EE 301 Signals and Systems I

Homework 4

(due Dec. 15, 2019)

4 questions.

Question 1

A signal $x[n]$ has F.T. $X(e^{j\Omega}) = \frac{e^{j\Omega}}{2+e^{-j\Omega}}$.

a) What is the total sum of the signal $x[n]$?

$$\sum_{n=-\infty}^{\infty} x[n] = \left(\sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} \right)\Bigg|_{\Omega=0} = \left(X(e^{j\Omega})\right)\big|_{\Omega=0} = X\left(e^{j0}\right) = \frac{1}{3}$$

b) Let $y[n]$ be the accumulation of $x[n]$, i.e, $y[n] = \sum_{k=-\infty}^{n} x[k]$. What is the total sum of the signal $y[n]$?

$$Y\left(e^{j\Omega}\right) = \frac{X(e^{j\Omega})}{1 - e^{-j\Omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$$

$$Y\left(e^{j\Omega}\right) = \frac{\frac{e^{j\Omega}}{2+e^{-j\Omega}}}{1 - e^{-j\Omega}} + \pi \frac{1}{3} \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k) = \frac{e^{j\Omega}}{2 - e^{-j\Omega} - e^{-j2\Omega}} + \pi \frac{1}{3} \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$$

$$\sum_{n=-\infty}^{\infty} y[n] = Y\left(e^{j0}\right) = \frac{1}{2-1-1} + \pi \frac{1}{3} \delta(0)$$

It is infinity.

c) What is the numerical value of $\left|X(e^{j\Omega})\right|$ at $\Omega = \frac{\pi}{2}$, $\Omega = \pi$? If $X(e^{j\Omega})$ is the frequency response of an LTI system, what would you say about its characteristics?

$$\left|X\left(e^{j\frac{\pi}{2}}\right)\right| = \left|\frac{e^{j\frac{\pi}{2}}}{2 + e^{-j\frac{\pi}{2}}}\right| = \left|\frac{j}{2-j}\right| = \frac{|j|}{|2-j|} = \frac{1}{\sqrt{5}}$$

$$\left|X\left(e^{j\pi}\right)\right| = \left|\frac{e^{j\pi}}{2 + e^{-j\pi}}\right| = \left|\frac{-1}{2-1}\right| = 1$$

It has high-pass characteristics because

$$\left|X\left(e^{j0}\right)\right| < \left|X\left(e^{j\frac{\pi}{2}}\right)\right| < \left|X\left(e^{j\pi}\right)\right|$$

d) As a last step, determine $x[n]$.

$$X\left(e^{j\Omega}\right) = \frac{e^{j\Omega}}{2 + e^{-j\Omega}} = \frac{1}{2} e^{j\Omega} \left(\frac{1}{1 - (-\frac{1}{2}) e^{-j\Omega}} \right) = \frac{1}{2} e^{j\Omega} F\left\{ \left(-\frac{1}{2}\right)^{n} u[n] \right\}$$

$$= \frac{1}{2} F\left\{ \left(-\frac{1}{2}\right)^{n+1} u[n+1] \right\} = F\left\{ \frac{1}{2} \left(-\frac{1}{2}\right)^{n+1} u[n+1] \right\}$$

$$x[n] = \frac{1}{2}\left(-\frac{1}{2}\right)^{n+1} u[n+1]$$

$$x[n] = -\left(-\frac{1}{2}\right)^{n+2} u[n+1]$$

Question 2

We defined periodic convolution in class and found the DTFS coefficients for the periodically convoluted signal. Alternatively, one can use DTFT properties and find the resultant DTFS coefficients.

Let the periodic signals $x_p[n]$ and $y_p[n]$ have period N and DTFS coefficients a_k and b_k, respectively.

a) Define $x_{sp}[n] = x_p[n]$ for $0 \le n < N$ and zero otherwise. Write down the DTFS coefficients a_k of $x_p[n]$ in terms of the DTFT of $x_{sp}[n]$?

$$X_{sp}(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_{sp}[n]\, e^{-j\Omega n} = \sum_{n=0}^{N-1} x_{sp}[n]\, e^{-j\Omega n}$$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p[n]\, e^{-j\frac{2\pi}{N}kn} = \frac{1}{N}\sum_{n=0}^{N-1} x_{sp}[n]\, e^{-j\frac{2\pi}{N}kn} = \frac{1}{N}\left(X_{sp}(e^{j\Omega})\right)\Big|_{\Omega=\frac{2\pi}{N}k}$$

b) Show that the periodic convolution of $x_p[n]$ and $y_p[n]$ equals $x_{sp}[n] * y_p[n]$ and is periodic with N.

$$z[n] \triangleq x_{sp}[n] * y_p[n]$$

$$z[n] = \sum_{k=-\infty}^{\infty} x_{sp}[k] y_p[n-k] = \sum_{k=0}^{N-1} x_{sp}[k] y_p[n-k] = \sum_{k=0}^{N-1} x_p[k] y_p[n-k]$$

$$z[n+kN] = \sum_{k=0}^{N-1} x_p[k] y_p[n+kN-k] = \sum_{k=0}^{N-1} x_p[k] y_p[n-k] = z[n], \qquad \forall k \in Z$$

c) What is the DTFT of $x_{sp}[n] * y_p[n]$?

$$Z(e^{j\Omega}) = X_{sp}(e^{j\Omega})\, Y_p(e^{j\Omega})$$

$$Y_p(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} b_k \delta\left(\Omega - \frac{2\pi}{N}k\right)$$

$$Z(e^{j\Omega}) = X_{sp}(e^{j\Omega})\, 2\pi \sum_{k=-\infty}^{\infty} b_k \delta\left(\Omega - \frac{2\pi}{N}k\right) = 2\pi \sum_{k=-\infty}^{\infty} X_{sp}\left(e^{j\frac{2\pi}{N}k}\right) b_k \delta\left(\Omega - \frac{2\pi}{N}k\right)$$

d) Based on part c, determine the DTFS coefficients of the periodically convoluted signal in terms of a_k and b_k.

$$a_k = \frac{1}{N}\left(X_{sp}(e^{j\Omega})\right)\Big|_{\Omega=\frac{2\pi}{N}k}, \qquad (part\ a)$$

$$Z(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} X_{sp}\left(e^{j\frac{2\pi}{N}k}\right) b_k \delta\left(\Omega - \frac{2\pi}{N}k\right), \qquad (part\ c)$$

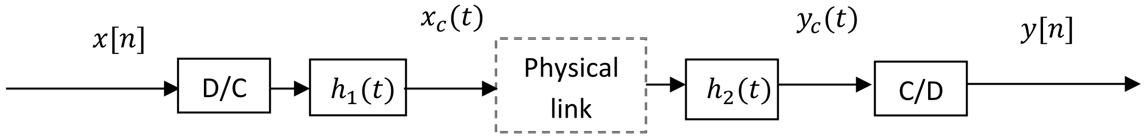
$$Z(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} N a_k b_k \delta\left(\Omega - \frac{2\pi}{N}k\right)$$

$$c_k = N a_k b_k$$

where c_k are DTFS coefficients of $z[n]$.

Question 3

We will investigate the conversion of a DT signal into CT and then back to DT in this question as complementary to our coverage in class.



Consider a DT signal $x[n]$ with $X(e^{j\Omega}) = \begin{cases} \pi, & -\pi \le \Omega < 0 \\ \pi - \Omega, & 0 \le \Omega < \pi \end{cases}$. The two CT filters have frequency responses

$$H_1(j\omega) = \begin{cases} 1 - \frac{|\omega| T_1}{2\pi}, & |\omega| \le \frac{2\pi}{T_1} \\ 0, & \text{o.w.} \end{cases} \quad \text{and} \quad H_2(j\omega) = \begin{cases} 1, & |\omega| \le \frac{2\pi}{T_1} \\ 0, & \text{o.w.} \end{cases}.$$

a) Define a signal $x_p(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT_1)$. Evaluate $X_p(j\omega)$ in terms of $X(e^{j\Omega})$ and sketch over $\omega \in [-3\pi/T_1, 3\pi/T_1]$.

$$X_p(j\omega) = \int_{-\infty}^{\infty} x_p(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT_1)\, e^{-j\omega t} dt$$

$$= \sum_{n=-\infty}^{\infty} x[n] \int_{-\infty}^{\infty} \delta(t - nT_1) e^{-j\omega t} dt = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n T_1}$$

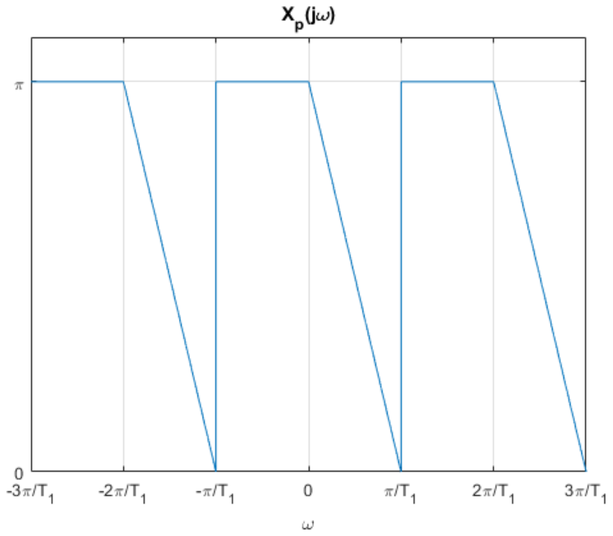
$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$$

$$X_p(j\omega) = \left(X(e^{j\Omega})\right)\Big|_{\Omega = \omega T_1}$$

$$X(e^{j\Omega}) = \begin{cases} \pi & -\pi \le \Omega < 0 \\ \pi - \Omega & 0 \le \Omega < \pi \end{cases}, \quad \text{and } X(e^{j(\Omega + k2\pi)}) = X(e^{j\Omega}), \quad \forall k \in Z$$

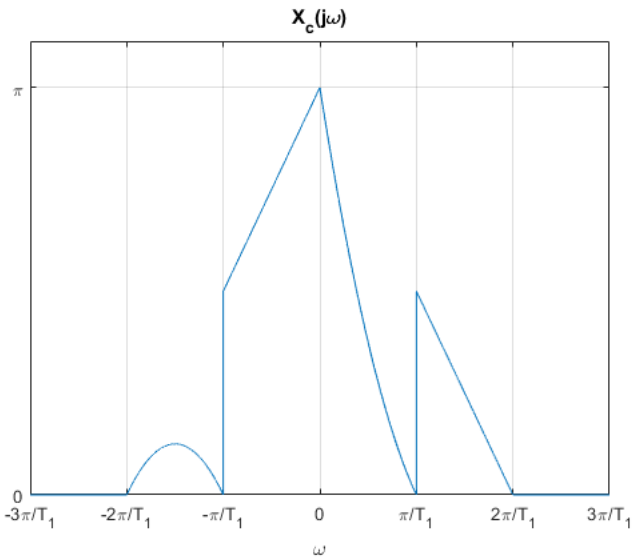
$$X_p(j\omega) = \begin{cases} \pi & -\pi \le \omega T_1 < 0 \\ \pi - \omega T_1 & 0 \le \omega T_1 < \pi \end{cases}, \quad \text{and } X_p\left(j\frac{\Omega + k2\pi}{T_1}\right) = X\left(j\frac{\Omega}{T_1}\right), \quad \forall k \in Z$$

$$\boxed{X_p(j\omega) = \begin{cases} \pi & -\frac{\pi}{T_1} \le \omega < 0 \\ \pi - \omega T_1 & 0 \le \omega < \frac{\pi}{T_1} \end{cases}}, \quad \text{and } \boxed{X_p\left(j\left(\omega + \frac{k2\pi}{T_1}\right)\right) = X(j\omega), \quad \forall k \in Z}$$



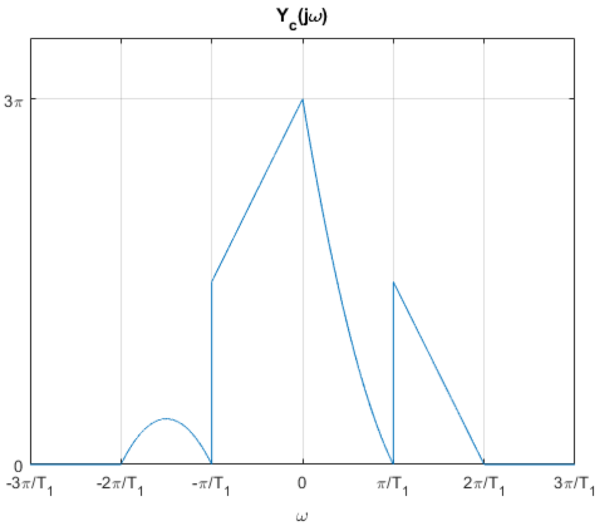
b) Find the FT of $x_c(t) = \sum_{n=-\infty}^{\infty} x[n]h_1(\mathrm{t} - \mathrm{n}T_1)$. Sketch it over $\omega \in [-3\pi/T_1, 3\pi/T_1]$.

$$\delta(\mathrm{t}) \rightarrow \boxed{h_1(t)} \rightarrow h_1(\mathrm{t})$$
$$\delta(\mathrm{t} - \mathrm{n}T_1) \rightarrow \boxed{h_1(t)} \rightarrow h_1(\mathrm{t} - \mathrm{n}T_1)$$
$$x_p(t) \rightarrow \boxed{h_1(t)} \rightarrow x_c(t)$$
$$X_c(j\omega) = X_p(j\omega)H_1(j\omega)$$
$$X_c(j\omega) = \begin{cases} (\pi - \omega T_1 - 2\pi)\left(1 - \dfrac{|\omega|T_1}{2\pi}\right) & -\dfrac{2\pi}{T_1} \leq \omega < -\dfrac{\pi}{T_1} \\ \pi\left(1 - \dfrac{|\omega|T_1}{2\pi}\right) & -\dfrac{\pi}{T_1} \leq \omega < 0 \\ (\pi - \omega T_1)\left(1 - \dfrac{|\omega|T_1}{2\pi}\right) & 0 \leq \omega < \dfrac{\pi}{T_1} \\ \pi\left(1 - \dfrac{|\omega|T_1}{2\pi}\right) & \dfrac{\pi}{T_1} \leq \omega < \dfrac{2\pi}{T_1} \\ 0 & otherwise \end{cases}$$



c) The physical link is also an LTI and have frequency response $H_l(j\omega) = 3$. Find and sketch $Y_c(j\omega)$.

$$Y_c(j\omega) = X_c(j\omega)H_l(j\omega)H_2(j\omega) = 3X_c(j\omega)H_2(j\omega) = 3X_c(j\omega)$$



d) What is the Nyquist rate for the sampling of $y_c(t)$ and the corresponding sampling period in terms of T_1?

$$Y_c(j\omega) = 0, for\ |\omega| > \frac{2\pi}{T_1}$$

$$\frac{2\pi}{T_s} > 2\frac{2\pi}{T_1}$$

$$T_s < \frac{T_1}{2}$$

e) We will now perform undersampling, that is, we will pick an insufficient period. Define a signal $y_{cp}(t) = \sum_{n=-\infty}^{\infty} y_c(nT_2)\delta(t - nT_2)$ for $T_2 = T_1$. Evaluate $Y_{cp}(j\omega)$ in terms of $Y_c(j\omega)$.

$$y_{cp}(t) = y_c(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_2)$$

$$Y_{cp}(j\omega) = \frac{1}{2\pi} Y_c(j\omega) \star F\left\{\sum_{n=-\infty}^{\infty} \delta(t - nT_2)\right\}$$

$$F\left\{\sum_{n=-\infty}^{\infty} \delta(t - nT_2)\right\} = \frac{2\pi}{T_2} \sum_{k=-\infty}^{\infty} \delta\left(j\left(\omega - k\frac{2\pi}{T_2}\right)\right)$$

$$Y_{cp}(j\omega) = \frac{1}{T_2} \sum_{k=-\infty}^{\infty} Y_c\left(j\left(\omega - k\frac{2\pi}{T_2}\right)\right)$$

f) Write the FT of $y[n] = y_c(nT_2)$. Simplify it until you have a summation only in terms of $X(e^{j\Omega})$ and $H_1(j\omega)$. Check this summation in $\Omega \in [-\pi, \pi]$ and draw $Y(e^{j\Omega})$.

$$Y(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} y[n]e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} y_c(nT_2)e^{-j\Omega n} = \int_{-\infty}^{\infty} y_c(t)e^{-j\Omega\frac{t}{T_2}} \sum_{n=-\infty}^{\infty} \delta(t - nT_2)\, dt$$

$$= \int_{-\infty}^{\infty} y_c(t)e^{-j\Omega\frac{t}{T_2}} \sum_{n=-\infty}^{\infty} \delta(t - nT_2)\, dt$$

$$= \int_{-\infty}^{\infty} y_{cp}(t)e^{-j\Omega\frac{t}{T_2}}\, dt = \left(Y_{cp}(j\omega)\right)\Big|_{\omega=\frac{\Omega}{T_2}}$$

$$Y(e^{j\Omega}) = \left(Y_{cp}(j\omega)\right)\Big|_{\omega=\frac{\Omega}{T_2}} = Y_{cp}\left(j\frac{\Omega}{T_2}\right)$$

$$Y_{cp}(j\omega) = \frac{1}{T_2} \sum_{k=-\infty}^{\infty} Y_c\left(j\left(\omega - k\frac{2\pi}{T_2}\right)\right)$$

$$Y(e^{j\Omega}) = \frac{1}{T_2} \sum_{k=-\infty}^{\infty} Y_c\left(j\left(\frac{\Omega}{T_2} - k\frac{2\pi}{T_2}\right)\right)$$

$$Y_c(j\omega) = 3X_c(j\omega)$$

$$Y(e^{j\Omega}) = \frac{3}{T_2} \sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\Omega}{T_2} - k\frac{2\pi}{T_2}\right)\right)$$

$$X_c(j\omega) = X_p(j\omega)H_1(j\omega)$$

$$Y(e^{j\Omega}) = \frac{3}{T_2} \sum_{k=-\infty}^{\infty} X_p\left(j\left(\frac{\Omega}{T_2} - k\frac{2\pi}{T_2}\right)\right) H_1\left(j\left(\frac{\Omega}{T_2} - k\frac{2\pi}{T_2}\right)\right)$$

$$X_p(j\omega) = \left(X\left(e^{j\Omega}\right)\right)\Big|_{\Omega=\omega T_1}$$
$$X\left(e^{j\Omega}\right) = X_p\left(j\frac{\Omega}{T_1}\right)$$
$$Y\left(e^{j\Omega}\right) = \frac{3}{T_2}\sum_{k=-\infty}^{\infty} X\left(e^{j(\Omega-k2\pi)T_1/T_2}\right) H_1\left(j\left(\frac{\Omega}{T_2} - k\frac{2\pi}{T_2}\right)\right)$$
$$T_1 = T_2$$
$$Y\left(e^{j\Omega}\right) = \frac{3}{T_1}X\left(e^{j\Omega}\right)\sum_{k=-\infty}^{\infty} H_1\left(j\left(\frac{\Omega}{T_1} - k\frac{2\pi}{T_1}\right)\right)$$

Check this summation in $\Omega \in [-\pi, \pi]$:

$$H_1(j\omega) = 0, \qquad for\ |\omega| > \frac{2\pi}{T_1}$$
$$H_1\left(j\left(\frac{\Omega}{T_1} - k\frac{2\pi}{T_1}\right)\right) = 0, \qquad for\ \Omega > k2\pi + 2\pi\ \ and\ \ \Omega < k2\pi - 2\pi$$
$$Y\left(e^{j\Omega}\right) = \frac{3}{T_1}X\left(e^{j\Omega}\right)\sum_{k=-\infty}^{\infty} H_1\left(j\left(\frac{\Omega}{T_1} - k\frac{2\pi}{T_1}\right)\right), \qquad \Omega \in [-\pi, \pi]$$
$$Y\left(e^{j\Omega}\right) = \frac{3}{T_1}X\left(e^{j\Omega}\right)\sum_{k=-1}^{1} H_1\left(j\left(\frac{\Omega}{T_1} - k\frac{2\pi}{T_1}\right)\right), \qquad \Omega \in [-\pi, \pi]$$
$$H_1(j\omega) = \begin{cases} 1 - \frac{|\omega|T_1}{2\pi}, & |\omega| \le \frac{2\pi}{T_1} \\ 0, & \text{o.w.} \end{cases}$$
$$H_1\left(j\frac{\Omega}{T_1}\right) = \begin{cases} 1 - \frac{\Omega}{2\pi}, & 0 < \Omega \le \pi \\ 1 + \frac{\Omega}{2\pi}, & -\pi \le \Omega \le 0 \end{cases}$$

$$H_1\left(j\left(\frac{\Omega}{T_1} - \frac{2\pi}{T_1}\right)\right) = \begin{cases} 1 - \frac{\Omega - 2\pi}{2\pi}, & 0 < \Omega \le \pi \\ 0, & -\pi \le \Omega \le 0 \end{cases} = \begin{cases} \frac{\Omega}{2\pi}, & 0 < \Omega \le \pi \\ 0, & -\pi \le \Omega \le 0 \end{cases}$$

$$H_1\left(j\left(\frac{\Omega}{T_1} + \frac{2\pi}{T_1}\right)\right) = \begin{cases} 0, & 0 < \Omega \le \pi \\ 1 - \frac{\Omega + 2\pi}{2\pi}, & -\pi \le \Omega \le 0 \end{cases} = \begin{cases} 0, & 0 < \Omega \le \pi \\ -\frac{\Omega}{2\pi}, & -\pi \le \Omega \le 0 \end{cases}$$

$$\sum_{k=-1}^{1} H_1\left(j\left(\frac{\Omega}{T_1} - k\frac{2\pi}{T_1}\right)\right) = 1, \qquad \Omega \in [-\pi, \pi]$$
$$In\ fact: \sum_{k=-\infty}^{\infty} H_1\left(j\left(\frac{\Omega}{T_1} - k\frac{2\pi}{T_1}\right)\right) = 1, \qquad \forall\Omega$$

$$Y\left(e^{j\Omega}\right) = \frac{3}{T_1}X\left(e^{j\Omega}\right)$$

g) Based on part f, what is the equivalent filter $H\left(e^{j\Omega}\right) = Y\left(e^{j\Omega}\right)/X\left(e^{j\Omega}\right)$ corresponding to all the D/C, C/D, and CT filtering operations?

$$H\left(e^{j\Omega}\right) = \frac{3}{T_1}$$

The output is a scaled version of the input. That is, the input is transmitted from one side to the other side (through a physical link) without any distortion. It can be recovered at the output.

h) Although sampling rate is lower than the Nyquist rate, it does not harm the application in this case as we observe in part g. This is a common trick used in communications. Please check whether the same outcome holds if the cutoff frequency of the second filter is set to $\frac{\pi}{T_1}$.

When it was $\frac{2\pi}{T_1}$, clipped parts of $Y_c(j\omega)$ in $\left[-\frac{\pi}{T_1}, +\frac{\pi}{T_1}\right)$ were being compensated with the parts at both sides (see the figure in part c). It is equivalent to the fact that

$$\sum_{k=-\infty}^{\infty} H_1\left(\mathrm{j}\left(\frac{\Omega}{T_1} - \mathrm{k}\frac{2\pi}{\mathrm{T}_1}\right)\right) = 1, \qquad \forall\Omega$$

from part f. In this expression, $H_2(j\omega)$ is omitted because it was ineffective in the nonzero region of $H_1(j\omega)$. Actually, the true expression to check is

$$\sum_{k=-\infty}^{\infty} H_1\left(\mathrm{j}\left(\frac{\Omega}{T_1} - \mathrm{k}\frac{2\pi}{\mathrm{T}_1}\right)\right) H_2\left(\mathrm{j}\left(\frac{\Omega}{T_1} - \mathrm{k}\frac{2\pi}{\mathrm{T}_1}\right)\right) = 1, \qquad \forall\Omega$$

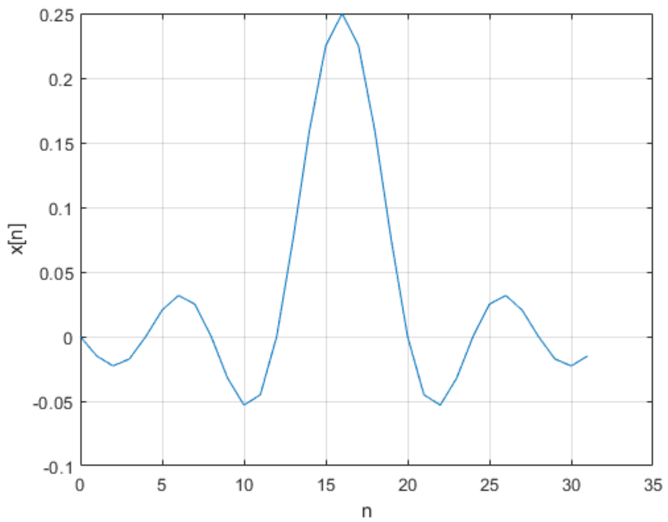
When the cutoff frequency of the second filter is set to $\frac{\pi}{T_1}$, this expression is not valid anymore. The parts at the both sides of the spectrum of $X_c(j\omega)$ are cancelled by the second filter, and clipped parts of $Y_c(j\omega)$ in $\left[-\frac{\pi}{T_1}, +\frac{\pi}{T_1}\right)$ cannot be compensated. Therefore, the output is not a scaled version of the input anymore, it is distorted now. That is, the input cannot be recovered at the output and transmission failed.

Question 4

Define the following DT signal in Matlab.

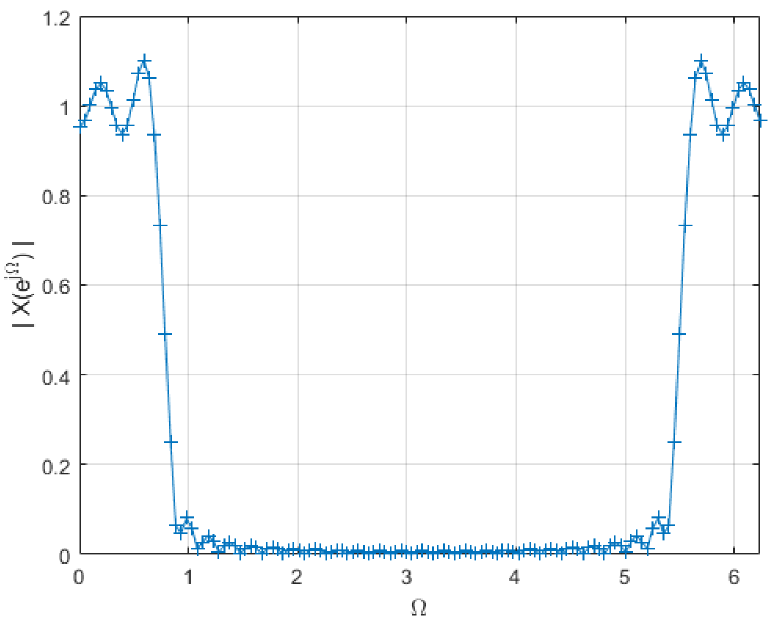
```
W=pi/4;
N=32;
n=0:(N-1);
x_n=W/pi*sinc(W*(n-N/2)/pi);
```

a) Plot `x_n` vs `n`.



b) Calculate the FT of `x_n` for $\Omega = \mathrm{k}\frac{2\pi}{128}, k = 0, \ldots, 127$, with the following code fragment. Plot $\left|X\left(e^{j\Omega}\right)\right|$ for the defined Ω.

```
N2=4*N;
Omega_seq=(0:(N2-1))*(2*pi/N2);
X_Omega=0*Omega_seq; % allocate memory for DTFT
for kk=1:length(Omega_seq)
    Omega=Omega_seq(kk);
    X_Omega(kk)=exp(-j*Omega*(0:(N-1)))*(x_n).'; % summation
end
figure,plot(Omega_seq,abs(X_Omega),'+-')
```



c) DFT of a signal can be calculated as we derived in class by the fft function in Matlab. Read the description of the fft function in Matlab. Use it to find the DFT of `x_n`.

Matlab fft( ):

$$fft(x[n]) = X_{FFT}[k] = \sum_{n=1}^{N} x[n]e^{-j\frac{2\pi}{N}(n-1)(k-1)}, \qquad k = 1, \dots, N$$

$$fft(x[n-1]) = X_{FFT-1}[k] = \sum_{n=1}^{N} x[n-1]e^{-j\frac{2\pi}{N}(n-1)(k-1)}$$

$$X_{DFT}[\mathrm{k}] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn} = \sum_{n=1}^{N} x[n-1]e^{-j\frac{2\pi}{N}k(n-1)}$$

$$X_{DFT}[\mathrm{k}] = X_{FFT-1}[k+1], \qquad k = 0, \dots, N-1$$

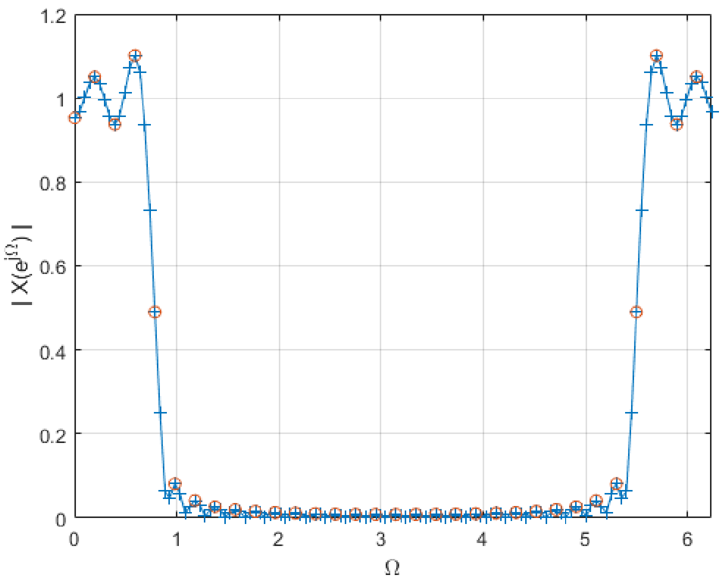
d) When the DFT analysis formula is considered, DFT's k^{th} element corresponds to what frequency?

$$X_{DFT}[\mathrm{k}] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

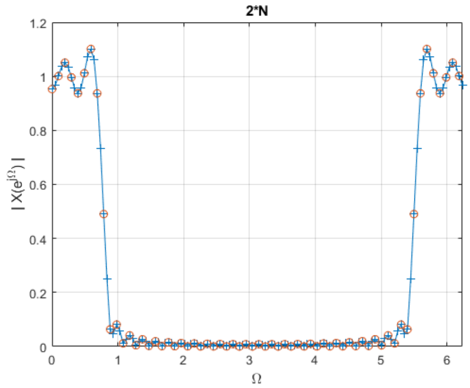
$$\Omega_{\mathrm{k}} = \frac{2\pi}{N}k$$

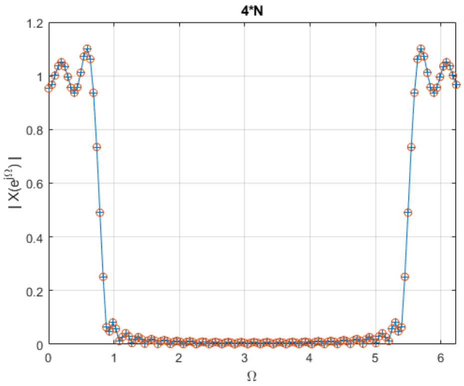
e) Draw the plot in part b and the absolute of DFT in part d vs $\Omega \in [-\pi, \pi)$. Verify the same values where you expect them to see.

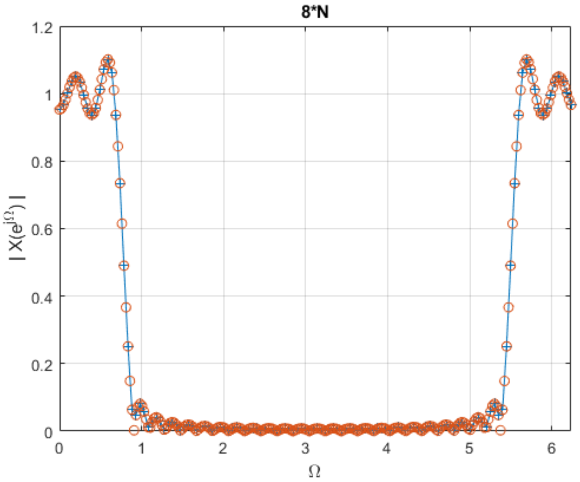
```
X_fft_1=fft(x_n); %signal length is 32.
plot((0:31)*(2*pi/32),abs(X_fft_1),'o');
```



f) The fft function can also give us a highly sampled DTFT. This is done by padding with trailing zeros to the length given to the fft function as an argument when the argument is larger than the DT signal's original length. Plot fft(x_n,2*N), fft(x_n,4*N), fft(x_n,8*N) with the corresponding Ω values.







g) There are many helpful functions in Matlab. One of them is the upsample function that inserts zeros in between signal samples. Use the following code fragment and draw the resultant DFT vs $\Omega \in [-\pi, \pi)$. Is this related to any of the properties discussed in class?

```
usr=3;
x_n_up=upsample(x_n,usr);
```