Perhitungan Perumusan Kovarian Gravitasi Teleparalel untuk Ruang-Waktu Vaidya

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Persiapan

In[ ]:=
```
Get["https://raw.githubusercontent.com/bshoshany/OGRe/master/OGRe.m"]
```

OGRe: An Object-Oriented General Relativity Package for Mathematica

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GitHub repository: https://github.com/bshoshany/OGRe

OGRe:
- To view the full documentation for the package, type TDocs[].
- To list all available modules, type ?OGRe`*.
- To get help on a particular module, type ? followed by the module name.
- To enable parallelization, type TSetParallelization[True].
- To disable automatic checks for updates at startup, type TSetAutoUpdates[False].

Mendefinisikan koordinat

In[ ]:=
```
TNewCoordinates["Eddington", {v, r, θ, ϕ}]
```

Out[ ]=

Eddington

Mendefinisikan metrik ruang singgung (Minkowski)

In[ ]:=
```
TShow@TNewMetric["TangentMetric", "Eddington", DiagonalMatrix[{1, -1, -1, -1}], "η"]
```

OGRe: TangentMetric: $\eta_{\mu\nu}(v, r, \theta, \phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

Mendefinisikan tensor metrik Vaidya

In[∘]:=
```
TShow@TNewTensor["Vaidya", "TangentMetric",
  "Eddington", {-1, -1}, {{Δ[v, r]^2, -ϵ, 0, 0}, {-ϵ, 0, 0, 0},
   {0, 0, -r^2, 0}, {0, 0, 0, -r^2 * Sin[θ]^2}}, "g"]
```

OGRe: Vaidya: $g_{\mu\nu}(v, r, \theta, \phi) = \begin{pmatrix} \Delta[v,r]^2 & -\epsilon & 0 & 0 \\ -\epsilon & 0 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$

Mendefinisikan tensor invers bagi metrik Vaidya

In[∘]:=
```
TShow@TNewTensor["InvVaidya", "TangentMetric",
  "Eddington", {1, 1}, Inverse[{{Δ[v, r]^2, -ϵ, 0, 0},
    {-ϵ, 0, 0, 0}, {0, 0, -r^2, 0}, {0, 0, 0, -r^2 * Sin[θ]^2}}], "g"]
```

OGRe: InvVaidya: $g^{\mu\nu}(v, r, \theta, \phi) = \begin{pmatrix} 0 & -\frac{1}{\epsilon} & 0 & 0 \\ -\frac{1}{\epsilon} & -\frac{\Delta[v,r]^2}{\epsilon^2} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{\mathrm{Csc}[\theta]^2}{r^2} \end{pmatrix}$

Mengecek apakah kontraksi metrik menghasilkan delta kronecker

In[∘]:=
```
TShow@TCalc["Vaidya"["ab"].“InvVaidya"["bc"]]
```

OGRe: Result: $\square_{\mu}{}^{\nu}(v, r, \theta, \phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Mendefinisikan delta kronecker

In[∘]:=
```
TShow@TNewTensor["Kronecker",
  "TangentMetric", "Eddington", {-1, 1}, IdentityMatrix[4], "δ"]
```

OGRe: Kronecker: $\delta_{\mu}{}^{\nu}(v, r, \theta, \phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Mendefinsikan tensor tetrad

In[ ]:=
```
TShow@TNewTensor["Tetrad h", "TangentMetric",
  "Eddington", {1, -1}, {{Sqrt[2] * Δ[v, r], -(Sqrt[2] + 1) ϵ / Δ[v, r], 0, 0},
   {Δ[v, r], -(Sqrt[2] + 1) ϵ / Δ[v, r], 0, 0}, {0, 0, r, 0}, {0, 0, 0, r * Sin[θ]}}, "h"]
```

OGRe: Tetrad h: $h^{\mu}{}_{\nu}(v, r, \theta, \phi) = \begin{pmatrix} \sqrt{2}\,\Delta[v,r] & -\frac{(1+\sqrt{2})\epsilon}{\Delta[v,r]} & 0 & 0 \\ \Delta[v,r] & -\frac{(1+\sqrt{2})\epsilon}{\Delta[v,r]} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r\,\mathrm{Sin}[\theta] \end{pmatrix}$

Mendefinsikan tensor inverse tetrad

In[ ]:=
```
TShow@TNewTensor["InvTetrad h", "TangentMetric", "Eddington",
  {-1, 1}, Inverse[Transpose[{{Sqrt[2] * Δ[v, r], -(Sqrt[2] + 1) ϵ / Δ[v, r], 0, 0},
     {Δ[v, r], -(Sqrt[2] + 1) ϵ / Δ[v, r], 0, 0}, {0, 0, r, 0}, {0, 0, 0, r * Sin[θ]}}]], "h"]
```

OGRe: InvTetrad h: $h_{\mu}{}^{\nu}(v, r, \theta, \phi) = \begin{pmatrix} \frac{1+\sqrt{2}}{\Delta[v,r]} & \frac{\Delta[v,r]}{\epsilon} & 0 & 0 \\ -\frac{1+\sqrt{2}}{\Delta[v,r]} & -\frac{\sqrt{2}\,\Delta[v,r]}{\epsilon} & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{Csc}[\theta]}{r} \end{pmatrix}$

Mengecek ortogonalitas tetrad

In[ ]:=
```
TShow@TCalc["Tetrad h"["ab"].“InvTetrad h"["ac"]]
```

OGRe: Result: $\square_{\mu}{}^{\nu}(v, r, \theta, \phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Mengecek apakah kontraksi tetrad dengan metrik ruang singgung menghasilkan metrik ruang waktu

In[ ]:=
```
TShow@TCalc["TangentMetric"["ab"].“Tetrad h"["aμ"].“Tetrad h"["bν"]]
```

OGRe: Result: $\square_{\mu\nu}(v, r, \theta, \phi) = \begin{pmatrix} \Delta[v,r]^2 & -\epsilon & 0 & 0 \\ -\epsilon & 0 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$

Mendefinisikan tensor tetrad referensi

In[•]:=
```
TShow@TNewTensor["Tetrad e", "TangentMetric",
  "Eddington", {1, -1}, {{Sqrt[2], -(Sqrt[2] + 1) ϵ, 0, 0},
   {1, -(Sqrt[2] + 1) ϵ, 0, 0}, {0, 0, r, 0}, {0, 0, 0, r * Sin[θ]}}, "e"]
```

OGRe: Tetrad e: $e^{\mu}{}_{\nu}(v, r, \theta, \phi) = \begin{pmatrix} \sqrt{2} & -\left(\left(1+\sqrt{2}\right)\epsilon\right) & 0 & 0 \\ 1 & -\left(\left(1+\sqrt{2}\right)\epsilon\right) & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r\,\mathrm{Sin}[\theta] \end{pmatrix}$

Mendefinisikan tensor invers tetrad referensi

In[•]:=
```
TShow@TNewTensor["InvTetrad e", "TangentMetric", "Eddington",
  {-1, 1}, Inverse[Transpose[{{Sqrt[2], -(Sqrt[2] + 1) ϵ, 0, 0},
     {1, -(Sqrt[2] + 1) ϵ, 0, 0}, {0, 0, r, 0}, {0, 0, 0, r * Sin[θ]}}]], "e"]
```

OGRe: InvTetrad e: $e_{\mu}{}^{\nu}(v, r, \theta, \phi) = \begin{pmatrix} 1+\sqrt{2} & \frac{1}{\epsilon} & 0 & 0 \\ -1-\sqrt{2} & -\frac{\sqrt{2}}{\epsilon} & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{Csc}[\theta]}{r} \end{pmatrix}$

Mengecek ortogonalitas tetrad referensi

In[•]:=
```
TShow@TCalc["Tetrad e"["ab"]."InvTetrad e"["cb"]]
```

OGRe: Result: $\square^{\mu}{}_{\nu}(v, r, \theta, \phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Menghitung potensial tera/tetrad gravitasi

In[•]:=
```
TShow@TCalc["Tetrad B", "Tetrad h"["ab"] - "Tetrad e"["ab"], "B"]
```

OGRe: Tetrad B: $B^{\mu}{}_{\nu}(v, r, \theta, \phi) = \begin{pmatrix} \sqrt{2}\,(-1+\Delta[v,r]) & \frac{\left(1+\sqrt{2}\right)\epsilon\,(-1+\Delta[v,r])}{\Delta[v,r]} & 0 & 0 \\ -1+\Delta[v,r] & \frac{\left(1+\sqrt{2}\right)\epsilon\,(-1+\Delta[v,r])}{\Delta[v,r]} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Menampilkan komponen tak lenyap tetrad gravitasi

In[ ]:=
```
TList["Tetrad B"]
```

OGRe:

Tetrad B:

$B^{v}{}_{v} = \sqrt{2}\,(-1+\Delta[v,r])$

$B^{v}{}_{r} = B^{r}{}_{r} = \frac{\left(1+\sqrt{2}\right)\epsilon\,(-1+\Delta[v,r])}{\Delta[v,r]}$

$B^{r}{}_{v} = -1+\Delta[v,r]$

Meghitung invers bagi tetrad gravitasi

In[ ]:=
```
TShow@TCalc["InvTetrad B", "InvTetrad h"["ab"] - "InvTetrad e"["ab"], "B"]
```

OGRe: InvTetrad B: $B_{\mu}{}^{\nu}(v,r,\theta,\phi) = \begin{pmatrix} -\frac{\left(1+\sqrt{2}\right)(-1+\Delta[v,r])}{\Delta[v,r]} & \frac{-1+\Delta[v,r]}{\epsilon} & 0 & 0 \\ \frac{\left(1+\sqrt{2}\right)(-1+\Delta[v,r])}{\Delta[v,r]} & -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{\epsilon} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Mengecek identitas $e^{a}{}_{\mu} + B^{a}{}_{\mu} = h^{a}{}_{\mu}$

In[ ]:=
```
TShow@TCalc["Tetrad e"["ab"] + "Tetrad B"["ab"] - "Tetrad h"["ab"]]
```

OGRe: Result: $\square^{\mu}{}_{\nu}(v,r,\theta,\phi) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Menghitung koneksi spin

Menghitung koefisien anholonomi dari tetrad referensi

$f^{c}{}_{a\,b} = -(\partial_{\mu} e^{c}{}_{\nu} - \partial_{\nu} e^{c}{}_{\mu})\, e_{a}{}^{\mu}\, e_{b}{}^{\nu}$

(urutan perkalian ruas kanan untuk semua perhitungan mungkin berbeda dengan yang ada di landasan teori untuk menyesuaikan urutan indeks)

In[∘]:=
```
TShow@TCalc["Anholonomy e",
  - (TPartialD["ν"]."Tetrad e"["cμ"] - TPartialD["μ"]."Tetrad e"["cν"]).
   "InvTetrad e"["aμ"]."InvTetrad e"["bν"], "f"]
```

OGRe: Anholonomy e: $f^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) = \begin{pmatrix} \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \\ \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \\ \begin{pmatrix}0\\0\\\frac{1}{r\epsilon}\\0\end{pmatrix} & \begin{pmatrix}0\\0\\-\frac{\sqrt{2}}{r\epsilon}\\0\end{pmatrix} & \begin{pmatrix}-\frac{1}{r\epsilon}\\\frac{\sqrt{2}}{r\epsilon}\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \\ \begin{pmatrix}0\\0\\0\\\frac{1}{r\epsilon}\end{pmatrix} & \begin{pmatrix}0\\0\\0\\-\frac{\sqrt{2}}{r\epsilon}\end{pmatrix} & \begin{pmatrix}0\\0\\0\\\frac{\mathrm{Cot}[\theta]}{r}\end{pmatrix} & \begin{pmatrix}-\frac{1}{r\epsilon}\\\frac{\sqrt{2}}{r\epsilon}\\-\frac{\mathrm{Cot}[\theta]}{r}\\0\end{pmatrix} \end{pmatrix}$

Menghitung koneksi spin

$$\omega^{a}{}_{b\,\mu} = \frac{1}{2}\left(-f^{a}{}_{b\,c} + f^{ba}{}_{c} + f^{ca}{}_{b}\right) e^{c}{}_{\mu}$$

In[∘]:=
```
TShow@TCalc["SpinConnection",
  1/2 * (-"Anholonomy e"["abc"] + "Anholonomy e"["bac"] + "Anholonomy e"["cab"]).
   "Tetrad e"["cμ"], "ω"]
```

OGRe: SpinConnection: $\omega^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) = \begin{pmatrix} \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\\frac{1}{\epsilon}\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\\frac{\mathrm{Sin}[\theta]}{\epsilon}\end{pmatrix} \\ \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\\frac{\sqrt{2}}{\epsilon}\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\\frac{\sqrt{2}\,\mathrm{Sin}[\theta]}{\epsilon}\end{pmatrix} \\ \begin{pmatrix}0\\0\\\frac{1}{\epsilon}\\0\end{pmatrix} & \begin{pmatrix}0\\0\\-\frac{\sqrt{2}}{\epsilon}\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\-\mathrm{Cos}[\theta]\end{pmatrix} \\ \begin{pmatrix}0\\0\\0\\\frac{\mathrm{Sin}[\theta]}{\epsilon}\end{pmatrix} & \begin{pmatrix}0\\0\\0\\-\frac{\sqrt{2}\,\mathrm{Sin}[\theta]}{\epsilon}\end{pmatrix} & \begin{pmatrix}0\\0\\0\\\mathrm{Cos}[\theta]\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \end{pmatrix}$

Komponen tak lenyap dari koneksi spin $\omega^{a}{}_{b\,\mu}$

In[ ]:=
```
TList["SpinConnection"]
```

SpinConnection:

OGRe:
$$\omega^{v}{}_{\theta\theta} = \omega^{\theta}{}_{v\theta} = \frac{1}{\epsilon}$$
$$\omega^{v}{}_{\phi\phi} = \omega^{\phi}{}_{v\phi} = \frac{\mathrm{Sin}[\theta]}{\epsilon}$$
$$\omega^{r}{}_{\theta\theta} = -\omega^{\theta}{}_{r\theta} = \frac{\sqrt{2}}{\epsilon}$$
$$\omega^{r}{}_{\phi\phi} = -\omega^{\phi}{}_{r\phi} = \frac{\sqrt{2}\,\mathrm{Sin}[\theta]}{\epsilon}$$
$$\omega^{\theta}{}_{\phi\phi} = -\omega^{\phi}{}_{\theta\phi} = -\mathrm{Cos}[\theta]$$

Torsi

$T^{a}{}_{\mu\nu}$ dihitung dengan tetrad referensi e

In[ ]:=
```
TShow@TCalc["Kronecker"["ac"].TPartialD["μ"]."Tetrad e"["aν"] -
  "Kronecker"["ac"].TPartialD["ν"]."Tetrad e"["aμ"] +
  "Kronecker"["ac"]."SpinConnection"["abμ"]."Tetrad e"["bν"] -
  "Kronecker"["ac"]."SpinConnection"["abν"]."Tetrad e"["bμ"]]
```

OGRe: Result:
$$\square^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) = \begin{pmatrix} \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \\ \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \\ \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \\ \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \end{pmatrix}$$

$T^a{}_{\mu\nu}$ dihitung dengan tetrad gravitasi B

In[ ]:=
```
TList@TCalc[-"Kronecker"["ac"].(TPartialD["μ"]."Tetrad B"["aν"] -
      TPartialD["ν"]."Tetrad B"["aμ"] + "SpinConnection"["abμ"]."Tetrad B"["bν"] -
      "SpinConnection"["abν"]."Tetrad B"["bμ"]), "T"]
```

Result:

OGRe:

$$T^{v}{}_{vr} = -T^{v}{}_{rv} = -\sqrt{2}\,\partial_r \Delta[v,r] + \frac{(1+\sqrt{2})\,\epsilon\,\partial_v \Delta[v,r]}{\Delta[v,r]^2}$$

$$T^{r}{}_{vr} = -T^{r}{}_{rv} = -\partial_r \Delta[v,r] + \frac{(1+\sqrt{2})\,\epsilon\,\partial_v \Delta[v,r]}{\Delta[v,r]^2}$$

$$T^{\theta}{}_{r\theta} = -T^{\theta}{}_{\theta r} = 1 - \frac{1}{\Delta[v,r]}$$

$$T^{\phi}{}_{r\phi} = \mathrm{Sin}[\theta] - \frac{\mathrm{Sin}[\theta]}{\Delta[v,r]}$$

$$T^{\phi}{}_{\phi r} = \mathrm{Sin}[\theta]\left(-1 + \frac{1}{\Delta[v,r]}\right)$$

$T^a{}_{\mu\nu}$ dihitung dari tetrad h

In[ ]:=
```
TList@TCalc[-1 * ("Kronecker"["ac"].TPartialD["μ"]."Tetrad h"["aν"] -
     "Kronecker"["ac"].TPartialD["ν"]."Tetrad h"["aμ"] +
     "Kronecker"["ac"]."SpinConnection"["abμ"]."Tetrad h"["bν"] -
     "Kronecker"["ac"]."SpinConnection"["abν"]."Tetrad h"["bμ"]), "T"]
```

Result:

OGRe:

$$T^{v}{}_{vr} = -T^{v}{}_{rv} = \sqrt{2}\,\partial_r \Delta[v,r] - \frac{(1+\sqrt{2})\,\epsilon\,\partial_v \Delta[v,r]}{\Delta[v,r]^2}$$

$$T^{r}{}_{vr} = -T^{r}{}_{rv} = \partial_r \Delta[v,r] - \frac{(1+\sqrt{2})\,\epsilon\,\partial_v \Delta[v,r]}{\Delta[v,r]^2}$$

$$T^{\theta}{}_{r\theta} = -T^{\theta}{}_{\theta r} = -1 + \frac{1}{\Delta[v,r]}$$

$$T^{\phi}{}_{r\phi} = \mathrm{Sin}[\theta]\left(-1 + \frac{1}{\Delta[v,r]}\right)$$

$$T^{\phi}{}_{\phi r} = \mathrm{Sin}[\theta] - \frac{\mathrm{Sin}[\theta]}{\Delta[v,r]}$$

Perhitungan koneksi Weitzenboeck

In[ ]:=
```
TShow@TCalc["Weitzenbock",
  ("SpinConnection"["abμ"]."Tetrad h"["bν"] + TPartialD["μ"]."Tetrad h"["aν"]).
   "Kronecker"["ρμ"], "Γ"]
```

OGRe: Weitzenbock: $\Gamma^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) =$

$$\begin{pmatrix} \begin{pmatrix} \sqrt{2}\,\partial_v \Delta[v,r] \\ \sqrt{2}\,\partial_r \Delta[v,r] \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{(1+\sqrt{2})\,\epsilon\,\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ \frac{(1+\sqrt{2})\,\epsilon\,\partial_r \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \frac{r}{\epsilon} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{r\,\mathrm{Sin}[\theta]^2}{\epsilon} \end{pmatrix} \\ \begin{pmatrix} \partial_v \Delta[v,r] \\ \partial_r \Delta[v,r] \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{(1+\sqrt{2})\,\epsilon\,\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ \frac{(1+\sqrt{2})\,\epsilon\,\partial_r \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \frac{\sqrt{2}\,r}{\epsilon} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{\sqrt{2}\,r\,\mathrm{Sin}[\theta]^2}{\epsilon} \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\Delta[v,r]} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -r\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta] \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{\mathrm{Sin}[\theta]}{\Delta[v,r]} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ r\,\mathrm{Cos}[\theta] \end{pmatrix} & \begin{pmatrix} 0 \\ \mathrm{Sin}[\theta] \\ r\,\mathrm{Cos}[\theta] \\ 0 \end{pmatrix} \end{pmatrix}$$

Menampilkan daftar koneksi Weitzenboeck yang tak lenyap

In[∘]:=
```
TList["Weitzenbock"]
```

Weitzenbock:

OGRe:

$$\Gamma^{v}{}_{vv} = \sqrt{2}\,\partial_v \Delta[v,r]$$
$$\Gamma^{v}{}_{vr} = \sqrt{2}\,\partial_r \Delta[v,r]$$
$$\Gamma^{v}{}_{rv} = \Gamma^{r}{}_{rv} = \frac{\left(1+\sqrt{2}\right)\epsilon\,\partial_v \Delta[v,r]}{\Delta[v,r]^2}$$
$$\Gamma^{v}{}_{rr} = \Gamma^{r}{}_{rr} = \frac{\left(1+\sqrt{2}\right)\epsilon\,\partial_r \Delta[v,r]}{\Delta[v,r]^2}$$
$$\Gamma^{v}{}_{\theta\theta} = \frac{r}{\epsilon}$$
$$\Gamma^{v}{}_{\phi\phi} = \frac{r\,\mathrm{Sin}[\theta]^2}{\epsilon}$$
$$\Gamma^{r}{}_{vv} = \partial_v \Delta[v,r]$$
$$\Gamma^{r}{}_{vr} = \partial_r \Delta[v,r]$$
$$\Gamma^{r}{}_{\theta\theta} = \frac{\sqrt{2}\,r}{\epsilon}$$
$$\Gamma^{r}{}_{\phi\phi} = \frac{\sqrt{2}\,r\,\mathrm{Sin}[\theta]^2}{\epsilon}$$
$$\Gamma^{\theta}{}_{r\theta} = \frac{1}{\Delta[v,r]}$$
$$\Gamma^{\theta}{}_{\theta r} = 1$$
$$\Gamma^{\theta}{}_{\phi\phi} = -r\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]$$
$$\Gamma^{\phi}{}_{r\phi} = \frac{\mathrm{Sin}[\theta]}{\Delta[v,r]}$$
$$\Gamma^{\phi}{}_{\theta\phi} = \Gamma^{\phi}{}_{\phi\theta} = r\,\mathrm{Cos}[\theta]$$
$$\Gamma^{\phi}{}_{\phi r} = \mathrm{Sin}[\theta]$$

Perhitungan torsi $T^{a}{}_{\mu\nu}$ dari koneksi Weitzenboeck

In[ ]:=
```
TShow@TCalc["Torsion mu nu", -1 * ("Weitzenbock"["aμν"] - "Weitzenbock"["aνμ"]), "T"]
```

OGRe: Torsion mu nu: $T^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) =$

$$\begin{pmatrix}
\begin{pmatrix} 0 \\ -\sqrt{2}\,\partial_r\Delta[v,r] + \frac{(1+\sqrt{2})\,\epsilon\,\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \sqrt{2}\,\partial_r\Delta[v,r] - \frac{(1+\sqrt{2})\,\epsilon\,\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ -\partial_r\Delta[v,r] + \frac{(1+\sqrt{2})\,\epsilon\,\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \partial_r\Delta[v,r] - \frac{(1+\sqrt{2})\,\epsilon\,\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 1 - \frac{1}{\Delta[v,r]} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ -1 + \frac{1}{\Delta[v,r]} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathrm{Sin}[\theta] - \frac{\mathrm{Sin}[\theta]}{\Delta[v,r]} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ \mathrm{Sin}[\theta]\left(-1 + \frac{1}{\Delta[v,r]}\right) \\ 0 \\ 0 \end{pmatrix}
\end{pmatrix}$$

Komponen torsi $T^{a}{}_{\mu\nu}$ dari koneksi Weitzenboeck

In[ ]:=
```
TList["Torsion mu nu"]
```

OGRe: Torsion mu nu:

$$T^{v}{}_{vr} = -T^{v}{}_{rv} = -\sqrt{2}\,\partial_r\Delta[v,r] + \frac{(1+\sqrt{2})\,\epsilon\,\partial_v\Delta[v,r]}{\Delta[v,r]^2}$$

$$T^{r}{}_{vr} = -T^{r}{}_{rv} = -\partial_r\Delta[v,r] + \frac{(1+\sqrt{2})\,\epsilon\,\partial_v\Delta[v,r]}{\Delta[v,r]^2}$$

$$T^{\theta}{}_{r\theta} = -T^{\theta}{}_{\theta r} = 1 - \frac{1}{\Delta[v,r]}$$

$$T^{\phi}{}_{r\phi} = \mathrm{Sin}[\theta] - \frac{\mathrm{Sin}[\theta]}{\Delta[v,r]}$$

$$T^{\phi}{}_{\phi r} = \mathrm{Sin}[\theta]\left(-1 + \frac{1}{\Delta[v,r]}\right)$$

Mengubah indeks torsi $T^{a}{}_{\mu u} \to T^{a}{}_{b\,c}$

In[ ]:=
```
TShow@
 TCalc["Torsion", "Torsion mu nu"["aμν"]."InvTetrad h"["bμ"]."InvTetrad h"["cν"], "T"]
```

OGRe: Torsion: $T^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) =$

$$\begin{pmatrix}
\begin{pmatrix} 0 \\ \frac{\sqrt{2}\,\partial_r \Delta[v,r]}{\epsilon} - \frac{(1+\sqrt{2})\,\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} -\frac{\sqrt{2}\,\partial_r \Delta[v,r]}{\epsilon} + \frac{(1+\sqrt{2})\,\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ \frac{\partial_r \Delta[v,r]}{\epsilon} - \frac{(1+\sqrt{2})\,\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} -\frac{\partial_r \Delta[v,r]}{\epsilon} + \frac{(1+\sqrt{2})\,\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{1-\Delta[v,r]}{r\,\epsilon} \\ \frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{1-\Delta[v,r]}{r\,\epsilon} \\ \frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \\ 0 \end{pmatrix}
\end{pmatrix}$$

Daftar komponen tak lenyap torsi $T^{a}{}_{b\,c}$

In[ ]:=
```
TList["Torsion"]
```

OGRe: Torsion:

$$T^{v}{}_{vr} = -T^{v}{}_{rv} = \frac{\sqrt{2}\,\partial_r \Delta[v,r]}{\epsilon} - \frac{(1+\sqrt{2})\,\partial_v \Delta[v,r]}{\Delta[v,r]^2}$$

$$T^{r}{}_{vr} = -T^{r}{}_{rv} = \frac{\partial_r \Delta[v,r]}{\epsilon} - \frac{(1+\sqrt{2})\,\partial_v \Delta[v,r]}{\Delta[v,r]^2}$$

$$T^{\theta}{}_{v\theta} = T^{\phi}{}_{v\phi} = \frac{-1+\Delta[v,r]}{r\,\epsilon}$$

$$T^{\theta}{}_{r\theta} = T^{\phi}{}_{r\phi} = -T^{\theta}{}_{\theta r} = -T^{\phi}{}_{\phi r} = -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon}$$

$$T^{\theta}{}_{\theta v} = T^{\phi}{}_{\phi v} = \frac{1-\Delta[v,r]}{r\,\epsilon}$$

Superpotential

$$S_a{}^{\rho\sigma} = \frac{1}{2}\,(T^{\sigma\rho}{}_a + T_a{}^{\rho\sigma} - T^{\rho\sigma}{}_a) - h_a{}^{\sigma}\,T^{\theta\rho}{}_{\theta} + h_a{}^{\rho}\,T^{\theta\sigma}{}_{\theta}$$

$$S_a{}^{b\,c} = \delta_a{}^{b}\,T^{d\,c}{}_d - \delta_a{}^{c}\,T^{d\,b}{}_d + \frac{1}{2}\left(T^{c\,b}{}_a + T_a{}^{b\,c} - T^{b\,c}{}_a\right) \quad \text{(S.1)}$$

Menghitung suku ketiga persamaan (s.1) (yg ada di dalam kurung)

In[ ]:=
```
TShow@TCalc["kurung", 1/2 * ("Torsion"["abc"] + "Torsion"["cba"] - "Torsion"["bca"])]
```

OGRe: kurung: $\square^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) =$

$$
\begin{pmatrix}
\begin{pmatrix} 0 \\ \frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} -\frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ \frac{\partial_r\Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} -\frac{\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{1-\Delta[v,r]}{r\,\epsilon} \\ \frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{1-\Delta[v,r]}{r\,\epsilon} \\ \frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \\ 0 \end{pmatrix}
\end{pmatrix}
$$

In[ ]:=
```
TShow@TChangeDefaultIndices["kurung", {-1, 1, 1}]
```

OGRe: kurung: $\square_{\mu}{}^{\nu\rho}(v, r, \theta, \phi) =$

$$
\begin{pmatrix}
\begin{pmatrix} 0 \\ -\frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ \frac{\partial_r\Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} -\frac{\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ \frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{1-\Delta[v,r]}{r\,\epsilon} \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{1-\Delta[v,r]}{r\,\epsilon} \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \\ 0 \end{pmatrix}
\end{pmatrix}
$$

Menghitung superpotensial $S_a{}^{bc}$

In[∘]:=
```
TShow@TCalc["superpotential", "Kronecker"["ab"]."Torsion"["dcd"] -
   "Kronecker"["ac"]."Torsion"["dbd"] + "kurung"["abc"], "S"]
```

OGRe: superpotential: $S_\mu{}^{\nu\rho}(v, r, \theta, \phi) =$

$$\begin{pmatrix} \begin{pmatrix} 0 \\ \frac{2\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} -\frac{2\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ \frac{2-2\,\Delta[v,r]}{r\,\epsilon} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{2\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ -\frac{-1+\Delta[v,r]+r\,\partial_r\Delta[v,r]}{r\,\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} - \frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{-1+\Delta[v,r]+r\,\partial_r\Delta[v,r]}{r\,\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} - \frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \end{pmatrix} \end{pmatrix}$$

Daftar komponen tak lenyap superpotensial $S_a{}^{bc}$

In[∘]:=
```
TList["superpotential"]
```

OGRe: superpotential:

$$S_v{}^{vr} = -S_v{}^{rv} = \frac{2\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon}$$

$$S_r{}^{vr} = \frac{2-2\,\Delta[v,r]}{r\,\epsilon}$$

$$S_r{}^{rv} = \frac{2\,(-1+\Delta[v,r])}{r\,\epsilon}$$

$$S_\theta{}^{v\theta} = S_\phi{}^{v\phi} = -S_\theta{}^{\theta v} = -S_\phi{}^{\phi v} = -\frac{-1+\Delta[v,r]+r\,\partial_r\Delta[v,r]}{r\,\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2}$$

$$S_\theta{}^{r\theta} = S_\phi{}^{r\phi} = -S_\theta{}^{\theta r} = -S_\phi{}^{\phi r} = -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} - \frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2}$$

Superpotensial dalam indeks ruang-waktu $S_\rho{}^{\mu\nu}$

In[ ]:=
```
TShow@TCalc["superpotential spacetime", "superpotential"["abc"].
   "Tetrad h"["aρ"]."InvTetrad h"["bμ"]."InvTetrad h"["cν"], "S"]
```

OGRe: superpotential spacetime: $S_\mu{}^{\nu\rho}(v, r, \theta, \phi) =$

$$\left(\begin{array}{ccc}
\begin{pmatrix} 0 \\ -\frac{2(-1+\Delta[v,r])\,\Delta[v,r]}{r\,\epsilon^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{2(-1+\Delta[v,r])\,\Delta[v,r]}{r\,\epsilon^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \\
\begin{pmatrix} 0 \\ \frac{2(-1+\Delta[v,r])}{r\,\epsilon\,\Delta[v,r]} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{2-2\,\Delta[v,r]}{r\,\epsilon\,\Delta[v,r]} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \\
\begin{pmatrix} 0 \\ 0 \\ \frac{-1+\Delta[v,r]+r\,\partial_r\Delta[v,r]}{r\,\epsilon\,\Delta[v,r]} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \frac{\Delta[v,r]\,(-1+\Delta[v,r]+r\,\partial_r\Delta[v,r])}{r\,\epsilon^2} - \frac{\partial_v\Delta[v,r]}{\epsilon\,\Delta[v,r]} \\ 0 \end{pmatrix} & \left(-\frac{-1}{\Delta[v,r]\,(-1+\Delta \cdots} \right. \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{-1+\Delta[v,r]+r\,\partial_r\Delta[v,r]}{r\,\epsilon\,\Delta[v,r]} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{\Delta[v,r]\,(-1+\Delta[v,r]+r\,\partial_r\Delta[v,r])}{r\,\epsilon^2} - \frac{\partial_v\Delta[v,r]}{\epsilon\,\Delta[v,r]} \end{pmatrix} &
\end{array}\right.$$

In[ ]:=
```
TList["superpotential spacetime"]
```

OGRe: superpotential spacetime:

$$S_v{}^{vr} = -S_v{}^{rv} = -\frac{2(-1+\Delta[v,r])\,\Delta[v,r]}{r\,\epsilon^2}$$

$$S_r{}^{vr} = \frac{2(-1+\Delta[v,r])}{r\,\epsilon\,\Delta[v,r]}$$

$$S_r{}^{rv} = \frac{2-2\,\Delta[v,r]}{r\,\epsilon\,\Delta[v,r]}$$

$$S_\theta{}^{v\theta} = S_\phi{}^{v\phi} = -S_\theta{}^{\theta v} = -S_\phi{}^{\phi v} = \frac{-1+\Delta[v,r]+r\,\partial_r\Delta[v,r]}{r\,\epsilon\,\Delta[v,r]}$$

$$S_\theta{}^{r\theta} = S_\phi{}^{r\phi} = -S_\theta{}^{\theta r} = -S_\phi{}^{\phi r} = \frac{\Delta[v,r]\,(-1+\Delta[v,r]+r\,\partial_r\Delta[v,r])}{r\,\epsilon^2} - \frac{\partial_v\Delta[v,r]}{\epsilon\,\Delta[v,r]}$$

Lagrangan

$$L = \frac{h}{16\,\pi\,G}\left(\frac{1}{4}\,T^\rho{}_{\mu\nu}\,T_\rho{}^{\mu\nu} + \frac{1}{2}\,T^\rho{}_{\mu\nu}\,T^{\nu\mu}{}_\rho - T^\rho{}_{\mu\rho}\,T^{\nu\mu}{}_\nu\right)$$

In[ ]:=

In[ ]:=
```
h = Det[{{Sqrt[2] * Δ[v, r], -(Sqrt[2] + 1) ϵ / Δ[v, r], 0, 0},
   {Δ[v, r], -(Sqrt[2] + 1) ϵ / Δ[v, r], 0, 0}, {0, 0, r, 0}, {0, 0, 0, r * Sin[θ]}}]
```

Out[ ]=

$-r^2\,\epsilon\,\mathrm{Sin}[\theta]$

In[∘]:=
```
TShow@TCalc["Torsion scalar", 1/4 * "Torsion"["abc"]."Torsion"["abc"] +
   1/2 * "Torsion"["abc"]."Torsion"["cba"] - "Torsion"["aba"]."Torsion"["cbc"], "T"]
```

OGRe: Torsion scalar: $T\ (v, r, \theta, \phi) = \dfrac{2\,(-1 + \Delta[v, r])\,\left(\Delta[v, r]^2\,(-1 + \Delta[v, r] + 2\,r\,\partial_r \Delta[v, r]) - 2\,r\,\epsilon\,\partial_v \Delta[v, r]\right)}{r^2\,\epsilon^2\,\Delta[v, r]^2}$

In[∘]:=
```
TShow@TCalc["Lagrangian", h/(2 * κ) * "Torsion scalar"[""], "ℒ"]
```

OGRe: Lagrangian: $\mathcal{L}\ (v, r, \theta, \phi) = \dfrac{\mathrm{Sin}[\theta]\,(-1 + \Delta[v, r])\,\left(-\Delta[v, r]^2\,(-1 + \Delta[v, r] + 2\,r\,\partial_r \Delta[v, r]) + 2\,r\,\epsilon\,\partial_v \Delta[v, r]\right)}{\epsilon\,\kappa\,\Delta[v, r]^2}$

In[∘]:=

Arus Noether

$$J_a{}^b = \frac{1}{\kappa}\, T^c{}_{\nu a}\, S_c{}^{\nu b} - \frac{\delta_a{}^b}{h}\, L + \frac{1}{\kappa}\, \omega^c{}_{a\sigma}\, S_c{}^{bd}\, h_d{}^\sigma$$

In[∘]:=
```
TShow@TCalc["Current", 1/κ * "Torsion"["cνa"]."superpotential"["cνb"] -
   1/h * "Kronecker"["ab"]."Lagrangian"[""] +
   1/κ "SpinConnection"["caσ"]."superpotential"["cbd"]."InvTetrad h"["dσ"], "J"]
```

OGRe: Current: $J_\mu{}^\nu(v, r, \theta, \phi) =$

$$\begin{pmatrix} \dfrac{-\Delta[v,r]\,(1+\Delta[v,r]\,(-4+3\,\Delta[v,r]+2\,r\,\partial_r\Delta[v,r]))+2\left(1+\sqrt{2}\right) r\,\epsilon\,\partial_v\Delta[v,r]}{r^2\,\epsilon^2\,\kappa\,\Delta[v,r]} & \dfrac{2\left(-\sqrt{2}\,\Delta[v,r]^3+\sqrt{2}\,\Delta[v,r]^2\,(1-r\,\partial_r\Delta[v,r])+\left(1+\sqrt{2}\right)\cdots\right.}{r^2\,\epsilon^2\,\kappa\,\Delta[v,r]} \\ \dfrac{2\,\sqrt{2}\left(\Delta[v,r]^3+\Delta[v,r]^2\,(-1+r\,\partial_r\Delta[v,r])-\left(1+\sqrt{2}\right) r\,\epsilon\,\partial_v\Delta[v,r]\right)}{r^2\,\epsilon^2\,\kappa\,\Delta[v,r]} & \dfrac{\Delta[v,r]\,(-1+\Delta[v,r]\,(-2+3\,\Delta[v,r]+4\,r\,\partial_r\Delta[v,r]))-2\left(2+\sqrt{\cdots}\right.}{r^2\,\epsilon^2\,\kappa\,\Delta[v,r]} \\ \dfrac{\mathrm{Cot}[\theta]\left(-\frac{-1+\Delta[v,r]+r\,\partial_r\Delta[v,r]}{r\,\epsilon}+\frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2}\right)}{r\,\kappa} & \dfrac{\mathrm{Cot}[\theta]\left(-\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon}-\frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon}+\frac{\left(1+\sqrt{2}\right)\partial_v\Delta\cdots}{\Delta[v,r]^2}\right.}{r\,\kappa} \\ 0 & 0 \end{pmatrix}$$

Contorsion

$$K^{c}{}_{b\,a} = \frac{1}{2}\left(-T^{c}{}_{b\,a} + T_{b}{}^{c}{}_{a} + T_{a}{}^{c}{}_{b}\right) \text{ (K.1)}$$

Menghitung suku pertama persamaan (K.1)

In[ ]:=
```
TShow@TCalc["depan", (1/2) * (-"Torsion"["cba"]), "K"]
```

OGRe: depan: $K^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) =$

$$\begin{pmatrix}
\begin{pmatrix} 0 \\ \frac{1}{2}\left(-\frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2}\right) \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{\partial_r\Delta[v,r]}{\sqrt{2}\,\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{2\,\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ \frac{1}{2}\left(-\frac{\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2}\right) \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{1}{2}\left(\frac{\partial_r\Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2}\right) \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ -\frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \\ -\frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \\ -\frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \\ 0 \\ 0 \end{pmatrix}
\end{pmatrix}$$

Menghitung suku kedua dan ketiga (K.1)

In[ ]:=
```
TShow@TCalc["mboh", 1/2 * "Kronecker"["dc"].("Torsion"["bca"] + "Torsion"["acb"]), "K"]
```

OGRe: mboh: $K_{\mu}{}^{\nu}{}_{\rho}(v, r, \theta, \phi) =$

$$
\begin{pmatrix}
\begin{pmatrix} 0 \\ \frac{\partial_r \Delta[v,r]}{\sqrt{2}\,\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v \Delta[v,r]}{2\,\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{1}{2}\left(-\frac{\sqrt{2}\,\partial_r \Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v \Delta[v,r]}{\Delta[v,r]^2}\right) \\ \frac{\partial_r \Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{r\,\epsilon} \end{pmatrix} \\
\begin{pmatrix} -\frac{\sqrt{2}\,\partial_r \Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ \frac{1}{2}\left(\frac{\partial_r \Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v \Delta[v,r]}{\Delta[v,r]^2}\right) \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{1}{2}\left(-\frac{\partial_r \Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v \Delta[v,r]}{\Delta[v,r]^2}\right) \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} -\frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \\ \frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} -\frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \\ \frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \\ 0 \\ 0 \end{pmatrix}
\end{pmatrix}
$$

In[ ]:= **TShow@TChangeDefaultIndices["mboh", {1, -1, -1}]**

OGRe: mboh: $K^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) =$

$$\begin{pmatrix}
\begin{pmatrix} 0 \\ \frac{\partial_r\Delta[v,r]}{\sqrt{2}\,\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{2\,\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{\partial_r\Delta[v,r]}{\sqrt{2}\,\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{2\,\Delta[v,r]^2} \\ -\frac{\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ \frac{1-\Delta[v,r]}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1-\Delta[v,r]}{r\,\epsilon} \end{pmatrix} \\
\begin{pmatrix} \frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ \frac{1}{2}\left(-\frac{\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2}\right) \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{1}{2}\left(-\frac{\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2}\right) \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ -\frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} -\frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \\ \frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} -\frac{-1+\Delta[v,r]}{2\,r\,\epsilon} \\ \frac{-1+\Delta[v,r]}{\sqrt{2}\,r\,\epsilon} \\ 0 \\ 0 \end{pmatrix}
\end{pmatrix}$$

Menghitung kontrosi $K^{c}{}_{b\,a} = \frac{1}{2}\left(-T^{c}{}_{b\,a} + T_{b}{}^{c}{}_{a} + T_{a}{}^{c}{}_{b}\right)$

In[ ]:=
```
TShow@TCalc["Contorsion", "depan"["abc"] + "mboh"["abc"], "k"]
```

OGRe: Contorsion: $k^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) =$

$$\begin{pmatrix}
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ -\frac{\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ \frac{1-\Delta[v,r]}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1-\Delta[v,r]}{r\,\epsilon} \end{pmatrix} \\
\begin{pmatrix} \frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ -\frac{\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ \frac{1-\Delta[v,r]}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ \frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1-\Delta[v,r]}{r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon} \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
\end{pmatrix}$$

Menampilkan komponen tak lenyap kontorsi $K^{c}{}_{b\,a}$

In[ ]:=
```
TList["Contorsion"]
```

OGRe: Contorsion:

$$k^{v}{}_{rv} = k^{r}{}_{vv} = \frac{\sqrt{2}\,\partial_r\Delta[v,r]}{\epsilon} - \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2}$$

$$k^{v}{}_{rr} = k^{r}{}_{vr} = -\frac{\partial_r\Delta[v,r]}{\epsilon} + \frac{\left(1+\sqrt{2}\right)\partial_v\Delta[v,r]}{\Delta[v,r]^2}$$

$$k^{v}{}_{\theta\theta} = k^{v}{}_{\phi\phi} = k^{\theta}{}_{v\theta} = k^{\phi}{}_{v\phi} = \frac{1-\Delta[v,r]}{r\,\epsilon}$$

$$k^{r}{}_{\theta\theta} = k^{r}{}_{\phi\phi} = -k^{\theta}{}_{r\theta} = -k^{\phi}{}_{r\phi} = -\frac{\sqrt{2}\,(-1+\Delta[v,r])}{r\,\epsilon}$$

Mengubah indeks kontorsi $K^{c}{}_{b\,a} \to K^{c}{}_{\mu\nu}$

In[ ]:=
```
TShow@TCalc["Contorsion spacetime",
  "Contorsion"["abc"]."Tetrad h"["bμ"]."Tetrad h"["cν"], "K"]
```

OGRe: Contorsion spacetime: $K^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) =$

$$\begin{pmatrix}
\begin{pmatrix} \frac{\Delta[v,r]^2\,\partial_r\Delta[v,r]}{\epsilon} - \partial_v\Delta[v,r] \\ -\partial_r\Delta[v,r] \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \left(1+\sqrt{2}\right)\left(-\partial_r\Delta[v,r] + \frac{\epsilon\,\partial_v\Delta[v,r]}{\Delta[v,r]^2}\right) \\ \frac{\left(1+\sqrt{2}\right)\epsilon\,\partial_r\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ \frac{r-r\,\Delta[v,r]}{\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{r\,\mathrm{Sin}[\theta]^2\,(-1+\Delta[v,r])}{\epsilon} \end{pmatrix} \\
\begin{pmatrix} \frac{\sqrt{2}\,\left(\Delta[v,r]^2\,\partial_r\Delta[v,r]-\epsilon\,\partial_v\Delta[v,r]\right)}{\epsilon} \\ -\sqrt{2}\,\partial_r\Delta[v,r] \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} \left(1+\sqrt{2}\right)\left(-\partial_r\Delta[v,r] + \frac{\epsilon\,\partial_v\Delta[v,r]}{\Delta[v,r]^2}\right) \\ \frac{\left(1+\sqrt{2}\right)\epsilon\,\partial_r\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ -\frac{\sqrt{2}\,r\,(-1+\Delta[v,r])}{\epsilon} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\sqrt{2}\,r\,\mathrm{Sin}[\theta]^2\,(-1+\Delta[v,r])}{\epsilon} \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ -1+\frac{1}{\Delta[v,r]} \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathrm{Sin}[\theta]\left(-1+\frac{1}{\Delta[v,r]}\right) \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
\end{pmatrix}$$

In[ ]:=
```
TList["Contorsion spacetime"]
```

OGRe:

Contorsion spacetime:

$$K^{v}{}_{vv} = \frac{\Delta[v,r]^2\,\partial_r\Delta[v,r]}{\epsilon} - \partial_v\Delta[v,r]$$

$$K^{v}{}_{vr} = -\partial_r\Delta[v,r]$$

$$K^{v}{}_{rv} = K^{r}{}_{rv} = \left(1+\sqrt{2}\right)\left(-\partial_r\Delta[v,r] + \frac{\epsilon\,\partial_v\Delta[v,r]}{\Delta[v,r]^2}\right)$$

$$K^{v}{}_{rr} = K^{r}{}_{rr} = \frac{\left(1+\sqrt{2}\right)\epsilon\,\partial_r\Delta[v,r]}{\Delta[v,r]^2}$$

$$K^{v}{}_{\theta\theta} = \frac{r-r\,\Delta[v,r]}{\epsilon}$$

$$K^{v}{}_{\phi\phi} = -\frac{r\,\mathrm{Sin}[\theta]^2\,(-1+\Delta[v,r])}{\epsilon}$$

$$K^{r}{}_{vv} = \frac{\sqrt{2}\,\left(\Delta[v,r]^2\,\partial_r\Delta[v,r]-\epsilon\,\partial_v\Delta[v,r]\right)}{\epsilon}$$

$$K^{r}{}_{vr} = -\sqrt{2}\,\partial_r\Delta[v,r]$$

$$K^{r}{}_{\theta\theta} = -\frac{\sqrt{2}\,r\,(-1+\Delta[v,r])}{\epsilon}$$

$$K^{r}{}_{\phi\phi} = -\frac{\sqrt{2}\,r\,\mathrm{Sin}[\theta]^2\,(-1+\Delta[v,r])}{\epsilon}$$

$$K^{\theta}{}_{r\theta} = -1+\frac{1}{\Delta[v,r]}$$

$$K^{\phi}{}_{r\phi} = \mathrm{Sin}[\theta]\left(-1+\frac{1}{\Delta[v,r]}\right)$$

Menghitung Koneksi Levi-Civita dari Weitzenboeck dan Kontorsi

In[ ]:=
```
TList@TCalc["Kartoffel Symbol",
  "InvTetrad h"["aρ"].("Weitzenbock"["aμν"] - "Contorsion spacetime"["aμν"]), "Γ"]
```

OGRe:

Kartoffel Symbol:

$$\Gamma^{v}{}_{vv} = -\Gamma^{r}{}_{vr} = -\Gamma^{r}{}_{rv} = \frac{\Delta[v,r]\,\partial_r \Delta[v,r]}{\epsilon}$$

$$\Gamma^{v}{}_{\theta\theta} = -\frac{r}{\epsilon}$$

$$\Gamma^{v}{}_{\phi\phi} = -\frac{r\,\text{Sin}[\theta]^2}{\epsilon}$$

$$\Gamma^{r}{}_{vv} = \frac{\Delta[v,r]^3\,\partial_r \Delta[v,r] - \epsilon\,\Delta[v,r]\,\partial_v \Delta[v,r]}{\epsilon^2}$$

$$\Gamma^{r}{}_{\theta\theta} = -\frac{r\,\Delta[v,r]^2}{\epsilon^2}$$

$$\Gamma^{r}{}_{\phi\phi} = -\frac{r\,\text{Sin}[\theta]^2\,\Delta[v,r]^2}{\epsilon^2}$$

$$\Gamma^{\theta}{}_{r\theta} = \Gamma^{\theta}{}_{\theta r} = \Gamma^{\phi}{}_{r\phi} = \Gamma^{\phi}{}_{\phi r} = \frac{1}{r}$$

$$\Gamma^{\theta}{}_{\phi\phi} = -\text{Cos}[\theta]\,\text{Sin}[\theta]$$

$$\Gamma^{\phi}{}_{\theta\phi} = \Gamma^{\phi}{}_{\phi\theta} = \text{Cot}[\theta]$$

Menghitung kelengkungan Weitzenboeck

In[ ]:=
```
TList@TCalc[
  TPartialD["ν"]."SpinConnection"["abμ"] - TPartialD["μ"]."SpinConnection"["abν"] +
   "SpinConnection"["aeν"]."SpinConnection"["ebμ"] -
   "SpinConnection"["aeμ"]."SpinConnection"["ebν"]]
```

OGRe:

Result:

$$\square_{\theta}{}^{\theta}{}_{\phi\phi} = \square_{\phi}{}^{\phi}{}_{\theta\theta} = \left(-1 + \frac{1}{\epsilon^2}\right)\text{Sin}[\theta]$$

$$\square_{\theta}{}^{\phi}{}_{\theta\phi} = \square_{\phi}{}^{\theta}{}_{\phi\theta} = \left(1 - \frac{1}{\epsilon^2}\right)\text{Sin}[\theta]$$

Menampilkan hasil-hasil dinyatakan dalam fungsi massa

In[ ]:=
```
Δ[v_, r_] := Sqrt[1 - (2 G * M[v, r])/r]
```

$T^{a}{}_{\mu\nu}$

In[ ]:= `TList["Torsion mu nu"]`

Torsion mu nu:

OGRe:

$$T^{v}{}_{vr} = -T^{v}{}_{rv} = -\frac{\frac{2\,G\,M[v,r]}{r^2} - \frac{2\,G\,\partial_r M[v,r]}{r}}{\sqrt{2}\,\sqrt{1-\frac{2\,G\,M[v,r]}{r}}} - \frac{\left(1+\sqrt{2}\right) G\,\epsilon\,\partial_v M[v,r]}{r\left(1-\frac{2\,G\,M[v,r]}{r}\right)^{3/2}}$$

$$T^{r}{}_{vr} = -T^{r}{}_{rv} = -\frac{\frac{2\,G\,M[v,r]}{r^2} - \frac{2\,G\,\partial_r M[v,r]}{r}}{2\,\sqrt{1-\frac{2\,G\,M[v,r]}{r}}} - \frac{\left(1+\sqrt{2}\right) G\,\epsilon\,\partial_v M[v,r]}{r\left(1-\frac{2\,G\,M[v,r]}{r}\right)^{3/2}}$$

$$T^{\theta}{}_{r\theta} = -T^{\theta}{}_{\theta r} = 1 - \frac{1}{\sqrt{1-\frac{2\,G\,M[v,r]}{r}}}$$

$$T^{\phi}{}_{r\phi} = \mathrm{Sin}[\theta] - \frac{\mathrm{Sin}[\theta]}{\sqrt{1-\frac{2\,G\,M[v,r]}{r}}}$$

$$T^{\phi}{}_{\phi r} = \left(-1 + \frac{1}{\sqrt{1-\frac{2\,G\,M[v,r]}{r}}}\right)\mathrm{Sin}[\theta]$$

$K^{c}{}_{b\,a}$

In[ ]:= `TList["Contorsion"]`

Contorsion:

OGRe:

$$k^{v}{}_{rv} = k^{r}{}_{vv} = \frac{\frac{2\,G\,M[v,r]}{r^2} - \frac{2\,G\,\partial_r M[v,r]}{r}}{\sqrt{2}\,\epsilon\,\sqrt{1-\frac{2\,G\,M[v,r]}{r}}} + \frac{\left(1+\sqrt{2}\right) G\,\partial_v M[v,r]}{r\left(1-\frac{2\,G\,M[v,r]}{r}\right)^{3/2}}$$

$$k^{v}{}_{rr} = k^{r}{}_{vr} = -\frac{\frac{2\,G\,M[v,r]}{r^2} - \frac{2\,G\,\partial_r M[v,r]}{r}}{2\,\epsilon\,\sqrt{1-\frac{2\,G\,M[v,r]}{r}}} - \frac{\left(1+\sqrt{2}\right) G\,\partial_v M[v,r]}{r\left(1-\frac{2\,G\,M[v,r]}{r}\right)^{3/2}}$$

$$k^{v}{}_{\theta\theta} = k^{v}{}_{\phi\phi} = k^{\theta}{}_{v\theta} = k^{\phi}{}_{v\phi} = \frac{1-\sqrt{1-\frac{2\,G\,M[v,r]}{r}}}{r\,\epsilon}$$

$$k^{r}{}_{\theta\theta} = k^{r}{}_{\phi\phi} = -k^{\theta}{}_{r\theta} = -k^{\phi}{}_{r\phi} = -\frac{\sqrt{2}\left(-1+\sqrt{1-\frac{2\,G\,M[v,r]}{r}}\right)}{r\,\epsilon}$$

$S_{a}{}^{b\,c}$

In[ ]:= `TList["superpotential"]`

superpotential:

OGRe:

$$S_{v}{}^{vr} = -S_{v}{}^{rv} = \frac{2\,\sqrt{2}\left(-1+\sqrt{1-\frac{2\,G\,M[v,r]}{r}}\right)}{r\,\epsilon}$$

$$S_{r}{}^{vr} = \frac{2-2\,\sqrt{1-\frac{2\,G\,M[v,r]}{r}}}{r\,\epsilon}$$

$$S_{r}{}^{rv} = \frac{2\left(-1+\sqrt{1-\frac{2\,G\,M[v,r]}{r}}\right)}{r\,\epsilon}$$

$$S_{\theta}{}^{v\theta} = S_{\phi}{}^{v\phi} = -S_{\theta}{}^{\theta v} = -S_{\phi}{}^{\phi v} = -\frac{-1+\sqrt{1-\frac{2\,G\,M[v,r]}{r}} + \frac{r\left(\frac{2\,G\,M[v,r]}{r^2} - \frac{2\,G\,\partial_r M[v,r]}{r}\right)}{2\,\sqrt{1-\frac{2\,G\,M[v,r]}{r}}}}{r\,\epsilon} - \frac{\left(1+\sqrt{2}\right) G\,\partial_v M[v,r]}{r\left(1-\frac{2\,G\,M[v,r]}{r}\right)^{3/2}}$$

$$S_{\theta}{}^{r\theta} = S_{\phi}{}^{r\phi} = -S_{\theta}{}^{\theta r} = -S_{\phi}{}^{\phi r} = -\frac{\sqrt{2}\left(-1+\sqrt{1-\frac{2\,G\,M[v,r]}{r}}\right)}{r\,\epsilon} - \frac{\frac{2\,G\,M[v,r]}{r^2} - \frac{2\,G\,\partial_r M[v,r]}{r}}{\sqrt{2}\,\epsilon\,\sqrt{1-\frac{2\,G\,M[v,r]}{r}}} - \frac{\left(1+\sqrt{2}\right) G\,\partial_v M[v,r]}{r\left(1-\frac{2\,G\,M[v,r]}{r}\right)^{3/2}}$$

Simbol Christoffel

In[ ]:=
```
TList["Kartoffel Symbol"]
```

OGRe:

Kartoffel Symbol:

$$\Gamma^{v}{}_{vv} = -\Gamma^{r}{}_{vr} = -\Gamma^{r}{}_{rv} = \frac{\frac{2\,G\,M[v,r]}{r^2} - \frac{2\,G\,\partial_r M[v,r]}{r}}{2\,\epsilon}$$

$$\Gamma^{v}{}_{\theta\theta} = -\frac{r}{\epsilon}$$

$$\Gamma^{v}{}_{\phi\phi} = -\frac{r\,\mathrm{Sin}[\theta]^2}{\epsilon}$$

$$\Gamma^{r}{}_{vv} = \frac{\frac{1}{2}\left(1-\frac{2\,G\,M[v,r]}{r}\right)\left(\frac{2\,G\,M[v,r]}{r^2} - \frac{2\,G\,\partial_r M[v,r]}{r}\right)+\frac{G\,\epsilon\,\partial_v M[v,r]}{r}}{\epsilon^2}$$

$$\Gamma^{r}{}_{\theta\theta} = -\frac{r\left(1-\frac{2\,G\,M[v,r]}{r}\right)}{\epsilon^2}$$

$$\Gamma^{r}{}_{\phi\phi} = -\frac{r\left(1-\frac{2\,G\,M[v,r]}{r}\right)\mathrm{Sin}[\theta]^2}{\epsilon^2}$$

$$\Gamma^{\theta}{}_{r\theta} = \Gamma^{\theta}{}_{\theta r} = \Gamma^{\phi}{}_{r\phi} = \Gamma^{\phi}{}_{\phi r} = \frac{1}{r}$$

$$\Gamma^{\theta}{}_{\phi\phi} = -\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]$$

$$\Gamma^{\phi}{}_{\theta\phi} = \Gamma^{\phi}{}_{\phi\theta} = \mathrm{Cot}[\theta]$$