Flexibly Modeling Shocks to Demographic and Health Indicators with Bayesian Shrinkage Priors

Herb Susmann[1] Leontine Alkema[2]

[1]NYU Grossman School of Medicine
[2]University of Massachusetts Amherst

PAA 2025
Washington, D.C., April 12, 2025

Slides:

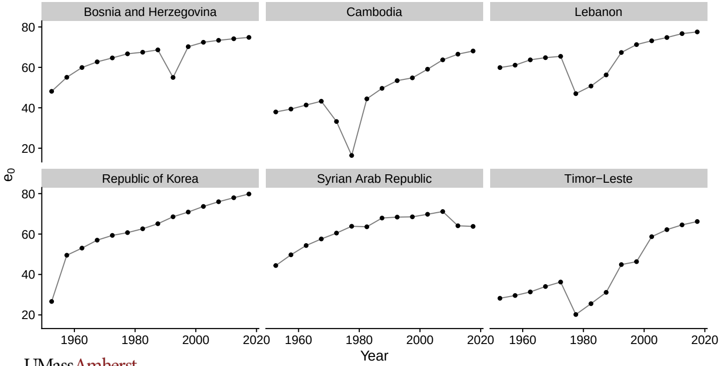
herbsusmann.com/paa2025

Preprint:

https://arxiv.org/abs/2410.09217

Motivation

- Goal: building statistical models for estimating and projecting demographic and health indicators.
- Many statistical models assume *smoothness* of the data.
- Statistical models that assume smoothness typically will not perform well when fit to data that exhibit shocks.
- **This talk:** we propose using Bayesian shrinkage priors as a practical way to build statistical models robust to shocks.



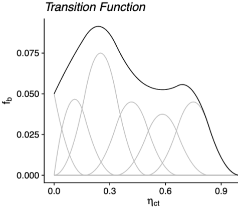
Smooth Transition Model

- Let $\eta_{c,t}$ be the true male period life expectancy at birth in country c and time t.
- Model change in $\eta_{c,t}$ as:

$$\eta_{c,t} = \eta_{c,t-1} + f(\eta_{c,t-1}, \boldsymbol{\beta}_c) + \epsilon_{c,t},$$

where
  - f: expected change with parameters $\boldsymbol{\beta}_c$
  - $\epsilon_{c,t}$: deviations from expected change

- We model f using B-splines (Susmann & Alkema 2025 JRSS-C).
  - Takeaway: f is flexible, but assumes change in life expectancy is smooth.



- Deviations typically modelled as ARIMA process; following Raftery et al. we use white noise for e_0:

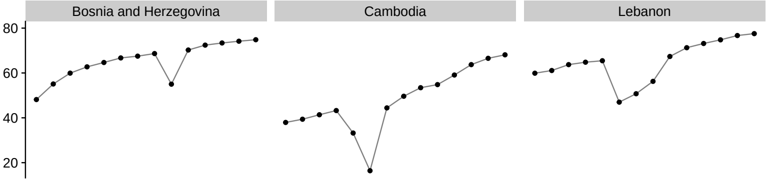
$$\epsilon_{c,t} | \tau_\epsilon \sim N(0, \tau_\epsilon^2).$$

Transition Model with Shocks

- Proposal: add an additional term to the transition model to handle shocks.

$$\eta_{c,t} = \eta_{c,t-1} + f(\eta_{c,t-1}, \boldsymbol{\beta}_c) - \delta_{c,t} + \epsilon_{c,t},$$

  where $\delta_{c,t} > 0$.
- We call $\delta_{c,t}$ the *shock term*.
- A-priori we do not think that $\delta_{c,t}$ will be large for most country-years.

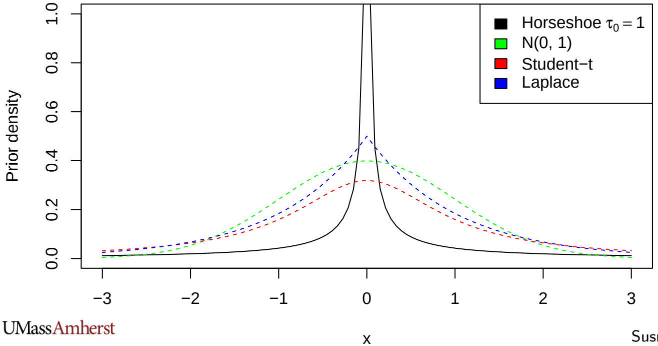


The horseshoe prior

- The *horseshoe prior* (Carvalho 2009 PMLR) is given by

$$\delta_{c,t} \mid \tau, \gamma_{c,t} \sim N(0, \tau^2 \gamma_{c,t}^2)$$
$$\gamma_{c,t} \sim C^+(0,1).$$

- Global scale parameter $\tau > 0$ shrinks all shocks to zero.
- Local scale parameters $\gamma_{c,t} > 0$ allow some shocks to escape shrinkage.



NYU Grossman School of Medicine UMass Amherst

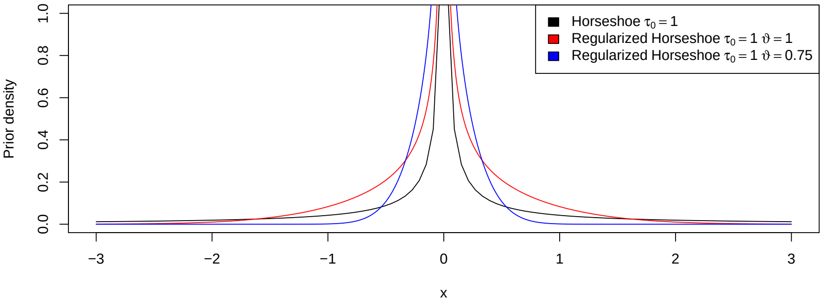
Regularized Horseshoe

- The *regularized horseshoe prior* (Piironen 2017 EJS) is given by

$$\begin{aligned}
\delta_{c,t} \mid \tau, \gamma_{c,t}, \vartheta &\sim N(0, \tau^2 \tilde{\gamma}^2_{c,t}), \\
\tilde{\gamma}^2_{c,t} &= \frac{\vartheta^2 \gamma^2_{c,t}}{\vartheta^2 + \tau^2 \gamma^2_{c,t}}, \\
\gamma_{c,t} &\sim C^+(0, 1), \\
\tau &\sim C^+(0, \tau_0^2).
\end{aligned}$$

- Global scale parameter $\tau > 0$ shrinks all shocks to zero.
- Local scale parameters $\gamma_{c,t}$ allow some shocks to escape shrinkage.
- "Slab scale" parameter $\vartheta > 0$ regularizes shocks that escape regularization.
  - For large $\delta_{c,t}$, prior approaches a Gaussian prior with variance ϑ^2

Regularized Horseshoe



Estimated shocks

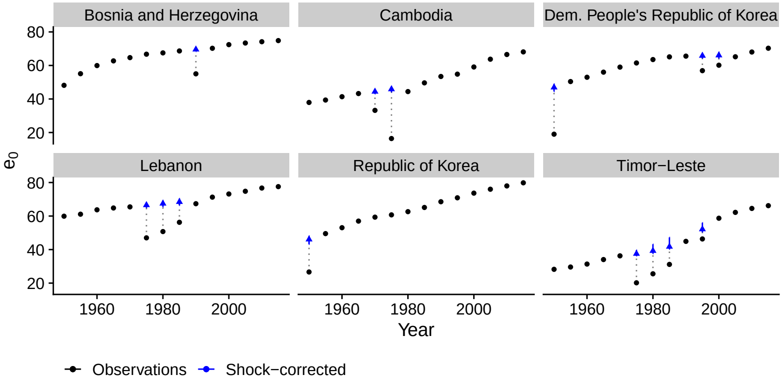


Figure: Six countries with the largest estimated detected shocks. Shocks are illustrated by plotting "shock-corrected" estimates, given by the observed e_0 minus the shock $\delta_{c,t}$

Example e_0 projections

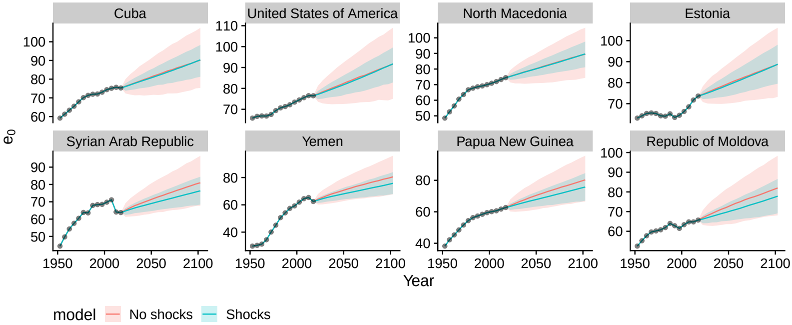


Figure: Projections of e_0 from the model with and without shocks, for countries with the smallest (top row) and largest (bottom row) differences in posterior median projected e_0 in 2095-2100.

Adding shocks has little effect on median projections, and reduces projection uncertainty

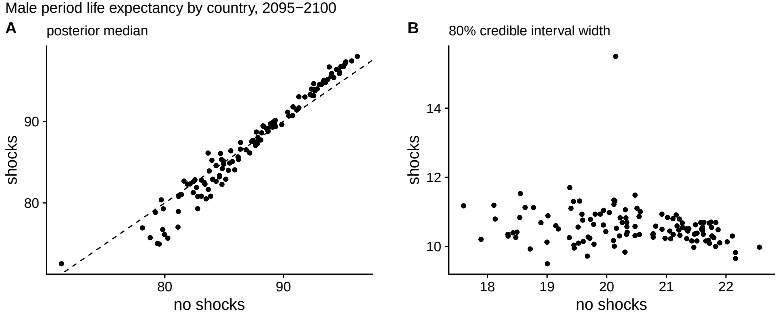


Figure: Posterior medians (A) and 80% projection interval widths (B) for male period life expectancy at birth by country in 2095-2100 for the model with and without shocks included.

Another example, mCPR: adding shocks improves historical estimates

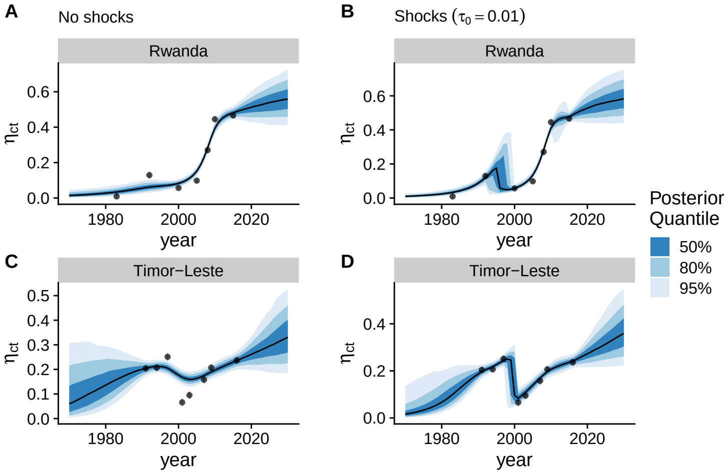


Figure: Modern Contraceptive Use Rate (mCPR) estimates with and without shocks.

Discussion

- Bayesian shrinkage priors provide a reasonable way to adapt models to handle demographic and health indicators that exhibit shocks.
- Important to carefully define the target indicator and goal of projections.
- Practically: regularized horseshoe prior available in `brms`, can be implemented in `Stan`.
- More details: `https://arxiv.org/abs/2410.09217`

Appendix: Tuning the prior

- Regularized horseshoe, full specification:

$$\begin{aligned}
\delta_{c,t} \mid \gamma_{c,t}, \tau, \vartheta &\sim N(0, \tau^2 \tilde{\gamma}_{c,t}^2), \\
\tilde{\gamma}_{c,t}^2 &= \frac{\vartheta^2 \gamma_{c,t}^2}{\vartheta^2 + \tau^2 \gamma_{c,t}^2}, \\
\gamma_{c,t} &\sim C^+(0, 1), \\
\tau &\sim C^+(0, \tau_0^2), \\
\vartheta^2 &\sim \text{Inv-Gamma}(\nu/2, \nu s^2/2).
\end{aligned}$$

- Calibrate priors by assuming
  - shock cannot exceed 100 life-expectancy years,
  - a 10% probability of shock exceeding 20 life-expectancy years,
  - and $P(\delta_{c,t} > \delta^*) \approx 0.5\%$.

Appendix: Defining shocks

- The distribution of the deviations $\epsilon_{c,t}$ characterize "regular" fluctuations in life-expectancy.
- We propose analyzing the shocks $\delta_{c,t}$ relative to the distribution of $\epsilon_{c,t}$.
- Define a *shock* as when the value of δ is greater than twice the marginal standard deviation of ϵ: $\delta > 2\mathrm{sd}(\epsilon) := \delta^*$

Appenix: Country with largest credible interval width

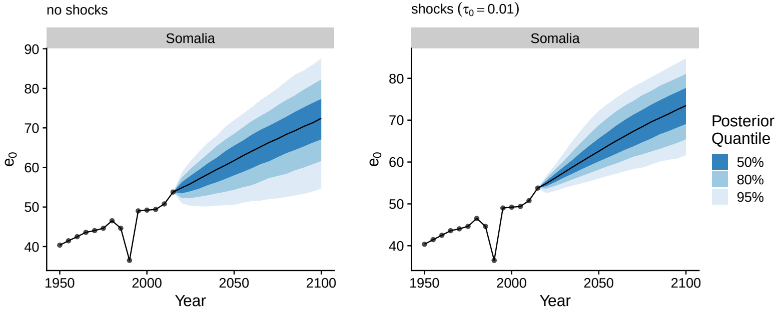


Figure: Projections of e_0 from the model with and without shocks for Somalia.

Appendix: Validations

- Out-of-sample model validations were used to compare predictive performance of the model with and without shocks.
- All observations after 2005-2010 held out, and fitted model used to predict observations from 2015-2020.

Appendix: Validation Results

Region	*n*	ME	MAE
Shocks			
Africa	11	0.49	1.14
Americas	25	-0.07	0.67
Asia & Oceania	50	0.62	0.86
Europe	35	1.10	1.10
Overall	*121*	*0.57*	*0.92*
No shocks			
Africa	11	0.42	1.27
Americas	25	-0.22	0.68
Asia & Oceania	50	0.67	1.02
Europe	35	0.76	0.78
Overall	*121*	*0.61*	*0.86*

Table: Validation results for the male period life expectancy at birth model with and without shocks. Included validation metrics are median error (ME), and median absolute error (MAE).

Appendix: Validation Results

Region	*n*	% Below	80% projection Interval % Included	% Above	PI Width
Shocks					
Africa	11	9.1%	72.7%	18.2%	3.12
Americas	25	12.0%	88.0%	0.0%	3.12
Asia & Oceania	50	4.0%	72.0%	24.0%	3.13
Europe	35	0.0%	71.4%	28.6%	3.08
Overall	*121*	*5.0%*	*75.2%*	*19.8%*	*3.11*
No shocks					
Africa	11	0.0%	100.0%	0.0%	7.55
Americas	25	0.0%	100.0%	0.0%	7.61
Asia & Oceania	50	4.0%	92.0%	4.0%	7.56
Europe	35	0.0%	100.0%	0.0%	7.64
Overall	*121*	*1.7%*	*96.7%*	*1.7%*	*7.60*

Table: Validation results for the male period life expectancy at birth model with and without shocks. Included validation metrics are the % of observations below, above, and included within the 80% projection interval, and the mean width of the 80% projection interval.