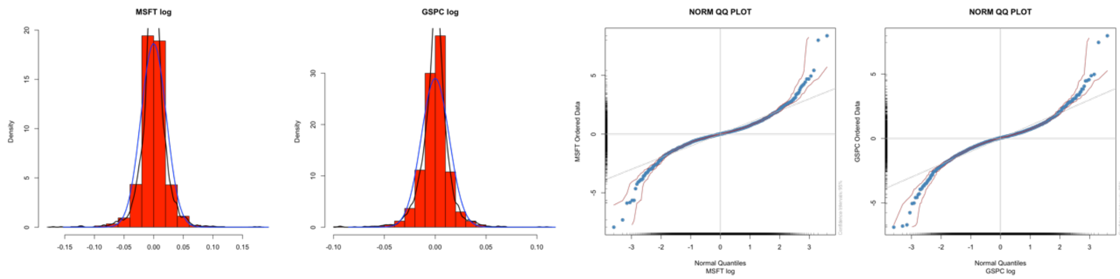
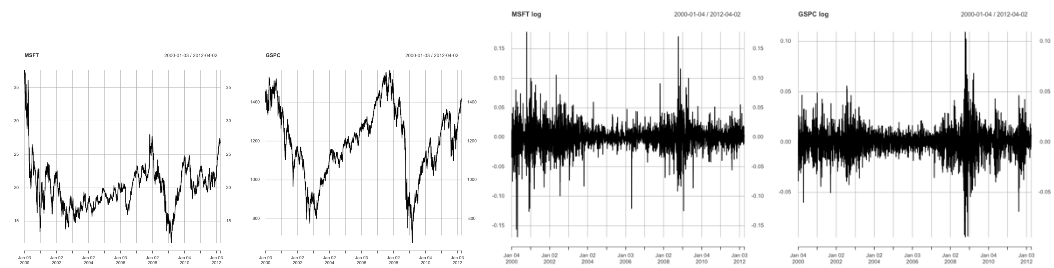
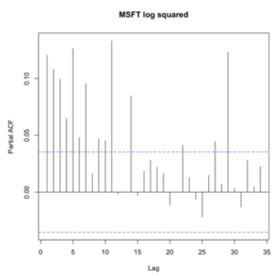
Steve Depp
413 – 55

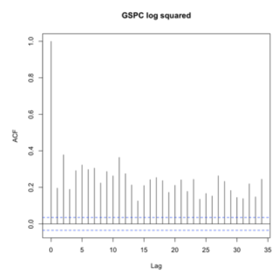
Assignment 9
26 August 2019



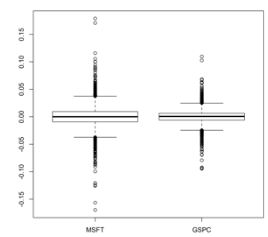





```
#######################
# KPSS Unit Root Test #
#######################

Test is of type: mu with 9 lags.

Value of test-statistic is: 2.7414

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```

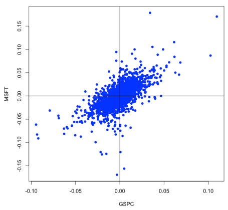



```
#######################
# KPSS Unit Root Test #
#######################

Test is of type: mu with 9 lags.

Value of test-statistic is: 0.9508

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```



	n	mean	sd	median	trimmed	min	max	range	skew	kurtosis
MSFT_ln	3081	-1.088848e-04	0.021388913	0.00000000	-3.324042e-05	-0.169576987	0.17869172	0.34826871	-0.0917266	8.5659247
GSPC_ln	3081	-8.171522e-06	0.013751547	0.00058855	1.774649e-04	-0.094695125	0.10957197	0.20426709	-0.1601957	7.1591387
MSFT_ln_sd	10	1.969796e-02	0.008850747	0.01623626	1.898500e-02	0.009228836	0.03587079	0.02664195	0.5504081	-1.2865145
GSPC_ln_sd	10	1.259973e-02	0.005820218	0.01251406	1.167151e-02	0.006417075	0.02620814	0.01979107	0.9827695	0.3169262

The **above table** presents sample size, mean, standard deviation, median, 20% trimmed mean, min, max, range, skewness, excess kurtosis for time series of MSFT and GSPC log returns and those log returns' standard deviations over 10 equal consecutive segments of the time series. The time series covers the period from the 1st trading day of January 2000 to 1st trading day of April 2012. **Time series and histograms above** appear to show zero means for both distributions. **QQ and time series plots and histogram above** exhibit fat tails and negative skew. MSFT outliers are slightly more skewed left than GSPC. **Time series and box plot** show longer spikes in both directions for MSFT than GSPC, and for more recent GSPC than more antique GSPC returns, but MSFT spiked regions on the right side of the **time series plot** are less thick than those for GSPC, seeming to suggest MSFT returns mean revert more quickly than GSPC returns. Tests of log returns cannot reject the <u>zero mean</u> null hypotheses but can reject the null hypothesis of zero <u>excess kurtosis</u> at the highest confidence level, and <u>zero skew</u> at the 99% and 95% confidence levels for MSFT and GSPC respectively. Augmented Dickey Fuller tests p-values = 0.01 for all return and return squared series allow rejection of null hypothesis of <u>unit roots</u> in these data. KPSS test statistics for all returns (not shown) are consistent with ADF tests allowing rejection of null hypothesis of no unit roots, but the same <u>tests for returns squared contradict ADF tests</u> allowing rejection of the null hypothesis of no <u>unit roots at the highest confidence level</u> (shown **above**). Repeated KPSS test for squared log returns suggested single differences. ACF plots **above** for the squared show significant but different patterns of auto correlation for the two stocks' log return series. Box-Ljung portmanteau p-values allow rejection of the hypothesis of no serial correlations at the highest of confidence levels for all series.




```
        Pearson's product-moment correlation

data:  MSFT.ret and GSPC.ret
t = 50.246, df = 3079, p-value < 2.2e-16
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.6513487 0.6901765
sample estimates:
      cor
0.6712227
```

```
Call:
lm(formula = msft_stdz ~ gspc_stdz)

Residuals:
    Min      1Q  Median      3Q     Max
-7.7641 -0.3065 -0.0113  0.3059  6.6921

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.004916   0.013357  -0.368    0.713
gspc_stdz    0.671223   0.013359  50.246   <2e-16 ***
```

Correlation between returns is apparent in the pair plot and measures 0.67 which is statistically significant at the highest of confidence levels, as shown **above**, and which matches the coefficent of the linear regression between the 2 log returns.
CCM and CCP
Arguably, the important difference between the BEKK models and the DCC models is that DCC models estimate parameters assuming a multivariate student-t distribution rather than a multivariate normal distribution.

1.1 First, estimate an ARCH(5) model for each series. What is the sum of the ARCH coefficients? What does the sum tell you?

All epsilon distributions are NID.

Model to be fitted:

$$r_t = \mu_t + a_t$$
$$a_t = \sigma_t \varepsilon_t$$
$$\sigma^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \alpha_2 a_{t-2}^2 + \alpha_3 a_{t-3}^2 + \alpha_4 a_{t-4}^2 + \alpha_5 a_{t-5}^2$$

Fitted model MSFT:

$$r_t = 0.000363 + a_t$$
$$a_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = 0.000116 + 0.160100a^2_{t-1} + 0.190518a^2_{t-2} + 0.141569a^2_{t-3} + 0.145010a^2_{t-4} + 0.201990a^2_{t-5}$$

Fitted model GSPC:

$$r_t = 0.000445 + a_t$$
$$a_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = 0.000037 + 0.054447a^2_{t-1} + 0.211255a^2_{t-2} + 0.178304a^2_{t-3} + 0.190940a^2_{t-4} + 0.189357a^2_{t-5}$$

Sum of coefficients

$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$
= 0.8394 for MSFT
= 0.8265 for GSPC

1.2 Next, estimate a GARCH(1,1) model for each series. What is the sum of the GARCH coefficients? Interpret and compare with the ARCH sum.

Model to be fitted:

$r_t = \mu_t + a_t$

$a_t = \sigma_t \varepsilon_t$

$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

Fitted model MSFT:

$r_t = 0.000343596 + a_t$

$a_t = \sigma_t \varepsilon_t$

$\sigma_t^2 = 0.000004795 + 0.069477030 a^2_{t-1} + 0.920093520 \sigma_{t-1}^2$

Fitted model GSPC:

$r_t = 0.000428987 + a_t$

$a_t = \sigma_t \varepsilon_t$

$\sigma_t^2 = 0.000001496 + 0.088282317 a^2_{t-1} + 0.902937178 \sigma_{t-1}^2$

Sum of coefficients:

$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$
= 0.9896 for MSFT
= 0.9896 for GSPC

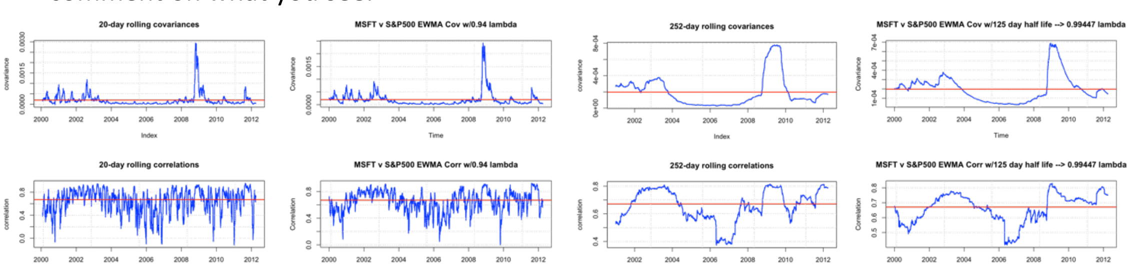
<u>Observations:</u>

a. <u>Means different</u>
Both the ARCH(5) and GARCH(1,1) specifications were run with ugarch (shown) and fGarch. All parameters and their significance were the same except one. The mean of the GSPC fit was the only parameter that differed by much, **4.28987e-4 shown above** from ugarch versus 8.172e-05 from fGARCH. The estimate from ugarch was significant while the fGarch parameter's test statistic yields a p-value that does not allow us to reject the null hypothesis of zero value.

b. The sum of ARCH(5) coefficients < 1 while those of GARCH(1,1) approximate 1 indicating that the GARCH model may be picking up patterns in the volatility that are less mean reverting than those in the ARCH(5) model. GARCH might be able to fit the data that has more extreme enduring moves away from the mean. I believe one might say there is a unit root available as indicated in the EDA **above** but I am not sure how this exactly translates to statements about mean reversion, yet.

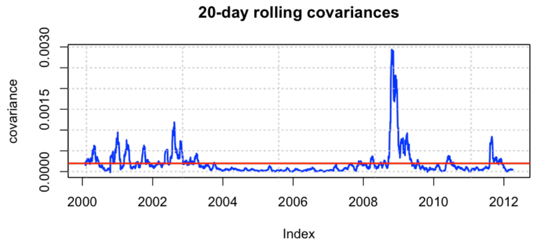
c. No residual analysis is performed for these models.

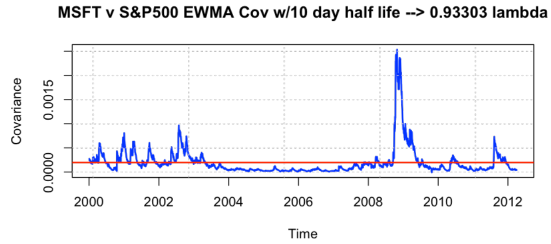
1.3 Using a 20-day moving window, compute and plot rolling covariances and correlations. Briefly comment on what you see.

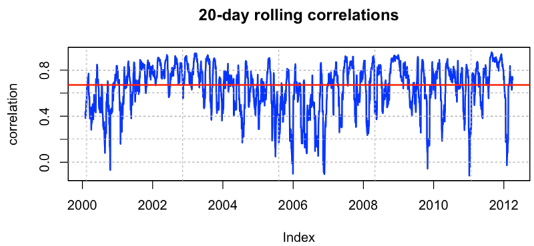


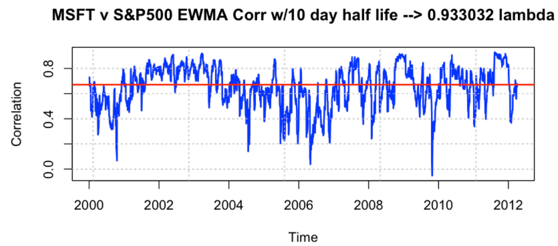
(a) Correlation is not constant. The variation in correlation would have an impact on variance estimation.

(b) The movement in covariance and correlation appear similar when modeled by 20-day rolling average compared with lambda=0.94 and when modeled with 252-day rolling averages compared with EWMA with 125-day half-life → Lambda of 99447.









Using similar rule for 20 day, a 10-day half-life → lambda = 0.93303, producing the plots on the right compared with the original 20-day rolling correlation plots on the left. Here the covariances appear to match up not as well as the corrrelations. There is more variability in the correlations modeled by 20 day rolling averages than correlations assembled by EWMA.

1.4 Let rt = $(r_{MSFT,t}, r_{GSPC,t})^T$. Using the dccfit() function from the rmgarch package estimate the normal-DCC(1,1) model. Briefly comment on the estimated coefficients and the fit of the model.

Model to be fitted:

$$r_{i,t} = \mu_{i,t} + a_{i,t}$$
$$a_{i,t} = \sigma_{i,t}\varepsilon_{i,t}$$
$$\sigma_{i,t}^2 = \alpha_{i,0} + \alpha_{i,1}a_{i,t-1}^2 + \beta_{i,1}\sigma_{i,t-1}^2$$

$$\sum_t \equiv [\sigma_{ij,t}] = D_t\,\rho_t\,D_t$$

where ρ_t is 2x2 conditional correlation matrix of a_t
and D_t is 2x2 conditional standard deviations of a_t

$$D_t = \text{diag}\{\sigma_{11,t}^{1/2}, \sigma_{22,t}^{1/2}\}$$
$$\rho_t = J_t\,Q_t\,J_t$$
$$Q_t = (1-\theta_1-\theta_2)Q^{bar} + \theta_1\varepsilon_{t-1}\varepsilon'_{t-1} + \theta_2 Q_{t-1}$$

where $0 < \theta_1 + \theta_2 < 1$

$$\varepsilon_{i,t} = \sum_t^{-1/2} a_{i,t}$$
$$Q^{bar} = \text{unconditional covariance matrix of } \varepsilon_t$$
$$J_t = \text{diag}\{q_{11,t}^{1/2}, q_{22,t}^{1/2}\}$$

Fitted model:

MSFT

$$r_{1,t} = 0.000343596 + a_{1,t}$$
$$a_{1,t} = \sigma_{1,t}\varepsilon_{1,t}$$
$$\sigma_{1,t}^2 = 0.000004795 + 0.069477030a_{1,t-1}^2 + 0.920093520\sigma_{1,t-1}^2$$

GSPC		
$r_{2,t}$	=	$0.000428987 + a_{2,t}$
$a_{2,t}$	=	$\sigma_{2,t}\varepsilon_{2,t}$
$\sigma_{2,t}^2$	=	$0.000001496 + 0.088282317a_{2,t-1}^2 + 0.902937178\sigma_{1,t-1}^2$
$\sum_t \equiv [\sigma_{ij,t}]$	=	$D_t\,\rho_t\,D_t$
		where ρ_t is 2x2 conditional correlation matrix of a_t
		and D_t is 2x2 conditional standard deviations of a_t
D_t	=	$\mathrm{diag}\{\sigma_{11,t}^{1/2}, \sigma_{22,t}^{1/2}\}$
ρ_t	=	$J_t\,Q_t\,J_t$
Q_t	=	$(1 - 0.024594 - 0.961329)Q^{bar}$
		$+ 0.024594\varepsilon_{t-1}\varepsilon'_{t-1} + 0.961329Q_{t-1}$
		where $0 < 0.024594 + 0.961329 = 0.985923 \sim 1.0$
ε_t	=	$\sum_t^{-1/2}a_{i,t}$
J_t	=	$\mathrm{diag}\{q_{11,t}^{1/2}, q_{22,t}^{1/2}\}$
Q^{bar}	=	$\begin{bmatrix} 1.0017 & 0.6303 \\ 0.6303 & 1.0009 \end{bmatrix}$
		… this doesn't seem right,
		but this is the output for "Qbar".

(a) There is some persistence to the model with theta's nearing 1. IGARCH behavior will mean that historical patterns will be more sustained by this model.

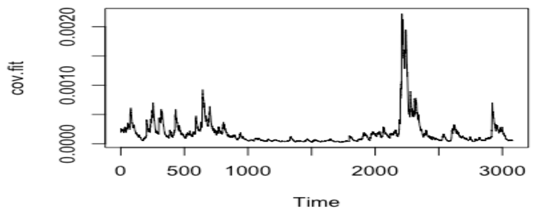
```
Optimal Parameters
-----------------------------------
                Estimate  Std. Error  t value Pr(>|t|)
[MSFT].mu       0.000344    0.000316  1.08676 0.277142
[MSFT].omega    0.000005    0.000013  0.37304 0.709122
[MSFT].alpha1   0.069477    0.014053  4.94404 0.000001
[MSFT].beta1    0.920094    0.037201 24.73308 0.000000
[GSPC].mu       0.000429    0.000167  2.57203 0.010110
[GSPC].omega    0.000001    0.000001  1.02831 0.303803
[GSPC].alpha1   0.088344    0.021669  4.07698 0.000046
[GSPC].beta1    0.902875    0.021161 42.66713 0.000000
[Joint]dcca1    0.024594    0.011369  2.16330 0.030518
[Joint]dccb1    0.961329    0.021460 44.79623 0.000000
```

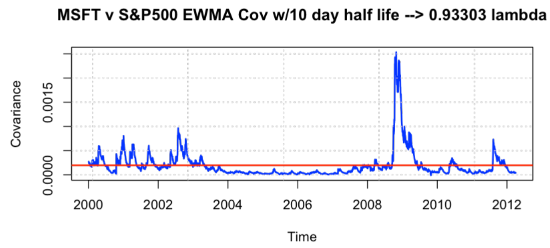
(b) p-values for 3 of 10 parameters ($\mu_{1,t}$ and $\alpha_{i,0}$) and would not allow rejection of null hypothesis of zero value. We could re estimate without mean values to see if other parameters are affected. Otherwise parameters are non-zero at the 95% or higher confidence level.

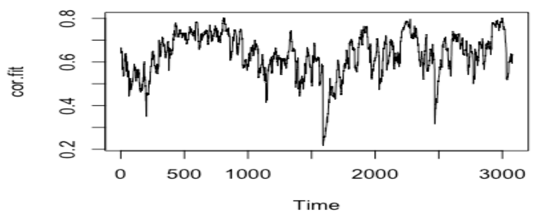
(c) The AIC of individual models are -5.2141 MSFT and -6.1981 GSPC versus -11.945 for the combined model (11 parameters).

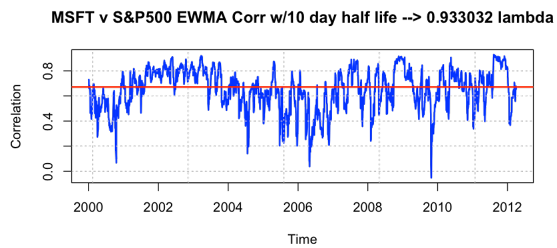
(d) Separate Box-Ljung portmanteau test of MSFT and GSPC standardized residuals yielded p-values 0.5 and 0.08. Thus the model is adequate for at least the separate residuals. Similar results were obtained for the squared standardized residuals.

1.5 Plot the estimated in-sample conditional covariances and correlations of the DCC model. Compare with the EWMA and rolling estimates.



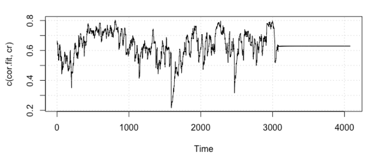


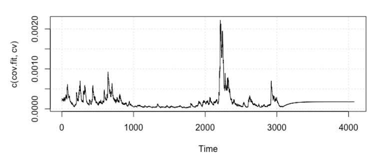




The DCC model covariance and correlation are shown **above on the left** and the EWMA covariance are replotted **above on the right**. Recall that the EWMA are half-life estimated lambdas for 20 days. The match for covariance appears better than for correlation. The same was true for comparing rolling 20-day to EWMA above: the covariances appeared to match up better than the correlations. The moves to sharply lower or higher levels are more sustained in the GARCH model than in the EWMA which might reflect the IGARCH behavior mentioned above.

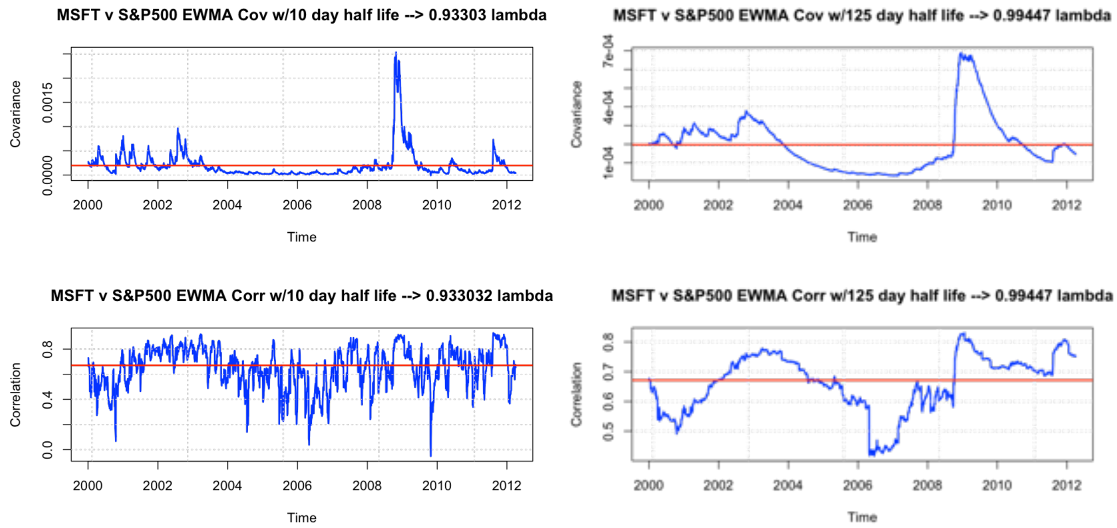
1.6 Using the Estimated DCC(1,1) model, compute (using dccforecast() function) and plot the first 100 h-step ahead forecasts of conditional covariance and correlation.





This is 1000 h-steps ahead forecasts so that one can see the movement. Correlation establishes forecasts at the current correlation mean and covariance is affected by the model prediction for conditional variance, and so it sloped upward until reaching its mean level.

2 Report (3.5 points) Write an executive summary of your MSFT and GSPC analysis.



The returns from holding MSFT and GSPC are on average zero. Predicting these returns individually is assisted by employing their volatility. Predicting their joint return is assisted by knowing their covariance. Predicting the volatility of holding these stocks can be also be assisted by knowing the covariance between returns which varies considerably whether your holding period is shorter (a month as shown on the LHS above) or longer (a year as shown on the RHS above). The models estimating joint returns demonstrate an improvement over models of individual returns largely because of an ability for model covariance. We should continue to look into modelling covariance.