matrix chain multiplication

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$p \times q$, $q \times r$

$p \to r$.

$((10 \times 100 \quad 100 \times 50) \quad 50 \times 5$ or $10 \times 100 \quad (100 \times 50 \quad 50 \times 5)$

$10 \times 50 \times 5$ or $10 \times 100 \times 5$

$= 4500$ ✓ 5000.

No of Paranthesization.

If $n=1$

(A_1)

If $N=2$

$(A_1 A_2)$

If $n=3$

$(P_1 P_2) P_3$

$P_1 (P_2 P_3)$

$P_1 P_2 + P_2 P_3$

$$P(n) = \begin{cases} 1 & ; \ n=1 \\ \sum_{k=1}^{n-1} (P_k)(P_{n-k}) & ; \ n \geq 2. \end{cases}$$

4 step sequence

i.) structure of optimal paranthesization

ii.) Recursively define the value of an opt. Sol

iii.) Compute the value of an optimal Sol

iv.) Construct an optimal solution from computed opt Soln.

Structure of OP (())

Recursively compute value of opt. solⁿ

solve.

Structure of Optimal Paranthesization.

$(A_1, A_2, A_3, \ldots A_K)(A_{K+1}, \ldots, A_n)$

optimal Substructure of a problem..

Recursive solution.

$$m[i,j] = \begin{cases} 0 & ; \ i=j \\ m[i][k] + m[k+1][j] + p_{i-1} p_j p_k & ; \ i<j \end{cases}$$

since k can have only possible values from i to $j-i$

$S[i,j]$ = value of k where chain is splitting.

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* we have relatively lower subproblems.
1 for each i and j, $1 \leq i \leq j \leq n$.

we have if $i < j$, $nC_2 = \frac{n(n-1)}{2}$

and when $i = j$, n.

$\therefore$ Total of $\frac{n(n-1)}{2} + n = \Theta(n^2)$ subproblems.

If we recursively call, we may encounter each subproblem many times,

This is called overlapping subproblem. (2^{nd} Hallmark)

i.) optimal substructure

ii.) overlapping subproblem.

Instead of recursion, use tabular approach (bottom up) using memoization.