EE 301 Signals and Systems I

Homework 4

(due Dec. 15, 2019, 23:55 via odtuclass.metu.edu.tr)
4 questions.

Question 1

A signal $x[n]$ has F.T. $X(e^{j\Omega}) = \frac{e^{j\Omega}}{2+e^{-j\Omega}}$.

a) What is the total sum of the signal $x[n]$?
b) Let $y[n]$ be the accumulation of $x[n]$, i.e, $y[n] = \sum_{k=-\infty}^{n} x[k]$. What is the total sum of the signal $y[n]$?
c) What is the numerical value of $|X(e^{j\Omega})|$ at $\Omega = \frac{\pi}{2}$, $\Omega = \pi$? If $X(e^{j\Omega})$ is the frequency response of an LTI system, what would you say about its characteristics?
d) As a last step, determine $x[n]$.

Question 2

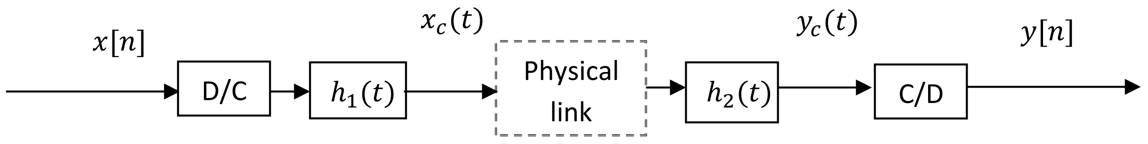
We defined periodic convolution in class and found the DTFS coefficients for the periodically convoluted signal. Alternatively, one can use DTFT properties and find the resultant DTFS coefficients.

Let the periodic signals $x_p[n]$ and $y_p[n]$ have period N and DTFS coefficients a_k and b_k, respectively.

a) Define $x_{sp}[n] = x_p[n]$ for $0 \leq n < N$ and zero otherwise. Write down the DTFS coefficients a_k of $x_p[n]$ in terms of the DTFT of $x_{sp}[n]$?
b) Show that the periodic convolution of $x_p[n]$ and $y_p[n]$ equals $x_{sp}[n] * y_p[n]$ and is periodic with N.
c) Express the DTFT of $x_{sp}[n] * y_p[n]$ in terms of DTFT's of $x_{sp}[n]$ and $y_p[n]$.
d) Based on previous parts, determine the DTFS coefficients of the periodically convoluted signal in terms of a_k and b_k.

Question 3

We will investigate the conversion of a DT signal into CT and then back to DT in this question as complementary to our coverage in class.



Consider a DT signal $x[n]$ with $X(e^{j\Omega}) = \begin{cases} \pi, & -\pi \leq \Omega < 0 \\ \pi - \Omega, & 0 \leq \Omega < \pi \end{cases}$. The two CT filters have frequency responses

$$H_1(j\omega) = \begin{cases} 1 - \frac{|\omega| T_1}{2\pi}, & |\omega| \leq \frac{2\pi}{T_1} \\ 0, & \text{o.w.} \end{cases} \quad \text{and} \quad H_2(j\omega) = \begin{cases} 1, & |\omega| \leq \frac{2\pi}{T_1} \\ 0, & \text{o.w.} \end{cases}.$$

a) Define a signal $x_p(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT_1)$. Evaluate $X_p(j\omega)$ in terms of $X(e^{j\Omega})$ and sketch over $\omega \in [-3\pi/T_1, 3\pi/T_1]$.
b) Find the FT of $x_c(t) = \sum_{n=-\infty}^{\infty} x[n]h_1(t - nT_1)$. Sketch it over $\omega \in [-3\pi/T_1, 3\pi/T_1]$.

c) The physical link is also an LTI and have frequency response $H_l(j\omega) = 3$. Find and sketch $Y_c(j\omega)$.

d) What is the Nyquist rate for the sampling of $y_c(t)$ and the corresponding sampling period in terms of T_1?

e) We will now perform undersampling, that is, we will pick an insufficient period. Define a signal $y_{cp}(t) = \sum_{n=-\infty}^{\infty} y_c(\mathrm{n}T_2)\delta(\mathrm{t}-\mathrm{n}T_2)$ for $T_2 = T_1$. Evaluate $Y_{cp}(j\omega)$ in terms of $Y_c(j\omega)$.

f) Write the FT of $y[n] = y_c(\mathrm{n}T_2)$. Simplify it until you have a summation only in terms of $X(e^{j\Omega})$ and $H_1(j\omega)$. Check this summation in $\Omega \in [-\pi, \pi]$ and draw $Y(e^{j\Omega})$.

g) Based on part f, what is the equivalent filter $H(e^{j\Omega}) = Y(e^{j\Omega})/X(e^{j\Omega})$ corresponding to all the D/C, C/D, and CT filtering operations?

h) Although sampling rate is lower than the Nyquist rate, it does not harm the application in this case as we observe in part g. This is a common trick used in communications. Please check whether the same outcome holds if the cutoff frequency of the second filter is set to $\frac{\pi}{T_1}$. You may answer this question just by checking $Y_c(j\omega)$.

Question 4

Define the following DT signal in Matlab.

```
W=pi/4;
N=32;
n=0:(N-1);
x_n=W/pi*sinc(W*(n-N/2)/pi);
```

a) Plot `x_n` vs `n`.

b) Calculate the FT of `x_n` for $\Omega = \mathrm{k}\frac{2\pi}{128}, k = 0, \dots, 127$, with the following code fragment. Plot $|X(e^{j\Omega})|$ for the defined Ω.

```
N2=4*N;
Omega_seq=(0:(N2-1))*(2*pi/N2);
X_Omega=0*Omega_seq; % allocate memory for DTFT
for kk=1:length(Omega_seq)
    Omega=Omega_seq(kk);
    X_Omega(kk)=exp(-j*Omega*(0:(N-1)))*(x_n).'; % summation
end
figure,plot(Omega_seq,abs(X_Omega),'+-')
```

c) DFT of a signal can be calculated as we derived in class by the fft function in Matlab. Read the description of the fft function in Matlab. Use it to find the DFT of `x_n`.

d) When the DFT analysis formula is considered, DFT's k$^{\text{th}}$ element corresponds to what frequency?

e) Draw the plot in part b and the absolute of DFT in part d vs $\Omega \in [-\pi, \pi)$. Verify the same values where you expect them to see.

f) The fft function can also give us a highly sampled DTFT. This is done by padding with trailing zeros to the length given to the fft function as an argument when the argument is larger than the DT signal's original length. Plot fft(x_n,2*N), fft(x_n,4*N), fft(x_n,8*N) with the corresponding Ω values.

g) There are many helpful functions in Matlab. One of them is the upsample function that inserts zeros in between signal samples. Use the following code fragment and draw the resultant DFT vs $\Omega \in [-\pi, \pi)$. Is this related to any of the properties discussed in class?

```
usr=3;
x_n_up=upsample(x_n,usr);
```

Do not forget to attach the plots to your solutions.