EE 301 Signals & Systems
Homework 2

Due: *Nov*. 3, 2019, 23:55 via `odtuclass.metu.edu.tr`

Problem 1: Consider the periodic signal $x(t)$ defined as

$$x(t) = \begin{cases} \cos(\pi t) & for \quad 0 \le t \le \frac{1}{2} \\ 0 & for \quad \frac{1}{2} \le t < 2 \end{cases}$$

over a single period. Compute the Fourier series coefficients of $x(t)$.

Problem 2: Let $x(t)$ be a periodic signal with the fundamental period T whose Fourier series coefficients are a_k. Determine the Fourier series coefficients of each of the following signals in terms of a_k:

a) $x(t-t_0)+x(t+t_0)$

b) $\dfrac{x(t)-x(t-T/2)}{2}$

c) $\boldsymbol{Ev}\{\, x(t) \,\}$

d) $\boldsymbol{Re}\{\, x(t) \,\}$

e) $\dfrac{d^2 x(t)}{dt^2}$

f) $x(3t-1)$

g) $x^2(t)$

Problem 3: Suppose we are given the following information about a signal $x(t)$ with Fourier series coefficients a_k:

I. $a_k = 0$ for $k = 0$ and $|k| > 2$.

II. $x(t)$ is a real signal.

III. a_1 is a positive and real number.

IV. $x(t)$ is periodic with period $T = 6$.

V. $x(t) = -x(t-3)$.

VI. $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 dt = \frac{1}{2}$.

Show that $x(t) = A\cos(Bt + C)$ and determine the values of constants *A*, *B* and *C*.

Problem 4: Assume both $x(t)$ and $y(t)$ are periodic signals with the same period T_0 whose Fourier series representations given as

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k\, e^{jk\omega_0 t} \quad \text{and} \quad y(t) = \sum_{k=-\infty}^{+\infty} b_k\, e^{jk\omega_0 t}$$

a) Show that the Fourier series coefficients of the signal

$$z(t) = x(t)\, y(t) = \sum_{k=-\infty}^{+\infty} c_k\, e^{jk\omega_0 t}$$

are given by discrete convolution $c_k = \sum_{n=-\infty}^{+\infty} a_n\, b_{k-n}$.

b) Assume that $y(t) = x^*(t)$. Expressing b_k in terms of a_k and using the result of part (a), prove the Parseval's relation for periodic signals, which is

$$\frac{1}{T_0}\int_{0}^{T_0} |x(t)|^2\, dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

EE 301 HW2 MATLAB Assignment

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We will examine Fourier series (FS) and some of its properties in this assignment. Throughout the assignment, a time-limited signal $x(t)$ and its periodic extension $x_p(t)$ with period T will be considered. Note the following relations:

$x(t) = 0$ for $t < 0$ and $t > L$. In addition, $x_p(t) = \sum_{k=-\infty}^{\infty} x(t - kT)$.

A MATLAB function `signal_generator.m` is already provided for you on `odtuclass.metu.edu.tr`. This function is executed in MATLAB by the command

```
[a_k, x_p] = signal_generator(signal_type,period,Length,figureonoff);
```

The input parameters of the function `signal_generator`:

`signal_type` : Selects one signal among three alternatives for $x(t)$.

- `signal_type = 1` : square wave
- `signal_type = 2` : triangular wave
- `signal_type = 3` : single period cosine

`period` : Determines the period T of periodic function $x_p(t)$.

`Length` : Determines the nonzero interval L of $x(t)$. Should be less than period T.

`figureonoff = 1` : Plot figures.

`figureonoff = 0` : Omit figures.

The output parameters of the function `signal_generator`:

`x_p` : Samples from $x_p(t)$ for $-T < t < 2T$.

`x_p(k)` $= x_p\left(\frac{(k-1)}{1000} - T\right)$

`a_k` : FS coefficients a_k for $x_p(t)$ for $k = 0,1,\ldots,100$.

`a_k(k)` $= a_{k-1}$

Assigned Questions:

Part 1:

i) Utilize `signal_generator` function to obtain periodic extension of square wave function with $L = 2$ and $T = 5$, and its FS coefficients. Obtain plots for $x(t)$, $x_p(t)$, its FS coefficients a_k (only magnitude).

ii) Observe $x_p(t)$ and obtain the expression for its FS coefficients a_k yourself, using Fourier series analysis equation. Calculate the values of a_k for $k = 0,1,2$ and compare with the output of the given MATLAB function.

iii) Write the Fourier series synthesis equation that constructs $x_p(t)$ from FS coefficients a_k. Then, write the equation that constructs the approximated signal $x_{p,M}(t)$ by using a_k for only $-M \leq 0 \leq M$. (In other words, ignore a_k for $|k| > M$ and reconstruct the signal.) Write a code that generates $x_{p,M}(t)$ for a given value of M. Your code should use the output `a_k` of the given MATLAB function.
($a_{-k} = a_k^*$, since $x_p(t)$ is real)

iv) Plot $x_{p,M}(t)$ for $-T < t < 2T$, using $M = 3,5,10,20$. Comment on the results.

v) Plot $x_{p,M}(t)$ for $-T < t < 2T$, using $M = 100$. Do you observe convergence to the square wave at the edges? Zoom to the edges and notice *Gibbs effect.* You can read more on *Gibbs phenomenon* from: http://en.wikipedia.org/wiki/Gibbs_phenomenon

Part 2: Repeat steps i) and iv) of Part 1 for single period cosine.

Part 3: Repeat steps i) and iv) of Part 1 for triangular wave. Compare especially the cases for $M = 10$ and $M = 20$. Extra 20 (10 per one side) coefficients are used in $M = 20$ case. How much difference did they cause? Do you expect any significant increase in the convergence for larger values of M? Comment. (Hint : Parseval's Relation)

Part 4: Express the FS coefficients b_k of $x_p(t - \Delta t)$ in terms of a_k. Execute MATLAB function for triangular wave with $L = 2$ and $T = 5$, obtain `a_k`, modify `a_k` to obtain `b_k` for $\Delta t = 1$ and $\Delta t = 4$. Then, use the code that you have written in step-iii of part-1 and plot the obtained signals for $-T < t < 2T$, using $M = 100$. Comment.

Part 5: Execute the given MATLAB function for triangular wave with $L = 2$ and $T = 5$, observe $x_p(t)$ and sketch $\frac{d}{dt}x_p(t)$. Express the FS coefficients c_k of $\frac{d}{dt}x_p(t)$ in terms of a_k. Modify the function output `a_k` to obtain `c_k`. Then use the code that you have written in step 3 of the Part 1 and plot the obtained signal for $-T < t < 2T$, using $M = 100$. Comment on the results.

Part 6: Execute the given MATLAB function for square wave with $L = 2$ and $T = 5$. Then, use again for $L = 4$ and $T = 10$. Compare the signals and FS coefficients. Comment on the results.