Graphs

Basic notation and definitions

Juan Antonio Díaz García

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Cardinality of a set¹

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Definition

Given a set $V = \{1, ..., n\}$, we denote by |V| = n, the cardinality (number of elements) of V (i.e., n is the number of elements of set V).

Definition

Given a set $V = \{1, ..., n\}$, we define the *power set* of V, denoted as 2^V , as the set of all subsets of V, including the empty set \emptyset and the set V itself.

Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

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Definition

A graph G = (V, U) is defined by a set $V = \{1, \ldots, n\}$ of vertices or nodes and a set $U \subseteq V \times V$ of unordered pairs $i, j \in V$ called edges. Therefore, both pairs $\{i, j\}$ or $\{j, i\}$ can be used to represent the same edge between $i, j \in V$ in U. A graph can also be referred to as an undirected graph.

Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

Representación gráfica de un grafo

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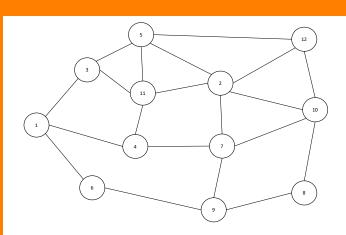
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$$V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$U = \{\{1,3\}, \{1,4\}, \{1,6\}, \{2,5\}, \{2,7\}, \{2,10\}, \{2,11\}, \{2,12\}, \{3,5\}, \{3,11\}, \{4,7\}, \{4,11\}, \{5,11\}, \{5,12\}, \{6,9\}, \{7,9\}, \{7,10\}, \{8,9\}, \{8,10\}, \{10,12\}\}$$

Complete graph³

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Definition

A graph G=(V,U) is said to be a *complete graph* or *complete undirected graph* if there is and edge between any two distinct nodes in V, that is, $\{i,j\} \in U$ for each pair $i,j \in V, i \neq j$.

Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

Path⁴

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Definition

Given a graph G=(V,U) a path $P_{st}(G)$ in an undirected graph G, from a node $s\in V$ to a node $t\in V$, is defined as a sequence of nodes $i_1,i_2,\ldots i_{q-1},i_q\in V$ where $i_1=s,\ i_q=t,$ and each $\{i_k,i_{k+1}\}\in U$, for $k=1,\ldots,q-1$. The number of edges in a path is given by q-1

^{*}Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

Connected⁵

Definition

Given a graph G = (V, U) we say that the graph is connected if there is at least one path $P_{st}(G)$ for every pair of distinct nodes $s, t \in V$.

⁵Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

Subgraph⁶

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Definition

Given a graph G = (V, U), a subgraph G' = (V', U') is such that for any pair of nodes $i, j \in V'$, edge $\{i, j\} \in U'$ if and only if $\{i, j\} \in U$, and therefore, $V' \subseteq V$ and $U' \subseteq U$.

Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

Spanning tree⁷

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Definition

A spanning tree of a graph G = (V, U) is a connected subgraph of G with the same node set V and whose edge set $U' \subseteq U$ has exactly n-1 edges, where n=|V|.

Induced graph⁸

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Definition

Given a graph G=(V,U) and a subset V' of its node set V, the graph G(V')=(V',U') induced in G by V' has $U'=\{\{i,j\}\in U:\{i,j\}\in V'\}$ as its edge set.

⁸Resende, M.G.C. and Ribeiro, C.C., *Optimization by GRASP: Greedy Randomized Adaptive Search Procedures*, Springer, 2013, Chapter 1

Clique⁹

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Definition

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A *clique* of graph G = (V, U) is a subset of nodes $C \subseteq V$ such that $\{i, j\} \in U$ for every nodes $i, j \in C$, with $i \neq j$. Alternatively, we can say that C is a *clique* if the graph G(C) induced in G by C is a complete graph. The *size* of a *clique* is

⁹ Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

Independent set¹⁰

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Definition

Given a graph G=(V,U), a subset $I\subseteq V$ of the nodes in G is said to be an *independent set* or a *stable set* if every two nodes in I are not directly connected by an edge, i.e., if $\{i,j\}\not\in U$ for all $i,j\in I$ such that $i\neq j$.

¹⁰ Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

Directed graph or digraph¹¹

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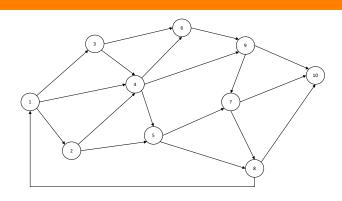
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Definition

A directed graph or digraph G = (V, A) is defined by a set $V = \{1, ..., n\}$ of nodes and a set $A \subseteq V \times V$ of ordered pairs (i, j) of nodes $i, j \in V$ called arcs.

¹¹ Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

Representación gráfica de un grafo



$$V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{(1,2), (1,3), (1,4), (2,4), (2,5), (3,4), (3,6), (4,5), (4,6), (4,9), (5,7), (5,8), (6,9), (7,8), (7,10), (8,1), (8,10), (9,10)\}$$

Directed path¹²

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Definition

Given a digraph G=(V,A), a directed path $P_{st}(G)$ in a directed graph G from $s\in V$ to $t\in V$ is defined as a sequence of nodes $i_1,i_2,\ldots,i_{q-1},i_q\in V$, where $i_1=s,i_q=t$, and each arc $(i_k,i_{k+1})\in A$, for any $k=1,\ldots,q-1$.

¹² Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

Strongly connected graph¹³

Graph

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Definition

A directed graph G = (V, A) is said to be a *strongly connected* graph if there is at least one path $P_{st}(G)$ connecting node s to node t and another path $P_{ts}(G)$ connecting node s to node t, for every pair of nodes $s, t \in V$.

¹³ Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1

Hamiltonian paths and cycles¹⁴

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Definition

A Hamiltonian path in a directed or undirected graph is a path between two nodes that visits each node of the graph exactly once. A Hamiltonian cycle in a directed or undirected graph is a Hamiltonian path that is also a cycle, i.e., its extremities coincide. Every Hamiltonian cycle corresponds to a circular permutation of the nodes of the graph. A Hamiltonian cycle is also known as a Hamiltonian tour or, simply, as a tour.

¹⁴ Resende, M.G.C. and Ribeiro, C.C., Optimization by GRASP: Greedy Randomized Adaptive Search Procedures, Springer, 2013, Chapter 1