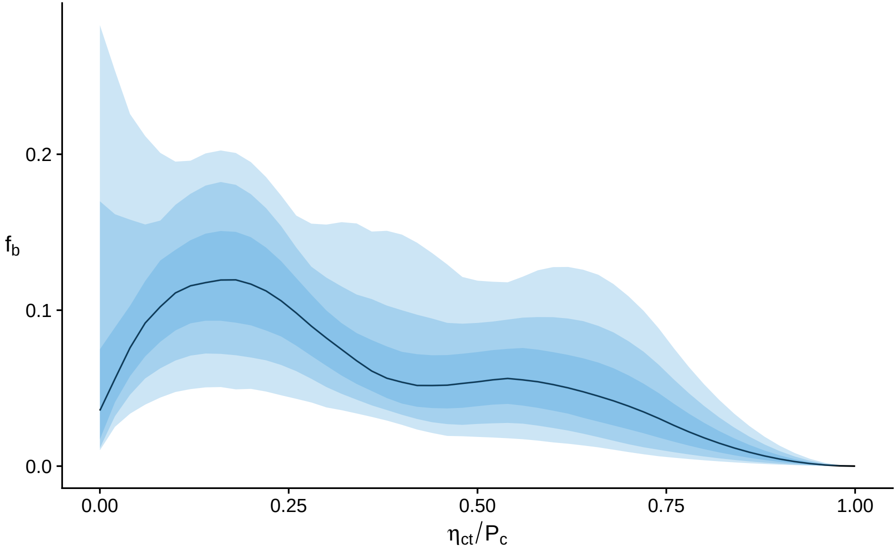
Illustrative Fits



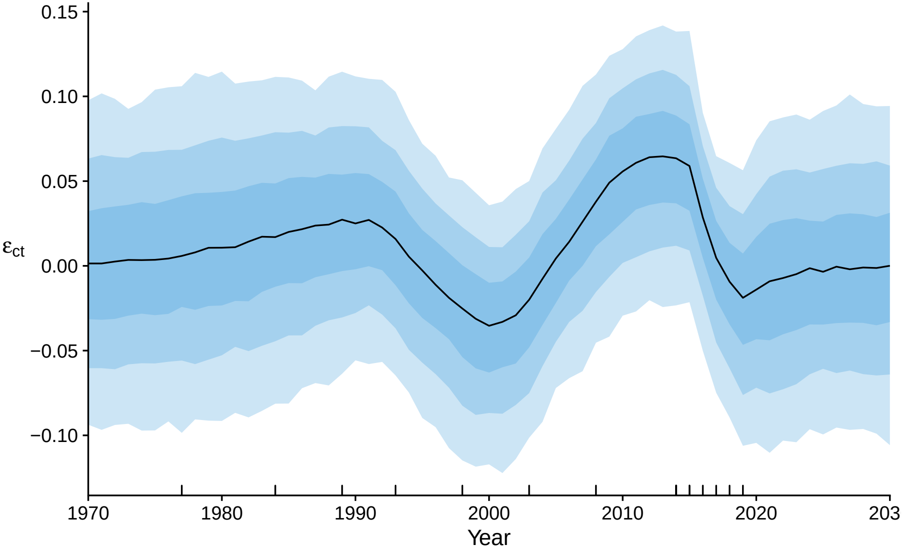
Kenya Raw Data



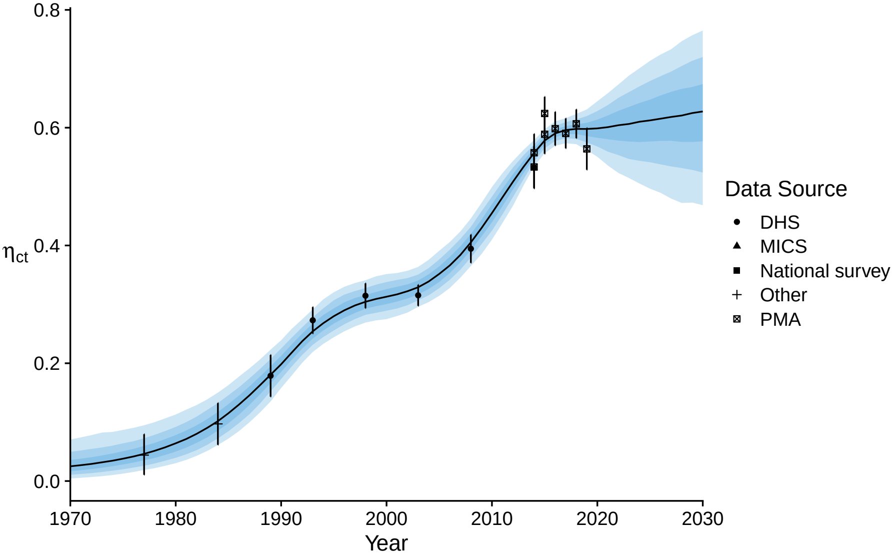
Kenya Transition Function



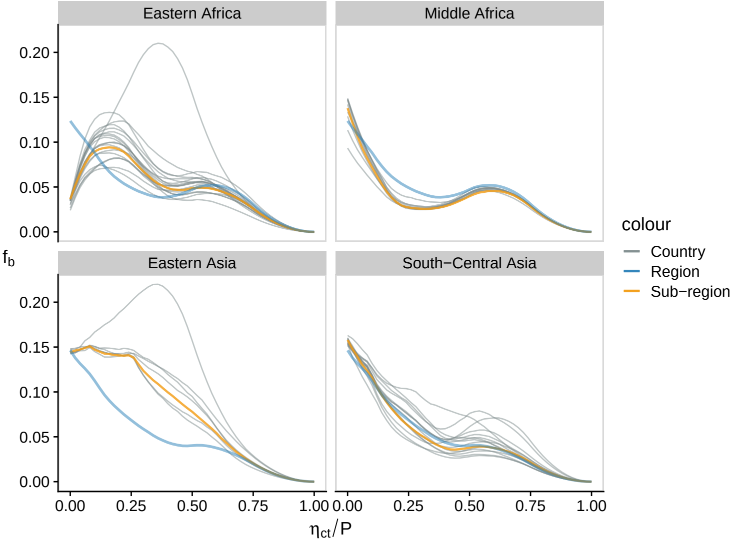
Kenya Smoothing Component



Kenya mCPR Estimates



Trends can be seen in regional and subregional transition functions



Summary

- Subclass of *Transition Models* for indicators that follow transitions.
- B-spline Transition Model: flexible modelling approach based on B-splines.
- Generated estimations and projections of mCPR in countries from 1970-2030.
- Found systematically different transitions in countries across regions.
- Flexible model framework that can be easily extended to new settings and use cases.

Choosing a spline specification

Validation exercise: hold out all observations after a cutoff year $L = 2010$.

	95% UI				Error	
	% Below	% Included	% Above	CI Width $\times 100$	ME $\times 100$	MAE $\times 100$
Model Check 2 ($L = 2010$), $n = 133$						
B-spline ($d = 2,\ K = 5$)	3.76%	94.7%	1.5%	32.0	-1.670	4.64
B-spline ($d = 2,\ K = 7$)	6.02%	91.7%	2.26%	31.5	-1.260	4.68
B-spline ($d = 3,\ K = 5$)	3.76%	94.7%	1.5%	32.4	-1.630	4.48
B-spline ($d = 3,\ K = 7$)	3.76%	94%	2.26%	31.6	-0.965	4.57

95% UI: 95% uncertainty interval. ME: median error. MAE: median absolute error.

Measures calculated using the last held-out observation within each area.

Sharing information on shape of transition function